**ASSESSMENT ITEM #1 (multiple choice)**

TAGS: Probability, Combinations, Compound Events, Counting Principle, Tree Diagram

COMMON CORE STANDARDS:

CCSS.MATH.CONTENT.7.SP.C.8 Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

CCSS.MATH.CONTENT.HSS.CP.B.9 Use permutations and combinations to compute probabilities of compound events and solve problems.

Tonysha has two bags. Each bag has three objects in it. The first bag has one dime, one nickel, and one penny. The second bag has one red button, one blue button, and one yellow button. Tonysha picks one object from each bag. How many possible combinations of two objects can she pick?

a) 3
b) 6
c) 9
d) 15

DESCRIPTION:

A multiple choice question in which students are asked to calculate the total number of combinations possible from two independent events.

EXPLANATION:

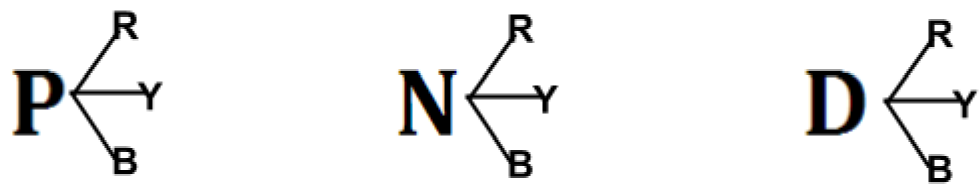
This problem is asking you to calculate the probability of a compound event. A compound event is made up of two or more independent events. In this case, the choice of a coin and the choice of a button. These events are considered independent because the choice of a coin has no impact on the choice of a button.

If we know all of the possible outcomes of each independent event we can calculate the number of possible combinations. This is known as the counting principle.

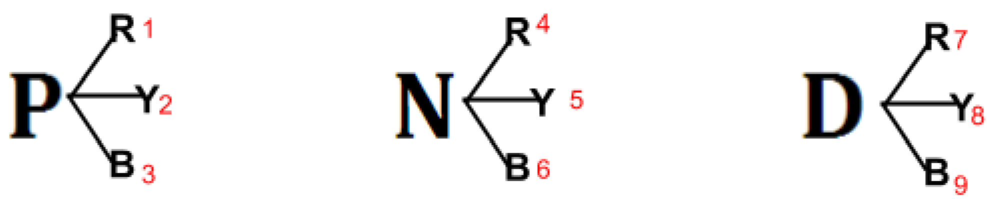
**Graphically** we can represent compound events in a tree diagram. To create a tree diagram we choose one event and draw each possibility as a "trunk". In this case let the "trunk"be the choice of a coin where **P** stands for penny, **D** stands for dime and **N** stands for nickel. (**note when calculating combinations of independent events it does not matter what order the events are graphed.*)

**P N D**

Next we create a "branch" from each of our trunks for all the possible outcomes of our second event, let **R** represent the red button, **B** the blue button and **Y** the yellow button.




If we count the total number of branches coming from all the trunks we will find that there are 9 possible combinations for these two events.




<u>**Mathematically**</u> we can find the answer without drawing a tree diagram. If we multiply the possible outcomes for each independent event we will get the number of combinations. We are told that there are 3 possible choices for coins, (penny, nickel, dime) and 3 choices for buttons, (red, blue, yellow), we can multiply 3 x 3 = 9.

**ASSESSMENT ITEM #2 (free response)**

TAGS: Division, Multiplication, Multiples, Least Common Multiple, LCM

COMMON CORE STANDARDS:

CCSS.MATH.CONTENT.4.OA.A.2 Multiply or divide to solve word problems involving multiplicative comparison.

CCSS.MATH.CONTENT.4.OA.A.3

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

CCSS.MATH.CONTENT.5.OA.B.3 Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane.

CCSS.MATH.CONTENT.6.NS.B.4 Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1-100 with a common factor as a multiple of a sum of two whole numbers with no common factor.

