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The Acceleration Effect and Gamma Factor in Asset Pricing

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The Acceleration effect and Gamma factor in Asset Pricing

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ABSTRACT

We report strong evidence that changes of momentum, i.e. "acceleration", defined as the first difference of successive returns, provide better performance and higher explanatory power than momentum. The corresponding Γ-factor explains the momentum-sorted portfolios entirely but not the reverse. Thus, momentum can be considered an imperfect proxy for acceleration, and its success can be attributed to its correlation to the predominant Γ-factor. Γ-strategies based on the "acceleration" effect are on average profitable and beat momentum-based strategies in two out of three cases, for a large panel of parameterizations. The "acceleration" effect and the Γ-factor profit from transient non-sustainable accelerating (upward or downward) log-prices associated with positive feedback mechanisms.

JEL classification: G01, G11, G12, G17

Keywords: Asset pricing, momentum, positive feedbacks, acceleration, investment strategies

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1 Introduction

Momentum, the tendency for rising asset prices to rise further, and falling prices to keep falling, enjoys a strong empirical support [Jegadeesh and Titman, 1993, Grinblatt, Titman, and Wermers, 1995, Jegadeesh and Titman, 2001, Grinblatt and Moskowitz, 2004] and provides an improved explanatory power in factor model regressions [Carhart, 1997, Fama and French, 2012a]. Momentum has been documented in the US, Europe and Asia Pacific [Fama and French, 2012b] as well as in different asset classes [Asness, Moskowitz, and Pedersen, 2013]. Additionally, it has withstood explanations based on industry effects, cross-correlation among assets, and data mining [Grundy and Martin, 2001, Jegadeesh and Titman, 2001]. Momentum is often attributed to investors' behavioral characteristics such as over-confidence, self-attribution and confirmation biases [Daniel, Hirschleifer, and Subrahmanyam, 1998], under-reaction and over-reaction [Barberis, Shleifer, and Vishny, 1998] as well as herding [Hoitash and Krishnan, 2008, Demirer, Lien, and Zhang, 2015]. It could also perhaps be accounted for by assuming efficient markets with rational investors in the presence of information noise [Crombez, 2001]. It is noteworthy that trend following strategies have a large number of followers among both individual and professional investors [Antonacci, 2014, Clenow, 2015].

In its simplest geometrical representation, momentum reflects the persistence of short-term returns, i.e. the existence of linear trends in the log-price processes, which can be referred to as "velocity" in the technical analysis language (see [Andersen, Gluzman, and Sornette, 2000] and references therein). Here, we report strong evidence that changes of momentum, i.e. "acceleration", defined specifically as the first difference of successive returns, provide better performance and higher explanatory power than momentum. We dub the corresponding factor Gamma (Γ), due to the curvature associated with the changes of trends that it captures, in analogy with the option Gamma defined as the second-order derivative of the option price with respect to the underlying stock price. In asset pricing tests, we find that momentum-sorted portfolios can be completely explained by the Γ-factor, but not the reverse. Thus, momentum can be considered an imperfect proxy for acceleration, and its success can be attributed to its correlation to the predominant Γ-factor.

We argue that the Γ-factor represents the existence of transient positive feedbacks influencing the price formation process. These procyclical mechanisms include the market impact of option hedging, insurance portfolio strategies [Ho, Cadle, and Theobald, 2010], market makers bid-ask spread in response to past volatility, learning of business networks, financing of firms by banks during boom compared to contracting times, algorithmic trading, asymmetric information on hedging strategies, stop-loss orders, portfolio execution optimization and order splitting, deregulation (e.g. the Gramm-Leach-Bliley act repelling the Glass-Steagall act), central banks easy monetary policies, as well as imitation, social influence and herding. When one or several of these mechanisms are at work, they tend to push the price further away from a sustainable fundamental price with constant return. The result is a finite lived accelerating upward swing or downward spiral, which is captured by the Γ-factor. While the price dynamics tends to mean reverse to a long term trend that is

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co-integrated with economic growth [El-Wassal, 2005, Sornette and Cauwels, 2014], at time scales up to a few years, prices do deviate by exhibiting finite-lived stochastic accelerating bursts, both upward and downward. A typical implication of a class of theoretical models of bubbles is that increasing rate of returns (positive acceleration) become necessary in order to sustain the bubble regime [Brunnermeier and Oehmke, 2013, Scherbina and Schlusche, 2014, Kaizoji, Leiss, Saichev, and Sornette, 2015]. This implication has started to be tested in empirical studies of financial bubbles [Phillips, Wu, and Yu, 2011, Leiss, Nax, and Sornette, 2015].

At the individual investor level, there is also physiological and psychological evidence that there is a decrease in response to a constant stimulus, a phenomenon known as desensitisation or habituation [Rankin et al., 2009]. A variation (acceleration/deceleration) is needed to create a new perceived stimulus. Hence, investors might react inordinately to change of trends [Andersen et al., 2000]. Since acceleration amounts to a change of the momentum that embodies the previous prevailing trend, then from a psychological point of view, the Γ-factor amounts to a breakdown of the status quo [Samuelson and Zeckhauser, 1988, Kahneman, Knetsch, and Thaler, 1991]. In a sense, the Γ-factor can be thought of as a premium for the psychological costs of the heightened decision difficulties and the increased uncertainty associated with a deviation from the status quo [Fleming, Thomas, and Dolan, 2010] that momentum represents.

The article is organized as follows. Section 2 presents our definition of "acceleration" leading to the specification of the Γ-factor. It also presents a number of standard asset pricing tests pitting our Γ-factor against the standard factors: market, size, book-to-value, and momentum. Section 3 develops Γ-based portfolio investment strategies that are compared to momentum-based strategies. It dissects the sources of the superior performance of the Γ-strategies. Section 4 concludes.

2 Asset pricing tests

2.1 Definition of the Γ-factor

Given the return $r_{i,t}(f) := (p_{i,t}/p_{i,t-f}) - 1$ (discrete approximation of the differences in log-prices) of stock i observed at time t (counted in months) over the time scale of f months, we define the intermediate variable $\Gamma_{i,t}(f)$ by

$$\Gamma_{i,t}(f) = r_{i,t}(f) - r_{i,t-f}(f) \ . \tag{1}$$

This expresses for $\Gamma_{i,t}(f)$ is nothing but the first-difference of successive returns in time steps of f months, i.e. a discrete approximation for log-price "acceleration".

