Biometrics Authentication: Formalization and Instantiation

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This report formalizes the biometric authentication scheme, including its structure, usage, and security analysis with a security game model.

1 Preliminaries

In this report, we assume

- λ is the security parameter.
- $[m]$ denotes the set of integers $\{1, 2, \cdots, m\}$.
- $\mathbb{Z}_q$ is the finite field modulo a prime number q.
- A function $f(n)$ is called *negligible* iff for any integer c, $f(n) < \frac{1}{n^c}$ for all sufficiently large n. We write it as $f(n) = \mathsf{negl}$, and we may also use negl to represent an arbitrary negligible function.
- poly is the class of polynomial funcions. We may also use poly to represent an arbitrary polynomial function.
- We write sampling a value r from a distribution $\mathcal{D}$ as $r \leftarrow_{\$} \mathcal{D}$. If S is a finite set, then $r \leftarrow_{\$} S$ means sampling r uniformly from S.
- The distribution $\mathcal{D}^t$ denotes t identical and independent distributions of $\mathcal{D}$.
- A PPT algorithm denotes a probabilistic polynomial time algorithm. Unless otherwise specified, all algorithms run in PPT.

We introduce three types of inner product functional encryption schemes: function hiding functional encryption, two-input functional encryption, and two-client functional encryption. We will instantiate our biometric authentication scheme using these primitives.

Definition 1 (Function Hiding Inner Product Functional Encryption)**.** A *function hiding inner product functional encryption* (fh-IPFE) scheme FE for a field $\mathbb{F}$ and input length k is composed of PPT algorithms $\mathsf{FE.Setup}$, $\mathsf{FE.KeyGen}$, $\mathsf{FE.Enc}$, and $\mathsf{FE.Dec}$:

- $\mathsf{FE.Setup}(1^\lambda) \to \mathsf{msk}, \mathsf{pp}$: It outputs the public parameter pp and the master secret key msk.
- $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}) \to f_\mathbf{x}$: It generates the functional decryption key $f_\mathbf{x}$ for an input vector $\mathbf{x} \in \mathbb{F}^k$.
- $\mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y}) \to \mathbf{c_y}$: It encrypts the input vector $\mathbf{y} \in \mathbb{F}^k$ to the ciphertext $\mathbf{c_y}$.
- $\mathsf{FE.Dec}(\mathsf{pp}, f_\mathbf{x}, \mathbf{c_y}) \to z$: It outputs a value $z \in \mathbb{F}$.

Correctness: The fh-IPFE scheme FE is *correct* if $\forall(\mathsf{msk}, \mathsf{pp}) \leftarrow \mathsf{FE.Setup}(1^\lambda)$ and $\mathbf{x}, \mathbf{y} \in \mathbb{F}^k$, we have

$$\mathsf{FE.Dec}(\mathsf{pp}, \mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}), \mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y})) = \mathbf{x}\mathbf{y}^T \in \mathbb{F}.$$

Instantiation using an fh-IPFE scheme is given in Section 2.3.

Definition 2 (Two-Input Inner Product Functional Encryption (adapted from [PP22]))**.** A *two-input inner product functional encryption* (2i-IPFE) scheme FE for a field $\mathbb{F}$ and input length k is composed of PPT algorithms $\mathsf{FE.Setup}$, $\mathsf{FE.KeyGen}$, $\mathsf{FE.Enc}$, and $\mathsf{FE.Dec}$:

- $\mathsf{FE.Setup}(1^\lambda) \to \mathsf{sk}, \mathsf{ek}_1, \mathsf{ek}_2$: It outputs a secret key sk and two encryption keys $\mathsf{ek}_1, \mathsf{ek}_2$.
- $\mathsf{FE.KeyGen}(\mathsf{sk}, \mathbf{A}) \to \mathsf{dk}_\mathbf{A}$: It generates the functional decryption key $\mathsf{dk}_\mathbf{A}$ for a diagonal matrix $\mathbf{A} \in \mathbb{F}^{k\times k}$,
- $\mathsf{FE.Enc}(\mathsf{ek}_i, \mathbf{x}) \to \mathbf{c_x}$: Given an encryption key, either ek_1 or ek_2, it encrypts the input vector $\mathbf{x} \in \mathbb{F}^k$ to the ciphertext $\mathbf{c_x}$.
- $\mathsf{FE.Dec}(\mathsf{dk}_\mathbf{A}, \mathbf{c_x}, \mathbf{c_y}) \to z$: It outputs a value $z \in \mathbb{F}$.

Correctness: The 2i-IPFE scheme FE is *correct* if $\forall(\mathsf{sk}, \mathsf{ek}_1, \mathsf{ek}_2) \leftarrow \mathsf{FE.Setup}(1^\lambda)$, $\mathbf{A} \in \mathbb{F}^{k\times k}$, and $\mathbf{x}, \mathbf{y} \in \mathbb{F}^k$, we have

$$\mathsf{FE.Dec}(\mathsf{FE.KeyGen}(\mathsf{sk}, \mathbf{A}), \mathsf{FE.Enc}(\mathsf{ek}_1, \mathbf{x}), \mathsf{FE.Enc}(\mathsf{ek}_2, \mathbf{y})) = \mathbf{x}\mathbf{A}\mathbf{y}^T \in \mathbb{F}.$$

Instantiation using a 2i-IPFE is given in Section 2.4.

Definition 3 (Two-Client Inner Product Functional Encryption (adapted from [PP22]))**.** A *two-client inner product functional encryption* (2c-IPFE) scheme FE for a field $\mathbb{F}$ and input length k is composed of PPT algorithms $\mathsf{FE.Setup}$, $\mathsf{FE.KeyGen}$, $\mathsf{FE.Enc}$, and $\mathsf{FE.Dec}$:

- $\mathsf{FE.Setup}(1^\lambda) \to \mathsf{sk}, \mathsf{ek}_1, \mathsf{ek}_2$: It outputs a secret key sk and two encryption keys $\mathsf{ek}_1, \mathsf{ek}_2$.
- $\mathsf{FE.KeyGen}(\mathsf{sk}, \mathbf{A}) \to \mathsf{dk}_\mathbf{A}$: It generates the functional decryption key $\mathsf{dk}_\mathbf{A}$ for a diagonal matrix $\mathbf{A} \in \mathbb{F}^{k\times k}$,

- $\mathsf{FE.Enc}(\ell, \mathsf{ek}_i, \mathbf{x}) \to \mathbf{c_x}$: Given a label ℓ and an encryption key, either ek_1 or ek_2, it encrypts the input vector $\mathbf{x} \in \mathbb{F}^k$ to the ciphertext $\mathbf{c_x}$.
- $\mathsf{FE.Dec}(\mathsf{dk}_\mathbf{A}, \mathbf{c_x}, \mathbf{c_y}) \to z$: It outputs a value $z \in \mathbb{F}$.

Correctness: The 2c-IPFE scheme FE is *correct* if $\forall(\mathsf{sk}, \mathsf{ek}_1, \mathsf{ek}_2) \leftarrow \mathsf{FE.Setup}(1^\lambda)$, $\mathbf{A} \in \mathbb{F}^{k\times k}$, label ℓ, and $\mathbf{x}, \mathbf{y} \in \mathbb{F}^k$, we have

$$\mathsf{FE.Dec}(\mathsf{FE.KeyGen}(\mathsf{sk}, \mathbf{A}), \mathsf{FE.Enc}(\ell, \mathsf{ek}_1, \mathbf{x}), \mathsf{FE.Enc}(\ell, \mathsf{ek}_2, \mathbf{y})) = \mathbf{xAy}^T \in \mathbb{F}.$$

Instantiation using a 2c-IPFE is given in Section 2.5.
We also consider an instantiation using a relational hash scheme.

Definition 4 (Relational Hash (adapted from [MR14]))**.** Let R_λ be a relation over sets X_λ, Y_λ, and Z_λ. A *relational hash* scheme RH for R_λ consists of PPT algorithms $\mathsf{RH.KeyGen}$, $\mathsf{RH.HASH}_1$, $\mathsf{RH.HASH}_2$, and $\mathsf{RH.Verify}$:

- $\mathsf{RH.KeyGen}(1^\lambda) \to \mathsf{pk}$: It outputs a public hash key pk.
- $\mathsf{RH.Hash}_1(\mathsf{pk}, \mathbf{x}) \to \mathbf{h_x}$: Given a hash key pk and $\mathbf{x} \in X_\lambda$, it outputs a hash $\mathbf{h_x}$.
- $\mathsf{RH.Hash}_2(\mathsf{pk}, \mathbf{y}) \to \mathbf{h_y}$: Given a hash key pk and $\mathbf{y} \in Y_\lambda$, it outputs a hash $\mathbf{h_y}$.
- $\mathsf{RH.Verify}(\mathsf{pk}, \mathbf{h_x}, \mathbf{h_y}) \to r \in \{0, 1\}$: Given a hash key pk, two hashes $\mathbf{h_x}$ and $\mathbf{h_y}$, and $\mathbf{z} \in Z_\lambda$, it verifies whether the relation among $\mathbf{x}, \mathbf{y}$ and $\mathbf{z}$ holds.

