Equity Duration

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Abstract

The concept of bond duration was originally introduced by Macaulay (1938) and nowadays is well-established in the fixed-income literature. In this paper, I lift the same concepts from the fixed-income asset class and apply them to equities. I derive three candidate models for estimating the duration of a stock. The models are vastly different in their theoretical underpinnings, yet there is strong empirical evidence of positive co-movements between all three models in my sample. Furthermore, I investigate the relationship between the equity duration factor and various common equity factors. Empirical evidence suggests that low-duration stocks are also high-value, high-profitability, low-investment and low-risk stocks. In particular, there is a strong link between duration and the classical value factor – both theoretically and empirically. Importantly, however, the correspondence between the two factors is not one-to-one in my sample. I perform numerous empirical tests suggesting that a duration strategy out-performed a value-strategy in the period following the Great Financial Crisis (2007–08).

Contents

1 Introduction

The concept of bond duration was originally introduced by Macaulay (1938) and nowadays is well-established in the fixed-income literature. In this paper, I lift the same concepts from the fixed-income asset class and apply them to equities. However, if one considers the defining characteristics of stocks and bonds, it becomes apparent that it is much more difficult to reliably apply the concept of Macaulay duration – not least because of a stock's stochastic future cash flows and stochastic lifespan. I address these characteristic difficulties and build three candidate models for measuring the duration of a stock.

The three candidate models are interesting in their own right because the underlying economic/behavioural/statistical theory differs substantially between them, yet they all attempt to capture the same variable. As such, I compare the three models along multiple pertinent dimensions – the construction procedure, the underlying mathematical/economic theory and the capacity to yield risk-adjusted excess returns.

Having constructed a suitable model, I then shift the focus to the relationship between equity duration and various other common equity factors (including value, momentum and profitability). The equity duration literature provides compelling evidence that low-duration stocks tend to out-perform high-duration stocks, *ceteris paribus*. As such, I investigate whether a duration strategy (which takes long positions in low-duration stocks and short positions in high-duration stocks) is being driven purely by well-researched, well-established risk factors. I show – theoretically and empirically – the close connection between a duration strategy and a value strategy.

The paper is ordered as follows: Section 2 provides an overview of the key equity factors that are well understood by academics and practitioners. In Section 3, I show how the traditional concept of Macaulay duration can be theoretically applied to stocks and highlight the practical challenges associated with this application. An equity duration literature review is provided in Section 4. Section 5 is devoted to the construction of three alternative models for equity duration and a comparison of portfolio performance under each of the three candidates. In Section 6, I investigate the relationship between equity duration and several common equity factors. In particular, I attempt to fully understand the co-dynamics of equity duration and the classical value factor. Section 7 introduces potential areas of further research and Section 8 concludes.

2 Factor Investing

If one examines historical stock returns, it is apparent that there is significant heterogeneity in stock performance – that is to say, some stocks perform better than others. The natural question that follows from such an observation is: what are the desirable characteristics of a stock that result in its above-average performance? The theory of factor-investing essentially attempts to answer this question by providing a means of systematic asset evaluation.

A large body of literature exists pertaining to stock characteristics or factors (both economic and statistical) that meaningfully contribute to investment return and risk. The foundations of factor investing stem from seminal papers by Treynor (1961), Sharpe (1964), Lintner (1965) and Jensen et al. (1972) in which it was argued that the broad market was the sole contributing factor to an asset's performance. Assets were deemed to have a certain level of sensitivity (or exposure) to broad market movements. The higher the exposure, the higher the asset's expected return as compensation for baring the risk of being exposed to market movements. This risk-return paradigm is the core premise behind the Capital Asset Pricing Model (CAPM). The central implication of the model can be seen in the mathematical representation from Jensen et al. (1972):

$$\mathbb{E}(R_i) \triangleq \mathbb{E}(r_i) - r_{rf} = \beta_{i,M}(\mathbb{E}(r_M) - r_{rf}), \tag{2.1}$$

$$\text{where } \beta_{i,M} = \frac{\sigma(r_i, r_M)}{\sigma^2(r_M)} = \rho(r_i, r_M)\frac{\sigma(r_i)}{\sigma(r_M)}$$

$\mathbb{E}(r_i)$ is the expected return of asset i, r_{rf} is the rate of return of some risk-free asset, $\mathbb{E}(r_M)$ is the expected return of the market. $\beta_{i,M}$ is the beta coefficient on asset i, defined as the product of the asset and market return correlation and the relative standard deviation of the asset and market return. $\beta_{i,M}$ is an important measure of the sensitivity of an asset's return to the broader market (often referred to as systematic risk). Crucially by Equation 2.1, expected return is proportional to sensitivity to market fluctuations, as measured by the beta coefficient.

The simplicity of the single-factor CAPM was very enticing for early practitioners of factor-investing, however, it quickly became evident that such an elementary model was deficient. Fama and French (2004) and the references therein, provide strong empirical evidence against the CAPM. In particular, the CAPM framework implies that the excess return on any stock is completely explained by the market risk premium. Accordingly, running a time-series regression of excess stock returns on the excess market return should yield a statistically insignificant intercept:

$$R_{it} = \alpha_i + \beta_{i,M} R_{Mt} + \epsilon_{it}, \quad \forall t \in T \tag{2.2}$$

However, there is strong empirical evidence of non-zero intercept (or 'alpha') across stocks and across time – with certain stocks generating positive alpha, i.e. returns above the CAPM predicted value due to market exposure.

For Equation 2.1 to hold theoretically, the early literature relied on a number of strong assumptions on the efficiency of the market as well as the rationality and homogeneity of investors operating within it. As a way of mitigating the pitfalls of the original CAPM, Merton (1973) introduced a variation in which investors were concerned about additional macroeconomic factors that posed a risk to their current and future consumption power. Merton's Intertemporal Capital Asset Pricing Model (ICAPM) accounted for additional time-varying market risks (inflation, business cycles and interest rates, for example) that investors want to hedge against. Ross (1976) developed a similar multifactor counterpart to the CAPM in which multiple sources of macroeconomic risk were deemed significant in the explanation of asset performance. Ross' Arbitrage Pricing Theory (APT) provided a formula for the expected return of an asset as a linear function of macroeconomic factors:

$$\mathbb{E}(r_i) = r_{rf} + \beta_{i,1}(\mathbb{E}(r_1) - r_{rf}) + \cdots + \beta_{i,k}(\mathbb{E}(r_k) - r_{rf}) \tag{2.3}$$

The variables are defined as in the CAPM formula (2.1), with the generalisation that $\mathbb{E}(r_j)$ is the expected return of the j^{th} macroeconomic risk factor. $\beta_{i,j}$ measures the exposure of asset i to the j^{th} factor.

With the CAPM theory debunked and following the publication of Merton's ICAPM and Ross' APT, there was a proliferation of academic papers investigating the risk factors that appeared to explain asset performance. As an example of a CAPM anomaly, Banz (1981) demonstrated that, historically, smaller firm stocks tended to consistently outperform larger firms – an anomaly often referred to as the 'size effect'. Fama and French (1992) provided empirical evidence indicating that stocks currently selling at a low market price relative to some measure of fundamental or intrinsic value (such as market-to-book or price-to-earnings ratio) tended to outperform more expensive (typically, high-growth) stocks – the so-called 'value effect'. Fama and French (1993) derived a three-factor model that was shown to more adequately capture the performance of US equities than the original CAPM. The three factors used were: the market factor (from the original CAPM), the size factor and the value factor. Running a three factor time-series regression (analogous to Equation 2.2) results in a general decrease in the statistical significance of alpha across stocks and across time.

Two additional factors were subsequently introduced, resulting in the Fama and French (2015) five-factor model. The first is the so-called profitability factor, which stems from the observation that highly-profitable firms have historically outperformed firms with lower profitability (see, for example Novy-Marx (2012)). The second is the investment factor, built upon the observation that firms operating a relatively conservative investment policy have historically outperformed their more aggressive counterparts. Other seminal equity factors include the momentum factor originally theorised by Jegadeesh and Titman (1993). The authors observed that stocks that outperformed the market average in the recent past tended to continue to outperform in the near future. Finally, the so-called betting-against-beta factor (Frazzini and Pedersen (2014)) relies on the shortcomings of the CAPM model. Equation 2.1 suggests that a stock's beta coefficient determines expected excess returns – with higher beta generating higher returns, *ceteris paribus*. Empirically however, it has been shown that stocks with relatively low beta, still generate excess returns above those hypothesised in the CAPM.

The literature pertaining to equity risk-factors is vast, implying the existence of numerous stock return anomalies that cannot be explained by the traditional CAPM. Although the examples above have yielded long-term excess returns (i.e. returns over and above those predicted by Equation 2.1), it is common for factors to suffer prolonged periods of under-performance. For example, the value factor (investing in stocks with relatively high book-to-market ratios) has performed badly since the Great Financial Crisis (2007–08). There are a number of competing theories about the root cause of the value factor's under-performance. Perhaps (as one of the most widely researched and traded risk factors) the factor's premium has been arbitraged away. Perhaps post-crisis economic conditions are contributing to value's under-performance (see Maloney and Moskowitz (2020) and Israel et al. (2020) for a discussion on the impact of low post-crisis rates on the value premium). As such, factors appear to be both numerous and temporal.

From the examples above, one might infer that investors should simultaneously identify small, high-value, highly-profitable, high-momentum, low-investment and low-CAPM-beta stocks. Such a task is arduous and it would certainly be advantageous to identify a unifying metric that captures much of the performance across factors (a problem often referred to as 'taming the factor zoo'). In the analysis that follows, I introduce the concept of equity duration and investigate its relationship with aforementioned common equity factors. I explore whether the introduction of a duration factor adds incremental explanatory power over the traditional CAPM/APT factor models.

3 Equity Duration

The risk characteristics or factors discussed in the previous section are most often applied in the context of equities. On the other hand, a seminal measure of risk for fixed-income securities is that of duration. The concept of bond duration was originally introduced by Macaulay (1938) and nowadays is a well-established concept in fixed-income literature. Consider a traditional bond that promises a series of cash flows over a finite horizon. The (Macaulay) duration of the bond is defined as:

$$D = \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{P} \tag{3.1}$$

where CF_t denotes the cash flow at time t, r denotes the yield to maturity and P denotes the bond price. Thus bond duration is a weighted-average of the time each promised cash flow is due to be received over the life of the bond. The weights are a function of the present value of the cash flow payments. The duration of a bond is usually specified in years and is often thought of as the average maturity of the bond's payments.

Equation 3.1 provides the necessary definition of bond duration, however the real significance of this characteristic becomes apparent when considering the bond price's sensitivity to changes in yield. Differentiating the present-value formula for a bond's price with respect to the yield gives the equation:

$$P = \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t} \quad \Longrightarrow \quad \frac{\partial P}{\partial r} = -\frac{1}{1+r} \sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t} = -\frac{DP}{1+r} \tag{3.2}$$

Thus for small changes in yield, a rearrangement gives the first-order Taylor approximation:

$$\frac{\Delta P}{P} \approx -\frac{D}{1+r} \Delta r + O(\Delta r^2) \tag{3.3}$$

Equation 3.3 shows that, provided duration $D > 0$ and yield $r > -1$, an increase in yield results in a decrease in bond price. For a one unit change in yield, the magnitude of the change in price is (linearly) approximated by the term $\frac{D}{1+r}$, often referred to as modified duration.

The price sensitivity of a bond to changes in yield is crucial in both theoretical and applied contexts. However, if one considers the defining characteristics of stocks and bonds, it becomes apparent that it is much more difficult to reliably apply the concept of Macaulay duration to stocks. Amongst a number of other problematic characteristics, the main difficulties that arise when considering the duration of stocks rather than bonds are:

- Bonds are typically much more structured in terms of the frequency and magnitude of future cash flows. Coupon payments are usually made at pre-determined intervals in pre-determined amounts. Future cash flows from stocks are uncertain, irregular and notoriously difficult to predict.

- A bond typically promises regular future cash flows up until a pre-determined maturity. There is no corresponding concept of maturity for stocks. Their lifespan is theoretically unbounded and stochastic.

- Stock discount rates are most likely a function of a number of unobservable variables (real interest rates, risk-premia, inflation expectations, etc.). These variables are likely to vary over time and other confounding variables may well vary across stocks.

These problems make the application of Equation 3.1 to stocks challenging. The first point implies that both CF_t and t itself are stochastic and the second point implies that T is stochastic. The final point implies that r is likely to be a function of unknown and/or unobservable economic variables.

With these challenges in mind, much of the analysis that follows pertains to generating accurate models for estimating stock duration. Numerous papers have proposed approximations to Equation 3.1 based on balance-sheet information and forecasted cash flows. Other papers suggest a link between equity duration and other measurable stock characteristics – such as forecasted growth rates and approximating how 'bond-like' a stock is. If a reliable model exists, another topic of practical and theoretical significance is the relationship between equity duration and the previously-discussed equity factors.

4 Literature Review

The literature pertaining to equity duration generally focuses on two concepts:

1. *Modelling equity duration.*

A significant contribution to equity duration literature comes from Dechow et al. (2004) and subsequently Weber (2018). In order to address the uncertain cash flows and maturity of stocks, Equation 3.1 is split into a finite forecasting horizon and a subsequent infinite horizon in which cash flows are assumed to behave as a level perpetuity. This partitioning of cash flow forecasts is common in equity valuation models (e.g. dividend discount models), where one attempts to determine the fair value of a stock via the discounted value of expected payoffs. Some algebraic manipulation of Equation 3.1

results in the following formula for equity duration:[1]

$$D = \underbrace{\frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{P}}_{\text{finite horizon duration}} + \underbrace{\left(T + \frac{1+r}{r}\right) \times \frac{P - \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}}{P}}_{\text{terminal duration}} \tag{4.1}$$

where P traditionally represents the current bond price, however in this context, P is now the current market capitalisation of the stock. T is the end of the forecasting period (typically 10–15 years) and cash flows in the intermediate years are estimated via forecasting functions of balance-sheet variables that equate to realized cash flows. Broughton and Lobo (2014) build a model for equity duration that shares a lot of similarities with the aforementioned cash flow model. The authors' residual income model also truncates the forecasting horizon but assumes a constant upward trend in terminal cash flows (based on long-run macroeconomic variables) rather than a level-perpetuity.

Rather than forecasting future cash flows, an alternative estimation approach is to link duration to another (more observable and highly-correlated) variable. For example, Gormsen and Lazarus (2019) use analysts' long-term forecasts for earnings/dividend growth as a proxy for duration. The hypothesis is that high duration stocks are expected to receive a large proportion of their cash flows in the distant future rather than near future (hence high expectations for long-term growth). Conversely, low duration stocks are expected to pay-out a large proportion of cash flow value in the near future with an implicit low long-term growth in earnings. Weber (2018) provides similar evidence of strong positive correlation between equity duration and long-term growth forecasts.