By construction, $\Gamma_{i,t}(f)$ has a quite strong correlation with $r_{i,t}(f)$. For instance, assuming i.i.d. Gaussian-distributed returns, their correlation is obtained as

$$\mathrm{Corr}(r_t(f), \Gamma_t(f)) = \frac{\sigma_{r_t(f)}}{\sigma_{r_t(f) - r_{t-f}(f)}} = \frac{1}{\sqrt{2}} \sim 0.71 \ . \tag{2}$$

To minimize this correlation, we define $\Gamma^*_{i,t}$ of stock i at time t, as a 6 month-shifted Γ at time scale $f = 6$ months, namely

$$\Gamma^*_{i,t} := \Gamma_{i,t-6}(6) = r_{i,t-6}(6) - r_{i,t-12}(6) \ . \tag{3}$$

Since momentum is typically constructed on yearly returns, we find that the shift of 6 months is a good compromise between the decrease of correlation with momentum and the retention of information of the acceleration at time t. The correlation between $\Gamma^*_{i,t}$ with momentum $r_{i,t}(6)$ is now reduced to 0.39, i.e. to almost one-half the correlation between $\Gamma_{i,t}(6)$ and $r_{i,t}(6)$.

Sorting stocks based on their $\Gamma^*_{i,t}$ given by (3), the 6 months shifted first difference of returns, we introduce 10 Γ-portfolios defined as the value-weighted portfolios of the stocks in each decile of their $\Gamma^*_{i,t}$-ranked values at the end of every month. Then, we define the Γ-factor as the portfolio that is

$$\Gamma\text{−factor} = \text{long the top } \Gamma^* \text{−ranked decile stocks and short the bottom } \Gamma^* \text{−ranked decile stocks} \ . \tag{4}$$

This Γ-factor is intended to capture the common risk premium associated with acceleration in the log-prices of stocks.

2.2 Setting up asset pricing tests

The explanatory returns that we use in our asset pricing tests are the market factor, the Fama-French book-to-value (HML) and size (SMB) factors, the momentum factor, and the above defined Γ factor (4).

The target portfolios that are supposed to be explained by these factors are the standard 25 book-to-value and size Fama and French (FF) portfolio, as well as the 12 industry portfolio based on the FF classification. Both sets of portfolios are recalculated directly by us and not extracted from the Fama-French website to prevent (minor) discrepancies due to differences in data processing. In addition, sorting stocks based on their momentum $r_t(12)$, the yearly stock return at the end of every month, we test on 10 momentum portfolios, denoted Δ-portfolios, defined as the value-weighted portfolios of the stocks in each decile of their momentum-ranked values at the end of each month. We shall also use the 10 Γ-portfolios defined above. Finally, we also construct 20 double-sorted $\Delta - \Gamma$ portfolios by combining the decile breakpoints associated with the construction of the 10 Δ-portfolios and 10 Γ-portfolios. The returns of all portfolios are value-weighted.

The data used in this work are all the common stocks in the Center for Research in Security Prices (CRSP) database (share code 10 and 11) between May 1963 and December 2013. We have limited the analysis to this period since the Fama-French factors require accounting data only available after 1963. Stocks with less than two years of existence (by time t) were discarded to control for survival bias. Following Fama and French [1992], the computation of the breakpoints of the portfolios and factors used in the asset pricing tests was done using stocks from the NYSE, the NASDAQ and the AMEX (exchange code 1, 2 and 3). The measures were then computed using stocks from all exchanges. Finally, to study the stock returns by industry, we employed the

Fama-French 12 industry classification, which in turn is based on the stock's SIC code.

Tables 1 and 2 report the summary statistics for the portfolios. Three observations can be made. The Δ, Γ, and 25 book-to-value and size portfolios exhibit considerable cross-sectional variation. Average excess monthly returns range from 0.263 to 0.881 for the Δ portfolios, 0.049 to 1.056 for the Γ portfolios, and 0.313 to 1.140 for the book-to-value and size portfolios. Industry portfolios present somewhat lower cross-sectional variation, with monthly returns ranging from 0.389 (Telco) to 0.668 (Energy). Second, excess returns are in general significant at a 5% confidence level, while exhibiting monthly Sharpe ratios typically around 0.1. Finally, Δ, Γ, and Industry portfolios are in general well diversified, with an average number of stocks in each portfolio typically above 100. In contrast, the average number of book-to-value and size portfolios varies substantially, from a minimum of 17 to a maximum 467, but this is a direct consequence of the rules employed to construct them.

2.3 Performance of the factors

We start by comparing the performance of the Γ-portfolio representing our Γ-factor with that of the 4 portfolios associated with the standard factors; market, size, book-to-value and momentum. The cumulative profits of the different portfolios can be seen in figure 1. We see that Γ clearly outperforms all the other factor portfolios including momentum. The better performance of the Gamma portfolio over the momentum portfolio is mainly observed in the second part of the time series, starting from 1992 and can be understood from two contributions: (i) a larger return from 1992 to 2003 than even the momentum portfolio and (ii) much smaller volatility and drawdowns from 2003 to 2013 compared with the other portfolios. The first contribution is particularly interesting since it coincides with a first period corresponding to a very strong super-exponential acceleration (the dot-com bubble) [Johansen and Sornette, 2000, Phillips et al., 2011] and a second period of more than two years during which the US market drops by 65% in a strongly nonlinear sequence of crashes and aborted rallies [Sornette and Zhou, 2002], for which the log-price exhibited strong deviations from a simple trend with marked convex and concave spells that can be associated with strong Γ signatures. This suggests that indeed Γ defined as the first-difference of r_t plays a more important role than momentum based on r_t in explaining asset returns.

This is confirmed by table 3 quantifying the over-performance of the portfolio representing the Γ factor in terms of annualised profits, volatility and Sharpe ratio.

The over-performance of Γ over the other factors and in particular over momentum is robust to the used data set. We computed both factors for different industries. The corresponding returns and volatility are presented in the form of summary descriptive statistics in table 4. One can observe that, for 12 sectors out of 13, the Γ portfolio outperforms the momentum portfolio in terms of profits and Sharpe ratios.

