# Technical Indicators and Cross-Sectional Expected Returns

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**Abstract**

This study shows that 14 widely documented technical indicators explain cross-sectional stock expected returns. The technical indicators have lower estimation errors than the three-factor Fama-French model and the historical mean. The long-short portfolios based on the cross-sectional estimated returns generate substantial profits consistently across the entire period. The well-known cross-sectional expected return determinants, including momentum, size, book-to-market, investment, and profitability, do not explain the explanatory power of the technical indicators. Our findings suggest that the technical indicators play an important role in determining the variation in cross-sectional expected returns in addition to the five-factor model.

**JEL Classification Codes:** C31, E32, G17
**Keywords**: Cross-sectional stock returns, technical indicators, three-factor Fama-French model, cross-sectional expected return determinants.

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## 1. Introduction

Numerous empirical studies document evidence on the time-series aggregate market predictability based on technical indicators (e.g., Brock, Lakonishok, and LeBaron, 1992; Metghalchi, Marcucci, and Chang, 2012; Neely, Rapach, Tu, and Zhou, 2014). However, much less is known about how technical indicators explain the cross-sectional equity returns. We contribute to the literature by applying 14 well documented technical indicators to determine the cross-section of stock returns beyond the well-known determinants such as momentum, size, book-to-market ratio, operating profits, and investment.

Neely, Rapach, Tu, and Zhou (2014) summarize four types of informationally inefficient market leaded theoretical models to support the efficacy of technical analysis. Numerous studies document evidence that technical analysis involving past prices or other past data could predict time series stock returns (Lo, Mamaysky, and Wang, 2000; Yamamoto, 2012; Zeng, Marshall, Nguyen, and Visaltanachoti, 2021). Relative to times-series stock returns prediction, research on cross-section stock returns explanatory by technical indicators have received significantly less attention. However, Neely, Rapach, Tu, and Zhou (2014) find that technical indicators significantly forecast the sentiment-changes index, while Baker and Wurgler (2006, 2007) show that investor sentiment measures help explain the cross-section of U.S. equity returns. We thus shed light on the explanation ability of technical indicators on the cross-section of stock returns.

Our study contributes to previous literature in two ways: First, we investigate the performance of 14 well-documented technical indicators in explaining the cross-sectional individual stock returns. We follow Neely, Rapach, Tu, and Zhou (2014) to construct the firm-level technical indicators and apply them jointly in cross-sectional analysis. This enables us to simultaneously incorporate all the information from the 14 technical indicators in estimating cross-sectional stock expected returns. To avoid the overfitting problem, we apply the smoothed OLS (SOLS) model introduced by Han, He, Rapach, and Zhou (2020), which averages the cross-sectional OLS estimated coefficients over time. We find that the SOLS model with joint 14 technical indicators generate positive cross-sectional and time-series out-of-sample $R^2_{OS}$ statistics, and the result is robust over time. Moreover, we compare its explanatory power with the widely used cross-sectional model of Fama French (1993) three-factor model. We find the SOLS model outperforms the Fama and French (1993) three-factor model with lower errors.

Second, we investigate the relative contribution of technical indicators in explaining the cross-sectional returns relative to the five well-known cross-sectional stock returns determinants such as momentum, size, book-to-market ratio, operating profits, and investment. We first examine the contribution of the momentum factors, which capture the trend of asset prices like many of the technical indicators. Jegadeesh and Titman (1993) introduce the momentum strategy. Conrad and Kaul (1998) argue that the momentum profits come from the cross-sectional differences in expected returns. Therefore, we examine whether the cross-sectional stock return determinants of the technical indicators are related to the momentum. In addition, we assess the cross-sectional contribution of the other widely used cross-sectional determinants[1] such as size, book-to-market ratio (BM), operating profit (OP), and investment (INV). We use the variance decomposition method following Hou and Loh (2016) to quantify the contribution of each candidate variable in explaining the cross-sectional expected return. We find the technical indicators provide independent information in explaining the cross-sectional stock returns, not shared by any of the five well-known firm characteristics.

[1] See Daniel, Titman, and Wei (2001) for the book to market value investigation; Ball, Gerakos, Linnainmaa, and Nikolaev (2016) for firm capitalization and operating profit analysis; Cooper, Gulen, and Schill (2008) for the investment characteristic examination.

The remainder of this paper is organized as follows. Section 2 describes the data. Section 3 is devoted to the method. Section 4 reports the empirical analysis results. We conclude in Section 5.

## 2. Data

### *2.1 Measuring Dependent and Independent Variables*

We source the monthly equity returns of all firms listed on NYSE, AMEX, and NASDAQ markets from CRSP. The monthly equity returns data span from January 1926 to December 2020, where January 1926 is the earliest month to obtain stock returns from CRSP. We remove firms with stock returns lower than -100% and exclude delisted firms. Our estimation is based on 60-month rolling regressions for the historical mean and smoothed OLS models. Therefore, we delete firms with fewer than 60 monthly return observations to ensure sufficient data in each regression. Since the construction of technical indicators needs past 12 months' observations, and we apply them in the 60-month rolling regression in the smoothed OLS model, our first estimation month is January 1932. The whole out-of-sample period is divided into three sub-periods of around 30 years each for the subsample analysis.

We also collect data on the Fama-French (1993) three-factor and Carhart (1997) four-factor models from Kenneth French's website[2]. Besides, we obtain data on the Fama-French (2016) five-factor model from Compustat to construct size, book-to-market value, operating profitability, and investment factors for all the individual firms. Because the book-to-market

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[2] Many appreciate for Kenneth French to provide the data on his website: https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html

value is robustly available in 1963, the estimation based on the Fama French three-factor model starts from January 1963 and ends in December 2020.

### *2.2 Firm-Level Technical Indicators*

The construction of the 14 firm-level technical indicators in this paper follows the method introduced by Neely, Rapach, Tu, and Zhou (2014), mainly based on three trend-following strategies (moving average, momentum, and volume-based indicators). The first strategy is based on the moving average (MA) rule, which forms the trading signals by comparing the two moving averages with different lengths:

$$B_{i,t} = \begin{cases} 1 & \text{if} \quad MA_{i,t}^{s} \geq MA_{i,t}^{l} \\ 0 & \text{if} \quad MA_{i,t}^{s} \leq MA_{i,t}^{l} \end{cases}, \tag{1a}$$

where $MA_{i,t}^{j} = \frac{1}{j}\sum_{h=0}^{j-1} P_{i,t-h}, \text{ for } j = s, l,$ (1b)

$P_{i,t-h}$ is the stock price level of stock $i$ in month $t-h$. $j = s$ ($j = l$) represents the length of the short (long) MA, and $s < l$. Thus, the MA indicator with MA lengths of $s$ and $l$ is denoted as MA $(s, l)$. We calculate monthly individual stock trading signals with $s = 1, 2, 3$ and $l = 9, 12$ months. $B_{i,t} = 1$ ($B_{i,t} = 0$) represents a buy (sell) signal when the short moving average $MA_{i,t}^{s}$ is higher (lower) than the long moving average $MA_{i,t}^{l}$.

The second strategy is based on the momentum (MOM) trading rule, which generates the trading signals by comparing the current stock price with its level *n* month periods ago as follows:

$$B_{i,t} = \begin{cases} 1 & \text{if } \; P_{i,t} \geq P_{i,t-n} \\ 0 & \text{if } \; P_{i,t} \leq P_{i,t-n} \end{cases}, \tag{2}$$

where $P_{i,t}$ is the current stock price of stock $i$ and $P_{i,t-n}$ is the stock price level $n$ months ago. $B_{i,t} = 1$ ($B_{i,t} = 0$) represents a buy (sell) signal when the current stock price level $P_{i,t}$ is higher (lower) than $P_{i,t-n}$, the price level *n* months ago.

The third strategy is based on the "on-balance" volume rule (e.g., Granville, 1963), which generates the trading signals by evaluating the changes in stock trading volume as follow:

$$B_{i,t} = \begin{cases} 1 & \text{if } \; MA_{i,t}^{OBV,s} \geq MA_{i,t}^{OBV,l} \\ 0 & \text{if } \; MA_{i,t}^{OBV,s} \leq MA_{i,t}^{OBV,l} \end{cases}, \tag{3a}$$

where

$$MA_{i,t}^{OBV,k} = \frac{1}{k} \sum_{h=0}^{k-1} OBV_{i,t-h}, \tag{3b}$$

$k = s, l$, and the 'on-balance' volume (OBV) is calculated as follow:

$$OBV_{i,t} = \sum_{m=1}^{t} VOL_{i,m} \times D_{i,m}, \tag{4}$$

where $VOL_{i,m}$ represents a measure of the trading volume during period $m$ and $D_{i,m}$ is a dummy variable that equals 1 if $P_{i,m} \geq P_{i,m-1}$, and -1 otherwise. $B_{i,t} = 1$ ($B_{i,t} = 0$) represents a buy (sell) signal, indicating a strong positive (negative) market trend evaluated by the volume-based strategy, which is generated by the relatively high (low) recent volume in conjunction with an increase (decrease) in the recent price. $k =$ s ($k = l$) represents the short length of the VOL, $s < l$, and we denote the volume indicator by VOL($s, l$). We compute the volume-based trading signals with lengths of $s = 1, 2, 3$ and $l = 9, 12$ months.

