

Masking Floating-Point Number Multiplication and Addition of Falcon

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- 2 Preliminaries
- 3 Masked Floating-Point Number Multiplication and Addition
- 4 Evaluation and Implementation
- 5 Conclusion

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Introduction

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- In 2022, four selected algorithms – CRYSTALS-Kyber, CRYSTALS-Dilithium, FALCON, and SPHINCS+ were expected to be part of NIST's post-quantum cryptographic standards.

Theoretical Security - Hardness of Mathematical Problems

In theory, these algorithms can base their security on some problems that are considered still hard given the advantage of quantum computing.

- 1 CRYSTALS-Kyber: Module Learning With Errors (MLWE)
- 2 CRYSTALS-Dilithium: Module Short Integer Solution (MSIS)
- 3 FALCON: NTRU Problem and SIS on NTRU lattices
- 4 SPHINCS+: Security of the used hash function families

Real-World Security – Side-Channel Attacks

In practice, the implementations of these algorithms can suffer side-channel attacks. Fortunately, there are countermeasures for them.

- ① CRYSTALS-Kyber: [[Bos+21](#); [Fri+22](#); [Hei+22](#)]
- ② CRYSTALS-Dilithium: [[Mig+19](#)]
- ③ FALCON: [[How+20](#); [Gue+22](#); [Zha+23](#)]
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Unfortunately, there are attacks on FALCON that have not been addressed.

Attacks on FALCON

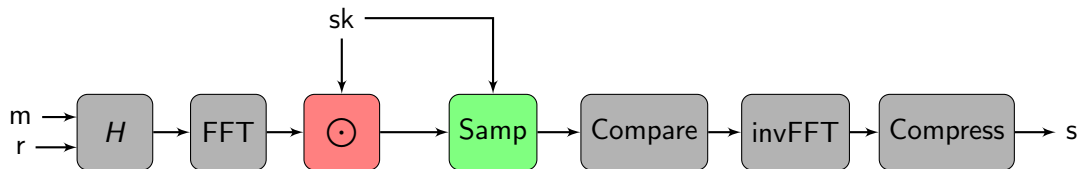


Figure: A graphical overview of FALCON.Sign.

	Attack	Countermeasure
Pre-image Vector Computation	[KA21; Gue+22]	None
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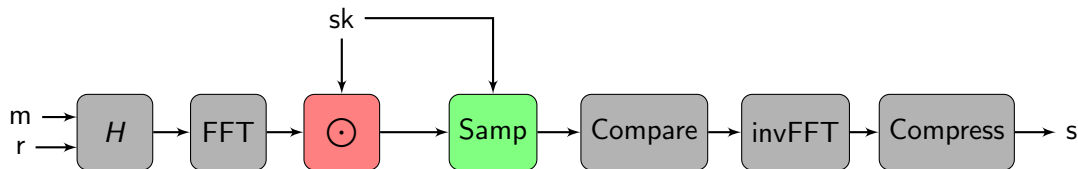


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- To test the practical leakage of our work, we conduct the Test Vector Leakage Assessment (TVLA) [[GJR+11](#)] experiments.

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- We verify the high-order security of our design in the probing model.
- To test the practical leakage of our work, we conduct the Test Vector Leakage Assessment (TVLA) [GJR+11] experiments.
- We also test the performance by comparing with the reference implementation of FALCON [Pre+20].

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- For a proposition P , $\llbracket P \rrbracket = 1$ if and only if P is true and 0 if otherwise.

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Introduction to FALCON

- A NIST-standardized digital signature
- Use the Gentry-Peikert-Vaikuntanathan (GPV) framework [GPV08] with NTRU lattices

KeyGen

Public Key: $\mathbf{A} \in \mathbb{Z}_q^{N \times M}$

Secret Key: Short $\mathbf{B} \in \mathbb{Z}_q^{M \times M}$

$$\mathbf{BA}^T = \mathbf{0} \bmod q$$

Sign(m)

A short \mathbf{s} s.t.

$$\mathbf{sA}^T = H(m) \bmod q$$

$$H : \{0, 1\}^* \rightarrow \{0, 1\}^N$$

Verify(m, s)

Check

① \mathbf{s} is short

② $\mathbf{sA}^T = H(m) \bmod q$

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- Apply the nearest plane algorithm to find an integer vector \mathbf{z} such that $(\mathbf{t} - \mathbf{z})\mathbf{B}$ is short.
- $\mathbf{s} \leftarrow (\mathbf{t} - \mathbf{z})\mathbf{B}$. Note that $\mathbf{s}\mathbf{A}^T = H(\mathbf{m}) \bmod q$

Introduction to FALCON

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- Short secret polynomials $f, g, F, G \in \mathbb{Z}[x]/(x^N + 1)$ where

$$fG - gF = q \quad \mathbf{B} = \left[\begin{array}{c|c} g & -f \\ \hline G & -F \end{array} \right]$$

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Moreover, FALCON applies the fast Fourier nearest plane algorithm [DP16] to speed up the signing process.

Introduction to FALCON

Sign (Simplified)

Input: Message m , secret key sk , bound $\lfloor \beta^2 \rfloor$

Output: Signature sig

- 1: Sample salt $r \leftarrow \{0, 1\}^{320}$ uniformly
- 2: $c \leftarrow H(r \| m)$
- 3: Compute the pre-image vector $\mathbf{t} \leftarrow \begin{bmatrix} c & 0 \end{bmatrix} \cdot \mathbf{B}^{-1}$
- 4: **repeat**
- 5: $\mathbf{z} = \text{ffSampling}(\mathbf{t}, sk)$
- 6: $\mathbf{s} = \begin{bmatrix} s_1 & s_2 \end{bmatrix} = (\mathbf{t} - \mathbf{z})\mathbf{B}$
- 7: **until** $\|\mathbf{s}\|^2 \leq \lfloor \beta^2 \rfloor$
- 8: $sig \leftarrow (r, s_2)$

Verify (Simplified)

Input: Message m , signature sig

Input: Bound $\lfloor \beta^2 \rfloor$

Output: Accept or Reject

- 1: $c \leftarrow H(r \| m)$
- 2: $s_1 \leftarrow c - s_2 h \bmod q$
- 3: **if** $\|(s_1, s_2)\|^2 \leq \lfloor \beta^2 \rfloor$ **then**
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- 2: $c \leftarrow H(r \| m)$
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- 4: **repeat**
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Fast-Fourier Transform

The pre-image vector computation includes polynomial multiplications

$$\mathbf{t} = \left[\begin{array}{c|c} c & 0 \end{array} \right] \cdot \mathbf{B}^{-1} = \frac{1}{q} \left[\begin{array}{c|c} c \cdot -F & c \cdot f \end{array} \right]$$

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$$\mathbf{t} = \begin{bmatrix} c & | & 0 \end{bmatrix} \cdot \mathbf{B}^{-1} = \frac{1}{q} \begin{bmatrix} c \cdot -F & | & c \cdot f \end{bmatrix}$$

To speed up and apply the fast Fourier nearest plane algorithm, the pre-image vector computation is performed in the Fourier domain:

$$\frac{1}{q} \begin{bmatrix} \text{FFT}(c) \odot \text{FFT}(-F) & | & \text{FFT}(c) \odot \text{FFT}(f) \end{bmatrix}$$

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Therefore, the pre-image vector computation is essentially coefficient-wise complex number multiplications.

Floating-Point Number

A complex number is represented by two 64-bit floating-point numbers (FPNs). An FPN is composed of sign bit s , exponent e , and mantissa \tilde{m}



Figure: A 64-bit Floating-Point Number

The value is $(-1)^s \cdot 2^{e-1023} \cdot \underbrace{(1 + \tilde{m} \cdot 2^{-52})}_{\times 2^{52} = m}$

For convenience, we may use (s, e, m) to represent an FPN.