The final approach to modelling equity duration stems from the literature pertaining to 'bond-like' stocks. The idea is based on the premise that bonds are structured assets with cash flows that are stable and predictable (at least relative to stocks). Baker and Wurgler (2009, 2012) and Koijen et al. (2017) provide empirical evidence that stocks behaving in a similar manner as bonds have yielded strong returns over the period 1963 – 2010. The authors consider a statistical approach to measuring the similarity of a stock to a long-term government bond and suggest that 'bond-like' stocks generally have relatively large and predictable near-term cash flows. Thus, there exists an implicit suggestion that 'bond-like' stocks may be closely related to low-duration stocks.

2. *Relationship between equity duration and other common risk factors.*

The second point pertains to investigating the relationship between equity duration and other common risk factors – assuming that one of the previous models accurately captures duration. If the magnitude of equity duration varies across stocks, is there evidence to suggest that investors should have a

[1] See Section 5.2 for full derivation.

preference for shorter or longer duration stocks? The seminal result from all of the aforementioned papers is that low duration stocks have, historically, generated higher risk-adjusted returns than high duration stocks. That is to say, on average, stocks with a larger proportion of their future cash flows tied up in the immediate future perform better than stocks with more distant cash flows. Van Binsbergen et al. (2013) and Van Binsbergen and Koijen (2017) provide excellent reviews of the equity duration literature. One popular economic argument for the existence of a (low) duration premium is that high duration stocks are expected to pay-out in the distant future, after experiencing substantial growth. Such growth stocks have, historically, been over-priced and have not performed to the level that investors expect. Low-duration and low-growth stocks with relatively stable cash flows, on the other hand, have generated positive risk-adjusted returns.

The suggested root causes of the (low) duration premium can generally be separated into two categories: 1) duration is an implicit proxy for other pre-existing alpha-generating risk factors, and 2) there exists behavioural biases that lead to systematic over (under) pricing of high (low) duration stocks.

In line with the first hypothesis, both Gonçalves (2018) and Gormsen and Lazarus (2019) provide empirical evidence that low duration stocks tend also to be high-value, high-profitability, low-investment and low-risk stocks. As mentioned in Section 2, stocks that exhibit such characteristics have tended to yield returns over and above those predicted by the CAPM – thus providing evidence of equity duration as a unifying framework for capturing common equity factors. As such, one possible explanation for the existence of the duration premium (i.e. low duration stocks outperforming high-duration stocks) is that the premium is captured by taking on exposure to well-known equity factors. Alternatively, Campbell and Vuolteenaho (2004) suggest that low-duration stocks are exposed to different risks than high-duration stocks. Considering the denominator in the summation of Equation 3.1, high duration stocks are exposed to substantial discount-rate risk (i.e. changes in yield, r), whereas low duration stocks are primarily exposed to cash flow risk (i.e. changes in expected cash flows, CF_t). If investors view these risks as being fundamentally different, then exposure to each risk will generate its own risk-premium and, thus, different prices for low/high duration assets. In particular, Santos and Veronesi (2005) and Gonçalves (2018) provide empirical evidence that investors demand a higher premium for exposure to cash flow risk versus discount rate risk and hence the existence of the duration risk premium. Other papers suggest that there are behavioural reasons for the short duration premium, e.g. investors tend to be overly-optimistic about long-term firm growth rates – resulting in underwhelming realized performance of high-duration firms (Lettau and Wachter (2007, 2011), Van Binsbergen et al. (2013), Van Binsbergen and Koijen (2017)).

Finally, it is also fruitful to examine equity duration not only across firms but along the time-series dimension. Evidence suggests that the short duration premium varies over time and appears to be positively correlated with both cross-sectional stock return volatility (Gonçalves (2018)) and investor sentiment (Weber (2018)). When cross-sectional volatility is high, it is more difficult to price high-duration, high-growth stocks relative to low-duration, value stocks. Indeed, Broughton and Lobo (2014) provide evidence that estimates for high-duration stocks are volatile relative to low-duration stocks. It is relatively more difficult to accurately forecast the distant cash flows from high-duration stocks, thus making such stocks more susceptible to mispricing. Similarly, when investor sentiment is high (that is, investors are relatively bullish), the duration premium also appears to increase. Baker and Wurgler (2009, 2012) show that high-sentiment periods result in over-optimistic evaluation of high-duration, high-growth stocks. Subsequent realised performance tends to be underwhelming – leading to higher risk-adjusted returns on low-duration, stable stocks.

5 Construction of three Equity Duration Models

As noted in the Literature Review, there are various competing theories on the best approach to modelling equity duration. Sections 5 & 6 are devoted to the construction and comparison of various equity duration models. In particular, I construct a cash flow duration model (Section 5.2), a growth forecast duration model (Section 5.3) and a 'bond-like' duration model (Section 5.4). In Section 6, I shift the focus to an in-depth comparison of the three models as well as investigation into the statistical/economic/behavioural driving-forces behind the (low) duration premium. In order to make reliable comparisons, I use identical sample periods and datasets across models.

5.1 Data

I study the monthly returns of US stocks between April 1998 and December 2019. Stock return data come from the Center for Research in Security Prices (CRSP) monthly stock file and the COMPUSTAT North America monthly securities dataset. In order to avoid survivorship bias and small firm effects,[2] I follow the literature by requiring stock prices to be above $5 and market capitalisation to be above $1 billion. Over the period 1998 – 2019, there are 6,886 stocks in the sample.

Stock-specific balance-sheet data is available annually from 1980 to 2019 and is obtained from the COMPUSTAT North America Annual Fundamentals file. Book Equity (BE) is constructed as per

[2]Oftentimes, anomalies are stronger for smaller firms due to their relative illiquidity. See, for example, Roll (1981).

Davis et al. (2000).[3] Market Equity (ME or P) is defined as the product of price-per-share and the total number of shares outstanding at fiscal year end. The book-to-market (BM) ratio for year t is then the book equity for the fiscal year ending in calendar year $t-1$ over the market equity as of December $t-1$. Return on equity (ROE) is the ratio of income before extraordinary items over lagged book equity. Sales growth (SG) is the percentage growth rate in net sales.

Daily US Treasury yield data and Treasury futures contract prices come from Bloomberg over the period April 1998 – December 2019. US Treasury data is available for a wide range of maturities (1 month – 30 years).

Where available, monthly (April 1998 – December 2019) long-run earnings-per-share growth forecasts come from the Institutional Brokers Estimates System (IBES) database. Growth rates are defined as the median analyst forecasted growth rate in earnings-per-share over the next 3–5 year business cycle. Corresponding realized growth rates also come from the IBES database.

Monthly investor sentiment data comes from the Sentiment Index in Baker and Wurgler (2006).[4] The data is in the form of a composite sentiment index that is based on the leading principal components of several underlying proxies for sentiment: the closed-end fund discount, NYSE share turnover, the number and average first-day returns on IPOs, the equity share in new issues, and the dividend premium.

Monthly returns (April 1998 – December 2019) for the value, size, profitability, investment and market factor are obtained from the Fama/French 5 Factor data library on Kenneth French's website.[5] Monthly returns for the momentum and betting-against-beta factor come from Factor/Style monthly datasets on the AQR website.[6]

[3]BE is defined as total shareholder's equity plus deferred taxes and investment tax credit minus the book value of preferred stock. Based on availability, I use the redemption value, liquidation value, or par value (in that order) for the book value of preferred stock. Following the literature, I first take the shareholders' equity number as reported by COMPUSTAT. If these data are not available, I calculate shareholders' equity as the sum of common and preferred equity. If neither of the two are available, I define shareholders' equity as the difference between total assets and total liabilities.

[4]http://people.stern.nyu.edu/jwurgler/

[5]https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html

[6]https://www.aqr.com/Insights/Datasets

5.2 Dechow et al. (2004), Weber (2018) Cash Flow Duration

The first duration model is derived from Dechow et al. (2004) and Weber (2018). The cash flow model takes the traditional measure of Macaulay duration (Equation 3.1) and attempts to make the definition more amenable to stocks by forcing some regularity on the dynamics and timing of future stochastic cash flows. In particular, cash flows are forecasted for a finite horizon and assumed to behave as a level perpetuity beyond this horizon. Mathematically, this implies the partitioning of Equation 3.1 into a finite forecasting horizon and a terminal infinite term:

$$D = \frac{\sum_{t=1}^{\infty} t \times \frac{CF_t}{(1+r)^t}}{P}$$

$$= \underbrace{\frac{\frac{\sum_{t=1}^{T} t\times \frac{CF_t}{(1+r)^t}}{P}}{\frac{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}}{P}} \times \frac{\sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}}{P}}_{\text{finite horizon duration}} + \underbrace{\frac{\frac{\sum_{t=T+1}^{\infty} t\times \frac{CF_t}{(1+r)^t}}{P}}{\frac{\sum_{t=T+1}^{\infty} \frac{CF_t}{(1+r)^t}}{P}} \times \frac{\sum_{t=T+1}^{\infty} \frac{CF_t}{(1+r)^t}}{P}}_{\text{terminal duration}} \tag{5.1}$$

where T now represents the horizon over which cash flows are forecasted. For $t > T$, cash flows are assumed to behave as a level perpetuity. The duration of such a perpetuity (beginning in T periods from now) is $T + \frac{1+r}{r}$.[7] Therefore, Equation 5.1 reduces to:

$$D = \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{P} + \left(T + {}^{1+r}/_{r}\right) \times \frac{\sum_{t=T+1}^{\infty} \frac{CF_t}{(1+r)^t}}{P}$$

$$= \frac{\sum_{t=1}^{T} t \times \frac{CF_t}{(1+r)^t}}{P} + \left(T + {}^{1+r}/_{r}\right) \times \frac{P - \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}}{P} \tag{5.2}$$

Thus, with T fixed and the current market capitalisation P being observable, what remains is to estimate future cash flows $CF_t, t = 1, \ldots, T$, and the expected return on equity r. The authors suggest

[7] The present value of a level perpetuity with constant cash flow, C, is:

$$PV_t = \sum_{t=1}^{\infty} \frac{C}{(1+r)^t}$$

Multiplying the result by $1 + r$, one finds that:

$$PV_t(1+r) - PV_t = \sum_{t=0}^{\infty} \frac{C}{(1+r)^t} - \sum_{t=1}^{\infty} \frac{C}{(1+r)^t} = C \qquad \Longrightarrow \qquad PV_t = \frac{C}{r}$$

The duration of a level perpetuity beginning at time T is (assuming $r > 0$):

$$D = \frac{\sum_{t=1}^{\infty} t \times \frac{C}{(1+r)^t}}{P} = \frac{\sum_{t=1}^{\infty} t \times \frac{C}{(1+r)^t}}{\frac{C}{r}} = r \times \sum_{t=1}^{\infty} t \times \frac{1}{(1+r)^t} = r \times \frac{\frac{1}{(1+r)}}{\left(1 - \frac{1}{(1+r)}\right)^2} = \frac{1+r}{r}$$

Therefore, the duration at time zero is $T + \frac{1+r}{r}$. $\square$

a methodology that balances accuracy with tractability.

Observe, firstly, that r is a cross-sectional constant across stocks. As we are primarily interested in ranking stocks from lowest to highest duration, the specific value of r does not impact the relative rankings. Therefore, without loss of generality, we can take the long-run estimated return on equity to be 12% as found empirically in Ibbotson and Chen (2003), Dechow et al. (2004) and Weber (2018). The authors show that results are robust to reasonable changes in r.

Furthermore, it is important to discuss the assumption that the terminal growth of cash flows behaves like a level perpetuity. One could alternatively assume that terminal cash flows grow at some long-term equilibrium rate such as the expected inflation rate, as per Claus and Thomas (2001) and Broughton and Lobo (2014). Note as before however, that in either case, the terminal duration is a cross-sectional constant across stocks. Therefore, the choice of terminal growth rate will not impact the sorting of portfolios based on implied duration.

Finally, the derivation of cash flow forecasts $CF_t,\ t = 1, \ldots, T$, relies on the so-called 'clean-surplus' accounting identity in which cash flows are partitioned in terms of earnings and changes in the book value of equity:

$$\begin{aligned} CF_t &= E_t - (BV_t - BV_{t-1}) \\ &= BV_{t-1} \times \Big(\frac{E_t}{BV_{t-1}} - \frac{BV_t - BV_{t-1}}{BV_{t-1}}\Big) \end{aligned} \tag{5.3}$$

where E_t is accounting earnings for period $(t-1, t)$ and BV_t is the book-value of equity for period $(t-1, t)$. Thus, in order to forecast future cashflows, one must forecast both ROE ($\frac{E_t}{BV_{t-1}}$) and growth in equity ($\frac{BV_t - BV_{t-1}}{BV_{t-1}}$). In other contexts, it has been well-established that both of these variables can be approximated by a mean-reverting autoregressive process of order one (Penman (1991), Nissim and Penman (2001), O'Brien (2017)). These papers also provide evidence that past sales growth is a better indicator of future equity growth than past equity growth. Accordingly, I follow the literature in using historical sale growth data when estimating equity growth dynamics. I estimate the autocorrelation coefficients for both variables using COMPUSTAT balance-sheet information over the period 1980–2019. Following the literature, I winsorize all variables at the 1% and 99% level in order to minimize the impact of outliers. I use an expanding window – updating the estimated coefficients as the pooled balance-sheet data becomes available.

At this juncture, I expand upon the simple models proposed in Dechow et al. (2004), Weber (2018) by allowing the dynamics of the autoregressive process to differ across industries. Previous models rely on the simplifying assumption that ROE and equity growth for all stocks follow the *same* autoregressive process. Fairfield et al. (2009) provides empirical evidence of intra-industry variation in these processes. As such, I split my stock sample based on GIC industry classification and empirically estimate the AR(1) process for *each* industry.

Across all industries, the average autocorrelation coefficients are 0.39 and 0.22 for ROE and equity growth, respectively. The industry-level autocorrelation coefficients are all bounded by zero and one – hence satisfying the necessary condition for stationarity and consistent conditional forecasts. Both Dechow et al. (2004) (ROE = 0.57, equity growth = 0.24) and Weber (2018) (ROE = 0.41, equity growth = 0.24) arrive at similar estimates – however, neither paper constructed rolling updates of model autocorrelation coefficients nor industry-level coefficients. The implicit simplifying assumption taken by the authors is that the coefficients remain stable over the sample period and across industries.

Following the previous discussion, I now have enough information to estimate equity duration from Equation 5.2. For each stock in the sample, I calculate the implied equity duration at year-end 1998–2019. Table 1 compares the summary statistics of my sample versus both Dechow et al. (2004), Weber (2018). Sample statistics are averaged across stocks and over time. My sample has a mean implied duration of about 19 years, with a standard deviation of over 3 years. In conjunction with the maximum and minimum statistics, there appears to be an observable variation in equity duration. Examining statistics individually across stocks and then across time (results not presented, for brevity), there is evidence of variation in duration across both dimensions. There is a natural variation in results across papers due to differing samples and sample periods – however, the summary statistics indicate, at least high-level, similarities.