We examine further the relationship between Γ and Δ by constructing double-sorted portfolios. This is not a formal asset pricing test, but provides some intuition about the relationship between the two dynamical factors. Table 5 shows the average return of those portfolios. Quite clearly, there is a strong tendency for returns to increase (decrease) for portfolios with increasing (decreasing) Γ_t. There is not such tendency for average returns along the Δ_t-sorted axis. We interpret this is as an indication that the returns of assets might be fundamentally determined by Γ_t and not by Δ_t, implying that momentum might be a manifestation of Γ.

2.4 Results of asset pricing tests

We regress the Γ, Δ, book-to-value and size, and industry portfolios on the CAPM, the Fama-French 3-factor model (FF), the FF model plus Δ ($FF+\Delta$), and the FF model plus Γ ($FF+\Gamma$). To determine if the models are able to explain the portfolios excess returns, we use the GRS statistic [Gibbons, Ross, and Shanken, 1989] with a 5% significance level. The GRS statistic tests the hypothesis that the intercepts ("alpha's") of our time series regressions on all portfolios are equal to 0. Rejecting the null hypothesis of $\alpha_i = 0, \forall i = 1..N$, with N the number of portfolios, implies that the model presents significant pricing errors and is unable to explain the returns of the portfolios. In addition, following the recommendation of Lewellen, Nagel, and Shanken [2010], we also report the Sharpe ratio $SR(a)$ of the intercepts of the models (the unexplained average returns), which is defined as $SR(a) = (a'S^{-1}a)^{1/2}$, where a is the column vector of all the intercepts, and S is the covariance matrix of regression residuals. As discussed by Fama and French [2012c], the advantage of $SR(a)$ as a summary statistic is that it combines the regression intercepts with the covariance matrix of the regression residuals, which is an important determinant of the precision of the alpha's. However, as $SR(a)$ combines information about both the magnitude of the intercepts and their precision, it is still useful to have the information about the two pieces provided by the average absolute intercept, the average and median R^2, and the average standard error of the intercepts.

Table 6 summarizes the results of the regressions and associated test statistics. The Γ portfolios cannot be explained by the standard factor models, CAPM, FF, and $FF + \Delta$. The GRS of all models but the $FF + \Gamma$ model is above the critical value, implying that the null hypothesis of all intercepts equal 0 can be soundly rejected. The $FF + \Gamma$ model exhibits also good performance in terms of R^2 and $SR(a)$, having correspondingly the highest and lowest values for these metrics. Since the success of the $FF + \Gamma$ model explaining the returns of the Γ portfolios might be due to the close relationship between the Γ factor and the Γ portfolios, we turn to the analysis of the other portfolios.

Similar to the Γ portfolios, we observe that the returns of the Δ portfolios cannot be explained by the standard models. On the contrary, the $FF + \Gamma$ model has a GRS statistic of 1.5784 (below the critical value) and a p-value of 0.109. The $FF + \Gamma$ models comes first in terms of the GRS statistic, average $|\alpha|$, and the $SR(a)$, but second to the $FF + \Delta$ model in terms of the average R^2. Thus, the $FF + \Gamma$ model successfully explain the momentum anomaly, but the fact that the $FF + \Delta$ model has a higher average R^2 suggests that this later model $FF + \Delta$ indeed benefits from the relationship between the quantity used to construct the portfolios and the respective factor.

As for explaining the book-to-value and size portfolios, the $FF + \Delta$ models comes first in terms of the GRS and $SR(a)$ values with 2.3749 and 0.3464, followed closely by the $FF + \Gamma$ model with 2.4341 and 0.3553. However, one should not over-emphasise this difference because the performance of all the models except CAPM is similar. The GRS statistic is in all cases above the critical value, rejecting the null hypothesis, while the average $|\alpha|$ is actually very small (0.009). Likewise, their average R^2s are all close to 0.9, very distant from the 0.72 value of CAPM.

The statistics obtained for the industry portfolios are also less convincing. The null hypothesis of all alphas equal to 0 is rejected in all models, as the GRS statistics and the $SR(a)$ reach values well above 3 (except for CAPM) and 0.8 respectively. The average $|\alpha|$s move above 0.0015 and the average R^2s are all close to 0.69 (except for CAPM). Hence, the models are in general not as good at explaining the industry portfolios as they are for the other portfolios.

The somewhat disappointing results obtained for the industry portfolios and the 25 book-to-value and size portfolios express a joint-hypothesis problem, namely testing for the relevance of our new Γ-factor is dependent on the choice of the other factors. A number of other studies have shown the importance of global factors needed to explain stock prices [Fama and French, 2012c, Asness et al., 2013] and important complementary factors seem to be needed to explain stock returns (such as the betting-against-beta [Frazzini and Pedersen, 2014], quality-minus-junk [Asness, Frazzini, and Pedersen, 2014], or profitability and investment factors [Fama and French, 2015]). Since we use the FF factors as the benchmark to quantify the added value of the Γ-factor, our results will be affected by the inadequacy of the FF factor in capturing these dimensions, which are also not accounted for by the Γ-factor.

3 Gamma strategies

3.1 Construction and performance

In a direct analogy with the strategies examined by Lewellen [2002] to analyze momentum, we consider trading strategies in which the weight $w^{\Gamma}_{i,t}(f)$ of an asset i in month t is determined by its Γ relative to the average Γ of the market at time $t-1$. The $w^{\Gamma}_{i,t}(f)$ of an asset i in month t is thus given by

$$w^{\Gamma}_{i,t}(f) = \frac{1}{N}(\Gamma_{i,t-1}(f) - \Gamma_{m,t-1}(f)) = \frac{1}{N}\left[(r_{i,t-1}(f) - r_{i,t-1-f}(f)) - (r_{m,t-1}(f) - r_{m,t-1-f}(f))\right] \tag{5}$$

where $\Gamma_{m,t-1}(f)$ and $r_{m,t-1}(f)$ correspond respectively to the Γ and returns of the equal-weighted index, and N is the total number of stocks (for notational convenience, we do not indicate the dependence of N with t). This strategy is long (short) on stocks with high (low) Γ. By construction,

