Some Observations on Trend Following: A Binomial Perspective

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revised June 12, 2019

I. Introduction

Managed Futures hedge funds and Commodity Trading Advisors (CTAs) have seen tremendous growth over recent decades, growing from roughly $300MM of assets under management (AUM) in 1980 to approximately $350BB of AUM in 2018. These investors actively manage futures and foreign exchange positions using a wide variety of different trading strategies. The vast majority of these AUM are managed by *systematic* investment strategies (~$300BB), which rely on quantitative, computer-driven models to invest. In contrast, *discretionary* investment strategies (~$45BB of AUM) tend to rely on more intuitive and heuristic approaches.[2] Systematic *Managed Futures* strategies predominantly employ *Trend Following*, also known as *time-series* momentum, in an attempt to exploit the empirical regularity that asset prices which have moved up historically tend to continue to go up, and asset prices which have declined over time on average continue to go down.[3]

Trend Following (managed futures) strategies go long futures markets where futures prices have risen and short markets where prices have fallen—usually diversifying risk across time horizons (of lookback windows), geography, and underlying asset classes. Historically, *Managed Futures* strategies have provided attractive returns to investors, returns that have largely been uncorrelated with global equity and bond markets and have exceeded the risk-free rate.[4] A number of papers, mainly written by financial economists, have suggested that the success of trend-following strategies results from market inefficiencies and the failure of the random walk hypothesis.[5] These papers argue that a wide variety of behavioral biases

[1] I am grateful to Harry Mamaysky for comments on an earlier draft. I would like to thank my former colleagues at AlphaSimplex Group LLC for many stimulating discussions on trend following and related finance topics. All mistakes reflect the author's shortcomings. Please address correspondence to david@qlspartners.com.

[2] Source: Assets under management (AUM) are taken from the BarclayHedge website.

[3] Excellent books on *trend following* include Carver (2015), Clenow (2013), and Greyserman and Kaminski (2014).

[4] Greyserman and Kaminski (2014), for instance, provide evidence on the attractiveness of trend following investing going back to the 1300s. Hurst, Ooi and Pedersen (2014), Lempérière, Deremble, Seager, Potters and Bouchaud (2014), and Geczy and Samanov (2017) present similar evidence going back over a century.

[5] See, for instance, Greyserman and Kaminski (2014) p. 66 ``Given that trend following is a method for systematically investing across asset classes following trends in historical prices, it is in direct conflict with even the weakest form of EMH.'' Moskowitz, Ooi and Pedersen (2012) p. 230 ``Our finding of time series momentum in virtually every instrument we examine seems to challenge the ``random walk'' hypothesis…''.

and non-profit seeking behavior by market participants (such as by central banks or due to risk management controls) are the sources of market inefficiencies and price trends.[6]

In this paper, I use a simple binomial framework to show (by counter example) that the existence of positive profits from a *trend-following* strategy (on a single asset), on its own, provides no *prima facie* evidence on the efficiency or inefficiency of markets. In addition, I explore the most important feature of *time series* momentum investment strategies: the *return shaping* impact of trend following through its *dynamic* positioning. In a stylized efficient market setting (with no transaction costs), I show that the *dynamic* nature of trend following *shapes* when profits and losses occur compared to a buy-and-hold strategy. There is, however, a *conservation of "mass"* in that gains and losses are shuffled across periods such that the unconditional distribution of profits is unaffected. In this sense, *trend following*, *by construction*, generates *crisis alpha* --- for crises where large losses occur over extended periods of time. I also show that *fast* and *slow* trend following strategies will shape returns in different ways. Due to its ability to shape when profit and losses occur, *trend following* can provide significant portfolio diversification and hedging potential for those investors with strategic risk-on exposures.

In Section II, I consider the profitability of a simple *trend-following* strategy in a stationary binomial framework that resembles coin tossing. The signal to go long or short is based on last period's "coin flip". If the last "coin flip" is heads (tails), the strategy goes long (short). The strategy makes money if you are long and the next flip is heads, or if you are short and the next flip is tails.[7] Profits follow a random walk in that they do not in any way depend on the past history of profits (or anything else), and the variance of profits is constant over time. The evolution of prices and profits are thus consistent with both weak- and strong-form market efficiency.[8] I consider scenarios in which: a) heads and tails are equally likely (Section II), and b) heads are more likely to occur than tails (Section V). In Section II, I demonstrate that: *trend following* will affect the timing of when profits occur and hence has the ability to generate "crisis alpha"—even if prices have no trend (i.e., heads and tails are equally likely) and *trend following* has zero expected profit.[9]

In section III, in a similar spirit, I show how buying a call option --- another dynamic strategy --- also changes the timing of when profits will occur. Section IV compares the performance of faster-moving and slower-moving trend strategies. Section V considers a similar coin-tossing binomial framework where the probability of a head exceeds that of a tails. In this environment, there is a structural expected profit to being long—and both long-only and trend-following strategies, on average, generate positive profits. Hence, the existence of positive profits from a trend-following strategy, on its own, provides no *prima facie* evidence

[6] See, for instance, Moskowitz, Ooi and Pedersen (2012) p. 229 and Greyserman and Kaminski (2014) Chapter 4.

