Tarea 2

Nombre: Joel Stalin Tinitana Carrion

CONJUNTO DE EJERCICIOS 1

Resuelva los siguientes ejercicios, tome en cuenta que debe mostrar el desarrollo completo del ejercicio.

1. **Calcule los errores absoluto y relativo en las aproximaciones de p por p*.**

a. $\boldsymbol{p = \pi, p^* = \frac{22}{7}}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = \left|\pi - \frac{22}{7}\right|$$

$$\boldsymbol{error_{abs}} = 1.2644893x10^{-3}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\pi - \frac{22}{7}}{\pi}\right|$$

$$\boldsymbol{error_{relativo}} = 4.0249943x10^{-4}$$

b. $\boldsymbol{p = \pi, p^* = 3.1416}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = |\pi - 3.1416|$$

$$\boldsymbol{error_{abs}} = 7.3464102x10^{-6}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\pi - 3.1416}{\pi}\right|$$

$$\boldsymbol{error_{relativo}} = 2.338434995x10^{-6}$$

c. $\boldsymbol{p = e, p^* = 2.718}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = |e - 2.718|$$

$$\boldsymbol{error_{abs}} = 2.8182846x10^{-4}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{e - 2.718}{e}\right|$$

$$\boldsymbol{error_{relativo}} = 1.03678896x10^{-4}$$

d. $\boldsymbol{p = \sqrt{2}, p^* = 1.414}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = \left|\sqrt{2} - 1.414\right|$$

$$\boldsymbol{error_{abs}} = 2.1356237x10^{-4}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\sqrt{2} - 1.414}{\sqrt{2}}\right|$$

$$\boldsymbol{error_{relativo}} = 1.5101140x10^{-4}$$

2. Calcule los errores absoluto y relativo en las aproximaciones de p por p*.

a. $\boldsymbol{p = e^{10}, p^* = 22000}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = |e^{10} - 22000|$$

$$\boldsymbol{error_{abs}} = 26.4657948$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{e^{10} - 22000}{e^{10}}\right|$$

$$\boldsymbol{error_{relativo}} = 1.2015452x10^{-3}$$

b. $\boldsymbol{p = 10^{\pi}, p^* = 1400}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = |10^{\pi} - 1400|$$

$$\boldsymbol{error_{abs}} = 14.5442686$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{10^{\pi} - 1400}{10^{\pi}}\right|$$

$$\boldsymbol{error_{relativo}} = 0.0104978$$

c. $\boldsymbol{p = 8!, p^* = 39900}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = |8! - 39900|$$

$$\boldsymbol{error_{abs}} = 420$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{8! - 39900}{8!}\right|$$

$$\boldsymbol{error_{relativo}} = 0.0104167$$

d. $\boldsymbol{p = 9!, p^* = \sqrt{18\pi}\left(\frac{9}{e}\right)^9}$

Cálculo

$$\boldsymbol{error_{abs}} = |\boldsymbol{p - p^*}|$$

$$\boldsymbol{error_{abs}} = \left|9! - \sqrt{18\pi}\left(\frac{9}{e}\right)^9\right|$$

$$\boldsymbol{error_{abs}} = 3343.127158$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p - p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{9! - \sqrt{18\pi}\left(\frac{9}{e}\right)^9}{9!}\right|$$

$$\boldsymbol{error_{relativo}} = 9.2127622x10^{-3}$$

3. **Encuentre el intervalo más largo en el que se debe encontrar p* para aproximarse a p con error relativo máximo de $\boldsymbol{10^{-4}}$ para cada valor de p.**

a. $\boldsymbol{\pi}$

Cálculo

$$\boldsymbol{error_{relativo}} = \left|\frac{p-p^*}{p}\right|, p \neq 0$$

$$\boldsymbol{error_{relativo}} = \left|\frac{p-p^*}{p}\right| \leq 10^{-4}$$

$$|p-p^*| \leq 10^{-4} \cdot |p|$$

$$p^*\ debe\ estar\ entre\ p-10^{-4}p\ y\ p+10^{-4}p$$

$$p-10^{-4}p\ hasta\ p+10^{-4}p$$

$$\boldsymbol{Longitud} = (p+10^{-4}p)-(p-10^{-4}p)$$

$$\boldsymbol{Longitud} = p+10^{-4}p-p+10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}(\pi)$$

