The Cryptographic Layer of Biometric Authentication

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- Possibly non-negligible false positive/negative rates.

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- $\mathbf{b} \leftarrow_{\$} \mathcal{B}$: Sample a biometric template $\mathbf{b}$ from $\mathcal{B}$.

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- $\mathsf{getEnroll}^{\mathcal{O}_\mathcal{B}}() \to \mathbf{b}$: Given oracle $\mathcal{O}_\mathcal{B}$, output a biometric template $\mathbf{b}$ for enrollment.

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- $\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}') \rightarrow s$: Given two templates $\mathbf{b}$ and $\mathbf{b}'$, output a score s.

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- $\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}') \to s$: Given two templates $\mathbf{b}$ and $\mathbf{b}'$, output a score s.
- $\mathsf{Verify}(s) \to r \in \{0, 1\}$: Determine whether this is a successful authentication $(r = 1)$ or not $(r = 0)$.

- $\mathcal{O}_{\mathcal{B}}$: When queried, return a biometric template $\mathbf{b} \leftarrow_{\$} \mathcal{B}$.

Cryptographic Layer

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- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c}_\mathbf{x}, \mathbf{c}_\mathbf{y}) \to s$: Compare the enrollment message $\mathbf{c}_\mathbf{x}$ and probe message $\mathbf{c}_\mathbf{y}$ and output a score s.

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Correctness

For any $\mathcal{B}, \mathcal{B}' \in \mathbb{B}$,

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$$\Pr\left[\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y}) = \mathsf{BioCompare}(\mathbf{b}, \mathbf{b}')\right] = 1 - \mathsf{negl}.$$

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Appendix - Usage Example

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Unforgeability

Let Π be a biometric authentication scheme. For an adversary $\mathcal{A}$, define $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$.

$\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$

1: $\mathcal{B} \leftarrow\$ \mathbb{B}, \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}$
2: $\mathsf{esk}, \mathsf{psk}, \mathsf{csk} \leftarrow \mathsf{Setup}(1^\lambda)$
3: $\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_\mathcal{B}}()$
4: $\mathbf{c_x} \leftarrow \mathsf{Enroll}(\mathsf{esk}, \mathbf{b})$
5: $\tilde{\mathbf{z}} \leftarrow \mathcal{A}(\mathsf{option})$
6: $s \leftarrow \mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \tilde{\mathbf{z}})$
7: **return** $\mathsf{Verify}(s)$

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7: **return** $\mathsf{Verify}(s)$

Π is called *option-unforgeable* (option-UF) if for any PPT adversary $\mathcal{A}$,

$$\mathsf{Adv}^{\mathsf{UF}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} := \Pr[\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A}) \to 1] = \mathsf{negl}.$$

Possible Choice of option

- Enrollment message: $\mathbf{c_x}$.

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Note that some combinations of option induce a winning probability *true positive rate* or *false positive rate*.

True/False Positive Rates

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$$\mathsf{TP} := \Pr\left[\begin{array}{l} \mathcal{B} \leftarrow\!\!\$\; \mathbb{B} \\ \mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}() \\ \mathbf{b}' \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}}}() \end{array} : \mathsf{Verify}(\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}')) = 1\right]$$

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$$\mathsf{FP} := \Pr\left[\begin{array}{l} \mathcal{B} \leftarrow_{\$} \mathbb{B}, \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}, \mathcal{B}' \leftarrow_{\$} \mathbb{B} \\ \mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}() \\ \mathbf{b}' \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}'}}() \end{array} : \mathsf{Verify}(\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}')) = 1\right]$$

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$\mathcal{A}_1^{\mathcal{O}_{\mathcal{B}}}(\mathsf{psk})$

1: $\mathbf{b} \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}}}()$
2: $\mathbf{c_y} \leftarrow \mathsf{Probe}(\mathsf{psk}, \mathbf{b})$
3: **return** $\mathbf{c_y}$

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$\mathcal{A}_1^{\mathcal{O}_{\mathcal{B}}}(\mathsf{psk})$

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3: **return** $\mathbf{c_y}$

$\mathcal{A}_2(\mathsf{psk})$

1: $\mathcal{B}' \leftarrow_{\$} \mathbb{B}$.
2: $\mathbf{b}' \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}'}}()$
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  - $\mathcal{A}_1$ has a winning probability TP.
- psk is given only when FP is negligible.
  - $\mathcal{A}_2$ has a winning probability FP.
- When $\mathcal{O}_{\text{Probe}}$ is given, we forbid the adversary to return an answer of $\mathcal{O}_{\text{Probe}}$.

$\mathcal{A}_1^{\mathcal{O}_{\mathcal{B}}}(\text{psk})$

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Indistinguishability

Let $t \geq 0$ be an integer. For an adversary $\mathcal{A}$, define $\mathsf{IND}_{\Pi,\mathbb{B}}(\mathcal{A})$.

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1: $b \leftarrow_{\$} \{0,1\}$
2: $\mathcal{B}^{(0)} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}^{(0)}$
3: $\mathcal{B}^{(1)} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}^{(1)}$
4: $\mathsf{esk}, \mathsf{psk}, \mathsf{csk} \leftarrow \mathsf{Setup}(1^\lambda)$
5: $\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}^{(b)}}}()$
6: $\mathbf{c_x} \leftarrow \mathsf{Enroll}(\mathsf{esk}, \mathbf{b})$
7: **for** $i = 1$ to t **do**
8: $\quad \mathbf{b}'^{(i)} \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}^{(b)}}}()$
9: $\quad \mathbf{c_y}^{(i)} \leftarrow \mathsf{Probe}(\mathsf{psk}, \mathbf{b}'^{(i)})$
10: $\tilde{b} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathcal{B}^{(0)}}, \mathcal{O}_{\mathcal{B}^{(1)}}}(\mathsf{csk}, \mathbf{c_x}, \{\mathbf{c_y}^{(i)}\}_{i=1}^{t})$
11: **return** $1_{\tilde{b}=b}$

Indistinguishability

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Indistinguishability

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- Π is called *indistinguishable* (IND) if for any PPT adversary $\mathcal{A}$,

$$\mathsf{Adv}^{\mathsf{IND}}_{\Pi,\mathbb{B},\mathcal{A}} := \left|\Pr[\mathsf{IND}_{\Pi,\mathbb{B}}(\mathcal{A}) \to 1] - \frac{1}{2}\right| = \mathsf{negl}.$$

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In our project, we also discuss an instantiation using *relational hash*.

