Comparison between various models of opinion spread dynamics

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In recent years the field of opinion spread dynamics has become very important. Public opinion makes a great difference to politics, economics and many other fields. The behavior of individual in the society is susceptible to the influence of public opinion. This opinion is triggered by social events, mass media, group leaders etc. Thus in order to control the spread of negative public opinion, we need to understand the mechanism of formation and propagation of public opinion. Each individual always hold his attitude or opinion for any event. His opinion might change after interaction with his peers. Here we will try to cover some classical models used by various socio physicist to model opinion spread dynamics, how the modeling parameters and communication mechanism affects the whole model and a detailed analysis of the results of every model.

INTRODUCTION

Agreement is one of the most important aspect of social dynamics.[2] Public opinion makes a great difference in the field of politics economics etc.Change in opinions is an important study in the field of social dynamics. Each individual has an opinion about something initially, which may change after the individual communicates with his neighbours, or when he sees some propogandas in T.V. or hear it on the radio or by any other means of mass media. The behaviour of an individual depends on his opinion about a matter, and is also susceptible to change depending on other's public opinion with whom he communicate at various social event, or even on one-on-one basis. This communication can trigger a change in his view about a matter and hence his opinion can sway in some other direction after some time.[4]In recent years, we have witnessed profound changes in social and political values in very short periods of time. In this work we aim to understand how new opinions propagate and when people come to an agreement.[7] In this article we have made a summary of various opinion spread models focusing on model parameters, how communication takes place among individuals and their network topology. We would like to see how an initial disorder among society changes to an order over the span of time. Order here means consensus, universality, agreement among individuals whereas disorder means fragmentation in the society and also disagreement. Repeated interactions among individual can lead to homogeneity in the society. In this article we have studied five different models i.e. Voter Model,Majority Rule Model,Deffuant Model, Hegselmann-Krause model, Sznajd model.

Cellular automata basically is a grid of various numerical values.Each point in the automata is called a cell.The value of the cell represents its state at a given time.We assign initial values to the entire grid and than see how this values changes with time depending on the rules of interaction.We determine the rules depending on the current state of the cell and its neighours. There are two ways to consider the neighbours. Moore neighbourhood in which there are eight neighbours and Neumann neighbourhood in which there are four neighours.

MODELS FOR OPINION SPREAD DYNAMICS

- **Voter Model** [2][3][5]

In this model, the complete neighbourhood influence is not considered as with most of the neighbour dependent opinion spread models. Here we choose one person at random, and then give him the opinion of only one of his neighbours. In 1-D model, we assign opinion to a cell equal to either next or previous cell. For 2-D model, we have used cellular automata with moore neighbourhood so the influencing neighbour is chosen amongst the eight neighbours. It is based on ising model. It is non-equilibrium stochastic process. Hence it sometimes results in unstable state i.e. some cells continuously changes their opinion.In a finite model, the system always reaches the absorbing state.

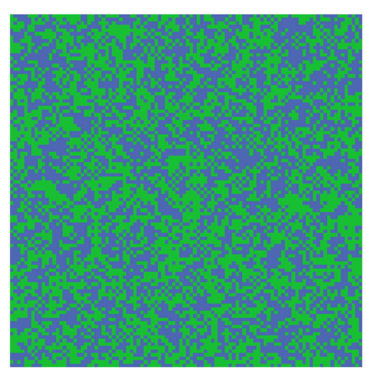

Figure 1. Opinion distribution for Voter model at time t = 1 and population size $100*100 = 10000$ where green color is for +1 and blue color is for -1 .+1,-1 two opinions in the system.

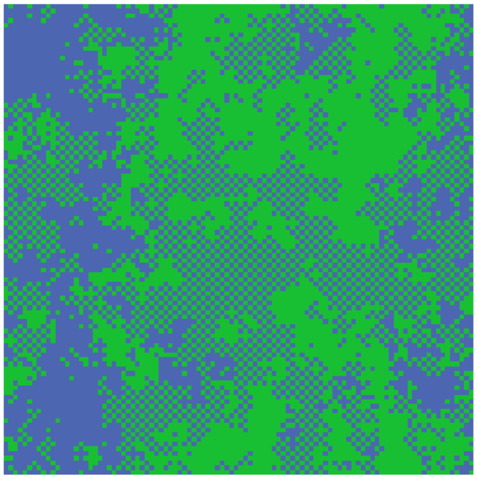

Figure 2. Opinion distribution for Voter model at time t = 1000 and population size $100 * 100 = 10000$ where green color is for +1 and blue color is for -1 .+1,-1 two opinions in the system.