## 3. Method

### *3.1 Smoothed OLS (SOLS) Model*

The standard framework to examine the cross-sectional expected stock returns that apply multiple technical indicators is generally based on the OLS regression model as follow:

$$r_{i,t} = \alpha_t + \sum_{j=1}^{J} \beta_{j,t} x_{i,j,t-1} + \varepsilon_{i,t} \text{ for } i = 1, \dots, N_t, \tag{5}$$

where $r_{i,t}$ is the equity return for stock *i* in month *t*, and $x_{i,j,t-1}$ is the *j*th technical indicator for stock *i* in month *t* - 1. *J* = 14, for we apply 14 technical indicators in each regression, and $N_t$ represents the number of available firms for month *t*.

The cross-sectional estimated return for stock *i* in the month *t* + 1 based on equation (5) is given by:

$$\hat{r}_{i,t+1}^{OLS} = \hat{\alpha}_t + \sum_{j=1}^{J} \hat{\beta}_{j,t} x_{i,j,t} \text{ for } i = 1, \dots, N_{t+1}, \tag{6}$$

where $\hat{\alpha}_t$ and $\hat{\beta}_{j,t}$ are the estimated coefficients of $\alpha_t$ and $\beta_{j,t}$ in equation (5). $J$ = 14 in equation (5) leads to overfitting concern. For mitigating this overfitting problem, we follow Han, He, Rapach, and Zhou (2020) to apply the smoothed OLS (SOLS) model[3] by taking the time-series average of the cross-sectional OLS estimated coefficients $\hat{\alpha}_t$ and $\hat{\beta}_{j,t}$ of equation (6) over a specific month period as follow:

$$\hat{r}_{i,t+1}^{SOLS} = \tilde{\alpha}_t + \sum_{j=1}^{J} \tilde{\beta}_{j,t} x_{i,j,t} \text{ for } i = 1, \dots, N_{t+1}, \tag{7a}$$

where

$$\tilde{\alpha}_t = \frac{1}{K} \sum_{k=0}^{K-1} \hat{\alpha}_{t-k}, \tag{7b}$$

$$\tilde{\beta}_{j,t} = \frac{1}{K} \sum_{k=0}^{K-1} \hat{\beta}_{j,t-k}, \tag{7c}$$

$K$ is the length of the smoothing window. The SOLS model reduces to the OLS model by taking $K$ = 1, and we apply a 60-month ($K$ = 60) smoothing window in this paper. The smoothed OLS model is a simple and efficient method to guard against the overfitting problem in the high-dimension OLS regression. It stabilizes the coefficients by smoothing the estimated coefficients over time and thus reduces the influence of estimation noise and helps to avoid the overfitting problem in the cross-sectional equity returns regression.

[3] This method is consistent with Haugen and Baker (1996), Lewellen (2015), and Green, Hand, and Zhang (2017).

The three-factor model (Fama French, 1993) is the widely used cross-sectional return model; we thus consider it as our benchmark:

$$r_{i,t} = a_t + b_t^{beta} x_{i,t-1}^{beta} + b_t^{cap} x_{i,t-1}^{cap} + b_t^{bm} x_{i,t-1}^{bm} + \varepsilon_{i,t} \quad \text{for } i = 1, \dots, N_t, \tag{8}$$

where $r_{i,t}$ is the equity return for stock $i$ in month $t$; $x_{i,t-1}^{beta}$ is the beta calculated by the covariance between individual stock returns and market returns divided by the variance of market returns in month $t$ - 1 by using the past sixty-month rolling window ($t$ - 61 to $t$ - 1). $x_{i,t-1}^{cap}$ is the market capitalization of firm $i$, calculated by stock price times shares outstanding in month $t$ - 1. $x_{i,t-1}^{bm}$ is the book-to-market value evaluated by the book value of stock $i$ divided by its market capitalization in month $t$ - 1. $N_t$ represents the number of available firms for month $t$. The estimated return of stock $i$ in month $t$ + 1 computed by the cross-sectional three-factor model based on equation (8) is:

$$\hat{r}_{i,t+1}^{FF3} = \hat{a}_t + \hat{b}_t^{beta} x_{i,t}^{beta} + \hat{b}_t^{cap} x_{i,t}^{cap} + \hat{b}_t^{bm} x_{i,t}^{bm}, \tag{9}$$

where $\hat{a}_t$, $\hat{b}_t^{beta}$, $\hat{b}_t^{cap}$, and $\hat{b}_t^{bm}$ are the estimated coefficients of $a_t$, $b_t^{beta}$, $b_t^{cap}$, and $b_t^{bm}$ in equation (8).

### *3.2 Evaluation*

To ascertain the performance of cross-sectional stock returns model, Han, He, Rapach, and Zhou (2020) introduce the cross-sectional out-of-sample $R^2$ ($R_{CSOS}^2$), which is analogous to the conventional time-series out-of-sample ($R_{TSOS}^2$) introduced by Campbell and Thompson

(2008) but provide insight into the cross-sectional stock return evaluation. We comprehensively compare both out-of-sample evaluation metrics in this section.

*3.2.1 Cross-Sectional Mean Squared Error (MSE) and Cross-sectional $R^2_{OS}$ ($R^2_{CSOS}$)*

The value-weighted cross-sectional mean squared error introduced by Han, He, Rapach, and Zhou (2020) is defined as follows:

$$MSE_i^h = \frac{1}{n_t}\sum_{i=1}^{n_t} w_{i,t}[(r_{i,t} - \bar{r}_t) - (\hat{r}_{i,t|t-1}^h - \bar{\hat{r}}_{t|t-1}^h)]^2 \text{ for } t = 1, \dots, T, \tag{10a}$$

where

$$\bar{r}_t = \sum_{i=1}^{n_t} w_{i,t} r_{i,t}, \tag{10b}$$

$$\bar{\hat{r}}_{t|t-1}^h = \sum_{i=1}^{n_t} w_{i,t} \hat{r}_{i,t|t-1}^h, \tag{10c}$$

and $w_{i,t} \geq 0$ is the weight for stock $i$ calculated by the proportional market capitalization of firm $i$ at the end of month $t-1$. $\hat{r}_{i,t|t-1}^h$ represents the estimated stock return by the SOLS model ($h$ = SOLS) from equation (7a) or the competing Fama French three-factor model ($h$ = FF3) from equation (9). In the cross-sectional context, instead of taking the historical mean model as a benchmark, Han, He, Rapach, and Zhou (2020) use the value-weighted cross-sectional mean value as the naïve benchmark:

$$\hat{r}_{i,t|t-1}^{Naive} = \bar{r}_{t-1} \text{ for } i = 1, \dots, n_t, \tag{11}$$

where $n_t$ is the total number of available firms in month *t*. Thus, for the historical mean expected return, $\hat{r}_{i,t|t-1}^{Naive} - \bar{\hat{r}}_{i,t|t-1}^{Naive} = 0$ for $i = 1, \ldots, n_t$, which translates into the value-weighted MSE of equation (10a) as follow:

$$MSE_t^{Naive} = \hat{\sigma}_{r,t}^2 = \frac{1}{n_t} \sum_{n=1}^{N_t} w_{i,t} (r_{i,t} - \bar{r}_t)^2 \text{ for } t = 1, \ldots, T, \tag{12}$$

where $\hat{\sigma}_{r,t}^2$ represents the simply value-weighted cross-sectional return variance. Evaluating the equal-weighted cross-sectional MSE, we set equal weights for equations (10a), (10b), (10c), and (12) by taking $w_{i,t} = \frac{1}{n_t}$ ($n_t$ is the number of available firms in month *t*). The value-weighted cross-sectional out-of-sample $R_{CSOS}^2$ in each month is defined based on comparing the value-weighted MSE of the SOLS model in equation (10a) and the value-weighted cross-sectional return variance in equation (12):

$$R_{CSOS,t}^2 = 1 - \frac{\sum_{i=1}^{n_t} w_{i,t}[(r_{i,t} - \bar{r}_t) - (\hat{r}_{i,t|t-1}^{SOLS} - \bar{\hat{r}}_{t|t-1}^{SOLS})]^2}{\sum_{n=1}^{n_t} w_{i,t} (r_{i,t} - \bar{r}_t)^2} \text{ for } t = 1, \ldots, T, \tag{13}$$

where T represents the total number of $R_{CSOS,t}^2$ statistics. Evaluating the equal-weighted cross-sectional $R_{CSOS,t}^2$, we compare the equal-weighted MSE of the SOLS model with the simple equal-weighted cross-sectional return variance by replacing the value weight $w_{i,t}$ with the equal weight $\frac{1}{n_t}$ in equation (13). After that, we take the time-series average of all the $R_{CSOS,t}^2$ statistics based on the Fama-MacBeth (1973) procedure:

$$R_{CSOS}^2 = \frac{1}{T} \sum_{t=1}^{T} R_{CSOS,t}^2, \tag{14}$$

Finally, we test $H_0: R^2_{CSOS} = 0$ against the $H_A: R^2_{CSOS} \neq 0$ using heteroskedasticity and autocorrelation consistent *t*-statistics (Newey and West, 1987).