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Sticky Bit

In floating-point arithmetic, when shifted right, the mantissa maintains a sticky bit

$$1001\textcolor{green}{0}\textcolor{red}{0100} \ggg 4 \rightarrow 1001 \underbrace{1}_{\text{Sticky}}$$

It indicates whether there exists any 1 after the least significant bit. In the above example,

$$\text{sticky bit} = \textcolor{green}{0} \vee \llbracket (\textcolor{red}{0100}) \neq 0 \rrbracket = \llbracket (\textcolor{green}{0}\textcolor{red}{0100}) \neq 0 \rrbracket$$

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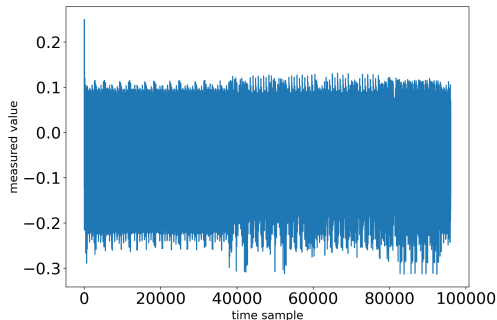


Figure: An Example of a Power Trace

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- Boolean Masking: A variable x is split into n shares (x_i) such that

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- Arithmetic Masking: A variable x is split into n shares (x_i) (when stored in a k -bit register) such that

$$x = \sum_{i=1}^n x_i \pmod{2^k}$$

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- All operations need to be operated via shares.

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 - SecFprUrsh
 - SecFprNorm64
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- B2A: $(y_i)_{1 \leq i \leq n} \mapsto (x_i)_{1 \leq i \leq n}$ such that $\bigoplus_{i=1}^n y_i = \sum_{i=1}^n x_i$

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 - Given (x_i) and (c_i) , right-shifting (x_i) by (c_i)
- Normalizing a secret value to $[2^{63}, 2^{64})$
 - Given (x_i) , left-shifting (x_i) until its 64th bit is set

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- SecNonzero (Algorithm 6): securely checking whether a secret value is nonzero.
- SecFprUrsh (Algorithm 7): securely right-shifting a secret value by another secret value
- SecFprNorm64 (Algorithm 8): securely normalizing a secret value to $[2^{63}, 2^{64})$

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We design novel gadgets for these three operations, including:

- SecNonzero (Algorithm 6): securely checking whether a secret value is nonzero.
- SecFprUrsh (Algorithm 7): securely right-shifting a secret value by another secret value
- SecFprNorm64 (Algorithm 8): securely normalizing a secret value to $[2^{63}, 2^{64})$

In addition, we make several improvements to reduce the cost.

Gadgets Used in Our Work

Algorithm	Description	Reference
SecAnd	AND of Boolean shares	[ISW03; Bar+16]
SecMult	Multiplication of arithmetic shares	[ISW03; Bar+16]
SecAdd	Addition of Boolean shares	[Cor+15; Bar+18]
A2B	Arithmetic to Boolean conversion	[Sch+19]
B2A	Boolean to arithmetic conversion	[BCZ18]
B2A _{Bit}	One-bit B2A conversion	[Sch+19]
RefreshMasks	t -NI refresh of masks	[Bar+16; BCZ18]
Refresh	t -SNI refresh of masks	[Bar+16]

Table: List of used gadgets in our work

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Algorithm	Description	Reference
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SecAdd	Addition of Boolean shares	[Cor+15; Bar+18]
A2B	Arithmetic to Boolean conversion	[Sch+19]
B2A	Boolean to arithmetic conversion	[BCZ18]
B2A _{Bit}	One-bit B2A conversion	[Sch+19]
RefreshMasks	t -NI refresh of masks	[Bar+16; BCZ18]
Refresh	t -SNI refresh of masks	[Bar+16]

Table: List of used gadgets in our work

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 - SecFprNorm64
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SecNonzero

We need a gadget that, given shares (x_i) , can derive one-bit shares (b_i) such that

$$\left[\bigoplus_{i=1}^n x_i \neq 0 \right] = \bigoplus_{i=1}^n b_i \quad \text{or} \quad \left[\sum_{i=1}^n x_i \neq 0 \right] = \bigoplus_{i=1}^n b_i$$

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For Boolean shares, our method is by considering OR-ing all the bits.

$$x = 0 \iff x^{(k)} \vee x^{(k-1)} \vee \dots \vee x^{(1)} = 0$$

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Now we turn to a gadget for secure OR operations.

SecOr: OR of Boolean Shares

SecOr

Input: Boolean shares $(x_i)_{1 \leq i \leq n}$ for value x

Input: Boolean shares $(y_i)_{1 \leq i \leq n}$ for value y

Output: Boolean shares $(z_i)_{1 \leq i \leq n}$ for value $z = x \vee y$

1: $(t_i)_{1 \leq i \leq n} \leftarrow (\neg x_1, x_2, \dots, x_n)$

2: $(s_i)_{1 \leq i \leq n} \leftarrow (\neg y_1, y_2, \dots, y_n)$

3: $(z_i) \leftarrow \text{SecAnd}((s_i), (t_i))$

4: $z_1 \leftarrow \neg z_1$

5: **return** (z_i)

It applies De Morgan's law and calls the AND algorithm SecAnd of shares as a subroutine.

$$x \vee y = \neg [(\neg x) \wedge (\neg y)]$$

SecNonzero

For arithmetic shares, instead of applying an n -shared A2B, we consider that

$$\sum_{i=1}^n x_i = 0 \iff \sum_{i=1}^{\frac{n}{2}} x_i = \sum_{i=\frac{n}{2}+1}^n (-x_i) \iff \sum_{i=1}^{\frac{n}{2}} x_i \oplus \sum_{i=\frac{n}{2}+1}^n (-x_i) = 0$$

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So we apply two $n/2$ -shared A2Bs to the first $n/2$ shares and negative of the second $n/2$ shares and use the same idea.

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So we apply two $n/2$ -shared A2Bs to the first $n/2$ shares and negative of the second $n/2$ shares and use the same idea.

In this way, we replace one n -shared A2B with two $n/2$ -shared A2Bs, which is usually more efficient.

SecNonzero

SecNonzero

Input: Shares $(x_i)_{1 \leq i \leq n}$ for value x , bitsize

Output: One-bit Boolean shares $(b_i)_{1 \leq i \leq n}$ where $\bigoplus_i b_i = 0 \Leftrightarrow x = 0$

```

1: if input  $(x_i)$  are arithmetic shares then
2:    $(t_i)_{1 \leq i \leq \frac{n}{2}} \leftarrow \text{A2B}((x_i)_{1 \leq i \leq \frac{n}{2}})$ 
3:    $(t_i)_{\frac{n}{2}+1 \leq i \leq n} \leftarrow \text{A2B}((-x_i)_{\frac{n}{2}+1 \leq i \leq n})$ 
4: else
5:    $(t_i)_{1 \leq i \leq n} \leftarrow (x_i)_{1 \leq i \leq n}$ 
6:  $\text{len} \leftarrow \text{bitsize}/2$ 
7: while  $\text{len} \geq 1$  do
8:    $(l_i) \leftarrow \text{Refresh}((t_i^{[2\text{len}:\text{len}]}) , \text{len})$ 
9:    $(r_i) \leftarrow (t_i^{[\text{len}:1]})$ 
10:   $(t_i) \leftarrow \text{SecOr}((l_i), (r_i))$ 
11:   $\text{len} \leftarrow \text{len} \gg 1$ 
12: return  $(t_i^{(1)})$ 

```

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SecFprUrsh

Given 64-bit shares (x_i) and 6-bit (c_i) , we need to derive shares (z_i) such that

$$\bigoplus_{i=1}^n z_i = \left(\left(\bigoplus_{i=1}^n x_i \right) \ggg \left(\sum_{i=1}^n c_i \bmod 2^6 \right) \right) \vee \left[\bigoplus_{i=1}^n x_i^{[c:1]} \neq 0 \right]$$

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- Both shifting and rotating can be operated share-wise.