$$\boldsymbol{Longitud} = 0.0006283$$

$$\boldsymbol{\pi} \approx 0.0006283$$

$$p^* \in [\,0.0003142-3.1415927, 0.0003142+3.1415927]$$

$$p^* \in [3.1412785, 3.1419069]$$

b. $\boldsymbol{e}$

Cálculo

$$\boldsymbol{error_{relativo}} = \left|\frac{p-p^*}{p}\right|, p \neq 0$$

$$\boldsymbol{error_{relativo}} = \left|\frac{p-p^*}{p}\right| \leq 10^{-4}$$

$$|p-p^*| \leq 10^{-4} \cdot |p|$$

$$p^*\ debe\ estar\ entre\ p-10^{-4}p\ y\ p+10^{-4}p$$

$$p-10^{-4}p\ hasta\ p+10^{-4}p$$

$$\boldsymbol{Longitud} = (p+10^{-4}p)-(p-10^{-4}p)$$

$$\boldsymbol{Longitud} = p+10^{-4}p-p+10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}(e)$$

$$\boldsymbol{Longitud} = 0.0005437$$

$$\boldsymbol{e} = 2.7182818$$

$$p^* \in [\,0.0002718-2.7182818, 0.0002718+2.7182818]$$

$$p^* \in [2.71801, 2.7185536]$$

c. $\sqrt{\mathbf{2}}$

Cálculo

$$\boldsymbol{error_{relativo}} = \left|\frac{p - p^*}{p}\right|, p \neq 0$$

$$\boldsymbol{error_{relativo}} = \left|\frac{p - p^*}{p}\right| \leq 10^{-4}$$

$$|p - p^*| \leq 10^{-4} \cdot |p|$$

$$p^* \; debe \; estar \; entre \; p - 10^{-4}p \; y \; p + 10^{-4}p$$

$$p - 10^{-4}p \; hasta \; p + 10^{-4}p$$

$$\boldsymbol{Longitud} = (p + 10^{-4}p) - (p - 10^{-4}p)$$

$$\boldsymbol{Longitud} = p + 10^{-4}p - p + 10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}(\sqrt{2})$$

$$\boldsymbol{Longitud} = 0.0002828$$

$$\sqrt{\mathbf{2}} = 1.4142136$$

$$p^* \in [\, 0.0001414 - 1.4142136, 0.0001414 + 1.4142136]$$

$$p^* \in [1.4140722, 1.414355]$$

d. $\sqrt[3]{\mathbf{7}}$

Cálculo

$$\boldsymbol{error_{relativo}} = \left|\frac{p - p^*}{p}\right|, p \neq 0$$

$$\boldsymbol{error_{relativo}} = \left|\frac{p - p^*}{p}\right| \leq 10^{-4}$$

$$|p - p^*| \leq 10^{-4} \cdot |p|$$

$$p^* \; debe \; estar \; entre \; p - 10^{-4}p \; y \; p + 10^{-4}p$$

$$p - 10^{-4}p \; hasta \; p + 10^{-4}p$$

$$\boldsymbol{Longitud} = (p + 10^{-4}p) - (p - 10^{-4}p)$$

$$\boldsymbol{Longitud} = p + 10^{-4}p - p + 10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}p$$

$$\boldsymbol{Longitud} = 2x10^{-4}(\sqrt[3]{7})$$

$$\boldsymbol{Longitud} = 0.0003826$$

$$\sqrt[3]{\mathbf{7}} = \ 1.9129312$$

$$p^* \in [0.0003826 - \ 1.9129312, 0.0003826 + \ 1.9129312]$$

$$p^* \in [1.9125486, 1.9133138]$$

4. Use la aritmética de redondeo de tres dígitos para realizar lo siguiente. Calcule los errores absoluto y relativo con el valor exacto determinado para por lo menos cinco dígitos.