*3.2.2 Time-series mean squared error (MSE) and Out-of-Sample $R^2$*

The most-documented Campbell and Thompson (2008) out-of-sample $R^2$ is calculated based on measuring the proportional reduction in the time-series mean squared error (MSE) for the smoothed OLS estimation vis-à-vis the historical average expected return, and the time-series mean squared error (MSE) measures accuracy based on the estimation deviations:

$$MSE_i^k = \frac{1}{T_i}\sum_{t=1}^{T_i}(r_{i,t} - \hat{r}_{i,t|t-1}^k)^2, \tag{15}$$

where $r_{i,t}$ is the actual return for stock *i* in month *t* and $\hat{r}_{i,t|t-1}^{SOLS}$ (*k* = SOLS), $\hat{r}_{i,t|t-1}^{HISM}$ (*k* = HISM), and $\hat{r}_{i,t|t-1}^{FF3}$ (*k* = FF3) are the expected return estimated by the SOLS, historical mean, and Fama French (1993) three-factor models, respectively. $T_i$ is the total number of out-of-sample period observations for stock *i*. The value-weighted time-series MSE is evaluated by:

$$\overline{MSE}^k = \sum_{i=1}^{N}\omega_i MSE_i^k, \tag{16}$$

$\omega_i \geq 0$ is the weight for stock $i$ calculated by the proportional averaged capitalization of firm *i*. For the equal-weighted MSE, we take $\omega_i = \frac{1}{N}$ in equation (16), where *N* is the total number of firms over the whole out-of-sample analysis. We calculate the positive proportion of the MSE difference between the historical mean model (Fama French three-factor model) and the SOLS model to evaluate the performance.

Finally, the conventional time-series out-of-sample $R^2$ ( $R^2_{TSOS}$ ) statistics take the historical mean model as a benchmark and compare the relative error of the SOLS estimate and the historical average expected return based on the mean squared error (MSE) as follow:

$$R^2_{TSOS,i} = 1 - \frac{MSE_i^{SOLS}}{MSE_i^{HISM}}, \tag{17}$$

where $MSE_i^{SOLS}$ ($MSE_i^{HISM}$) is the value, or equal-weighted time-series mean squared error of the smoothed OLS model (historical mean model) for stock *i* computed by equation (13). To assess the general firm expected return estimation, we take the simple average of the out-of-sample $R^2_{TSOS,i}$ statistic for all the firms:

$$\bar{R}^2_{TSOS} = \frac{1}{N}\sum_{i=1}^{N} R^2_{TSOS,i}, \tag{18}$$

where $N$ represents the total number of firms, a positive value of $\bar{R}^2_{TSOS}$ indicates that the SOLS estimate outperforms the historical average estimation overall. In contrast, the negative value suggests an opposite role.

### *3.3 Profit-Making Strategy*

We measure the economic value of the cross-sectional expected return by evaluating the profitability of the value (equal) weighted long-short portfolios constructed based on the ranking of estimated return for each stock. At the end of each month, we sort all the stocks into ten value-weighted (equal-weighted) portfolios based on their estimated returns for the next month, and we select firms with the highest (lowest) expected returns into the top (bottom) investment portfolio. We then buy stocks in the top portfolio and sell stocks in the bottom portfolio, holding this position for one month and rebalance the strategy monthly. For comparison, we take the equal-weighted market portfolio as the benchmark. After obtaining the monthly return of the constructed investment strategy, we compute the risk-adjusted returns by applying Carhart’s (1997) four-factor model.

### *3.4 Decomposition of the Cross-Sectional Determinants*

This section applies Hou and Loh's (2016) decomposition method to test whether the five cross-sectional stock returns determinants (momentum, size, book-to-market ratio, operating profit, and investment) contribute to the cross-sectional determination captured by the technical indicators. Momentum and the technical indicators capture the trend-following price movement. Thus, we raise the first question: whether momentum shares the cross-sectional determinant with the 14 technical indicators. Book-to-market ratio, size, operating profit, and investment are the most-documented determinants of cross-section stock returns which are also known as the pricing factors of the Fama French (2016) five-factor model. The second question is whether the cross-sectional explanatory power captured by the technical indicators is related to these four factors.

The decomposition methodology is based on the Fama-MacBeth (1973) cross-sectional regression. We first regress univariate cross-sectional regression between the individual stock returns and technical indicators:

$$r_{i,t} = \alpha_{j,t} + \theta_{j,t} x_{i,j,t-1} + \varepsilon_{i,j,t} \quad \text{for } i = 1, \dots, N_t, \tag{19}$$

where $r_{i,t}$ is the equity return for stock *i* in month *t*, and $x_{i,j,t-1}$ is the *j*th technical indicator for stock *i* at month $t$ - 1. Next, we investigate the relationship between the technical indicators and the five candidate variables by regressing the individual technical indicator $x_{i,j,t-1}$ on each of the five selected candidate variables as follow:

$$x_{i,j,t-1} = a_{j,h,t-1} + \eta_{j,h,t-1} V_{i,h,t-1} + \epsilon_{i,j,h,t-1} \quad \text{for } h = \text{MOM, BM, Size, OP, INV}, \tag{20}$$

where $V_{i,h,t-1}$ represents the five candidate variables of each firm *i*: momentums (*h* = MOM), the book to market ratio (*h* = BM), size (*h* = Size), operating profit (*h* = OP), and investment (*h* = INV). We apply four firm-level momentums based on the past 3, 6, 9, and 12 months. According to Fama and French (2016), the four firm-level factors are used. After that, we decompose $x_{i,j,t-1}$ into two orthogonal components based on the regression coefficients from equation (20) by following Hou and Loh (2016) as follow:

$$\begin{aligned}\theta_{j,h,t} &= \frac{Cov(r_{i,t},\ x_{i,j,t-1})}{Var(x_{i,j,t-1})} = \frac{Cov[r_{i,t},\ (a_{j,h,t-1}+\eta_{j,h,t-1}V_{i,h,t-1}+\epsilon_{i,j,h,t-1})]}{Var(x_{i,j,t-1})} \\ &= \frac{Cov(r_{i,t},\ \eta_{j,h,t-1}V_{i,h,t-1})}{Var(x_{i,j,t-1})} + \frac{Cov(r_{i,t},\ a_{j,h,t-1}+\epsilon_{i,j,h,t-1})}{Var(x_{i,j,t-1})} \\ &= \theta^C_{j,h,t} + \theta^R_{j,h,t}.\end{aligned} \tag{21}$$

where $\eta_{j,h,t-1}V_{i,h,t-1}$ $(a_{j,h,t-1} + \epsilon_{i,j,h,t-1})$ is the related (residual) component of $x_{i,j,t-1}$. We then use $\frac{\theta^C_{j,h,t}}{\theta_{j,h,t}}$ $(\frac{\theta^R_{j,h,t}}{\theta_{j,h,t}})$ to calculate the explained (residual) fractions for each of the 14 technical indicators by each of the five factors: momentums (*h* = MOM), size (*h* = Size), the book to market ratio (*h* = BM), operating profit (*h* = OP), and investment (*h* = INV). After that, we estimate the mean and variance of the fractions over the whole regression periods as:

$$\hat{E}\left(\frac{\theta^C_{j,h,t}}{\theta_{j,h,t}}\right) \approx \frac{\bar{\theta}^C_{j,h,t}}{\bar{\theta}_{j,h,t}},\ \hat{E}\left(\frac{\theta^R_{j,h,t}}{\theta_{j,h,t}}\right) \approx \frac{\bar{\theta}^R_{j,h,t}}{\bar{\theta}_{j,h,t}}, \tag{22a}$$