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We observe that

- Right-shifting and right-rotating by a value c only differ by the most c significant bits.
- Both shifting and rotating can be operated share-wise.
- Right-rotating x by a value c is equal to right-rotating x by a value $c \bmod 64$.

SecFprUrsh

Hence, our idea is to right-rotate all (x_i) by c_1, c_2, \dots, c_n sequentially.

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Some high bits are redundant, so we use an index $m = (1 \ll 63)$ to indicate the first meaningful bit of the result. To clear the redundant high bits, consider

$$m' := m \ggg c = \underbrace{(0, \dots, 0)}_{c \text{ bits}}, 1, 0, \dots, 0)$$

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$$m' := m \ggg c = \underbrace{(0, \dots, 0, 1, 0, \dots, 0)}_{c \text{ bits}}$$

$$m'' := m' \oplus (m' \ggg 1) \oplus \dots \oplus (m' \ggg 63) = \underbrace{(0, \dots, 0, 1, 1, \dots, 1)}_{c \text{ bits}}$$

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$$m'' := m' \oplus (m' \ggg 1) \oplus \dots \oplus (m' \ggg 63) = (\underbrace{0, \dots, 0}_{c \text{ bits}}, 1, 1, \dots, 1)$$

By an AND operation with m'' , we can clear useless bits. Moreover, these redundant bits actually form the sticky bit.

SecFprUrsh

SecFprUrsh

Input: 64-bit Boolean shares $(x_i)_{1 \leq i \leq n}$

Input: 6-bit arithmetic shares $(c_i)_{1 \leq i \leq n}$

Output: Boolean shares $(z_i)_{1 \leq i \leq n}$ for value
 $z = x \ggg c$ with the sticky bit preserved

1: $(m_i)_{1 \leq i \leq n} \leftarrow ((1 \lll 63), 0, \dots, 0)$

2: **for** $j = 1$ to n **do**

3: Right-rotate (x_i) by c_j

4: $(x_i) \leftarrow \text{RefreshMasks}((x_i))$

5: Right-rotate (m_i) by c_j

6: $(m_i) \leftarrow \text{RefreshMasks}((m_i))$

7: $\text{len} \leftarrow 1$

8: **while** $\text{len} \leq 32$ **do**

9: $(m_i) \leftarrow (m_i \oplus (m_i \ggg \text{len}))$

10: $\text{len} \leftarrow \text{len} \lll 1$

11: $(y_i) \leftarrow \text{SecAnd}((x_i), (m_i))$

12: $(z_i) \leftarrow (y_i \oplus x_i \oplus y_i^{(1)})$

13: $(b_i) \leftarrow \text{SecNonzero}((z_i))$

14: $(z_i) \leftarrow (y_i^{[64:2]} \vee b_i)$

15: **return** (z_i)

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SecFprNorm64

Given 64-bit shares (x_i) and 16-bit shares (e_i) , we need to derive new (x'_i) and (e'_i) such that if c is the smallest integer such that $((\oplus_{i=1}^n x_i) \ll c) \in [2^{63}, 2^{64})$

$$\text{then } (\oplus_{i=1}^n x'_i) = ((\oplus_{i=1}^n x_i) \ll c) \text{ and } \sum_{i=1}^n e'_i = \left(\sum_{i=1}^n e_i\right) - c$$

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We can repeatedly check whether $(x_i^{(64)}) = 0$, conditionally shift by 1 bit, and then decrease (e_i) by $\llbracket (x_i^{(64)}) = 0 \rrbracket$.

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To improve efficiency, we consider sequentially checking $x^{[64:64-2^j]} = 0$ for $j = 5, 4, \dots, 0$.

SecFprNorm64

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We can repeatedly check whether $(x_i^{(64)}) = 0$, conditionally shift by 1 bit, and then decrease (e_i) by $\llbracket (x_i^{(64)}) = 0 \rrbracket$.

To improve efficiency, we consider sequentially checking $x^{[64:64-2^j]} = 0$ for $j = 5, 4, \dots, 0$.

In addition, we first decrease (e_i) by 63 and later add $\llbracket (x_i^{[64:64-2^j]}) \neq 0 \rrbracket \cdot 2^j$ to it.

SecFprNorm64

SecFprNorm64

Input: 64-bit Boolean shares $(x_i)_{1 \leq i \leq n}$

Input: 16-bit arithmetic shares $(e_i)_{1 \leq i \leq n}$

Output: Normalized $(x_i)_{1 \leq i \leq n}$ in $[2^{63}, 2^{64})$ and $(e_i)_{1 \leq i \leq n}$ with shift added

- 1: $e_1 \leftarrow e_1 - 63$
- 2: **for** $j = 5$ to 0 **do**
- 3: $(t_i) \leftarrow (x_i \oplus (x_i \lll 2^j))$
- 4: $(n_i) \leftarrow (x_i \ggg (64 - 2^j))$
- 5: $(b_i) \leftarrow \text{SecNonzero}((n_i))$
- 6: $(b'_i) \leftarrow (-b_i)$
- 7: $(t_i) \leftarrow \text{SecAnd}((t_i), (\neg b'_1, b'_2, \dots, b'_n))$
- 8: $(x_i) \leftarrow (x_i \oplus t_i)$
- 9: $(b_i) \leftarrow \text{B2A}_{\text{Bit}}((b_i))$
- 10: $(e_i) \leftarrow (e_i + (b_i \lll j))$
- 11: **return** $(x_i), (e_i)$

Wrapping-up

Utilizing these new gadgets `SecNonzero`, `SecFprUrsh`, and `SecFprNorm64`, we design the following algorithms:

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Utilizing these new gadgets SecNonzero, SecFprUrsh, and SecFprNorm64, we design the following algorithms:

- SecFPR: Secure FPR by masking.
- SecFprMul: Secure FprMul by masking.
- SecFprAdd: Secure FprAdd by masking.

We leave the details of the implementations and several tricks for improvements in Appendix.

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Probing Model

To theoretically evaluate the security of our design, we consider the probing model [[ISW03](#)].

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Probing Model

To theoretically evaluate the security of our design, we consider the probing model [ISW03].

- The t -probing model assumes that an adversary is able to peek any t intermediate values in the algorithm.
- To be secure in t -probing model, $n \geq t + 1$, and any share cannot be combined with each other.
- It is complicated to prove t -probing security for a large composition of small gadgets. The concept of non-interference is convenient in this case.

Non-Interference Security

t -Non-Interference (t -NI) Security (from [Bar+16])

A gadget is t -Non-Interference (t -NI) secure if every set of t intermediate values can be simulated by no more than t shares of each of its inputs.

t -Strong Non-Interference (t -SNI) Security (from [Bar+16])

A gadget is t -Strong-Non-Interference (t -SNI) secure if for every set of t_I internal intermediate values and t_O of its output shares with $t_I + t_O \leq t$, they can be simulated by no more than t_I shares of each of its inputs.

Non-Interference Security

For $t = n - 1$, if a gadget is t -NI or t -SNI secure, and if any $n - 1$ input shares are uniformly and independently distributed, then it is t -probing secure.