[7] Since the current position only depends on last period's "coin flip", I refer to this as *fast trend following* in subsequent sections.

[8] This framework thus abstracts from our real world experience, where asset prices have had time-varying and stochastic trends, volatilities, and correlations.

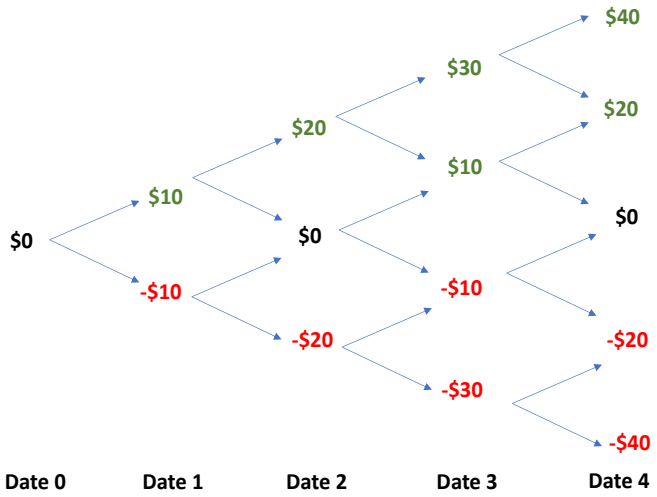
[9] Martin and Zhou (2012) make a related point and show that even if returns are independently and identically distributed, trend-following strategies can generate positive skewness. In a similar vein, Potters and Bouchaud (2008) demonstrate that trend-following strategies can affect the probability of winning trades but that this does not provide any information on the efficacy of the strategy.

on the efficiency or inefficiency of markets.[10] Section VI concludes. One important conclusion is that regardless of the efficiency of asset markets, trend-following strategies are one way to customize and alter the conditional return distribution of portfolios from that which could be earned by long-only investment strategies. In a forthcoming companion piece (Modest [2019]), I use a toy trend following model to explore how much of trend following profits arise structural / passive (in the sense of Lo [2008]) exposures and how much arise from tactical / dynamic positioning. Trend following strategies are often touted for their ability to go short as well as long. The companion piece also explores whether this perceived benefit is borne out by the data.

II. *Trend Following* in a World without Trend

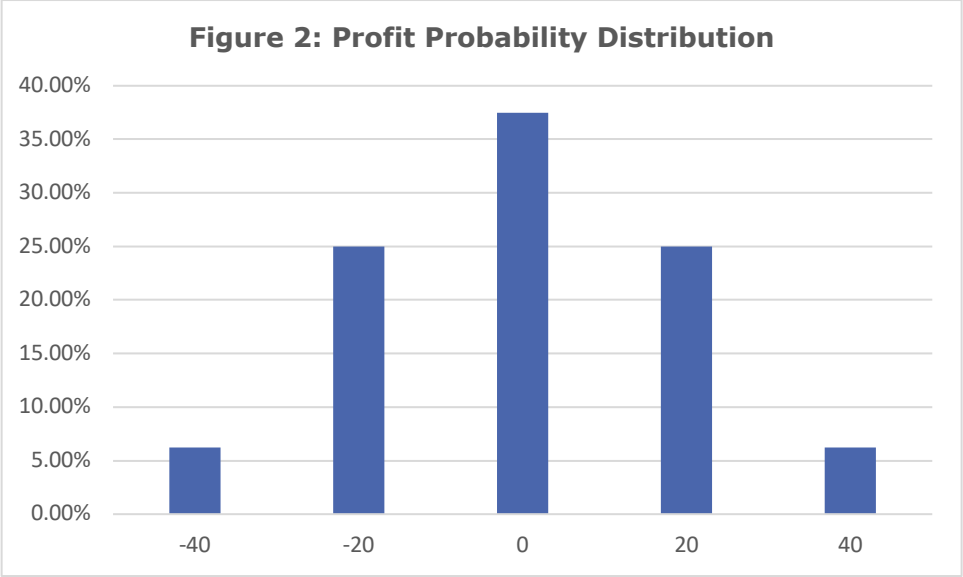
Consider a futures investment environment that resembles "coin-tossing" with a fair coin.[11] There is a fifty percent probability of a head and fifty percent probability of a tail. Upon flipping a head, the investor who is long the futures contract earns a $10 profit, and upon flipping a tail the long investor loses $10.[12] Assume there are four "coin tosses" (i.e., four periods), and the riskless rate of interest is zero. The potential cumulative dollar profits to a long futures investor, at each date, are summarized in Figure 1.

Figure 1: Cumulative Hypothetical Profits to Long-Only Investor



[10] While the existence of positive profits from trend-following strategies provides no evidence on the efficiency of markets, the magnitude of the profits will be related to the efficiency of markets and the martingale assumptions usually associated with financial markets.
[11] Often referred to as a binomial framework.
[12] Conceptually, one can think of the notional investment being altered each period to keep the dollar risk fixed. Short positions earn the above profits with a negative sign.