$$\widehat{Var}(\frac{\theta^C_{j,h,t}}{\theta_{j,h,t}}) \approx \frac{1}{T}\left(\frac{\overline{\theta}^C_{j,h,t}}{\overline{\theta}_{j,h,t}}\right)^2 (\frac{\sigma^2_{\theta^C_{j,h,t}}}{\overline{\theta}^{C\ 2}_{j,h,t}} + \frac{\sigma^2_{\theta_{j,h,t}}}{\overline{\theta}_{j,h,t}^{\ 2}} - 2\frac{\widehat{\rho}_{\theta^C_{j,h,t},\theta_{j,h,t}}\sigma^2_{\theta^C_{j,h,t}}\sigma^2_{\theta_{j,h,t}}}{\overline{\theta}^C_{j,h,t}\overline{\theta}_{j,h,t}}), \tag{22b}$$

$$\widehat{Var}(\frac{\theta^R_{j,h,t}}{\theta_{j,h,t}}) \approx \frac{1}{T}\left(\frac{\overline{\theta}^R_{j,h,t}}{\overline{\theta}_{j,h,t}}\right)^2 (\frac{\sigma^2_{\theta^R_{j,h,t}}}{\overline{\theta}^{R\ 2}_{j,h,t}} + \frac{\sigma^2_{\theta_{j,h,t}}}{\overline{\theta}_{j,h,t}^{\ 2}} - 2\frac{\widehat{\rho}_{\theta^R_{j,h,t},\theta_{j,h,t}}\sigma^2_{\theta^R_{j,h,t}}\sigma^2_{\theta_{j,h,t}}}{\overline{\theta}^R_{j,h,t}\overline{\theta}_{j,h,t}}), \tag{22c}$$

and,

$$t_{\frac{\overline{\theta}^C_{j,h,t}}{\overline{\theta}_{j,h,t}}} = \frac{\frac{\overline{\theta}^C_{j,h,t}}{\overline{\theta}_{j,h,t}}}{\sigma(\frac{\overline{\theta}^C_{j,h,t}}{\overline{\theta}_{j,h,t}})}, \ t_{\frac{\overline{\theta}^R_{j,h,t}}{\overline{\theta}_{j,h,t}}} = \frac{\frac{\overline{\theta}^R_{j,h,t}}{\overline{\theta}_{j,h,t}}}{\sigma(\frac{\theta^R_{j,h,t}}{\theta_{j,h,t}})}, \tag{22d}$$

We refer to Hou and Loh’s (2016) paper to see more details.

## 4. Empirical Results

### *4.1 Cross-Sectional Performance of Technical Indicators*

Table 1 reports the value- and equal-weighted cross-sectional $R^2$ statistics and the mean squared errors for the SOLS model and Fama French three-factor (FF3) estimation over the whole period and three (two) subsamples. The results suggest that technical indicators based on the SOLS model show strong ability in explaining the cross-sectional individual stock expected returns and significantly outperform the benchmark Fama French three-factor (FF3) models consistently over time. The value-weighted and equal-weighted cross-sectional out-of-sample $R^2$ statistics in Panel A and Panel B are positive and significant at the 1% level. The

SOLS model has a smaller averaged MSE ($\overline{MSE}_{SOLS}$) than the naïve benchmark[4] ($\overline{MSE}_{NAIVE}$) for the full out-of-sample and the three subsamples.

In addition, we take the estimated results of the Fama French three-factor model as another benchmark and present the value-weighted and equal-weighted cross-sectional MSE results in Panel C and D, respectively. We can see that the SOLS model generates significantly smaller errors than the well-documented Fama French three-factor (FF3) model: the MSE differences between the SOLS model and FF3 model are all positive and significant at a 1% level in the last row of Panels C and D.

Table 2 further assesses the performance of the SOLS, historical mean, and Fama French (1993) three-factor models by applying the traditional value- and equal-weighted time-series out-of-sample $R^2$ statistics and mean squared errors. Panels A, B, and C (D, E, and F) show the cross-sectional performance comparison between the SOLS model and historical mean (FF3) model over the entire sample 1932:01 (1963:01) to 2020:12 and three (two) subsamples: 1932:01 to 1959:12, 1960:01 to 1989:12, and 1990:01 to 2020:12 (1963:01 to 1989:12, and 1990:01 to 2020:12)[5]. The positive and significant value- and equal-weighted out-of-sample $R^2$ statistics ($\bar{R}^2_{TSOS}$) in the first row of Panels A and B suggest that the SOLS model outperforms the historical mean model in explaining the cross-sectional stock expected returns over the whole out-of-sample and three subsamples.

Besides, the SOLS model generates a lower error than the historical mean model. The value- and equal-weighted mean squared errors for the SOLS model ($\overline{MSE}_{SOLS}$) in the second row of Panels A and B are smaller than that for the historical mean model ($\overline{MSE}_{HISM}$) over the entire sample and three subsamples. Moreover, we evaluate the proportion of positive MSE

[4] Han, He, Rapach, and Zhou (2020) consider a naïve benchmark predictive model in the cross-sectional forecast, and its cross-sectional MSE is simply the cross-sectional return variance. Besides, the magnitude of $R^2_{CSOS}$ is small in our analysis but relatively larger than the $R^2_{CSOS}$ calculated by Han, He, Rapach, and Zhou (2020) in their study.
[5] The cross-sectional period of the Fama French three-factor model covers January 1963 to December 2020 because the book-to-market value only becomes robustly available in 1963. We divide it into two subsamples: 1963:01 to 1989:12 and 1990:01 to 2020:12.

difference ($MSE_{HISM}$ - $MSE_{SOLS}$) between the historical mean model and the SOLS model in Panel C. The results show that 87.93% (80.01%, 87.43%, and 82.31%) firms produce lower MSE by using the SOLS model than the historical mean model over the entire sample (three subsamples) from 1932:01 to 2020:12 (1932:01 to 1959:12, 1960:01 to 1989:12, and 1990:01 to 2020:12).

The first and second rows of Panel D (E) show the value- (equal)-weighted MSE for the SOLS and FF3 models. We report the positive percentage of their difference in Panel F. The results show that the SOLS model outperforms the Fama French three-factor model over the entire sample and the two subsamples by producing lower error. We can observe that all the value- and equal-weighted MSEs are lower for the SOSL model than those for the FF3 model. Moreover, the P_MSEF in Panel F shows that 87.90% (82.76%, and 86.40%) firms produce lower MSE by using the SOLS model than the FF3 model over the entire sample (two subsamples) from 1932:01 to 2020:12 (1963:01 to 1989:12, and 1990:01 to 2020:12).

Figure 1 provides the time variation in out-of-sample cross-sectional returns performance based on the SOLS model that utilizes the 14 joint technical indicators. The figure portrays the cumulative value-weighted cross-sectional mean squared error difference between the SOLS and the historical mean estimation. To evaluate the difference between SOLS and the historical mean model over time, we first calculate the MSE difference between the Han, He, Rapach, and Zhou (2020) defined value-weighted cross-sectional mean squared error of these two models. We then cumulate the MSE difference over the entire out-of-sample period from 1932:01 to 2020:12. An increasing trend of the line implies a better performance of the SOLS model, while a decreasing trend suggests a stronger determinant of the prevailing mean model.

The cumulative squared error difference displays a solid upward trend throughout the out-of-sample period, indicating that the SOLS model appears to generate consistently significant out-of-sample cross-sectional equity expected returns over time. We can see that the cumulative squared error difference steadily increases to the value of approximately 2% from 1932 to 1973, and decrease to 1.3% in 1986, then keep increasing at a faster rate to around 5.8% in 2005 and keep growing at a fluctuating increase rate to around 6.2% at the end of the estimation period (2020).

### *4.2. Economic Value*

Table 3 provides the summary statistics for the monthly profits of the value- and equal-weighted long-short portfolios based on the SOLS estimates and simple equal-weighted market portfolios over the whole sample and three subsamples (1932:01 to 1959:12, 1960:01 to 1989:12, and 1990:01 to 2020:12). Mean, STD, Skewness, Kurtosis, and Ann.SR represents the monthly mean, standard deviation, skewness, kurtosis, and annual Sharpe ratio, respectively. It is evident that the equal-weighted long-short portfolio (in Panel A) and the value-weighted long-short portfolio (in Panel B) based on the investment strategy that buys the best past performing firms and sells the worst past performing firms generate a remarkably higher monthly profit and lower standard deviation than the naïve market portfolio returns (in Panel C).

The equal- (value-) weighted long-short portfolio for the SOLS expected return in the first row of Panel A (B) has an average return of 4.22% (2.96%) over the total investment period, which is over four (two) times higher than the monthly average return of 1.02% for the simple market portfolio in Panel C. However, the relative monthly standard deviation is lower for the equal- and value-weighted long-short portfolios (4.71% and 3.40%) than that for the market portfolio (5.23%), implying a higher annualized Sharpe ratio of 2.90 and 2.73 for the equal- and value-weighted long-short portfolios than the 0.50 Sharpe ratio for the market portfolio.