Correctness: The relational hash scheme RH is *correct* if $\forall \mathbf{x}, \mathbf{y}, \mathbf{z} \in X_\lambda \times Y_\lambda \times Z_\lambda$,

$$\Pr\left[\begin{cases}\mathsf{pk} \leftarrow \mathsf{RH.KeyGen}(1^\lambda) \\ \mathbf{h_x} \leftarrow \mathsf{RH.Hash}_1(\mathsf{pk}, \mathbf{x}) \\ \mathbf{h_y} \leftarrow \mathsf{RH.Hash}_2(\mathsf{pk}, \mathbf{y})\end{cases} : \mathsf{RH.Verify}(\mathsf{pk}, \mathbf{h_x}, \mathbf{h_y}, \mathbf{z}) = R(\mathbf{x}, \mathbf{y}, \mathbf{z})\right] = 1 - \mathsf{negl}.$$

Instantiation using a relational hash is given in Section 2.6.

2 Formalization

In general, an authentication shceme Π associated with a family of biometric distributions $\mathbb{B}$ is composed of the following algorithms.

- $\mathsf{Setup}(1^\lambda) \to \mathsf{esk}, \mathsf{psk}, \mathsf{csk}$: It outputs the enrollment secret key esk, probe secret key psk, and compare secret key csk.
- $\mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}() \to \mathbf{x}$: Given an oracle $\mathcal{O}_\mathcal{B}$, which samples biometric data from the distribution $\mathcal{B} \in \mathbb{B}$, it encodes biometric samples as $\mathbf{x}$, the input format for enrollment.
- $\mathsf{Enroll}(\mathsf{esk}, \mathbf{x}) \to \mathbf{c_x}$: It outputs the enrollment message $\mathbf{c_x}$ from $\mathbf{x}$.

- $\mathsf{encodeProbe}^{\mathcal{O}_{\mathcal{B}}}() \rightarrow \mathbf{y}$: Given an oracle $\mathcal{O}_{\mathcal{B}}$, which samples biometric data from the distribution $\mathcal{B} \in \mathbb{B}$, it encodes biometric samples as $\mathbf{y}$, the input format for probe.
- $\mathsf{Probe}(\mathsf{psk}, \mathbf{y}) \rightarrow \mathbf{c_y}$: It outputs the probe message $\mathbf{c_y}$ from $\mathbf{y}$.
- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y}) \rightarrow s$: It compares the enrollment message $\mathbf{c_x}$ and probe message $\mathbf{c_y}$ and outputs a score s.
- $\mathsf{Verify}(s) \rightarrow r \in \{0, 1\}$: It is a deterministic algorithm that reads the comparison score s and determines whether this is a successful authentication ($r = 1$) or not ($r = 0$).

We discuss two usage models that employs the authentication scheme Π.

2.1 Usage Model – Device-of-User

In the model described in Figure 1 (an overview), Figure 2 (on enrollment), and Figure 3 (on authentication), users authenticate themselves to a server through their own devices and biometric scanners that are shared among different users. A key distribution service distributes keys for them. In practice, this model applies to the situation when the users access an online service run by the server.

- User: The user who enrolls its biometric data and authenticates itself to the server. We assume the user's biometric distribution is $\mathcal{B} \in \mathbb{B}$.
- Scanner: A machine to extract the user's biometric data by querying the oracle $\mathcal{O}_{\mathcal{B}}$.
- Device: A device belonging to the user. In practice, it can be a desktop or a mobile phone. It processes the Enroll and Probe functions for User with keys esk and psk. It queries $\mathcal{O}_{\mathcal{B}}$ for biometric data through the Scanner.
- KDS: A key distribution service. It runs Setup to generate keys and distribute them to Device and Server.
- Server: The server responsible for authenticating the user. It stores the comparison key csk and the user's enrollment message $\mathbf{c_x}$. On authentication, it compares the probe message with the registered enrollment message and returns the result.

The Device-of-User model, when instantiated by an fh-IPFE scheme (Section 2.3), is analogous to the use case presented in [EM23]. In their model, a user possesses a personal device, such as a smartphone or laptop, and a secure hardware device that runs an initial setup and stores all the keys, which corresponds to our KDS. On enrollemnt and authentication, the user inputs biometric templates onto the device, which corresponds to our Scanner. Subsequently, the device transmits the template to the secure hardware for the enrollment or probing processes, which are equivalent to our Device. In addition, they incorporate a two-factor authentication mechanism. The secure hardware also executes a digital signature scheme and sign the probe message on authentication.

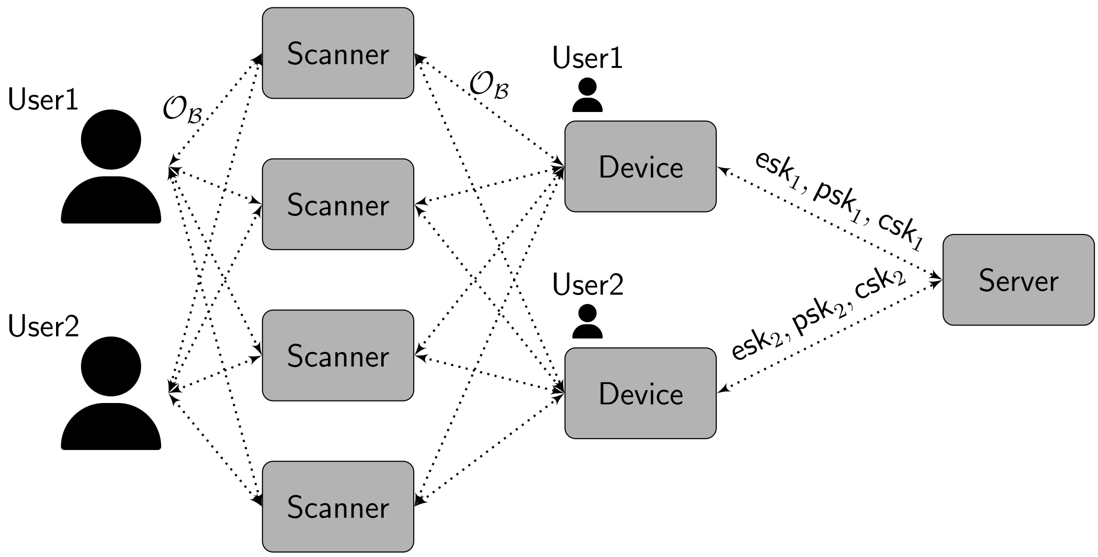

Figure 1: An Overview of the Device-of-User Usage Model

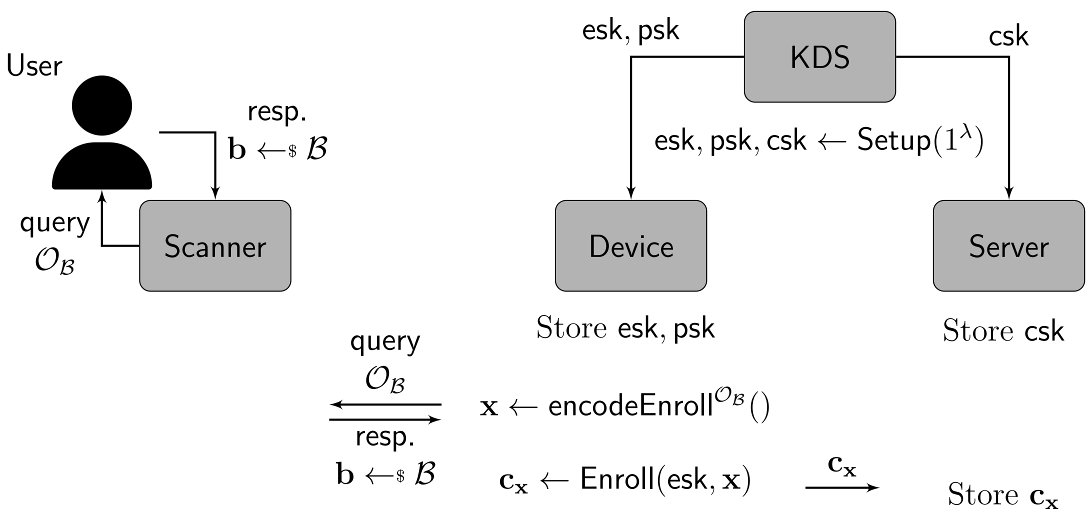

Figure 2: Device-of-User Usage Model on Enrollment

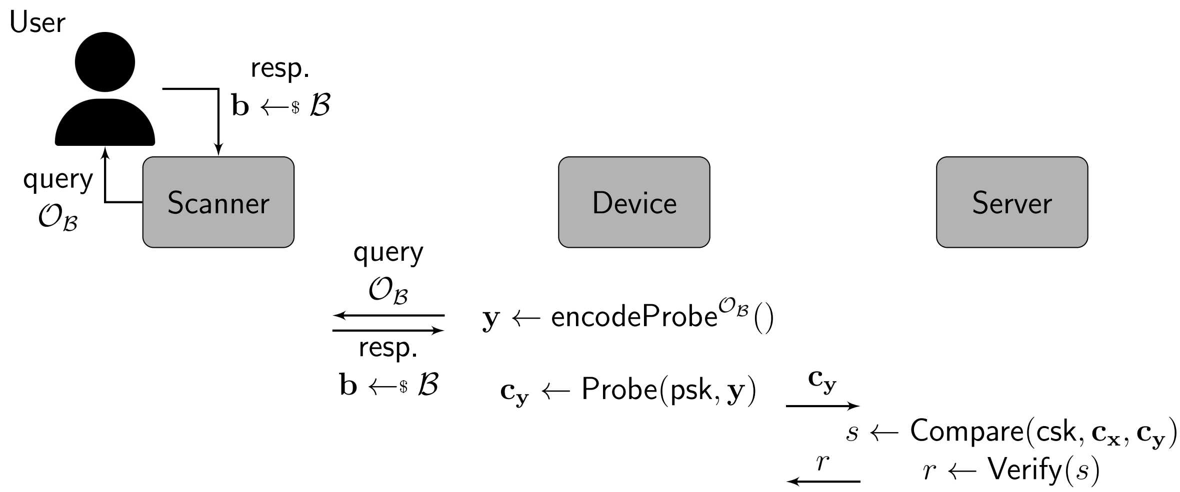

Figure 3: Device-of-User Usage Model on Authentication

2.2 Usage Model – Device-of-Domain

In the model described in Figure 4 (an overview), Figure 5 (on enrollment), and Figure 6 (on authentication), users first enroll themselves at an enrollment station and then authenticate themselves to a server through devices that belong to a domain. A key distribution service distributes enrollment keys to the enrollment station, probe keys to the domain, and comparison keys to the server. In practice, a domain can be a department in an organization, and this models applies to the situation when a user wants to access a public service of a department, such as a restricted area or instruments.