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Function-Hiding Inner Product Functional Encryption (adapted from [Kim+16])

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- $\mathsf{FE.Dec}(\mathsf{pp}, f_{\mathbf{x}}, \mathbf{c}_{\mathbf{y}})$: Output a value $z \in \mathbb{F}$ or an error symbol $\perp$.

fh-IPFE

Correctness

FE is *correct* if $\forall \mathbf{x}, \mathbf{y} \in \mathbb{F}^k$, let $(\mathsf{msk}, \mathsf{pp}) \leftarrow \mathsf{FE.Setup}(1^\lambda)$, we have

$$\Pr\left[\mathsf{FE.Dec}(\mathsf{pp}, \mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}), \mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y})) = \mathbf{x}\mathbf{y}^T\right] = 1 - \mathsf{negl}.$$

Instantiation using fh-IPFE [EM23]: Biometric Layer

- getEnroll$^{\mathcal{O}_\mathcal{B}}()$, getProbe$^{\mathcal{O}_\mathcal{B}}()$: Output vectors in $\{0, 1, \cdots, m\}^k$ for all $\mathcal{B} \in \mathbb{B}$.

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- $\mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$, $\mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}}}()$: Output vectors in $\{0, 1, \cdots, m\}^k$ for all $\mathcal{B} \in \mathbb{B}$.
- $\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}') \to \|\mathbf{b} - \mathbf{b}'\|^2$.

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- $\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}') \to \|\mathbf{b} - \mathbf{b}'\|^2$.
- For a pre-defined real number $\tau \geq 0$,

$$\mathsf{Verify}(s) \to \begin{cases} 1 & \text{if } \sqrt{s} \leq \tau \\ 0 & \text{if } \sqrt{s} > \tau \end{cases}.$$

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  - Encode $\mathbf{b}'$ as $\mathbf{y} = (y_1, y_2, \cdots, y_{k+2}) = (-2b'_1, -2b'_2, \cdots, -2b'_k, \|\mathbf{b}'\|^2, 1)$.
  - Run and output $\mathbf{c_y} \leftarrow \mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y})$.

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  - Run and output $\mathbf{c_y} \leftarrow \mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y})$.
- $\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y})$: Run and output $s \leftarrow \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c_x}, \mathbf{c_y})$.

Instantiation using fh-IPFE [EM23]

By the correctness of FE,

$$s = \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}_{\mathbf{x}}, \mathbf{c}_{\mathbf{y}}) = \mathbf{x}\mathbf{y}^T = \sum_{i=1}^{k} -2b_i b_i' + \|\mathbf{b}\|^2 + \|\mathbf{b}'\|^2 = \|\mathbf{b} - \mathbf{b}'\|^2.$$

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Note that in this instantiation,

- $\mathsf{esk} = \mathsf{psk}$.
- $\mathsf{csk} = \mathsf{pp}$, the public parameter of FE. Therefore, we offer csk for all adversaries.

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fh-IND of fh-IPFE

Given an fh-IPFE scheme FE, define the fh-IND game [Kim+16].

$\mathsf{fh\text{-}IND}_{\mathsf{FE}}(\mathcal{A})$

1: $b \leftarrow_\$ \{0, 1\}$
2: $\mathsf{msk}, \mathsf{pp} \leftarrow \mathsf{FE.Setup}(1^\lambda)$
3: $\tilde{b} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathsf{KeyGen}}, \mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$
4: **return** $1_{\tilde{b}=b}$

- $\mathcal{O}_{\mathsf{KeyGen}}(\cdot, \cdot)$: On input pair $(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$, output $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}^{(b)})$.
- $\mathcal{O}_{\mathsf{Enc}}(\cdot, \cdot)$: On input pair $(\mathbf{y}^{(0)}, \mathbf{y}^{(1)})$, output $\mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y}^{(b)})$.

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A trivial adversary can ask $\mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$ and $\mathcal{O}_{\mathsf{Enc}}(\mathbf{y}^{(0)}, \mathbf{y}^{(1)})$ such that

$$\mathbf{x}^{(0)}\mathbf{y}^{(0)\,T} \neq \mathbf{x}^{(1)}\mathbf{y}^{(1)\,T}.$$

fh-IND of fh-IPFE

Admissible Adversary

Let $\mathcal{A}$ be an adversary in an fh-IND game, and let $(\mathbf{x}_1^{(0)}, \mathbf{x}_1^{(1)}), \cdots, (\mathbf{x}_{Q_K}^{(0)}, \mathbf{x}_{Q_K}^{(1)})$ be its queries to $\mathcal{O}_{\mathsf{KeyGen}}$ and $(\mathbf{y}_1^{(0)}, \mathbf{y}_1^{(1)}), \cdots, (\mathbf{y}_{Q_E}^{(0)}, \mathbf{y}_{Q_E}^{(1)})$ be its queries to $\mathcal{O}_{\mathsf{Enc}}$.

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We say $\mathcal{A}$ is *admissible* if $\forall i \in [Q_K], \forall j \in [Q_E]$,

$$\mathbf{x}_i^{(0)} {\mathbf{y}_j^{(0)}}^T = \mathbf{x}_i^{(1)} {\mathbf{y}_j^{(1)}}^T.$$

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fh-IND Security

FE is called *fh-IND* secure if for any admissible adversary $\mathcal{A}$,

$$\mathsf{Adv}_{\mathsf{FE},\mathcal{A}}^{\text{fh-IND}} := \left| \Pr[\text{fh-IND}_{\mathsf{FE}}(\mathcal{A}) \to 1] - \frac{1}{2} \right| = \text{negl.}$$

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- However, fh-IND security is not sufficient for the UF security of Π.
    - We found that the instantiation Π using [Kim+16] is not option-UF for any option.
- For this, we define another extra security notion of FE: *RUF Security*.

RUF of fh-IPFE

Let $\gamma \geq 0$ be a real number and $\mathbb{F} = \mathbb{Z}_q$ for a prime number q. Define the RUF game.

$\mathsf{RUF}_{\mathsf{FE}}^{\mathcal{O},\gamma}(\mathcal{A})$

1: $\mathbf{r} \leftarrow\!\!\$\ \mathbb{F}^k$
2: $\mathsf{msk}, \mathsf{pp} \leftarrow \mathsf{FE.Setup}(1^\lambda)$
3: $\mathbf{c} \leftarrow \mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{r})$
4: $\tilde{\mathbf{z}} \leftarrow \mathcal{A}^{\mathcal{O}}(\mathsf{pp}, \mathbf{c})$
5: $s \leftarrow \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}, \tilde{\mathbf{z}})$
6: **return** $1_{s \leq \gamma}$

Oracle $\mathcal{O}$ can be nothing or include

- $\mathcal{O}'_{\mathsf{KeyGen}}(\cdot)$: On input $\mathbf{x}'$, output $\mathsf{FE.KeyGen}(\mathsf{msk}, \mathsf{pp}, \mathbf{x}')$.
- $\mathcal{O}'_{\mathsf{Enc}}(\cdot)$: On input $\mathbf{y}'$, output $\mathsf{FE.Enc}(\mathsf{msk}, \mathsf{pp}, \mathbf{y}')$.

