High-Frequency ETF Pairs Trading

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Abstract

In this paper we examine the effectiveness of modeling a paris-traded ETF portfolio as an Ornstein-Uhlenbeck process. Using ETF pairs that have similar references indexes, we apply maximum likelihood estimation to historical data in order to optimize trading sginals for two strategies. Using this information, we test the optimal trading rules using intraday price observations over a variety of trading periods ranging from 5 days to 42 days. Our results have shown that the sample of ETF pairs-traded portfolios selected exhibit mean-reversion properties that are well modeled as an Ornstein-Uhlenbeck process. We have found that while higher total trading returns were correlated with shorter optimization and trading periods, they also carried considerable risk and as such more stable results were found using longer optimization and trading windows.

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Overview

My strategy project will focus on statistical arbitrage trading via pairs-trading. It is well understood in finance that numerous instruments exhibit mean-reversion properties (see Elliott, Hoek, and Malcolm (2005), Leung and Li (2016), Avellaneda and Lee (2010), Gatev, Goetzmann, and Rouwenhorst (2006), Yadav and Pope (1992), and MacKinlay and Ramaswamy (1988)). Single securities are often too volatile or non-stationary, but by taking positions in two cointegrated securities we can construct a portfolio that displays mean-reversion properties with greater reliability. Rather than testing for cointegration, I will instead assume that two ETFs that are designed to track the same index or commodity are highly cointegrated, and as such a long-short portfolio of them will create a stationary time series, herein modeled as an Ornstein-Uhlenbeck (OU) process (see Leung and Li (2015)). The intuition behind the strategy is that two ETFs tracking the same index or commodity should be priced the same (have the same proportional price movements), and so any mis-pricing of the assets can be exploited for statistical arbitrage (Gatev, Goetzmann, and Rouwenhorst (2006) conclude that the robustness of their results from a pairs trading strategy indicate that pairs trading profits from temporary mispricing of assets). These assumptions also imply that the market direction shouldn't affect the success of the strategy and, theoretically, there is no exposure to the broader market regardless of global market trends. Thus, the strategy is based on relative value pricing between the two assets in each portfolio.

The pairs portfolios will be composed of two assets, with respective positions α_{t-1}^i and β_{t-1}^i

$$P_t^i = \alpha_{t-1}^i S_t^1 - \beta_{t-1}^i S_t^2$$

where P_t^i is the i^{th} portfolio during trading period t, with optimal weights found via maximum likelihood estimation (MLE) during period $t-1$. Finding the optimal investment weights in $t-1$ will also provide the optimal long-run mean for the portfolio during that trading period, and the hypothesis is that the portfolio will continue to revert back to this same mean during out-of-sample testing. Market entry/exit signals are also found during period $t-1$ based on which buy/sell levels maximize total returns. From these indicators and signals, we either buy the portfolio (buy α_{t-1}^i shares of S^1 and sell/short β_{t-1}^i shares of S^2) or sell/short the portfolio.

Based on the model above, I test two trading strategies for each portfolio: 1) Hold positions over night and trade exclusively on the price movements and the interaction with indicators, signals, and rules process, and 2) trade during the day based on indicators, signals, and rules, but with the additional rule where we liquidate all positions at the end of the trading date regardless of portfolio value. Each strategy will be tested for trading periods of length 5 days, 11 days, 21 days, 31 days, and 42 days (total trading periods = total observations/(trading days x 78), where 78 is the number of observations per day based on 5-minute pricing).

The success of the strategy will be measured based on total returns compounded over the total year's worth of trading data (12 months, with the first month serving as the optimization period for the first actual trading month), where simple returns are given by

$$r_t = \frac{P_t - P_{t-1}}{P_{t-1}}$$

for any observation t, and where total returns over period t to $t+i$ are given by

$$TR_t = \prod_{i=1}^{n} (1 + r_i) - 1$$

I will also use Sharpe Ratio to evaluate each portfolio by using the mean return and average volatility over the 12 month period, but 'success' is ascribed based on total returns.

Hypotheses

The central hypothesis of this paper is that the closeout trading strategy – where positions are liquidated at the end of every trading day – outperforms the non-closeout strategy – positions are held overnight and only adjusted based on the spread and portfolio price – as assessed by total returns when both are traded based on their respective optimal market entry/exit points. A secondary but related hypothesis is that it is not necessary to trade optimally (in terms of pure returns per trade) given a high enough

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trading frequency. The closeout strategy should trade more frequently during a given trading period, and so consistently higher overall returns with smaller returns per trade would support this hypothesis.

Another underlying assumption is that the optimal parameters found in trading period $t-1$ will also be optimal during period t, which will be easy to test based on simple comparison. However, the overall validity of this assumption can be tested via the mean-squared error between the optimized parameters in $t-1$ and the optimized parameters in t.

Key Techniques

While not a hypothesis test, a key technique required for my strategy is to estimate the parameters of the OU model using the maximum likelihood estimation procedure outlined by Leung and Li (2015). Under the OU model, the conditional probability density of the process (portfolio) $\{X_{t_i}\}_0^n$ at any given time t_i given $X_{t_{i-1}} = x_{t_{i-1}}$ is

$$f(x_i|x_{i-1};\theta,\mu,\sigma) = \frac{1}{2\pi\tilde{\sigma}^2} exp\left(-\frac{(x_i - x_{i-1}e^{-\mu\Delta t} - \theta(1-e^{-\mu\Delta t}))^2}{2\tilde{\sigma}^2}\right)$$

where

$$\tilde{\sigma}^2 = \sigma^2 \frac{1-e^{-2\mu\Delta t}}{2\mu}$$

After observing n values $x_t^{\alpha,\beta}$ for the process we maximize the average log-likelihood defined by

$$\begin{aligned}
l(\theta,\mu,\sigma|x_0^{\alpha,\beta}, x_1^{\alpha,\beta}..., x_n^{\alpha,\beta}) &= \frac{1}{n}\sum_{i=1}^{n} \log f(x_i|x_{i-1};\theta,\mu,\sigma) \\
&= -\frac{1}{2}\log(2\pi) - \log(\tilde{\sigma}) - \frac{1}{2n\tilde{\sigma}^2}\sum_{i=1}^{n}(x_i - x_{i-1}e^{-\mu\Delta t} - \theta(1-e^{-\mu\Delta t}))^2
\end{aligned}$$