- User: The user who enrolls its biometric data at an enrollment station and authenticates itself to the server. We assume the user's biometric distribution is $\mathcal{B} \in \mathbb{B}$.
- Domain: A domain that owns several devices, all of which share one enrollment key esk, one probe key psk and one comparison key csk. Only the probe key is stored at each device of a domain. The enrollment key is stored at the enrollment station, and the comparison key is stored at the server. In practice, a domain can be a department, and users enroll and authenticate themselves before accessing a restricted service of this department.
- Scanner: A machine to extract the user's biometric data by querying the oracle $\mathcal{O}_\mathcal{B}$.
- Station: An enrollment station responsible for collecting the user's biometric data to enroll them for a domain on the server.
- Device: A device belonging to a domain. In practice, it can be a device checking identities for a restricted area or an instrument. It owns a probe key psk and processes the Probe function for enrolled users of this domain.

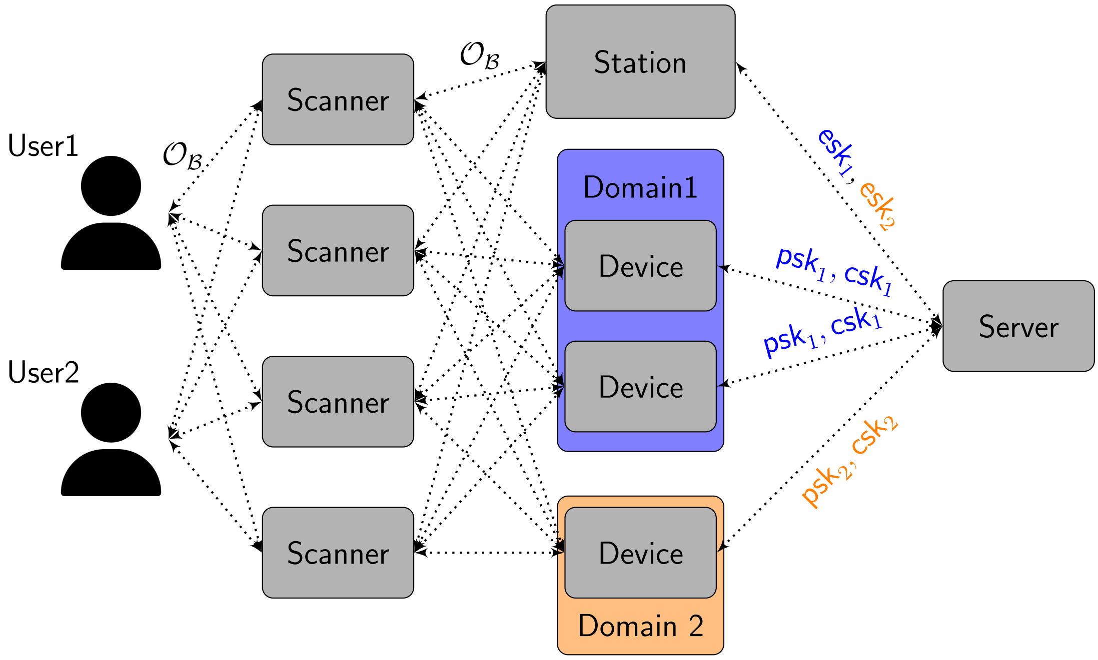

Figure 4: An overview of the Device-of-Domain Usage Model

- KDS: A key distribution service. It runs Setup to generate keys and distribute them to Station, Domain, and Server.
- Server: The server responsible for authenticating the user. It stores the comparison key csk for each domain and the user's enrollment message $\mathbf{c_x}$. On authentication, it compares the probe message with the registered enrollment message and returns the result.

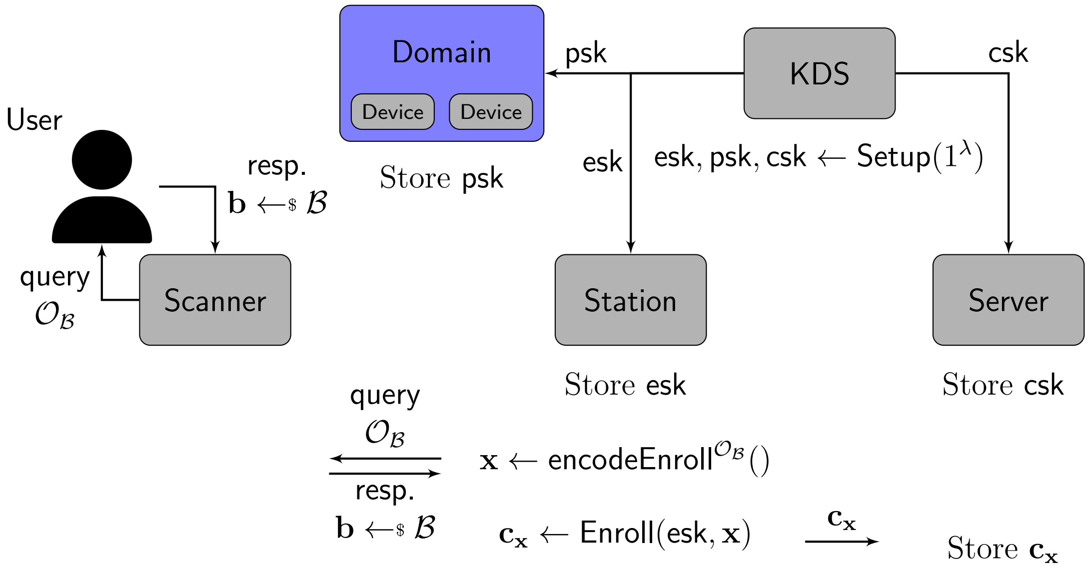

Figure 5: Device-of-Domain Usage Model on Enrollment

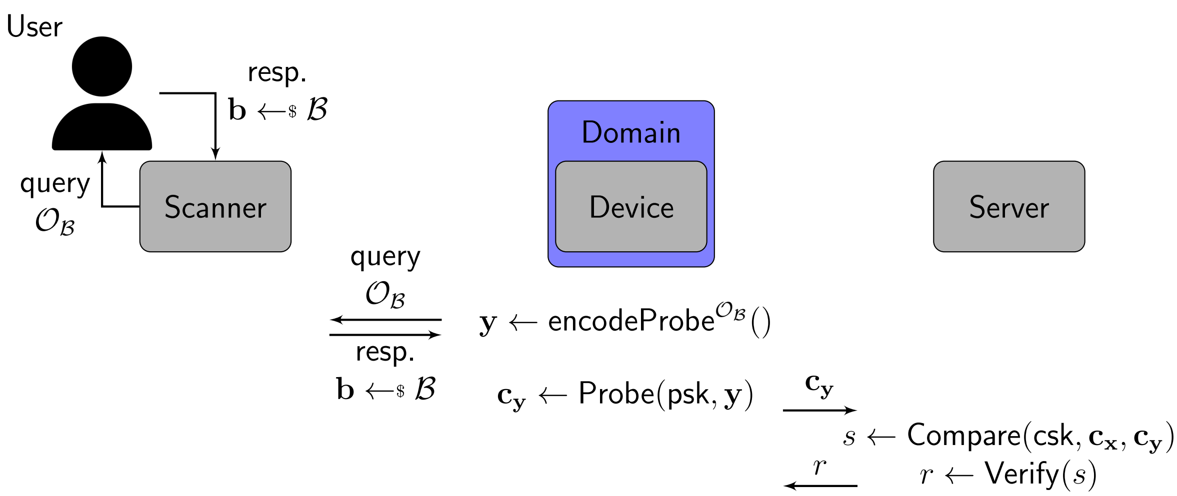

Figure 6: Device-of-Domain Usage Model on Authentication

2.3 Instantiation with an fh-IPFE Scheme

Let $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ be an fh-IPFE scheme we defined in Definition 1. Following [EM23], we can instantiate a biometric authentication scheme using FE with the distance metric the Euclidean distance. Let the biometric distribution $\mathcal{B} \subseteq [m]^k$, and let the associated field of FE be $\mathbb{Z}_q$ where q is a prime number larger than the maximum possible Euclidean distance $m^2 \cdot k$. The scheme is instantiated as follows.