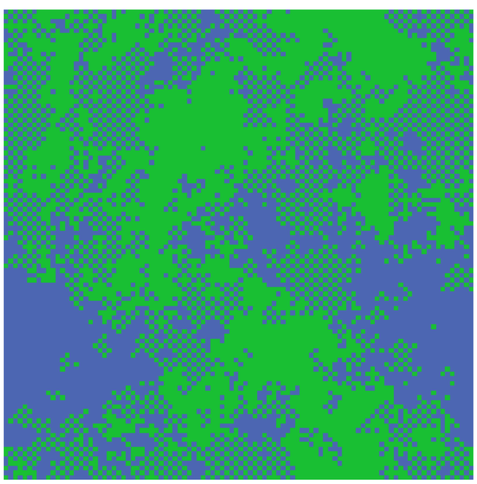

Figure 3. Opinion distribution for Voter model at time t = 2000 and population size $100 * 100 = 10000$ where green color is for +1 and blue color is for -1 .+1,-1 two opinions in the system.

Now we can see from the above figures that initially (Figure 1) the opinion is scattered with very small cluster of same opinion. Now if we see in the next two figures (Figure 2,3) , the cluster are starting to form. Now there are cluster of larger size. If we continue this for a much larger time, we would see the cluster vanishing and everyone would have the same opinion.

- **Majority Rule Model** [2][3][5]

  There are N agents in this model. In our 2-D model N = nxm where n is the number of rows and m is the number of columns. There are binary opinion among the individuals. A fraction p+ of agents has initial opinion +1 whereas a fraction p- = 1 - p+ of agents has initial opinion as -1. Now here we assume that each of the agents can communicate with each of the other agent in the system, hence there is a complete graph. Now at each iteration, we take a small group of people of size r(odd). The group is selected as random and is called the discussion group. We find the majority opinion in the group. Now after the interaction all the member of the discussion group take the majority opinion of the group. We took r as odd so that there is always a majority among the group. The entire group of agents (i.e. all the N agents) will reach to a consensus(agreement) in the long run. So the fragments(small groups of different opinion) which we see initially will be removed and there will be a single opinion in the whole dynamics. Now we can also choose r to be even in which case there may b a tie among the group members chosen. Each opinion is supported by r/2 agents . A way to resolve this tie is to be biased towards a opinion and hence that opinion vote will be r/2 + 1 which will then become a majority and the group of people will accept that opinion. This is because in a case of tie, people are reluctant to change their opinion as there is no clear majority and hence we include a bias towards one of the opinion.

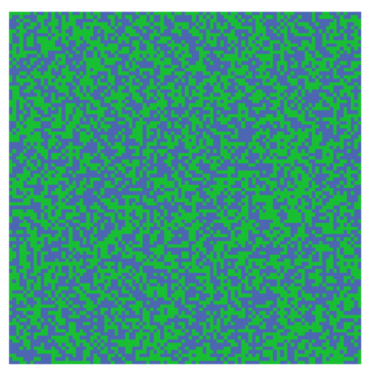

Figure 4. Opinion distribution for Majority rule model at time t = 1 and population size $100 * 100 = 10000$ where green color is for +1 and blue color is for -1 .+1,-1 two opinions in the system.

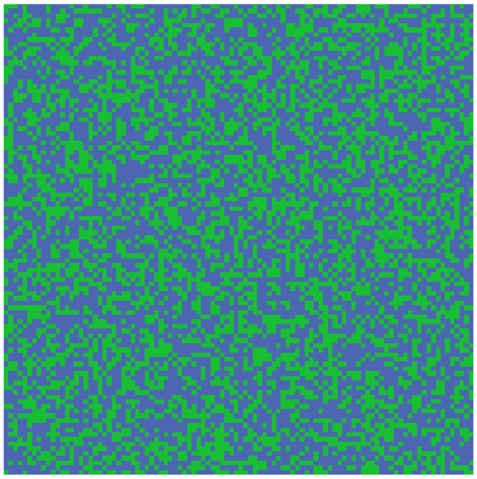

Figure 5. Opinion distribution for Majority rule model at time t = 1500 and population size $100 * 100 = 10000$ where green color is for +1 and blue color is for -1 .+1,-1 two opinions in the system.