$\sum_{i=1}^{N} w_{i,t}^{\Gamma} = 0$. The profit π_t of holding such portfolio for $h-$months is thus

$$\begin{aligned}\pi_t^{\Gamma}(f,h) &= \sum_{i=1}^{N} w_{i,t}^{\Gamma}(f) r_{i,t}(h) \\ &= \frac{1}{N}\sum_{i=1}^{N}(\Gamma_{i,t-1}(f) - \Gamma_{m,t-1}(f)) r_{i,t}(h) \\ &= \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1}(f) r_{i,t}(h)\right) - \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1-f}(f) r_{i,t}(h)\right) - \frac{1}{N}\left[r_{m,t-1}(f) - r_{m,t-1-f}(f)\right] r_{m,t}(h) \ .\end{aligned} \tag{6}$$

We compare the strategies defined by equations (5) and (6) with their Δ analogs, i.e., trading strategies in which the weight $w_{i,t}^{\Delta}(f)$ of an asset i in month t is determined by its momentum relative to the average momentum of the market at time $t-1$. Thus, the weight of asset i in month t determined by the Δ-strategy is given by

$$w_{i,t}^{\Delta}(f) = \frac{1}{N}(r_{i,t-1}(f) - r_{m,t-1}(f)) \ . \tag{7}$$

The corresponding $h-$months profit is

$$\begin{aligned}\pi_t^{\Delta}(f,h) &= \sum_{i=1}^{N} w_{i,t}^{\Delta}(f) r_{i,t}(h) \\ &= \frac{1}{N}\sum_{i=1}^{N}(r_{i,t-1}(f) - r_{m,t-1}(f)) r_{i,t}(h) = \frac{1}{N}\left(\sum_{i=1}^{N} r_{i,t-1}(f) r_{i,t}(h)\right) - \frac{1}{N} r_{m,t-1}(f) r_{m,t}(h) \ .\end{aligned} \tag{8}$$

Table 7 presents the profits for 36 different Γ and Δ-strategies. In addition to exploring the dependence of the results as a function of f and h, we introduce the additional parameter s to explore the sensitivity with respect to delays in implementing the positions in the portfolios. Specifically, a strategy is implemented at time t, based on the estimation of the latest returns at time $t-1-s$. Expressions (5) and (7) correspond to $s = 0$. For $s > 0$, they are replaced respectively by $w_{i,t}^{\Gamma}(f) = \frac{1}{N}(\Gamma_{i,t-1-s}(f) - \Gamma_{m,t-1-s}(f))$ and $w_{i,t}^{\Delta}(f) = \frac{1}{N}(r_{i,t-1-s}(f) - r_{m,t-1-s}(f))$. We find 30 Γ-strategies with positive returns and 20 of them are statistically significant. 6 out of the 7 strategies with negative profits correspond to a $f = 3$ months formation period. The most successful Γ-strategy selects stocks based on their returns over the previous 6 months and then holds the portfolio for 6 months, with the value $1 + s = 2$ months of delay between return estimation and implementation. In comparison, Δ-strategies exhibit positive profit for 29 out of the 36 examined strategies, 18 of them statistically significant. Their negative profits mostly correspond to a formation period $f = 12$ months (4 cases). Comparing Γ and Δ-strategies, the profits and Sharpe ratios of Γ-strategies are higher respectively in 25 and 24 cases, i.e. for more than two-third of the 36 strategies. Hence, Γ-strategies are not only profitable, but tend to outperform Δ-strategies under a robust set of parametrizations.

3.2 Sources of Gamma profit

Having established that Γ-strategies are on average profitable and beat Δ-strategies in two out of three cases, following Lewellen [2002], we now investigate their sources of profitability, and identify further differences with Δ-strategies. For this, we first note that $\pi_t^{\Gamma}(f,h)$ can be expressed as

$$\pi_t^{\Gamma}(f,h) = \pi_t^{\Delta}(f,h) - \pi_t^{2\Delta}(f,h) \ , \tag{9}$$

where $\pi_t^{\Delta}(f,h)$ is defined by (8) and

$$\pi_t^{2\Delta}(f,h) := \sum_{i=1}^{N}(r_{i,t-1-f}(f) - r_{m,t-1-f}(f))r_{t,i}(h) = \left(\sum_{i=1}^{N} r_{i,t-1-f}(f) r_{i,t}(h)\right) - r_{m,t-1-f}(f) r_{t,m}(h) \tag{10}$$

has the same structure as expression (8), but derives from it with $t-1$ replaced by $t-1-f$. The performance of Γ-strategies is thus directly related to that $(\pi_t^{\Delta}(f,h))$ of Δ-strategies. But there is another important term $\pi_t^{2\Delta}(f,h)$ that makes the story richer.

In order to unravel its meaning, assuming that the unconditional mean returns of individuals stocks are constants and making the holding period equal to the formation period to simplify the analysis (i.e. $h = f$), we can decompose the Γ profits into four components by taking the expectation of equation (6):

$$E\left[\pi_t^{\Gamma}(f)\right] = O_1 - O_2 - C_1 + C_2 \ , \tag{11}$$

where

$$O_1 := \frac{N-1}{N^2}\sum_{i=1}^{N} Cov\left[r_{i,t-1}(f), r_{i,t}(f)\right] \ , \tag{12}$$

$$O_2 := \frac{N-1}{N^2}\sum_{i=1}^{N} Cov\left[r_{i,t-1-f}(f), r_{i,t}(f)\right] \ , \tag{13}$$

$$C_1 := Cov\left[r_{m,t-1}(f), r_{m,t}(f)\right] - \frac{1}{N^2}\sum_{i=1}^{N} Cov\left[r_{i,t-1}(f), r_{i,t}(f)\right] \ , \tag{14}$$

$$C_2 := Cov\left[r_{m,t-1-f}(f), r_{m,t}(f)\right] - \frac{1}{N^2}\sum_{i=1}^{N} Cov\left[r_{i,t-1-f}(f), r_{i,t}(f)\right] \ . \tag{15}$$

The first two terms O_1 and O_2 depend on the auto-covariances of the returns of the constituting stocks, respectively between $t-1$ and t and between $t-1-f$ and t. The last two terms C_1 and C_2 provide measures of the cross-sectional diversity of the covariance of the returns of the constituting stocks compared with that of the equally-weighted index, again between $t-1$ and t and between

$t-1-f$ and t, respectively. Γ-strategies perform well when stocks are positively first-order auto-correlated and negatively second-order auto-correlated (over the time scale of $f-$months). This implies that, if firms with a positive return today have a positive return in the next period, but a lower or negative return two periods into the future, then Γ-strategies can exploit this. On the other hand, cross-serial covariances should be negative with respect to the last period but positive with respect to the period before the last one, for Γ-strategies to profit. In other words, the average one lag (resp. two lags) return covariance over all stocks should be larger (resp. smaller) than the equi-weighted index return covariance in order to contribute positively to the performance of Γ-strategies.