At the end of the first period (date 1), the long futures investor has either made or lost $10, each with a 50% probability. At the end of two periods (date 2), there are three possible outcomes for the long investor: +$20 (25% probability), $0 (50% probability), and −$20 (25% probability). Similarly, at the end of four periods (date 4), there are five possible outcomes: $40, $20, $0, −$20, and −$40 profit. There are multiple paths that can be taken to reach most outcomes in the binomial tree depicted in Figure 1.[13] We can think of the state of the world at date 4 where profits are −$40 as a *crisis*, in that the long futures investor suffers the worst outcome (a −$40 drawdown).[14] The expected cumulative profit over the four periods is zero, the (four-period) variance of profits is 400,[15] profits follow a random walk, and the market is efficient in that all information about future payoffs is known with certainty. Figure 2 displays the payoff distribution of cumulative profits. As can be seen in this figure, the "boom" and "bust" outcomes occur 6.25% of the time, and the most likely outcome is $0 profit (37.5%).



Let us now consider a (fast) trend-following strategy where the trend follower starts long the market.[16] If heads is tossed, the trend follower stays long, but if tails is tossed the trend follower goes short. This is a *fast* trend following strategy in that next period's position only

[13] The exception are the "boom'' outcome, which is only reached by tossing four heads in a row and the "crisis" outcome, which is only reached by tossing four tails' in a row.

[14] This framework precludes sharp short-term drawdowns such as occurred on Black Monday (October 19, 1987) and the Flash Crash (May 6, 2010). Large drawdowns must result from a series of negative draws over an extended period of time.

[15] The one-period variance of profits is 100 and the corresponding standard deviation is $10. The four-period variance of profits is 400 and the corresponding standard deviation is $20. Variances increase linearly with time whereas standard deviations only increase with the square root of time.

[16] Below I also consider examples where the *trend follower* starts with a neutral position (i.e. out of the market) and only puts on positions once a trend is established.

depends on the outcome of this period's coin toss. As depicted in Table 1 below, there are sixteen possible outcomes of this four-period investment. The outcomes have been colored coded where green indicates a profitable investment for the f*ast trend* follower and red represents unprofitable investments. The maximum profit for both the long-only and fast-trend investors is $40 and the maximum loss is -$40. The maximum loss for the long-only investor occurs when four tails are thrown in a row (path #16) and the maximum loss for the trend investor occurs when the path has maximum mean reversion (relative to the starting point; path #10). The expected profit from both strategies are zero and the variance of profits is 400.[17] The unconditional distributions of profits are identical for the two strategies: 6.25%

Table 1: Hypothetical Outcomes and Payoffs

	Trend Outcomes				Cumulative Profits	
Path	Date 1	Date 2	Darte 3	Date 4	Long Only Profits	Fast Trend Profits
1	u	u	u	u	40	40
2	u	u	u	d	20	20
3	u	u	d	u	20	0
4	u	d	u	u	20	0
5	d	u	u	u	20	0
6	u	u	d	d	0	20
7	u	d	u	d	0	-20
8	u	d	d	u	0	0
9	d	u	u	d	0	-20
10	d	u	d	u	0	-40
11	d	d	u	u	0	0
12	u	d	d	d	-20	20
13	d	u	d	d	-20	-20
14	d	d	u	d	-20	-20
15	d	d	d	u	-20	0
16	d	d	d	d	-40	20

of the time the strategies deliver $40 of profit, 25% of the time the strategies earns $20 profit, 37.5% of the time $0 profit, 25% of the time −$20 profit, and 6.25% the strategies both earn −$40.[18] However, the *dynamic* nature of the trend-following strategy changes when the profits and losses occur. It generates $20 of profit during the market crisis (i.e., the states where the cumulative profit from the long-only strategy is −$40). Hence, in this crisis state, the trend-following strategy generates $60 of crisis alpha! This leads to observation #1.[19]

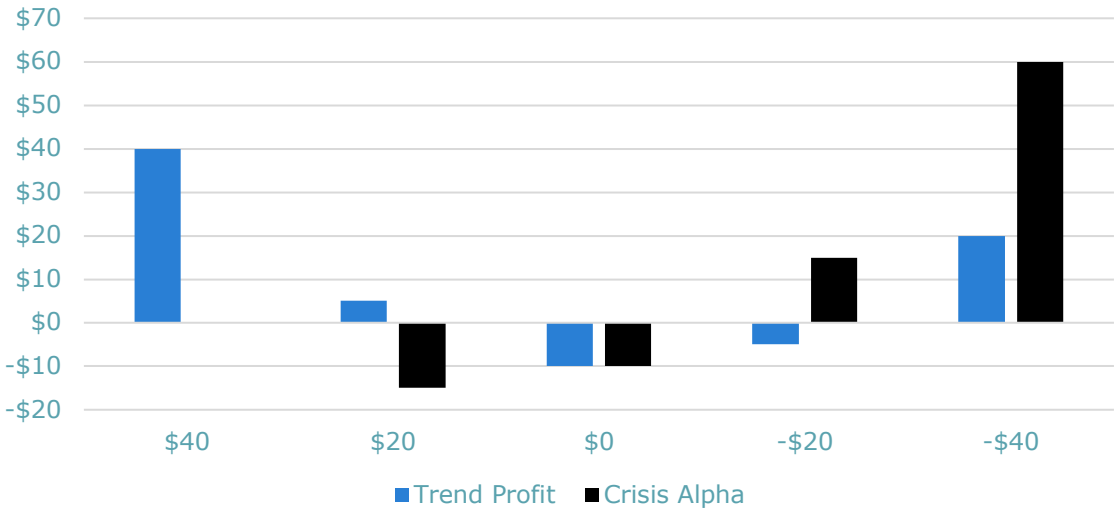
[17] Acar (1993) shows more generally that when prices follow a Gaussian random walk, the expected profit to any binary trading rule is zero and the variance of profits will equal the variance of the underlying return distribution—independent of the trading rule.
[18] In this four-period binomial example, there are sixteen different possible profit paths.
[19] In this example, the long-only and trend-following strategies have identical unconditional distributions of profit and loss. However, the conditional distributions (conditional on the realization of the underlying asset) differ substantially.