The equal (value)-weighted long-short portfolio based on the SOLS estimate in Panel A (B) generates a monthly average return of 1.42%, 2.93% and 7.98% (1.27%, 2.33%, and 5.08%), and standard deviation of 3.54%, 2.41% and 4.88% (2.68%, 2.64%, and 3.53%) for the three subsamples, respectively, which translate to an annual Sharpe ratio of 0.74, 1.96 and 3.11 (0.87, 1.34, and 2.69). As a benchmark, the market portfolio in Panel C of Table 2 has much lower average returns (1.26%, 0.91%, and 0.92%) and higher standard deviations (6.87%, 4.33%, and 4.23%) for the first and second subsamples, which produce lower Sharpe ratios (0.34, 0.18, and 0.32) than that for the equal and value-weighted long-short portfolios.

The skewness and kurtosis in the fourth and fifth column of Table 3 suggest that the equal- and value-weighted long-short portfolios based on the SOLS estimation have higher probabilities of generating extremely positive returns than the market portfolio. The skewness is 1.59 and 1.04 for the equal (value)-weighted long-short portfolio over the entire sample, higher than the market portfolio skewness of 0.66. Besides, the kurtosis in Panel A (B) is 5.76 (3.41) for the equal (value)-weighted long-short portfolio over the whole sample, which is much lower than for the market portfolio kurtosis of 10.72 in Panel C. Both cases suggest that the equal- and value-weighted long-short portfolios' returns are more likely to follow a normal distribution.

Next, we examine whether the well-documented Carhart (1997) four-factor model can explain the profits generated by equal- and value-weighted long-short portfolios based on the SOLS estimation. Table 4 reports the estimates of the monthly alphas and the four factors' exposures, which include market (MKT), size (SMB), value (HML), and momentum (WML) of the Carhart (1997) model over the entire out-of-sample period (1932:01 to 2020:12) and the subsamples from 1932:01 to 1959:12, 1960:01 to 1989:12, and 1990:01 to 2020:12. The equal-and value-weighted long-short portfolios based on the SOLS model consistently generate a sizable risk-adjusted return over time. The second column of Panel A (B) shows that the monthly alpha is 4.24% (3.02%) for the equal (value)-weighted portfolio over the full sample and 1.25%, 2.91%, and 7.87% (1.25%, 2.36%, and 5.31%) for the three subsample periods, respectively, all of which are statistically significant at the 1% level.

Besides, both the equal- and value-weighted long-short portfolios exhibits positive but essentially zero exposure to the market (MKT) factor. However, the equal-weighted long-short portfolio in Panel A produces significant positive exposures to the size (SMB) factor and significant negative exposure to the momentum (WML) factor. The value-weighted long-short portfolio in Panel B exhibits insignificant exposure to the size (SMB) and value (HML) factors but show negative and significant exposure to the momentum (WML) factor for the full and three subsamples, all of which are statistically significant at 1% level. Moreover, the last column of Panel A indicates that the Carhart (1997) four-factor model can explain 7.50% (3.91%) of the equal (value)-weighted long-short portfolio return movement for the whole out-of-sample estimation, and 20.15%, 6.94%, and 13.68% (8.01%, 6.30%, and 5.53%) for the three subsamples, respectively.

### *4.3. Decomposition Analysis*

We use Hou and Loh's (2016) covariance decomposition approach to explore whether the well-known factors (momentum, size, book-to-market ratio, operating profit, and investment) share the information of individual technical indicators in explaining the cross-sectional expected stock returns.

Table 5 measures the fraction of the cross-sectional expected return determinant of technical indicators captured by the four momentums, evaluated based on individual firm returns over the past 3, 6, 9, and 12 months. The 14 technical variables and the four momentums are constructed based on individual-level trend-following rules. However, we can see that all the four momentums fail to contribute to the explanatory power of the 14 technical variables in explaining cross-sectional equity expected returns. The explained fractions, *E(C/Y)*, attributable to MOM3, MOM6, and MOM9 in Panels A, B, and C are primarily negative and insignificantly related to the cross-sectional determinant captured by the 14 technical variables, except for the technical variable VOL (1,9), which is positive but statistically insignificant. Moreover, the results in Panel D show that 13 out of 14 of the fractions are small and insignificantly positive, and the fraction for VOL (1,9) is negative and insignificant, which suggest that MOM12 also fails to share the explanatory information of all the technical indicators in explaining cross-sectional stock expected returns.

We further investigate the well-documented four characteristics: market capitalization, book-to-market ratio, investment, and operating profit. Table 6 shows that none of the four candidate variables explain the cross-sectional determinant obtained by the technical indicators. The explained fractions attributable to the book-to-market ration, size, and investment are either insignificantly negative or negligibly small, and statistically insignificant. Panel D reports a similar result that operating profit is uncorrelated with the cross-sectional stock returns determinant provided by the technical indicators. Therefore, we conclude that the well-known four factors have an insignificant contribution to all the 14 technical variables in explaining cross-sectional stock expected returns.

## 5. Conclusion

We apply 14 technical indicators into the smoothed OLS model to estimate individual stock expected returns in the cross-section, using the Fama French three-factor model and the historical average as benchmarks. Our results show that technical indicators generate lower estimation error than the Fama French three-factor model and exhibit statistically significant out-of-sample explanatory power in determining cross-sectional equity expected returns, and the result is significant over time. The traditional time-series out-of-sample $R^2_{TSOS}$ and the cross-sectional out-of-sample $R^2_{CSOS}$ defined by Han, He, Rapach, and Zhou (2020) are positive and significant. Moreover, we measure the economic value of the cross-sectional model by constructing the value- and equal-weighted long-short portfolios based on the estimated returns of the SOLS model. We find that both the value- and equal-weighted long-short portfolios generate a sizable monthly profit, much higher than the simple market portfolio returns. Lastly, we show that the four well-known determinants of cross-sectional stock returns (momentum, market capitalization, book-to-market ratio, operating profit, and investment) fail to explain the cross-sectional determinant captured by the technical indicators.

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**Table 1: Cross-Sectional Out-of-Sample $R^2$ and Averaged Mean Squared Error**

| | Full Sample | Subsamples | | |
|---|---|---|---|---|
| | 1932:01-2020:12 | 1932:01-1959:12 | 1960:01-1989:12 | 1990:01-2020:12 |
| Panel A: Value-Weighted $R^2_{CSOS}$ and MSE | | | | |
| $R^2_{CSOS}$ | 0.594% [4.36]*** | 0.472% [3.18]*** | 0.451% [2.36]*** | 0.841% [2.65]*** |
| $\overline{MSE}_{SOLS}$ | 0.647% | 0.609% | 0.556% | 0.778% |
| $\overline{MSE}_{NAIVE}$ | 0.653% | 0.603% | 0.559% | 0.790% |
| $\overline{MSE}_{D\iota f}$ | 0.006% [5.53]*** | 0.002% [3.11]*** | 0.003% [3.25]*** | 0.012% [4.32]*** |
| Panel B: Equal-Weighted $R^2_{CSOS}$ and MSE | | | | |
| $R^2_{CSOS}$ | 1.121% [19.27]*** | 0.361% [4.61]*** | 0.892% [10.09]*** | 2.026% [18.63]*** |
| $\overline{MSE}_{SOLS}$ | 2.011% | 1.442% | 1.513% | 3.005% |
| $\overline{MSE}_{NAIVE}$ | 2.035% | 1.445% | 1.523% | 3.063% |
| $\overline{MSE}_{D\iota f}$ | 0.024% [15.43]*** | 0.003% [3.44]*** | 0.009% [9.51]*** | 0.058% [15.44]*** |
| | 1963:01-2020:12 | | 1963:01-1989:12 | 1990:01-2020:12 |
| Panel C: Value-Weighted MSE (Fama French Three-Factor Model) | | | | |
| $\overline{MSE}_{SOLS}$ | 0.662% | | 0.548% | 0.761% |
| $\overline{MSE}_{FF3}$ | 0.722% | | 0.599% | 0.828% |
| $\overline{MSE}_{D\iota f}$ | 0.060% [7.08]*** | | 0.051% [6.76]*** | 0.0672% [4.70]*** |
| Panel D: Equal-Weighted MSE (Fama French Three-Factor Model) | | | | |
| $\overline{MSE}_{SOLS}$ | 2.275% | | 1.430% | 3.006% |
| $\overline{MSE}_{FF3}$ | 2.371% | | 1.481% | 3.141% |
| $\overline{MSE}_{D\iota f}$ | 0.096% [8.29]*** | | 0.051% [6.41]*** | 0.135% [6.66]*** |

Table 1 reports the cross-sectional out-of-sample $R^2$ statistics ($R^2_{CSOS}$) and averaged mean squared errors (MSE) for the SOLS model ($\overline{MSE}_{SOLS}$) and Fama French three-factor estimation ($\overline{MSE}_{FF3}$). Panels A and B (C and D) reports the cross-sectional performance for the SOLS (Fama French three-factor) model, and we report the results for the three (two) subsamples from the third (fourth) to the fifth column. $R^2_{CSOS}$ in the first row of Panel A (B) shows the value (equal)-weighted out-of-sample $R^2$ statistics. $\overline{MSE}_{SOLS}$ in the second (first) rows of Panels A and B (C and D) represent the value-weighted and equal-weighted mean squared errors for the SOLS model based on 14 joint technical indicators. $\overline{MSE}_{NAIVE}$ in the third row of Panels A and B is the naïve benchmark value and equal-weighted mean squared errors introduced by Han, He, Rapach, and Zhou (2020), which is simply the cross-sectional return variance. $\overline{MSE}_{FF3}$ in the second row of Panels C and D are the value-weighted and equal-weighted mean squared errors based on the estimation of the Fama French three-factor model. We present the averaged MSE difference ($\overline{MSE}_{D\iota f}$) between the SOLS model and the competing model in the last row of each panel. *** indicates statistical significance at the 1% level.