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Non-Interference Security

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Moreover,

- t -SNI is stronger than t -NI by definition.
- A composition of t -NI gadgets may not be t -NI, so we insert t -SNI gadgets to make it t -NI or t -SNI.

All the gadgets/algorithms in our paper are proven either t -NI or t -SNI secure.

Gadgets/Algorithms in Our Work

Algorithm	Security	Algorithm	Security
SecAnd	t -SNI	SecOr	t -SNI
SecMult	t -SNI	SecNonzero	t -SNI
SecAdd	t -NI	SecFprUrsh	t -SNI
A2B	t -SNI	SecFprNorm64	t -NI
B2A	t -SNI	SecFPR	t -SNI
B2A _{Bit}	t -SNI	SecFprMul	t -SNI
RefreshMasks	t -NI	SecFprAdd	t -SNI
Refresh	t -SNI		

Table: List of gadgets/algorithms in our work with $n = t + 1$ shares

Test Vector Leakage Assessment (TVLA)

Probing model validates the security theoretically.

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By convention, we consider the leakage is significant if the t -value exceeds ± 4.5 .

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By convention, we consider the leakage is significant if the t -value exceeds ± 4.5 .

For traces with a large number of points, we refer to [Din+17] alter this threshold to avoid false positives.

Experiment Setup

We implement our algorithms in the following setting:

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- Using ChipWhisperer with target board STM32F303 with an Arm Cortex-M4 MCU

Experiment Setup

We implement our algorithms in the following setting:

- Plain-C code
- Compiled by `arm-none-eabi-gcc 10.3.1`
- Using ChipWhisperer with target board STM32F303 with an Arm Cortex-M4 MCU
- We compare the result with the reference implementation of the NIST Round-3 Submission of FALCON [[Pre+20](#)].

TVLA

The TVLA results of floating-point number multiplication (FprMul, SecFprMul).

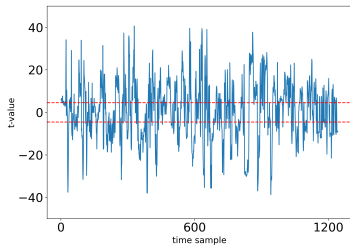


Figure: 1,000 traces, unmasked FprMul

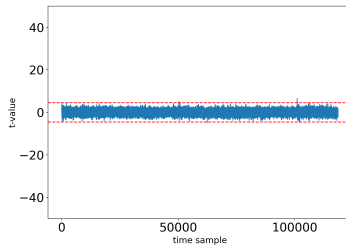


Figure: 10,000 traces, 2-shared SecFprMul

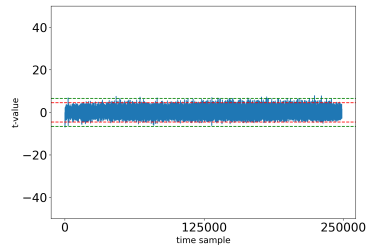


Figure: 100,000 traces, 3-shared SecFprMul

TVLA

The TVLA results of floating-point number addition (FprAdd, SecFprAdd).

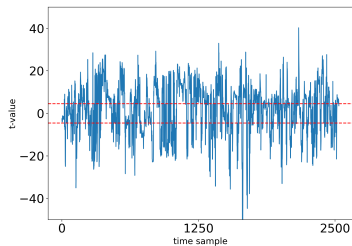


Figure: 1,000 traces, unmasked FprAdd

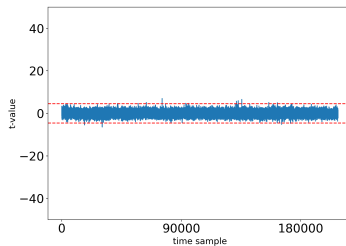


Figure: 10,000 traces, 2-shared SecFprAdd

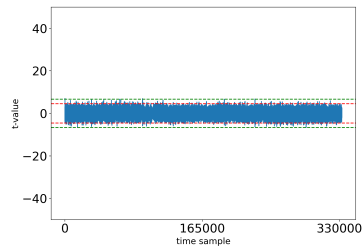


Figure: 100,000 traces, 3-shared SecFprAdd

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Performance Evaluation on ARM Cortex-M4

Algorithm		Cycles		
		Unmasked	2 Shares	3 Shares
SecFprMul	Total	308	7134 (23×)	36388 (118×)
	128-bit A2B	-	1619	19253
	64-bit SecNonzero	-	389	1350
	Two 16-bit SecNonzero	-	662	2012
	SecFPR	-	3362	10813
	#randombytes	-	333	2005
SecFprAdd	Total	487	17154 (35×)	48291 (99×)
	Three 64-bit SecAdd	-	6990	16956
	Two 16-bit B2A	-	88	332
	16-bit A2B	-	146	2267
	SecFprUrsh	-	1112	3214
	SecFprNorm64	-	2846	7270
	SecFPR	-	3362	10813
	#randombytes	-	849	2691

Performance Evaluation on General Purpose CPU

We also test the time for signing one message on Intel-Core i9-12900 KF.

Security Level	Unmasked	2 Shares	3 Shares
Falcon-512	246.56	1905.55 ($7.7\times$)	6137.25 ($24.9\times$)
Falcon-1024	501.62	3819.76 ($7.6\times$)	12287.29 ($24.5\times$)

Table: Time (in microseconds) for signing a message on Intel-Core i9-12900KF CPU.

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In this paper,

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- We present the first masking algorithm for floating-point number multiplication and addition to protect the pre-image vector computation.
- We design novel gadgets SecNonzero, SecFprUrsh, and SecFprNorm64 to mask the algorithms.

Conclusion

In this paper,

- We present the first masking algorithm for floating-point number multiplication and addition to protect the pre-image vector computation.
- We design novel gadgets SecNonzero, SecFprUrsh, and SecFprNorm64 to mask the algorithms.
- All our masked algorithms are proven t -NI or t -SNI secure – they are t -probing secure.

Conclusion

In this paper,

- We present the first masking algorithm for floating-point number multiplication and addition to protect the pre-image vector computation.
- We design novel gadgets SecNonzero, SecFprUrsh, and SecFprNorm64 to mask the algorithms.
- All our masked algorithms are proven t -NI or t -SNI secure – they are t -probing secure.
- The TVLA result shows no leakage in the 2-shared version in 10,000 traces, and no leakage in the 3-shared version in 100,000 traces.

Conclusion

In this paper,

- We present the first masking algorithm for floating-point number multiplication and addition to protect the pre-image vector computation.
- We design novel gadgets SecNonzero, SecFprUrsh, and SecFprNorm64 to mask the algorithms.
- All our masked algorithms are proven t -NI or t -SNI secure – they are t -probing secure.
- The TVLA result shows no leakage in the 2-shared version in 10,000 traces, and no leakage in the 3-shared version in 100,000 traces.
- Our countermeasure when compared to the unmasked reference implementation is much slower. Improved SecAdd and A2B can reduce the cost.