Leung and Li (2015) define the following to express the parameter values that maximize the average log-likelihood given above

$$\begin{aligned}
X_x &= \sum_{i=1}^{n} x_{i-1}^{\alpha,\beta} \\
X_y &= \sum_{i=1}^{n} x_i^{\alpha,\beta} \\
X_{xx} &= \sum_{i=1}^{n} (x_{i-1}^{\alpha,\beta})^2 \\
X_{xy} &= \sum_{i=1}^{n} x_{i-1}^{\alpha,\beta} x_i^{\alpha,\beta} \\
X_{yy} &= \sum_{i=1}^{n} (x_i^{\alpha,\beta})^2
\end{aligned}$$

Now, the optimal parameter estimates are given explicitly by

$$\begin{aligned}
\hat{\theta} &= \frac{X_y X_{xx} - X_x X_{xy}}{n\left(X_{xx} - X_{xy}\right) - (X_x^2 - X_x X_y)} \\
\hat{\mu} &= -\frac{1}{\Delta t}\log\frac{X_{xy} - \hat{\theta}X_x - \hat{\theta}X_y + n\hat{\theta}^2}{X_{xx} - 2\hat{\theta}X_x + n\hat{\theta}^2} \\
\hat{\sigma}^2 &= \frac{2\hat{\mu}}{n(1-e^{-2\hat{\mu}\Delta t})}(X_{yy} - 2e^{-\hat{\mu}\Delta t}X_{xy} + e^{-2\hat{\mu}\Delta t}X_{xx} \\
&\quad - 2\hat{\theta}(1-e^{-\hat{\mu}\Delta t})(X_y - e^{-\hat{\mu}\Delta t}X_x) + n\hat{\theta}^2(1-e^{-\hat{\mu}\Delta t})^2)
\end{aligned}$$

So, letting $\hat{l}(\hat{\theta}, \hat{\mu}, \hat{\sigma})$, we fix α to be the amount of initial capital divided by the initial value of the long asset at time $t = 0$ and then choose β^* where

$$\beta^* = \underset{\beta}{\text{argmax}} \;\; \hat{l}(\hat{\theta}, \hat{\mu}, \hat{\sigma} | x_0^{\alpha,\beta}, x_1^{\alpha,\beta}, ..., x_n^{\alpha,\beta})$$

so that our portfolios are given by

$$P_t^i = \alpha S_t^1 - \beta^* S_t^2$$

With the long-run mean estimate $\hat{\theta}$ in hand, we back-test entry/exit spreads based on dollar-difference from the long-run mean and isolate the spread that maximizes returns (i.e., test $\pm\$1, \pm\$2$, etc. as the entry/exit spreads so that we are buying the portfolio if it is undervalued and selling the portfolio if it is overvalued). At every price observation, the strategy will either buy, sell, or remain neutral 1-share of the portfolio, where buying one share is buying α shares of asset one and shorting β shares of asset two, and vice versa for selling (in this paper, one trade refers to either buying or selling the *portfolio* once). So, when a position is closed in the closeout strategy, the number of trades is the absolute value of the number of shares of the portfolio held just prior to the end of the trading day (another way to state this is that the number of trades executed in a trading period is the sum of the absolute number of shares bought or sold of the portfolio itself). This method is a 'brute force' approach rather than solving stopping problems or using statistical moments. Whatever the market entry/exits points are discovered for trading period $t - 1$ are the ones used to trade in period t. This will act as a rolling estimation and trading window where signals (parameters) are 'rebalanced' every x days for $x \in \{5, 11, 21, 31, 42\}$.

Filters – Indicators, Signals, and Rules

Indicators

The indicators necessary for implementation of both strategies that I will be testing are based on relative value pricing of the two assets in each pairs portfolio, along with OU parameters estimated via maximum likelihood estimation (MLE). Speed of mean reversion ($\mu > 0$) and volatility ($\sigma > 0$) will not be included as indicators in the strategy, although incorporating these could provide material for an extension of this paper. The primary indicator will be the estimate of the long-run mean of the process θ, where the process is given by

$$\Delta P_t^i = \mu_{t-1}^i \left(\theta_{t-1}^i - P_t^i \right) \Delta dt - \sigma_{t-1}^i \Delta W_t^i$$

and W_t^i is a standard Brownian motion.

The indicator θ_{t-1}^i for pairs portfolio P_t^i is derived from MLE applied to market data over trading period $t - 1$. Given an initial dollar value A invested in the long position, we optimize the dollar value B invested in the short position (and by extension, the number of shares invested in each asset) by testing B values for $B/A = 0.001, 0.002, ..., 1$ based on the maximum log-likelihood value obtained via MLE. Thus, the positions taken in each asset during trading period t (trading based on market data in period t), are given by the optimized dollar amounts A and B from trading period $t - 1$. The portfolio at time t is

$$P_t^i = \alpha_{t-1}^i S_t^{i,1} - \beta_{t-1}^i S_t^{i,2}$$

where $S_t^{i,1}$ and $S_t^{i,2}$ are time-series of market data for each asset traded in portfolio i. The primary indicator, however, is θ_{t-1}^i, which we will take positions in the asset based on the value of P_t^i relative to θ_{t-1}^i.

The underlying hypothesis about the indicators in the strategy is that the value θ_{t-1}^i will be an accurate measure of the long-run mean for trading period t. In theory, the value of the portfolio should fluctuate around the same mean no matter what the broader market conditions are since the portfolio is composed of optimized shares invested in each asset. If θ_{t-1}^i is a good indicator, then the absolute total returns calculated in trading period t should be maximal when trading based on the optimal signals derived from θ_{t-1}^i, discussed below. The indicator θ_{t-1}^i will be evaluated based on MSE between periods.