RUF of fh-IPFE

- We forbid the adversary to return $\tilde{\mathbf{z}}$ that is an answer of $\mathcal{O}'_{\mathsf{Enc}}(\cdot)$.
  - Otherwise, returning $\mathcal{O}'_{\mathsf{Enc}}(\mathbf{0})$ wins with probability 1.

RUF of fh-IPFE

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  - Otherwise, returning $\mathcal{O}'_{\mathsf{Enc}}(\mathbf{0})$ wins with probability 1.

RUF Security

FE is called $\mathcal{O}$-RUF secure for a real number γ if for any adversary $\mathcal{A}$,

$$\mathsf{Adv}^{\mathsf{RUF},\mathcal{O},\gamma}_{\mathsf{FE},\mathcal{A}} := \Pr[\mathsf{RUF}^{\mathcal{O},\gamma}_{\mathsf{FE}}(\mathcal{A}) \to 1] = \mathsf{negl}.$$

We say FE is RUF secure if it is $\{\mathcal{O}'_{\mathsf{KeyGen}}, \mathcal{O}'_{\mathsf{Enc}}\}$-RUF secure.

RUF of fh-IPFE

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If FE is fh-IND, and if the RUF adversary can only return $\tilde{\mathbf{z}}$ that is an encryption of a nonzero vector, then FE is $\mathcal{O}'_{KeyGen}$-RUF for any $\gamma \leq \|\mathbb{F}\|$.

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Given an sEUF-CMA digital signature scheme Sig and any fh-IPFE FE, we can obtain an fh-IPFE FE′ that is RUF for any γ.

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We provide details in Appendix - Achievability of RUF Security.

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For any distribution family $\mathbb{B}$, if FE is fh-IND and $\mathcal{O}'_{KeyGen}$-RUF for a $\gamma \geq \tau^2$, then Π is $\{\mathbf{c_x}, csk, \mathcal{O}_{\mathcal{B}}, \mathcal{O}_{Enroll}\}$-UF.

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Theorem

For any distribution family $\mathbb{B}$ satisfying some "reasonable conditions", if FE is fh-IND, then Π is IND.

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For the rest of this section, let Π be a biomtric authentication scheme instantiated by an fh-IPFE FE. In our project, we show

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Proof Sketch.

Given an adversary $\mathcal{A}$ in the $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

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- If the challenge bit $b = 0$, $\mathcal{R}$ simulates a $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$ game.

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- Advantage of $\mathcal{R}$ is bounded by the difference between advantages of $\mathcal{A}$ and $\mathcal{A}'$,

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- Advantage of $\mathcal{R}$ is bounded by the difference between advantages of $\mathcal{A}$ and $\mathcal{A}'$, and

$$\mathsf{Adv}^{\mathsf{UF}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} \leq 4 \cdot \mathsf{Adv}^{\mathsf{fh\text{-}IND}}_{\mathsf{FE},\mathcal{R}} + \mathsf{Adv}^{\mathsf{RUF},\mathcal{O}'_{\mathsf{KeyGen}},\gamma}_{\mathsf{FE},\mathcal{A}'} = \mathsf{negl}.$$

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- $\mathcal{O}_{\mathsf{Enroll}}$ is "encoding + FE.KeyGen". We can simulate $\mathcal{O}_{\mathsf{Enroll}}$ by $\mathcal{O}_{\mathsf{KeyGen}}$ in fh-IND game and $\mathcal{O}'_{\mathsf{KeyGen}}$ in RUF game.

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- $\mathcal{R}$ never calls $\mathcal{O}_{\mathsf{Enc}}$, so it is an admissible adversary.

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Given an adversary $\mathcal{A}$ in the $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

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- If the challenge bit $b = 0$, $\mathcal{R}$ simulates a $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}(\mathcal{A})$ game.
- If the challenge bit $b = 1$, $\mathcal{R}$ simulates a $\mathsf{RUF}_{\mathsf{FE}}^{\mathcal{O}'_{\mathsf{Enc}},\gamma}(\mathcal{A}')$ game, for some $\mathcal{A}'$.

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- To let $\mathcal{R}$ be admissible, we cannot directly simulate $\mathcal{O}_{\mathsf{Probe}}$ by $\mathcal{O}_{\mathsf{Enc}}$.
  - $\mathcal{R}$ uses $\mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}, \mathbf{r})$, $\mathbf{x}$ from $\mathcal{O}_{\mathcal{B}}$ and $\mathbf{r} \leftarrow \mathbb{Z}_q^{k+2}$, to prepare $\mathbf{c}$ for $\mathcal{A}$ and $\mathcal{A}'$.
  - For any query $\mathcal{O}_{\mathsf{Enc}}(\mathbf{y}', \mathbf{y}'')$, it must satisfy $\mathbf{x}{\mathbf{y}'}^T = \mathbf{x}{\mathbf{y}''}^T$.

$\{\mathbf{c}_{\mathbf{x}}, \mathsf{csk}, \mathcal{O}_{\mathcal{B}}, \mathcal{O}_{\mathsf{Probe}}\}$-UF

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Given an adversary $\mathcal{A}$ in the $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

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  - $\mathcal{R}$ uses $\mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}, \mathbf{r})$, $\mathbf{x}$ from $\mathcal{O}_{\mathcal{B}}$ and $\mathbf{r} \leftarrow \mathbb{Z}_q^{k+2}$, to prepare $\mathbf{c}$ for $\mathcal{A}$ and $\mathcal{A}'$.
  - For any query $\mathcal{O}_{\mathsf{Enc}}(\mathbf{y}', \mathbf{y}'')$, it must satisfy $\mathbf{x}{\mathbf{y}'}^T = \mathbf{x}{\mathbf{y}''}^T$.
- We simulate $\mathcal{O}_{\mathsf{Probe}}(\mathsf{psk}, \mathbf{b}')$ by
  - Encode $\mathbf{b}'$ to $\mathbf{y}' = (-2b'_1, \cdots, -2b'_k, \|\mathbf{b}'\|^2, 1)$
  - Compute $d \leftarrow \mathbf{x}{\mathbf{y}'}^T$ and find a vector $\mathbf{y}''$ such that $\mathbf{r}{\mathbf{y}''}^T = d$
  - $\mathcal{O}_{\mathsf{Enc}}(\mathbf{y}', \mathbf{y}'')$.