It is instructive to contrast this decomposition (11) with the equivalent representation for Δ-strategies:

$$E\left[\pi_t^{\Delta}(f)\right] = O_1 - C_1 + \sigma^2(\mu) \ , \tag{16}$$

where O_1 and C_1 are given by (12) and (14) respectively and

$$\sigma^2(\mu) := \sum_{i=1}^{N} \left[\mu_i(f) - \mu_m(f)\right]^2 \ . \tag{17}$$

The meaning of the first term O_1 is obvious: a positive serial covariance of stock returns provides the simplest metric of return persistance, which can be directly exploited to obtain the optimal prediction for the next return based on the linear Wiener filter. The negative contribution of the second term C_1 means that the average one lag return covariance over all stocks should be larger than the equi-weighted index return covariance to contribute positively to the performance of Δ-strategies. The last term $\sigma^2(\mu)$ quantifies cross-sectional variation in mean returns. If stock prices follow random walks with vanishing first-order auto-covariances, then Δ-strategies can still profit from the cross-sectional variation in mean returns, in absence of any time series predictability [Conrad and Kaul, 1998].

Compared with Δ-strategies, Γ-strategies do not depend on the cross-sectional variation in mean returns that contributes to $E\left[\pi_t^{\Delta}(f)\right]$. However, Γ-strategies can profit from the second order terms O_2 (13) and C_2 (15). Note that the second order terms contribute to the performance of Γ-strategies with the sign opposite to the first order terms. The existence of these contributions suggests that stock returns tend to be positively second order cross-serially correlated, implying that positive returns of others today will have a positive influence of the stock returns in the future. Moreover, stock returns tend to be negatively second order serially correlated, implying that individual winners (losers) today will tend to be losers (winners) two periods into the future.

Table 8 quantifies the contribution of these terms expressed respectively in (11) and (16) to the performance of Δ-strategies and Γ-strategies, for forming period $f=6$ and different delays $s=1$, $s=3$, and $s=6$ months to implement the positions in the portfolios. The contribution of the first-order covariance are negative for all three cases $s=1$, $s=3$, and $s=6$. The positive profit of the strategies stems from the other terms in the decomposition. Δ-strategies benefit mainly from the cross-sectional variance, which suggest that behavioral models of momentum are far from being the full story. The Γ-strategies mainly benefit from the second order terms.

From the evidence of superior performance of Γ-strategies, we have hinted at the fact that stock returns tend to be negatively second order serially correlated, implying that individual winners (losers) today will tend to be losers (winners) two periods into the future. Together with the positive correlation between first-order returns, this implies a short-term persistence in returns and a reversal over longer time horizon. This observation seems to be contradicting the short-term reversal effect of individual stock returns documented by Jegadeesh [1990], Lehmann [1990], Lo and McKinley [1990] and the return continuation for individual stocks in the medium-run [Jegadeesh and Titman, 1993] as well as their combination at the weekly time scales [Gutierrez and Kelley, 2008, Huehn and Scholz, 2015]. Our results are more in agreement with the evidence provided by Moskowitz, Ooi, and Pedersen [2012] of persistence in returns for one to 12 months that partially reverses over longer horizons. Thus, it is likely that the acceleration effect we identify is the most effective at time scales of several months, as shown in Table 7.

4 Concluding remarks

We have analyzed the effect of "acceleration" of log-prices on the predictability of stock returns. By proposing a simple quantification of acceleration based on the first difference of returns, we constructed a new Γ-factor, which is superior overall to the standard momentum factor according to standard asset pricing tests. Specifically, it explains better than momentum, when added to the three Fama-French factor model, the cross-section of stock returns. We have also built Γ-strategies for portfolio investments based on the "acceleration" effect and compared them with standard momentum-based strategies. We found that Γ-strategies are on average profitable and beat momentum-based strategies in two out of three cases, for a large panel of parameterizations. Intuitively, "acceleration" corresponds to a change of momentum. Its high significance suggests deep relations with procyclical mechanisms and psychological effects, such as desensitisation or habituation and the influence of the breakdown of the status quo associated with heightened decision difficulties and the increased uncertainty. The "acceleration" effect and Γ-factor makes more explicit and help elucidate many previous reports of transient non-sustainable accelerating (upward or downward) log-prices associated with positive feedback mechanisms [Johansen, Ledoit, and Sornette, 2000, Sornette, 2003, Johansen and Sornette, 2010, Jiang, Zhou, Sornette, Woodard, Bastiaensen, and Cauwels, 2010, Corsi and Sornette, 2014, Leiss et al., 2015], which are now being used routinely for advanced bubble warning signals [Sornette and Cauwels, 2015]. We thus interpret our Γ-factor to be the expression of the ubiquity of such regimes in stock market dynamics. We argue that the Γ-factor represents the existence of transient positive feedbacks influencing the price formation process.

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Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Avg#	Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Avg#
		10 Delta portfolios						10 Gamma portfolios			
Low	0.263	8.199	0.032	0.783	523	Low	0.049	5.934	0.008	0.201	423
2	0.299	6.403	0.047	1.139	304	2	0.351	5.092	0.069	1.680	260
3	0.630	5.508	0.114	2.791	252	3	0.441	4.833	0.091	2.226	232
4	0.477	4.935	0.097	2.361	226	4	0.378	4.574	0.083	2.020	221
5	0.405	4.490	0.090	2.201	214	5	0.593	4.524	0.131	3.200	217
6	0.347	4.417	0.079	1.919	207	6	0.544	4.480	0.121	2.963	217
7	0.446	4.468	0.100	2.437	209	7	0.621	4.533	0.137	3.346	220
8	0.644	4.598	0.140	3.418	221	8	0.662	4.923	0.134	3.283	233
9	0.584	4.820	0.121	2.958	248	9	0.811	5.157	0.157	3.838	261
High	0.881	6.070	0.145	3.545	404	High	1.056	6.131	0.172	4.205	422

Table 1: The table reports summary statistics for the 10 Γ and 10 Δ-portfolios over the period from May 1963 to December 2013. At the end of every month t, we compute decile breakpoints by sorting the NYSE stocks using $\Gamma_{i,t}$ defined by expression (3). Individual stocks from NYSE, AMEX, and Nasdaq are then allocated in the corresponding decile according to their respective $\Gamma_{i,t}$. The 10 Δ-portfolios are constructed in a similar manner, but using the one year return $r_t(12)$ to compute the decile breakpoints. We report the average excess size-weighted return (over the 1-Month U.S. T-bill-rate), standard deviation of returns, Sharpe ratio SR, t-statistic of the average return for each portfolio, and average number of firms of each portfolio.

Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Avg#	Portfolio	$\bar{r}$	$\sigma(r)$	SR	t-stat	Avg#
12 Industry portfolios						25 Book-to-value and size portfolios					
No Dur.	0.634	4.400	0.144	3.520	270	Small: Low	0.313	8.287	0.038	0.921	330
Durables	0.661	6.483	0.102	2.491	76	2	0.849	7.266	0.117	2.852	230
Manufact.	0.476	5.437	0.087	2.136	515	3	0.964	6.745	0.143	3.488	232
Energy	0.668	5.443	0.123	2.995	159	4	0.991	6.239	0.159	3.876	269
Chemicals	0.468	4.656	0.101	2.456	86	High	1.140	6.444	0.177	4.318	467
B. Equip.	0.550	6.632	0.083	2.023	486	2: Low	0.475	7.288	0.065	1.590	108
Telco	0.389	4.891	0.080	1.941	55	2	0.699	6.349	0.110	2.690	86
Utilities	0.416	4.118	0.101	2.467	147	3	0.924	5.921	0.156	3.812	79
Retailers	0.567	5.301	0.107	2.612	386	4	0.842	5.456	0.154	3.767	66
Healthcare	0.648	4.940	0.131	3.204	235	High	1.000	6.031	0.166	4.049	50
Other	0.440	5.495	0.080	1.954	516	3: Low	0.550	6.787	0.081	1.978	84
						2	0.704	5.782	0.122	2.972	62
						3	0.790	5.232	0.151	3.688	51
						4	0.761	4.964	0.153	3.741	43
						High	1.120	5.489	0.204	4.983	32
						4: Low	0.592	5.998	0.099	2.408	73
						2	0.543	5.318	0.102	2.491	53
						3	0.699	5.180	0.135	3.296	41
						4	0.717	4.890	0.147	3.580	36
						High	0.831	5.360	0.155	3.787	24
						Big: Low	0.404	4.682	0.086	2.108	85
						2	0.406	4.600	0.088	2.157	45
						3	0.406	4.378	0.093	2.263	33
						4	0.578	4.402	0.131	3.204	28
						High	0.441	5.063	0.087	2.128	17

Table 2: The table contains summary statistics for the 25 book-to-value and size portfolios and the 12 industry portfolios over the period from May 1963 to December 2013, which are constructed using the rules described by [Fama and French, 1996]. We report the average excess size-weighted return (over the 1-Month U.S. T-bill-rate), standard deviation of returns, Sharpe ratio SR and, t-statistic of the average return for each portfolio, and average number of firms of each portfolio.

	Γ	Δ	size	book-to-value	market
annualized average return	0.12	0.07	0.05	0.06	0.06
annualized volatility	0.15	0.25	0.18	0.15	0.15
annualized sharpe ratio	0.46	0.09	-0.01	0.05	0.08

Table 3: Performance of the portfolios representing the different factors in terms of annualized profits, volatility and Sharpe ratio. The Sharpe ratio was computed by taking into account the risk-free rate. This table confirms the visual information in figure 1, namely the existence of a significant over-performance of the Γ portfolio with respect to the others.

	$\mu(\Gamma)$	$\mu(\Delta)$	$\sigma(\Gamma)$	$\sigma(\Delta)$	$\frac{\mu}{\sigma}(\Gamma)$	$\frac{\mu}{\sigma}(\Delta)$
utilities	**0.001**	0.000	0.048	0.065	**0.029**	0.007
telecom	-0.001	**0.008**	0.122	0.171	-0.010	**0.048**
retail	**0.014**	0.007	0.065	0.106	**0.218**	0.064
non financial	**0.012**	0.008	0.054	0.089	**0.224**	0.095
manufacturing	**0.010**	0.004	0.067	0.093	**0.144**	0.041
healthcare	**0.006**	0.000	0.083	0.127	**0.068**	0.003
finance	**0.011**	0.009	0.062	0.103	**0.182**	0.083
energy	**0.006**	0.003	0.083	0.107	**0.075**	0.026
cons. non-durable	**0.007**	0.006	0.074	0.109	**0.089**	0.051
cons. durable	**0.015**	0.014	0.101	0.128	**0.144**	0.107
chemical	**0.005**	0.003	0.094	0.114	**0.053**	0.024
business	**0.012**	-0.000	0.078	0.114	**0.148**	-0.003
all exchanges	**0.013**	0.012	0.051	0.092	**0.250**	0.128

Table 4: Performance of the Γ and momentum portfolios across sectors. Bold figures emphasize which of the Γ or momentum had the highest profits or Sharpe ratio respectively. Out of 13 sectors, Γ outperforms momentum in 12 cases.

		Δ decile portfolios									
		smallest	2	3	4	5	6	7	8	9	biggest
Γ decile portfolios	smallest	0.004	0.004	0.006	0.004	0.006	0.005	0.004	0.006	0.005	0.010
	2	0.007	0.007	0.010	0.007	0.006	0.005	0.008	0.008	0.010	0.008
	3	0.009	0.009	0.012	0.010	0.010	0.010	0.008	0.008	0.012	0.006
	4	0.011	0.011	0.011	0.010	0.010	0.009	0.007	0.010	0.011	0.011
	5	0.012	0.012	0.011	0.012	0.011	0.010	0.011	0.011	0.011	0.013
	6	0.017	0.012	0.013	0.011	0.010	0.010	0.009	0.008	0.009	0.015
	7	0.011	0.009	0.012	0.014	0.009	0.010	0.009	0.012	0.012	0.010
	8	0.018	0.017	0.011	0.012	0.014	0.010	0.010	0.012	0.011	0.013
	9	0.015	0.015	0.011	0.011	0.011	0.011	0.014	0.014	0.012	0.012
	biggest	0.013	0.019	0.015	0.015	0.017	0.017	0.015	0.013	0.015	0.015

Table 5: Average returns of the Γ/momentum double sorted portfolios. While a tendency of increasing returns can be observed for increasing Γ deciles, no such tendency can be observed along the momentum axis, suggesting that Γ explains momentum.

