Q1. (8 points) Using propositional resolution, show the following propositional sentence is unsatisfiable.

```
(p | q | -r) & ((-r | q | p) -> ((r | q) & -q & -p))
```

$(p \vee q \vee \neg r) \wedge [(\neg r \vee q \vee p) \rightarrow ((r \vee q) \wedge \neg q \wedge \neg p)]$

$\rightarrow (p \vee q \vee \neg r) \wedge [\neg(\neg r \vee q \vee p) \vee ((r \vee q) \wedge \neg q \wedge \neg p)]$

$\rightarrow (p \vee q \vee \neg r) \wedge [(r \wedge \neg q \wedge \neg p) \vee ((r \vee q) \wedge \neg q \wedge \neg p)]$

$\rightarrow (p \vee q \vee \neg r) \wedge \{[(r \wedge \neg q \wedge \neg p) \vee (r \vee q)] \wedge$
$[(r \wedge \neg q \wedge \neg p) \vee \neg q] \wedge [(r \wedge \neg q \wedge \neg p) \vee \neg p]\}$

$\rightarrow (p \vee q \vee \neg r) \wedge \{[(r \vee q \vee r) \wedge (r \vee q \vee \neg q) \wedge (r \vee q \vee \neg p)]$
$\wedge [(\neg q \vee r) \wedge (\neg q \vee \neg q) \wedge (\neg q \vee \neg p)]$
$\wedge [(\neg p \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg p \vee \neg p)]$

$\rightarrow (p \vee q \vee \neg r) \wedge \{[(r \vee q) \wedge (r) \wedge (r \vee q \vee \neg p)]$
$\wedge [(\neg q \vee r) \wedge (\neg q) \wedge (\neg q \vee \neg p)]$
$\wedge [(\neg p \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg p)]$

$\rightarrow (p \vee q \vee \neg r) \wedge (r \vee q) \wedge r \wedge (r \vee q \vee \neg p) \wedge (\neg q \vee r)$
$\wedge \neg q \wedge (\neg q \vee \neg p) \wedge (\neg p \vee r) \wedge (\neg p \vee \neg q) \wedge (\neg p)$

$\rightarrow \{p, q, \neg r\}, \{r, q\}, \{r\}, \{r, q, \neg p\}, \{\neg q, r\}, \{\neg q\},$
$\{\neg q, \neg p\}, \{\neg p, r\}, \{\neg p, \neg q\}, \{\neg p\}$

1) $\{p, q, \neg r\}$

2) $\{\neg p, q, r\}$

3) $\{q, r\}$

4) $\{\neg q, r\}$

5) $\{\neg p, \neg q\}$

6) $\{\neg p, r\}$

7) $\{\neg p\}$

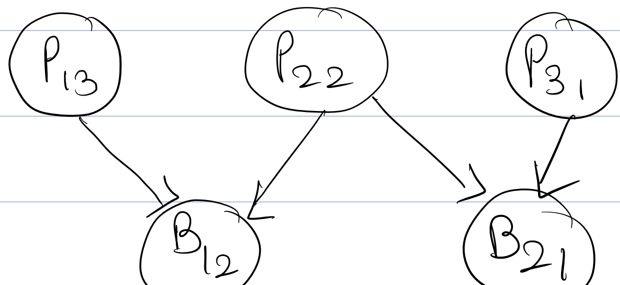
8) $\{\neg q\}$

9) $\{r\}$

10) $\{p, Q\}$ (1, 3)

11) $\{\ \}$ (10, 5)

Q3. (8 pts) Redo the probability calculation for pits in [1,3], [2,2] assuming that each square contains a pit with probability 0.01, independent of the other squares.



P_{13}	P_{22}	$P(B_{12} \mid P_{13}, P_{22})$
t	t	1
t	f	1
f	t	1
f	f	0

P_{31}	P_{22}	$P(b_{21} \mid P_{31}, P_{22})$
t	t	1
t	f	1
f	t	1
f	f	0

$$P(P_{13} \mid b_{12}, b_{21}) = \alpha \sum_{P_{22}} \sum_{P_{31}} \left[P(b_{12} \mid P_{13}, P_{22}) \cdot P(b_{21} \mid P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) \right]$$

$$= \alpha \big[P(b_{12} \mid P_{13}, P_{22}) \cdot P(b_{21} \mid P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12} \mid P_{13}, \neg P_{22}) \cdot P(b_{21} \mid P_{31}, \neg P_{22}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12} \mid P_{13}, P_{22}) \cdot P(b_{21} \mid \neg P_{31}, P_{22}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) +$$
$$P(b_{12} \mid P_{13}, \neg P_{22}) \cdot P(b_{21} \mid \neg P_{31}, \neg P_{22}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(\neg P_{31}) \big]$$