- $\mathcal{R}$ is now admissible, but then we have to simulate this tweaked $\mathcal{O}_{\mathsf{Probe}}$ in $\mathsf{RUF}_{\mathsf{FE}}^{\mathcal{O}'_{\mathsf{Enc}},\gamma}(\mathcal{A}')$.

$\square$

- $\mathcal{R}$ is now admissible, but then we have to simulate this tweaked $\mathcal{O}_{\mathsf{Probe}}$ in $\mathsf{RUF}_{\mathsf{FE}}^{\mathcal{O}'_{\mathsf{Enc}},\gamma}(\mathcal{A}')$.
- Advantage of $\mathcal{R}$ is bounded by the difference between advantages of $\mathcal{A}$ and $\mathcal{A}'$, and

$$\mathsf{Adv}^{\mathsf{UF}}_{\Pi,\mathbb{B},\mathcal{A},\mathsf{option}} \leq 4 \cdot \mathsf{Adv}^{\mathsf{fh\text{-}IND}}_{\mathsf{FE},\mathcal{R}} + \mathsf{Adv}^{\mathsf{RUF},\mathcal{O}'_{\mathsf{Enc}},\gamma}_{\mathsf{FE},\mathcal{A}'} + \frac{k+2}{q^{k+2}-1} + \frac{1}{q^{k+2}} = \mathsf{negl}.$$

$\square$

Security of Instantiation using fh-IPFE

For the rest of this section, let Π be a biomtric authentication scheme instantiated by an fh-IPFE FE. In our project, we show

Theorem

For any distribution family $\mathbb{B}$, if FE is fh-IND and $\mathcal{O}'_{KeyGen}$-RUF for a $\gamma \geq \tau^2$, then Π is $\{\mathbf{c_x}, csk, \mathcal{O}_{\mathcal{B}}, \mathcal{O}_{Enroll}\}$-UF.

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Theorem

For any distribution family $\mathbb{B}$ satisfying some "reasonable conditions", if FE is fh-IND, then Π is IND.

IND

Assumption 1

Let t be an integer. Assume that for any distribution $\mathcal{B} \in \mathbb{B}$, the following distribution is identical.

$$\mathcal{D}_{\mathcal{B}}(t) := \left(\mathsf{BioCompare}(\mathbf{b}, \mathbf{b}^{(1)}), \mathsf{BioCompare}(\mathbf{b}, \mathbf{b}^{(2)}), \cdots, \mathsf{BioCompare}(\mathbf{b}, \mathbf{b}^{(t)})\right)$$

where $\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$ and $\mathbf{b}^{(i)} \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}}}()$ for all $i \in [t]$.

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where $\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$ and $\mathbf{b}^{(i)} \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}}}()$ for all $i \in [t]$.

Note that indistinguishability of $\mathcal{D}_{\mathcal{B}}(t)$ for $\mathcal{B} \in \mathbb{B}$ is a necessary condition to achieve IND security because

$$\left(\mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y}^{(1)}), \cdots, \mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y}^{(t)})\right) = \mathcal{D}_{\mathcal{B}^{(b)}}(t)$$

where b is the challenge bit.

IND

Theorem

For any distribution family $\mathbb{B}$ satisfying Assumption 1 and having a true positive rate $TP > \frac{1}{\text{poly}}$, if FE is fh-IND, then Π is IND.

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For any distribution family $\mathbb{B}$ satisfying Assumption 1 and having a true positive rate $TP > \frac{1}{\text{poly}}$, if FE is fh-IND, then Π is IND.

Proof Sketch.

Given an adversary $\mathcal{A}$ in the $\text{IND}_{\Pi,\mathbb{B}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

IND

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Proof Sketch.

Given an adversary $\mathcal{A}$ in the $\text{IND}_{\Pi,\mathbb{B}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

- $\mathcal{R}$ first samples $\mathcal{B}^{(0)}$ and $\mathcal{B}^{(1)}$.

IND

Theorem

For any distribution family $\mathbb{B}$ satisfying Assumption 1 and having a true positive rate $TP > \frac{1}{\text{poly}}$, if FE is fh-IND, then Π is IND.

Proof Sketch.

Given an adversary $\mathcal{A}$ in the $\text{IND}_{\Pi,\mathbb{B}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

- $\mathcal{R}$ first samples $\mathcal{B}^{(0)}$ and $\mathcal{B}^{(1)}$.
- $\mathcal{R}$ then calls $\mathbf{c}_{\mathbf{x}} \leftarrow \mathcal{O}_{\text{KeyGen}}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$, where $\mathbf{x}^{(0)}$ and $\mathbf{x}^{(1)}$ are created from $\mathcal{B}^{(0)}$ and $\mathcal{B}^{(1)}$.

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For any distribution family $\mathbb{B}$ satisfying Assumption 1 and having a true positive rate $TP > \frac{1}{\text{poly}}$, if FE is fh-IND, then Π is IND.

Proof Sketch.

Given an adversary $\mathcal{A}$ in the $\text{IND}_{\Pi,\mathbb{B}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

- $\mathcal{R}$ first samples $\mathcal{B}^{(0)}$ and $\mathcal{B}^{(1)}$.
- $\mathcal{R}$ then calls $\mathbf{c_x} \leftarrow \mathcal{O}_{\text{KeyGen}}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$, where $\mathbf{x}^{(0)}$ and $\mathbf{x}^{(1)}$ are created from $\mathcal{B}^{(0)}$ and $\mathcal{B}^{(1)}$.
- $\mathcal{R}$ prepares $\mathbf{y}^{(0)}$ and $\mathbf{y}^{(1)}$ from $\mathcal{B}^{(0)}$ and $\mathcal{B}^{(1)}$ in a way that, $\mathbf{x}^{(0)}{\mathbf{y}^{(0)}}^T = \mathbf{x}^{(1)}{\mathbf{y}^{(1)}}^T$, and calls $\mathbf{c_y} \leftarrow \mathcal{O}_{\text{Enc}}(\mathbf{y}^{(0)}, \mathbf{y}^{(1)})$.

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Theorem

For any distribution family $\mathbb{B}$ satisfying Assumption 1 and having a true positive rate $TP > \frac{1}{\mathsf{poly}}$, if FE is fh-IND, then Π is IND.

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Given an adversary $\mathcal{A}$ in the $\mathsf{IND}_{\Pi,\mathbb{B}}$ game, we build a reduction adversary $\mathcal{R}$ in the fh-IND game such that:

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- By Assumption 1, $\mathbf{y}^{(0)}$ and $\mathbf{y}^{(1)}$ still follow the correct distribution.