Reference I

- [ISW03] Yuval Ishai, Amit Sahai, and David Wagner. “Private Circuits: Securing Hardware against Probing Attacks”. In: *CRYPTO 2003*. Ed. by Dan Boneh. Vol. 2729. LNCS. Springer, Heidelberg, Aug. 2003, pp. 463–481. DOI: [10.1007/978-3-540-45146-4_27](https://doi.org/10.1007/978-3-540-45146-4_27).
- [GPV08] Craig Gentry, Chris Peikert, and Vinod Vaikuntanathan. “Trapdoors for hard lattices and new cryptographic constructions”. In: *40th ACM STOC*. Ed. by Richard E. Ladner and Cynthia Dwork. ACM Press, May 2008, pp. 197–206. DOI: [10.1145/1374376.1374407](https://doi.org/10.1145/1374376.1374407).
- [Ber+10] Guido Bertoni et al. “Building power analysis resistant implementations of Keccak”. In: *Second SHA-3 candidate conference*. Vol. 142. Citeseer. 2010.
- [GJR+11] Benjamin Jun Gilbert Goodwill, Josh Jaffe, Pankaj Rohatgi, et al. “A testing methodology for side-channel resistance validation”. In: *NIST non-invasive attack testing workshop*. Vol. 7. 2011, pp. 115–136.
- [Bel+13] Sonia Belaïd et al. “Differential power analysis of HMAC SHA-2 in the Hamming weight model”. In: *2013 International Conference on Security and Cryptography (SECRYPT)*. 2013, pp. 1–12.
- [Cor+15] Jean-Sébastien Coron et al. “Conversion from Arithmetic to Boolean Masking with Logarithmic Complexity”. In: *FSE 2015*. Ed. by Gregor Leander. Vol. 9054. LNCS. Springer, Heidelberg, Mar. 2015, pp. 130–149. DOI: [10.1007/978-3-662-48116-5_7](https://doi.org/10.1007/978-3-662-48116-5_7).
- [Bar+16] Gilles Barthe et al. “Strong Non-Interference and Type-Directed Higher-Order Masking”. In: *ACM CCS 2016*. Ed. by Edgar R. Weippl et al. ACM Press, Oct. 2016, pp. 116–129. DOI: [10.1145/2976749.2978427](https://doi.org/10.1145/2976749.2978427).

Reference II

- [DP16] Léo Ducas and Thomas Prest. “Fast fourier orthogonalization”. In: *Proceedings of the ACM on International Symposium on Symbolic and Algebraic Computation*. 2016, pp. 191–198.
- [Din+17] A. Adam Ding et al. “Towards Sound and Optimal Leakage Detection Procedure”. In: *Smart Card Research and Advanced Applications - 16th International Conference, CARDIS 2017, Lugano, Switzerland, November 13-15, 2017, Revised Selected Papers*. Ed. by Thomas Eisenbarth and Yannick Teglia. Vol. 10728. Lecture Notes in Computer Science. Springer, 2017, pp. 105–122. DOI: [10.1007/978-3-319-75208-2_7](https://doi.org/10.1007/978-3-319-75208-2_7). URL: https://doi.org/10.1007/978-3-319-75208-2_7.
- [Bar+18] Gilles Barthe et al. “Masking the GLP Lattice-Based Signature Scheme at Any Order”. In: *EUROCRYPT 2018, Part II*. Ed. by Jesper Buus Nielsen and Vincent Rijmen. Vol. 10821. LNCS. Springer, Heidelberg, Apr. 2018, pp. 354–384. DOI: [10.1007/978-3-319-78375-8_12](https://doi.org/10.1007/978-3-319-78375-8_12).
- [BCZ18] Luk Bettale, Jean-Sébastien Coron, and Rina Zeitoun. “Improved High-Order Conversion From Boolean to Arithmetic Masking”. In: *IACR TCHES 2018.2* (2018). <https://tches.iacr.org/index.php/TCHES/article/view/873>, pp. 22–45. ISSN: 2569-2925. DOI: [10.13154/tches.v2018.i2.22-45](https://doi.org/10.13154/tches.v2018.i2.22-45).
- [Mig+19] Vincent Migliore et al. “Masking Dilithium - Efficient Implementation and Side-Channel Evaluation”. In: *ACNS 19*. Ed. by Robert H. Deng et al. Vol. 11464. LNCS. Springer, Heidelberg, June 2019, pp. 344–362. DOI: [10.1007/978-3-030-21568-2_17](https://doi.org/10.1007/978-3-030-21568-2_17).

Reference III

- [Sch+19] Tobias Schneider et al. “Efficiently Masking Binomial Sampling at Arbitrary Orders for Lattice-Based Crypto”. In: *PKC 2019, Part II*. Ed. by Dongdai Lin and Kazue Sako. Vol. 11443. LNCS. Springer, Heidelberg, Apr. 2019, pp. 534–564. DOI: [10.1007/978-3-030-17259-6_18](https://doi.org/10.1007/978-3-030-17259-6_18).
- [How+20] James Howe et al. “Isochronous Gaussian Sampling: From Inception to Implementation”. In: *Post-Quantum Cryptography - 11th International Conference, PQCrypto 2020*. Ed. by Jintai Ding and Jean-Pierre Tillich. Springer, Heidelberg, 2020, pp. 53–71. DOI: [10.1007/978-3-030-44223-1_5](https://doi.org/10.1007/978-3-030-44223-1_5).
- [Pre+20] Thomas Prest et al. *FALCON*. Tech. rep. available at <https://csrc.nist.gov/projects/post-quantum-cryptography/post-quantum-cryptography-standardization/round-3-submissions>. National Institute of Standards and Technology, 2020.
- [Bos+21] Joppe W. Bos et al. “Masking Kyber: First- and Higher-Order Implementations”. In: *IACR TCHES 2021.4* (2021). <https://tches.iacr.org/index.php/TCHES/article/view/9064>, pp. 173–214. ISSN: 2569-2925. DOI: [10.46586/tches.v2021.i4.173-214](https://doi.org/10.46586/tches.v2021.i4.173-214).
- [KA21] Emre Karabulut and Aydin Aysu. “FALCON Down: Breaking FALCON Post-Quantum Signature Scheme through Side-Channel Attacks”. In: *2021 58th ACM/IEEE Design Automation Conference (DAC)*. 2021, pp. 691–696. DOI: [10.1109/DAC18074.2021.9586131](https://doi.org/10.1109/DAC18074.2021.9586131).
- [Fri+22] Tim Fritzmann et al. “Masked Accelerators and Instruction Set Extensions for Post-Quantum Cryptography”. In: *IACR TCHES 2022.1* (2022), pp. 414–460. DOI: [10.46586/tches.v2022.i1.414-460](https://doi.org/10.46586/tches.v2022.i1.414-460).

Reference IV

- [Gue+22] Morgane Guerreau et al. “The Hidden Parallelepiped Is Back Again: Power Analysis Attacks on Falcon”. In: *IACR TCHES* 2022.3 (2022), pp. 141–164. DOI: [10.46586/tches.v2022.i3.141-164](https://doi.org/10.46586/tches.v2022.i3.141-164).
- [Hei+22] Daniel Heinz et al. *First-Order Masked Kyber on ARM Cortex-M4*. Cryptology ePrint Archive, Report 2022/058. <https://eprint.iacr.org/2022/058>. 2022.
- [Zha+23] Shiduo Zhang et al. “Improved Power Analysis Attacks on Falcon”. In: *EUROCRYPT 2023, Part IV*. Ed. by Carmit Hazay and Martijn Stam. Vol. 14007. LNCS. Springer, Heidelberg, Apr. 2023, pp. 565–595. DOI: [10.1007/978-3-031-30634-1_19](https://doi.org/10.1007/978-3-031-30634-1_19).
- [Inc] NewAE Technology Inc. *ChipWhisperer-Pro (Complete Level 3 Starter Kit)*. <https://store.newae.com/chipwhisperer-pro-complete-level-3-starter-kit/>. URL: <https://store.newae.com/chipwhisperer-pro-complete-level-3-starter-kit/>.