- $\mathsf{Setup}(1^\lambda)$: It calls $\mathsf{FE.Setup}(1^\lambda) \to \mathsf{msk}, \mathsf{pp}$ and outputs $\mathsf{esk} \leftarrow (\mathsf{msk}, \mathsf{pp})$, $\mathsf{psk} \leftarrow (\mathsf{msk}, \mathsf{pp})$ and $\mathsf{csk} \leftarrow \mathsf{pp}$.
- $\mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}()$: For a template vector $\mathbf{b} = (b_1, b_2, \cdots, b_k)$ sampled from $\mathcal{O}_\mathcal{B}$, the function encodes it as $\mathbf{x} = (x_1, x_2, \cdots, x_{k+2}) = (b_1, b_2, \cdots, b_k, 1, \|\mathbf{b}\|^2)$.
- $\mathsf{Enroll}(\mathsf{esk}, \mathbf{x})$: It calls $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}) \to f_\mathbf{x}$ and outputs $\mathbf{c_x} \leftarrow f_\mathbf{x}$.
- $\mathsf{encodeProbe}^{\mathcal{O}_\mathcal{B}}()$: For a template vector $\mathbf{b}' = (b'_1, b'_2, \cdots, b'_k)$ sampled from $\mathcal{O}_\mathcal{B}$, the function encodes it as $\mathbf{y} = (y_1, y_2, \cdots, y_{k+2}) = (-2b'_1, -2b'_2, \cdots, -2b'_k, \|\mathbf{b}'\|^2, 1)$.
- $\mathsf{Probe}(\mathsf{psk}, \mathbf{y})$: It calls $\mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y}) \to \mathbf{c_y}$ and outputs $\mathbf{c_y}$.
- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y})$: It calls $\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c_x}, \mathbf{c_y}) \to s$ and outputs the value s.
- $\mathsf{Verify}(s)$: If $\sqrt{s} < \tau$, a pre-defined threshold for comparing the closeness of two templates, then it outputs $r = 1$; otherwise, it outputs $r = 0$.

By the correctness of the functional encryption scheme FE, we have

$$s = \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c_x}, \mathbf{c_y}) = \mathbf{x}\mathbf{y}^T = \sum_{i=1}^{k} -2b_ib'_i + \|\mathbf{b}\|^2 + \|\mathbf{b}'\|^2 = \|\mathbf{b} - \mathbf{b}'\|^2.$$

which is the square of the Euclidean distance between two templates $\mathbf{b}$ and $\mathbf{b}'$. Therefore, if two templates $\mathbf{b}$ and $\mathbf{b}'$ are close enough such that $\|\mathbf{b} - \mathbf{b}'\| < \tau$, the scheme results in $r = 1$, a successful authentication.

Instantiated with an fh-IPFE scheme in this way, the comparison secret key csk is public, and the enrollment secret key esk and probe secret key psk are the same. Anyone with access to the enrollment message $\mathbf{c_x}$ and either one of esk, psk, or a probe oracle $\mathsf{Probe}(\mathsf{psk}, \cdot)$ can probe some $\mathbf{y}' \in \mathbf{F}^{k+2}$ and find $\mathbf{x}\mathbf{y}'^T$ to get partial or full information about $\mathbf{x}$. Even if the adversary can only sample random ciphertexts $\mathbf{c_y}$ without knowing $\mathbf{y}$, if the field size q is not large enough, one can find a forged $\mathbf{c}_{\mathbf{y}^*}$ such that $\mathbf{x}\mathbf{y}^{*T} < \tau$ to impersonate the user by sampling many times offline.

Therefore, Server must store $\mathbf{c_x}$ securely, to avoid such an attack from an adversary who can access the probe oracle; Device must protect its probe function, to avoid such an attack from a malicious Server.

In the Device-of-Domain model, we assume the probe oracle is public, just as everyone can try accessing a public service. A malicious $\mathsf{Station}$ or Server, who has the enrollment message $\mathbf{c_x}$, can utilize this attack to retrieve information about User.

2.4 Instantiation with a 2i-IPFE Scheme

Let $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ be a 2i-IPFE scheme we defined in Definition 2. Following the scheme in Section 2.3, we can instantiate a biometric authentication scheme using FE.

- $\mathsf{Setup}(1^\lambda)$: It calls $\mathsf{FE.Setup}(1^\lambda) \to \mathsf{sk}, \mathsf{ek}_1, \mathsf{ek}_2$, $\mathsf{FE.KeyGen}(sk, \mathbf{I}_{k+2}) \to \mathsf{dk}_\mathbf{I}$, where $\mathbf{I}_{k+2}$ is an identity matrix of size $(k+2) \times (k+2)$. It outputs $\mathsf{esk} \leftarrow \mathsf{ek}_1$, $\mathsf{psk} \leftarrow \mathsf{ek}_2$, and $\mathsf{csk} \leftarrow \mathsf{dk}_\mathbf{I}$
- $\mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}()$, $\mathsf{encodeProbe}^{\mathcal{O}_\mathcal{B}}()$: The same as the scheme in 2.3.
- $\mathsf{Enroll}(\mathsf{esk}, \mathbf{x})$: It calls $\mathsf{FE.Enc}(\mathsf{ek}_1, \mathbf{x}) \to \mathbf{c_x}$ and outputs $\mathbf{c_x}$.
- $\mathsf{Probe}(\mathsf{psk}, \mathbf{y})$: It calls $\mathsf{FE.Enc}(\mathsf{ek}_2, \mathbf{y}) \to \mathbf{c_y}$ and outputs $\mathbf{c_y}$.
- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y})$: It calls $\mathsf{FE.Dec}(\mathsf{dk}_\mathbf{I}, \mathbf{c_x}, \mathbf{c_y}) \to s$ and outputs the value s.
- $\mathsf{Verify}(s)$: If $\sqrt{s} < \tau$, a pre-defined threshold for comparing the closeness of two templates, then it outputs $r = 1$; otherwise, it outputs $r = 0$.

By the correctness of the functional encryption scheme FE, we have

$$s = \mathsf{FE.Dec}(\mathsf{dk}_\mathbf{I}, \mathbf{c_x}, \mathbf{c_y}) = \mathbf{x}\mathbf{I}_{k+2}\mathbf{y}^T = \mathbf{x}\mathbf{y}^T = \|\mathbf{b} - \mathbf{b}'\|^2.$$

just as the scheme in Section 2.3

Unlike the previous scheme, instantiated with a 2i-IPFE scheme in this way, the comparison secret key csk is now secret, and the enrollment secret key esk and probe secret key psk are distinct. Without csk, one cannot compare an enrollment message $\mathbf{c_x}$ and a probe message $\mathbf{c_y}$. We can also transmit $\mathbf{c_x}$ in a public channel and store it in a public storage, under necessary security requirements of the 2i-IPFE scheme, such as indistinguishability of $\mathbf{c_x}$.

In the Device-of-Domain model, the indistinguishability of $\mathbf{c_x}$ is against an adversary who has a probe oracle $\mathsf{Probe}(\mathsf{psk}, \cdot)$. If Server is malicious, then it can use csk to distinguish $\mathbf{c_x}$ enrolled by different samples. Therefore, we must limit the adversary's ability. For example, we can require the adversary to distinguish biometric vectors sampled from distributions in a pre-defined pool, and the adversary can only probe vectors randomly sampled from a distribution in the pool. We can also limit the rate of the probe oracle.

2.5 Instantiation with a 2c-IPFE Scheme

Note that if labels remain constant, a 2c-IPFE scheme is reduced to a 2i-IPFE scheme. Therefore, we can consider utilizing the label to represent each domain in the Device-of-Domain model. Let $\mathsf{FE} = (\mathsf{FE.Setup}, \mathsf{FE.KeyGen}, \mathsf{FE.Enc}, \mathsf{FE.Dec})$ be a 2c-IPFE scheme we defined in Definition 3. Following the scheme in Section 2.4, we can instantiate a biometric authentication scheme using FE.

- $\mathsf{Setup}(1^\lambda)$: It calls $\mathsf{FE.Setup}(1^\lambda) \rightarrow \mathsf{sk}, \mathsf{ek}_1, \mathsf{ek}_2$, $\mathsf{FE.KeyGen}(sk, \mathbf{I}_{k+2}) \rightarrow \mathsf{dk}_\mathbf{I}$, where $\mathbf{I}_{k+2}$ is an identity matrix of size $(k+2) \times (k+2)$. For keys used for Domain ℓ, it outputs $\mathsf{esk} \leftarrow (\ell, \mathsf{ek}_1)$, $\mathsf{psk} \leftarrow (\ell, \mathsf{ek}_2)$, and $\mathsf{csk} \leftarrow \mathsf{dk}_\mathbf{I}$.

  Note that when the previous 2i-IPFE-based scheme in Section 2.4 is applied to a Device-of-Domain model, we assume that Setup is run once for each domain to generate different $\mathsf{esk}, \mathsf{psk}, \mathsf{csk}$. In the scheme in this section, however, Setup is run only once for all the domains, and each domain shares the same csk and the same $\mathsf{esk}, \mathsf{psk}$ except different labels.

- $\mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}(), \mathsf{encodeProbe}^{\mathcal{O}_\mathcal{B}}()$: The same as the scheme in 2.4.