Table 1 also highlights the fact that Equation 5.2 can yield negative values for equity duration. Such values can result from the numerator in the right-most term of Equation 5.2 being sufficiently negative. Observe that if the present value of the cash flows over the forecast horizon is sufficiently larger than the current market value, then a negative duration value will follow. Although initially surprising, negative duration may occur – for example – if the stock is under-priced and its market price is too low relative to the value of future expected cash flows, i.e. $P < \sum_{t=1}^{T} \frac{CF_t}{(1+r)^t}$.[8]

[8] As discussed in the Literature Review, low-duration stocks yield positive risk-adjusted returns. Hence, there is no contradiction in negative-duration stocks being potentially under-priced – indeed, the results in the Literature Review would actually imply the under-pricing.

As previously highlighted in the Literature Review, there exists evidence of correlation between the duration factor and the value factor: high (low) duration stocks pay-out a large (small) proportion of cash flows in the distant-future and are thus typically growth (value) firms. Dechow et al. (2004) provides a theoretical link between the two risk factors – albeit with overly-restrictive assumptions. As a first step, assume that the future cash flows in Equation 5.2 take the form of a level annuity, A. The duration and present value of such an annuity are: [9]

$$D_A = \frac{1+r}{r} - \frac{T}{(1+r)^T - 1}, \qquad PV_A = A \times \frac{1 - {}^1\!/_{(1+r)^T}}{r} \tag{5.4}$$

Plugging the above formulae into Equation 5.2 gives:

$$D = T + \frac{1+r}{r} - \frac{A}{P} \times \frac{T}{r} \tag{5.5}$$

Depending on our assumptions on the dynamics of A, we can derive a formula for equity duration in terms of two common measures of the value factor – earnings-to-price ratio and book-to-market ratio. For the first scenario, consider Equation 5.3 and assume that growth in equity is zero and that ROE is constant (i.e. $BV_t - BV_{t-1} = 0$ and ${}^{E_t}\!/_{BV_{t-1}} = {}^{E_0}\!/_{BV_{t-1}}, \forall t = 1, \ldots T$). In this case, Equation 5.3 implies that the value of the annuity equals current earnings ($A = E_0$) and Equation 5.5 reads:

$$D = T + \frac{1+r}{r} - \frac{E_0}{P} \times \frac{T}{r} \tag{5.6}$$

Alternatively, assume again that $BV_t - BV_{t-1} = 0, \forall t = 1, \ldots T$, but now we take the opposite extreme in assuming that forecasted ROE reverts immediately back to the long-term expectation (${}^{E_t}\!/_{BV_{t-1}} = r, \forall t = 1, \ldots T$). In this case, Equation 5.3 implies that the value of the annuity reduces to $A = rBV_0$ and Equation 5.5 reads:

$$D = T + \frac{1+r}{r} - \frac{BV_0}{P} \times T \tag{5.7}$$

Thus Equations 5.6 and 5.7 imply a negative linear relationship between duration and earnings-to-price/book-to-market (under restrictive assumptions). It is overly-optimistic to suggest a one-to-one correspondence, but there does appear to be some veritability to the hypothesis that low-duration stocks tend to be high-value stocks and *vice versa*.

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[9] PV of annuity, A:

$$PV_A = \sum_{t=1}^{T} \frac{A}{(1+r)^t} = \frac{A}{1+r} \times \sum_{t=0}^{T-1} \frac{1}{(1+r)^t} = \frac{A}{1+r}\left(\frac{1-(1+r)^{-T}}{1-(1+r)^{-1}}\right) = A \times \frac{1 - {}^1\!/_{(1+r)^T}}{r}. \quad \square$$

Duration, D_A, follows by plugging the above in to Equation 3.1.

Furthermore, note that Equations 5.6 and 5.7 are derived from the two extreme possibilities for mean-reversion of the AR(1) process that defines ROE dynamics. The negative relationship between equity duration and earnings-to-price is derived under the assumption of perfect persistence of the AR(1) process, whereas the negative relationship between equity duration and book-to-market is derived under the assumption of immediate mean-reversion of the AR(1) process. In practice, we expect that the true dynamics of the AR process are likely to lie somewhere in-between the two extremes. Thus it is interesting to note that the dynamics of the original model are bounded (in some sense) by two equations that yield perfect negative correlation between equity duration and common measures of value.

To mathematically access the relationship between the two duration approximations and the original model, I re-estimate equity duration for the 6,886 stocks using the IBES balance-sheet information over the period 1980–2019, as before. Thus, every year over the sample period, all stocks yield three alternative measures of equity duration (Equations 5.2, 5.6 and 5.7). Table 2 shows that the three duration measures yield exclusively positive pairwise correlations, using pooled stock data. There exists particularly strong correlation between the original duration estimator and the book-to-market proxy (Spearman: 0.92, Pearson: 0.62). It is informative to consider rank correlations in order to mitigate the impact of outliers when evaluating the relationship between the three estimators. Thus there is strong evidence that the two alternative proxies do roughly capture equity duration dynamics – lending credence to the hypothesised relationship between equity duration and value. However, to suppose a strictly one-to-one negative relationship between equity duration and value would be a vast over-simplification. Empirically investigating the true relationship is an important issue that will be further discussed in Section 6.

Reverting to the original duration model proposed by Dechow et al. (2004) and Weber (2018) (i.e. Equation 5.2), I construct duration-sorted portfolios and compare performance over the observation period. Following the literature, I construct quintile portfolios sorted on duration – with the convention that the first quintile portfolio, $Q1$, contains the lowest 20% of stocks as measured by equity duration and each subsequent quintile portfolio contains stocks of increasing duration. The duration statistics are updated annually at the end of December and (as per the literature) portfolios are re-balanced at the end of March the following year. This 3-month delay reflects the time until the balance-sheet information can realistically be built in to the model. I construct both equally-weighted and market-capitalisation-weighted portfolios. Stocks with insufficient return data or duration data are omitted.

Panel A of Table 3 shows the result of regressing the quintile portfolios on the market risk premium (i.e. the CAPM regression in Equation 2.2). I re-run the regression procedure using data from the full sample period (April 1998 – December 2019), post-crisis sample period (April 2011 – December 2019) and pre-crisis sample period (April 1998 – April 2007). As expected, alpha values are larger for low duration portfolios over the full sample period. Furthermore, there is strong evidence of CAPM betas (systematic risk) increasing with duration. In the pre-crisis sample, again there is evidence of CAPM alpha decreasing with duration. As a result, the $Q1 - Q5$ long/short duration factor generates statistically significant positive alpha over 1998–2007. CAPM betas are again monotonically increasing in duration. The post-crisis sample generates similar results – albeit without the same monotonicity of betas. Re-running the tests using market-capitalisation-weighted portfolios generates broadly similar results.

5.3 Gormsen and Lazarus (2019) Growth Forecast Duration

The second approach to modelling equity duration somewhat stems from an interesting link between duration and forecasted long-term growth (LTG) in earnings, established in Weber (2018). Over the sample period 1981–2014, the author provides empirical evidence that LTG forecasts increase monotonically and significantly with duration. As forecasts are averaged over increasing horizons (1 – 5 years), LTG forecasts remain largely stable for low duration stocks but decrease substantially for high duration stocks. That is to say, LTG forecasts are largely static over time for low-duration stocks, whereas LTG forecasts for high-duration stocks tend first to be over-optimistic and subsequently revised downwards. Before discussing some of the suggested mechanisms driving this observation, I examine whether the aforementioned results hold in my specific sample, when using the previous duration model.

I determine the average analyst long-term forecasted EPS growth within each of the five duration-sorted portfolios. In addition to cross-sectional comparison of the quintile portfolios, I also investigate how the forecasts evolve as we average over 1–5 years. Table 4 provides the results. Considering first the rows, we see that expected long-term growth is monotonically increasing from $Q1 \approx 6\%$ to $Q5 \approx 10\%$, and this result is consistent across 1–5 year averaging. In addition, Figure 1 plots the 5-year average growth forecasts for the quintile portfolios over time. The plot highlights the fact that the monotonic relationship between duration and growth forecasts is largely consistent across the entire sample period – in both economic expansions and contractions. Thus, in this sample, high-duration firms consistently have higher forecasted long-term EPS growth than low-duration firms. Furthermore, the second comment made by Weber (2018) is visible when examining the columns in Table 4. The decrease in LTG forecasts when averaging over longer periods is more drastic for high-duration firms, with a decrease in $Q5$ that is almost three times as large as in $Q1$.

Thus there appears to be a strong relationship between equity duration and forecasted growth rates. The relationship does appear to be somewhat different at the short and long end of the duration spectrum, however. Both Weber (2018) and Gormsen and Lazarus (2019) investigate the economic and/or behavioural explanations for the correlation between the two variables. A plausible connection between duration and forecasted growth stems from the fact that low-duration firms pay-out a large portion of cash flows in the immediate future and therefore analysts typically expect low long-term growth in EPS. Analogously, high-duration firms are expected to substantially increase cash flows in the distant future, yielding high long-term growth forecasts. In Section 6.4, I will empirically examine the various economic/behavioural hypothesis raised in the aforementioned papers that link equity duration to growth forecasts. As an initial step, however, I derive the forecasted growth model for equity duration from Gormsen and Lazarus (2019) using my dataset.

In an attempt to ensure reliable comparisons across equity duration models, I make the datasets, sample period and model construction procedure as homogeneous as possible across models. As before, I consider monthly returns for 6,886 large-cap US stocks over the period April 1998 – December 2019. Monthly long-term EPS forecasts are from the IBES database. Following Weber (2018) and Gormsen and Lazarus (2019), I use the median forecast. I winsorize all variables at the 1% and 99% level. Detailed forecasts are only provided for the largest stocks. In my sample, 4,890 stocks have sufficient forecast data – a capture rate of slightly over 71%.

Unlike the previous cash flow model for equity duration, Gormsen and Lazarus (2019) adopt a forecasted growth model – justified by the preceding discussion on the relationship between the two variables. The equity duration for stock $j \in J$ at time $t \in T$ is simply approximated by:

$$D_{j,t} = LTG_{j,t}, \quad \forall\ j \in J, t \in T \tag{5.8}$$

One immediate implication of the above specification is that we can no longer interpret the value of equity duration as we did in the previous cash flow model. That is to say, we cannot take results from Equation 5.8 and interpret them as indicating the average maturity (in years) of a stock's payments. Thus this model focuses on reliability and simplicity, but at the cost of some loss in interpretability. Importantly, as noted in the previous model, if we are primarily focused on ranking stocks from lowest to highest duration, then the interpretability issue isn't particularly important.

As a relevant aside, Gormsen and Lazarus (2019) consider contemporaneous growth forecasts, whereas Weber (2018) opt for 1–5 year rolling averages. In my dataset, I find stronger results (in terms of statistical significance) when considering rolling-averages. As a reasonable compromise between the two papers, I report the results from taking 1-year rolling averages of IBES growth forecasts.

As before, I construct quintile portfolios sorted from lowest to highest duration as measured by Equation 5.8. Portfolios are rebalanced at the end of each month over the period April 1998 – December 2019. Panel B of Table 3 shows the result of regressing the quintile portfolios on the market risk premium (i.e. the CAPM regression in Equation 2.2) over the same three sample periods as the cash flow duration model. As expected, lower duration quintiles generate statistically significant positive alpha, whereas the higher duration portfolios do not over the full sample period. Furthermore, CAPM betas are monotonically increasing with duration. The same broad conclusions can be made about the pre-crisis sample – CAPM alphas are monotonically decreasing in duration and CAPM betas are monotonically increasing. A simple $Q1 - Q5$ long/short duration factor generates positive alpha over 1998–2007, with the alpha coming from both the long and short leg. The post-crisis sample generates a few eye-catching results. Firstly, CAPM alphas are negative for low duration portfolios and the betas show no strong pattern. This result is contradictory to the findings in Dechow et al. (2004) over the period 1961–1999 and hints at a potential structural shift in the duration premium post-crisis. Indeed, if one refers back to the link between duration and value (established in the same paper) and one simultaneously considers the value factor's under-performance post-crisis (Maloney and Moskowitz (2020)) then perhaps the results are to be expected. That said, it is important not to necessarily draw causal relationships from a potentially coincidental correlation. The link between duration and value will continue to be explored in the coming sections.

5.4 Baker and Wurgler (2009, 2012) 'Bond-like' Duration

The final model to be considered comes from two papers that do not address the concept of equity duration directly. Unlike previous models, there is no requirement to forecast future stochastic cash flows nor rely on analysts' forecast for long-term growth in earnings. Instead, the goal is to generate a measure of how 'bond-like' a stock is (i.e. estimate 'bond-beta'). As discussed in Sections 3 & 4, it is substantially more difficult to measure Macaulay duration (Equation 3.1) for stocks than for bonds – not least because of their uncertain future cash flows and lifespan. Therefore, the hypothesis that I wish to test is whether or not there is a relationship between 'bond-like' stocks and low duration stocks. All else equal, a low-duration stock (with a large proportion of cash flow value expected in the near-term) may have somewhat more regular and predictable cash flows versus high duration stocks and hence be relatively 'bond-like'. Thus, I build a measure of 'bond-likeliness' in order to draw comparisons to the previous duration models and test some of the hypothesis in Baker and Wurgler (2009, 2012) and Koijen et al. (2017).

The first step in my analysis requires an estimator for 'bond-like' stocks. Rather than rely on economic arguments, I opt for a statistical/empirical approach as per Baker and Wurgler (2009, 2012). For each stock in the sample, I run a 5-year rolling regression of monthly excess stock returns on monthly excess returns on a 10-year US Treasury futures contract and monthly stock market excess returns:

$$r_{j,t} - r_{rf,t} = \alpha_j + \beta_j(r_{mkt} - r_{rf,t}) + b_j(r_{TY_{10}} - r_{rf,t}) + \epsilon_{j,t}, \quad \forall t = \tau - T, \dots, \tau. \tag{5.9}$$

where $T = 5$ years, τ is the current time and b_j is the bond-beta or 'bond-likeliness' of stock j. It is important to include excess returns to the stock market in Equation 5.9 in order to distinguish between stock-specific bond-beta (which is the variable we are attempting to capture) and general correlation between common fluctuations in the broad stock market and bond market.

Using a 5-year rolling window, I apply Equation 5.9 to each stock in the sample over the period April 1998 – December 2019. For stock j, b_j measures how 'bond-like' a stock is and is regarded as a proxy for low duration under the null-hypothesis. To maintain consistency across models, I build five equally-weighted portfolios, sorted on the bond-beta measure. I winsorize all variables at the 1% and 99% level. Portfolios are rebalanced at the end of each month.