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7 Appendix – Details of Our Design

Floating-Point Number Packing and Rounding

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

- 1: $e \leftarrow e + 1076$
- 2: $b \leftarrow \llbracket e < 0 \rrbracket$
- 3: $z \leftarrow z \wedge (b - 1)$
- 4: $b \leftarrow \llbracket z \neq 0 \rrbracket$
- 5: $e \leftarrow e \wedge (-b)$
- 6: $x \leftarrow ((s \ll 63) \vee (z \gg 2)) + e \ll 52$
- 7: $f \leftarrow 0XC8 \gg z^{[3:1]}$
- 8: $x \leftarrow x + f^{(1)} \{ \text{increment if } z^{[3:1]} \text{ is } 011, 110 \text{ or } 111 \}$
- 9: **return** x

Floating-Point Number Multiplication

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

1: $s \leftarrow sx \oplus sy$

2: $e \leftarrow ex + ey - 2100$

3: $z \leftarrow mx \times my$

4: $b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$

5: $z \leftarrow z^{[106:51]} \vee b$

6: $z' \leftarrow (z \ggg 1) \vee z^{(1)}$

7: $w \leftarrow z^{(106)}$

8: $z \leftarrow z \oplus (z \oplus z') \wedge (-w)$

9: $e \leftarrow e + w$

10: $bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$

11: $b \leftarrow bx \wedge by$

12: $z \leftarrow z \wedge (-b)$

13: **return** FPR(s, e, z)

Floating-Point Number Addition

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$
- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

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- SecFprAdd: Secure FprAdd

Simple Tricks

If we want to operate the following operations:

```
1: if  $a = 0$  then  
2:    $b \leftarrow 0$ 
```

Suppose a is one-bit, we may write it as

```
1:  $b \leftarrow b \wedge (-a)$ 
```

We also apply the same idea for Boolean-shared values in our design

```
1:  $(b_i) \leftarrow \text{SecAnd}((b_i), (-a_i))$ 
```

We utilize that $\bigoplus_{i=1}^n -a_i = -a$, which is not true for general k -bit a .

Simple Tricks

Similarly, for operations

```
1: if  $a = 1$  then  
2:    $b \leftarrow 0$ 
```

Suppose a is one-bit, we may write it as

```
1:  $b \leftarrow b \wedge (\neg(-a))$ 
```

For Boolean-shared values,

```
1:  $(c_i) \leftarrow (-a_i)$   
2:  $(b_i) \leftarrow \text{SecAnd}((b_i), (\neg c_1, c_2, \dots, c_n))$ 
```

Simple Tricks

Moreover, for operations,

```
1: if  $a = 1$  then  
2:    $b \leftarrow c$ 
```

Suppose a is one-bit, we may write it as

```
1:  $d \leftarrow b \oplus c$   
2:  $b \leftarrow b \oplus (d \wedge (-a))$ 
```

For Boolean-shared values,

```
1:  $(d_i) \leftarrow (b_i \oplus c_i)$   
2:  $(d_i) \leftarrow \text{SecAnd}((d_i), (-a_i))$   
3:  $(b_i) \leftarrow (b_i \oplus d_i)$ 
```

Simple Tricks in Masking FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

- 1: $e \leftarrow e + 1076$
- 2: $b \leftarrow \llbracket e < 0 \rrbracket$
- 3: $z \leftarrow z \wedge (b - 1)$
- 4: $b \leftarrow \llbracket z \neq 0 \rrbracket$
- 5: $e \leftarrow e \wedge (-b)$
- 6: $x \leftarrow ((s \lll 63) \vee (z \ggg 2)) + e \lll 52$
- 7: $f \leftarrow 0XC8 \ggg z^{[3:1]}$
- 8: $x \leftarrow x + f^{(1)} \{ \text{increment if } z^{[3:1]} \text{ is } 011, 110 \text{ or } 111 \}$
- 9: **return** x

Simple Tricks in Masking FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \text{return FPR}(s, e, z)$$

Simple Tricks in Masking FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$
- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

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- SecFPR: Secure FPR
- SecFprMul: Secure FprMul
- SecFprAdd: Secure FprAdd

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \lll 63) \vee (z \ggg 2)) + e \lll 52$ 
7:  $f \leftarrow 0XC8 \ggg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

We now show how we mask the floating-point number rounding and packing algorithm FPR.

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \ll 63) \vee (z \gg 2)) + e \ll 52$ 
7:  $f \leftarrow 0XC8 \gg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

Recall that FPR is the last subroutine of FprMul and FprAdd.

By our masking design of FprMul and FprAdd, (s_i) is Boolean-masked, (e_i) is 16-bit arithmetic-masked, and (z_i) is Boolean-masked.

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \ll 63) \vee (z \gg 2)) + e \ll 52$ 
7:  $f \leftarrow 0XC8 \gg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

This is by adding to any one share.

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \lll 63) \vee (z \ggg 2)) + e \lll 52$ 
7:  $f \leftarrow 0XC8 \ggg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

This is equivalent to

```

1: if  $e < 0$  then
2:    $z \leftarrow 0$ 

```

and is done by an A2B, taking the MSB, and the simple trick.

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \lll 63) \vee (z \ggg 2)) + e \lll 52$ 
7:  $f \leftarrow 0XC8 \ggg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

This is done by SecNonzero and the simple trick.

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \ll 63) \vee (z \gg 2)) + e \ll 52$ 
7:  $f \leftarrow 0XC8 \gg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

Shift, OR, and a SecAdd.

We add (e_i) and the 55th bit of (z_i) in advance instead of adding (e_i) to a 64-bit value. That is, use a 16-bit SecAdd to save a 64-bit SecAdd

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \ll 63) \vee (z \gg 2)) + e \ll 52$ 
7:  $f \leftarrow 0XC8 \gg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

If the least 3 bits of (z_i) are 011, 110, and 111, $f^{(1)} = 1$.
 We OR $(z_i^{(1)})$ and $(z_i^{(3)})$ by SecOr, and then AND $(z_i^{(2)})$ by SecAnd. The result is then added to (x_i) by SecAdd.

SecFPR: Secure FPR

FPR

Input: Sign bit s , exponent e , and 55-bit mantissa z

Output: FPN x packed by s, e, z

```

1:  $e \leftarrow e + 1076$ 
2:  $b \leftarrow \llbracket e < 0 \rrbracket$ 
3:  $z \leftarrow z \wedge (b - 1)$ 
4:  $b \leftarrow \llbracket z \neq 0 \rrbracket$ 
5:  $e \leftarrow e \wedge (-b)$ 
6:  $x \leftarrow ((s \lll 63) \vee (z \ggg 2)) + e \lll 52$ 
7:  $f \leftarrow 0XC8 \ggg z^{[3:1]}$ 
8:  $x \leftarrow x + f^{(1)}$ 
9: return  $x$ 

```

Done.

The returned value is a 64-bit Boolean-masked (x_i).

