P1-Q1 (10 points)

Rotation Gates

The unitary matrix $U_{ROT_k} = \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^k}} \end{bmatrix}$ for $k \in \mathbb{N}$.

a) Determine $U_{ROT_3}(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle))$. Write your answer in Dirac notation.

b) Apply U_{ROT_k} to the general single-qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and in your own words, describe what U_{ROT_k} does.

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a) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^3}} \end{bmatrix} \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$

$= \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{bmatrix} \left(\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right)$

$= \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{1}{\sqrt{2}} e^{i\pi/4} \begin{bmatrix} \phi \\ 0 \end{bmatrix}$

$= \frac{1}{\sqrt{2}} \left(|0\rangle + e^{i\pi/4} |1\rangle \right)$

b) $U_{Rot_K}(\alpha|0\rangle + \beta|1\rangle)$

$$= \begin{bmatrix} 1 & 0 \\ 0 & e^{\frac{2\pi i}{2^K}} \end{bmatrix} \left(\alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right)$$

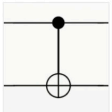
$$= \alpha \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \beta e^{\frac{2\pi i}{2^K}} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Changes Phase of quantum State
but the keeps the probability of measurement
the Same

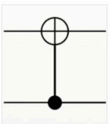
P1-Q2 (16 points)

Two qubit gates

Recall from lecture that the definition for the standard Controlled-NOT gate is $C_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, where the first (upper) qubit is the control and the second (lower) qubit is the target. (See reference image below.)



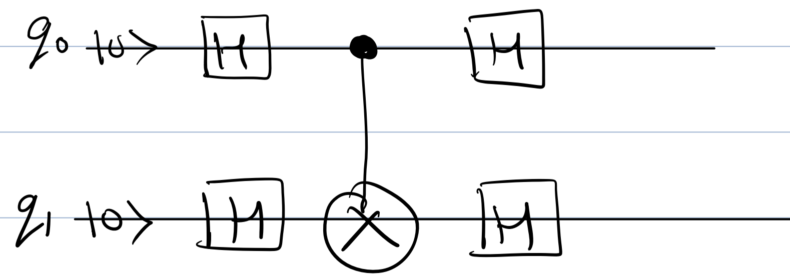
a) Give the matrix of a C_X gate when the control and target qubits are swapped. We will refer to this as the "reverse" C_X gate for the rest of this exercise. (See reference image below.)



b) Design a circuit using only the standard C_X gate and H (Hadamard) gate(s) which produces the same outcome as the "reverse" C_X gate.

(a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

(b)



P1-Q3 **(20 points)**

Teleportation and Superdense coding protocol

a) In brief bullet-point form, describe the following for the Quantum Teleportation algorithm in your own words:

- the goal
- the members
- each member's function (ie. what actions they must perform and why, 1-2 sentences max)

b) Read about the Superdense coding algorithm: https://qiskit.org/textbook/ch-algorithms/superdense-coding.html. In your own words, compare the goal of Superdense coding protocol to the goal of the Quantum Teleportation algorithm and describe how they differ. (Note: 2-4 sentences is good.)

a)
- Moving a qubit state from one qubit to another qubit

- Alice wants to send quantum information to Bob. Uses a third party Charlie to send them entangled pair of qubit

- Charlie creates an entangled pair of qubit and sends one to Alice and one to Bob

Alice applies CNot gate and H gate. Measures the 2 qubit she has and sends the classical info. to Bob

Bob depending on the classical info it receives, performs some operation to receive Alice's qubit

b) Quantum teleportation starts with an entangled qubit shared between two people and uses 2 classical bits to transmit qubit state. While in superdense coding we also start with an entangled qubit shared between 2 people, but uses a quantum state to basically transfer 2 classical bits of information