a. $\frac{\frac{13}{14}-\frac{5}{7}}{2e-5.4}$

Cálculo

$$\boldsymbol{p} = \ 5.860620418$$

$$\boldsymbol{p}^* \approx ?$$

$$\mathbf{13} \ \oslash \mathbf{14} \ \approx 0.929$$

$$\mathbf{5} \ \oslash \mathbf{7} \approx 0.714$$

$$(\mathbf{13} \ \oslash \mathbf{14}\ ) \ominus (\mathbf{5} \ \oslash \mathbf{7}) \approx \ 0.215$$

$$\mathbf{2} \otimes \boldsymbol{e} \approx 5.437$$

$$(\mathbf{2} \otimes \boldsymbol{e}) \ominus \mathbf{5.4} \approx 0.037$$

$$((\mathbf{13} \ \oslash \mathbf{14}\ )(\mathbf{5} \ \oslash \mathbf{7})) \oslash ((\mathbf{2} \otimes \boldsymbol{e}) \ominus \mathbf{5.4}) \approx 5.811$$

$$\boldsymbol{p}^* \approx 5.811$$

$$\boldsymbol{error_{abs}} = |\boldsymbol{p} - \boldsymbol{p}^*|$$

$$\boldsymbol{error_{abs}} = |5.860620418 \ - 5.811|$$

$$\boldsymbol{error_{abs}} \approx 0.05$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p} - \boldsymbol{p}^*}{\boldsymbol{p}}\right|, \boldsymbol{p} \neq \mathbf{0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{5.860620418 \ - 5.811}{5.860620418}\right|$$

$$\boldsymbol{error_{relativo}} \approx 0.008$$

b. $-\mathbf{10}\,\boldsymbol{\pi} + \mathbf{6}\boldsymbol{e} - \frac{\mathbf{3}}{\mathbf{61}}$

Cálculo

$$\boldsymbol{p} = \ -15.15541589$$

$$\boldsymbol{p}^* \approx ?$$

$$-\boldsymbol{10} \otimes \boldsymbol{\pi} \approx -31.416$$

$$\boldsymbol{6} \otimes \boldsymbol{e} \approx 16.31$$

$$(\boldsymbol{3} \oslash \boldsymbol{61}) \approx 0.049$$

$$(-\boldsymbol{10} \otimes \boldsymbol{\pi}) \oplus (\boldsymbol{6} \otimes \boldsymbol{e}) \approx -15.106$$

$$((-\boldsymbol{10} \otimes \boldsymbol{\pi}) \oplus (\boldsymbol{6} \otimes \boldsymbol{e})) \ominus (\boldsymbol{3} \oslash \boldsymbol{61}) \approx -15.155$$

$$\boldsymbol{p}^* \approx -15.155$$

$$\boldsymbol{error_{abs}} = |\boldsymbol{p} - \boldsymbol{p}^*|$$

$$\boldsymbol{error_{abs}} = |-15.15541589 + 15.155|$$

$$\boldsymbol{error_{abs}} \approx 4.159x10^{-4}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p} - \boldsymbol{p}^*}{\boldsymbol{p}}\right|, \boldsymbol{p} \neq \boldsymbol{0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{-15.15541589 + 15.155}{-15.15541589}\right|$$

$$\boldsymbol{error_{relativo}} \approx 2.744x10^{-5}$$

c. $\left(\frac{\boldsymbol{2}}{\boldsymbol{9}}\right) \cdot \left(\frac{\boldsymbol{9}}{\boldsymbol{11}}\right)$

Cálculo

$$\boldsymbol{p} = 0.1818181818$$

$$\boldsymbol{p}^* \approx ?$$

$$(\boldsymbol{2} \oslash \boldsymbol{9}) \approx 0.222$$

$$(\boldsymbol{9} \oslash \boldsymbol{11}) \approx 0.818$$

$$(\boldsymbol{2} \oslash \boldsymbol{9}) \otimes (\boldsymbol{9} \oslash \boldsymbol{11}) \approx 0.182$$

$$\boldsymbol{p}^* \approx 0.182$$

$$\boldsymbol{error_{abs}} = |\boldsymbol{p} - \boldsymbol{p}^*|$$

$$\boldsymbol{error_{abs}} = |0.1818181818 - 0.182|$$

$$\boldsymbol{error_{abs}} = 1.818x10^{-4}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p} - \boldsymbol{p}^*}{\boldsymbol{p}}\right|, \boldsymbol{p} \neq \boldsymbol{0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{0.1818181818 - 0.182}{0.1818181818}\right|$$