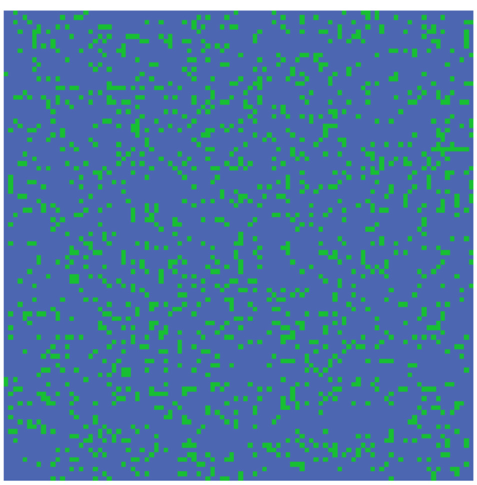

Figure 6. Opinion distribution for Majority rule model at time t = 3000 and population size $100 * 100 = 10000$where green color is for +1 and blue color is for -1 .+1,-1 two opinions in the system.

The abouve figures shows how the opinion in an area changes with time. Initially (Figure 4 both the opinions have almost the same number of people. Now the next two figures (Figure 5,6 shows the change. We can see that after a long time one opinion will dominate and if e run it for further time,all the person would have the same opinion irrespective of what opinion they had initially.

- **Deffuant Model**[2][3][5]

The continuous opinion models reflect the amount of confidence each individuals have in a particular opinion. In models of two extreme opinions, all the values between the two extreme values indicates the amount of confidence an individual posses towards a particular opinion. These models try to reflect the ever changing dynamics of human mind towards general issues and various opinions. These models are known as bounded confidence models.

Deffuant model is a bounded confidence model. Here we randomly assigned real values between 0 to 1 to a population of N individuals. Here 0 and 1 represent two extreme opinions. We have used a network of N nodes to simulate this model. Each node corresponds to an individual and all the nodes connected to it represents its neighbours. At each instant we choose a random node. Then we choose one of its neighbours. Then both of these nodes will change their values according to the following equations only

if $|x_i(t) - x_j(t)| < \epsilon$

$$x_i(t+1) = x_i(t) + \mu * (x_j(t) - x_i(t))$$
$$x_j(t+1) = x_j(t) + \mu * (x_i(t) - x_j(t))$$

Here ϵ is a threshold parameter which indicates that two interacting individuals can influence one another only if their difference of opinions is considerably less. So threshold parameter represents the extent of interactions occurring between individuals. u is called the convergence parameter which indicates the amount of influence each individual has on the person he is interacting. It can be different for different individuals and so is assigned at the beginning. However in our implementation, we have taken u to be constant throughout the population. For any value of and u, the average opinion of the agent's' pair is the same before and after the interaction, so the global average opinion (half in our case) of the population is an invariant of Deffuant dynamics. A stable state is easily reached in deffuant model since opinion groups will be formed with opinion of boundary individuals will be within threshold parameter and would gradually increase as we move towards the center. Thus in stable state, almost all the neighbours will have opinion differences within threshold parameter. Hence the value of threshold parameter affects the time to reach stable state:- lower the parameter, higher the time required to reach the stable state and vice versa.

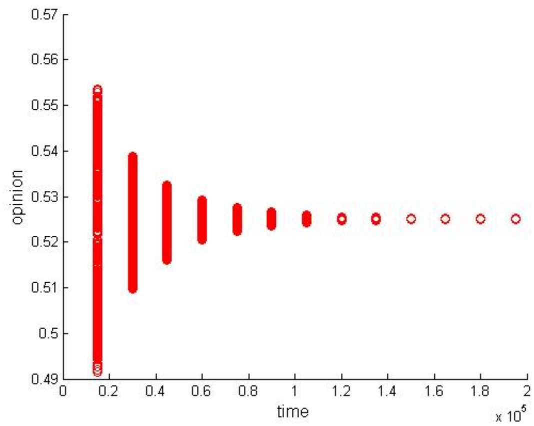

Figure 7. Opinion distribution for grid based Deffuant model for *population-size* $= 1000$, $\epsilon = 0.5$ and $\mu = 0.5$