**Table 2: Time-Series Out-of-Sample $R^2$ and Averaged Mean Squared Error**

| | Full Sample | Subsamples | | |
|---|---|---|---|---|
| | 1932:01-2020:12 | 1932:01-1959:12 | 1960:01-1989:12 | 1990:01-2020:12 |
| Panel A: Value-Weighted $R^2_{TSOS}$ and MSE (Historical Mean) | | | | |
| $\bar{R}^2_{TSOS}$ | 4.89% | 2.69% | 3.20% | 3.98% |
| $\overline{MSE}_{SOLS}$ | 1.49% | 0.84% | 1.02% | 1.31% |
| $\overline{MSE}_{HISM}$ | 1.58% | 0.87% | 1.09% | 1.38% |
| Panel B: Equal -Weighted $R^2_{TSOS}$ and MSE (Historical Mean) | | | | |
| $\bar{R}^2_{TSOS}$ | 4.24% | 3.74% | 2.91% | 3.70% |
| $\overline{MSE}_{SOLS}$ | 3.68% | 2.37% | 1.86% | 3.34% |
| $\overline{MSE}_{HISM}$ | 3.86% | 2.41% | 1.95% | 3.49% |
| Panel C: Positive percentage of MSE-F | | | | |
| P_MSEF | 87.93% | 80.01% | 87.43% | 82.31% |
| | Full Sample | Subsamples | | |
| | 1963:01-2020:12 | | 1963:01-1989:12 | 1990:01-2020:12 |
| Panel D: Value-Weighted MSE (Fama French Three-Factor Model) | | | | |
| $\overline{MSE}_{SOLS}$ | 1.39% | | 0.97% | 1.28% |
| $\overline{MSE}_{FF3}$ | 1.72% | | 1.21% | 1.62% |
| Panel E: Equal-Weighted MSE (Fama French Three-Factor Model) | | | | |
| $\overline{MSE}_{SOLS}$ | 3.82% | | 2.79% | 5.62% |
| $\overline{MSE}_{FF3}$ | 4.15% | | 3.02% | 5.98% |
| Panel F: Positive percentage of MSE-F | | | | |
| P_MSEF | 87.90% | | 82.76% | 86.40% |

Table 2 reports the traditional time-series out-of-sample $R^2$ statistics and averaged mean squared errors for the SOLS model ($\overline{MSE}_{SOLS}$), historical mean model ($\overline{MSE}_{HISM}$), and Fama French three-factor estimation ($\overline{MSE}_{FF3}$). Panels A, B, and C (D, E, and F) show the cross-sectional performance comparison between the SOLS model and historical mean (Fama French three-factor) model over the full sample 1932:01(1963:01) to 2020:12 and three (two) subsamples: 1932:01 to 1959:12, 1960:01 to 1989:12, and 1990:01 to 2020:12 (1963:01 to 1989:12, and 1990:01 to 2020:12). $\bar{R}^2_{TSOS}$ in the first row of Panel A (B) shows the value (equal)-weighted time-series out-of-sample $R^2$ statistics. $\overline{MSE}_{SOLS}$ in the second (first) row of Panels A and B (E and F) represent the equal-weighted and value-weighted mean squared errors for the SOLS model based on 14 joint technical indicators. $\overline{MSE}_{HISM}$ ($\overline{MSE}_{FF3}$) in the third (second) row of Panels A and B (E and F) is the value-weighted and equal-weighted mean squared errors of the historical mean (Fama French three-factor) model. P_MSEF in Panel C (F) represents the positive percentage of MSE difference between the historical mean (Fama French three-factor) model and the SOLS model.

**Table 3: Long-Short Performance**

| Period | Mean | STD | Skewness | Kurtosis | Ann. SR |
|---|---|---|---|---|---|
| Panel A: Equal-Weighted Long-Short Portfolio | | | | | |
| 1932:01-2020:12 | 4.22% | 4.71% | 1.59 | 5.76 | 2.90 |
| 1932:01-1959:12 | 1.42% | 3.54% | 3.47 | 31.38 | 0.74 |
| 1960:01-1989:12 | 2.93% | 2.41% | 0.48 | 0.67 | 1.96 |
| 1990:01-2020:12 | 7.98% | 4.88% | 1.28 | 3.84 | 3.11 |
| Panel B: Value-Weighted Long-Short Portfolio | | | | | |
| 1932:01-2020:12 | 2.96% | 3.40% | 1.04 | 3.41 | 2.73 |
| 1932:01-1959:12 | 1.27% | 2.68% | 0.71 | 3.45 | 0.87 |
| 1960:01-1989:12 | 2.33% | 2.64% | 0.58 | 0.94 | 1.34 |
| 1990:01-2020:12 | 5.08% | 3.53% | 1.27 | 4.20 | 2.69 |
| Panel C: Market Portfolio | | | | | |
| 1932:01-2020:12 | 1.02% | 5.23% | 0.66 | 10.72 | 0.50 |
| 1932:01-1959:12 | 1.26% | 6.87% | 1.09 | 10.10 | 0.34 |
| 1960:01-1989:12 | 0.91% | 4.33% | -0.28 | 2.35 | 0.18 |
| 1990:01-2020:12 | 0.92% | 4.23% | -0.55 | 1.22 | 0.32 |

Table 3 reports the summary statistics, including mean, standard deviation (STD), skewness, kurtosis, and annual Sharpe ratio (Ann. SR) of the monthly profits for equal (value)-weighted long-short portfolio constructed from the out-of-sample estimation of cross-sectional stock returns based on the SOLS model and market portfolio. We present the equal (value)-weighted long-short portfolio results based on the SOLS estimate in Panel A (B) and the results for the market portfolio in Panel C. The first row of each panel shows the summary statistics for the full 1932:01 to 2020:12 period, and the second to the last row reports the results for the three subsamples periods (1932:01 to 1959:12, 1960:01-1989:12, and 1990:01-2020:12).

**Table 4: Alphas and Factor Exposure**

| Period | alpha | MKT | SMB | HML | WML | $R^2$ |
|---|---|---|---|---|---|---|
| Panel A: Equal-Weighted Portfolio Risk-Adjusted Returns | | | | | | |
| 1932.01-2020.12 | 4.24% | 0.07% | 0.21% | -0.07% | -0.10% | 7.50% |
| | [29.31]*** | [2.26]** | [4.57]*** | [-1.52] | [-5.00]*** | |
| 1932.01-1959.12 | 1.25% | 0.04% | 0.26% | 0.09% | -0.05% | 20.15% |
| | [6.90]*** | [0.97] | [4.54]*** | [1.56] | [-1.92]** | |
| 1960.01-1989.12 | 2.91% | 0.03% | 0.19% | 0.04% | -0.03% | 6.94% |
| | [21.54]*** | [0.87] | [3.96]*** | [0.68] | [-1.36] | |
| 1990.01-2020.12 | 7.87% | 0.15% | 0.29% | -0.00% | -0.14% | 13.68% |
| | [31.67]*** | [2.50]*** | [3.63]*** | [-0.04] | [-4.34]*** | |
| Panel B: Value-Weighted Portfolio Risk-Adjusted Returns | | | | | | |
| 1932.01-2020.12 | 3.02% | 0.00% | 0.04% | -0.00% | -0.08% | 3.91% |
| | [28.37]*** | [0.27] | [1.31] | [-0.05] | [-5.26]*** | |
| 1932.01-1959.12 | 1.25% | 0.00% | 0.03% | 0.06% | -0.05% | 8.01% |
| | [8.49]*** | [0.16] | [0.69] | [1.26] | [-2.66]*** | |
| 1960.01-1989.12 | 2.36% | 0.07% | 0.06% | 0.13% | -0.08% | 6.30% |
| | [15.87]*** | [2.04]** | [1.09] | [2.17]** | [-3.06]*** | |
| 1990.01-2020.12 | 5.31% | -0.06% | 0.12% | -0.03% | -0.10% | 5.53% |
| | [26.81]*** | [-1.13] | [1.90]* | [-0.46] | [-4.07]*** | |

