In [1]:

```
from qiskit import QuantumCircuit, QuantumRegister
from qiskit.quantum_info import Statevector
from qiskit.visualization import plot_histogram
from qiskit.circuit.library.standard_gates import C3XGate
from qiskit.circuit.library.standard_gates import ZGate
```

# Part a)

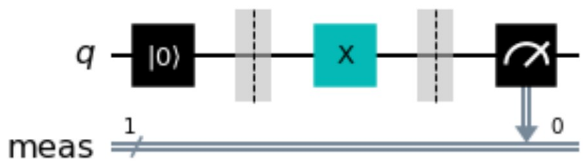
## Increment

Starting with $|0\rangle$ apply a X gate to increment the state to $|1\rangle$

In [2]:

```
qc = QuantumCircuit(1)
qc.reset(0)
qc.barrier()
qc.x(0)
qc.measure_all()
qc.draw('mpl')
```

Out[2]:

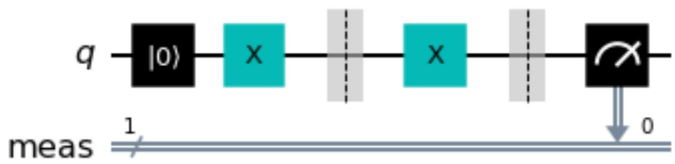



Starting with $|1\rangle$ apply a X gate to increment the state to $|0\rangle$

In [3]:

```
qc = QuantumCircuit(1)
qc.reset(0)
qc.x(0)
qc.barrier()
qc.x(0)
qc.measure_all()
qc.draw('mpl')
```

Out[3]:




## Decrement

Decrement in single qubit case is the same as increment. All we have to do is apply a X gate to the qubit state.

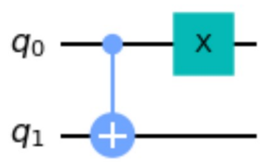
# Part b)

The following is the curcuit to increment any 2 qubit state

In [4]:

```
qc = QuantumCircuit(2)
qc.cnot(0,1)
qc.x(0)
qc.draw('mpl')
```

Out[4]:

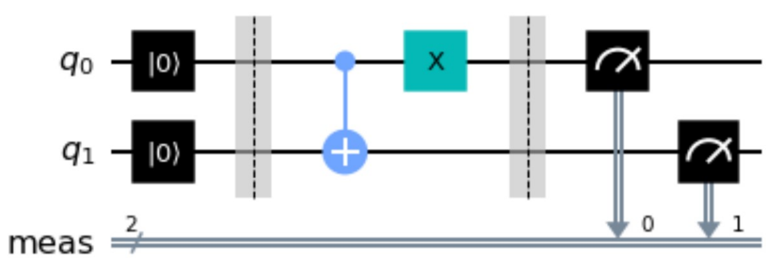



Increment $|00\rangle$ to $|01\rangle$

In [5]:

```
qc = QuantumCircuit(2)
qc.reset(0)
qc.reset(1)
qc.barrier()
qc.cnot(0,1)
qc.x(0)
qc.measure_all()
qc.draw('mpl')
```

Out[5]:

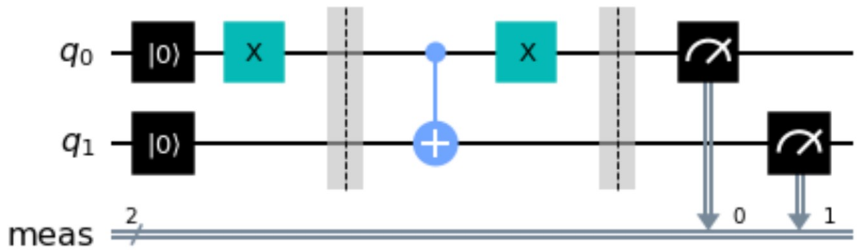



Increment state $|01\rangle$ to $|10\rangle$

In [6]:

```
qc = QuantumCircuit(2)
qc.reset(0)
qc.reset(1)
qc.x(0)
qc.barrier()
qc.cnot(0,1)
qc.x(0)
qc.measure_all()
qc.draw('mpl')
```

Out[6]:

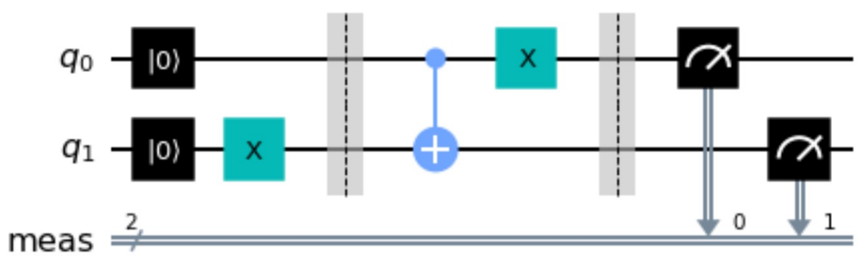



Increment $|10\rangle$ to $|11\rangle$

In [7]:

```
qc = QuantumCircuit(2)
qc.reset(0)
qc.reset(1)
qc.x(1)
qc.barrier()
qc.cnot(0,1)
qc.x(0)
qc.measure_all()
qc.draw('mpl')
```