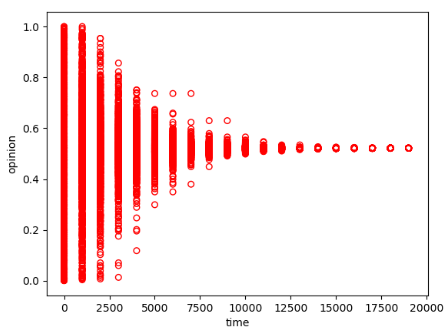

Figure 8. Opinion distribution for complete graph based Deffuant model for *population-size* $= 1000$, $\epsilon = 0.5$ and $\mu = 0.5$

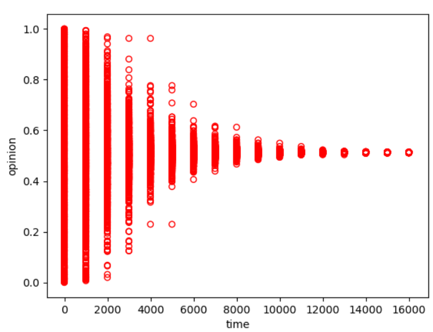

Figure 9. Opinion distribution for Erdos-Renyi graph based Deffuant model for *population-size* $= 1000$, $p = 0.4$, $\epsilon = 0.5$ and $\mu = 0.5$

The figures (7,8,9) shows the distribution of opinion with time. We can see that iniially lots of opinion prevail, but with time they tend to take up a common opinion and with the passage of time only one opinion will prevail which can be seen at a large value of time

- **Hegselmann-Krause model (or HK model)**[2][3][7]

  This model is similar to deffuant model except for consideration of neighbours and update rule. Here all the neighbours are taken into consideration. The average of all its neighbours with $|x_i(t) - x_j(t)| < \epsilon$. is assigned to the chosen node. So the update rule is as follows

  $x_i(t+1) = mean(x_j(t))$
  for all neighbours j such that $|x_i(t) - x_j(t)| < \epsilon$.

  Here however convergence parameter is absent which implies that a person will converge to cumulative average opinion of his interacting neighbours regardless of his own opinion. This is rather a crude extension to deffuant model for real world opinion spread dynamics.

  Deffuant model is used to simulate opinion spread dynamics over a large population where some people are not influenced while other influence and are influenced by their neighbours. HK model takes into consideration a small faction of discussion group where opinion of all the individuals present affect the cumulative opinion. It considers only the interaction within groups and not of individuals. Hence it can only be applied to a small fraction of population because a person does not gather everyday with his peers only to discuss a particular opinion. As in Deffuant model, HK model also reaches consensus.

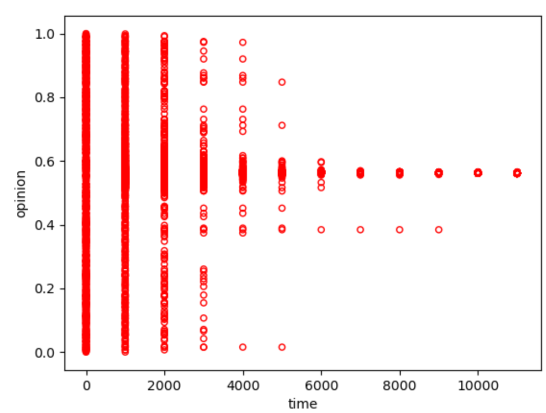

Figure 10. Opinion distribution for Erdos-Renyi graph based Hegselmann-Krause model for *population-size* $= 1000$, $p = 0.4$, $\epsilon = 0.5$ and $\mu = 0.5$

Here we can see a similar distribution of opinion as Deffuant. Initially there are a lot of opinions but with time they move toward a one common opinion.