Panel C of Table 3 shows the result of regressing the quintile portfolios on the market risk premium (i.e. the CAPM regression in Equation 2.2). As expected, alpha is decreasing with duration over the full sample period. Furthermore, CAPM betas are monotonically increasing with duration. The broad conclusions are the same if one considers the pre-crisis or post-crisis period. In this model, a simple $Q1 - Q5$ long/short duration factor generates positive alpha over the full sample, with the alpha mainly coming from the $Q5$ short leg.

Insummary, all three models are in alignment with the literature in providing evidence of the superior performance of low-duration portfolios and the monotone increasing relationship between duration and CAPM beta. There are no significant discrepancies between equally-weighted and value-weighted portfolios.

6 Comparative performance of duration models & the connection to common equity risk factors

The previous sections were devoted to the construction of three equity duration models:

- Cash flow model, as per Dechow et al. (2004) and Weber (2018).
- Growth forecast model, as per Gormsen and Lazarus (2019).
- Bond-beta model, as per Baker and Wurgler (2009, 2012).

Interestingly, the underlying economic/behavioural/statistical theory differs substantially between the candidate models, yet they all attempt to capture the same variable. Returning once again to Table 3, one can see similarities between the models as evidenced by the dynamics of duration-sorted portfolios. In particular, CAPM alpha is decreasing with duration, whereas CAPM beta is increasing. However, the results are somewhat weaker in the aftermath of the crisis – with the $Q1-Q5$ long/short duration factor no longer generating positive alpha. Quintile portfolios aside, I now investigate the statistical/economic relationship between the candidate models and, in particular, perform a more quantitative examination of the driving forces behind the three models' performance.

6.1 Constructing a duration risk-factor

It is often argued that comparative tests based on individual stock-level data may be adversely subject to stock-specific noise. Indeed, seminal papers by Friend and Blume (1970), Black et al. (1972), Fama and MacBeth (1973) prove that regression/correlation-based empirical tests can often be substantially stronger at the portfolio-level – particularly when noise at the stock-level is largely independent in the cross-section of stock returns.[10] Accordingly, I now consider aggregation to the portfolio level. This aggregation is also of practical use as we are typically interested in constructing and analysing the performance of portfolios based on some (supposedly alpha-generating) factor – rather than necessarily focusing on stock-specific minutiae.[11]

In this vein, I derive three instances of the duration factor – one for each model. As discussed in Sections 2–4, classical equity risk factors are generally constructed as per Fama and French (1993). Accordingly, stocks are sorted into six portfolios based on size and duration. Note that this double-sort approach to portfolio construction is different to the single-sort quintile construction used thus

[10] A quick proof of this is provided in Section 6.3.

[11] Note that portfolio aggregation is a linear operation on individual stock returns and does not adversely impact the reliability of linear regression tests. As shown in Fama and MacBeth (1973) for example, the CAPM beta of a portfolio is simply the weighted sum of the individual CAPM betas of the constituent assets. Thus portfolio-level linear regression tests essentially require one additional prior linear operation and can often reduce the noise in comparison to stock-level regression.

far (i.e. $Q1-$Q5 in Table 3). Stocks are split first into two portfolios: 'small' and 'big', based on median market-capitalisation. For each of these size portfolios, stocks are then sub-sorted into three portfolios on equity duration. Portfolios are rebalanced in April using data available at the previous year end. The final duration factor return is defined as:

$$R_{D,t} = \frac{1}{2}(R_{\text{ small, low duration, } t} + R_{\text{ big, low duration, } t}) \\ - \frac{1}{2}(R_{\text{ small, high duration, } t} + R_{\text{ big, high duration, } t}) \tag{6.1}$$

where '$R_{\text{ small, low duration, } t}$' refers to the time t return on the portfolio containing the 33% of stocks with the lowest duration within the 'small' size portfolio, for example.

Thus, from Equation 6.1 I can now construct daily returns on the three candidate duration factors over April 1998 – December 2019: cash flow duration factor, D_{CF}, forecasted growth duration factor, D_{EPS} and bond-beta duration factor, D_{BB}. Given the potential close link between equity duration and value, I also include the Fama-French value factor (hml) in the comparative tests. Figure 2 plots the one-year rolling average (annualised) return and volatility on the three candidate duration factors as well as the Fama-French value factor over the period 1999–2019. At least visually, the returns on the four factors appear to be strongly correlated up until the financial crisis. During and after the crisis, the time-series correlation between the factors becomes less clear-cut. The bond-beta duration factor is consistently the most volatile during and after the crisis. In addition, the value factor and bond-beta duration factor appear to shift from a positive relationship to a strongly negative one post-crisis. Thus there appears to be (at least anecdotal) evidence that factor correlations are time-dependent and can change sharply from the pre-crisis to post-crisis regime.

Figure 3 plots OLS regression line for each of the pairwise combinations of the four factors. Scanning each row from left-to-right, we see strong pairwise exposures for each combination of cash flow duration, growth forecast duration and value. The data points are typically clustered closely around the line of best fit, which yields an approximate 45 degree angle with the horizontal. In strong contrast, the third row of Figure 3 suggests that the linear relationship between the bond-beta duration factor and the other three factors is relatively weak. The line of best fit is noticeably inferior due to the erratic data points, however, the upward slope is an indication of some level of positive pairwise correlation.

As the previous plots suggest, Table 5 shows that all three candidate factors are positively correlated over the three sample periods. Panel C indicates that the general strength in correlation between the three measures was strongest in the pre-crisis period. The cash flow and growth forecast duration factors yield a positive (Pearson) correlation that is consistently around 0.7 over the three sample periods. The bond-beta and growth forecast duration factors are also strongly correlated over the three sample periods, although the correlation has reduced somewhat post-crisis. The final insight from Table 5 relates to the value factor. We see that both the cash flow and growth forecast duration factors are consistently (and strongly) correlated with value. The bond-beta duration factor initially co-moved with value, however the relationship has inverted post-crisis – leaving a (mild) net-negative correlation with value over 1998–2019.

Thus both Figure 2/3 and Table 5 provide strong evidence of a positive relationship between cash flow duration, growth forecast duration and the value factor. That said, the interpretation of the bond-beta model is more complex. We find that, although the bond-beta duration factor is consistently correlated with the other two models, it also has a negative (albeit weak) correlation with the value factor. There are many possibilities as to why this dynamic is present in the data. For example, in forming the hypothesis that bond-beta is a proxy for low-duration, there may be a number of other confounding factors at play that disrupt the relationship between variable and proxy. A full investigation into the bond-beta model is beyond the scope of this paper. Instead, I return my focus to examining the driving forces behind the equity duration risk factor. The bond-beta model is still useful for this examination as it can be insightful to understand where the model diverges from the cash-flow/growth-forecast models.

6.2 Equity duration and the connection to common equity risk factors

The previous section was primarily devoted to understanding the relationship between the three candidate duration models as well as the construction of a duration risk-factor. In order to better understand the root causes of the duration premium, I now examine the relationship between equity duration and common equity risk factors (value, size, momentum, investment, profitability and betting-against-beta).

As noted in Section 2, the CAPM asserts that expected return on a stock is a linear function of market beta – with no other variable lending marginal explanatory power (Equation 2.1). However, Section 2 also provides numerous empirical examples of CAPM anomalies, in which this simple relationship breaks down. Indeed, small, high-value, high-momentum, high-profitability, low-investment and low-market-beta stocks have all exhibited risk-adjusted return (or alpha) over and above those predicted by the CAPM.[12] By considering suitable long/short portfolios (as per Fama and French

[12]It is important to note that these anomalies have not existed across all sample periods. The under-performance of certain risk factors for long periods of time is an important issue that will be discussed in the proceeding sections.

(1993) and Section 5.5, for example), one can construct synthetic risk factor returns. Accordingly, I investigate the temporal performance of the duration factor and follow the literature by incorporating the factor into an ICAPM/APT-style asset-pricing model (Equation 2.3).

With the above equity risk factors in mind and having constructed the duration factor in Section 5.5, we now examine their co-movement – cross-sectionally and over time. In particular, the aim is to gain insight into the question: is the duration premium being driven purely by these factors or does it capture unique risk-adjusted returns?

Although originally used to compare performance of the three duration models, Table 3 is again relevant to this discussion. Considering the duration-sorted portfolios over the entire sample period, we see that CAPM alphas are generally decreasing and CAPM betas increasing with duration. Thus, low duration stocks appear to yield higher risk-adjusted returns and lower systematic risk than high duration stocks. However, the regression results are highly dependent on the sample period and there is a stark shift in performance pre and post-crisis. In particular, note that the post-crisis performance of the low duration portfolios is noticeably worse than pre-crisis. This result indicates that the duration premium may well be time-dependent, with lower risk-adjusted returns post-crisis. Furthermore, the $Q1 - Q5$ duration factor generates positive alpha across all three models pre-crisis, however there is no statistical significance post-crisis. The natural question that follows is: what conditions have changed post-crisis that have instigated the inferior performance of low duration stocks? The key to answering this question is to investigate the dynamic relationship between equity duration and common risk factors.

Following closely the empirical tests in Baker and Wurgler (2009, 2012), Weber (2018) and Gormsen and Lazarus (2019), I assess the aforementioned relationship via a series of time-series regressions. In particular, I regress monthly returns for each of the common equity factors on the excess market return, the size factor and the duration factor (constructed in Section 5.5):

$$\begin{aligned} r_{i,t} - r_{rf,t} = \alpha_i + \beta_{i,mkt}(r_{mkt,t} - r_{rf,t}) + \beta_{i,smb}(r_{smb,t} - r_{rf,t}) \\ + \beta_{i,dur}(r_{dur,t} - r_{rf,t}) + \epsilon_{i,t} \quad \forall t \in T \end{aligned} \tag{6.2}$$

where $r_{i,t}$ is the monthly return on the i^{th} risk factor. To gain a more complete insight into the temporal co-movement of factors, I run the regressions over the entire sample period, the post-crisis period (2011–2019) and the pre-crisis period (1998–2007). I also perform the three sub-sample tests for each of the three duration models – thus generating a 3×3 matrix of results. The factors I consider are value (*hml*), momentum (*umd*), profitability (*wmr*), investment (*cma*) and betting-against-beta (*bab*). The reader should refer to the Section 2 and/or the Literature Review for an overview of these risk factors.

Table 6 shows the results of applying Equation 6.2 to the data over the three sample periods and using the three alternative measures of equity duration. The key results can be summarised as follows:

- Consider, first, the cash flow and growth forecast duration models (left and centre columns, respectively). Over the three sample periods, both models generate very similar results. In the full sample, value, profitability, investment and betting-against-beta all load positively on the (low) duration factor – thus indicating that low duration stocks are also high-value, high-profitability, low-investment and low-risk stocks. Value and investment retain statistically significant positive exposure to duration over the pre and post-crisis period. Profitability and betting-against-beta are positively exposed to duration pre-crisis but there is no statistically significant exposure post-crisis. Momentum appears to be largely unexposed to duration over any of the three sample periods. Thus value/investment yield the most robust relationship with duration, with momentum being unrelated and profitability/betting-against-beta appearing to have correlation with duration that has weakened in recent years.

- The second observation pertains to the contrast between the cash-flow/forecasted-growth models and the bond-beta model. This contrast re-hashes a previous discussion on the bond-beta model. Whereas the cash flow and forecasted growth models attempt to capture duration directly, the bond-beta model focuses on 'bond-likeliness' – without a specific emphasis on low or high duration. The right-most column of Table 6 provides the results for the bond-beta duration model. In contrast to the other models, we see statistically significant exposure to the momentum factor across all three sample periods. A further contrast is the negative exposure to value over the full sample and post-crisis sample. Profitability and betting-against-beta are positively exposed to duration across all three samples – roughly aligning with the previous two models. Investment is largely unrelated to duration (with negative exposure in the post-crisis sample). The key point remains that the bond-beta duration model yields surprising results if we are to believe that it does indeed capture duration and not some other stock characteristic. Both the literature pertaining to equity duration and the previous two models suggest a weak (if any) relationship with momentum and a strong positive relationship with value. This discrepancy provides further evidence that the bond-beta model is likely to be capturing dynamics beyond duration.

6.3 Deep dive into value versus duration

Section 5.2 provides a strong theoretical connection between equity duration and common measures of value (book-to-market and earnings-to-price). In addition, Sections 6.1 & 6.2 complement this theoretical link with empirical results across the three duration models. With these results in mind, I now turn my attention to dis-entangling the duration premium from the value premium. I aim to first investigate whether the duration factor and/or duration-sorted portfolios generate returns that are significantly different from the value factor and/or book-to-market (BM) sorted portfolios. Secondly, I examine the performance of various factors that are double-sorted on value and duration. As mentioned in Section 2, the value factor has performed poorly post-crisis (particularly since 2017) and I look at the effect of neutralising value's exposure to duration.

In order to address the aforementioned hypotheses, I consider two broad empirical tests: 1) Statistically significant differences between portfolios sorted on value, duration and a combination of the two, and 2) Fama and MacBeth (1973) regression tests which are the archetypal method for estimating the risk-premia of the individual factors within a larger multi-factor model (i.e. Equation 2.3).

6.3.1 Returns to various value/duration-sorted portfolios

The first test relies on using the methodology from Fama and French (1993) and Section 6.1 to create value/duration double-sorted risk factors – using my usual sample of stocks. Specifically, I create four variants of duration/value factors. Firstly, consider a value factor that is neutral to duration by splitting stocks into three portfolios from lowest to highest duration. I subsequently split the stocks within each portfolio into a further three portfolios from low to high book-to-market – generating nine portfolios in total. To create a duration-neutral value factor, the following combination of portfolios is selected:

$$
\begin{aligned}
R_{\text{duration-neutral value, t}} &= \frac{1}{2}(R_{\text{ low duration, high BM, } t} + R_{\text{ high duration, high BM, } t}) \\
&- \frac{1}{2}(R_{\text{low duration, low BM, } t} + R_{\text{high duration, low BM, } t})
\end{aligned} \tag{6.3}
$$

where '$R_{\text{ low duration, high BM, t}}$' refers to the time t return on the portfolio containing the 33% of stocks with the highest book-to-market ratio within the portfolio containing the lowest 33% stocks sorted by duration, for example. Portfolios are equally-weighted and rebalanced in April of each year (using BM/duration data from the end of the previous calendar year). The duration-neutral value factor has a low and high duration portfolio in the long end and short end – hence duration-neutral.

Analogously, I produce a value-neutral duration factor by reversing the roles of BM and duration in the previous construction. The final two factor constructions are a simple single-sort on BM (generating a pure value factor) and, separately, a single-sort on duration (generating a pure duration factor).