Signals

The signals for trading during period t will be based on the indicator θ^i_{t-1}. After optimizing our positions in each asset during $t-1$ based on MLE, we test each strategy based on a widening buy/sell spread built around the value θ^i_{t-1}. Each portfolio will be traded using the buy/sell signals $\pm 0.5, \pm 1.5, ..., \{-0.5 - min(k, |P^i_{t-1}(0) - min(P^i_{t-1})|), 0.5 + min(k, |max(P^i_{t-1}) - P^i_{t-1}(0))|)\}$, for $k \in [-(P^i_{t-1}(0) - min(P^i_{t-1})), max(P^i_{t-1}) - P^i_{t-1}(0)]$. Put more simply, we expand the market entry/exit signals by $1 around the spread until we hit the maximum absolute difference between the maximum and minimum value of the portfolio in $t-1$ and its initial value. Our trading signal in t is the spread that maximizes total returns over period $t-1$, and so our signals can change from one trading period to another and evolve on a rolling basis. The rules that will be based on the spread are to sell/short the portfolio for each pricing time (every 5 minutes) that the portfolio value P^i_t is above $\theta^i_{t-1} + sell_{i-1}$ and to buy when it is at or below $\theta^i_{t-1} - buy_{i-1}$. The choice of $1 is arbitrary, and there is possible room for improvement here by decreasing the amount the spread changes over each iteration or by doing this based on a relationship to the initial portfolio value (additionally, the signals are also not necessarily symmetric about the mean).

As such, the signals will be difficult to test for since there is not a fixed relationship to the portfolio value or any other parameters. Rather, they are found via brute force search and are based on portfolio value relative to its long run mean and not measured via standard deviation, for example. Other authors have developed different market timing signals, such as Gatev, Goetzmann, and Rouwenhorst (2006) who study entry/exit levels based on ± 1 standard deviation of the price from the mean, Elliott, Hoek, and Malcolm (2005) who model market timing signals by the first passage of an OU process, and Leung and Li (2015) who generate market timing signals by solving an optimal double stopping problem. These approaches would be easier to test, but the signals derived in this paper are somewhat *ad hoc*. This is an inherent weakness in the strategy, and so being able to extend the project to develop more rigorous signals would be a valuable addition.

Rules

My strategy has only a few, simple rules. We are not automating the strategy (i.e. via *quantstrat*), and the strategy doesn't incorporate any information about the spread, so the orders are not aggressive but rather made at the value of the pairs portfolio at the time of execution. The only rules for the non-closeout strategy is to buy the portfolio when its value during period t drops below the optimal mean and threshold $\theta_{t-1} - buy_{t-1}$, and to sell/short the portfolio when it is priced above $\theta_{t-1} + sell_{t-1}$. The strategies do nothing when the value falls in the interval between the buy and sell threshold. The only additional rule is for the closeout strategy, where all positions are liquidated at the end each trading day, regardless of the value of the portfolio.

These rules are based on optimization from the prior trading period, and as such they are certainly overfitted for the trading period in which they are derived. However, given the assumption that the portfolio will continue to revert to the mean found in $t-1$, then this shouldn't affect profitability, but any change in the portfolio dynamics or asset weights will increase the likelihood that the rules are no longer optimal.

Inherently, the strategy is a walk-forward analysis. It optimizes parameters during in-sample period $t-1$ and then evaluates the performance of the indicators, signals, and rules based on these parameters using out-of-sample data in period t. As we roll this window forward, the in-sample and out-of-sample data roll forward as well to t and $t+1$ respectively.

Literature Review

In their 2015 paper, Leung and Li develop an analytical approach to finding optimal market timing signals by solving a double stopping problem subject to transaction costs over a finite horizon. Their approach, which models pairs portfolio as an OU process and utilizes maximum likelihood estimation, is the one I've adopted above as my initial assumptions. In Leung and Li (2015) and Guo and Leung (2015), they have applied the methodology to the pairs trading of commodity ETFs.

Gatev, Goetzmann, and Rouwenhorst (2006) examine the risk and returns characteristics of pairs trading using daily data from US equity markets over 1962-2002. They don't model their portfolios by a stochastic process, but rather assume mean reversion for their data based on the theory of the Law of One Price (LOP). They filter by choosing pairs based on the criterion that they have had the same or nearly the same historical state prices and test for cointegration of the residuals of their pairs processes.

They further sort their pairs portfolios by choosing a partner asset for each stock that minimizes the sum of squared deviations between the two normalized price series. They traded multiple strategies based on different levels of standard deviation of the pairs from their historical mean (i.e. $\pm1, \pm2$), and close out once they have reverted. In addition, they close all positions at the end of the trading period.

Their approach incorporates low-frequency institutional risk factors, such as bankruptcy, as well as short term factors such as bid-ask bounce, short selling costs, and transaction costs. They found that some short term factors affect the magnitude of excess returns, but that pairs trading remains profitable using reasonable assumption over the long term. They argue that even though their pairs strategy benefits from short-term mispricing of assets, profits are not caused by simple mean-reversion (they found an average annualized return in excess of 11%). Additionally, their risk assesment finds that their profits are uncorrelated to the S&P 500, which supports the hypothesis that the pairs strategy tested here is similarly not, or minimally, exposed to broader market movements.

Hogan et al. (2004) test a dollar-neutral strategy that holds a position α in an equity and β in a money-market account (risk free asset) such that their initial investment is $0 (they also argue that the risky asset they invest in can be represented by long-short positions in various assets). While they examine different strategies, such as momentum and value strategies, they also show that long-short portfolios can be used for statistical arbitrage purposes.