Alice and Carl each have the same total number of marbles. Alice put her marbles into groups of 4 with none left over. Carl put his marbles into groups of 10 with none left over. What is the least total number of marbles that Alice and Carl can each have?

DESCRIPTION: A free response question that asks students to recognize the relationship between division and multiplication and identify the least common multiple of numbers less than 12.

EXPLANATION:
In order to answer this question you must understand what is being asked by paying close attention to the wording. Phrases that include "*put into groups"* and *"none left over*" are usually associated with division, however multiplication will be more helpful in solving this problem.

Numbers that can be divided by a number with nothing left over are known as multiples. Multiples are basically the times tables of a number, for example:

**4 x 1 = 4**
**4 x 2 = 8**
**4 x 3 = 12**
**4 x 4 = 16**
**4 x 5 = 20**

This means that 4, 8, 12, 16, and 20 are multiples of 4. They are not the only multiples, just like the times tables for a number can keep going forever there is no end to a numbers multiples.

Since the number of marbles Alice has can be divided by 4 with nothing left over we know that the number of marbles she has is a multiple of 4. The number of marbles Carl has can be divided by 10 with nothing left over, therefore we know that the amount of marbles he has is a multiple of 10. If we compare both sets of multiples we can find that some numbers appear in both lists.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **Alice** | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 | 36 | 40 |
| **Carl** | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90 | 100 |

*if we kept up the pattern we would see that 60, 80 and 100 are also common multiples of 4 and 10.

Since this problem asks us what the **least** number of marbles that Alice and Carl can each have that means you are looking for the least common multiple (LCM), or the smallest number that can be divided by 4 and 10 with nothing left over.  Looking at the chart above we can see that the LCM is 20.

This means that the least total number of marbles that Alice and Carl can each have is 20.

**ASSESSMENT ITEM #3 (multiple choice)**

TAGS: Integers, Positive, Negative, Subtracting, Number Line

COMMON CORE STANDARDS:

CCSS.MATH.CONTENT.6.NS.C.5

Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.

CCSS.MATH.CONTENT.6.NS.C.6

Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.

The temperature at midnight was 2°F. At sunrise the temperature was 5°F lower. What was the temperature at sunrise?

A) $-7^0$F
B) $-3^0$F
C) $3^0$F
D) $7^0$F

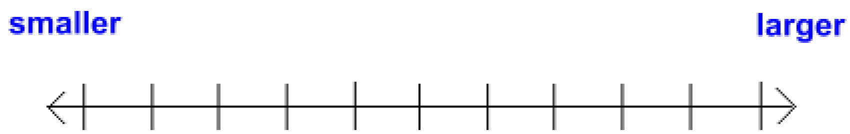
DESCRIPTION:

A multiple choice question that asks students to subtract two integers resulting in a negative value.

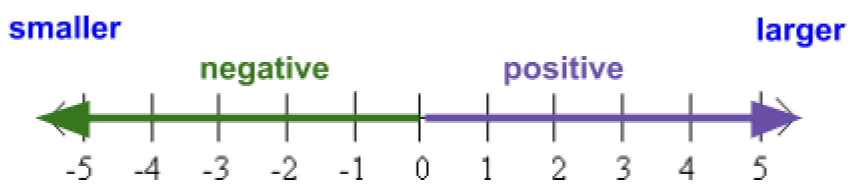
EXPLANATION:

This question gives you a temperature at one time and asks you what the temperature will be after a change.  First we must recognise that if the temperature at sunrise is 5°F **LOWER** that means that the temperature has gone down, or that it will be **LESS** than it was at midnight.  With that understanding alone you can eliminate answer choices C and D, because both 3°F and 7°F are **GREATER** or **MORE** than the original temperature of 2°F.