Observation #1:

In this simplified binomial setting with no trend in asset prices, the trend-following strategy has the same expected profits and variance of profits as a long-only investment strategy. However, the *dynamic* nature of the trend-following strategy shapes the return distribution and substantially changes WHEN the profits and losses occur.

Figure 3 shows the average profit and (crisis) alpha[20] of the (fast) trend-following strategy, conditional on the dollar profit of the long-only strategy.

Figure 3: Average Trend Profit and Crisis Alpha Conditional on Long-Only Profit



The trend-following strategy, on average, makes its largest profit when the market "booms" ($40 profit) or "busts" ($20 profit)—i.e., during the largest trends. On average, the *trend-following* strategy loses money when profits are equal to zero (−$10 average profit) and has its worst loss of −$40 when the long-only investor would earn $0.[21] As is well known, *trend following* does its worse in choppy markets; in this example, that corresponds to a price path of: tails, heads, tails, and heads. This leads to observation #2.

Observation #2:

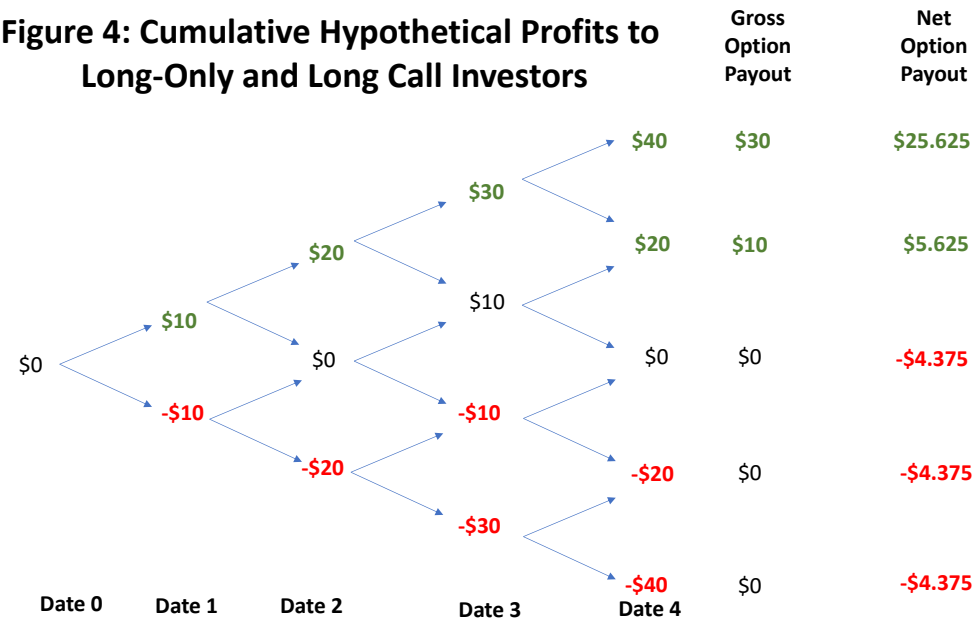
Although there is nothing "smart" about this simple trend-following strategy, it is able to generate "crisis alpha" and prevent losses in those states of the world when the long-only investor is most seriously affected—despite the absence of predictable trends in asset prices.

[20] Simply defined as the difference between the *trend-following* profit and long-only profit.
[21] In this setting, there are six different ways the *long-only* investor can earn $0 profit.

III. Dynamic Trading Strategies and Return Shaping

A *trend-following* strategy is not the only type of dynamic strategy that can alter the distribution of profits—holding the expected profit and variance of profits constant. Let us now consider a strategy of buying one call option on the four-period cumulative dollar profits with exercise price equal to $10. It is straightforward to show that the no-arbitrage value of this option is $4.375. At date 4, there are five possible cumulative profits ($40, $20, 0, −$20, and −$40) and the option only ends up in-the-money (i.e., profits greater than the exercise price) in two of these states. The gross and net (subtracting out the $4.375 option premium) cash flows for the long call option strategy are depicted in Figure 4.