Asset pricing model	GRS F-Stat	$p-value$	$SR(a)$	Average $\|\alpha\|$	$\sigma(\|\alpha\|)$	Average R^2	Median R^2
25 Book-to-value and size portfolios							
Mkt(CAPM)	3.5259	0.0000	0.4023	0.0027	0.0025	0.7104	0.7236
Mkt-RF+HML+SMB	2.5929	0.0000	0.3593	0.0009	0.0014	0.8830	0.8930
Mkt-RF+HML+SMB+Delta	2.3749	0.0002	0.3464	0.0009	0.0013	0.8859	0.9003
Mkt-RF+HML+SMB+Gamma	2.4341	0.0002	0.3553	0.0009	0.0013	0.8866	0.9013
Gamma portfolios							
Mkt(CAPM)	4.1250	0.0000	0.2777	0.0022	0.0027	0.8258	0.8371
Mkt-RF+HML+SMB	3.1717	0.0006	0.2476	0.0020	0.0025	0.8361	0.8403
Mkt-RF+HML+SMB+Delta	2.2730	0.0131	0.2115	0.0015	0.0018	0.8440	0.8465
Mkt-RF+HML+SMB+Gamma	0.8141	0.6151	0.1281	0.0006	0.0006	0.8769	0.8707
Delta portfolios							
Mkt(CAPM)	3.3615	0.0003	0.2483	0.0017	0.0022	0.7631	0.7913
Mkt-RF+HML+SMB	2.7865	0.0023	0.2298	0.0019	0.0027	0.7798	0.7979
Mkt-RF+HML+SMB+Delta	1.8024	0.0573	0.1864	0.0008	0.0010	0.8733	0.8588
Mkt-RF+HML+SMB+Gamma	1.5784	0.1094	0.1784	0.0008	0.0010	0.8007	0.8113
12 Industry portfolios							
Mkt(CAPM)	1.9050	0.0362	0.1961	0.0013	0.0013	0.6451	0.6794
Mkt-RF+HML+SMB	3.7150	0.0000	0.2786	0.0016	0.0020	0.6904	0.6971
Mkt-RF+HML+SMB+Delta	3.3490	0.0002	0.2693	0.0016	0.0018	0.6937	0.6977
Mkt-RF+HML+SMB+Gamma	3.1427	0.0004	0.2643	0.0015	0.0019	0.6918	0.6969

Table 6: The table reports summary statistics for regressions of the excess monthly returns of the 10 Γ-portfolios, the 10 Δ-portfolios, the 25 B/M-size and 12 industry portfolios on the CAPM, FF, $FF+\Delta$, $FF+\Gamma$ factors, constructed from May 1963 to December 2013. The GRS statistic tests whether all intercepts are zero. The $SR(a)$ equals $(a'S^{-1}a)^{1/2}$, where a is the column vector of all the intercepts, and S is the covariance matrix of regression residuals; it corresponds to the Sharpe ratio for the intercepts (unexplained average returns) of a model.

Formation period f	Skipping months s	Holding period h $\mu[\pi(f,h)]$ $(\sigma[\pi(f,h)])$ Δ	Γ	$\frac{\mu[\pi(f,h)]}{\sigma[\pi(f,h)]}$ (t-stat) Δ	Γ
		$h = 1$			
3	1	0.004 (0.395)	−0.016 (0.386)	0.011 (0.262)	−0.043(−1.019)
	3	0.022 (0.353)	0.011 (0.380)	0.061 (1.465)	0.029(0.686)
	6	0.011 (0.289)	−0.047 (0.488)	0.038 (0.920)	−0.095(−2.274)
6	1	0.034 (0.569)	−0.020 (0.634)	0.059 (1.417)	−0.032(−0.764)
	3	0.040 (0.506)	0.031 (0.630)	0.080 (1.911)	0.049(1.180)
	6	0.072 (0.460)	0.139 (0.528)	0.157 (3.761)	0.263(6.275)
12	1	0.088 (0.831)	0.173 (0.837)	0.106 (2.527)	0.207(4.938)
	3	0.047 (0.793)	0.127 (0.821)	0.060 (1.424)	0.155(3.692)
	6	0.001 (0.699)	0.058 (0.735)	0.001 (0.033)	0.078(1.866)
		$h = 3$			
3	1	0.062 (0.740)	0.015 (0.813)	0.084 (2.006)	0.018(0.435)
	3	0.056 (0.622)	0.001 (0.676)	0.090 (2.143)	0.002(0.036)
	6	0.053 (0.546)	−0.046 (0.722)	0.097 (2.309)	−0.064(−1.534)
6	1	0.126 (0.919)	0.035 (1.018)	0.137 (3.267)	0.035(0.830)
	3	0.129 (0.881)	0.158 (0.934)	0.146 (3.492)	0.169(4.037)
	6	0.155 (0.909)	0.331 (1.040)	0.170 (4.072)	0.319(7.601)
12	1	0.213 (1.447)	0.464 (1.538)	0.147 (3.515)	0.301(7.189)
	3	0.088 (1.362)	0.310 (1.494)	0.065 (1.549)	0.207(4.949)
	6	−0.037 (1.237)	0.120 (1.308)	−0.030 (−0.713)	0.092(2.198)
		$h = 6$			
3	1	0.099 (1.092)	−0.032 (1.103)	0.090 (2.160)	−0.029(−0.690)
	3	0.099 (1.015)	−0.039 (1.052)	0.097 (2.329)	−0.037(−0.888)
	6	0.132 (0.841)	0.154 (0.933)	0.157 (3.762)	0.165(3.926)
6	1	0.260 (1.475)	0.300 (1.450)	0.176 (4.211)	0.207(4.937)
	3	0.258 (1.462)	0.458 (1.509)	0.176 (4.221)	0.304(7.246)
	6	0.091 (1.231)	0.376 (1.306)	0.074 (1.771)	0.288(6.865)
12	1	0.196 (2.116)	0.639 (2.175)	0.093 (2.214)	0.294(7.006)
	3	0.022 (2.063)	0.393 (2.177)	0.011 (0.258)	0.180(4.302)
	6	−0.200 (1.707)	0.048 (1.976)	−0.117 (−2.802)	0.024(0.580)
		$h = 12$			
3	1	0.150 (1.587)	0.155 (1.559)	0.095 (2.266)	0.100(2.376)
	3	0.049 (1.488)	0.087 (1.527)	0.033 (0.794)	0.057(1.363)
	6	−0.048 (1.150)	0.049 (1.303)	−0.042 (−0.997)	0.038(0.900)
6	1	0.144 (2.124)	0.354 (2.209)	0.068 (1.624)	0.160(3.820)
	3	0.002 (2.015)	0.315 (2.222)	0.001 (0.027)	0.142(3.386)
	6	−0.174 (1.719)	0.229 (1.950)	−0.101 (−2.426)	0.117(2.800)
12	1	−0.101 (3.018)	0.498 (3.079)	−0.033 (−0.801)	0.162(3.857)
	3	−0.373 (2.789)	0.110 (2.861)	−0.134 (−3.203)	0.039(0.919)
	6	−0.623 (2.354)	−0.317 (2.618)	−0.265 (−6.334)	−0.121(−2.892)

