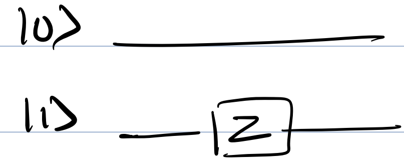
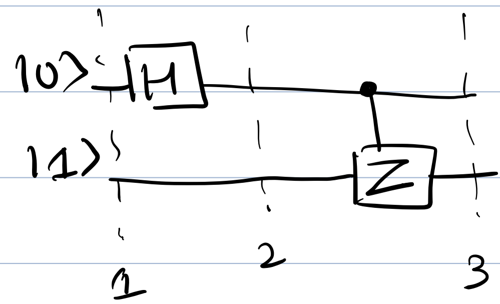
Q-1

a)



Phasekick back is the ability to move a global phase applied to one of the kets in a tensor product, to any other ket in that tensor product.
In this example $|0\rangle \otimes |1\rangle \longrightarrow |0\rangle \otimes -|1\rangle$, can be written as $|01\rangle \rightarrow -|01\rangle \rightarrow -|0\rangle \otimes |1\rangle$

(b)



At ① $|0\rangle \otimes |1\rangle$

At ② $|+\rangle \otimes |1\rangle$ $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |1\rangle$

$= \frac{1}{\sqrt{2}} |01\rangle + |11\rangle$

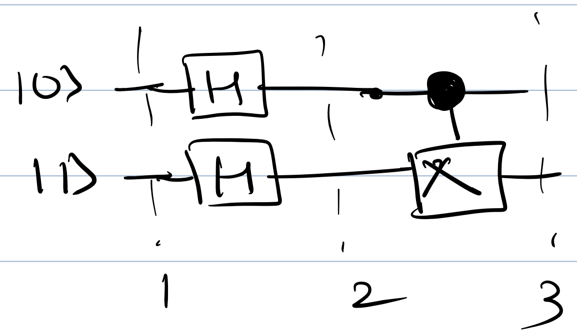
At ③ $\frac{1}{\sqrt{2}} |01\rangle \underbrace{- |11\rangle}_{\text{phase applied}}$

$= \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle) \otimes |1\rangle$

$= |-\rangle \otimes |1\rangle$

The Phase is applied to the bottom qubit at State $|1\rangle$ but with Phase kickback it can be used to Convert $|+\rangle$ to $|-\rangle$.

(c)



<u>At 1</u> $|0\rangle \otimes |1\rangle$ or $|01\rangle$

<u>At 2</u> $|+\rangle \otimes |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$= \frac{1}{2}(|00\rangle - |01\rangle + |10\rangle - |11\rangle)$

<u>At 3</u>

$\frac{1}{2}(|00\rangle - |01\rangle + |11\rangle - |10\rangle)$ (After C_x)

$= \frac{1}{2}(|00\rangle - |10\rangle - |01\rangle + |11\rangle)$

$= \frac{1}{2}\left[(|0\rangle - |1\rangle)|0\rangle - (|0\rangle - |1\rangle)|1\rangle\right]$

$$= \frac{1}{2}\left[(|0\rangle - |1\rangle)(|0\rangle - |1\rangle)\right)$$

$$= |-\rangle \otimes |-\rangle$$

In this circuit, Controlled not should have been applied to qubit with state $|-\rangle$ at position 2, but instead with phase kickback we converted $|+\rangle$ to $|-\rangle$.

Q-2

(a) $H^{\otimes n}|0\rangle^{\otimes n} = a\left(\sum_{x=0}^{2^n-1} b\right)$

$$H^{\otimes n}|0\rangle^{\otimes n} = \left(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\right)^{n^2}$$

$$= \frac{1}{(\sqrt{2})^{n^2}}(|0\rangle + |1\rangle)^{n^2}$$

where,
$|x\rangle = \{|0\rangle, |1\rangle\}$

$$= \underbrace{\frac{1}{(\sqrt{2})^{n^2}}}_{a} \sum_{x=0}^{2^n-1} \underbrace{|x\rangle}_{b}$$

b) At stage 1,

$$\frac{1}{(\sqrt{2})^{n^2}} \sum_{x=0}^{2^n-1} |x\rangle \overbrace{\frac{(|0\rangle - |1\rangle)}{\sqrt{2}}}^{H|1\rangle}$$

$$= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0}^{2^n-1} |x\rangle(|0\rangle - |1\rangle)$$