SecFPR: Secure FPR

SecFPR

Input: 1-bit Boolean shares $(s_i)_{1 \leq i \leq n}$

Input: 16-bit arithmetic shares $(e_i)_{1 \leq i \leq n}$

Input: 55-bit Boolean shares $(z_i)_{1 \leq i \leq n}$

Output: Boolean shares $(x_i)_{1 \leq i \leq n}$

1: $e_1 \leftarrow e_1 + 1076$

2: $(e_i) \leftarrow \text{A2B}((e_i))$

3: $(b_i) \leftarrow (-e_i^{(16)})$

4: $(z_i) \leftarrow \text{SecAnd}((z_i), (\neg b_1, b_2, \dots, b_n))$

5: $(e_i) \leftarrow \text{SecAnd}((e_i), (-z_i^{(55)}))$

6: $(e_i) \leftarrow \text{SecAdd}((e_i), (z_i^{(55)}))$

7: $(e_i) \leftarrow \text{Refresh}((e_i))$

8: $(s_i) \leftarrow \text{Refresh}((s_i))$

9: $(x_i) \leftarrow ((s_i^{(1)} \ll 63) \vee (e_i^{[11:1]} \ll 52) \vee (z_i^{[54:3]}))$

10: $(f_i) \leftarrow \text{SecOr}(\text{Refresh}(z_i^{(1)}), (z_i^{(3)}))$

11: $(f_i) \leftarrow \text{SecAnd}((f_i), (z_i^{(2)}))$

12: $(x_i) \leftarrow \text{SecAdd}((x_i), (f_i))$

13: **return** (x_i)

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SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \textbf{return FPR}(s, e, z)$$

We show how we mask the floating-point number multiplication algorithm FprMul.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \textbf{return FPR}(s, e, z)$$

We assume (sx_i) and (sy_i) are Boolean shares, (ex_i) and (ey_i) are 16-bit arithmetic shares, and (mx_i) and (my_i) are 128-bit arithmetic shares, which can load the product of two 53-bit values.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \text{return FPR}(s, e, z)$$

These can be operated share-wise.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

1: $s \leftarrow sx \oplus sy$

2: $e \leftarrow ex + ey - 2100$

3: $z \leftarrow mx \times my$

4: $b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$

5: $z \leftarrow z^{[106:51]} \vee b$

6: $z' \leftarrow (z \ggg 1) \vee z^{(1)}$

7: $w \leftarrow z^{(106)}$

8: $z \leftarrow z \oplus (z \oplus z') \wedge (-w)$

9: $e \leftarrow e + w$

10: $bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$

11: $b \leftarrow bx \wedge by$

12: $z \leftarrow z \wedge (-b)$

13: **return** FPR(s, e, z)

This is done by SecMult. For further operations, we then apply an A2B to turn them to Boolean shares.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

1: $s \leftarrow sx \oplus sy$

2: $e \leftarrow ex + ey - 2100$

3: $z \leftarrow mx \times my$

4: $b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$

5: $z \leftarrow z^{[106:51]} \vee b$

6: $z' \leftarrow (z \ggg 1) \vee z^{(1)}$

7: $w \leftarrow z^{(106)}$

8: $z \leftarrow z \oplus (z \oplus z') \wedge (-w)$

9: $e \leftarrow e + w$

10: $bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$

11: $b \leftarrow bx \wedge by$

12: $z \leftarrow z \wedge (-b)$

13: **return** FPR(s, e, z)

Conditional shift by 50 bits and 51 bits, depending on $z^{(106)}$, while preserving the sticky bit. These can be done by SecNonzero and SecOr.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

1: $s \leftarrow sx \oplus sy$

2: $e \leftarrow ex + ey - 2100$

3: $z \leftarrow mx \times my$

4: $b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$

5: $z \leftarrow z^{[106:51]} \vee b$

6: $z' \leftarrow (z \ggg 1) \vee z^{(1)}$

7: $w \leftarrow z^{(106)}$

8: $z \leftarrow z \oplus (z \oplus z') \wedge (-w)$

9: $e \leftarrow e + w$

10: $bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$

11: $b \leftarrow bx \wedge by$

12: $z \leftarrow z \wedge (-b)$

13: **return** FPR(s, e, z)

We observe that we can save one SecOR.

- When shifted by 50 bits, we OR the last bit with $z^{[50:1]}$.
- When shifted by 51 bits, we OR the last bit with $z^{[51:1]}$.

We can simply OR the the last bit with $z^{[51:1]}$, regardless of the conditional shift result.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \textbf{return FPR}(s, e, z)$$

This is by adding to any share.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \text{return FPR}(s, e, z)$$

This is by SecNonzero and SecAnd, and applying the simple trick.

SecFprMul: Secure FprMul

FprMul

Input: FPN $x = (sx, ex, mx)$

Input: FPN $y = (sy, ey, my)$

Output: FPN product of x and y

$$1: s \leftarrow sx \oplus sy$$

$$2: e \leftarrow ex + ey - 2100$$

$$3: z \leftarrow mx \times my$$

$$4: b \leftarrow \llbracket z^{[50:1]} \neq 0 \rrbracket$$

$$5: z \leftarrow z^{[106:51]} \vee b$$

$$6: z' \leftarrow (z \ggg 1) \vee z^{(1)}$$

$$7: w \leftarrow z^{(106)}$$

$$8: z \leftarrow z \oplus (z \oplus z') \wedge (-w)$$

$$9: e \leftarrow e + w$$

$$10: bx \leftarrow \llbracket ex \neq 0 \rrbracket, by \leftarrow \llbracket ey \neq 0 \rrbracket$$

$$11: b \leftarrow bx \wedge by$$

$$12: z \leftarrow z \wedge (-b)$$

$$13: \text{return } \text{FPR}(s, e, z)$$

Now it calls FPR to return a 64-bit Boolean-masked FPN.

SecFprMul: Secure FprMul

SecFprMul

Input: Shares $(sx_i)_{1 \leq i \leq n}, (ex_i)_{1 \leq i \leq n}, (mx_i)_{1 \leq i \leq n}$

Input: Shares $(sy_i)_{1 \leq i \leq n}, (ey_i)_{1 \leq i \leq n}, (my_i)_{1 \leq i \leq n}$

Output: Boolean shares for the FPN product.

$$1: (s_i) \leftarrow (sx_i \oplus sy_i)$$

$$2: (e_i) \leftarrow (ex_1 + ey_1 - 2100, ex_2 + ey_2, \dots)$$

$$3: (p_i) \leftarrow \text{SecMult}((mx_i), (my_i))$$

$$4: (p_i) \leftarrow \text{A2B}((p_i))$$

$$5: (b_i) \leftarrow \text{SecNonzero}((p_i^{[51:1]}))$$

$$6: (z_i) \leftarrow (p_i^{[105:51]})$$

$$7: (z'_i) \leftarrow (p_i^{[105:51]} \oplus p_i^{[106:52]})$$

$$8: (w_i) \leftarrow (p_i^{(106)})$$

$$9: (z'_i) \leftarrow \text{SecAnd}((z'_i), \text{Refresh}((-w_i)))$$

$$10: (z_i) \leftarrow (z'_i \oplus z_i)$$

$$11: (z_i) \leftarrow \text{SecOr}((z_i), (b_i))$$

$$12: (w_i) \leftarrow \text{B2ABit}((w_i))$$

$$13: (e_i) \leftarrow (e_i + w_i)$$

$$14: (bx_i) \leftarrow \text{SecNonzero}((ex_i))$$

$$15: (by_i) \leftarrow \text{SecNonzero}((ey_i))$$

$$16: (d_i) \leftarrow \text{SecAnd}((bx_i), (by_i))$$

$$17: (z_i) \leftarrow \text{SecAnd}((z_i), (-d_i^{(1)}))$$

$$18: \textbf{return SecFPR}((s_i), (e_i), (z_i))$$

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SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

We show how we mask the floating-point number addition algorithm FprAdd.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

By the output of SecFprMul, we assume the input shares (x_i) and (y_i) are 64-bit Boolean-masked FPNs.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

```

1:  $d \leftarrow x^{[63:1]} - y^{[63:1]}$ 
2:  $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$ 
3:  $m \leftarrow (x \oplus y) \wedge (-cs)$ 
4:  $x \leftarrow x \oplus m, y \leftarrow y \oplus m$ 
5: Extract  $(sx, ex, mx)$  and  $(sy, ey, my)$ 
   from  $x, y$ , respectively.
6:  $mx \leftarrow mx \lll 3, my \leftarrow my \lll 3$ 
7:  $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$ 
8:  $c \leftarrow ex - ey$ 