Avellaneda and Lee (2010) demonstrated that the residuals of a portfolio composed on long positions in two cointegrated assets can be successfully modeled as a mean-reverting process (i.e. shown to be a stationary process). The residuals in this case are the results of a long-short poortfolio of two assets, and so this lends support to the assumption of a mean-reverting model for a pairs traded portfolio.

There is considerable literature on the subject of pairs trading and mean-reversion of long-short portfolios, and the consensus seems to be that long-short portfolios of significantly cointegrated (or correlated) assets exhibit mean-reversion and can be used for statistical arbitrage using a variety of trading indicators, signals, and rules.

Data

This paper will trade six pairs portfolios composed of the following asset pairs and their respective reference index

SPY/VOO	S&P 500
SPY/IVV	S&P 500
VOO/IVV	S&P 500
IWM/SLY	Russell 2000
$VIOO/VTWO$	S&P Small-Cap 600
USO/OIL	Crude Oil

The instruments were chose to provide a small sample of ETFs that track US equities, with some diversity coming from the different indexes themselves. The crude oil ETFs were chosen as an example of how this strategy might apply to commodity ETFs, and hopefully this could be extended to a broader array of commodity or US equity sector ETFs.

We have collected 5-min intraday observations for all ETFs from December 31, 2015 to January 31, 2016, which is roughly 272 trading days. With 78 observations per day, this makes for approximately 21,200 observations for each instrument. The prices are listed for each observation as the 'Close' price and so the bid/ask spread in not considered.

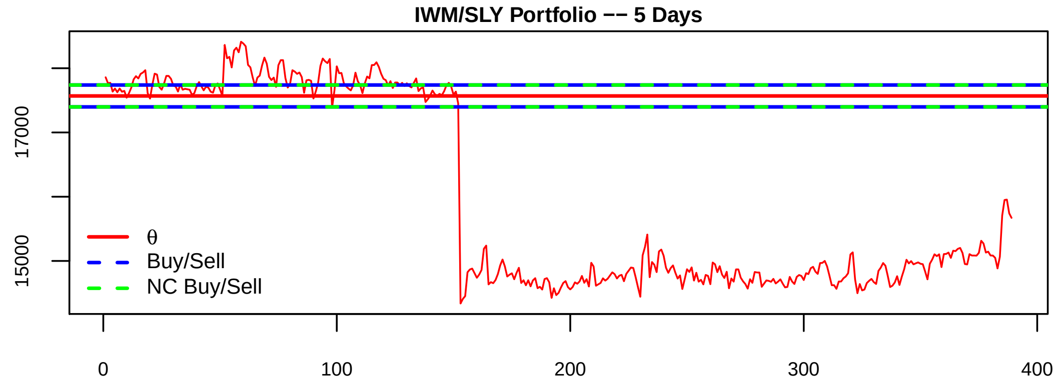
Constraints, Benchmarks, and Objectives

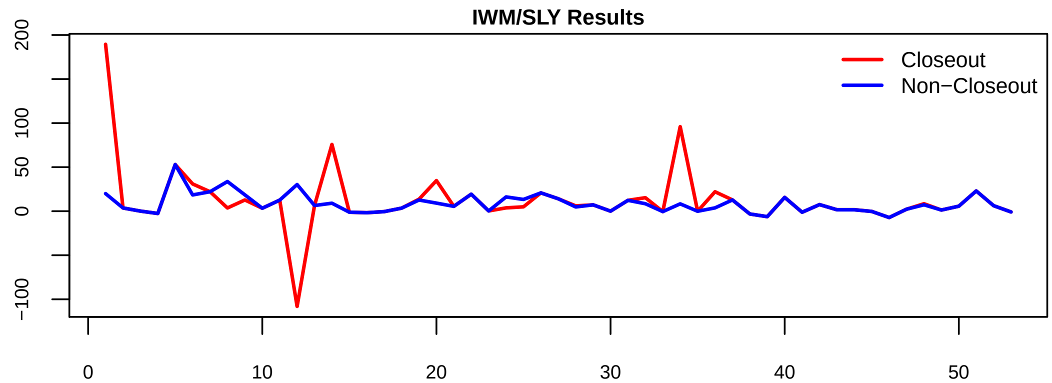
The main objective is to 1) show that both strategies are profitable overall, 2) show that the closeout strategy outperforms the non-closeout strategy over the 12-month window, and 3) demonstrate that trading at optimal profit levels are not necessary for maximal overall returns. The last objective really is to show that although the closeout strategy will likely trade at a lower profit-per-trade, the higher number of trades will still lead to higher overall returns than the non-closeout for any given portfolio or trading window.

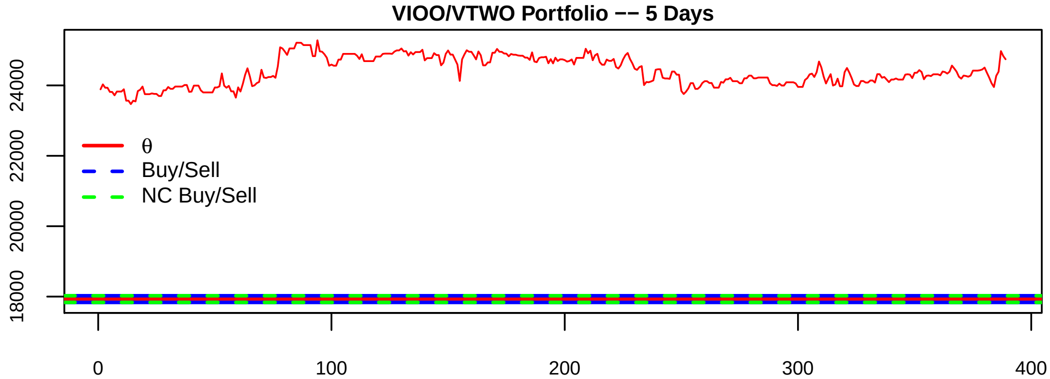
There is no true benchmark to compare the strategy to, aside from perhaps looking at the overall returns for each ETF during the 12-month period. Comparing total returns of a portfolio vs the ETFs that make up that portfolio may give an idea as to the success of the strategy versus a buy-and-hold strategy, but this is not very relevant to a statistical arbitrage strategy. In theory, the pairs portfolios will have no market exposure and can be profitable whether or not any given instrument being traded systematically increases or decreases in value over the whole investment window. As such, the best benchmark here is the measure of overall total returns.