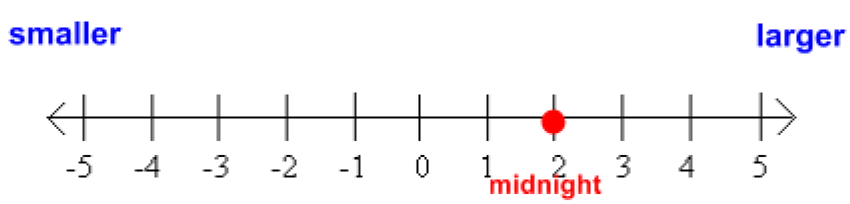
To illustrate what is happening in this problem it is helpful to draw a number line. A number line is a line that extends infinitely to both the left and the right and can be used for comparing numbers. The most important rule of the number line is when you go to the right on the number line the numbers get larger, when you go to the left they get smaller.



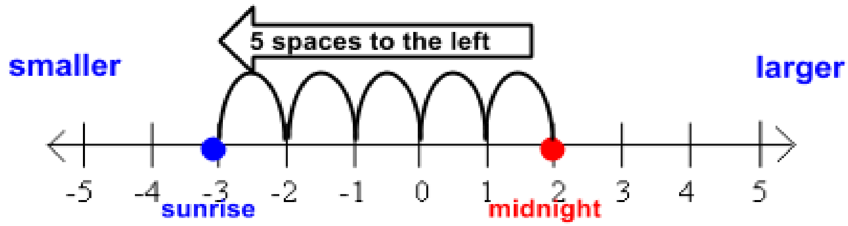

The number line extends forever in both directions and doesn't ever stop, not even at zero. Numbers smaller than zero, or to the left of zero on the number line, are known as negative numbers. All the numbers greater than zero and to the right of zero on the number line are positive numbers.




In this problem we are told that at midnight the temperature is 2°F, let's mark that on our number line.




Next we are told that the temperature at sunrise is 5°F lower than the temperature at midnight. Remembering that lower means smaller and smaller is left on the number line we know that if we start at 2°F, the temperature at midnight, and move 5 spaces to the left we will land on the temperature at sunrise.




The number line tells us that the temperature at sunrise is -3°F.

**<u>Mathematically</u>** you can solve this problem if you are familiar with integers. Integers are all the positive whole numbers, all the negative whole numbers and zero. When the problem says that the temperature is 5°F lower than 2°F that is telling you to take away 5°F from 2°F, take away is another way to say subtraction.

**2°F - 5°F = ?**

It is important that you write the number as they are described in the problem and in the correct order. Since 5°F is larger than 2°F your answer is going to be less than zero or a negative number.

**2°F - 5°F = -3°F**

**ASSESSMENT ITEM #4 (multiple choice)**

TAGS: Parts of a Whole, Fractions, Equivalent Fractions, Numerator, Denominator, Data Table, Tally Mark, Greatest Common Factor, GCF COMMON CORE STANDARDS:

CCSS.MATH.CONTENT.1.MD.C.4 Organize, represent, and interpret data with up to three categories; ask and answer questions about the total number of data points, how many in each category, and how many more or less are in one category than in another.

CCSS.MATH.CONTENT.3.NF.A.1 Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into $b$ equal parts; understand a fraction $a/b$ as the quantity formed by $a$ parts of size $1/b$.

CCSS.MATH.CONTENT.3.NF.A.3.B Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

Diane tossed a coin 20 times. She recorded whether the coin landed heads up or tails up. The results are shown in the table below.

What fraction of the coin tosses landed tails up?

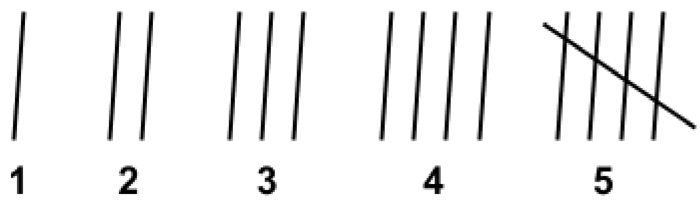
a) 1/8
b) 2/3
c) 2/5
d) 3/5

**COIN TOSSES**

| Position | Number of Times |
| --- | --- |
| Heads up | 𝍸 𝍸 𝍷𝍷 |
| Tails up | 𝍸 𝍷𝍷𝍷 |

DESCRIPTION:
A multiple choice question that asks students to read data from a tally chart, create an appropriate fraction and then reduce that fraction to its simplest form.