Figure 4: Cumulative Hypothetical Profits to Long-Only and Long Call Investors



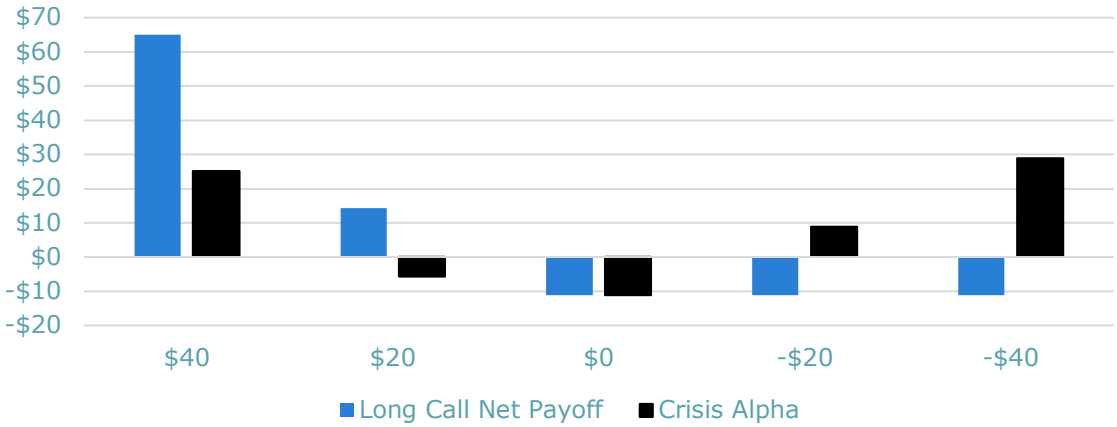
The option ends up in-the-money when cumulative profits are $40 or $20. In these two outcomes, a single option will pay off—on a gross basis—either $30 ($40–$10) or $10 ($20–$10). Given a cost of $4.375 for the option, the net profits will be either $25.625 or $5.625.[22] If cumulative profits end up being $0, −$20, or −$40, the call options are not exercised and the purchaser loses the call premium of $4.375. The variance of profits from buying one call option is 62.11.

The purchase of one call option has substantially less variance of profits than the long-only investment strategy (62.11 versus 400). In order to make these two investment strategies more comparable, it makes more sense to compare the long-only investment strategy with

[22] Since we have assumed a zero interest rates, we do not need to discount the cash flows paid at date 4 with the premium paid at date zero to put them on a comparable basis.

the purchase of 2.54 call options. Now both strategies have the same expected profit ($0) and the same variance of profits ($400). Figure 5 shows the average profit and crisis alpha of the long-call strategy, conditional on the dollar profit of the long-only futures strategy (plotted on the X axis).

Figure 5: Average *Long-Call* Profit and Crisis Alpha Conditional on *Long-Only* Profit



While both strategies have the same expected profit and variance of profits, the *conditional* distribution of profits is significantly different. The dynamic strategy of buying a call option only generates a profit 31.25% of the time and loses a moderate amount (the call premium) 68.75% of the time. While the long call option strategy loses money over two-thirds of the time, it benefits from having a strong positive skew. The long-only strategy only loses money 31.25% of the time, and the distribution of profits is symmetrical (i.e., skew = 0). Note that the call option strategy produces crisis alpha of $35.625 in that it only loses $4.375 when the "market" is losing $40. This leads to observation #3.

Observation #3:

Buying (and selling) options is an example of another dynamic investment strategy that can shape the *conditional* distribution of profits—i.e., when profits occur—even though the first two moments of the unconditional distribution are unaffected.

IV. Crisis Alpha and Lookback Horizons

One of the important decisions in practical implementations of trend following strategies is the horizon chosen for the trend signal (also known as the lookback window). The trend following

strategy examined above is *fast* in the sense that next period's position only depends on the current period profit. Let us now consider a *slower* trend strategy where, as above, the investor is either long or short the market (i.e. the investor is never out of the market), and where the investor only reverses position when they suffer losses for two periods. Table 2 summarizes the sixteen possible outcomes of this four-period investment example. The color coding in Table 2 refers to the profitability of the slow *trend following* strategy. Green squares indicate profitable investments. Red squares represent losses. As above, assume that your initial position is long the futures contract.[23] The far-right column of Table 2 shows the profit to the *slower* trend strategy. It also has a maximum (minimum) profit of $40 (-$40). Whereas the *fast* trend strategy loses the maximum in path #10, the *slow* trend strategy loses the maximum amount in path #11.

