

Statistical Inference Course Project Part 1

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10/13/2020

Overview

The purpose is to investigate the exponential distribution in R and comparing it with the Central Limit Theorem. The mean of exponential distribution is $1/\lambda$ and the standard deviation is also $1/\lambda$. with $\lambda = 0.2$ for all the simulations. The investigation compares the distribution of averages of 40 exponentials over a thousand simulations

Simulations

For the 1000 simulations, λ is 0.2 and sample size is 40

```
#Set seed to ensure reproducibility
set.seed(6666)

#Set lambda
lambda <- 0.2

#Set exponentials
n <- 40

#Simulating the exponential distribution and getting the mean of 1 simulation
simul <- rexp(n,lambda)
simul

## [1] 3.54431668 1.53150748 0.32929821 3.27650114 2.69959882 8.74659368
## [7] 3.56559497 0.62182882 1.61579815 1.29026995 4.73752971 1.46355046
## [13] 13.40054884 2.06984195 0.35475479 4.45554234 8.60265237 12.09602350
## [19] 1.07535294 1.39329189 1.28706599 1.57589110 7.72572449 7.00650160
## [25] 5.11022532 3.02797012 2.19538448 9.61173891 6.33892175 4.12758995
## [31] 0.01988841 11.82788231 18.65521784 1.99198980 8.48981051 14.90360759
## [37] 8.14093665 9.86643793 4.81952719 5.16277506

expMean <- mean(simul)
expMean

## [1] 5.218887
```

Sample Mean versus Theoretical Mean

```
#Getting the means of each simulations
simul1000 <- as.data.frame(replicate(1000,mean(rexp(n, lambda))))
names(simul1000) <- c("sample.mean")

#Calculating the mean of the means of 1000 simulation
mean1000 <- mean(simul1000$sample.mean)
mean1000
```

```
## [1] 5.062025
```

```
#The theoretical mean
theoMean <- 1/lambda
theoMean
```

```
## [1] 5
```

The theoretical mean and the experimental mean have a very similar value therefore the center of distribution of sample means of 40 exponential is close to the theoretical center of the distribution

Sample Variance versus Theoretical Variance

```
# Calculating the variance of this simulation
var1000 <- var(simul1000$sample.mean)
var1000
```

```
## [1] 0.621038
```

```
# The theoretical variance
theoVariance <- ((1/lambda)^2)/40
theoVariance
```

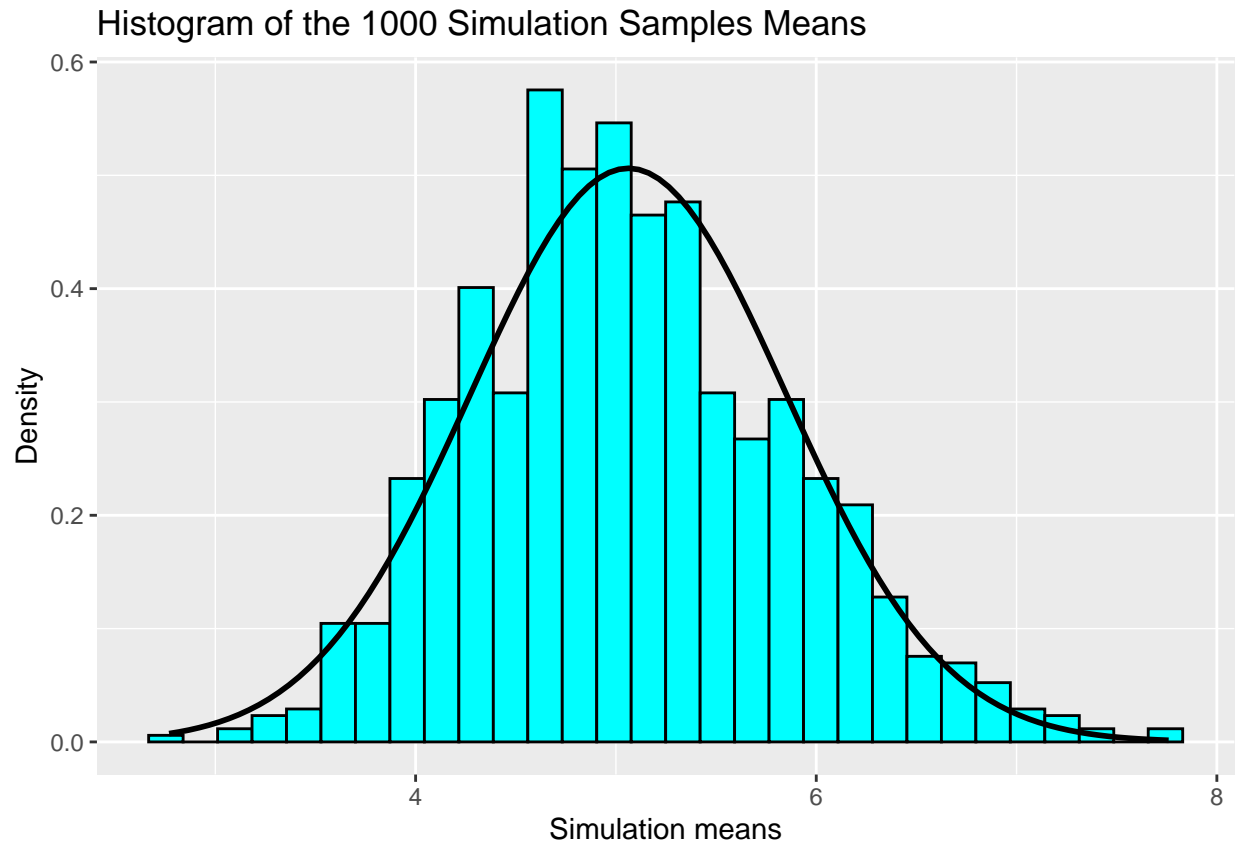
```
## [1] 0.625
```

The variance of the distribution of means have a very similar value to the theoretical variance which is calculated by squaring the standard deviation and dividing by the sample size

Distribution

```
library(ggplot2)
ggplot(simul1000, aes(x=sample.mean)) +
  geom_histogram(aes(y = ..density..), colour="black", fill="cyan")+
  stat_function(fun=dnorm, args=list( mean=mean1000, sd=sqrt(var1000)), geom="line", color = "black", size=1) +
  ggtitle("Histogram of the 1000 Simulation Samples Means") +
  scale_x_continuous("Simulation means")+
  ylab("Density")
```

```
## 'stat_bin()' using 'bins = 30'. Pick better value with 'binwidth'.
```



The black curve represents the normal distribution which is used to compare with the histogram. The central limit theorem states that the sample means would become that of a standard normal distribution as the sample size increases whilst meeting the two conditions of independence ($n < 10\%$) and normal, or if skewed distribution, that $n > 30$.