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- We show that how an fh-IPFE-based instantiation can be $\{\mathbf{c}_\mathbf{x}, \mathsf{csk}, \mathcal{O}_\mathcal{B}, \mathcal{O}_{\mathsf{Enroll}}\}$-UF, $\{\mathbf{c}_\mathbf{x}, \mathsf{csk}, \mathcal{O}_\mathcal{B}, \mathcal{O}_{\mathsf{Probe}}\}$-UF, and IND.

Discussion and Future Work

- We can consider other two oracles in option in the $\mathsf{UF}_{\Pi,\mathbb{B},\mathsf{option}}$ game:

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  Some of them have different structures from our framework, such as a challenge-based protocol.

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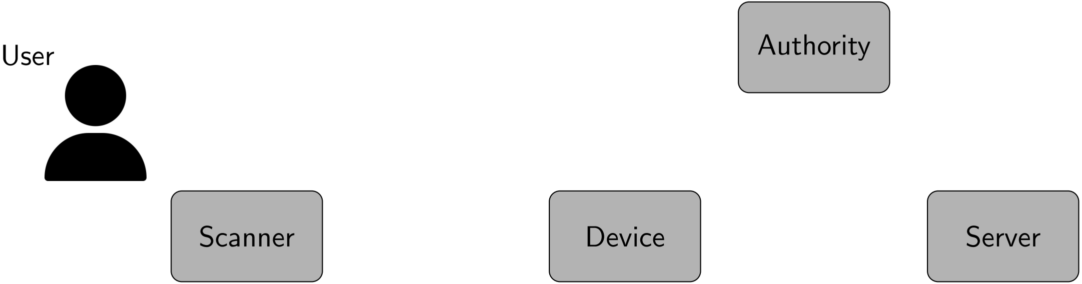
Usage Example: Enrollment



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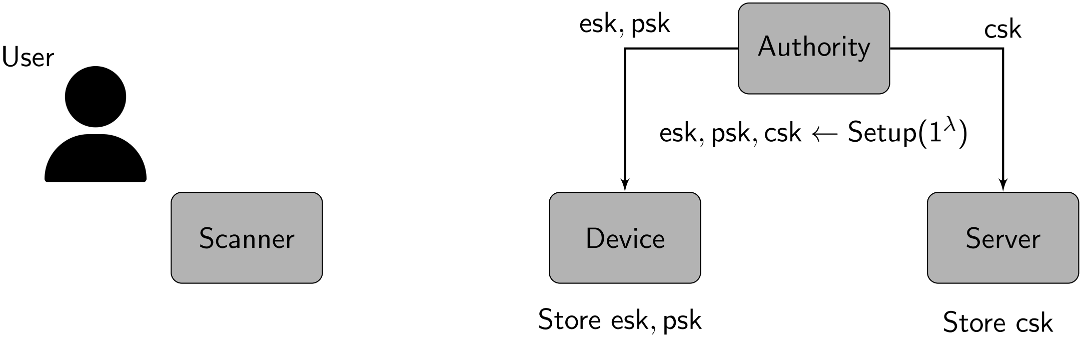

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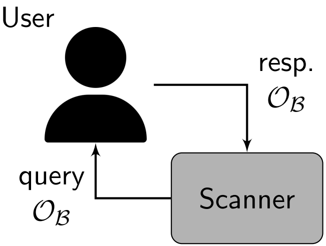
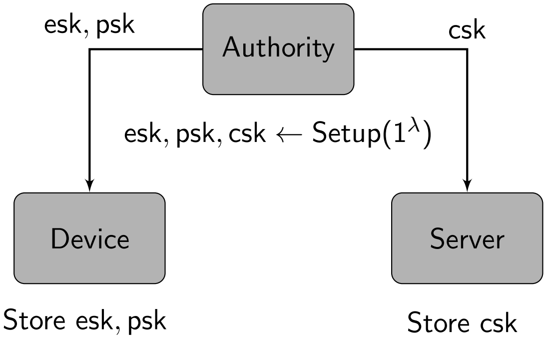


$\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$



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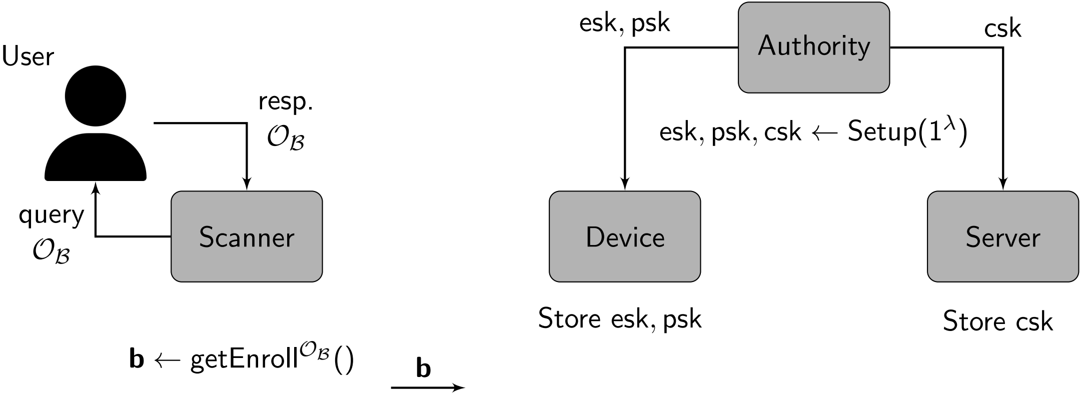

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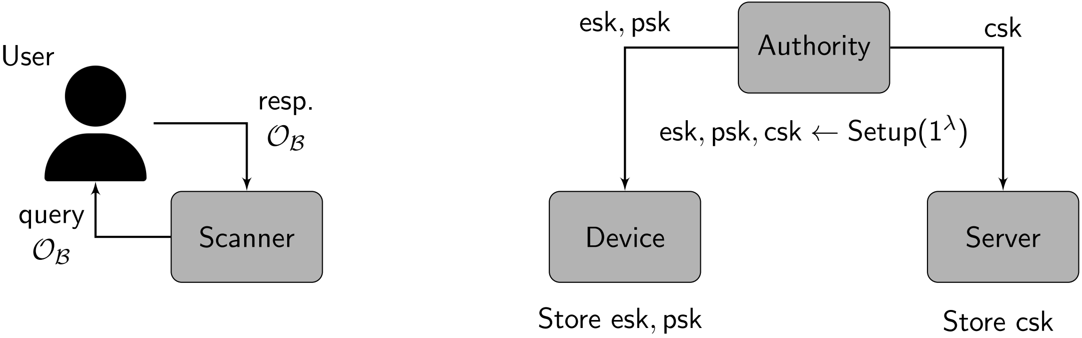

