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Knowledge Base:

```
qux :: [Int] -> (Int, Int)
qux [] = (0, 0)              Q1
qux x = (ham x, 1)           Q2

ham :: [Int] -> Int
ham (h:_) = h                H1

foo :: [Int] -> Int
foo [] = 0                   F1
foo [x] = 1                  F2
foo (h:t) = 1 + (foo t)      F3

xyz :: (Int, Int) -> Int
xyz (x, 1) = x * (x + 1)     X1
xyz x = 0                    X2

bar :: [Int] -> [Int]
bar [] = []                             B1
bar (h:t) = (foo (h:t)) : bar t         B2
```

Inductive Case (IA): xyz (qux (bar (h:t))) = = (foo (h:t)) * (foo (h:t) + 1)

Prove this by structural induction: xyz (qux (bar x)) = = (foo x) * (foo x + 1)

Base Case: xyz (qux (bar [])) = = (foo []) * (foo [] + 1)

LHS of base case: xyz (qux (bar []))

by B1 = xyz (qux (0))

by Q2 = xyz (ham 0, 1)

by H1 = xyz (0,1)

by Q1 = 0 * (0 + 1)

By MATHZ = 0 * (1)

By MATHZ = 0

RHS of base case: (foo []) * (foo [] + 1)

By F1 = 0 * (foo [] + 1)

By F1 = 0 * (0 + 1)

By MATHZ = 0 * (1)

By MATHZ = 0

0 == 0 Q.E.D

Inductive Assumption (IA): xyz (qux (bar t)) = = (foo t) * (foo t + 1)

Inductive Case (IC): xyz (qux (bar (h:t))) = = (foo (h:t)) * (foo (h:t) + 1)

LHS of IC: xyz (qux (bar (h:t)))

by B2 = xyz (qux ((foo (h:t)) : bar t))

by F3 = xyz (qux (1 + (foo t) : bar t))

by Q2 = xyz (ham (1 + (foo t) : bar t), 1)

by X1 = ham (1 + (foo t) : bar t) * (ham (1 + (foo t) : bar t) + 1)

by H1= (1 + (foo t)) * (ham (1 + (foo t) : bar t) + 1)

by H1= (1 + (foo t)) * ((1 + (foo t)) + 1)

by MATHZ = (1 + (foo t)) * (2 + (foo t))

RHS of IC: (foo (h:t)) * (foo (h:t) + 1)

by F3 = (1 + (foo t)) * (foo (h:t) + 1)

by F3 = (1 + (foo t)) * ((1 + (foo t)) + 1)

by MATHZ = (1 + (foo t)) * (2 + (foo t))

(1 + (foo t)) * (2 + (foo t)) == (1 + (foo t)) * (2 + (foo t)) Q.E.D