Figure 4 shows the cumulative monthly return on the aforementioned four factors over the full sample and pre/post-crisis. As is common in the literature, I have scaled the returns on each factor to 10% (annualised) volatility in order to improve comparability. In addition, Table 7 provides statistics on the correlation between the four factors. An immediate result is the strong co-movement of all four factors across all periods. The pure duration and pure value factors are extremely correlated (0.94 pre-crisis, 0.87 post-crisis), as well as pure value and duration-neutral value (0.59 pre-crisis, 0.76 post-crisis). Interestingly, this jump in correlation post-crisis is by far the largest across all pairwise combinations and provides some evidence that the relationship between value and duration may be time-dependent. Specifically, neutralising the value factor's exposure to duration appears to have less differentiating effect to a pure value factor post-crisis.

Aside from the strong correlations, it is insightful to look at the relative performance of the factors, as seen in Figure 4. The two duration factors (pure and value-neutral) under-perform relative to the two value factors in the full sample, however, the relationship has flipped post-crisis. It is interesting to note that the duration-neutral value factor appears to magnify the performance of the pure value factor. That is to say, neutralising exposure to duration yields higher returns when value performs well (pre-crisis) and lower returns when value performs poorly (post-crisis). One noticeable exception is the duration-neutral value factor's strong performance during the late 1990s tech bubble. It has been well documented in the literature that (individually) value and duration strategies performed badly during this period (see, e.g. Van Binsbergen et al. (2013) or Van Binsbergen and Koijen (2017)).[13] However, neutralising value's exposure to duration appears to have acted as a successful hedge against exposure to the pure value/duration under-performance – generating positive cumulative returns throughout this crisis period.

6.3.2 Fama MacBeth Regression Analysis

The previous factor analysis provides some indication (in terms of correlations and cumulative returns) that the duration premium and value premium are closely related. However, the performance of the double-sorted factors somewhat muddles the story. In particular, the duration-neutral value factor

[13] During the Dot.Com bubble, prices for high-growth technology stocks soared – despite generally weak justification in terms of profitability or balance-sheet structure. As such, the value (and duration) strategy performed poorly due to the short positions in such technology stocks. After the Dot.Com era, both value and duration generated strong returns as valuations for high-growth stocks reverted back to more justifiable levels.

performed best over the entire sample period and remained resilient throughout Dot.Com bubble in the late 90s – when pure value and duration exhibited large losses. Furthermore, value-neutral duration achieved the highest cumulative return post-crisis – albeit very similar to the pure duration factor.

When one is tasked with disentangling one risk factor from another, a popular statistical/empirical approach is to conduct a two-stage regression procedure which was first proposed by Fama and MacBeth (1973). The procedure is used to estimate the risk-premia for the factors within an APT-style model. In this context, we are interested in testing whether duration yields a statistically significant risk-premium, even when value is included in the multi-factor model. Since the initial publication, many authors have expanded upon the statistical properties of this procedure (see, for example Chen et al. (1986), Cochrane (2009) and Petersen (2009)). Before summarising the specific numerical results from my analysis, it is first insightful to discuss the mathematics that underpin the Fama-MacBeth procedure.

Originally, Fama and MacBeth (1973) employed a two-step regression procedure in order to empirically test the CAPM linear risk-return relationship (Equation 2.1). In doing so the authors examined variations of the following multi-factor equation:

$$R_{it} = \alpha_i + \beta_{i,1} R_{Mt} + \beta_{i,2} R_{Mt}^2 + \ldots + \epsilon_{it}, \quad \forall\ i \in I,\ t \in T \tag{6.4}$$

where R_{it} is the excess return on asset i at time t and the quadratic market-return term is used to test the linearity of the CAPM. As in OLS regression, the error term, ϵ_{it}, is assumed to be zero mean and independent across time and across stocks. ϵ_{it} is also assumed to be independent of all regressors. Strictly speaking, returns are assumed to be normally distributed, however this assumption can be relaxed without affecting the consistency of OLS/Fama-MacBeth estimators.

In the context of understanding the connection between duration, value and stock returns, the two-step regression procedure is as follows:

- Given the time-series return for each stock and each risk factor, we regress individual stock returns on the risk factor returns. That is to say, for stock i:

$$R_{it} = \alpha_i + \beta_{i,1} F_{1,t} + \beta_{i,2} F_{2,t} + \ldots + \beta_{i,K} F_{K,t} + \epsilon_{i,t}, \quad \forall\ t \in T \tag{6.5}$$

  where $F_{k,t}$ is the excess return on the k^{th} risk-factor at time t. This generates estimates for the factor exposures (or loadings) for each of the individual stocks, $\hat{\beta}_{i,k},\ k = 1, \ldots, K$.

- For the second step, we compute cross-sectional regressions of stock returns on the estimated factor loadings from above. Therefore, for fixed time t:

$$R_{it} = \alpha_i + \gamma_{i,1}\hat{\beta}_{i,1} + \gamma_{i,2}\hat{\beta}_{i,2} + \ldots + \gamma_{i,K}\hat{\beta}_{i,K} + \eta_{i,t}, \quad \forall\ i \in I \tag{6.6}$$

This generates estimated risk-premia for each risk factor at the fixed time t: $\hat{\gamma}_{k,t},\ k = 1, \ldots, K$. The final estimate for the risk premium on the k^{th} factor over the sample period is the time-series average over $t = 1, \ldots, T$. Thus, the final risk-premium estimate and corresponding test statistic are:

$$\hat{\gamma}_k := \frac{\sum_{t=1}^{T} \hat{\gamma}_{k,t}}{T}, \tag{6.7}$$

$$\widehat{\sigma(\gamma_k)} := \sqrt{\frac{\sum_{t=1}^{T} (\hat{\gamma}_{k,t} - \hat{\gamma}_k)^2}{T}}, \qquad t(\hat{\gamma}_k) := \frac{\hat{\gamma}_k}{\widehat{\sigma(\gamma_k)}/\sqrt{T}} \tag{6.8}$$

The goal of the procedure outlined above is to estimate the expected premium attained by taking a unit exposure to a risk factor over time – as given by Equation 6.7. Before applying this procedure to the duration factor, there are a couple of pertinent points to be made in regards to Equation 6.8.

Firstly, for the standard errors to be consistent, we require the residuals, $\eta_{i,t}$, in Equation 6.6 to be homoskedastic and serially-uncorrelated.[14] Failing these requirements, it is important to substitute the usual standard errors with Newey-West heteroscedasticity and autocorrelation robust (HAC) standard errors.[15] This result is not specific to Fama-MacBeth regressions, but holds for OLS regression in general (see Petersen (2009) for a thorough analysis of Fama-MacBeth standard error correction procedures).

Another important issue is the so-called 'errors-in-variables' problem that necessarily arises in the two-stage regression procedure. In running the second stage (Equation 6.6), the right-hand-side regressors, $\hat{\beta}_{i,k},\ k = 1, \ldots, K$, are empirically estimated exposures from the first stage – hence, they are

[14] It is well-documented that stock returns are fairly weakly correlated across time, but heteroscedasticity or 'volatility clustering' is much more commonly seen in data (Cochrane (2009)). In this particular data set, approximately 77% of the stock return process fail to reject the null that returns are uncorrelated across time. The Ljung-Box test is a commonly used statistical test for autocorrelation. A simple plot of the time-series of stock returns shows clear heteroscedasticity – particularly during the crisis period.

[15] As shown in Verbeek (2008), for each risk factor, one needs to multiply the usual standard error by the correction term $\sqrt{N}$, defined via:

$$N := 1 + \sum_{t=1}^{T} \left(1 - \frac{t}{T+1}\right)\rho_i \tag{6.9}$$

where ρ_i is the i-lagged autocorrelation for the factor returns and T is the period beyond which there is no statistically significant correlation.

estimated with error: $\delta_{i,k} = \beta_{i,k} - \hat{\beta}_{i,k},\ k = 1, \ldots, K$. If one considers a large collection of stocks – for each of which multiple factor loadings are estimated – there can be a substantial amount of noise in the estimates. Following this observation, Fama and MacBeth (1973) proposed grouping the large number of stocks into a smaller set of portfolios in an attempt to generate more realistic standard errors in the second stage. To see why, consider a simple one-factor model, i.e. the CAPM in Equation 2.1. For any portfolio, p, that contains N assets with corresponding weights $w_i, i = 1, \ldots, N$, the portfolio market exposure is the weighted-average market exposure of the individual assets:

$$\beta_p := \frac{\sigma(R_p, R_M)}{\sigma^2(R_M)} = \sum_{i=1}^{N} w_i \times \frac{\sigma(R_i, R_M)}{\sigma^2(R_M)} = \sum_{i=1}^{N} w_i \times \beta_i \tag{6.10}$$

If we analogously consider the estimators, $\hat{\beta}_i$, the error in $\hat{\beta}_p$ can be much less than the individual errors for $\hat{\beta}_i$. One can consult Fama and MacBeth (1973) for a more in-depth discussion but the result follows by virtue of the summation in Equation 6.10 and the assumption of independent errors across $\hat{\beta}_i$. All else equal, taking the sum over a large portfolio reduces the average error of the factor exposure estimates.

In constructing the portfolios, I follow closely the procedure in Fama and MacBeth (1973). Using the first 4 years of monthly return data, 20 portfolios are formed on the basis of ranked $\hat{\beta}_i$, for individual securities. The following 5 years of data are then used to recompute the $\hat{\beta}_i$, and these are averaged across securities within portfolios to obtain 20 initial portfolio $\hat{\beta}_{pt}$ for the risk-return tests. The subscript t is added to indicate that each month of the following four years these $\hat{\beta}_{pt}$ are recomputed as simple averages of individual security $\hat{\beta}_i$, thus adjusting the portfolio $\hat{\beta}_{pt}$ month by month to allow for delisting of securities. The component $\hat{\beta}_i$ for securities are themselves updated yearly.

Returning to the problem at hand of understanding the dynamics of the duration factor, I consider seven different multifactor models. They are variations of some of the most commonly used models in the literature – the CAPM, Fama-French three factor model and Fama-French three factor model + momentum. As usual, all regressions are performed using monthly returns for the large-cap US stock sample and Fama-French factors – estimated risk premia are subsequently annualised. I perform full sample period regressions as well as pre/post-cirisis regressions, with a summary of the main results in Table 8. In the analysis that follows, it is also useful to keep in mind the results from Section 6.3.1 and Figure 4.

It is more insightful to first consider the pre-crisis results in Table 8 (Panel C). The first two columns show the estimated risk premia when pure value and pure duration are included in a standard CAPM framework. Both the market and value/duration yield statistically significant risk premia and this is expected if one considers the strong performance of these factors in the pre-crisis sample of Figure 4. The third column indicates that adding both pure value and duration into the standard CAPM framework yields a statistically significant risk premium on value but not for duration. Columns 4 & 5 repeat the previous test with using a Fama-French three factor model and subsequently adding momentum. In these higher order models, neither value nor duration yield a statistically significant risk premium. Finally, columns 6 & 7 look at combinations of pure and mixed value/duration factors under a CAPM framework – with column 6 including pure value and value-neutral duration. Analogously, column 7 includes pure duration and duration-neutral value. In the pre-crisis period, there is only a statistically significant risk premium on the pure factors.

Panel B considers the same seven multi-factor models using post-crisis data. Columns 1–3 indicate that the risk-premia for value/duration are now negative. This is expected if one considers value/duration under-performance, particularly in the recent past (Figure 4). Again, columns 4 & 5 indicate that the risk premia become insignificant in larger multi-factor models. The most striking result in this investigation is in column 6. If one adds pure value and value-neutral duration into the CAPM, value-neutral duration earns a statistically significant positive risk-premium. That is to say, during an extended poor period for pure value/duration strategies, neutralising duration's exposure to value yields positive excess returns.

Finally, Panel A shows the results for the entire sample period. The general statistical significance of results is reduced in comparison to the previous two sub-periods. This is likely due to the regime shift pre/post crisis. As previously discussed (and visible in Figure 4), there is a strong shift from positive to negative performance for value/duration strategies during the crisis. Therefore, running a regression over the two conflicting periods is likely to reduce the statistical significance due to the inherent linearity of the test. As in the two sub-periods, the market generally yields statistically significant positive risk-premium, with columns 1 & 2 indicating that value/duration do too when added to the CAPM. For the remaining columns the sign of the results are largely as expected, however there is not enough evidence for statistically significant conclusions.

In conclusion, both the cumulative returns and Fama-MacBeth tests provide strong empirical evidence that duration co-moves closely with value. Accordingly, if an investor is to follow a duration strategy, they must take on a large exposure to the value factor. However, the previous two tests also suggest that the correlation between duration and value is not one-to-one. Indeed, Figure 4 shows that duration has out-performed value post-crisis and Table 8 also suggests that value-neutral duration yields a statistically significant premium post-crisis. Thus, the duration factor appears to be more robust to the post-crisis economic environment – yielding positive cumulative returns and statistically significant positive risk-premia. Importantly, the robust performance occurred over a period where the classical value factor performed poorly.

6.4 Behavioural driving forces behind the equity duration premium

To conclude my analysis, I investigate a potential behavioural driver of the the duration premium. Now that there is evidence of less than perfect co-movement of equity duration and value, it is important to understand these drivers. Hypotheses considered in the literature include:

- High-duration/high-growth stocks are more difficult to price due to the distant cash flows. If investors are over-optimistic about the growth prospects of such firms, they may be systematically over-priced, relative to lower duration stocks. Thus, market participants may be operating in a sub-optimal manner and, as a result, generating risk-return opportunities that contradict the CAPM (Equation 2.1). See Pedersen (2015) for a thorough investigation into market inefficiencies and the impact on asset pricing models.

- Rather than investors operating sub-optimally and aggregating the impact of this across the market, there are certainly also market frictions that can force even an optimal investor to make sub-optimal choices. For example, the betting-against-beta strategy (Frazzini and Pedersen (2014)) is predicated on the fact that the market efficiency under-pinning the CAPM is absent in the empirical data. The authors suggest that short-selling and leverage constraints inhibit investors' capacity to operate efficiently and ensure the relationship imposed by Equation 2.1. In particular, there is strong evidence that low-risk stocks (as measured by market exposure) generate excess returns over and above those implied by the CAPM.