$\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_\mathcal{B}}()$ $\xrightarrow{\mathbf{b}}$ $\mathbf{c_x} \leftarrow \mathsf{Enroll}(\mathsf{esk}, \mathbf{b})$

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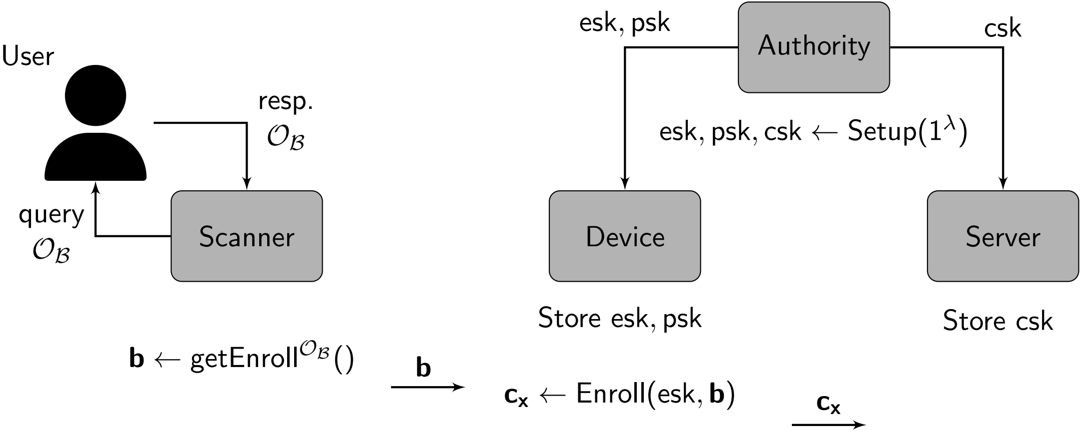

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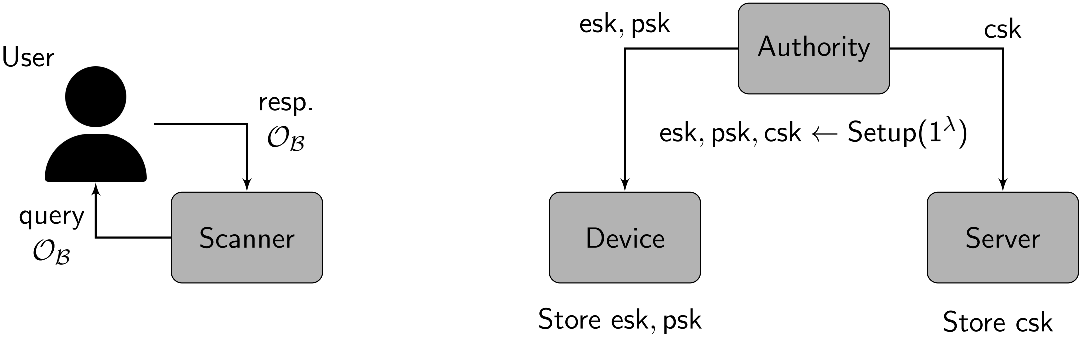
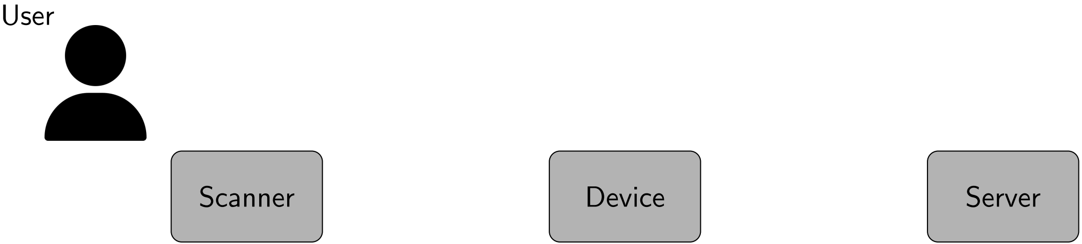


$\mathbf{b} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$ $\xrightarrow{\mathbf{b}}$ $\mathbf{c_x} \leftarrow \mathsf{Enroll}(\mathsf{esk}, \mathbf{b})$ $\xrightarrow{\mathbf{c_x}}$ Store $\mathbf{c_x}$

Usage Example: Authentication



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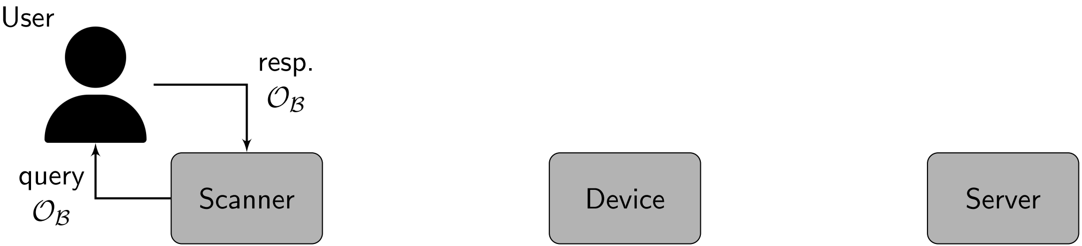

$\mathbf{b} \leftarrow \text{getProbe}^{\mathcal{O}_{\mathcal{B}}}()$

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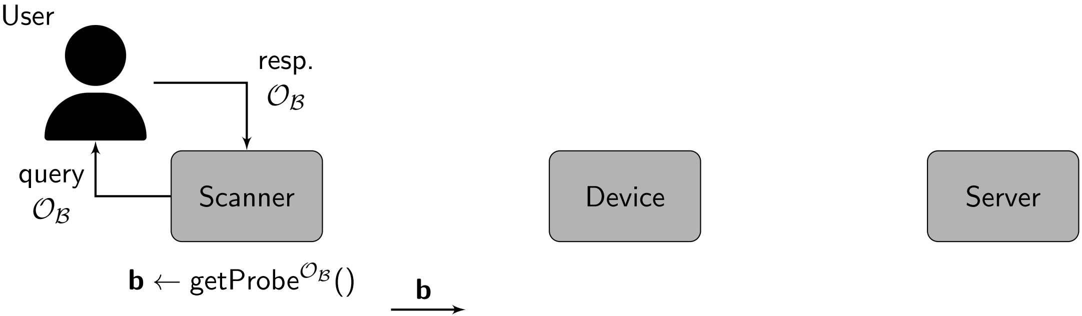

