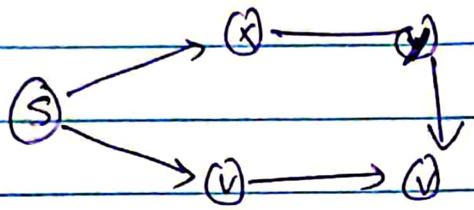
Q-2 In the Proof, we saw that Dijkstra's algorithm finds the shortest path $P_v$, from source $s$ to vertex $v$ for all vertices in $S$

Now, Path $P_v$ is the algorithm's shortest path after the addition of vertex $u$ in $S$




Consider, an alternative path $P_v'$

$P_v'$ has its first edge $(x,y)$ that crosses the cut $(S, V \setminus S)$.

$\therefore$ Now, $w(P_v') \geq \delta(s,x) + w(x,y)$

$= d[x] + w(x,y)$

$\geq d[u] + w(u,v)$

$= \delta(s,u) + w(u,v)$

Here, we see the algorithm will abandon the $x \to y \to v$ route as the route from $u \to v$ has a smaller dist[] value that $(x,y)$ route

But in case of negative edge weights; if the edge $(y,v)$ has a large enough negative edge weight such that :—

~~Path s → x → y → v has~~

$$d[v] + w(y,v) < d[u] + w[u,v],$$

then the algorithm will have failed to find the correct path, as it has already abandoned this path

Q-3 To Prove: Every even graph decomposes into cycles

Proof (By Induction): Let the graph be denote $G(V,E)$

- Base case: $E=0$. For 0 edges it forms a trivial case. Thus cycle decomposition does exists

- IH: Suppose, for all even graphs for $< k$ ($E<K$) edges there exists a cycle decomposition

- IS: Now, G has $k$ edges. ($E=K$)

Now, suppose $H$ is a subgraph of $G$ such that $\deg(V(H)) > 0$. i.e. ignore all isolated vertices.

$\therefore$ $H$ is also even graph with all vertices with degree $\geq 2$.

$\therefore$ There is atleast a cycle $C$ in $H$.

Take, $G' = \underbrace{G}_{\text{even}} \setminus \underbrace{C}_{\text{even}}$ $\therefore$ $G'$ is also an even graph.

However, $G'$ is an even graph with $< k$ edges.

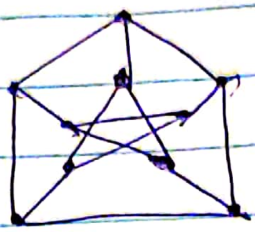
$\therefore$ By IH, $G'$ has a cycle decomposition

$\therefore$ $G = G' \cup C$ is also a cycle decomposition

Thus, by PMI, Every even graph decomposes into cycles. $\blacksquare$

Q-4 a) As the subgraph induced is a bipartite graph, the chromatic number of G is 2

b)



The graph is not 2-colorable as it's not a bipartite graph.

It's a 3-colourable graph