It is, however, important to emphasize that a necessary condition for a source of market inefficiency to develop into a source of alpha, is that the inefficiency is systematic. If markets are inefficient but in a random manner (that is to say, asset prices come from a symmetric distribution centered about their fair value), then one cannot expect to follow an investment policy that consistently capitalises on stochastic mis-pricings. To put this condition on firmer mathematical ground, consider again Equation 2.2. Random market inefficiencies would result in an increase in the variance of the white-noise term (ϵ) – but would not affect its zero (conditional) mean. On the other hand, a systematic inefficiency does not manifest itself in the white-noise term but rather implies a non-zero alpha. If this is the case, then the incentive for the investor (as before) is to consider a multi-factor model (Equation 2.3), in which the additional factor(s) capture the alpha

Following this discussion, I return to an interesting behavioural bias previously discussed in Section 5.3 that is closely linked to the duration premium. One should consult Table 4 and Figure 1 for the relevant statistics. The results indicate (as per Weber (2018)) that long-term forecasts for earnings growth are significantly more volatile for high-duration stocks. This is likely to be reflective of the difficulty in predicting distant cash flows relative to near-future ones. Thus, a strategy that is long low-duration stocks and short high-duration stocks will perform well if the relative difficulty in forecasting distant cash flows leads to a systematic over-pricing of high-duration stocks.

To formally test this hypothesis, I determine the relationship between investor sentiment and the duration premium in a similar manner to Baker and Wurgler (2012). In particular, I examine whether periods of high sentiment lead to a general investor over-optimism that manifests itself in irrational prices for high-duration stocks. A more general argument is made by Stambaugh et al. (2015) in regards to the hypothesis that CAPM anomalies should be larger in periods of high-sentiment if the anomaly is driven by the mis-pricing. Following the literature, I use the monthly data for the sentiment index constructed in Baker and Wurgler (2006) – a popular metric for academics and practitioners. I perform a statistical test similar to Weber (2018) and a variation of those considered in Stambaugh et al. (2015). In doing so, I define a period of high-sentiment as one in which the sentiment index is above its previous 6-18 month rolling average.[16] I construct monthly returns on equally-weighted quintile portfolios sorted on cash flow duration (as per Section 5.2) and run regressions of the general form:

$$r_{i,t} - r_{rf,t} = \alpha_{i,H} D_{H,t} + \alpha_{i,L} D_{L,t} + \beta_{i,mkt}(r_{mkt,t} - r_{rf,t}) + \ldots + \epsilon_{i,t} \quad \forall t \in T \tag{6.11}$$

where $D_{H,t}, D_{L,t}$ are dummy variables for high and low sentiment months, respectively. The other variables on the right-hand-side are rotated to examine performance under three common multi-factor models – CAPM, Fama-French three-factor model and Fama-French five-factor model. The results are provided in Table 9.

The table suggests a negative relationship between duration and excess returns during high sentiment periods – with the strongest pattern emerging post-crisis. Over the full sample, lower duration portfolios generate larger excess returns in periods of high sentiment and the result is statistically significant under the CAPM and three-factor model. This result also induces increased excess return to the $Q1 - Q5$ duration factor. There is essentially no statistically significant relationship between

[16] In using this lagged rolling average, I diverge from the methodology in Weber (2018) and instead opt for the methodology in Baker and Wurgler (2012). Mindful of Occam's razor, the authors suggest a reasonable lag to allow for sticky prices and a suitable rolling average to reduce the noise of a single sentiment value. In both their tests and mine, this approach increases the general statistical significance of results versus a non-lagged, single-period observation.

duration and excess return in low-sentiment months. The same broad conclusions hold when one looks at the pre-crisis data. That is to say, low duration portfolios generate positive excess return in periods of high sentiment with strong results for the CAPM and three-factor model. Finally, the post-crisis data suggests that high-duration portfolios generate negative risk-adjusted returns under the CAPM and three-factor model. Across the board, there is little evidence to suggest that excess return varies across duration-sorted stocks during periods of low investor sentiment. The combination of these results goes some way in confirming that behavioural biases (in the form of over-optimism of high-duration/high-growth stocks) are a contributing factor to the duration premium.

7 Potential Areas of Further Research

The results provided in this paper inspire a number of potential topics of further research:

- The behaviour of the bond-beta model is markedly different to both the cash flow and growth forecast models – thus suggesting that 'bond-like' stocks are not necessarily low-duration stocks. It would be advantageous to investigate the drivers (mathematical or economic) that cause the bond-beta model to diverge from the other two. Furthermore, the performance of the bond-beta model is robust to the post-crisis economic environment and, therefore, it would be interesting to further investigate the alpha-generating potential of a 'bond-like' stocks investment strategy.

- As has been shown theoretically and empirically, there is a strong correlation between the duration factor and the classical value factor. It would, however, be insightful to further investigate the relationship between duration and both the profitability and investment factors. Potentially, one may find that the duration factor is more closely aligned with, say, a linear combination of the value, profitability and investment factors.

- Finally, it would be interesting to merge the cash flow and growth forecast models. As previously mentioned, both models use vastly different mathematical/economic arguments, yet they attempt to capture the same variable. As a possible combination of both models, one could consider incorporating EPS growth forecasts in to the future cash flow predictions – rather than assuming that balance-sheet variables simply follow an AR(1) process.

8 Concluding Remarks

In this paper, I have investigated the topic of equity duration along two broad dimensions. Firstly, I derive three candidate models for estimating the duration of a stock. The models are vastly different in their theoretical underpinnings, yet there is strong empirical evidence of positive co-movements between all three models in my sample. In my sample, low duration portfolios out-performed high-duration portfolios – however, CAPM alphas become largely statistically insignificant post-crisis. Indeed, if one considers the theoretical link between duration and value (established in Section 5.2) and one simultaneously considers the value factor's under-performance post-crisis, then the results are unsurprising.

Secondly, I investigate the relationship between the equity duration factor and various common equity factors. I provide empirical evidence that low-duration stocks are also high-value, high-profitability, low-investment and low-risk stocks. In particular, there is a strong link between duration and the classical value factor. I construct various value/duration double-sorted factors and demonstrate that the correspondence between the two factors is less than one-to-one. In particular, I perform numerous empirical tests that suggest that a duration strategy out-performed a value-strategy in the period following the Great Financial Crisis. In addition, both value-neutral duration and duration-neutral value factors are shown to be robust to the Dot.Com bubble, in which pure value/duration strategies performed very poorly.

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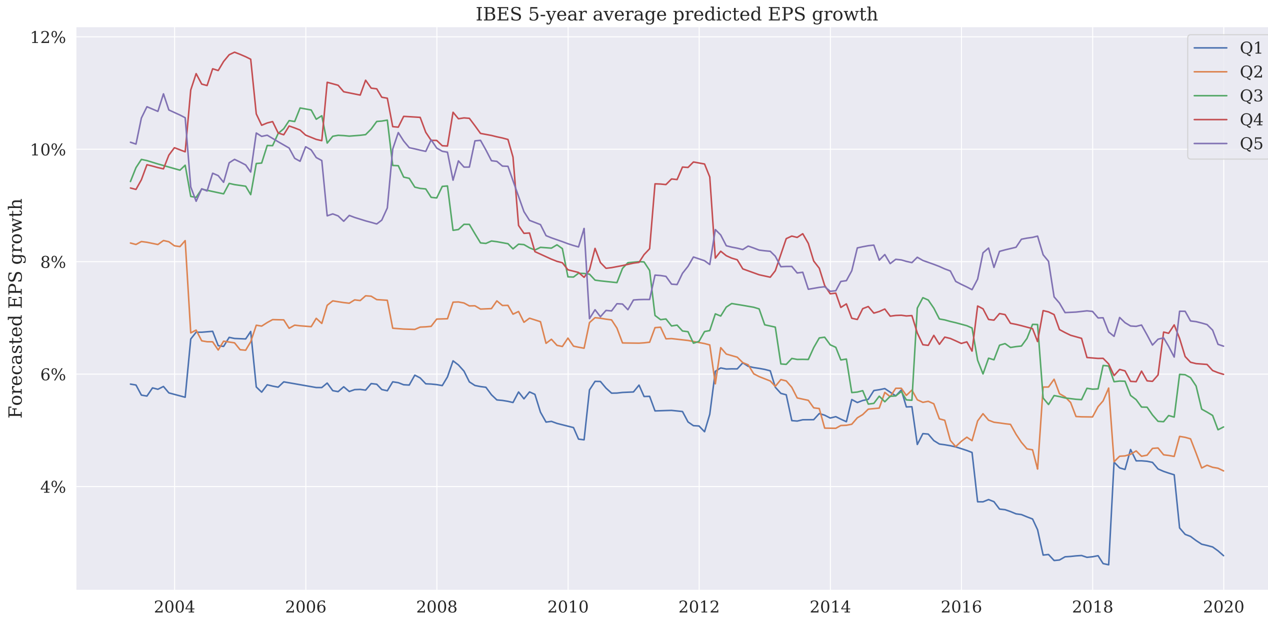

Figure 1: IBES 5-year average predicted EPS growth for cash flow duration-sorted quintile portfolios.

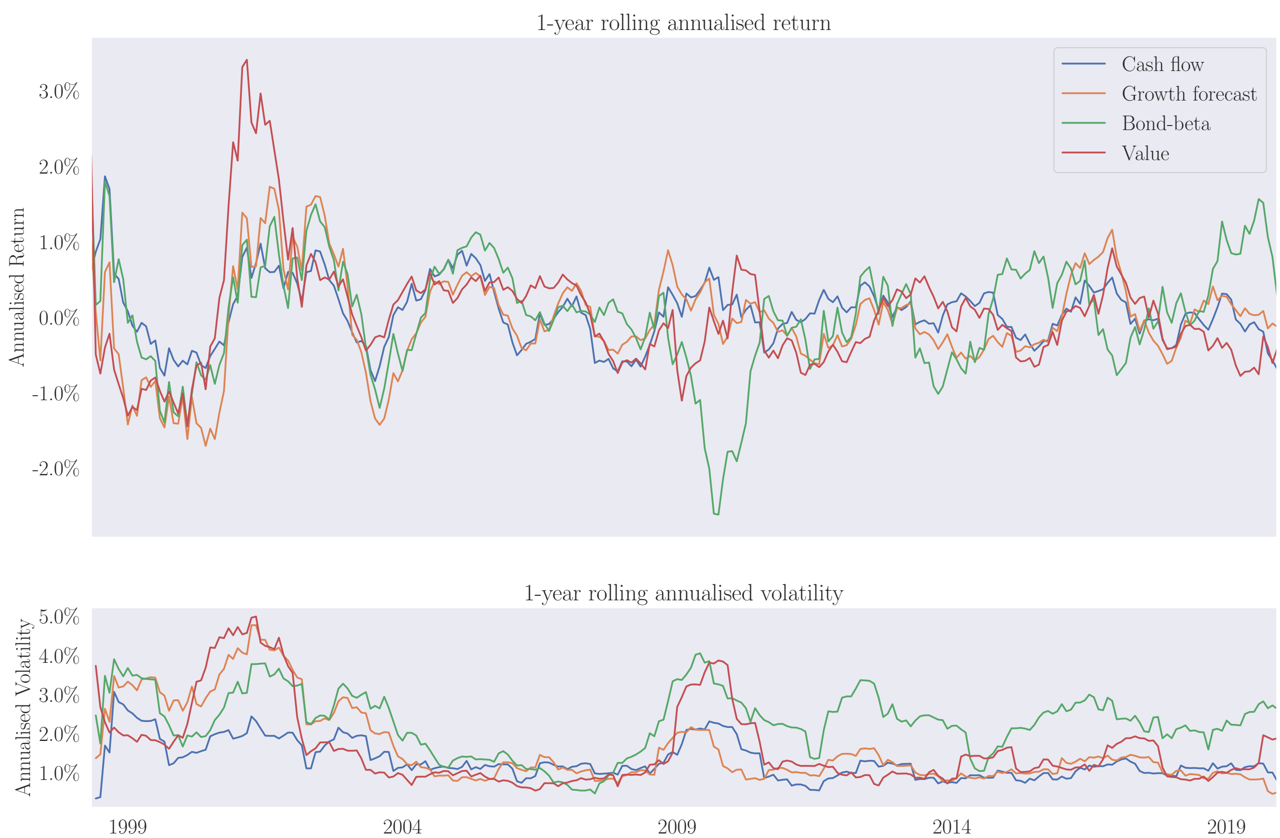

Figure 2: One-year rolling average annualised return and volatility for cash flow duration, growth forecast duration, bond-beta duration and Fama-French (hml) factor.

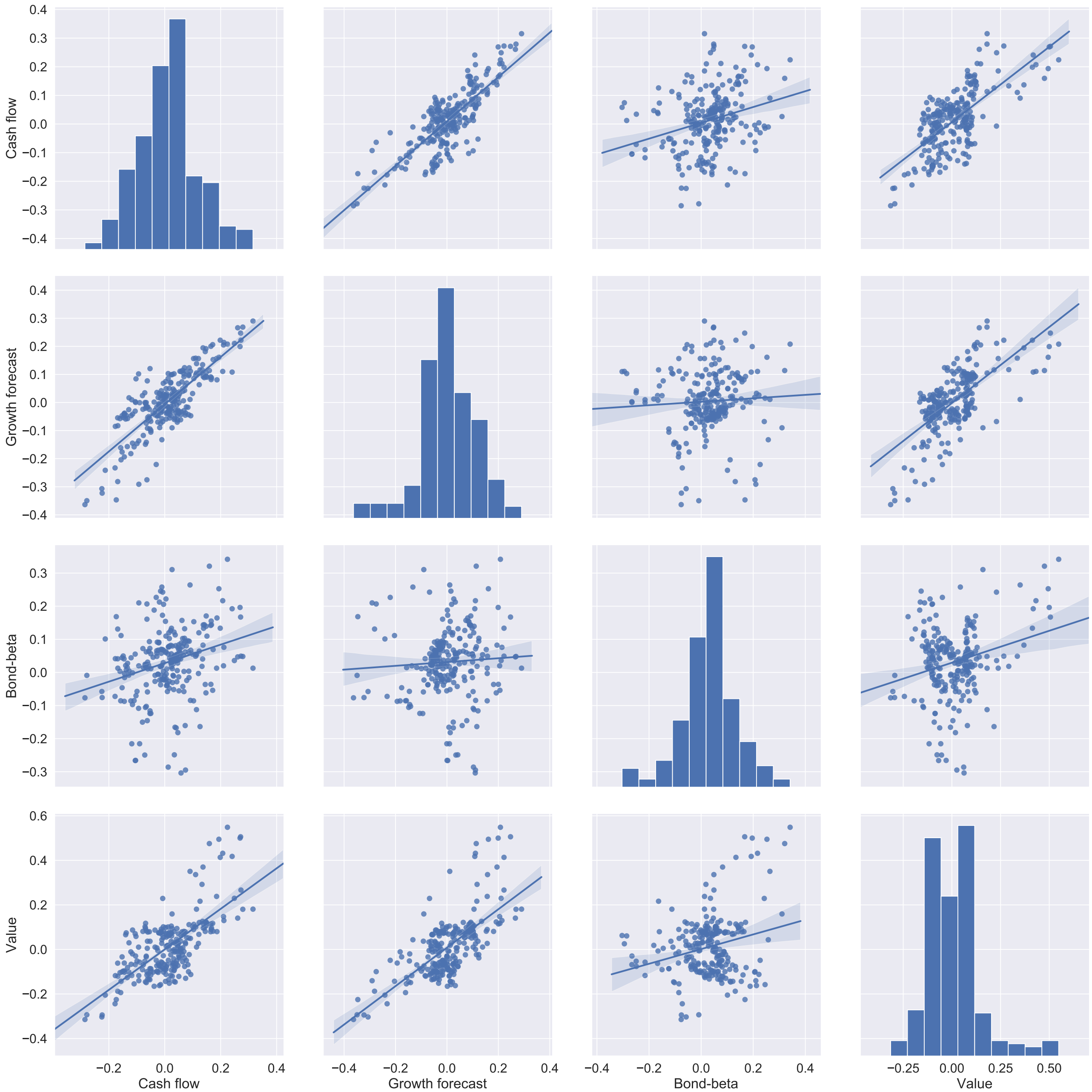

Figure 3: Pairwise simple linear regression plots for cash flow duration, growth forecast duration, bond-beta duration and Fama-French (*hml*) factor.