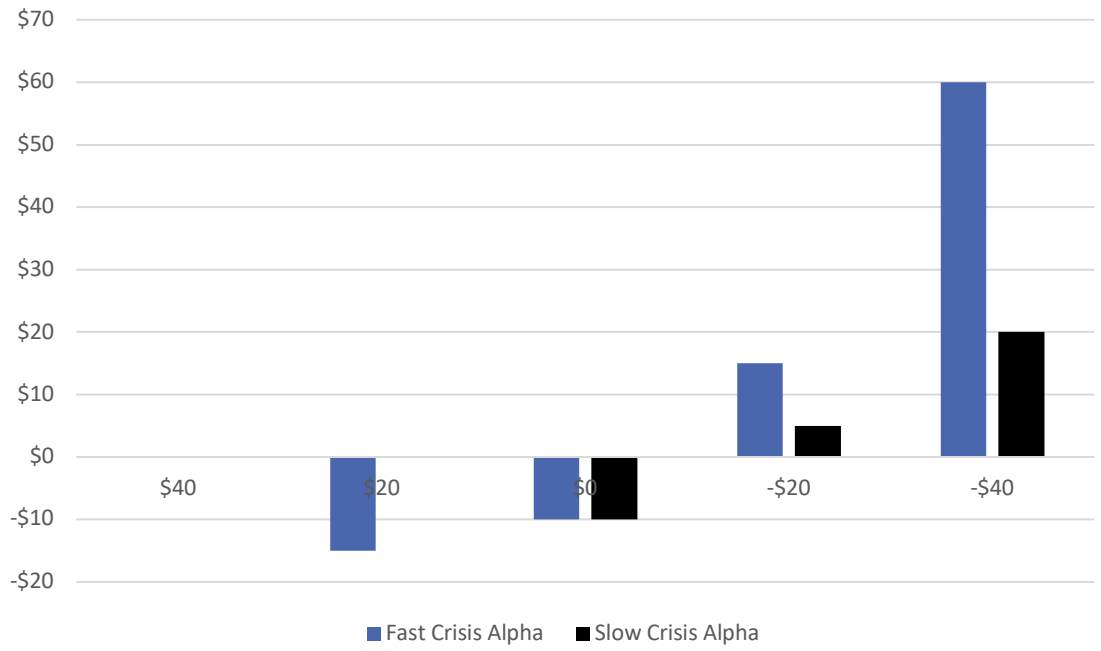
Table 2: Hypothetical Outcomes and Payoffs

	Outcomes							
Path	Date 1	Date 2	Darte 3	Date 4	Long Only Profits	Fast Trend Profits	Slow Trend Profits
1	u	u	u	u	$40	$40	$40
2	u	u	u	d	$20	$20	$20
3	u	u	d	u	$20	$0	$20
4	u	d	u	u	$20	$0	$20
5	d	u	u	u	$20	$0	$20
6	u	u	d	d	$0	$20	$0
7	u	d	u	d	$0	-$20	$0
8	u	d	d	u	$0	$0	$0
9	d	u	u	d	$0	-$20	$0
10	d	u	d	u	$0	-$40	$0
11	d	d	u	u	$0	$0	-$40
12	u	d	d	d	-$20	$20	$0
13	d	u	d	d	-$20	-$20	-$20
14	d	d	u	d	-$20	-$20	-$20
15	d	d	d	u	-$20	$0	-$20
16	d	d	d	d	-$40	$20	$0

As the slower strategy is less responsive to recent outcomes (i.e. it takes longer to adjust), the *slower* trend strategy provides less crisis alpha as depicted in Figure 6. In this example, it only provides $40 of crisis alpha during an equity market crash whereas the fast strategy provides $60 of crisis alpha--- conditional on the observation period, the length of time required for the strategy to react to market movements, and initial positioning.

[23] In the next section, we consider examples where the *trend follower* starts in a neutral position and only establishes a position once a trend is established.

Figure 6: Average *Fast and Slow* Crisis Alpha Conditional on *Long-Only* Profit



This leads to observation #4.

Observation #4:

The *faster* the trend signal (i.e. the shorter the lookback window), the more responsive the strategy will be to market turns and hence more able to provide crisis alpha. The greater responsiveness leads to higher turnover and, in a world with transaction costs, increased trading costs.

In practice, many trend followers take a barbell approach to lookback horizons: with outsized exposures in the < 3-month and > 8-month lookback horizons and smaller exposures around 6 months.

V. *Trend Following* in a World with Price Trends

In the examples discussed above, futures prices follow a particular type of martingale called a random walk in that: $E_t(\tilde{p}_{t+1}) = p_t$. In words, this simply states that the expected price at date t+1, conditional on all information available at date t, is equal to the current price. Hence, all changes in prices consist of random and unpredictable steps, and the expected change in

asset prices is zero. This is unlikely to hold in equilibrium in most financial markets, as investors typically require a positive rate of return on capital. U.S. equities, for instance, historically have gone up more than down—earning positive returns in over 70% of the years and an average annual return of 9.49% between 1928 and 2018.[24] Similarly, U.S. government bonds have had positive returns in over 80% of the years and an average annual return of 4.83% over the same period. It is thus more appropriate to think of the prices of most financial assets as following a sub-martingale where:

$$E_t(\tilde{p}_{t+1}) > p_t .^{25}$$

Let us therefore now consider an investing environment where prices follow a sub-martingale and investors earn positive expected profits from a long position. Under these assumptions, asset *prices* have a trend.[26] Assume the probability of a heads is 65%, the probability of a tails is 35%, and, as above, the long-only investor earns $10 on heads and loses $10 on tails. The expected profit over one period for an investor who is long the market is: $3 (= {.65*$10}−{.35*-$10}) and hence the expected profit over four periods is $12 (as we assume a fixed notional is invested each period). Here we consider only the fast *trend following* strategy where the investor goes long or short depending on whether the previous "coin toss" was either heads or tails. As the probability of heads exceeds a tail, it is unfair to assume that the *trend follower* starts in a long position. Instead, I assume that 65% of the time the *trend follower* starts in a long position and 35% of the time in a short position.