$$\boldsymbol{error_{relativo}} = 0.001$$

d. $\frac{\sqrt{13}+\sqrt{11}}{\sqrt{13}-\sqrt{11}}$

Cálculo

$$\boldsymbol{p} = 23.95826074$$

$$\boldsymbol{p}^* \approx ?$$

$$\sqrt{\mathbf{13}} \approx 3.606$$

$$\sqrt{\mathbf{11}} \approx 3.317$$

$$\sqrt{\mathbf{13}} \oplus \sqrt{\mathbf{11}} \approx 6.923$$

$$\sqrt{\mathbf{13}} \ominus \sqrt{\mathbf{11}} \approx 0.289$$

$$(\sqrt{\mathbf{13}} \oplus \sqrt{\mathbf{11}}) \oslash (\sqrt{\mathbf{13}} \ominus \sqrt{\mathbf{11}}) \approx 23.955$$

$$\boldsymbol{p}^* \approx 23.955$$

$$\boldsymbol{error_{abs}} = |\boldsymbol{p} - \boldsymbol{p}^*|$$

$$\boldsymbol{error_{abs}} = |23.95826074 - 23.955|$$

$$\boldsymbol{error_{abs}} = 0.003$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p} - \boldsymbol{p}^*}{\boldsymbol{p}}\right|, \boldsymbol{p} \neq \mathbf{0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{23.95826074 - 23.955}{23.95826074}\right|$$

$$\boldsymbol{error_{relativo}} = 1.361x10^{-4}$$

5. Los primeros tres términos diferentes a cero de la serie de Maclaurin para la función arcotangente son: $\boldsymbol{x} - \left(\frac{1}{3}\right)\boldsymbol{x}^3 + \left(\frac{1}{5}\right)\boldsymbol{x}^5$. Calcule los errores absoluto y relativo en las siguientes aproximaciones de $\boldsymbol{\pi}$ mediante el polinomio en lugar del arcotangente:

a. $\mathbf{4}\left[\mathbf{arctan}\left(\frac{1}{2}\right) + \mathbf{arctan}\left(\frac{1}{3}\right)\right]$

Cálculo

$$\boldsymbol{x_1} = \frac{1}{2}\ ;\ \boldsymbol{x_2} = \frac{1}{3}$$

$$\boldsymbol{Formula} = x - \left(\frac{1}{3}\right)x^3 + \left(\frac{1}{5}\right)x^5$$

$$\boldsymbol{arctan}\ (\boldsymbol{x_1}) = \left(\frac{1}{2}\right) - \left(\frac{1}{3}\right)\left(\frac{1}{2}\right)^3 + \left(\frac{1}{5}\right)\left(\frac{1}{2}\right)^5$$

$$\left(\frac{1}{2}\right) \approx 0.5$$

$$\left(\frac{1}{3}\right) \approx 0.333$$

$$\left(\frac{1}{5}\right) \approx 0.2$$

$$\left(\frac{1}{2}\right) \otimes \left(\frac{1}{2}\right) \approx 0.25$$

$$fl\left(\left(\frac{1}{2}\right)^{2}\right) \otimes \left(\frac{1}{2}\right) \approx 0.125$$

$$fl\left(\left(\frac{1}{2}\right)^{3}\right) \otimes \left(\frac{1}{2}\right) \approx 0.063$$

$$fl\left(\left(\frac{1}{2}\right)^{4}\right) \otimes \left(\frac{1}{2}\right) \approx 0.032$$

$$fl\left(\left(\frac{1}{2}\right)^{3}\right) \otimes \left(\frac{1}{3}\right) \approx 0.042$$

$$\left(\frac{1}{2}\right) \ominus fl\left(\left(\frac{1}{2}\right)^{3} \otimes \left(\frac{1}{3}\right)\right) \approx 0.458$$

$$\left(\frac{1}{5}\right) \otimes fl\left(\left(\frac{1}{2}\right)^{5}\right) \approx 0.006$$

$$fl\left(\left(\frac{1}{2}\right) \ominus \left(\left(\frac{1}{2}\right)^{3} \otimes \left(\frac{1}{3}\right)\right)\right) \oplus fl\left(\left(\frac{1}{5}\right) \otimes \left(\frac{1}{2}\right)^{5}\right) \approx 0.464$$

$$\boldsymbol{arctan}\ (\boldsymbol{x_2}) = \left(\frac{1}{3}\right) - \left(\frac{1}{3}\right)\left(\frac{1}{3}\right)^{3} + \left(\frac{1}{5}\right)\left(\frac{1}{3}\right)^{5}$$