At stage 2,

$$\boxed{\begin{array}{ccc} x & \xrightarrow{\quad} & x \\ & U_f & \\ y & \xrightarrow{\quad} & y \oplus f(x) \end{array}}$$

$$|x, y\rangle \longrightarrow |x, y \oplus f(x)\rangle$$

Now, $|x,0\rangle \rightarrow \begin{cases} |x,0\rangle, & f(x)=0 \\ |x,1\rangle, & f(x)=1 \end{cases}$ i.e. $|x,0\rangle \rightarrow |x, f(x)\rangle$

& $|x,1\rangle \rightarrow \begin{cases} |x,1\rangle, & f(x)=0 \\ |x,0\rangle, & f(x)=1 \end{cases}$ i.e. $|x,1\rangle \rightarrow |x, \overline{f(x)}\rangle$

$$\therefore \; U_f \; \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0} |x\rangle(|0\rangle - |1\rangle)$$

$|x\rangle \overline{f(x)}$

$|x\rangle f(x)$

$$= \frac{1}{\sqrt{2^{n+1}}} \sum_{x=0} |x\rangle \left[f(x) - \overline{f(x)} \right]$$

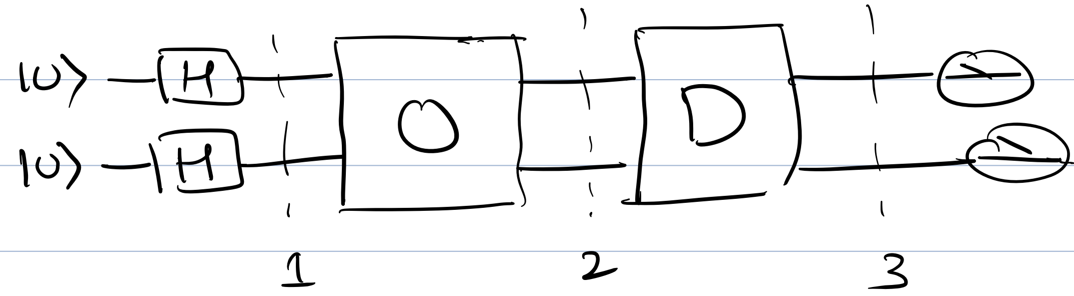
(c) The output of the oracle on the input $|x, y\rangle$ is $|x, y \oplus f(x)\rangle$. Now, on $y=0$, the output $\underbrace{y}_{0} \oplus \underbrace{f(x)}_{\{0,1\}} = \underbrace{f(x)}_{\{0,1\}}$

while on $y=1$ the output $\underbrace{y}_{1} \oplus \underbrace{f(x)}_{\{0,1\}} = \underbrace{\overline{f(x)}}_{\{1,0\}}$

$$\therefore |x\rangle[|0\rangle - |1\rangle] = |x\rangle[f(x) - \overline{f(x)}]$$

Q-3

(a)



At pos 1

$$|+\rangle \otimes |+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$$

$$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)$$

At pos 2

$$= \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle - |11\rangle)$$

$$= \frac{1}{2}\left(\begin{bmatrix}1\\0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\1\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\1\\0\end{bmatrix} - \begin{bmatrix}0\\0\\0\\1\end{bmatrix}\right)$$

$$= \frac{1}{2}\begin{bmatrix}1\\1\\1\\-1\end{bmatrix}$$

At Pos 3

$$\frac{1}{2}\begin{bmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{bmatrix}_{4\times 4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}_{4\times 1}$$

$$= \frac{1}{2}\begin{bmatrix} -1+1+1-1 \\ 1-1+1-1 \\ 1+1-1-1 \\ 1+1+1+1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 0 \\ 0 \\ 0 \\ 4 \end{bmatrix}$$

$$= 2\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 2.|11\rangle$$

Amplitude of $|00\rangle = 0$

$|01\rangle = 0$

$|10\rangle = 0$

$|11\rangle = 2$

(c) Probability to measure $|11\rangle$ is 100% or 1.