Table 3 summarizes the outcomes from this thought experiment. In this four-period binomial example, as above, there are sixteen-possible price/profit paths to analyze. As the expected profit is positive each period and does not depend on past outcomes, the long-only investment strategy clearly maximizes expected profits with an expected profit of $12. The expected profit from the fast trend-following strategy is $3.60 --- with a $5.70 expected profit if you start with a long position and a -$0.30 loss if you start with a short position. Assuming that the initial positions are randomized such that, you start long 65% of the time, the expected profit from the fast *trend-following* strategy is $3.60. Note that regardless the initial position, the maximum gain ($40) and the maximum loss (-$40) are identical for the long-only and *trend-following* strategies. While the range of outcomes are identical, the timing of when profits and losses occur differ, and the variance of profits are no longer identical. The variance of the long-only strategy is maximized (at 400) when the coin is fair (probability of a heads equals 50%) and diminishes as the probability of a heads or tails approaches one. For instance, when there is a 65% probability of a heads (i.e. the market going up), the variance of the long-only strategy is 364. The variance of the long-only strategy diminishes to 300 when the probability of heads increases to 75%.

While the range of outcomes are identical for the long-only and *trend following* strategies, the dynamic positioning increases the variance --- relative to the long-only strategy --- to 445.90.

[24] S&P 500 and bond returns obtained from:
www.stern.nyu.edu/~adamodar/pc/datasets/histretSP.xls. The mean returns are geometric.

[25] Appendix A discusses the conditions under which prices would be expected to follow a martingale.

[26] Although returns may or may not have a trend.

The strategy thus produces crisis alpha, but at a cost of a lower expected return and higher variance.

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Table 3: Hypothetical Outcomes and Payoffs with Unfair Coin

	Fast Trend Outcomes Initial Position: Long				Fast Trend Outcomes Initial Position: Short					Fast Trend Cumulative Profits	
Path	Date 1	Date 2	Darte 3	Date 4	Date 1	Date 2	Darte 3	Date 4	Long Only Profits	Initial Position Long	Initial Position Short
1	u	u	u	u	u	u	u	u	$40	$40	$20
2	u	u	u	d	u	u	u	d	$20	$20	$0
3	u	u	d	u	u	u	d	u	$20	$0	-$20
4	u	d	u	u	u	d	u	u	$20	$0	-$20
5	d	u	u	u	d	u	u	u	$20	$0	$20
6	u	u	d	d	u	u	d	d	$0	$20	$0
7	u	d	u	d	u	d	u	d	$0	-$20	-$40
8	u	d	d	u	u	d	d	u	$0	$0	-$20
9	d	u	u	d	d	u	u	d	$0	-$20	$0
10	d	u	d	u	d	u	d	u	$0	-$40	-$20
11	d	d	u	u	d	d	u	u	$0	$0	$20
12	u	d	d	d	u	d	d	d	-$20	$20	$0
13	d	u	d	d	d	u	d	d	-$20	-$20	$0
14	d	d	u	d	d	d	u	d	-$20	-$20	$0
15	d	d	d	u	d	d	d	u	-$20	$0	$20
16	d	d	d	d	d	d	d	d	-$40	$20	$40
									Expected Profit		
	Probability of Heads = 50%								$0	$0	$0
	Probability of Heads = 65%								$12.00	$5.70	-$0.30

This illustrates a more general point:

Observation #5:

In general, trend-following strategies will earn positive profits—even in efficient markets—as long as asset prices have a consistently positive (or negative) trend (i.e. expected returns are non-zero). The dynamic positioning of the strategy enables it to earn positive profits during sustained and predictable market moves in either direction --- as long as the moves play out over a sufficient period of time.

VI. Conclusions

Trend following, also known as time-series momentum, has generated a huge amount of intellectual and investment interest. Its early practitioners have parlayed its success into personal fortunes, including ownership of professional baseball and soccer teams and world-class golf courses. The idea behind *trend following* is a remarkably simple one: prices that have gone up will continue rising and prices that have gone down will continue to fall. That this simple idea has generated robust profits, in a highly competitive investment world, has perplexed many. Nobel Prize winner and ardent Efficient Markets proponent, Eugene Fama, for instance, said of momentum: ``Of all the things that I think are potential embarrassments to market efficiency that is the primary one.[27]''

In this paper, I have demonstrated that the most noteworthy aspect of *trend following* is its ability to *shape* the return distribution of *when* profits and losses occur --- similar to other dynamic investment strategies such as option buying and selling. I have used a simple binomial setting to illustrate this point. Other researchers have made this point in more general settings. Martin and Zhou (2012), for instance, prove that *trend following* (even in a setting where returns are independently and identically distributed) generates positive skewness. In a similar spirit, Dao, Nguyen, Deremble, Lempérière, Bouchaud and Potters (2016) show that even when returns are perfectly uncorrelated, trend strategies generate positive convexity and positive skewness. They further show the connection between trend following and an infinite collection of (continuously rebalanced) strangles.