The strategy is constrained by the fact that, as it is coded right now, it doesn't incorporate any information from the bid/ask spread. This information is important for high-frequency trading, and even though 5 minute intervals is often considered slow by HFT standards, lacking order book information is likely a limiting factor in the realism of the results, as well as the fact that we have ignored transaction costs for the sake of simplicity. Additionally, the strategy assumes that one can take fractional positions in ETFs, and so it is trading non-integer share values/weights, which is not a realistic assumption.

Results and Analysis






VIOO/VTWO Portfolio -- 5 Days
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Buy/Sell
NC Buy/Sell
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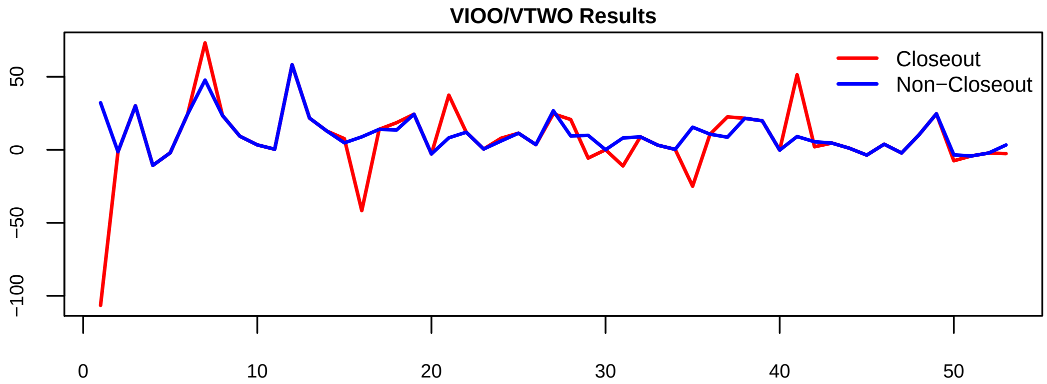
20000
22000
24000
0
100
200
300
400


VIOO/VTWO Results
Closeout
Non−Closeout
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−50
0
50
0
10
20
30
40
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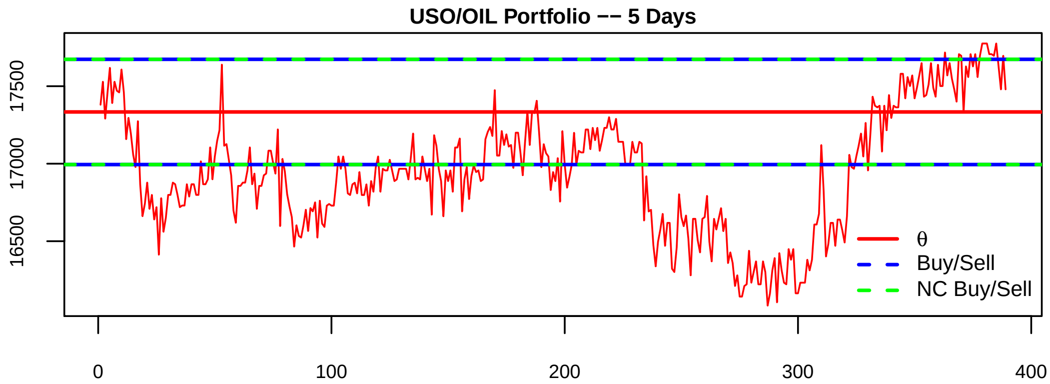
USO/OIL Portfolio -- 5 Days
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Buy/Sell
NC Buy/Sell
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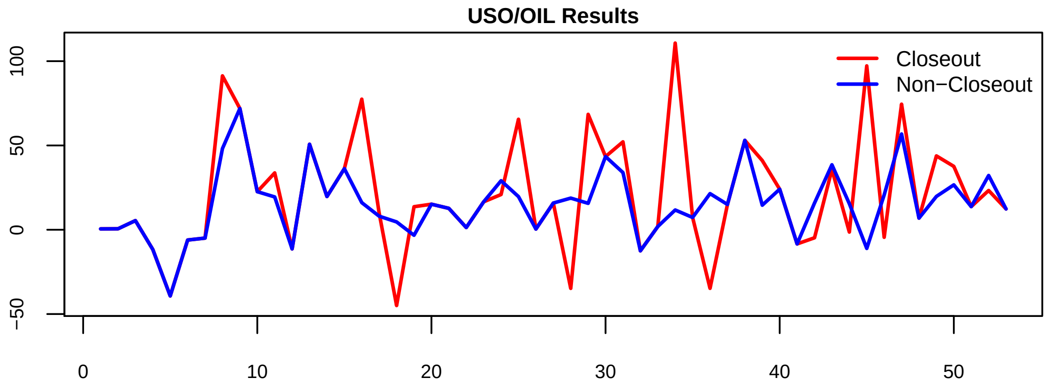
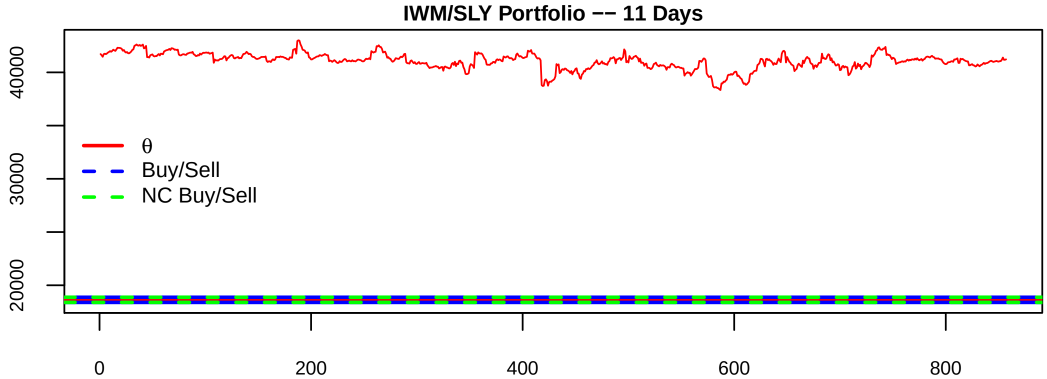
17000
17500
0
100
200
300
400