$$\left(\frac{1}{3}\right) \approx 0.333$$

$$\left(\frac{1}{5}\right) \approx 0.2$$

$$\left(\frac{1}{3}\right) \otimes \left(\frac{1}{3}\right) \approx 0.111$$

$$fl\left(\left(\frac{1}{3}\right)^{2}\right) \otimes \left(\frac{1}{3}\right) \approx 0.037$$

$$fl\left(\left(\frac{1}{2}\right)^{3}\right) \otimes \left(\frac{1}{3}\right) \approx 0.012$$

$$fl\left(\left(\frac{1}{3}\right)^{4}\right)\otimes\left(\frac{1}{3}\right)\approx 0.004$$

$$fl\left(\left(\frac{1}{3}\right)^{3}\right)\otimes\left(\frac{1}{3}\right)\approx 0.012$$

$$\left(\frac{1}{3}\right)\ominus fl\left(\left(\frac{1}{3}\right)^{3}\otimes\left(\frac{1}{3}\right)\right)\approx 0.321$$

$$\left(\frac{1}{5}\right)\otimes fl\left(\left(\frac{1}{2}\right)^{5}\right)\approx 0.001$$

$$fl\left(\left(\frac{1}{3}\right)\ominus\left(\left(\frac{1}{3}\right)^{3}\otimes\left(\frac{1}{3}\right)\right)\right)\oplus fl\left(\left(\frac{1}{5}\right)\otimes\left(\frac{1}{3}\right)^{5}\right)\approx 0.322$$

Resultados

$$\arctan(x_1)\approx 0.464$$

$$\arctan(x_2)\approx 0.322$$

$$\boldsymbol{Formula}=4[\arctan\left(\frac{1}{2}\right)+\arctan\left(\frac{1}{3}\right)]$$

$$\boldsymbol{fl}\,(\mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)\oplus\mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{3}}\right))\approx 0.786$$

$$\mathbf{4}\otimes(\mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{2}}\right)\oplus\mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{3}}\right))\approx 3.144$$

$$\boldsymbol{p}^{*}\approx 3.144$$
$$\boldsymbol{p}\approx\pi$$

$$\boldsymbol{error_{abs}}=|\boldsymbol{p}-\boldsymbol{p}^{*}|$$

$$\boldsymbol{error_{abs}}=|\pi-3.144|$$

$$\boldsymbol{error_{abs}}\approx 0.002$$

$$\boldsymbol{error_{relativo}}=\left|\frac{\boldsymbol{p}-\boldsymbol{p}^{*}}{\boldsymbol{p}}\right|,\boldsymbol{p}\neq\mathbf{0}$$

$$\boldsymbol{error_{relativo}}=\left|\frac{\pi-3.144}{\pi}\right|$$

$$\boldsymbol{error_{relativo}}\approx 7.663x10^{-4}$$

b. $\mathbf{16\,arctan}\left(\frac{\mathbf{1}}{\mathbf{5}}\right)-\mathbf{4\,arctan}\left(\frac{\mathbf{1}}{\mathbf{239}}\right)$

Cálculo

$$\boldsymbol{x_1}=\frac{1}{5}\,;\boldsymbol{x_2}=\frac{1}{239}$$

$$\boldsymbol{Formula} = x - \left(\frac{1}{3}\right)x^3 + \left(\frac{1}{5}\right)x^5$$

$$\boldsymbol{arctan}\,(\boldsymbol{x_1}) = \left(\frac{1}{5}\right) - \left(\frac{1}{3}\right)\left(\frac{1}{5}\right)^3 + \left(\frac{1}{5}\right)\left(\frac{1}{5}\right)^5$$

$$\left(\frac{1}{3}\right) \approx 0.333$$

$$\left(\frac{1}{5}\right) \approx 0.2$$

$$\left(\frac{1}{5}\right) \otimes \left(\frac{1}{5}\right) \approx 0.04$$

$$fl\left(\left(\frac{1}{5}\right)^2\right) \otimes \left(\frac{1}{5}\right) \approx 0.008$$

$$fl\left(\left(\frac{1}{5}\right)^3\right) \otimes \left(\frac{1}{5}\right) \approx 0.002$$

$$fl\left(\left(\frac{1}{5}\right)^4\right) \otimes \left(\frac{1}{5}\right) \approx 0.0004$$

$$fl\left(\left(\frac{1}{5}\right)^3\right) \otimes \left(\frac{1}{3}\right) \approx 0.003$$

