Supplementary File 4: Calculating goodness of fit statistics

* 1. **Pearson residuals**

The squared Pearson residual is defined as:

where is an observed response variable, , and is the variance function (McCullagh and Nelder 1989, equation 2.11).

**1.1.1 Occupancy Models**

Occupancy models (sensu MacKenzie et al. 2017) are defined by the following joint distribution of , the partially observed latent state, and , detection / non-detections of the underlying state:

This can alternatively be written as:

Since we only fully observe , we marginalize over to obtain the distribution of :

To calculate squared Pearson's residuals, we first obtain as:

To obtain , we first obtain the variance of as:

Since we can now write a squared Pearson's residual as:

**Literature Cited**

MacKenzie, D., J. Nichols, J. Royle, K. Pollock, L. Bailey, and J. Hines. 2017. Occupancy Estimation and Modeling: Inferring Patterns and Dynamics of Species Occurrence, 2nd Edition. Academic Press, London, UK.

McCullagh, P., and J.A. Nelder. 1989. Generalized Linear Models, Second Edition. Chapman & Hall, Boca Raton, Florida, USA.