EXPLANATION:
The information that is given for this problem is given in a data table, data tables are ways that we can share information. In this data table information is recorded as tally marks, tally marks are a way of counting. Each vertical slash equals one as you count you continue to make vertical slashes until you get to five. When you get to five you "bundle" your tallies with a diagonal slash.




If you count the tallies for heads you see two bundles and two slashes which adds up to 12. For tails up there is one bundle and three slashes which adds up to 8.

If you look at the answer choices for this problem you will see that they are all fractions. Fractions represent parts of a whole where the part we are interested in is in the numerator, or top of the fraction and the number of parts in the whole is written in the denominator, or bottom. In this case the whole is going to be the total number of coin flips and the since the questions asks about tails the part we are interested in is going to be the number of tails.

$$\frac{\text{Number of tails}}{\text{Total number of coin flips}}$$

To find the total number of parts in the whole we need to add up all of the outcomes, all of the tails and all of the heads. We can see from our slashes that there are 12 heads and 8 tails, if we add these numbers up we can see that there are 20 total coin flips. That makes our denominator 20.

$$\frac{\text{Number of tails}}{20}$$

Our numerator is going to be the number of tails, which after counting our tallies we know to be 8.

$$\frac{8}{20}$$

Unfortunately 8/20 is not an answer choice. In order to match our answer with one of the choices we need to find an equivalent fraction to 8/20. An equivalent fraction is a fraction that has the same value another fraction but has a different numerator and denominator. One way to find an equivalent fraction is to identify the greatest common factor (GCF), or largest number that can divide both the numerator and denominator evenly and divide each by that number. With a numerator of 8 and a denominator of 20 the greatest common factor is 4. Since 8÷ 4 = 2 and 20 ÷ 4 = 5, we know that:

$$\frac{8 \div 4}{20 \div 4} = \frac{2}{5}$$

Which is the correct answer to this problem is C $\frac{2}{5}$.

**ASSESSMENT ITEM #5 (multiple choice)**

TAGS: Imaginary Unit, Complex Numbers, Distributing Binomials, Combine Like Terms

COMMON CORE STANDARDS:

CCSS.MATH.CONTENT.HSN.CN.A.1 Know there is a complex number *i* such that $i^2 = -1$, and every complex number has the form *a* + *bi* with *a* and *b* real.

CCSS.MATH.CONTENT.HSN.CN.A.2 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

What is the product of $5 + \sqrt{-36}$ and $1 - \sqrt{-49}$, expressed in simplest $a + bi$ form?

(1) $-37 + 41i$ (3) $47 + 41i$

(2) $5 - 71i$ (4) $47 - 29i$

DESCRIPTION:

A multiple choice question where students are asked to distribute binomials containing imaginary numbers and simplify by combining like terms.

EXPLANATION:

This problem is asking you to find the product of, or result of multiplying two complex numbers. A complex number is a number that can be expressed in the form a + bi, where a and b are real numbers and i is the imaginary unit, which satisfies the equation $i^2 = -1$.

Since all of our answer choices are in a + bi form it is helpful to convert both factors to that form. If $i^2 = -1$ then $\sqrt{-1} = i$. Using the following steps we can convert our factors into a+ bi form:

| | | **1. factor $\sqrt{-1}$ out of the radical** | | **2. convert $\sqrt{-1}$ to *i*** | | **3. simplify the radical** | | **4. rewrite in a+b*i* form** |
|---|---|---|---|---|---|---|---|---|
| $5+\sqrt{-36}$ | = | $5+\sqrt{36}\times\sqrt{-1}$ | = | $5+\sqrt{36}\times i$ | = | $5+6\times i$ | = | $5+6i$ |
| $1-\sqrt{-49}$ | = | $1-\sqrt{49}\times\sqrt{-1}$ | = | $1+\sqrt{49}\times i$ | = | $1-7\times i$ | = | $1-7i$ |