$$\left(\frac{1}{5}\right) \ominus fl\left(\left(\frac{1}{5}\right)^3 \otimes \left(\frac{1}{3}\right)\right) \approx 0.197$$

$$\left(\frac{1}{5}\right) \otimes fl\left(\left(\frac{1}{2}\right)^5\right) \approx 0.00008$$

$$fl\left(\left(\frac{1}{2}\right) \ominus \left(\left(\frac{1}{2}\right)^3 \otimes \left(\frac{1}{3}\right)\right)\right) \oplus fl\left(\left(\frac{1}{5}\right) \otimes \left(\frac{1}{2}\right)^5\right) \approx 0.197$$

$$\boldsymbol{arctan}\,(\boldsymbol{x_2}) = \left(\frac{1}{239}\right) - \left(\frac{1}{3}\right)\left(\frac{1}{239}\right)^3 + \left(\frac{1}{5}\right)\left(\frac{1}{239}\right)^5$$

$$\left(\frac{1}{239}\right) \approx 0.004$$

$$\left(\frac{1}{3}\right) \approx 0.333$$

$$\left(\frac{1}{5}\right) \approx 0.2$$

$$\left(\frac{1}{239}\right) \otimes \left(\frac{1}{239}\right) \approx 1.6x10^{-5}$$

$$fl\left(\left(\frac{1}{239}\right)^2\right)\otimes\left(\frac{1}{239}\right)\approx 6.4x10^{-8}$$

$$fl\left(\left(\frac{1}{239}\right)^3\right)\otimes\left(\frac{1}{239}\right)\approx 2.56x10^{-10}$$

$$fl\left(\left(\frac{1}{239}\right)^4\right)\otimes\left(\frac{1}{239}\right)\approx 1.024x10^{-12}$$

$$fl\left(\left(\frac{1}{239}\right)^3\right)\otimes\left(\frac{1}{3}\right)\approx 0.0000000213$$

$$\left(\frac{1}{3}\right)\ominus fl\left(\left(\frac{1}{3}\right)^3\otimes\left(\frac{1}{3}\right)\right)\approx 0.333$$

$$\left(\frac{1}{5}\right)\otimes fl\left(\left(\frac{1}{2}\right)^5\right)\approx 0.000000000000205$$

$$fl\left(\left(\frac{1}{3}\right)\ominus\left(\left(\frac{1}{3}\right)^3\otimes\left(\frac{1}{3}\right)\right)\right)\oplus fl\left(\left(\frac{1}{5}\right)\otimes\left(\frac{1}{3}\right)^5\right)\approx 0.333$$

Resultados

$$\arctan(x_1)\approx 0.197$$

$$\arctan(x_2)\approx 0.333$$

$$\boldsymbol{Formula}=16\arctan\left(\frac{1}{5}\right)-4\arctan\left(\frac{1}{239}\right)$$

$$\mathbf{16}\otimes\boldsymbol{fl}\left(\mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{5}}\right)\right)\approx 3.152$$

$$\mathbf{4}\otimes\boldsymbol{fl}\left(\mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{239}}\right)\right)\approx 1.332$$

$$\boldsymbol{fl}\,(\mathbf{16}\otimes\ \mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{5}}\right))\ \ominus\boldsymbol{fl}\,(\mathbf{4}\otimes\ \mathbf{arctan}\left(\frac{\mathbf{1}}{\mathbf{239}}\right))\ \approx 1.82$$

$$\boldsymbol{p}^*\approx 1.82$$
$$\boldsymbol{p}=\pi$$

$$\boldsymbol{error_{abs}}=|\boldsymbol{p}-\boldsymbol{p}^*|$$

$$\boldsymbol{error_{abs}}=|\pi-1.82|$$

$$\boldsymbol{error_{abs}}=1.322$$

$$\boldsymbol{error_{relativo}}=\left|\frac{\boldsymbol{p}-\boldsymbol{p}^*}{\boldsymbol{p}}\right|,\boldsymbol{p}\neq\mathbf{0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\pi - 1.322}{\pi}\right|$$

$$\boldsymbol{error_{relativo}} = 0.421$$

6. **El número e se puede definir por medio de $\boldsymbol{e = \sum_{n=0}^{\infty}\left(\frac{1}{n!}\right)}$, donde $\boldsymbol{n! = n(n-1)\cdots\ 2\cdot 1}$ para $\boldsymbol{n \neq 0}$ y 0! = 1. Calcule los errores absoluto y relativo en la siguiente aproximación de e:**