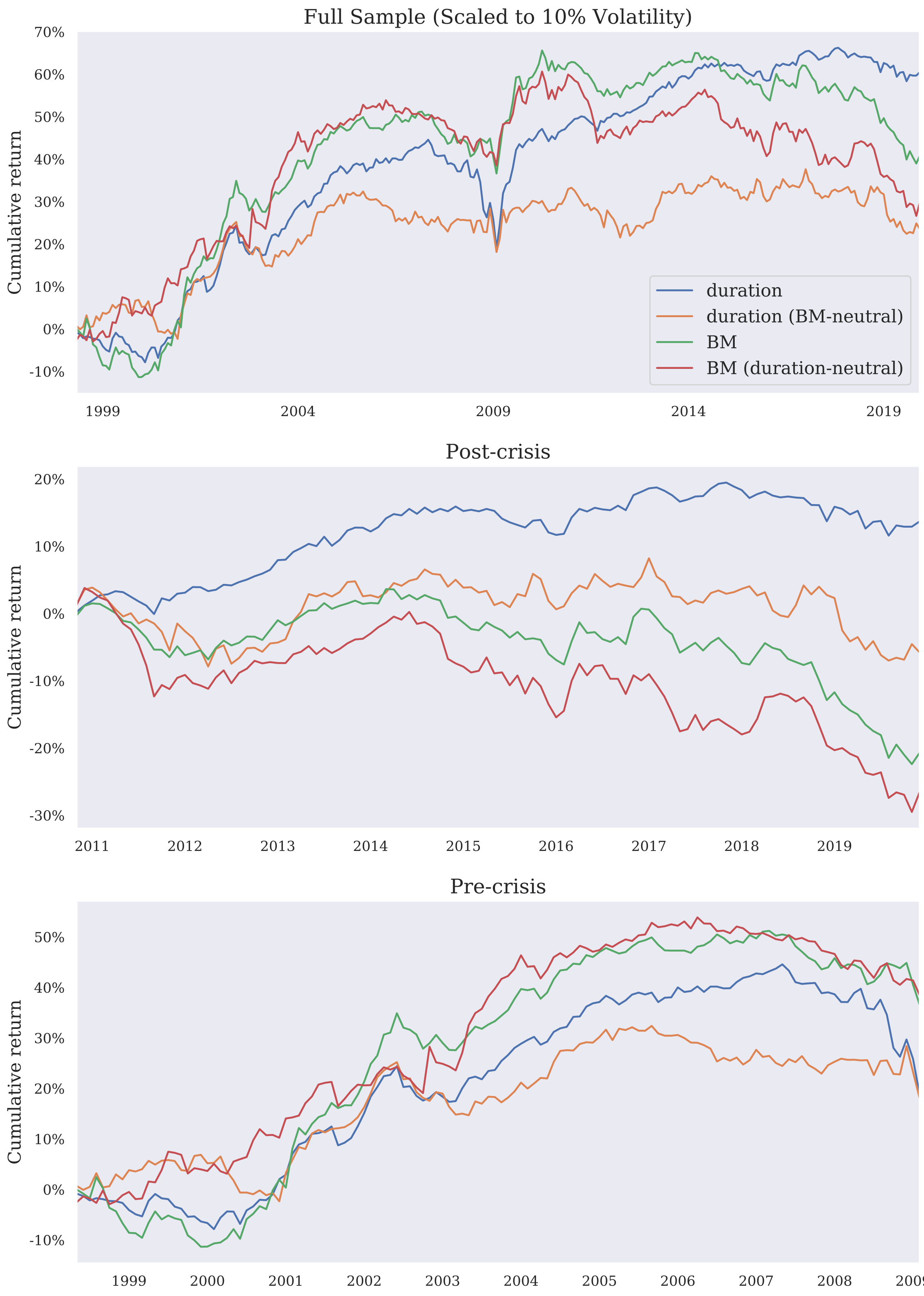

Figure 4: Cumulative return on four alternative value/duration factors. Factors considered: pure value, pure duration, duration-neutral value and value-neutral duration. Cumulative returns are plotted for the full sample 1998 – 2019 (top), post-crisis 2011 – 2019 (center), pre-crisis 1998 – 2007 (bottom).

	Summary Statistics		
Parameter	My Sample	Dechow et al. (2004)	Weber (2018)
Sample Period	1998–2019	1961–1999	1981–2014
Obs.	66,792	126,870	—
Mean	18.93	15.13	18.77
Std. Dev.	3.27	4.09	5.37
Min.	-17.67	-16.75	—
Max.	39.65	31.97	—

Table 1: Comparative summary statistics (where available) for cash flow duration estimators. Statistics are averaged across stocks and across time. Left: 1077 large-cap US sample, 1998–2019. Centre: Dechow et al. (2004) US sample, 1961–1999. Right: Weber (2018) US sample, 1981–2014.

	Spearman Correlation			Pearson Correlation		
	Original	E/P proxy	B/M proxy	Original	E/P proxy	B/M proxy
Original	1	0.50	0.92	1	0.65	0.62
E/P proxy	—	1	0.36	—	1	0.22
B/M proxy	—	—	1	—	—	1

Table 2: Spearman (left) and Pearson (right) correlation between the original cash flow duration estimator (Equation 5.2), earnings-to-price proxy (Equation 5.6) and book-to-market proxy (Equation 5.7). Correlations are determined using panel data over the period 1998–2019. Each of the three duration measures are calculated annually for each of the stocks in the sample.

	Full sample CAPM regression 1998–2019						Post-crisis CAPM regression 2011–2019						Pre-crisis CAPM regression 1998–2007					
Panel A	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
α_{CAPM}	0.016*	0.019**	0.003	-0.017*	0.004	0.012	0.008	0.018**	0.012	-0.021**	-0.022	0.049	0.054***	0.024*	-0.002	-0.006	0.001	0.053*
	(0.008)	(0.008)	(0.008)	(0.010)	(0.012)	(0.019)	(0.009)	(0.009)	(0.009)	(0.010)	(0.017)	(0.024)	(0.020)	(0.013)	(0.010)	(0.016)	(0.018)	(0.028)
β_{mkt}	0.825***	0.976***	1.035***	1.105***	1.115***	-0.289***	0.901***	0.972***	1.015***	1.111***	1.046***	-0.144***	0.603***	0.975***	1.001***	1.031***	1.213***	-0.610***
	(0.027)	(0.018)	(0.018)	(0.022)	(0.028)	(0.043)	(0.020)	(0.019)	(0.020)	(0.020)	(0.036)	(0.051)	(0.054)	(0.035)	(0.028)	(0.042)	(0.047)	(0.076)
R-squared	0.78	0.91	0.92	0.91	0.85	0.15	0.95	0.96	0.96	0.96	0.88	0.06	0.54	0.88	0.92	0.85	0.86	0.37
No. observations	266	266	266	266	266	266	121	121	121	121	121	121	110	110	110	110	110	110

Panel B	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
α_{CAPM}	0.022**	0.014	0.005	-0.002	- 0.013	0.035	-0.012	-0.014*	-0.015**	0.003	0.015	-0.026	0.065***	0.031**	0.028**	-0.013	-0.048*	0.113**
	(0.011)	(0.010)	(0.009)	(0.008)	(0.014)	(0.020)	(0.011)	(0.008)	(0.007)	(0.008)	(0.013)	(0.019)	(0.018)	(0.015)	(0.012)	(0.011)	(0.028)	(0.044)
β_{mkt}	0.959***	1.078***	1.177***	1.129***	1.165***	-0.206***	0.871***	0.959***	1.029***	0.983***	0.962***	-0.091**	0.595***	0.741***	0.891***	1.072***	1.320***	-0.725***
	(0.030)	(0.026)	(0.024)	(0.023)	(0.038)	(0.055)	(0.023)	(0.018)	(0.015)	(0.017)	(0.028)	(0.040)	(0.058)	(0.048)	(0.038)	(0.036)	(0.091)	(0.142)
R-squared	0.81	0.88	0.91	0.91	0.80	0.06	0.92	0.96	0.97	0.96	0.91	0.04	0.56	0.74	0.87	0.91	0.71	0.24
No. observations	266	266	266	266	266	266	121	121	121	121	121	121	110	110	110	110	110	110

Panel C	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
α_{CAPM}	0.019	-0.003	-0.012**	-0.026***	-0.039*	0.058*	0.014	0.011	-0.005	-0.027***	-0.045**	0.059	0.018	0.014	-0.012	-0.026*	-0.085***	0.103*
	(0.017)	(0.008)	(0.005)	(0.008)	(0.020)	(0.036)	(0.018)	(0.008)	(0.005)	(0.008)	(0.021)	(0.038)	(0.024)	(0.012)	(0.010)	(0.013)	(0.030)	(0.052)
β_{mkt}	0.684***	0.729***	0.827***	0.930***	1.153***	-0.469***	0.621***	0.766***	0.891***	1.007***	1.193***	-0.572***	0.811***	0.734***	0.807***	0.904***	1.205***	-0.394***
	(0.028)	(0.013)	(0.009)	(0.013)	(0.033)	(0.059)	(0.032)	(0.015)	(0.010)	(0.015)	(0.038)	(0.068)	(0.046)	(0.022)	(0.018)	(0.025)	(0.058)	(0.100)
R-squared	0.73	0.94	0.97	0.95	0.84	0.22	0.76	0.96	0.99	0.97	0.89	0.37	0.80	0.93	0.96	0.94	0.85	0.17
No. observations	260	260	260	260	260	260	121	121	121	121	121	121	104	104	104	104	104	104

Table 3: CAPM α and β for equally-weighted, quintile portfolios sorted on three alternative measures of equity duration. Columns labelled $Q1 - Q5$ indicate results for portfolios that are long $Q1$ and short $Q5$. Regressions are run over the entire sample period (left), the post-crisis period (center) and the pre-crisis period (right). Monthly returns are used for all regressions and results are annualised. Panel A: Dechow et al. (2004), Weber (2018) cash flow model. Panel B: Gormsen and Lazarus (2019) forecasted growth model. Panel C: Baker and Wurgler (2009, 2012) bond-beta model.

Monthly rolling average	Low Duration			High Duration	
	Q1	Q2	Q3	Q4	Q5
1-year	6.41	8.00	9.62	10.33	11.33
2-year	5.99	7.42	9.04	9.73	10.61
3-year	5.76	6.87	8.53	9.24	9.89
4-year	5.54	6.55	8.00	8.79	9.17
5-year	5.39	6.21	7.48	8.30	8.64

Table 4: Mean IBES forecasted long-term EPS growth across duration-sorted quintile portfolios, constructed as per Dechow et al. (2004), Weber (2018). Forecasts are averaged over increasing time periods (1–5 years). EPS growth forecasts are provided monthly over the period 1998–2019, with approximately 71% of the total stock sample included.

Panel A	Spearman Correlation				Pearson Correlation			
	D_{CF}	D_{EPS}	D_{BB}	hml	D_{CF}	D_{EPS}	D_{BB}	hml
D_{CF}	1	0.63	0.18	0.57	1	0.69	0.18	0.58
D_{EPS}	—	1	0.37	0.52	—	1	0.36	0.63
D_{BB}	—	—	1	-0.09	—	—	1	-0.07
hml	—	—	—	1	—	—	—	1

Panel B	Spearman Correlation				Pearson Correlation			
	D_{CF}	D_{EPS}	D_{BB}	hml	D_{CF}	D_{EPS}	D_{BB}	hml
D_{CF}	1	0.64	0.09	0.52	1	0.71	0.08	0.57
D_{EPS}	—	1	0.28	0.35	—	1	0.39	0.41
D_{BB}	—	—	1	-0.34	—	—	1	-0.36
hml	—	—	—	1	—	—	—	1

Panel C	Spearman Correlation				Pearson Correlation			
	D_{CF}	D_{EPS}	D_{BB}	hml	D_{CF}	D_{EPS}	D_{BB}	hml
D_{CF}	1	0.70	0.44	0.63	1	0.73	0.46	0.64
D_{EPS}	—	1	0.49	0.75	—	1	0.46	0.79
D_{BB}	—	—	1	0.31	—	—	1	0.27
hml	—	—	—	1	—	—	—	1

Table 5: Panel A: Full sample (1998–2019) correlation between cash flow, growth forecast, bond-beta duration risk factors (constructed as per Fama and French (1993)) and Fama-French value factor. Correlations are calculated using monthly factor returns. Panel B: Post-crisis sample (2011–2019). Panel C: Pre-crisis sample (1998–2007).