- $\mathsf{Enroll}(\mathsf{esk}, \mathbf{x})$: It calls $\mathsf{FE.Enc}(\ell, \mathsf{ek}_1, \mathbf{x}) \rightarrow \mathbf{c_x}$ and outputs $\mathbf{c_x}$.

- $\mathsf{Probe}(\mathsf{psk}, \mathbf{y})$: It calls $\mathsf{FE.Enc}(\ell, \mathsf{ek}_2, \mathbf{y}) \rightarrow \mathbf{c_y}$ and outputs $\mathbf{c_y}$.

- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y})$: It calls $\mathsf{FE.Dec}(\mathsf{dk}_\mathbf{I}, \mathbf{c_x}, \mathbf{c_y}) \rightarrow s$ and outputs the value s.

- $\mathsf{Verify}(s)$: If $\sqrt{s} < \tau$, a pre-defined threshold for comparing the closeness of two templates, then it outputs $r = 1$; otherwise, it outputs $r = 0$.

By the correctness of the functional encryption scheme FE, if the labels of $\mathbf{c_x}$ and $\mathbf{c_y}$ are the same (they are of the same domain), we have

$$s = \mathsf{FE.Dec}(\mathsf{dk}_\mathbf{I}, \mathbf{c_x}, \mathbf{c_y}) = \mathbf{x}\mathbf{I}_{k+2}\mathbf{y}^T = \|\mathbf{b} - \mathbf{b}'\|^2.$$

just as the scheme in Section 2.4

When the Device-of-Domain model is instantiated with a 2c-IPFE scheme in this way, the enrollment secret key esk and probe secret key psk are now shared among all the devices, regardless of their domains. Therefore, to let a malicious or broken Domain not threaten other honest ones, one needs to make sure given esk or psk, $\mathbf{c_x}$ still does not leak information about $\mathbf{x}$. This is different from the scheme in Section 2.4, where we only need seurity against an adversary who has a probe oracle $\mathsf{Probe}(\mathsf{psk}, \cdot)$.

If Server and Domain are both malicious, then the adversary can use csk to distinguish $\mathbf{c_x}$ and even recover $\mathbf{x}$. Therefore, we assume at most one party of them can be malicious at the same time. Note that this is the same as the 2i-IPFE-based scheme, where only one of Server and Domain can be malicious.

2.6 Instantiation with a Relational Hash Scheme

Let $\mathsf{RH} = (\mathsf{RH.KeyGen}, \mathsf{RH.Hash}_1, \mathsf{RH.Hash}_2, \mathsf{RH.Verify})$ be a relational hash scheme we defined in Definition 4 for the relation R of Hamming distance proximity parametrized by a constant τ.

$$R = \{(\mathbf{x}, \mathbf{y}) \mid \mathsf{HD}(\mathbf{x}, \mathbf{y}) \leq \tau \wedge \mathbf{x}, \mathbf{y} \in \{0,1\}^k\}$$

Note that here we ignore the third parameter Z. Following [EM23] and [MR14], we can instantiate a biometric authentication scheme using RH. Let the biometric distribution $\mathcal{B} \subseteq \{0,1\}^k$.

- $\mathsf{Setup}(1^\lambda)$: It calls $\mathsf{RH.Setup}(1^\lambda) \to \mathsf{pk}$ and outputs $\mathsf{esk} \leftarrow \mathsf{pk}$, $\mathsf{psk} \leftarrow \mathsf{pk}$, and $\mathsf{csk} \leftarrow \mathsf{pk}$.
- $\mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}()$: For a template vector $\mathbf{b}$ sampled from $\mathcal{O}_\mathcal{B}$, it direclty outputs $\mathbf{x} \leftarrow \mathbf{b}$.
- $\mathsf{Enroll}(\mathsf{esk}, \mathbf{x})$: It calls $\mathsf{RH.Hash}_1(\mathsf{pk}, \mathbf{x}) \to \mathbf{h_x}$ and outputs $\mathbf{c_x} \leftarrow \mathbf{h_x}$.
- $\mathsf{encodeProbe}^{\mathcal{O}_\mathcal{B}}()$: For a template vector $\mathbf{b}'$ sampled from $\mathcal{O}_\mathcal{B}$, it directy outputs $\mathbf{y} \leftarrow \mathbf{b}'$.
- $\mathsf{Probe}(\mathsf{psk}, \mathbf{y})$: It calls $\mathsf{RH.Hash}_2(\mathsf{pk}, \mathbf{y}) \to \mathbf{h_y}$ and outputs $\mathbf{c_y} \leftarrow \mathbf{h_y}$.
- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y})$: It calls $\mathsf{RH.Verify}(\mathsf{pk}, \mathbf{h_x}, \mathbf{h_y}) \to s$ and outputs the value s.
- $\mathsf{Verify}(s)$: It direclty returns $r \leftarrow s$.

By the correctness of the relational hash scheme RH, we have (except for a negligible probability),

$$r = 1 \Leftrightarrow (\mathbf{x}, \mathbf{y}) \in R \Leftrightarrow \mathsf{HD}(\mathbf{b}, \mathbf{b}') \leq \tau$$

3 Security Games

From now on, we consider a family of biometric distributions $\mathbb{B}$. Removing a person $\mathcal{B}$ from $\mathbb{B}$ is written as $\mathbb{B} \setminus \mathcal{B}$. We presume that $\mathbb{B}$ has an excessively large size for an adversary to enumerate. To model the knowledge about the biometric distributions, we offer an oracle $\mathcal{O}_{\mathsf{samp}}(\cdot)$ to all adversaries in this section.

- $\mathcal{O}_{\mathsf{samp}}(\cdot)$: On input an index i,
  - If i was not queried before, it first samples a biometric distribution $\mathcal{B}_i \in \mathbb{B}$ and then outputs a biometric sample $\mathbf{b} \leftarrow_\$ \mathcal{B}_i$.
  - If i has been queried before, it outputs a biometric sample $\mathbf{b} \leftarrow_\$ \mathcal{B}_i$.

3.1 Unforgeability against Leakage of Biometrics (UF-LoB)

In the game of Unforgeability against Leakage of Biometrics, we model the ability of an adversary who has access to User's biometrics and tries to impersonate User. The adversary $\mathcal{A}$ is given the enrollment message $\mathbf{c_x}$, oracles $\mathcal{O}_\mathcal{B}$ and $\mathcal{O}^q_{\mathsf{auth}}$, and auxiliary information option, which depends on our threat model. The adversary tries to find a valid probe message $\tilde{\mathbf{z}}$. The whole game $\mathsf{UF\text{-}LoB}$ is defined in Algorithm 1.

Algorithm 1 $\mathsf{UF\text{-}LoB}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$

```
B ←$ 𝔹, 𝔹 ← 𝔹 \ B
esk, psk, csk ← Setup(1^λ)
x ← encodeEnroll^{O_B}()
c_x ← Enroll(esk, x)
z̃ ← A^{O_samp, O_B, O^q_auth}(c_x, option)
if z̃ equals to any output of O_Probe then
    return ⊥
end if
s ← Compare(csk, c_x, z̃)
return Verify(s)
```

The given oracle is defined as follows:

- $\mathcal{O}_\mathcal{B}$: It outputs a biometric sample $\mathbf{b} \leftarrow_\$ \mathcal{B}$.
- $\mathcal{O}^q_{\mathsf{auth}}(\mathsf{csk}, \cdot, \cdot)$: This is a resource-limited oracle. If it has been queried over q times in total, it aborts. Otherwise, on input $\tilde{\mathbf{c_x}}, \tilde{\mathbf{c_y}}$, it outputs $\mathsf{Verify}(\mathsf{Compare}(\mathsf{csk}, \tilde{\mathbf{c_x}}, \tilde{\mathbf{c_y}}))$.

The auxiliary information option can be nothing or include esk or the following oracles:

- $\mathcal{O}_{\mathsf{Enroll}}(\mathsf{esk}, \cdot)$: On input $\mathbf{x}'$, it outputs the enrollment message $\mathsf{Enroll}(\mathsf{esk}, \mathbf{x}')$.
- $\mathcal{O}_{\mathsf{Probe}}(\mathsf{psk}, \cdot)$: On input $\mathbf{y}'$, it outputs the probe message $\mathsf{Probe}(\mathsf{psk}, \mathbf{y}')$.

To avoid trivial attacks, if the probe oracle $\mathcal{O}_{\mathsf{Probe}}$ is given, the adversary is asked to return some $\tilde{\mathbf{z}}$ that is not returned by the probe oracle.

To consider potential false positives of biometrics match, we consider the plain UF game in Algorithm 2, in which the adversary has only public information.

Algorithm 2 $\mathsf{UF}_{\Pi,\mathbb{B}}(\mathcal{A}')$

```
B ←$ 𝔹, 𝔹 ← 𝔹 \ B
esk, psk, csk ← Setup(1^λ)
x ← encodeEnroll^{O_B}()
c_x ← Enroll(esk, x)
z̃ ← A'^{O_samp, O^q_log}()
s ← Compare(csk, c_x, z̃)
return Verify(s)
```

- $\mathcal{O}^q_{\mathsf{log}}(\mathsf{csk}, \mathbf{c_x}, \cdot)$: This is a resource-limited oracle. If it has been queried over q times in total, it aborts. Otherwise, on input $\mathbf{z}$, it outputs $\mathsf{Verify}(\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{z}))$.