Even in a+b*i* form our factors are binomials, meaning they have 2 terms separated by addition or subtraction. In order to multiply them we must follow the procedure of distributing binomials.

$$(5+6i)(1-7i) = 5(1-7i) + 6i(1-7i)$$

$$5 - 35i + 6i - 42i^2$$ and since $i^2 = -1$

$$5 - 35i + 6i - 42(-1)$$

or

$$5 - 35i + 6i + 42$$

To combine like terms we combine everything that has the same variable raised to the same power.

$$\overbrace{5 - \underbrace{35i + 6i}_{-35i+6i} + 42}^{5+42}$$

After simplifying we are left with the answer D $47 - 29i$.

**ASSESSMENT ITEM #6 (multiple choice)**

TAGS: Ellipse, Major Axis, Geometric Equation

COMMON CORE STANDARDS:

CCSS.MATH.CONTENT.HSG.GPE.A.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

A commercial artist plans to include an ellipse in a design and wants the length of the horizontal axis to equal 10 and the length of the vertical axis to equal 6. Which equation could represent this ellipse?

(1) $9x^2 + 25y^2 = 225$ (3) $x^2 + y^2 = 100$

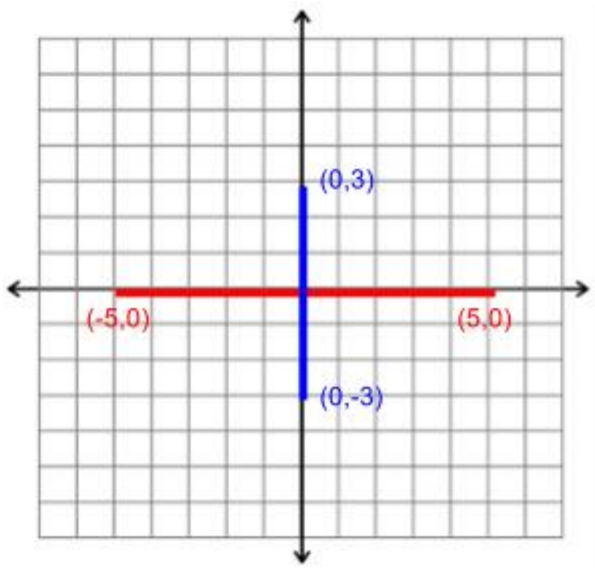
(2) $9x^2 - 25y^2 = 225$ (4) $3y = 20x^2$

DESCRIPTION:

A multiple choice question that gives students the lengths of the major and minor axis of an ellipse and asks them to derive the equation of the ellipse that corresponds to the given measurements.

EXPLANATION:

To solve this problem it is helpful to graph the given information. By centering the ellipse at the origin you can get valuable information about this problem by making a simple sketch. A horizontal line that is 10 units long and is centered at the origin will stretch from (-5,0) to (5,0) and a vertical line centered at the origin and measuring 6 units the origin will go from (0,3) to (0,-3).




Now we can see that the major axis, or the longer axis is horizontal. The formula for any point on an ellipse with a horizontal major axis centered at the origin is given by the geometric equation:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Where $a$ is half the length of the major axis and $b$ is half the length of the minor axis. In this case $a = \frac{10}{2}$ or 5 and $b = \frac{6}{2}$ or 3. This makes our equation:

$$\frac{x^2}{5^2} + \frac{y^2}{3^2} = 1$$

or

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

We can simplify this equation by multiplying both sides by the denominators of the fractions:

$$25\left(\frac{x^2}{\cancel{25}} + \frac{y^2}{9}\right) = 25(1)$$

$$9\left(x^2 + \frac{25y^2}{\cancel{9}}\right) = 9(25)$$

$$9x^2 + 25y^2 = 225$$

That makes A $9x^2 + 25y^2 = 225$ the equation for this ellipse.