$$= \alpha \big[1 * 1 * 0.01 * 0.01 * 0.01 + 1 * 1 * 0.01 * 0.99 * 0.01$$
$$+ 1 * 1 * 0.01 * 0.01 * 0.99 + 0 \big]$$

$$= \alpha [0.000001 + 0.000099 + 0.000099] = \alpha [0.000199]$$

$$P(\neg P_{13} \mid b_{12}, b_{21}) = \alpha \sum_{P_{22}} \sum_{P_{31}} \left[P(b_{12} \mid \neg P_{13}, P_{22}) \cdot P(b_{21} \mid P_{31}, P_{22}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) \right]$$

$$= \alpha \big[P(b_{12} \mid \neg P_{13}, P_{22}) \cdot P(b_{21} \mid P_{31}, P_{22}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12} \mid \neg P_{13}, \neg P_{22}) \cdot P(b_{21} \mid P_{31}, \neg P_{22}) \cdot P(\neg P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12} \mid \neg P_{13}, P_{22}) \cdot P(b_{21} \mid \neg P_{31}, P_{22}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) +$$
$$P(b_{12} \mid \neg P_{13}, \neg P_{22}) \cdot P(b_{21} \mid \neg P_{31}, \neg P_{22}) \cdot P(\neg P_{13}) \cdot P(\neg P_{22}) \cdot P(\neg P_{31}) \big]$$

$$= \alpha \left[1 * 1 * 0.99 * 0.01 * 0.01 + 0 + 1 * 1 * 0.99 * 0.01 * 0.99 + 0 \right]$$
$$= \alpha [0.000099 + 0.009801] = \alpha [0.0099]$$

$$\alpha [0.000199 + 0.0099] = 1$$
$$\alpha [0.010099] = 1$$
$$\therefore \alpha = \frac{1}{0.010099}$$

$$\therefore P(P_{13} \mid b_{12}, b_{21}) = \alpha \cdot 0.000199 = \frac{0.000199}{0.010099}$$

$$\approx 0.0197$$

$$P(P_{22}|b_{12},b_{21}) = \alpha \sum_{P_{13}} \sum_{P_{31}} \left[P(b_{12}|P_{13},P_{22}) \cdot P(b_{21}|P_{22},P_{31}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) \right]$$

$$= \alpha \big[P(b_{12}|P_{13},P_{22}) \cdot P(b_{21}|P_{22},P_{31}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12}|\neg P_{13},P_{22}) \cdot P(b_{21}|P_{22},P_{31}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12}|P_{13},P_{22}) \cdot P(b_{21}|P_{22},\neg P_{31}) \cdot P(P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) +$$
$$P(b_{12}|\neg P_{13},P_{22}) \cdot P(b_{21}|P_{22},\neg P_{31}) \cdot P(\neg P_{13}) \cdot P(P_{22}) \cdot P(\neg P_{31}) \big]$$

$$= \alpha [1 * 1 * 0.01 * 0.01 * 0.01 + 1 * 1 * 0.99 * 0.01 * 0.01$$
$$+ 1 * 1 * 0.01 * 0.01 * 0.99 + 1 * 1 * 0.99 * 0.01 * 0.99]$$
$$= \alpha [0.000001 + 0.000099 + 0.000099 + 0.009801] = \alpha [0.01]$$

$$P(\neg P_{22}|b_{12},b_{21}) = \alpha \sum_{P_{13}} \sum_{P_{31}} \left[P(b_{12}|P_{13},\neg P_{22}) \cdot P(b_{21}|\neg P_{22},P_{31}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) \right]$$

$$= \alpha \big[P(b_{12}|P_{13},\neg P_{22}) \cdot P(b_{21}|\neg P_{22},P_{31}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12}|\neg P_{13},\neg P_{22}) \cdot P(b_{21}|\neg P_{22},P_{31}) \cdot P(\neg P_{13}) \cdot P(\neg P_{22}) \cdot P(P_{31}) +$$
$$P(b_{12}|P_{13},\neg P_{22}) \cdot P(b_{21}|\neg P_{22},\neg P_{31}) \cdot P(P_{13}) \cdot P(\neg P_{22}) \cdot P(\neg P_{31}) +$$
$$P(b_{12}|\neg P_{13},\neg P_{22}) \cdot P(b_{21}|\neg P_{22},\neg P_{31}) \cdot P(\neg P_{13}) \cdot P(\neg P_{22}) \cdot P(\neg P_{31}) \big]$$
$$= \alpha [1 * 1 * 0.01 * 0.99 * 0.01 + 0 + 0 + 0] = \alpha [0.000099]$$

$$\therefore \alpha [0.01 + 0.000099] = 1 \qquad \therefore \alpha = \frac{1}{0.010099}$$

$$\therefore P(P_{22}|b_{12},b_{21}) = \alpha [0.01] = \frac{0.01}{0.010099} \approx 0.99$$