Many authors have noted the benefits of *trend following* in inefficient markets where prices do not follow a random walk.[28] There is no disputing this assertion. However, it has also often been erroneously claimed that the profitability of trend-following strategies is *prima facie* evidence against the efficient market hypothesis. As demonstrated above with a counter-example, this is simply not true. In a simple example, where tossing "heads" is more likely than tossing "tails", I have shown that a simple trend-following strategy of following the last outcome produces a positive profit, has a low correlation (0.25) with the long-only investment strategy, and produces crisis alpha.[29]

[27] Remarks to the 65th Annual CFA Conference.

[28] These include Moskowitz, Ooi, and Pedersen (2012); Balta and Kosowski (2013); and Greyserman and Kaminski (2014).

[29] Acar (1993) provides a general formula for the expected profit to a general class of trend-following strategies when the underlying (log) prices follow a linear Gaussian process with drift. He shows that expected profits are positively related to the drift (i.e., higher drift leads to higher trend profits) and the degree of serial correlation in asset returns.

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VII. Appendix: Positive Expected Returns as a Structural Source of Trend Following Profits

What determines the behavior of expected asset prices? Should asset prices be trendless in an efficient market? Why are efficient markets associated with the notion of a random walk? In this Appendix, I briefly consider the theoretical underpinning to these questions.

The central tenet underlying most of economics, including financial economics, is the idea that economic agents (i.e., households), at the margin, make decisions that are in their best interests. Those decisions might include: how much to consume today and how much to save, should I buy Stock A or Stock B, and am I better off taking the highway or back roads. Economists, however, disagree on how agents make those decisions. Traditionally, economists have viewed economic agents as optimizing using all available information and brainpower—ideas articulated and widely accepted by the rational expectations (e.g. Lucas (1972)) and efficient markets (e.g. Fama (1970)) proponents. More recently, behavioral and other economists (e.g. Kahneman and Teversky (1979), Lo (2017)) have argued that households are more likely to find solutions that are "good enough," but do not necessarily meet all of the requirements of full rationality and optimization.

How would asset prices behave in a hyper-rational world with efficient markets? The idea that consumers/investors trade off consumption today for consumption tomorrow, and optimize their investment decisions is captured in the following equation that underlies all modern equilibrium asset pricing models, including the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT):[30]

$$p_t = E_t(\tilde{m}_{t+1}\tilde{p}_{t+1}) \qquad (1.1)$$

It provides a theory of how an asset's price, p_t , should be determined in a frictionless world with rational economic agents. The theory says that today's price should equal the expected value of tomorrow's price—with an adjustment for risk and households' time impatience. In the above expression, $\tilde{p}_{t+1}$ is the price of the asset at date t+1 (the tilde denotes that it is stochastic from the perspective of date t), $\tilde{m}_{t+1}$ is referred to in the academic literature as the stochastic discount factor (or pricing kernel), and E_t denotes the expectation operator using information available at date t . Equation (1.1) can be re-written as:

[30] This equation is the first-order condition for utility maximization and is known as the stochastic Euler equation. In the above, for simplicity of notation I assume that the asset has no payouts. Cochrane (2001) and Campbell, Lo and MacKinlay (1997) provide excellent discussions of the stochastic Euler equation and its empirical implications.

$$E_t[\tilde{p}_{t+1}] = \frac{p_t - \text{cov}_t(\tilde{m}_{t+1}, \tilde{p}_{t+1})}{E_t[\tilde{m}_{t+1}]} \qquad (1.2)$$

This differs from the martingale/random walk[31] model of security prices, which would require $E_t(\tilde{p}_{t+1}) = p_t$. Clearly, in general, prices would only be expected to follow a martingale if $\tilde{m}_{t+1}$ always equals 1 and is non-stochastic, such that the covariance term in equation (1.2) is equal to zero.[32] Equation (1.2) can also be written in return form:[33]

$$E_t[\tilde{r}_{t+1}] = r^f_{t+1} - \text{cov}_t\left\{\tilde{m}_{t+1}, \tilde{r}_{t+1}\right\}\left(1 + r^f_{t+1}\right) \qquad (1.3)$$

where $\tilde{r}_{t+1}$ is the return on the asset between dates t and t+1, and r^f_{t+1} is the return on the *riskless* asset which pays $1 at date t and is defined by: $E_t[\tilde{m}_{t+1}] = 1/(1 + r^f_{t+1})$ [34] The two necessary conditions, given above, for prices to follow a martingale (i.e., $\tilde{m}_{t+1}$ always equals 1 and is non-stochastic) clearly also imply that expected returns (including the riskless rate) are zero: $E_t[\tilde{r}_{t+1}] = 0$.

The conditions for prices to follow a martingale and expected returns to equal zero could, in principle, be met. However, we have centuries of data suggesting that investors require compensation for deferring consumption today (i.e., positive riskless rates) and investing in risky assets (i.e., risk premia above the riskless rate). Empirical evidence thus suggests asset prices generally follow a sub-martingale: $E_t(\tilde{p}_{t+1}) > p_t$, prices trend, and trend-following strategies will generate positive expected profits.

[32] The "in general" in this sentence is meant to indicate that there are other narrow assumptions that could be made under which prices would follow a random walk or martingale.
[33] Campbell, Lo, and Mackinlay (1997) provide a more complete derivation in Chapter 8.
[34] In the absence of a riskless asset, this can be thought of as the return on a zero-beta asset.