USO/OIL Results
Closeout
Non−Closeout
−50
0
50
100
0
10
20
30
40
50


IWM/SLY Portfolio -- 11 Days
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Buy/Sell
NC Buy/Sell
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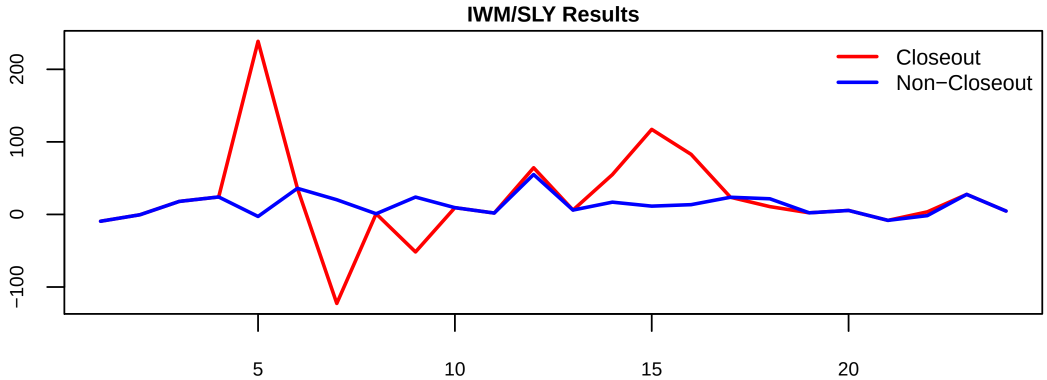
30000
20000
0
200
400
600
800


IWM/SLY Results
Closeout
Non−Closeout
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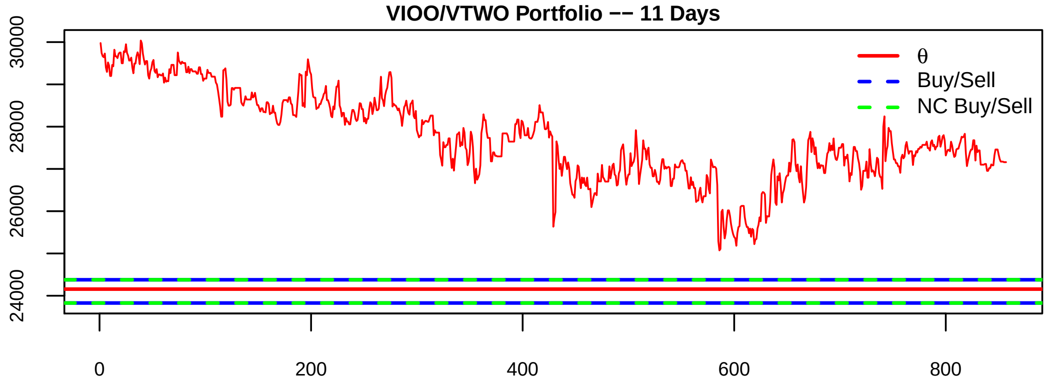
−100
5
10
15
20