Panel A: Full Sample

	Cash flow					Growth forecast					Bond-beta				
	hml	umd	rmw	cma	bab	hml	umd	rmw	cma	bab	hml	umd	rmw	cma	bab
α	0.007	0.077**	0.055***	0.036***	0.116***	0.005	0.079**	0.055***	0.034***	0.116***	0.028	0.049	0.053***	0.043***	0.113***
	(0.020)	(0.037)	(0.009)	(0.013)	(0.020)	(0.019)	(0.037)	(0.009)	(0.012)	(0.020)	(0.023)	(0.031)	(0.009)	(0.015)	(0.019)
β_{mkt}	0.009	-0.475***	-0.103***	-0.146***	-0.190***	0.123***	-0.506***	-0.081***	-0.077***	-0.144***	-0.231***	0.060	-0.023	-0.194***	-0.042
	(0.039)	(0.072)	(0.018)	(0.026)	(0.038)	(0.039)	(0.077)	(0.019)	(0.025)	(0.040)	(0.057)	(0.076)	(0.021)	(0.036)	(0.047)
β_{smb}	-0.165***	0.298***	-0.134***	-0.062*	-0.089*	-0.013	0.263**	-0.107***	0.024	-0.023	-0.298***	0.530***	-0.104***	-0.096**	-0.041
	(0.054)	(0.098)	(0.024)	(0.035)	(0.051)	(0.052)	(0.103)	(0.025)	(0.034)	(0.054)	(0.063)	(0.084)	(0.024)	(0.039)	(0.052)
β_{dur}	0.831***	-0.046	0.118***	0.354***	0.458***	0.908***	-0.156	0.151***	0.473***	0.425***	-0.189***	0.701***	0.123***	-0.009	0.267***
	(0.084)	(0.153)	(0.038)	(0.055)	(0.080)	(0.072)	(0.142)	(0.035)	(0.047)	(0.074)	(0.049)	(0.066)	(0.018)	(0.031)	(0.041)
R-squared	0.34	0.16	0.32	0.32	0.27	0.44	0.16	0.34	0.44	0.27	0.13	0.42	0.40	0.21	0.30
No. observations	260	260	260	260	260	260	260	260	260	260	260	260	260	260	260

Panel B: Post-crisis

	hml	umd	rmw	cma	bab	hml	umd	rmw	cma	bab	hml	umd	rmw	cma	bab
α	-0.066***	0.101***	0.051***	-0.011	0.123***	-0.049*	0.097***	0.050***	-0.007	0.121***	0.008	0.034	0.030**	0.007	0.099***
	(0.025)	(0.036)	(0.013)	(0.011)	(0.016)	(0.025)	(0.035)	(0.013)	(0.010)	(0.016)	(0.026)	(0.033)	(0.012)	(0.012)	(0.015)
β_{mkt}	0.154***	-0.410***	-0.074**	-0.025	-0.125***	0.124**	-0.389***	-0.064**	-0.013	-0.090**	-0.249***	0.007	0.053	-0.113***	0.043
	(0.059)	(0.086)	(0.031)	(0.027)	(0.038)	(0.060)	(0.084)	(0.031)	(0.025)	(0.037)	(0.073)	(0.094)	(0.034)	(0.035)	(0.042)
β_{smb}	0.011	-0.137	-0.136***	-0.074*	-0.086	0.038	-0.132	-0.125**	-0.069*	-0.096	-0.138	0.093	-0.045	-0.101**	-0.004
	(0.091)	(0.134)	(0.049)	(0.042)	(0.059)	(0.094)	(0.132)	(0.048)	(0.039)	(0.058)	(0.097)	(0.125)	(0.045)	(0.046)	(0.055)
β_{dur}	0.720***	-0.385**	-0.041	0.216***	-0.113	0.622***	-0.302	0.035	0.303***	0.048	-0.300***	0.386***	0.138***	-0.055**	0.159***
	(0.118)	(0.174)	(0.064)	(0.055)	(0.076)	(0.130)	(0.182)	(0.066)	(0.054)	(0.080)	(0.054)	(0.070)	(0.025)	(0.026)	(0.031)
R-squared	0.27	0.23	0.17	0.22	0.15	0.18	0.21	0.16	0.32	0.13	0.23	0.37	0.34	0.14	0.30
No. observations	105	105	105	105	105	108	108	108	108	108	108	108	108	108	108

Panel C: Pre-crisis

	hml	umd	rmw	cma	bab	hml	umd	rmw	cma	bab	hml	umd	rmw	cma	bab
α	0.083***	0.098	0.055***	0.067**	0.145***	0.078***	0.102	0.054***	0.059**	0.143***	0.097***	0.072	0.053***	0.073**	0.148***
	(0.031)	(0.066)	(0.016)	(0.028)	(0.034)	(0.029)	(0.067)	(0.016)	(0.024)	(0.033)	(0.034)	(0.057)	(0.015)	(0.028)	(0.034)
β_{mkt}	-0.248***	-0.417***	-0.148***	-0.193***	-0.326***	-0.120*	-0.429***	-0.106***	-0.046	-0.216***	-0.367***	0.111	-0.064*	-0.214***	-0.218***
	(0.061)	(0.134)	(0.031)	(0.054)	(0.066)	(0.068)	(0.158)	(0.036)	(0.056)	(0.077)	(0.083)	(0.138)	(0.035)	(0.068)	(0.082)
β_{smb}	-0.243***	0.458***	-0.124***	-0.072	-0.027	-0.116*	0.471***	-0.081**	0.053	0.093	-0.299***	0.570***	-0.115***	-0.095*	-0.033
	(0.061)	(0.135)	(0.032)	(0.054)	(0.066)	(0.065)	(0.151)	(0.035)	(0.053)	(0.074)	(0.069)	(0.115)	(0.029)	(0.057)	(0.068)
β_{dur}	0.514***	0.295	0.199***	0.316***	0.598***	0.662***	0.172	0.237***	0.573***	0.677***	0.035	0.809***	0.185***	0.090	0.366***
	(0.098)	(0.216)	(0.050)	(0.086)	(0.106)	(0.107)	(0.247)	(0.057)	(0.087)	(0.121)	(0.074)	(0.124)	(0.032)	(0.061)	(0.074)
R-squared	0.56	0.19	0.53	0.38	0.53	0.59	0.18	0.53	0.50	0.53	0.44	0.41	0.59	0.31	0.50
No. observations	108	108	108	108	108	108	108	108	108	108	108	108	108	108	108

Table 6: Regression of common equity factors on excess stock market return, size factor and duration factor. The factors I consider are value (*hml*), momentum (*umd*), profitability (*wmr*), investment (*cma*) and betting-against-beta (*bab*). Regressions are run over the entire sample period (Panel A), the post-crisis period (Panel B) and the pre-crisis period (Panel C), using monthly return data. All return statistics are annualised. Left Column: Dechow et al. (2004), Weber (2018) cash flow model. Centre Column: Gormsen and Lazarus (2019) forecasted growth model. Right Column: Baker and Wurgler (2009, 2012) bond-beta model.

Panel A	Spearman Correlation				Pearson Correlation			
	D_P	D_N	BM_P	BM_N	D_P	D_N	BM_P	BM_N
D_P	1	0.51	0.88	0.49	1	0.57	0.91	0.51
D_N	—	1	0.49	0.48	—	1	0.48	0.45
BM_P	—	—	1	0.70	—	—	1	0.73
BM_N	—	—	—	1	—	—	—	1

Panel B	Spearman Correlation				Pearson Correlation			
	D_P	D_N	BM_P	BM_N	D_P	D_N	BM_P	BM_N
D_P	1	0.52	0.82	0.45	1	0.51	0.87	0.49
D_N	—	1	0.44	0.52	—	1	0.43	0.50
BM_P	—	—	1	0.72	—	—	1	0.76
BM_N	—	—	—	1	—	—	—	1

Panel C	Spearman Correlation				Pearson Correlation			
	D_P	D_N	BM_P	BM_N	D_P	D_N	BM_P	BM_N
D_P	1	0.55	0.91	0.41	1	0.63	0.94	0.42
D_N	—	1	0.48	0.45	—	1	0.54	0.45
BM_P	—	—	1	0.58	—	—	1	0.59
BM_N	—	—	—	1	—	—	—	1

Table 7: Panel A: Full sample (1998–2019) correlation between single-sort pure duration factor (D_P), single-sort pure value factor (BM_P), double-sort value-neutral duration factor (D_N) and double-sort duration-neutral value factor (BM_N). Correlations are calculated using monthly factor returns. Panel B: Post-crisis sample (2011–2019). Panel C: Pre-crisis sample (1998–2007).

Panel A	**Full Sample 1998–2019**						
	1	2	3	4	5	6	7
γ_α	-0.08	-0.42	-0.31	0.16	-0.72	-1.21	-0.45
	(-0.04)	(-0.20)	(-0.22)	(0.11)	(-0.47)	(-0.78)	(-0.27)
γ_{mkt}	6.71*	7.46*	8.04*	6.65	9.02**	9.27***	7.94**
	(1.79)	(1.77)	(1.87)	(1.56)	(1.95)	(2.25)	(1.95)
γ_{smb}				7.80	6.83		
				(1.53)	(1.36)		
γ_{umd}					5.72		
					(0.76)		
γ_{V_P}	4.48*		2.68	0.98	1.32	3.08	
	(1.75)		(1.57)	(0.44)	(0.45)	(1.45)	
γ_{D_P}		3.89*	2.22	1.12	2.15		3.14
		(1.77)	(1.09)	(0.52)	(0.65)		(1.09)
γ_{D_N}						4.16	
						(1.33)	
γ_{V_N}							2.90
							(0.80)
Observations	260	260	260	260	260	260	260

Table 8: Fama and MacBeth (1973) regression results for multi-factor models of the general form: $R_{it} = \alpha_i + \beta_{i,1}R_{Mt} + \beta_{i,2}F_{it} + \ldots + \beta_{i,K}F_{Kt} + \epsilon_{it}$. The models follow closely either the CAPM, Fama-French three factor model or Fama-French three factor model + momentum. Regressions are run using monthly returns on the factors and the 20 portfolios constructed as per Section 6.3.2 (or, equivalently, Fama and MacBeth (1973)). Panel A: Full sample 1998 – 2019. Panel B: Post-crisis 2011 – 2019. Panel C: Pre-crisis 1998 – 2007.

Panel B	**Post-crisis 2011–2019**						
	1	2	3	4	5	6	7
γ_α	1.39	1.34	1.32	0.62	0.62	2.38***	1.54
	(1.51)	(1.38)	(1.37)	(0.82)	(0.82)	(2.18)	(1.49)
γ_{mkt}	9.14*	7.40	8.79*	10.46***	10.47	6.94	8.42
	(1.75)	(1.40)	(1.69)	(2.00)	(0.93)	(1.31)	(1.61)
γ_{smb}				-9.80*	-9.59		
				(-1.74)	(-1.54)		
γ_{umd}					1.91		
					(0.28)		
γ_{V_P}	-14.95***		-15.43***	-7.41	-7.39	-6.11	
	(-2.48)		(-2.47)	(-1.24)	(-1.23)	(-1.59)	
γ_{D_P}		-14.52***	-13.38***	-8.17	-8.18		-12.93***
		(-2.43)	(-2.22)	(-1.36)	(-1.37)		(-2.13)
γ_{D_N}						3.72**	
						(1.75)	
γ_{V_N}							-15.19***
							(-2.34)
Observations	108	108	108	108	108	108	108

Panel C	**Pre-crisis 1998–2007**						
	1	2	3	4	5	6	7
γ_α	-3.82	-4.44	-2.12	3.15	-3.75	-4.19	-5.05
	(-0.91)	(-1.00)	(-0.73)	(0.77)	(-0.90)	(-1.30)	(-1.23)
γ_{mkt}	12.12*	13.72*	10.77	5.98	13.91	12.99**	14.93***
	(1.81)	(1.80)	(1.57)	(0.91)	(1.39)	(1.96)	(1.99)
γ_{smb}				18.33***	9.82***		
				(3.86)	(4.02)		
γ_{umd}					-1.99		
					(-0.25)		
γ_{V_P}	7.23*		4.85*	0.77	4.62	6.23*	
	(1.80)		(1.78)	(0.12)	(0.72)	(1.85)	
γ_{D_P}		5.63*	2.14	-1.89	0.48		6.03*
		(1.81)	(0.78)	(-0.27)	(0.07)		(1.83)
γ_{D_N}						3.58	
						(0.96)	
γ_{V_N}							3.88
							(0.38)
Observations	108	108	108	108	108	108	108

Panel A

	Full sample 1998–2019						Post-crisis 2011–2019						Pre-crisis 1998–2007					
	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
α_L	-0.012	-0.019	-0.005	0.014	0.049	-0.061	0.027	0.025	0.023	0.026	0.026	0.001	-0.050	-0.059*	-0.059	-0.064	-0.025	-0.026
	(0.020)	(0.020)	(0.023)	(0.026)	(0.040)	(0.039)	(0.021)	(0.017)	(0.022)	(0.026)	(0.048)	(0.053)	(0.037)	(0.033)	(0.036)	(0.039)	(0.055)	(0.044)
α_H	0.041**	0.045**	0.017	0.005	-0.029	0.070*	-0.012	-0.017	-0.037*	-0.053**	-0.083*	0.072	0.103***	0.104***	0.076**	0.092**	0.064	0.039
	(0.020)	(0.020)	(0.023)	(0.026)	(0.040)	(0.039)	(0.021)	(0.018)	(0.022)	(0.027)	(0.049)	(0.053)	(0.035)	(0.031)	(0.034)	(0.037)	(0.052)	(0.042)

Panel B

	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
α_L	-0.012	-0.020	-0.007	0.010	0.042	-0.054	0.026	0.027	0.025	0.025	0.024	0.002	-0.028	-0.036*	-0.035	-0.043	0.001	-0.029
	(0.019)	(0.015)	(0.017)	(0.020)	(0.032)	(0.034)	(0.021)	(0.017)	(0.021)	(0.023)	(0.037)	(0.039)	(0.029)	(0.021)	(0.026)	(0.031)	(0.046)	(0.044)
α_H	0.037*	0.038**	0.007	-0.007	-0.048	0.085**	-0.013	-0.018	-0.034	-0.043*	-0.057	0.044	0.049*	0.049**	0.018	0.036	-0.010	0.059
	(0.019)	(0.015)	(0.017)	(0.020)	(0.032)	(0.034)	(0.022)	(0.018)	(0.021)	(0.023)	(0.037)	(0.040)	(0.028)	(0.021)	(0.025)	(0.030)	(0.045)	(0.042)

Panel C

	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5	Q1	Q2	Q3	Q4	Q5	Q1 - Q5
α_L	-0.013	-0.020	-0.007	0.010	0.043	-0.056*	0.022	0.024	0.021	0.023	0.018	0.004	-0.028	-0.036*	-0.035	-0.043	-0.001	-0.027
	(0.019)	(0.015)	(0.017)	(0.020)	(0.032)	(0.033)	(0.019)	(0.016)	(0.020)	(0.022)	(0.035)	(0.035)	(0.029)	(0.021)	(0.026)	(0.031)	(0.044)	(0.040)
α_H	0.022	0.025	-0.004	-0.011	-0.027	0.049	-0.030	-0.026	-0.036*	-0.034	-0.030	-0.000	0.041	0.048**	0.016	0.036	0.015	0.027
	(0.019)	(0.015)	(0.018)	(0.020)	(0.033)	(0.034)	(0.020)	(0.017)	(0.021)	(0.024)	(0.036)	(0.037)	(0.028)	(0.021)	(0.025)	(0.031)	(0.044)	(0.039)

Table 9: Excess return following periods of high and low sentiment as measured by Baker and Wurgler (2006) sentiment index. Equally-weighted quintile portfolios are sorted on cash flow equity duration. Regressions are run over the entire sample period (left), the post-crisis period (center) and the pre-crisis period (right). Monthly returns are used for all regressions and results are annualised. Panel A: CAPM excess returns. Panel B: Fama-French three-factor model excess returns. Panel C: Fama-French five-factor model excess returns.