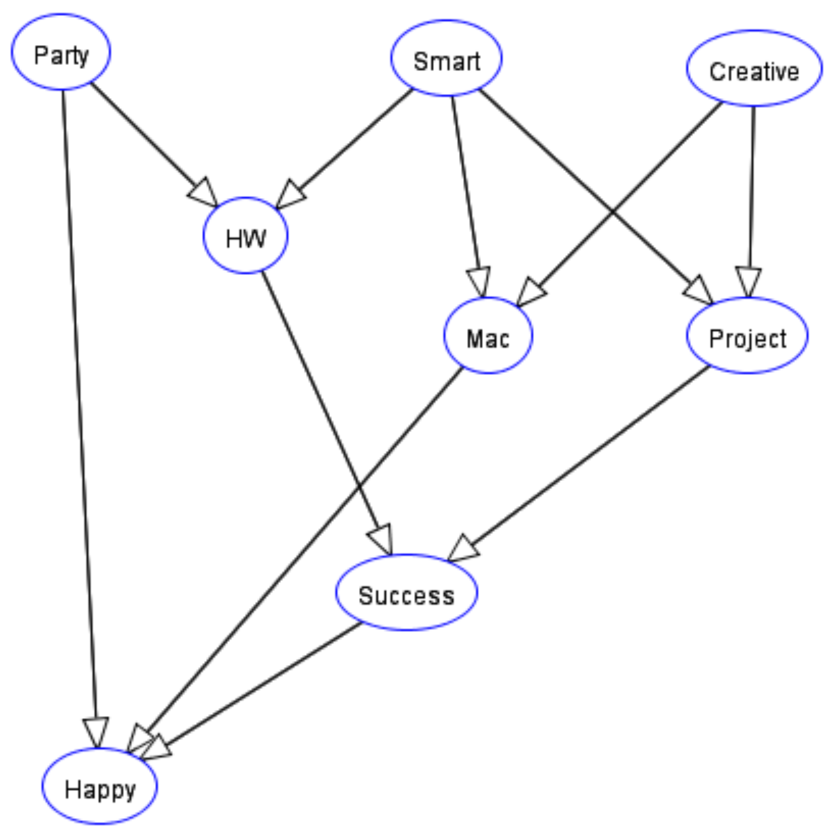
Q4. (9 pts) [Adapted from a CMU machine learning assignment]
As part of a comprehensive study of the role of CMU 10-601 (Machine Learning) on people's happiness, CMU has been collecting data from graduating students. In an optional survey, the following questions were asked:

- Do you party frequently [Party: Yes/No]?
- Are you wicked smart [Smart: Yes/No]?
- Are you very creative [Creative: Yes/No]? (Please only answer Yes or No)
- Did you do well on all your homework assignments? [HW: Yes/No]
- Do you use a Mac? [Mac: Yes/No]
- Did your course project succeed? [Project: Yes/No]
- Did you succeed in your most important class (which is 10-601)? [Success: Yes/No]
- Are you currently Happy? [Happy: Yes/No]

You can obtain the comma-separated survey results from the accompanying file.
Each row in *students.csv* corresponds to the responses of a separate student.
The columns in *students.csv* correspond to each question (random variable) in the order Party, Smart, Creative, HW, Mac, Project, Success, and Happy.
The entries are either zero, corresponding to a No response, or one, corresponding to a Yes response.
After consulting a behavioral psychologist they obtained the following complete set of conditional relationships:

- HW depends only on Party and Smart
- Mac depends only on Smart and Creative
- Project depends only on Smart and Creative
- Success depends only on HW and Project
- Happy depends only on Party, Mac, and Success

1. (1 pt) Draw the Bayesian network.



2. (2 pt) Estimate the probabilities of the conditional probability tables using the data provided

Probability Table for Party

	P(Party=T)	P(Party=F)
Prior Probability	0.6022	0.3978

No observed value for this node.

OK

Probability Table for Smart

	P(Smart=T)	P(Smart=F)
Prior Probability	0.7048	0.2952

No observed value for this node.