- **Sznajd Model** [2][1][8][5][6]

An empirical understanding of opinion spread model would be that an individual would believe in a certain opinion only if two or more people around him holds that opinion. This was clear from all the previous model. In all those models we can certainly say that rate of opinion spread increased with increasing group size. Hence convincing somebody is easier for two or more people than for a single individual. This is the basic principle behind sznajd model. We have implemented the 1-D version of this model. We arranged the population of size N in sequential order. All the individuals we assigned 0 or 1 value. We chose a node i at random. Then we compared it with its neighbour i+1. Based on this comparison, we update the opinion value of i-1 and i+1 as follows:-
if
$s_i(t) = s_{i+1}(t)$ then
$s_i(t) = s_{i+1}(t) = s_{i-1}(t) = s_{i+2}(t)$

else
$s_{i-1}(t) = s_{i+1}(t) and s_{i+2}(t) = s_i(t)$

Hence as seen above, in 1D Sznajd model, the flow of information through network is represented. 1D Sznajd model also reaches stable state. The entire array of nodes forms an approximate partition of each opinion i.e. almost all individuals in that partition will have same opinion. For simulation part first we define the magnetization of our spin system, which in this case corresponds to the general opinion of the society. We give the definition both in the language of the voting society and in magnetic language:

m = *(number of A votes - number of B votes)* / N
= 1 / N $\sum_{n=i}^{N} S_i$

Where N is number of people in society. Here we ultimately interested in whether opinion A or opinion B dominates the society. It is clear that after infinite time the magnetization should evolve to either -1,0,1, Since final state of system in either all spin down, all spin up or equal number of up and down spin. In other words, system should evolve to one of the fixed point.

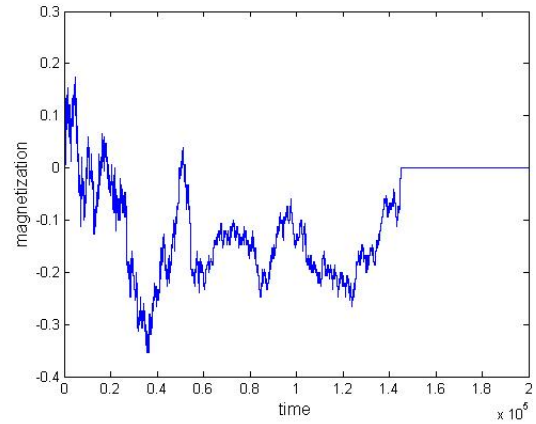

Figure 11. Time evolution of consensus in terms of magnetization

Now another point to think about is that 'Why would an individual oppose its neighbouring opinion just because its neighbouring does not match with its own neighbour?' So it becomes more logical to assume an individual stays with its opinion in such a case. So we get a new set of rules.
if
$s_i(t) = s_{i+1}(t)$ then
$s_i(t) = s_{i+1}(t) = s_{i-1}(t) = s_{i+2}(t)$

else
do nothing.

In the above image, the y-axis shows the polarization i.e. the difference between the number of opinion with respect to time. We see that initially there is some difference between the number of opinion but as we move ahead in time, it tends to change depending on the interacting condition we have taken, and after some time there will be equal number of both the opinions. This will happen when none of the neighbours have the same opinion. The 1-d lattice will look like this A,B,A,B,.. where A and B are two different opinion.

COMPARISON BETWEEN VARIOUS MODELS OF OPINION SPREAD

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- **Communication Method**

  Voter model is based on communication with the neighbours for a particular person. We choose a person whose opinion and then select one of its neighbour, and now the person has the same opinion as his/her neighbour. In deffuant model the person interacts with its neighbour as well, but their opinions change in a different manner. A person selects one of its neighbour and then sees how close their opinions are, they would then change their opinion only if this difference is below a threshold. Hegselmann-Krause model is similar to deffuant in terms of how it communicate except one small difference that instead of communicating with just one neighbour, it communicates with all the neighbours whose opinion varies from his opinion below a threshold. It then takes up the mean of all this neighbors' opinions as its own. In Sznajd , the opinion of a person changes depending on whether the opinion is hold by one or more people around it. More the people with same opinion, more easily a person will come round to that opinion. In the majority rule model we form a small group of people, who then see which opinion is in majority among the group, and then all of the member of the group take up the majority opinion.

