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Exercise 1:

Beta reduction with last 2 digits of student number (37):

Original expression: $(\lambda n.\ (\lambda q.\ q))\ (\lambda o.\ (\lambda r.\ o))\ (\lambda p.\ (\lambda s.\ s))$

Beta Reduction:

Parameter, Body, Argument

$(\lambda n.\ (\lambda q.\ q))\ (\lambda o.\ (\lambda r.\ o))\ (\lambda p.\ (\lambda s.\ s))$

$= (\lambda q.\ q)\ (\lambda p.\ (\lambda s.\ s))$

$= (\lambda p.\ (\lambda s.\ s))$

= (λf. (λx. x)) church encoding of zero

$= zero$

Fully Beta Reduced Expression/Answer: $(\lambda p.\ (\lambda s.\ s))$ or $zero$

Exercise 2:

Beta reduction of λ-calculus expression with largest digit of student number 7:

Original expression: $\lambda a.\ (\lambda b.\ (\lambda c.\ (\lambda d.\ ((a\ c)\ ((b\ c)\ d)))))$ seven $zero$

Beta Reduction:

Parameter, Body, Argument

$\lambda a.\ (\lambda b.\ (\lambda c.\ (\lambda d.\ ((a\ c)\ ((b\ c)\ d)))))$ seven $zero$

$= \lambda b.\ (\lambda c.\ (\lambda d.\ ((\text{seven}\ c)\ ((b\ c)\ d))))\ zero$

$= \lambda c.\ (\lambda d.\ ((\text{seven}\ c)\ ((zero\ c)\ d)))$

$= \lambda c.\ (\lambda d.\ ((\lambda f.\ (\lambda x.f^7x)\ c)\ ((\lambda g.\ (\lambda y.\ y)\ c)\ d)))$

$= \lambda c.\ (\lambda d.\ ((\lambda f.\ (\lambda x.f^7x)\ c)\ ((\lambda g.\ (\lambda y.\ y)\ c)\ d)))$

$= \lambda c.\ (\lambda d.\ (\lambda x.(c^7x)\ ((\lambda g.\ (\lambda y.\ y)\ c)\ d)))$

$= \lambda c.\ (\lambda d.\ (\lambda x.(c^7x)\ (\lambda y.\ (y)\ d)))$

$= \lambda c.\ (\lambda d.\ (\lambda x.(c^7x)\ d))$

$= \lambda c.\ (\lambda d.\ (c^7d))$

= λf. (λx. f^7x) church encoding of seven

= seven

Fully Beta Reduced Expression/Answer: $\lambda c.\ (\lambda d.\ (c^7d))$ or seven as this is the function for addition of two nonnegative integers which returns the sum of the arguments:

seven + zero = seven.