OK

Probability Table for Creative

	P(Creative=T)	P(Creative=F)
Prior Probability	0.6994	0.3006

No observed value for this node.

OK

Probability Table for HW

Party	Smart	P(HW=T)	P(HW=F)
T	T	0.803	0.197
T	F	0.094	0.906
F	T	0.898	0.102
F	F	0.305	0.695

No observed value for this node.

OK

Probability Table for Project

Smart	Creative	P(Project=T)	P(Project=F)
T	T	0.905	0.095
T	F	0.794	0.206
F	T	0.403	0.597
F	F	0.106	0.894

No observed value for this node.

OK

Probability Table for Mac

Smart	Creative	P(Mac=T)	P(Mac=F)
T	T	0.686	0.314
T	F	0.413	0.587
F	T	0.897	0.103
F	F	0.122	0.878

No observed value for this node.

OK

Probability Table for Happy

Party	Mac	Success	P(Happy=T)	P(Happy=F)
T	T	T	0.96	0.04
T	T	F	0.492	0.508
T	F	T	0.722	0.278
T	F	F	0.42	0.58
F	T	T	0.358	0.642
F	T	F	0.205	0.795
F	F	T	0.307	0.693
F	F	F	0.094	0.906

No observed value for this node.

OK

Probability Table for Success

HW	Project	P(Success=T)	P(Success=F)
T	T	0.897	0.103
T	F	0.306	0.694
F	T	0.207	0.793
F	F	0.05	0.95

No observed value for this node.

OK

3. (2 pts) What is the probability of being happy given that you party often, are wicked smart, but not very creative? Show details of computation.

$$P(h \mid p, s, \neg c) = \alpha^* \sum_{hw, mac, Proj, Succ} \left[P(hw \mid P, S) . P(mac \mid P, S) . P(Proj \mid S, \neg c) . P(Succ \mid hw, Proj)\, P(P) . P(S) . P(\neg C) . P(h) \right]$$

$$P(\neg h \mid P, S, \neg c) = \alpha^* \sum_{hw, mac, Proj, Succ} \left[P(hw \mid P, S) . P(mac \mid P, S) . P(Proj \mid S, \neg c) . P(Succ \mid hw, Proj)\, P(P) . P(S) . P(\neg C) . P(\neg h) \right]$$

$$\alpha = \frac{1}{P(h \mid P, S, \neg c) + P(\neg h \mid P, S, \neg c)}$$

Query Results

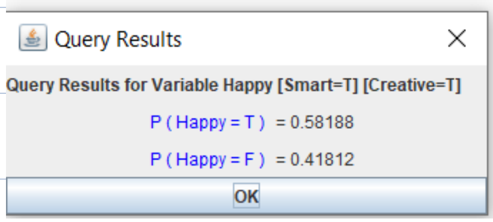
Query Results for Variable Happy [Party=T] [Smart=T] [Creative=F]

P (Happy = T) = 0.69317

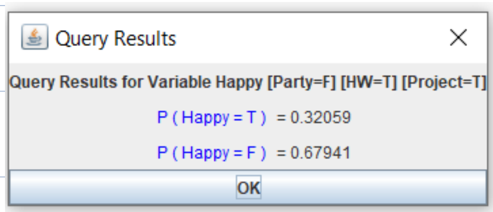
P (Happy = F) = 0.30683

OK

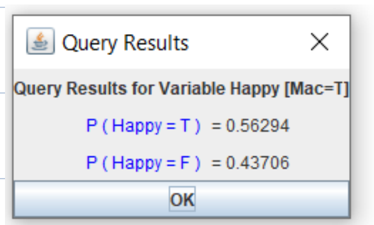
4. (2 pts) What is the probability of being happy given that you are wicked smart and very creative? No details required. Use the AIspace tool.



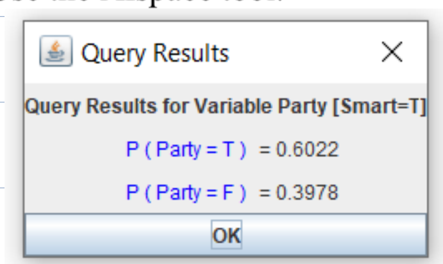
5. (0.5 pts) What is the probability of being happy given you do not party, and do well on all your homework and class project? No details required. Use the AIspace tool.



6. (0.5 pts) What is the probability of being happy given you own a mac?
No details required. Use the AIspace tool.



7. (0.5 pts) What is the probability that you party often given you are wicked smart?
No details required. Use the AIspace tool.



8. (0.5 pts) What is the probability that you party often given you are wicked smart and happy?
No details required. Use the AIspace tool.