Table 7: The table reports the profits $\pi_t^{\Delta}(f,h)$ and $\pi_t^{\Gamma}(f,h)$ and simple derived statistics (standard deviation σ, Sharpe ratio and its t-statistic) for momentum Δ and acceleration Γ-strategies based on different formation time scales f, holding periods h, and skipping periods s, computed over the period from May 1963 to December 2013. Γ-strategies invest every month t on asset i proportional to $w_{i,t}^{\Gamma} = \frac{1}{N}(\Gamma_{i,t-1}(f) - \Gamma_{m,t-1}(f))$, where $\sum_{i=0}^{N} w_{i,t}^{\Gamma} = 0$ and holds this portfolio for h months. Δ-strategies invest every month t on asset i proportional to $w_{i,t}^{\Delta} = \frac{1}{N}(r_{i,t-1}(f) - r_{m,t-1}(f))$, where $\sum_{i=0}^{N} w_{i,t}^{\Delta} = 0$ and holds this portfolio for h months. All profit estimates are multiplied by 100.

	$\mu_\Delta[\pi]$	$\mu_\Gamma[\pi]$	$\mu_\Delta[\pi*]$	$\mu_\Gamma[\pi*]$	$-C1+O1$	$C2-O2$	$\sigma^2(\mu)$	ϵ_Δ	ϵ_Γ
Skipping months									
1	0.260	0.300	0.282	0.388	-3.299	3.687	3.581	0.022	0.087
3	0.258	0.458	0.259	0.504	-3.860	4.364	4.119	0.001	0.045
6	0.091	0.376	0.048	0.344	-5.311	5.654	5.359	0.043	0.032

Table 8: The table contains the decomposition of the average profits of momentum Δ and Γ-strategies for forming period $f = 6$, skipping the last $s = 1$, $s = 3$, and $s = 6$ months as explained in the text. To simplify the analysis, we only consider strategies with $h = f = 6$ months. Expected profits are estimated directly via equations (8) and (6). The estimation errors of using formulas (16) and (11) correspond respectively to ϵ_Δ and ϵ_Γ. All measures are multiplied by 100.

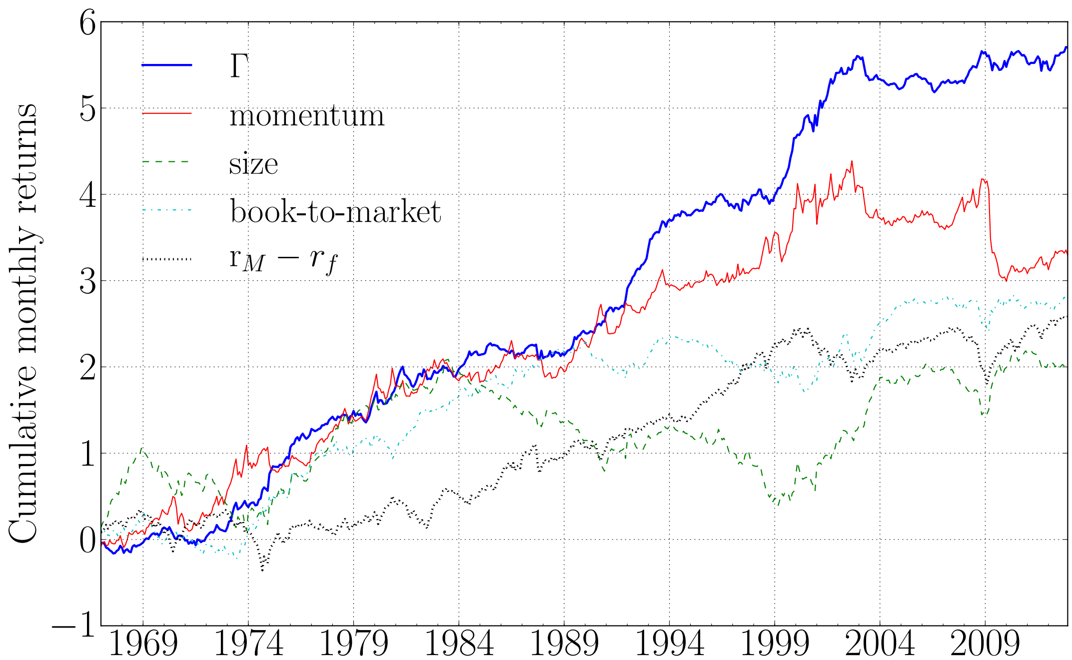

Figure 1: Cumulative monthly profits of the portfolios representing the Γ, momentum, size, book-to-market and market factors. For the factors to be comparable, each of them were constructed on the difference in returns between the top and the bottom decile portfolios, except for the market factor. Notice that this is not the standard way to compute the size and the book-to-market factors [Fama and French, 1996]. The cumulative monthly profits show the strongest performance for the Γ factor.

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