We define the advantage of an adversary $\mathcal{A}$ with option in the $\mathsf{UF\text{-}LoB}$ game of a scheme Π associated with a family of distributions $\mathbb{B}$ as

$$\mathbf{Adv}^{\mathsf{UF\text{-}LoB}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} := \Pr[\mathsf{UF\text{-}LoB}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A}) \to 1] - \sup_{\text{PPT } \mathcal{A}'} \Pr[\mathsf{UF}_{\Pi,\mathbb{B}}(\mathcal{A}') \to 1].$$

An authentication scheme Π associated with a family of distributions $\mathbb{B}$ is called option-*unforgeable against leakage of biometrics (UF-LoB)* if for any PPT adversary $\mathcal{A}$,

$$\mathbf{Adv}^{\mathsf{UF\text{-}LoB}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} = \mathsf{negl}.$$

Note that if csk is an empty or public string, then the scheme cannot achieve UF-LoB security when the false positive rate is not negligible, as the adversary can run the $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \cdot)$ over q times to boost the false positive rates.

For the rest of this report, if the scheme, the family distribution, and the auxiliary information option are clear from context, we omit the subscript and write the game as $\mathsf{UF\text{-}LoB}(\mathcal{A})$. This abbreviation also holds for all other games.

3.2 Unforgeability against Leakage of Keys (UF-LoK)

In the game of Unforgeability against Leakage of Keys, we model the ability of an adversary who has access to keys and tries to impersonate User. The adversary $\mathcal{A}$ is given the enrollment message $\mathbf{c_x}$, oracles $\mathcal{O}^q_{\mathsf{auth}}$, and auxiliary information option, which depends on our threat model. The adversary tries to find a valid probe message $\tilde{\mathbf{z}}$. The whole game $\mathsf{UF\text{-}LoK}$ is defined in Algorithm 3.

Algorithm 3 $\mathsf{UF\text{-}LoK}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$

1: $\mathcal{B} \leftarrow_\$ \mathbb{B}, \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}$
2: **for** $i = 1$ to L **do**
3: $\quad \mathsf{esk}_i, \mathsf{psk}_i, \mathsf{csk}_i \leftarrow \mathsf{Setup}(1^\lambda)$ ▷ Keys for Domain $i \in [L]$
4: **end for**
5: $\mathbf{x} \leftarrow \mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}()$
6: $\mathbf{c_x} \leftarrow \mathsf{Enroll}(\mathsf{esk}_1, \mathbf{x})$
7: $\tilde{\mathbf{z}} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{samp}}, \mathcal{O}^q_{\mathsf{auth}}}(\mathbf{c_x}, \mathsf{option})$
8: $s \leftarrow \mathsf{Compare}(\mathsf{csk}_1, \mathbf{c_x}, \tilde{\mathbf{z}})$
9: **return** $\mathsf{Verify}(s)$

The auxiliary information option can be nothing or include esk_1, $\mathcal{O}_{\mathsf{Enroll}}(\mathsf{esk}_1, \cdot)$, psk_1, $\mathcal{O}_{\mathsf{Probe}}(\mathsf{psk}_1, \cdot)$, csk_1, or the following oracles.

- $\mathcal{O}^{(i)}_{\mathsf{Enroll}}(\mathsf{esk}_i, \cdot)$: On input $\mathbf{x}'$, it outputs the enrollment message $\mathsf{Enroll}(\mathsf{esk}_i, \mathbf{x}')$.
- $\mathcal{O}^{(i)}_{\mathsf{Probe}}(\mathsf{psk}_i, \cdot)$: On input $\mathbf{y}'$, it outputs the probe message $\mathsf{Probe}(\mathsf{psk}_i, \mathbf{y}')$.

- $\mathcal{O}'_{\mathsf{Enroll}}(\cdot)$: On input esk', it first samples $\mathbf{x}' \leftarrow\!\$\ \mathsf{encodeEnroll}^{\mathcal{O}_{\mathcal{B}}}()$ and outputs $\mathsf{Enroll}(\mathsf{esk}', \mathbf{x}')$.
- $\mathcal{O}'_{\mathsf{Probe}}(\cdot)$: On input psk', it first samples $\mathbf{y}' \leftarrow\!\$\ \mathsf{encodeProbe}^{\mathcal{O}_{\mathcal{B}}}()$ and outputs $\mathsf{Probe}(\mathsf{psk}', \mathbf{y}')$. This oracle and psk_1 should not be given at the same time.

To consider potential false positives of biometrics match, we also consider the plain UF game in Algorithm 2, in which the adversary has only public information.

We define the advantage of an adversary $\mathcal{A}$ with option in the $\mathsf{UF\text{-}LoK}$ game of a scheme Π associated with a family of distributions $\mathbb{B}$ as

$$\mathbf{Adv}^{\mathsf{UF\text{-}LoK}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} := \Pr[\mathsf{UF\text{-}LoK}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A}) \to 1] - \sup_{\text{PPT } \mathcal{A}'} \Pr[\mathsf{UF}_{\Pi,\mathbb{B}}(\mathcal{A}') \to 1].$$

An authentication scheme Π associated with a family of distributions $\mathbb{B}$ is called *option-unforgeable against leakage of keys (UF-LoK)* if for any PPT adversary $\mathcal{A}$,

$$\mathbf{Adv}^{\mathsf{UF\text{-}LoK}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} = \mathsf{negl}.$$

3.3 Indistinguishable against Malicious Server (IND-MSV)

In the game of Indistinguishable against Malicious Server, we model the ability of a malicious Server who tries to identify the user. The adversary $\mathcal{A}$ is given oracles to two biometric distributions $\mathcal{B}^{(0)}, \mathcal{B}^{(1)}$, the comparison key csk, an enrollment message $\mathbf{c_x}$, and a list of t probe messages $\{\mathbf{c_y}^{(i)}\}_{i=1}^{t}$. It tries to guess from either $\mathcal{B}^{(0)}$ or $\mathcal{B}^{(1)}$ these messages are generated. The whole game is defined in Algorithm 4.

Algorithm 4 $\mathsf{IND\text{-}MSV}_{\Pi,\mathbb{B}}(\mathcal{A})$

```
b ←$ {0,1}
B^(0) ←$ 𝔹,    𝔹 ← 𝔹 \ B^(0)
B^(1) ←$ 𝔹,    𝔹 ← 𝔹 \ B^(1)
esk, psk, csk ← Setup(1^λ)
x ← encodeEnroll^{O_{B^(b)}}()
c_x ← Enroll(esk, x)
for i = 1 to t do
    y^(i) ← encodeProbe^{O_{B^(b)}}()
    c_y^(i) ← Probe(psk, y^(i))
end for
In Device-of-User Model:
    b̃ ← A^{O_{B^(0)}, O_{B^(1)}}(csk, c_x, {c_y^(i)}_{i=1}^t)
In Device-of-Domain Model:
    b̃ ← A^{O_{B^(0)}, O_{B^(1)}, O^samp_Probe}(csk, c_x, {c_y^(i)}_{i=1}^t)
return 1_{b̃=b}
```

Note that in Device-of-Domain model, a probe oracle is given to the adversary.

- $\mathcal{O}^{\mathsf{samp}}_{\mathsf{Probe}}(\cdot)$: On input an index i, it first samples $\mathbf{y}' \leftarrow\!\$\ \mathsf{encodeProbe}^{\mathcal{O}_{\mathsf{samp}}(i)}$, which uses $\mathcal{O}_{\mathsf{samp}}(i)$ to answer biometric queries, and outputs $\mathsf{Probe}(\mathsf{psk}, \mathbf{y}')$.

We provide $\mathcal{O}^{\mathsf{samp}}_{\mathsf{Probe}}(\cdot)$ instead of $\mathcal{O}_{\mathsf{Probe}}(\mathsf{psk}, \cdot)$. This is to avoid the trivial attack where the adversary probes samples from the oracles $\mathcal{O}_{\mathcal{B}^{(0)}}$ and $\mathcal{O}_{\mathcal{B}^{(1)}}$ and compare the results with $\mathbf{c}_{\mathbf{x}}$.

We define the advantage of an adversary $\mathcal{A}$ in the $\mathsf{IND\text{-}MSV}$ game of a scheme Π associated with a family of distributions $\mathbb{B}$ as

$$\mathbf{Adv}^{\mathsf{IND\text{-}MSV}}_{\Pi,\mathbb{B},\mathcal{A}^{\mathcal{O}}} := \left|\Pr[\mathsf{IND\text{-}MSV}_{\Pi}(\mathcal{A}^{\mathcal{O}}) \to 1] - \frac{1}{2}\right|.$$

An authentication scheme Π associated with a family of distributions $\mathbb{B}$ is called *indistinguishable against malicious server (IND-MSV)* if for any PPT adversary $\mathcal{A}$,

$$\mathbf{Adv}^{\mathsf{IND\text{-}MSV}}_{\Pi,\mathbb{B},\mathcal{A}} = \mathsf{negl}.$$

4 Security Analysis

Given an fh-IPFE scheme FE, we define the IND game in algorithm 5.

Algorithm 5 $\mathsf{IND}_{\mathsf{FE}}(\mathcal{A})$

1: $b \leftarrow_{\$} \{0,1\}$
2: $\mathsf{msk}, \mathsf{pp} \leftarrow \mathsf{FE.Setup}(1^{\lambda})$
3: $\tilde{b} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{KeyGen}}, \mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$
4: **return** $1_{\tilde{b}=b}$

- $\mathcal{O}_{\mathsf{KeyGen}}(\cdot,\cdot)$: On input pair $(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$, it outputs $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}^{(b)})$.
- $\mathcal{O}_{\mathsf{Enc}}(\cdot,\cdot)$: On input pair $(\mathbf{y}^{(0)}, \mathbf{y}^{(1)})$, it outputs $\mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y}^{(b)})$.