- **Types of Model**

  - Discrete: Models in which there are only a finite number of opinions i.e. true,false,-1,+1,-1,0,1,Left,Center,Right etc. We have modeled the Voter Model,Majority Voter Model and Sznajd as discrete models
  - Continuous: Models in which the value varies between two points, and cn take any value between them i.e. [0,1] , [-1,1].

    We have modeled the deffuant and In Hegselmann-Krause model as continuous models.

    Models of changes in discrete variables are invariably stochastic because changes by jumps almost dictate reference to probabilities. By contrast, continuous-time models of changes in metric variables are typically deterministic. Continuous model does not follow some of the concepts of discrete models like majority of an opinion, equality of an opinion. In discrete opinions a lot of the agents start with the same opinion, where as in continuous models most of the agents have a different starting value.

- **Topology Used**

  - Voter Model: 2-D Grid. Voter model is one of the basic model that exist . Neighbour interact with one of its neighbour and form its opinion depending on its neighbours opinion. Hence to implement this scenario we have taken a 2-D model where the neighbour can communicate with each of its eight neighbours
  - Majority Voter Model: 2-D Grid. In this model a small group of random size is formed, and all the members take up the opinion which is the majority in the group. WE have not used 1-D Grid because it becomes analytically insolvable , and the probability to reach a consensus depends on the initial distribution of opinion. There is also a chance that a minority wins.

    Both this 2-D Grid are the basic version for the models mentioned above.
  - Sznajd Model : 1-D Grid . A very basic model is implemented here and that is why we have used a 1-D Grid for this model. Since the agent i and i+1 influence the opinion of i-1 and i+2, it becomes more reasonable to consider a 1-D Grid for the model.
  - Deffuant:
    * 2-D Grid. This is one of the basic models for deffuant. Here the number of neighbours are fixed and an agent can only react with a given number of people.
    * Complete Graph: This is one of the variations in the deffuant model. Here each person can interact with each of the other person. This type of model is generally used for populations where there is a small community and they form a small cluster where each person is connected with each of the other person.
    * Erdős–Rényi model: This is also a network model. This is similar to a random graph where there is an equal probability of each of the edge being present or absent. There are a fix number of edges in the graph. We use probability to find the presence or absence of an edge. All the possible graphs for a given number of vertex and a given number of edge are equally likely. This is used when some people in a network has more connections while some in the network have a few connections. This is use to denote varying number of available interactions for a particular agent. For example, A highly social person will have a large number of connections where as a person living in

a very remote area will hardly have any connections.

– Hegselmann-Krause Model: Erdős–Rényi model: To provide different number of interactions to different people we have used this model.

- **Time to reach consensus**

In voter model only two agents interact at a given time and only one agent among them changes its opinion. So this process is quite slow. Same is the case for deffuant model, as only two person interact in this model as well. Hence this being a slow process as well. Now in majority rule model, we manually define small group of people who interacts among the group and take up the opinion which is the majority. Hence a large number of person change opinion at a given time. In Hegselmann-Krause model an agent interact with all its neighbours at a given time and hence it expedites the process of reaching the consensus. Hence Voter model , deffuant takes much more time to reach a consensus with respect to Majority rule model and In Hegselmann-Krause model.

CONCLUSION

We have used two implementation methods - Grids and Networks, to implement these opinion model. Grids have a fixed number of neighbours for each node. Networks can have different number of neighbours for each nodes. Most of these models reached stability and hence can be used to model simple opinion spread dynamics. However when a model reaches stability, it has been assumed that it will remain in stable state for infinite time. This is however not the case in real world because opinions change continuously in the information age. These models do not take into consideration change and emergence of new opinions. So these models can be used to model short-span opinion spread dynamics. The result obtained from continuous and discrete models are almost similar for a particular scenario of opinion spread. However continuous model more clearly determines confidence of an individual towards an opinion. Such models cannot be used in election opinions but can be used in modelling rumor spread across internet. It is due to this very reason that we have used static networks to analyse opinion spread across the internet. As interactions of a person on internet with his peers does not changes erratically we have used static networks. Dynamic networks have been used for modelling opinion spread through group interactions. Here interactions of a person at various places have been considered. As complexity increases, more and more real world constraints can be included. As we move from simple models like voter and Majority Rule to complex network models like HK and Deffuant we can make more comprehensive analysis of opinion spread dynamics in real world.

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