Out[7]:

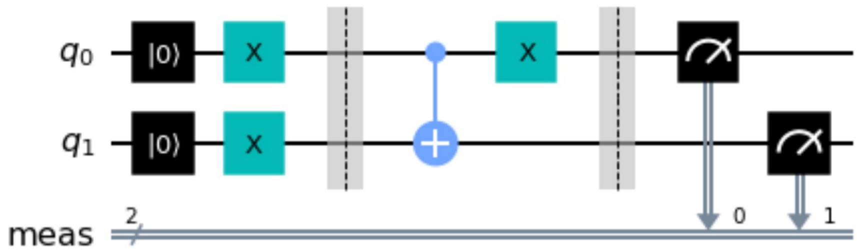



Increment $|11\rangle$ to $|00\rangle$

In [8]:

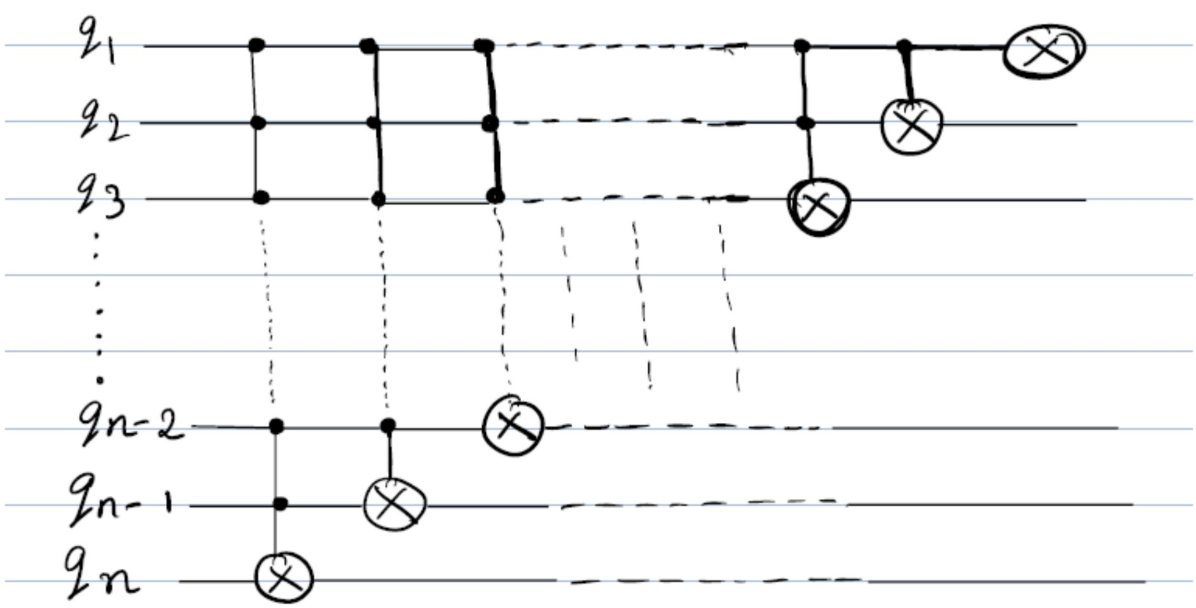
```
qc = QuantumCircuit(2)
qc.reset(0)
qc.reset(1)
qc.x(1)
qc.x(0)
qc.barrier()
qc.cnot(0,1)
qc.x(0)
qc.measure_all()
qc.draw('mpl')
```

Out[8]:




# Part c)

The following is the curcuit to increment an n qubit state




# Part d)

$$\frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle$$ Original State,

$$n=1, \quad \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right) \overset{inc}{=} \frac{1}{\sqrt{2}}\left(|1\rangle + |0\rangle\right)$$

$$n=2, \quad \frac{1}{2}\left(\underset{0}{|00\rangle} + \underset{1}{|01\rangle} + \underset{2}{|10\rangle} + \underset{3}{|11\rangle}\right)$$

$$\overset{inc}{=} \frac{1}{2}\left(\underset{1}{|01\rangle} + \underset{2}{|10\rangle} + \underset{3}{|11\rangle} + \underset{0}{|00\rangle}\right)$$

$$n=3, \quad \frac{1}{2\sqrt{2}}\left(\overset{0}{|000\rangle} + \overset{1}{|001\rangle} + \overset{2}{|010\rangle} + \overset{3}{|011\rangle} + \overset{4}{|100\rangle} + \underset{5}{|101\rangle} + \underset{6}{|110\rangle} + \underset{7}{|111\rangle}\right)$$

$$\overset{inc}{=} \frac{1}{2\sqrt{2}}\left(\overset{1}{|001\rangle} + \overset{2}{|010\rangle} + \overset{3}{|011\rangle} + \overset{4}{|100\rangle} + \overset{5}{|101\rangle} + \underset{6}{|110\rangle} + \underset{7}{|111\rangle} + \underset{0}{|000\rangle}\right.$$

After applying n-qubit incrementer to the original state, there is no change as the the qubit state as we do not make use of carry. Only the order gets jumbled which does not affect the state in any way.