To avoid trivial attacks, we consider *admissible adversaries.*

Definition 5 (Admissible Adversary)**.** Let $\mathcal{A}$ be an adversary in an IND game, and let $(\mathbf{x}_1^{(0)}, \mathbf{x}_1^{(1)}), \cdots, (\mathbf{x}_{Q_K}^{(0)}, \mathbf{x}_{Q_K}^{(1)})$ be its queries to $\mathcal{O}_{\mathsf{KeyGen}}$ and $(\mathbf{y}_1^{(0)}, \mathbf{y}_1^{(1)}), \cdots, (\mathbf{y}_{Q_E}^{(0)}, \mathbf{y}_{Q_E}^{(1)})$ be its queries to $\mathcal{O}_{\mathsf{Enc}}$. We say $\mathcal{A}$ is *admissible* if $\forall i \in [Q_K], \forall j \in [Q_E]$,

$$\mathbf{x}_i^{(0)^T} \mathbf{y}_j^{(0)} = \mathbf{x}_i^{(1)^T} \mathbf{y}_j^{(1)}$$

Definition 6 (IND Security)**.** An fh-IPFE scheme FE is called IND secure if for any admissible adversary $\mathcal{A}$, the advantage of $\mathcal{A}$ in the IND game in Algorithm 5 is

$$\mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{A}} := \left|\Pr[\mathsf{IND}_{\mathsf{FE}}(\mathcal{A}) \to 1] - \frac{1}{2}\right| = \mathsf{negl}.$$

Algorithm 6 $\mathsf{UF\text{-}LoB}^*_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$

$\mathcal{B} \leftarrow_{\$} \mathbb{B}, \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}$
$\mathsf{esk}, \mathsf{psk}, \mathsf{csk} \leftarrow \mathsf{Setup}(1^\lambda)$
$\mathbf{x} \leftarrow \mathsf{encodeEnroll}^{\mathcal{O}_\mathcal{B}}()$
$\mathbf{c_x} \leftarrow \mathsf{Enroll}(\mathsf{esk}, \mathbf{x})$
$\tilde{\mathbf{z}} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{samp}}, \mathcal{O}^q_{\mathsf{auth}}}(\mathbf{c_x}, \mathsf{option})$
$s \leftarrow \mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \tilde{\mathbf{z}})$
return $\mathsf{Verify}(s)$

4.1 IND Security and UF-LoB Security

Let Π be the authentication scheme instantiated by an fh-IPFE scheme FE as in Section 2.3. We see a relation between the IND security of FE and the UF-LoB security of Π. To show this, first we define a middle game for UF-LoB security in Algorithm 6. The difference is that the adversary $\mathcal{A}$ has no access to $\mathcal{O}_\mathcal{B}$.

Theorem 1. *For any distribution family $\mathbb{B}$, if FE is IND secure, then $\forall \mathcal{A}$ in the $\mathsf{UF\text{-}LoB}$ game where the auxiliary information option is either nothing or the oracle $\mathcal{O}_{\mathsf{Enroll}}$, there exists an adversary $\mathcal{A}^*$ in the $\mathsf{UF\text{-}LoB}^*$ game such that*

$$\Pr[\mathsf{UF\text{-}LoB}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF\text{-}LoB}^*_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A}^*) \to 1] = \mathsf{negl}.$$

Proof. We only prove the case when option includes the oracle $\mathcal{O}_{\mathsf{Enroll}}$. The proof can be directly applied when option is empty. Given an adversary $\mathcal{A}$ in the $\mathsf{UF\text{-}LoB}$ game, consider the reduction adversary $\mathcal{R}$ in Algorithm 7 which plays the IND game. $\mathcal{R}$ runs $\mathcal{A}$ and simulates each oracle in the following way.

- $\mathcal{O}_{\mathsf{samp}}(i)$: This is sampled by $\mathcal{O}_{\mathsf{samp}}(i)$ of $\mathcal{R}$.
- $\mathcal{O}_\mathcal{B}$: This is simulated by the oracle $\mathcal{O}_{\mathcal{B}^{(0)}}$
- $\mathcal{O}^q_{\mathsf{auth}}(\mathsf{csk}, \tilde{\mathbf{c_x}}, \tilde{\mathbf{c_y}})$: This is simulated by calling $\mathsf{Verify}(\mathsf{FE.Dec}(\mathsf{pp}, \tilde{\mathbf{c_x}}, \tilde{\mathbf{c_y}}))$.
- $\mathcal{O}_{\mathsf{Enroll}}(\mathsf{esk}, \mathbf{x}')$: This is simulated by $\mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}', \mathbf{x}')$ given in the IND game.

Note that since $\mathcal{R}$ never calls $\mathcal{O}_{\mathsf{Enc}}$, it is an admissible adversary.

If the challenge bit $b = 0$, then $\mathcal{R}$ perfectly simulates a $\mathsf{UF\text{-}LoB}$ game for $\mathcal{A}$. Therefore, the probability that $\mathsf{Verify}(s) = 1$ in Line 8 is $\Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1]$.

If the challenge bit $b = 1$, then $\mathcal{B}^{(1)}$ is never seen by $\mathcal{A}$, and $\mathcal{B}^{(0)}$ has the same distribution as any $\mathcal{B}^* \in \mathbb{B}$ that has never been queried before in the view of $\mathcal{A}$. Let $\mathcal{A}^*$ be an adversary in the $\mathsf{UF\text{-}LoB}^*$ game which runs $\mathcal{A}$ and simulates oracle $\mathcal{O}_\mathcal{B}$ by $\mathcal{O}_{\mathsf{samp}}(i^*)$ for some index i^* that is never queried by $\mathcal{A}$ in $\mathcal{O}_{\mathsf{samp}}(\cdot)$. The probability that $\mathsf{Verify}(s) = 1$ in Line 8 is $\Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1]$.

Algorithm 7 $\mathcal{R}^{\mathcal{O}_{\mathsf{KeyGen}},\mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$

$\mathcal{B}^{(0)} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}^{(0)}$
$\mathcal{B}^{(1)} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}^{(1)}$
$\mathbf{x}^{(0)} \leftarrow \mathsf{encodeEnroll}^{\mathcal{O}_{\mathcal{B}^{(0)}}}()$
$\mathbf{x}^{(1)} \leftarrow \mathsf{encodeEnroll}^{\mathcal{O}_{\mathcal{B}^{(1)}}}()$
$\mathbf{c}_{\mathbf{x}} \leftarrow \mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$
$\tilde{\mathbf{z}} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{samp}},\mathcal{O}_{\mathcal{B}^{(0)}},\mathcal{O}^{q}_{\mathsf{auth}},\mathcal{O}_{\mathsf{Enroll}}}(\mathbf{c}_{\mathbf{x}})$
$s \leftarrow \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}_{\mathbf{x}}, \tilde{\mathbf{z}})$
if $\mathsf{Verify}(s) = 1$ **then**
  return $\tilde{b} = 0$
else
  return $\tilde{b} \leftarrow_{\$} \{0, 1\}$
end if

In conclusion,

$$\begin{aligned}
\Pr[\mathsf{IND}(\mathcal{R}) \to 1] &= \Pr[b = 0] \cdot \left(\Pr[\mathsf{Verify}(s) = 1 \mid b = 0] + \frac{1}{2} \Pr[\mathsf{Verify}(s) = 0 \mid b = 0] \right) \\
&\quad + \Pr[b = 1] \cdot \frac{1}{2} \Pr[\mathsf{Verify}(s) = 0 \mid b = 1] \\
&= \frac{1}{2} \cdot \left(\Pr[\mathsf{Verify}(s) = 1 \mid b = 0] + \frac{1}{2}(1 - \Pr[\mathsf{Verify}(s) = 1 \mid b = 0]) \right) \\
&\quad + \frac{1}{4} \cdot (1 - \Pr[\mathsf{Verify}(s) = 1 \mid b = 1]) \\
&= \frac{1}{2} + \frac{1}{4} (\Pr[\mathsf{Verify}(s) = 1 \mid b = 0] - \Pr[\mathsf{Verify}(s) = 1 \mid b = 1]) \\
&= \frac{1}{2} + \frac{1}{4} (\Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1])
\end{aligned}$$

Since $\mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}} = \left| \Pr[\mathsf{IND}(\mathcal{R}) \to 1] - \frac{1}{2} \right| = \mathsf{negl}$,

$$\Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1] = 4 \cdot \mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}} = \mathsf{negl}.$$

□

For the next theorem, we consider two assumptions for the fh-IPFE scheme FE.

Assumption 1. Assume that there exists a simulator $\mathcal{S}$ such that given pp, the statistical distance between $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{r})$ and $\mathcal{S}(\mathsf{pp})$, where $(\mathsf{msk}, \mathsf{pp}) \leftarrow \mathsf{FE.Setup}(1^\lambda)$ and $\mathbf{r} \leftarrow_{\$} \mathbb{F}^{k+2}$, is bounded by δ. That is, let $(\mathsf{msk}, \mathsf{pp}) \leftarrow \mathsf{FE.Setup}(1^\lambda)$ and $\mathbf{r} \leftarrow_{\$} \mathbb{F}^{k+2}$,

$$\Delta\left((\mathsf{pp}, \mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{r})), (\mathsf{pp}, \mathcal{S}(\mathsf{pp}))\right) \leq \delta.$$

The distribution of $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{r})$ is taken over $\mathsf{FE.Setup}$, $\mathsf{FE.KeyGen}$, and $\mathbf{r}$.