Table 4 reports the monthly risk-adjusted returns of the Carhart four-factor model for the equal and value-weighted long-short portfolios constructed by the out-of-sample estimate of cross-sectional stock expected returns based on the SOLS model. MKT is the “market excess return” factor; SMB is the “small firm size minus big firm size” factor; HML is the “high firm value minus low firm value” factor; WML is the “winner minus loser” momentum factor. The first row of each panel shows the full sample estimation results for the equal and value-weighted long-short portfolios, and the third to the fifth rows report the results for the three subsamples. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

**Table 5: Decomposition: Momentum Factors**

| Indicators | E(C/Y) | E(R/Y) | E(C/Y) | E(R/Y) | E(C/Y) | E(R/Y) | E(C/Y) | E(R/Y) |
|---|---|---|---|---|---|---|---|---|
| | Panel A: MOM3 | | Panel B: MOM6 | | Panel C: MOM9 | | Panel D: MOM12 | |
| MA(1, 9) | -0.70 [-1.81]* | 1.70 [4.38]*** | -0.38 [-1.17] | 1.37 [4.26]*** | -0.13 [-0.58] | 1.13 [4.99]*** | 0.01 [0.09] | 0.99 [6.61]*** |
| MA(1, 12) | -0.55 [-1.97]** | 1.55 [5.52]*** | -0.33 [-1.25] | 1.33 [5.08]*** | -0.12 [-0.60] | 1.12 [5.48]*** | 0.01 [0.07] | 0.99 [6.96]*** |
| MA(2, 9) | -0.60 [-1.98]** | 1.60 [5.29]*** | -0.36 [-1.26] | 1.36 [4.77]*** | -0.12 [-0.59] | 1.12 [5.48]*** | 0.02 [0.13] | 0.98 [7.29]*** |
| MA(2, 12) | -0.49 [-2.10]** | 1.49 [6.36]*** | -0.32 [-1.34] | 1.32 [5.51]*** | -0.12 [-0.62] | 1.12 [5.76]*** | 0.01 [0.09] | 0.98 [7.31]*** |
| MA(3, 9) | -0.54 [-1.92]* | 1.54 [5.46]*** | -0.35 [-1.19] | 1.35 [4.56]*** | -0.12 [-0.53] | 1.12 [5.14]*** | 0.04 [0.27] | 0.96 [6.91]*** |
| MA(3, 12) | -0.42 [-2.09]** | 1.42 [7.07]*** | -0.31 [-1.33] | 1.31 [5.64]*** | -0.12 [-0.61] | 1.12 [5.76]*** | 0.02 [0.19] | 0.98 [7.28]*** |
| MOM(9) | -0.70 [-1.68]* | 1.70 [4.08]* | -0.48 [-1.17] | 1.47 [3.65]*** | -0.24 [-0.67] | 1.24 [3.52]*** | 0.07 [0.37] | 0.93 [4.84]*** |
| MOM(12) | -0.61 [-1.95]* | 1.61 [5.16]*** | -0.41 [-1.34] | 1.41 [4.62]*** | -0.20 [-0.76] | 1.2019 [4.52]*** | 0.04 [0.18] | 0.96 [4.64]*** |
| VOL(1, 9) | 16.11 [0.20] | -15.10 [-0.18] | 10.06 [0.18] | -9.06 [-0.16] | 2.95 [0.28] | -1.95 [-0.19] | -0.36 [-0.08] | 1.36 [0.31] |
| VOL(1, 12) | -8.27 [-0.31] | 9.27 [0.35] | -4.64 [-0.30] | 5.64 [0.37] | -2.06 [-0.26] | 3.06 [0.39] | 0.42 [0.38] | 0.58 [0.53] |
| VOL(2, 9) | -2.14 [-1.13] | 3.14 [1.65]* | -1.27 [-0.94] | 2.27 [1.68]* | -0.59 [-0.67] | 1.59 [1.80] | 0.03 [0.07] | 0.97 [2.69]*** |
| VOL(2, 12) | -1.44 [-1.41] | 2.44 [2.38]*** | -0.88 [-1.10] | 1.88 [2.35]*** | -0.40 [-0.72] | 1.40 [2.50]*** | 0.06 [01.9] | 0.94 [3.26]*** |
| VOL(3, 9) | -1.09 [-1.71]* | 2.09 [3.28]*** | -0.70 [-1.27] | 1.70 [3.09]*** | -0.31 [-0.79] | 1.31 [3.34]*** | 0.05 [0.26] | 0.95 [4.573]*** |
| VOL(3, 12) | -0.85 [-1.89]* | 1.85 [4.11]*** | -0.54 [-1.33] | 1.54 [3.79]*** | -0.24 [-0.75] | 1.24 [3.88]*** | 0.08 [0.40] | 0.92 [4.85]*** |

Table 5 reports the fraction of the four-firm characteristics that explain the cross-sectional variation captured by the 14 technical indicators. We apply the variance decomposition method of Hou and Loh (2016) to decompose each of the 14 technical indicators into the explained component and residual component. First, we apply the univariate cross-sectional regression as follow:

$r_{i,t} = \alpha_{i,t} + \theta_{j,t} x_{i,j,t-1} + \varepsilon_{i,t}$,

where $r_{i,t}$ ($x_{i,j,t-1}$) is the equity return (*j*th technical indicator) of stock *i* at month *t*. Second, we exam the relationship between the each technical indicator and each of the four momentum factors:

$x_{i,j,t-1} = a_{j,m,t-1} + \eta_{j,m,t-1} V_{i,m,t-1} + \epsilon_{i,j,m,t-1}$ for *m* = MOM3, MOM6, MOM9, MOM12,

Third, we decompose $x_{i,j,t-1}$ into the explained component and residual component based on the regression coefficients $\eta_{j,m,t-1}$:

$$\theta_{j,m,t} = \frac{Cov(r_{i,t},\ x_{i,j,t-1})}{Var(x_{i,j,t-1})} = \frac{Cov(r_{i,t},\ \eta_{j,m,t-1} V_{i,m,t-1})}{Var(x_{i,j,t-1})} + \frac{Cov(r_{i,t},\ a_{j,m,t-1} + \epsilon_{i,j,m,t-1})}{Var(x_{i,j,t-1})} = \theta^C_{j,m,t} + \theta^R_{j,m,t},$$

where $\eta_{j,m,t-1} V_{i,m,t-1}$ ($a_{j,m,t-1} + \epsilon_{i,j,m,t-1}$) is the related (residual) component of $x_{i,j,t-1}$. The 'E(C/Y)' ('E(R/Y)') column corresponds to the explained (residual) fraction $\frac{\theta^C_{j,m,t}}{\theta_{j,m,t}}$ ($\frac{\theta^R_{j,m,t}}{\theta_{j,m,t}}$). MOM3 (MOM6, MOM9, and MOM12) is the momentum calculated by the cumulative returns over past 3 (6, 9, and 12) months. ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