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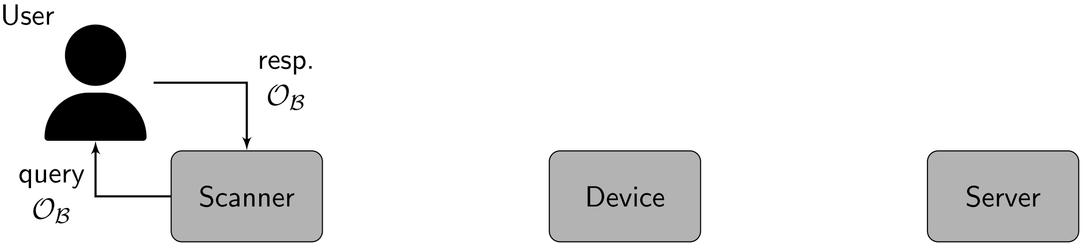

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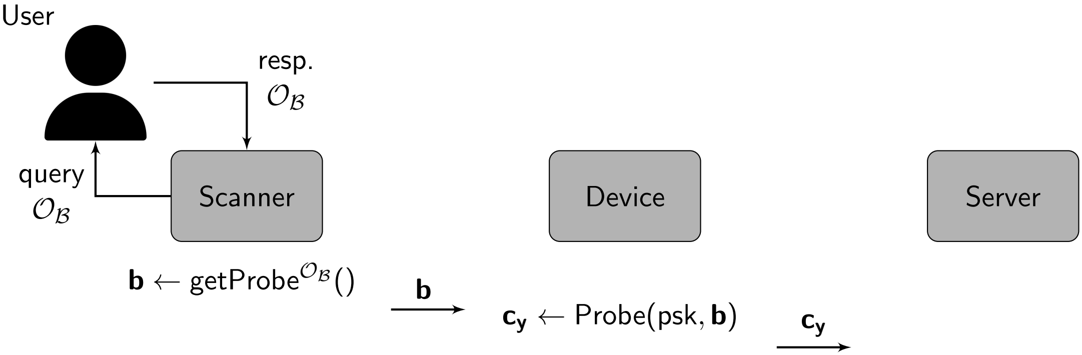

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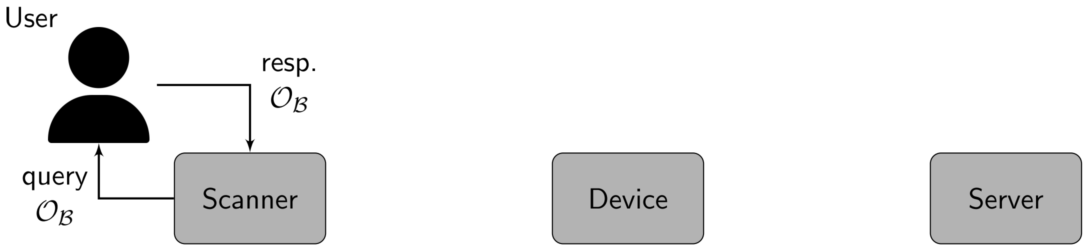

$\mathbf{b} \leftarrow \mathsf{getProbe}^{\mathcal{O}_\mathcal{B}}()$ $\xrightarrow{\mathbf{b}}$ $\mathbf{c_y} \leftarrow \mathsf{Probe}(\mathsf{psk}, \mathbf{b})$ $\xrightarrow{\mathbf{c_y}}$

$s \leftarrow \mathsf{Compare}(\mathsf{csk}, \mathbf{c_x}, \mathbf{c_y})$

$r \leftarrow \mathsf{Verify}(s)$

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fh-IND almost implies $\mathcal{O}'_{\mathsf{KeyGen}}$-RUF

Theorem

If FE is fh-IND, and if the RUF adversary can only return $\tilde{\mathbf{z}}$ that is an encryption of a nonzero vector, then FE is $\mathcal{O}'_{\mathsf{KeyGen}}$-RUF for any $\gamma \leq \|\mathbb{F}\|$.

fh-IND almost implies $\mathcal{O}'_{\mathsf{KeyGen}}$-RUF

Theorem

If FE is fh-IND, and if the RUF adversary can only return $\tilde{\mathbf{z}}$ *that is an encryption of a nonzero vector, then FE is* $\mathcal{O}'_{\mathsf{KeyGen}}$*-RUF for any* $\gamma \leq \|\mathbb{F}\|$.

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5. If $\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}_i, \tilde{\mathbf{z}}) \leq \gamma$ for all i, return $\tilde{b} = 0$. Otherwise, return $\tilde{b} \leftarrow\!\!\$\; \{0, 1\}$.

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6. If $b = 0$ and $\mathcal{A}$ wins, $\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}_i, \tilde{\mathbf{z}}) \leq \gamma$ for all i. Otherwise, $\mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}_i, \tilde{\mathbf{z}}) \leq \gamma$ is a random number in $\{0, 1, \cdots, q-1\}$.

Upgrading FE to RUF FE′

Theorem

Given an sEUF-CMA digital signature scheme Sig and any fh-IPFE FE, we can obtain an fh-IPFE FE′ that is RUF for any γ.

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- $\mathsf{FE}'.\mathsf{Setup}(1^\lambda)$: Run $\mathsf{FE}.\mathsf{Setup}(1^\lambda) \to (\mathsf{msk}, \mathsf{pp})$ and $\mathsf{Sig}.\mathsf{KeyGen}(1^\lambda) \to (\mathsf{sk}_{\mathsf{Sig}}, \mathsf{pk}_{\mathsf{Sig}})$. Output $\mathsf{msk}' = (\mathsf{msk}, \mathsf{sk}_{\mathsf{Sig}})$ and $\mathsf{pp}' = (\mathsf{pp}, \mathsf{pk}_{\mathsf{Sig}})$.

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- $\mathsf{FE}'.\mathsf{KeyGen}(\mathsf{msk}', \mathbf{x})$: Run and output $f_{\mathbf{x}} \leftarrow \mathsf{FE}.\mathsf{Enc}(\mathsf{msk}, \mathbf{x})$.

