

Chapter 4 Kan extensions

4.1 The definition of Kan extensions: their expressibility by limits and colimits.

$K_- : \mathcal{A} \rightarrow \mathcal{C}$ は \mathcal{V} -functor として与えられ、
 $\hat{K} : \mathcal{C}^{\text{op}} \rightarrow [\mathcal{A}, \mathcal{V}]$, $\tilde{K} : \mathcal{C} \rightarrow [\mathcal{A}^{\text{op}}, \mathcal{V}]$

$$\hat{K} ? = \mathcal{C}(?, K_-) \quad , \quad \tilde{K} ? = \mathcal{C}(K_-, ?)$$

ここで、このとき、

$$F : \mathcal{C} \rightarrow \mathcal{V} \quad \text{と与え、} \quad (\hat{K} : \mathcal{C}^{\text{op}} \rightarrow [\mathcal{A}, \mathcal{V}] \quad \text{と与え、})$$

$$F \star \hat{K} \in [\mathcal{A}, \mathcal{V}].$$

いま、

$$H : \mathcal{A} \rightarrow \mathcal{V} \quad \text{と与え、}$$

$$[\mathcal{A}, \mathcal{V}] (F \star \hat{K}, H)$$

$$\cong [\mathcal{C}^{\text{op}}, \mathcal{V}] (F, [\mathcal{A}, \mathcal{V}] (\hat{K} -, H))$$

$$\cong \int_{\mathcal{A}} [\mathcal{C}^{\text{op}}, \mathcal{V}] (F, \mathcal{V}((\hat{K} -)A, HA))$$

の def

関手圏の def
 $\int_{\mathcal{A}}$ の def

$$\cong \int_A V(F? \star (\hat{K}?)A, HA)$$

\star a def.

$$\cong [A, V](F? \star (\hat{K}?)-, H-)$$

yoneda b's,

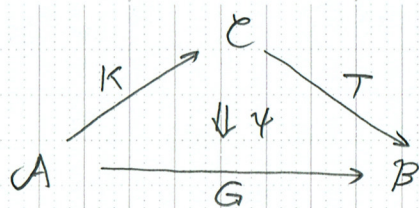
$$(F \star \hat{K})A \cong F? \star (\hat{K}?)A$$

$$\cong (\hat{K}?)A \star F?$$

$$= \mathcal{C}(?, KA) \star F?$$

$$\cong FKA$$

$$\begin{aligned} (\hat{K}?)A &: \mathcal{C}^p \rightarrow V \\ F? &: \mathcal{C} \rightarrow V \end{aligned}$$



$$c \quad F: c \longrightarrow v \text{ is a c.t.,}$$

$$[c, v] (F, \beta(B, T-)) \xrightarrow[\hat{\tau}_B]{\text{pre-composite } [K, -]} [A, B] (FK, \beta(B, \tau K-))$$

$$\begin{aligned} & \xrightarrow[\hat{\tau}_{KB}]{[A, v] (1, \beta(1, \psi-))} [A, B] (FK, \beta(B, G-)) \end{aligned}$$

$$\text{よ} \quad [c, v] (F, \beta(B, T-)) \cong \beta(B, \{F, T\})$$

$$[A, B] (FK, \beta(B, G-)) \cong \beta(B, \{FK, G\})$$

$B = \{F, T\}$ に對して $I \xrightarrow{j} \beta(\{F, T\}, \{F, T\}) \longrightarrow \beta(\{F, T\}, \{FK, G\})$ を得る.

これに,

$$(K, \psi)_* : \{F, T\} \longrightarrow \{FK, G\}$$

とす.