```

```

9:  $b \leftarrow \llbracket c < 60 \rrbracket$ 
10:  $my \leftarrow my \wedge (-b)$ 
11:  $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$ 
12:  $s \leftarrow sx \oplus sy$ 
13:  $z \leftarrow mx + (-1)^s my$ 
14: Normalize  $z, ex$  to make the 64th bit of
    $z$  set
15:  $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$ 
16:  $ex \leftarrow ex + 9$ 
17: return FPR( $sx, ex, z$ )

```

The subtraction of two Boolean-masked values can be operated by considering $x^{[63:1]} - y^{[63:1]} = x^{[63:1]} + (\neg y^{[63:1]}) + 1$, which takes two SecAdds.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

But since we only need $(x^{[63:1]} - y^{[63:1]})^{(64)}$, we only compute $x^{[63:1]} + (\neg y^{[63:1]})$ and then check the boundary conditions. This saves us one SecAdd.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

That is, $\llbracket u - v < 0 \rrbracket = \llbracket u + (\neg v) < 0 \rrbracket \oplus \llbracket u + (\neg v) = -1 \rrbracket \oplus \llbracket u + (\neg v) = 2^{63} - 1 \rrbracket$

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

Moreover, we apply $u + (\neg v) \neq -1 \Leftrightarrow \neg(u + (\neg v)) \neq 0$ and $u + (\neg v) \neq 2^{63} - 1 \Leftrightarrow (u + (\neg v)) \oplus (1 \ll 63) \neq -1 \Leftrightarrow \neg((u + (\neg v)) \oplus (1 \ll 63)) \neq 0$

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$
- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

Therefore, these operations can be computed by SecNonzero, SecAnd, and SecOr.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: **Extract** (sx, ex, mx) and (sy, ey, my)
from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: **Normalize** z, ex to make the 64th bit of
 z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

Share-wise operations, two B2As to convert (ex_i) and (ey_i) to arithmetic shares, and subtractions to any shares.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

Subtraction to any share of c by 60 and an A2B to get the MSB of c . Then apply the simple trick.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

This is by our gadget SecFprUrsh.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

A Share-wise operation.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \lll 3, my \leftarrow my \lll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

Considering the simple trick with $my + (my \oplus (-my)) \wedge -s$, where $-my = (\neg my) + 1$ is derived by an SecAdd. Then add the result to mx .

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: **Normalize z, ex to make the 64th bit of z set**
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

This is by our gadget SecFprNorm64.

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$

- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** FPR(sx, ex, z)

A Share-wise operation and a SecNonzero. Add 9 to any share of ex .

SecFprAdd: Secure FprAdd

FprAdd

Input: FPNs x and y

Output: FPN sum of x and y

- 1: $d \leftarrow x^{[63:1]} - y^{[63:1]}$
- 2: $cs \leftarrow d^{(64)} \vee ((1 - (-d)^{(64)}) \wedge x^{(64)})$
- 3: $m \leftarrow (x \oplus y) \wedge (-cs)$
- 4: $x \leftarrow x \oplus m, y \leftarrow y \oplus m$
- 5: Extract (sx, ex, mx) and (sy, ey, my) from x, y , respectively.
- 6: $mx \leftarrow mx \ll 3, my \leftarrow my \ll 3$
- 7: $ex \leftarrow ex - 1078, ey \leftarrow ey - 1078$
- 8: $c \leftarrow ex - ey$
- 9: $b \leftarrow \llbracket c < 60 \rrbracket$
- 10: $my \leftarrow my \wedge (-b)$
- 11: $my \leftarrow (my \ggg c) \vee \llbracket my^{[c:1]} \neq 0 \rrbracket$
- 12: $s \leftarrow sx \oplus sy$
- 13: $z \leftarrow mx + (-1)^s my$
- 14: Normalize z, ex to make the 64th bit of z set
- 15: $z \leftarrow (z \ggg 9) \vee \llbracket z^{[9:1]} \neq 0 \rrbracket$
- 16: $ex \leftarrow ex + 9$
- 17: **return** $\text{FPR}(sx, ex, z)$

Finally, it calls FPR to return a 64-bit Boolean-masked FPN.

SecFprAdd: Secure FprAdd

SecFprAdd

Input: Boolean shares $(x_i)_{1 \leq i \leq n}$

Input: Boolean shares $(y_i)_{1 \leq i \leq n}$

Output: Boolean shares for the FPN sum

- 1: $(xm_i) \leftarrow (x_i^{[63:1]})$
- 2: $(ym_i) \leftarrow (\neg y_1^{[63:1]}, y_2^{[63:1]}, \dots, y_n^{[63:1]})$
- 3: $(d_i) \leftarrow \text{SecAdd}((xm_i), (ym_i))$
- 4: $(b_i) \leftarrow \text{SecNonzero}(\neg d_1, d_2, \dots, d_n)$
- 5: $(b'_i) \leftarrow \text{SecNonzero}(\neg(d_1 \oplus (1 \ll 63)), d_2, \dots, d_n)$
- 6: $(cs_i) \leftarrow \text{SecAnd}((\neg b_1, b_2, \dots, b_n), (x_i^{(64)}))$
- 7: $(cs_i) \leftarrow \text{SecOr}((cs_i), (d_i^{(64)} \oplus b_i \oplus b'_i))$
- 8: $(m_i) \leftarrow \text{SecAnd}((x_i \oplus y_i), (\neg cs_i))$
- 9: $(x_i) \leftarrow (x_i \oplus m_i), (y_i) \leftarrow (y_i \oplus m_i)$
- 10: Extract $(sx_i), (ex_i), (mx_i)$ and $(sy_i), (ey_i), (my_i)$ from (x_i) and (y_i) , respectively.
- 11: $(mx_i) \leftarrow (mx_i \ll 3), (my_i) \leftarrow (my_i \ll 3)$
- 12: $(ex_i) \leftarrow \text{B2A}((ex_i)), (ey_i) \leftarrow \text{B2A}((ey_i))$
- 13: $ex_1 \leftarrow ex_1 - 1078, ey_1 \leftarrow ey_1 - 1078.$

- 14: $(c_i) \leftarrow (ex_i - ey_i)$
- 15: $(c'_i) \leftarrow \text{A2B}((c_1 - 60, c_2, \dots, c_n))$
- 16: $(my_i) \leftarrow \text{SecAnd}((my_i), (\neg(c'_i)^{(16)})))$
- 17: $(my_i) \leftarrow \text{SecFprUrsh}((my_i), (c_i^{[6:1]}))$
- 18: $(my'_i) \leftarrow (\neg my_1, my_2, \dots, my_n)$
- 19: $(my'_i) \leftarrow \text{SecAdd}((my'_i), (1, 0, \dots, 0))$
- 20: $(s_i) \leftarrow (\neg(sx_i \oplus sy_i))$
- 21: $(my_i) \leftarrow \text{Refresh}((my_i))$
- 22: $(my'_i) \leftarrow \text{SecAnd}((my_i \oplus my'_i), (s_i))$
- 23: $(my_i) \leftarrow (my_i \oplus my'_i)$
- 24: $(z_i) \leftarrow \text{SecAdd}((mx_i), (my_i))$
- 25: $(z_i), (ex_i) \leftarrow \text{SecFprNorm64}((z_i), (ex_i))$
- 26: $(b_i) \leftarrow \text{SecNonzero}((z_i^{[10:1]}))$
- 27: $(z_i) \leftarrow (z_i \gg 9)$
- 28: $(z_i^{(1)}) \leftarrow (b_i)$
- 29: $ex_1 \leftarrow ex_1 + 9$
- 30: **return** $\text{SecFPR}(\text{Refresh}((sx_i)), (ex_i), (z_i))$