VIOO/VTWO Portfolio -- 11 Days
θ
Buy/Sell
NC Buy/Sell
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26000
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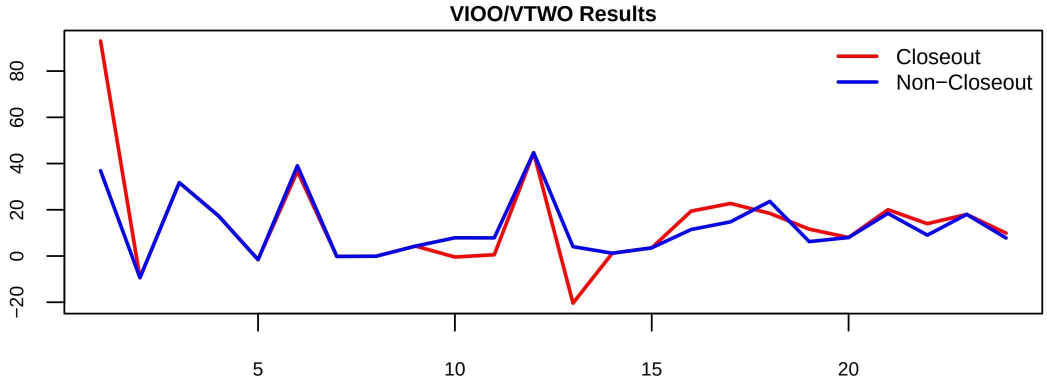
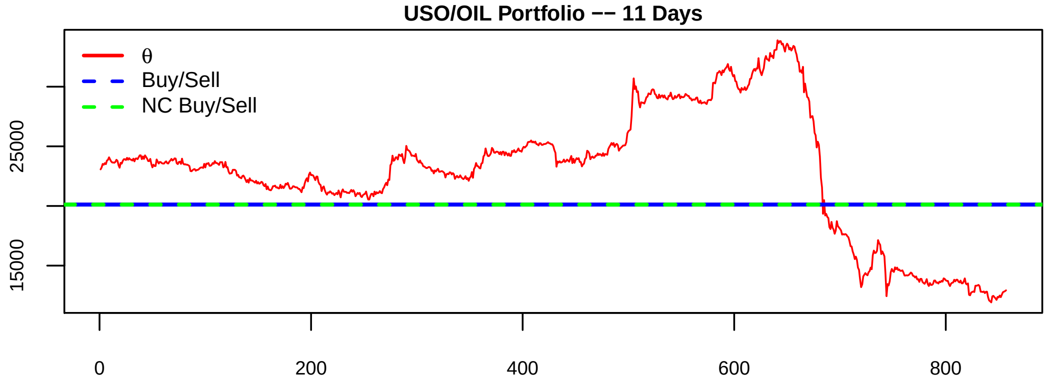
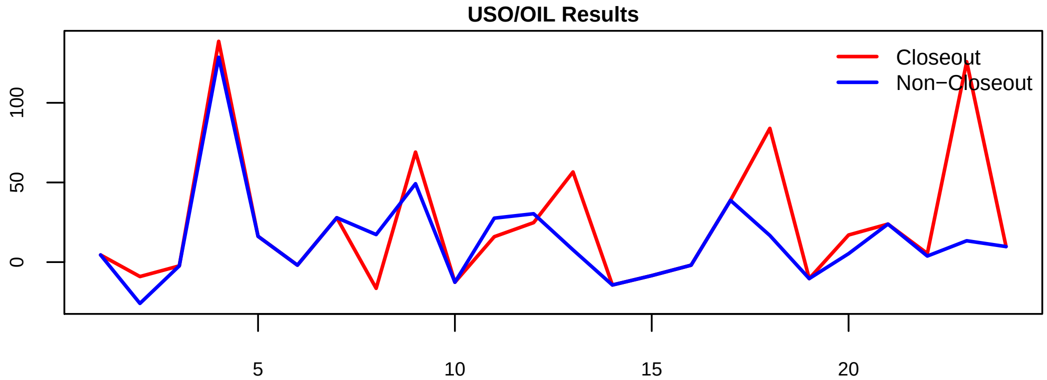
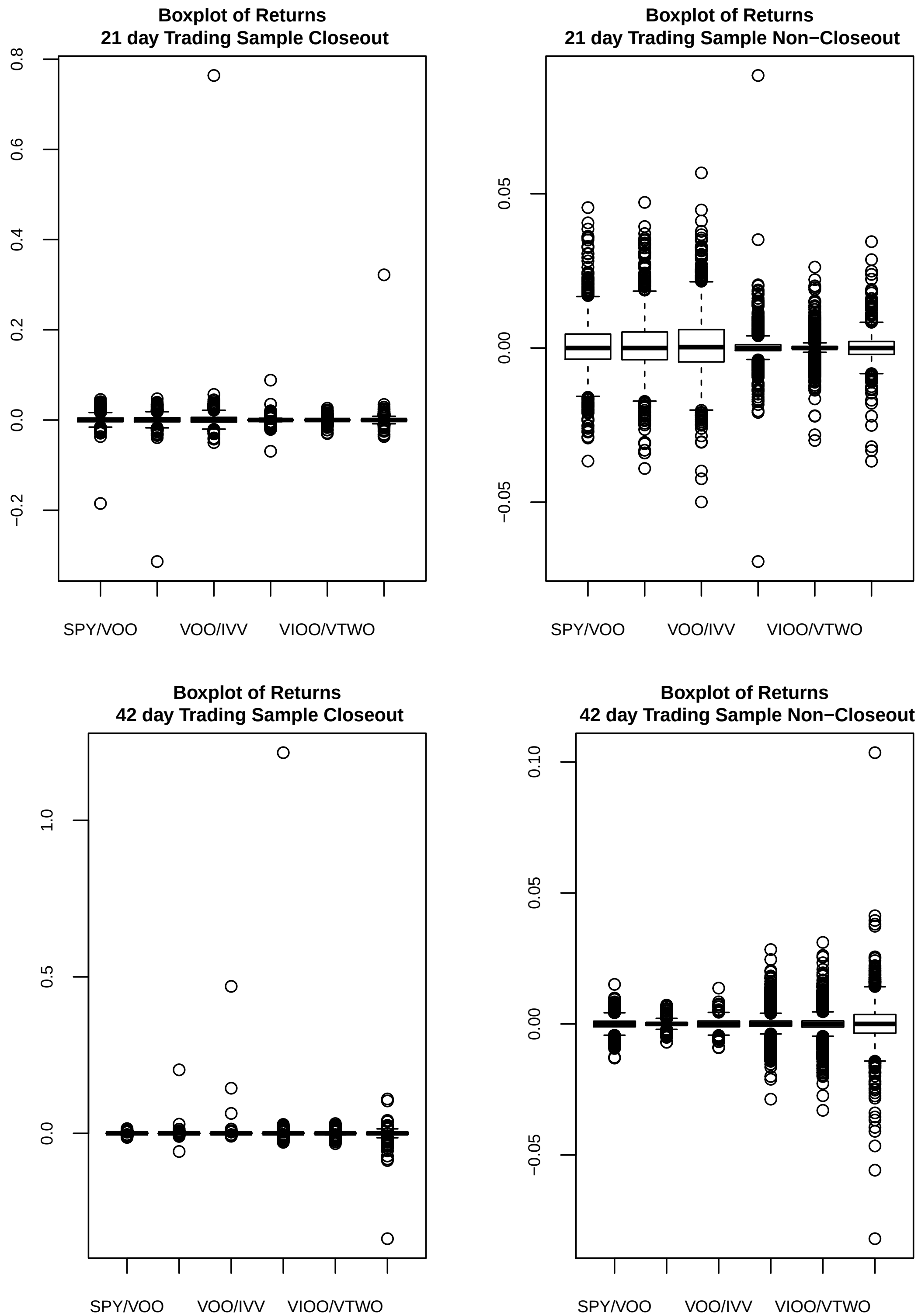