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- $\mathsf{FE}'.\mathsf{KeyGen}(\mathsf{msk}', \mathbf{x})$: Run and output $f_{\mathbf{x}} \leftarrow \mathsf{FE}.\mathsf{Enc}(\mathsf{msk}, \mathbf{x})$.
- $\mathsf{FE}'.\mathsf{Enc}(\mathsf{msk}', \mathbf{y})$: Run $\mathsf{FE}.\mathsf{Enc}(\mathsf{msk}, \mathbf{y}) \to \mathbf{c_y}$ and sign $\mathbf{c_y}$ by $\mathsf{Sig}.\mathsf{Sign}(\mathsf{sk}_{\mathsf{Sig}}, \mathbf{c_y}) \to \sigma$. Output $\mathbf{c_y}' = (\mathbf{c_y}, \sigma)$.

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- $\mathsf{FE}'.\mathsf{Dec}(\mathsf{pp}', f_{\mathbf{x}}, \mathbf{c_y}')$: Output $\mathsf{FE}.\mathsf{Dec}(\mathsf{pp}, f_{\mathbf{x}}, \mathbf{c_y})$ if $\mathsf{Sig}.\mathsf{Verify}(\mathsf{pk}_{\mathsf{Sig}}, \mathbf{c_y}, \sigma) \to 1$. Otherwise, output $\perp$.

Upgrading FE to RUF FE$'$

Theorem

Given an sEUF-CMA digital signature scheme Sig and any fh-IPFE FE, we can obtain an fh-IPFE FE$'$ that is RUF for any γ.

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If an adverary can find $\tilde{\mathbf{z}}$ such that $\mathsf{FE}'.\mathsf{Dec}(\mathsf{pp}', \mathbf{c}, \tilde{\mathbf{z}}) \neq \perp$, it can forge a signature.

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Reduction on proving $\{\mathbf{c_x}, \mathsf{csk}, \mathcal{O}_{\mathcal{B}}, \mathcal{O}_{\mathsf{Enroll}}\}$-UF

$\mathcal{R}^{\mathcal{O}_{\mathsf{KeyGen}}, \mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$

$\mathcal{B} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}$
$\mathbf{b} = (b_1, \cdots, b_k) \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$
$\mathbf{x} \leftarrow (b_1, \cdots, b_k, 1, \|\mathbf{b}\|^2)$
$\mathbf{r} \leftarrow_{\$} \mathbb{F}^{k+2}$
$\mathbf{c} \leftarrow \mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}, \mathbf{r})$
$\tilde{\mathbf{z}} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathcal{B}}, \mathcal{O}_{\mathsf{Enroll}}}(\mathbf{c}, \mathsf{pp})$
$s \leftarrow \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}, \tilde{\mathbf{z}})$
if $\mathsf{Verify}(s) = 1$ **then**
    return $\tilde{b} = 0$
else
    return $\tilde{b} \leftarrow_{\$} \{0, 1\}$

Reduction on proving $\{\mathbf{c}_{\mathbf{x}}, \mathsf{csk}, \mathcal{O}_{\mathcal{B}}, \mathcal{O}_{\mathsf{Probe}}\}$-UF

$\mathcal{R}^{\mathcal{O}_{\mathsf{KeyGen}}, \mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$

1: $\mathcal{B} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}$
2: $\mathbf{b} = (b_1, \cdots, b_k) \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}}}()$
3: $\mathbf{x} \leftarrow (b_1, \cdots, b_k, 1, \|\mathbf{b}\|^2)$
4: $\mathbf{r} \leftarrow_{\$} \mathbb{F}^{k+2}$
5: $\mathbf{c} \leftarrow \mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}, \mathbf{r})$
6: $\tilde{\mathbf{z}} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathcal{B}}, \mathcal{O}_{\mathsf{Probe}}}(\mathbf{c}, \mathsf{pp})$
7: **if** $\tilde{\mathbf{z}}$ is equal to any output of $\mathcal{O}_{\mathsf{Probe}}$ **then**
8:     **return** $\perp$
9: $s \leftarrow \mathsf{FE.Dec}(\mathsf{pp}, \mathbf{c}, \tilde{\mathbf{z}})$
10: **if** $\mathsf{Verify}(s) = 1$ **then**
11:     **return** $\tilde{b} = 0$
12: **else**
13:     **return** $\tilde{b} \leftarrow_{\$} \{0, 1\}$

Reduction on proving IND

$\mathcal{R}^{\mathcal{O}_{\mathsf{KeyGen}},\mathcal{O}_{\mathsf{Enc}}}(\mathsf{pp})$

$\mathcal{B}^{(0)} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}^{(0)}$

$\mathcal{B}^{(1)} \leftarrow_{\$} \mathbb{B}, \quad \mathbb{B} \leftarrow \mathbb{B} \setminus \mathcal{B}^{(1)}$

$\mathbf{b}^{(0)} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}^{(0)}}}()$

$\mathbf{x}^{(0)} \leftarrow (b_1^{(0)}, \cdots, b_k^{(0)}, 1, \|\mathbf{b}^{(0)}\|^2)$

$\mathbf{b}^{(1)} \leftarrow \mathsf{getEnroll}^{\mathcal{O}_{\mathcal{B}^{(1)}}}()$

$\mathbf{x}^{(1)} \leftarrow (b_1^{(1)}, \cdots, b_k^{(1)}, 1, \|\mathbf{b}^{(1)}\|^2)$

$\mathbf{c_x} \leftarrow \mathcal{O}_{\mathsf{KeyGen}}(\mathbf{x}^{(0)}, \mathbf{x}^{(1)})$

for $i = 1$ to t **do**

  $\mathbf{b}'^{(0)} \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}^{(0)}}}()$

  $\mathbf{y}^{(0)} \leftarrow (-2b_1'^{(0)}, \cdots, -2b_k'^{(0)}, \|\mathbf{b}'^{(0)}\|^2, 1)$

  repeat

    $\mathbf{b}'^{(1)} \leftarrow \mathsf{getProbe}^{\mathcal{O}_{\mathcal{B}^{(1)}}}()$

    $\mathbf{y}^{(1)} \leftarrow$
    $(-2b_1'^{(1)}, \cdots, -2b_k'^{(1)}, \|\mathbf{b}'^{(1)}\|^2, 1)$

  until $\mathbf{x}^{(0)}{\mathbf{y}^{(0)}}^T = \mathbf{x}^{(1)}{\mathbf{y}^{(1)}}^T$

  ${\mathbf{c_y}}^{(i)} \leftarrow \mathcal{O}_{\mathsf{Enc}}(\mathbf{y}^{(0)}, \mathbf{y}^{(1)})$

$\tilde{b} \leftarrow \mathcal{A}^{\mathcal{O}_{\mathcal{B}^{(0)}},\mathcal{O}_{\mathcal{B}^{(1)}}}(\mathsf{pp}, \mathbf{c_x}, \{{\mathbf{c_y}}^{(i)}\}_{i=1}^t)$

return $\tilde{b}$