a. $\boldsymbol{\sum_{n=0}^{5}\left(\frac{1}{n!}\right)}$

Calcular

$$\sum_{n=0}^{5}\left(\frac{\boldsymbol{1}}{\boldsymbol{n}!}\right) = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} = 2.717$$

$$\boldsymbol{p^*} \approx 2.717$$

$$\boldsymbol{p} = e$$

$$\boldsymbol{error_{abs}} = |\boldsymbol{p} - \boldsymbol{p^*}|$$

$$\boldsymbol{error_{abs}} = |e - 2.717|$$

$$\boldsymbol{error_{abs}} = 0.001$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p} - \boldsymbol{p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{e - 2.717}{e}\right|$$

$$\boldsymbol{error_{relativo}} = 4.715x10^{-4}$$

b. $\boldsymbol{\sum_{n=0}^{10}\left(\frac{1}{n!}\right)}$

Calcular

$$\sum_{n=0}^{5}\left(\frac{\boldsymbol{1}}{\boldsymbol{n}!}\right) = \frac{1}{0!} + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \frac{1}{5!} + \frac{1}{6!} + \frac{1}{7!} + \frac{1}{8!} + \frac{1}{9!} + \frac{1}{10!} = 2.718$$

$$\boldsymbol{p^*} \approx 2.717$$

$$\boldsymbol{p} = e$$

$$\boldsymbol{error_{abs}} = |\boldsymbol{p} - \boldsymbol{p^*}|$$

$$\boldsymbol{error_{abs}} = |e - 2.718|$$

$$\boldsymbol{error_{abs}} = 2.818x10^{-4}$$

$$\boldsymbol{error_{relativo}} = \left|\frac{\boldsymbol{p} - \boldsymbol{p^*}}{\boldsymbol{p}}\right|, \boldsymbol{p \neq 0}$$

$$error_{relativo} = \left|\frac{e - 2.718}{e}\right|$$

$$error_{relativo} = 1.037x10^{-4}$$

7. **Suponga que dos puntos (x_0, y_0) y (x_1, y_1) se encuentran en línea recta con $y_1 \neq y_0$. Existen dos fórmulas para encontrar la intersección x de la línea:**

$$x = \frac{x_0y_1 - x_1y_0}{y_1 - y_0} \text{ y } x = x_0 - \frac{(x_1 - x_0)y_0}{y_1 - y_0}$$

a. **Use los datos $(x_0, y_0) = (1.31, 3.24)$ y $(x_1, y_1) = (1.93, 5.76)$ y la aritmética de redondeo de tres dígitos para calcular la intersección con x de ambas maneras. ¿Cuál método es mejor y por qué?**

Resolución

Primera formula

$$x = \frac{x_0y_1 - x_1y_0}{y_1 - y_0}$$

Aritmética de redondeo de tres dígitos

$$x_0 \otimes y_1 \approx 7.546$$

$$x_1 \otimes y_0 \approx 6.253$$

$$(x_0 \otimes y_1) \ominus (x_1 \otimes y_0) \approx 1.293$$

$$y_1 \ominus y_0 \approx 2.52$$

$$\frac{(x_0 \otimes y_1) \ominus (x_1 \otimes y_0)}{y_1 \ominus y_0} \approx 0.513$$

Segunda formula

$$x = x_0 - \frac{(x_1 - x_0)y_0}{y_1 - y_0}$$

Aritmética de redondeo de tres dígitos

$$x_1 \ominus x_0 \approx 0.62$$

$$(x_1 \ominus x_0) \otimes y_0 \approx 2.009$$

$$y_1 \ominus y_0 \approx 2.52$$

$$\frac{(x_1 \ominus x_0) \otimes y_0}{y_1 \ominus y_0} \approx 0.797$$

$$x_0 \ominus \frac{(x_1 \ominus x_0) \otimes y_0}{y_1 \ominus y_0} \approx 0.513$$

La segunda fórmula es más estable numéricamente cuando los productos $x_0 y_1$ y $x_1 y_0$ son muy cercanos, ya que evita la resta de dos números similares que puede causar pérdida de cifras significativas (error de cancelación), lo que hace que no sea menos estable.