**Table 6: Decomposition: BM, Capitalization, Investment, and Operating Profit**

| Indicators | E(C/Y) | E(R/Y) | E(C/Y) | E(R/Y) | E(C/Y) | E(R/Y) | E(C/Y) | E(R/Y) |
|---|---|---|---|---|---|---|---|---|
| | Panel A: BM | | Panel B: Size | | Panel C: Investment | | Panel D: Operating Profit | |
| MA(1, 9) | -0.94 [-0.25] | 1.94 [0.52] | -0.01[-0.55] | 1.01 [58.80]*** | -0.04 [-0.70] | 1.04 [20.26]*** | 0.00 [0.00] | 1.00 [29.29]*** |
| MA(1, 12) | -0.39 [-0.38] | 1.39 [1.35] | -0.01 [-0.41] | 1.01 [58.97]*** | -0.01 [-0.47] | 1.01 [43.72]*** | 0.00 [0.16] | 1.00 [47.88]*** |
| MA(2, 9) | -0.29 [-0.46] | 1.29 [2.05]** | 0.00 [0.10] | 1.00 [63.77]*** | -0.01 [-0.72] | 1.01 [56.39]*** | 0.00 [0.19] | 1.00 [75.36]*** |
| MA(2, 12) | -0.20 [-0.50] | 1.20 [3.02]*** | 0.00 [0.08] | 1.00 [69.40]*** | -0.00 [-0.30] | 1.00 [82.08]*** | 0.00 [0.24] | 1.00 [81.71]*** |
| MA(3, 9) | -0.34 [-0.52] | 1.34 [2.04]** | 0.03 [1.14] | 0.97 [44.14]*** | -0.01 [-0.61] | 1.01 [51.91]*** | 0.01 [0.65] | 0.99 [84.71]*** |
| MA(3, 12) | -0.26 [-0.59] | 1.26 [2.89]*** | 0.01 [0.56] | 0.99 [70.12]*** | -0.00 [-0.05] | 1.00 [94.52]*** | 0.01 [0.88] | 0.99 [120.91]*** |
| MOM(9) | -0.34 [-0.52] | 1.34 [2.02]** | -0.03 [-0.85] | 1.03 [31.42]*** | -0.01 [-0.54] | 1.01 [44.49]*** | 0.00 [0.24] | 1.00 [57.76]*** |
| MOM(12) | -0.03 [-0.21] | 1.03 [6.95]*** | -0.00 [-0.16] | 1.00 [45.36]*** | -0.00 [-0.22] | 1.00 [62.99]*** | 0.03 [1.24] | 0.97 [35.22]*** |
| VOL(1, 9) | 0.49 [-0.90] | 0.50 [0.93] | 0.88 [0.08] | 0.12 [0.01] | 0.16 [0.52] | 0.84 [2.73]*** | -0.01 [-0.13] | 1.01 [13.10]*** |
| VOL(1, 12) | 1.13 [0.41] | -0.13 [-0.05] | -0.25 [-0.37] | 1.25 [1.82]* | -0.50 [-0.11] | 1.50 [0.33] | 0.53 [0.07] | 0.47 [0.06] |
| VOL(2, 9) | -0.76 [-0.27] | 1.76 [0.63] | -0.01 [-0.25] | 1.01 [20.67]*** | -0.07 [-0.74] | 1.07 [11.92]*** | 0.03 [0.78] | 0.97 [22.63]*** |
| VOL(2, 12) | -0.40 [-0.43] | 1.40 [1.51] | -0.03 [-0.85] | 1.03 [25.36]*** | -0.03 [-0.83] | 1.03 [27.38]*** | 0.03 [1.07] | 0.97 [37.93]*** |
| VOL(3, 9) | -0.79 [-0.34] | 1.79 [0.76] | 0.001 [0.28] | 0.99 [33.08]*** | -0.02 [-0.83] | 1.02 [36.13]*** | -0.00 [-0.07] | 1.00 [46.03]*** |
| VOL(3, 12) | -0.54 [-0.48] | 1.54 [1.37] | -0.01 [-0.24] | 1.01 [40.87]*** | -0.01 [-0.54] | 1.01 [56.10]*** | 0.01 [0.44] | 0.99 [61.93]*** |

Table 6 reports the fraction of the four-firm characteristics: BM, Size, Investment (INV), and operating profit (OP) that explain the cross-sectional variation in expected returns captured by the 14 technical indicators. We apply the variance decomposition method of Hou and Loh (2016) to decompose each of the 14 technical indicators into the explained component. First, we apply the univariate cross-sectional regression as follow:

$r_{i,t} = \alpha_{i,t} + \theta_{j,t} x_{i,j,t-1} + \varepsilon_{i,t}$,

where $r_{i,t}$ ($x_{i,j,t-1}$) is the equity return (*j*th technical indicator) of stock *i* at month *t*. Second, we exam the relationship between the technical indicator and the four momentums:

$x_{i,j,t-1} = a_{j,h,t-1} + \eta_{j,h,t-1} V_{i,h,t-1} + \epsilon_{i,j,h,t-1}$ for *h* = BM, Size, INV, OP,

Third, we decompose $x_{i,j,t-1}$ into the explained component $\theta^C_{h,t}$ and residual component $\theta^R_{j,h,t}$ based on the regression coefficients $\eta_{j,h,t-1}$:

$\theta_{j,h,t} = \frac{Cov(r_{i,t},\ x_{i,j,t-1})}{Var(x_{i,j,t-1})} = \frac{Cov(r_{i,t},\ \eta_{j,h,t-1} V_{i,h,t-1})}{Var(x_{i,j,t-1})} + \frac{Cov(r_{i,t},\ a_{j,h,t-1} + \epsilon_{i,j,h,t-1})}{Var(x_{i,j,t-1})} = \theta^C_{j,h,t} + \theta^R_{j,h,t}$,

where $\eta_{j,h,t-1} V_{i,h,t-1}$ ($a_{j,h,t-1} + \epsilon_{i,j,h,t-1}$) is the related (residual) component of $x_{i,j,t-1}$. The 'E(C/Y)' ('E(R/Y)') column corresponds to the explained (residual) fraction $\frac{\bar{\theta}^C_{j,h,t}}{\bar{\theta}_{j,h,t}}$($\frac{\bar{\theta}^R_{j,h,t}}{\bar{\theta}_{j,h,t}}$). ***, **, and * indicate statistical significance at the 1%, 5%, and 10% levels, respectively.

**Figure 1**

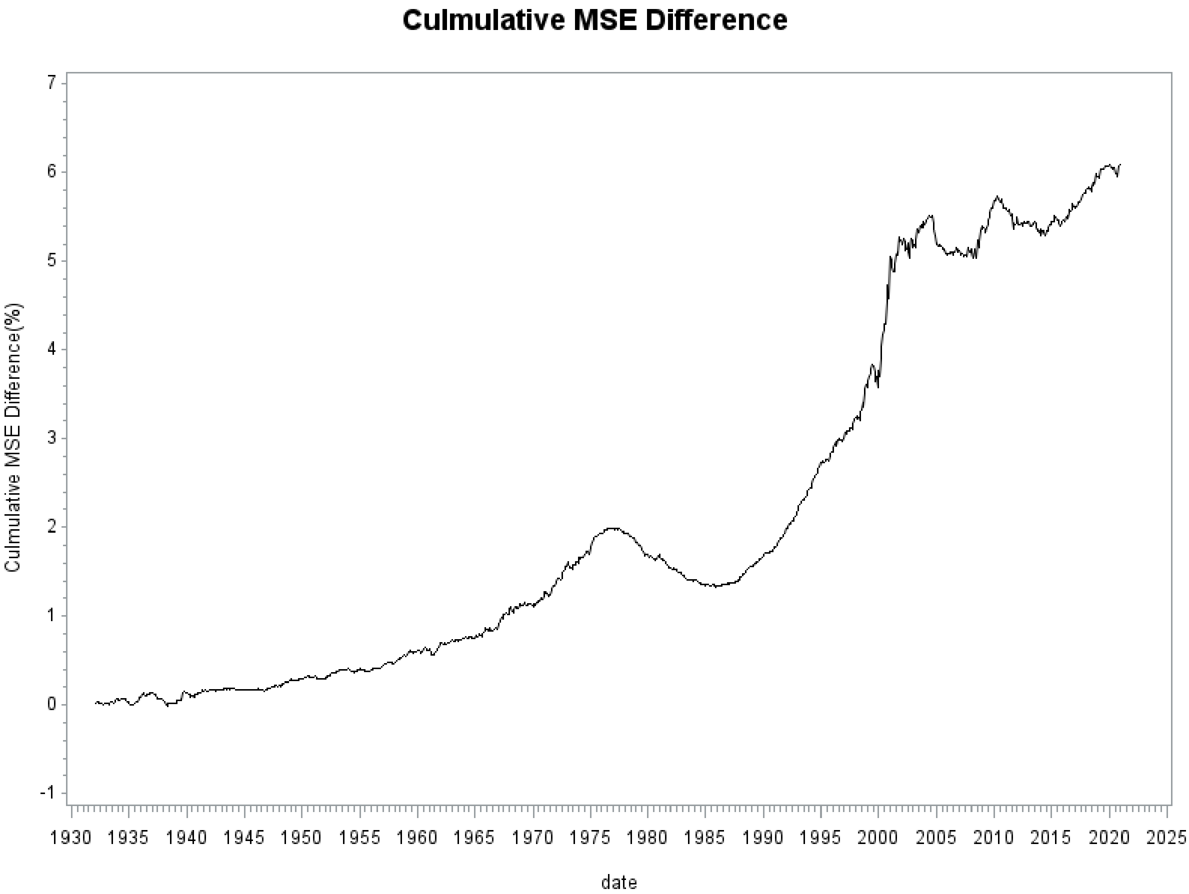


**Figure 1: Cumulative value-weighted cross-sectional mean squared error (MSE) difference between the historical mean model and the SOLS model, 1932:01–2020:12**

The figure depicts the out-of-sample performance of the cross-sectional SOLS regression model by using the cumulative value-weighted cross-sectional mean squared error (MSE) difference between the SOLS model and the historical mean model. An increasing trend of the line implies a better performance of the SOLS model, while a decreasing trend suggests a stronger ability of the prevailing mean model. The whole out-of-sample period spans from January 1932 to December 2020.