Assumption 2. Assume that $\forall \mathbf{z}$ and $\forall \mathbf{x} \leftarrow_\$ \mathcal{B} \in \mathbb{B}$, functional decryption results of two invocations of $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x})$ and $\mathbf{z}$ are the same with a probability bounded below by $1 - \epsilon$. That is, let $(\mathsf{msk}, \mathsf{pp}) \leftarrow \mathsf{FE.Setup}(1^\lambda)$, and let $\mathbf{c_x}, \mathbf{c_x}'$ be the results of two invocations of $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x})$,

$$\Pr\left[\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c_x}, \mathbf{z}) = \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c_x}', \mathbf{z})\right] \geq 1 - \epsilon$$

The probability is taken over $\mathsf{FE.Setup}$ and $\mathsf{FE.KeyGen}$.

Theorem 2. *Given Assumption 1 with a negligible δ and Assumption 2 with a negligible ϵ. For any distribution family $\mathbb{B}$, if FE is IND secure, then $\forall \mathcal{A}^*$ in the UF-LoB* game where the auxiliary information option is either nothing or the oracle $\mathcal{O}_{\mathsf{Enroll}}$, there exists an adversary $\mathcal{A}'$ in the UF game such that*

$$\Pr[\mathit{UF\text{-}LoB}^*_{\Pi,\mathbb{B},\mathit{option}}(\mathcal{A}^*) \rightarrow 1] - \Pr[\mathit{UF}_{\Pi,\mathbb{B}}(\mathcal{A}') \rightarrow 1] = \mathsf{negl}.$$

Proof. We only prove the case when option includes the oracle $\mathcal{O}_{\mathsf{Enroll}}$ since the case when option is empty directly follows the result. Given an adversary $\mathcal{A}^*$ in the $\mathsf{UF\text{-}LoB}^*$ game, consider the reduction adversary $\mathcal{R}$ in Algorithm 8 which plays the IND game. $\mathcal{R}$ runs $\mathcal{A}^*$ and simulates each oracle in the following way.

- $\mathcal{O}_{\mathsf{samp}}(i)$: This is sampled by $\mathcal{O}_{\mathsf{samp}}(i)$ of $\mathcal{R}$.
- $\mathcal{O}^q_{\mathsf{auth}}(\mathsf{csk}, \tilde{\mathbf{c_x}}, \tilde{\mathbf{c_y}})$: This is simulated by calling $\mathsf{Verify}(\mathsf{FE.Dec}(\mathsf{pp}, \tilde{\mathbf{c_x}}, \tilde{\mathbf{c_y}}))$.
- $\mathcal{O}_{\mathsf{Enroll}}(\mathsf{esk}, \mathbf{x}')$: This is simulated by $\mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}', \mathbf{x}')$ given in the IND game.

Note that since $\mathcal{R}$ never calls $\mathcal{O}_{\mathsf{Enc}}$, it is an admissible adversary.

Algorithm 8 $\mathcal{R}^{\mathcal{O}_{\mathsf{KeyGen}}, \mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$

```
1: B ←$ 𝔹,   𝔹 ← 𝔹 \ B
2: x ← encodeEnroll^{O_B}()
3: r ←$ F^{k+2}
4: c ← O_KeyGen(x, r)
5: z̃ ← A*^{O_samp, O^q_auth, O_Enroll}(c)
6: c_x ← O_KeyGen(x, x)
7: s ← FE.Dec(pp, c_x, z̃)
8: if Verify(s) = 1 then
9:     return b̃ = 0
10: else
11:    return b̃ ←$ {0, 1}
12: end if
```

If the challenge bit $b = 0$, then $\mathcal{R}$ perfectly simulates a $\mathsf{UF\text{-}LoB}^*$ game for $\mathcal{A}^*$. Therefore, the probability that $\mathsf{Verify}(\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}, \tilde{\mathbf{z}})) = 1$ is $\Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \rightarrow 1]$. By Assumption 2, the probability that $\mathsf{Verify}(s) = 1$ in Line 8 is at least

$$\begin{aligned} & \Pr\left[s = \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}, \tilde{\mathbf{z}})\right] \cdot \Pr\left[\mathsf{Verify}(\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}, \tilde{\mathbf{z}})) = 1\right] \\ \geq \quad & (1 - \epsilon) \cdot \Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \rightarrow 1] \end{aligned}$$

If the challenge bit $b = 1$, then the statistical distance between $\mathbf{c}$ in Line 4 and the output of $\mathcal{S}$ from Assumption 1 is within δ. Let $\mathcal{A}'$ be an adversary in the UF game which runs $\mathcal{A}^*$ and simulates $\mathbf{c}_\mathbf{x}$ by $\mathcal{S}(\mathsf{pp})$ from Assumption 1 and $\mathcal{O}^q_{\mathsf{auth}}$ by running $\mathsf{Verify}(\mathsf{FE.Dec}(\mathsf{pp}, \cdot, \cdot))$. The probability that $\mathsf{Verify}(s) = 1$ in Line 8 is at most $\Pr[\mathsf{UF}(\mathcal{A}') \to 1] + \delta$.

In conclusion,

$$\begin{aligned}\Pr[\mathsf{IND}(\mathcal{R}) \to 1] = \Pr[b = 0] \cdot &\left(\Pr[\mathsf{Verify}(s) = 1 \mid b = 0] + \frac{1}{2}\Pr[\mathsf{Verify}(s) = 0 \mid b = 0]\right) \\ &+ \Pr[b = 1] \cdot \frac{1}{2}\Pr[\mathsf{Verify}(s) = 0 \mid b = 1] \\ &= \frac{1}{2} + \frac{1}{4}\left(\Pr[\mathsf{Verify}(s) = 1 \mid b = 0] - \Pr[\mathsf{Verify}(s) = 1 \mid b = 1]\right) \\ &\geq \frac{1}{2} + \frac{1}{4}\left((1 - \epsilon) \cdot \Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1] - \Pr[\mathsf{UF}(\mathcal{A}') \to 1] - \delta\right)\end{aligned}$$

Since $\mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}} = \left|\Pr[\mathsf{IND}(\mathcal{R}) \to 1] - \frac{1}{2}\right| = \mathsf{negl}$, $\epsilon = \mathsf{negl}$, and $\delta = \mathsf{negl}$,

$$\Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1] - \Pr[\mathsf{UF}(\mathcal{A}') \to 1] \leq 4 \cdot \mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}} + \epsilon + \delta = \mathsf{negl}.$$

□

Corollary 1. *Given Assumption 1 with a negligible δ and Assumption 2 with a negligible ϵ. For any distribution family $\mathbb{B}$, if FE is IND secure, then $\forall \mathcal{A}$ in the $\mathsf{UF\text{-}LoB}$ game where the auxiliary information option is either nothing or the oracle $\mathcal{O}_{\mathsf{Enroll}}$, there exists an adversary $\mathcal{A}'$ in the UF game such that*

$$\Pr[\mathsf{UF\text{-}LoB}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF}_{\Pi,\mathbb{B}}(\mathcal{A}') \to 1] = \mathsf{negl}.$$

As a result, $\mathbf{Adv}^{\mathsf{UF\text{-}LoB}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} = \mathsf{negl}$. The authentication scheme is option-UF-LoB secure.

Proof. Given an adversary $\mathcal{A}$ in the $\mathsf{UF\text{-}LoB}$ game, from Theorem 1, we know there exists a reduction $\mathcal{R}_1$ and an adversary $\mathcal{A}^*$ such that

$$\Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1] = 4 \cdot \mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}_1}$$

With $\mathcal{A}^*$, from Theorem 2, we know there exists a reduction $\mathcal{R}_2$ and an adversary $\mathcal{A}'$ such that

$$\Pr[\mathsf{UF\text{-}LoB}^*(\mathcal{A}^*) \to 1] - \Pr[\mathsf{UF}(\mathcal{A}') \to 1] \leq 4 \cdot \mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}_2} + \epsilon + \delta.$$

Hence,

$$\Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF}(\mathcal{A}') \to 1] \leq 4 \cdot \left(\mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}_1} + \mathbf{Adv}^{\mathsf{IND}}_{\mathsf{FE},\mathcal{R}_2}\right) + \epsilon + \delta$$

Since FE is IND secure and both ϵ and δ are negligible,

$$\Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF}(\mathcal{A}') \to 1] = \mathsf{negl}.$$

In particular, since $\mathbf{Adv}^{\mathsf{UF\text{-}LoB}}_{\Pi,\mathbb{B},\mathcal{A}} \leq \Pr[\mathsf{UF\text{-}LoB}(\mathcal{A}) \to 1] - \Pr[\mathsf{UF}(\mathcal{A}') \to 1]$,

$$\mathbf{Adv}^{\mathsf{UF\text{-}LoB}}_{\Pi,\mathbb{B},\mathcal{A}} = \mathsf{negl}.$$

This holds for all PPT adversaries $\mathcal{A}$, so Π is UF-LoB secure.

□

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