VIOO/VTWO Results
Closeout
Non−Closeout
80
60
40
20
0
−20
5
10
15
20





The graphical results above are taken from the first trading period during each trial (i.e. first 5 trading days, first 11 trading days, etc.). It is evident that the portfolios composed of S&P 500 ETFs exhibit strong mean reversion properties, but the portfolios also experience significant jumps in value. The jumps, likely due to a market shock (event) that quickly affects the price of one of the assets, can distort the value of the long-run mean being estimated during that period. By looking at the periods between shocks, the portfolio exhibits strong mean reversion as well, but over the entire sample these shocks can distort the strategy. The other equity ETF portfolios, IWM/SLY and VIOO/VTWO, also exhibit mean reversion properties but don't experience as many jumps in value. The commodities portfolio, USO/OIL, appears the most stable and least affected by market shocks, although the graphical results above are only one sample of numerous trading periods. Significant jumps, however, can lead to immense profit (or immense losses) if the jump forces the portfolio value to cross the mean, as evident in the results displayed for VOO/IVV 5-day. The more a portfolio actually fluctuates around θ^i_{t-1} – that is, the more accurate the parameters, indicators, and signals are – leads to more stable trading positions and more realistic returns. On the whole, all portfolios exhbit relatively strong mean-reversion properties, indicating that the initial assumption that they follow an Ornstein-Uhlenbeck process is not a bad one.



The hypothesis that the optimized parameter θ^i_{t-1} and asset weights $\alpha^i_{t-1}, \beta^i_{t-1}$ are good estimates for those that will be optimal in period t is not well supported based on the graphical results. As is evident, the parameter θ^i_{t-1} can be significantly different from the center of the portfolio value (the actual θ^i_t), and while this can be distorted visually due to the high value of initial investment in asset S^i_{t-1} ($100,000), the same proportional difference would hold for any value (this strategy can be applied without loss of generality for any initial investment). As a result, the trading signals are not optimal either, and the rule evaluation will lead to signifcant risk exposure due to a strong buildup of long or short positions (i.e., all trades in a given period will be long or short, depending on value relative to signals derived in period $t-1$).

Table 1: SPY/VOO Results – 21 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
1351.3861	1.3348	-5.5164	-5.5164
8375.6873	3.4629	124.7517	175.7842
140.9095	1.2540	5.0724	8.0807
20.7851	1.2006	17.3391	15.5101
1512.7264	1.9718	100.0619	146.4475
279.7298	1.4983	38.5646	58.0860
49.1597	1.4110	-3.4742	-3.4742
418.3291	2.1980	5.6424	5.6424
1123.6501	2.8160	98.3935	98.3935
27.5363	0.9847	14.7993	7.7533
1062.7822	2.6094	24.1964	24.1964
273.4777	1.0817	-5.6699	-5.6699

Table 2: SPY/IVV Results – 21 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
851.7897	0.9325	-5.4027	-5.4027
8602.7797	3.3437	64.6954	139.9642
105.2500	0.9847	0.7521	2.4389
20.2147	1.3119	5.8759	5.8759
1099.2949	1.4797	101.6881	135.7196
41.8183	0.5688	144.5117	54.7237
35.8406	1.3168	-3.5792	-3.5792
294.0475	1.8448	29.5247	29.5247
1058.8664	2.3987	108.2592	108.2592
24.2462	0.9987	-6.8826	-2.0168
1877.8309	1.9271	40.2809	40.2809
99.2004	0.7618	-8.2035	-8.2035

Table 3: VOO/IVV Results – 21 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
18.4096	1.6471	-5.3087	-5.3087
14199.8832	4.8298	599.9426	296.8292
1368.8518	1.9600	23.0455	42.1307

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
21.4188	1.0945	9.2492	5.2783
1435.0349	2.3303	754.3923	259.3429
1230.7869	1.6029	482.2632	217.6264
1166.9499	1.9203	-4.1636	-4.1636
1233.1007	2.7407	291.0836	137.9169
1336.6803	2.8643	122.2621	122.2621
25.9015	0.9770	-2.7794	-0.0347
3546.3245	3.4477	2.1150	31.1466
2512.6150	2.1876	136.3007	136.3007

Table 4: IWM/SLY Results – 21 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
228.8433	9.7223	-3.1284	-3.1284
190.2907	29.2628	30.1758	30.1758
31.0168	9.2528	177.0822	28.4727
130.0104	18.4011	6.7461	6.7461
95.6024	9.1141	-20.7147	18.2701
39.1939	8.7619	47.8760	47.8760
103.8787	18.3538	-1.1366	-1.1366
186.8911	11.3947	41.7931	34.5469
48.8327	7.5936	70.1187	51.3109
52.4010	14.1326	16.5936	16.1480
26.1153	5.9949	-9.7537	-9.7537
71.5612	10.2514	-69.3762	36.8122

Table 5: VIOO/VTWO Results – 21 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
78.5660	12.9692	-14.8355	-14.8355
194.2216	32.6433	9.2523	7.8184
102.7762	7.9479	6.6920	17.3322
175.0710	9.8979	15.8111	10.7277
201.6917	7.3756	-12.3833	21.1518
12.6378	3.3336	55.4694	41.8481
110.2811	12.2778	-7.1080	-7.1080
36.2419	7.6322	7.3837	9.7532

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
49.9270	8.4669	53.4678	49.9596
57.0693	11.2076	11.1613	3.6088
36.3835	6.3285	22.1028	8.5000
74.5576	8.6297	69.4613	25.2950

Table 6: USO/OIL Results – 21 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
256.4495	8.9119	9.4906	-9.0097
100.6074	35.6315	54.6218	15.5704
414.3429	21.8632	127.1948	65.0515
48.2734	9.1959	88.9755	37.7511
316.3661	12.5105	-2.8587	-2.8587
152.2507	10.0727	14.0940	37.3339
127.7878	8.0227	-10.4672	-10.4672
66.5327	4.6731	19.5501	23.5112
547.4493	8.5441	64.8338	87.3296
222.5042	7.3958	3.6085	3.6085
71.7631	5.7910	-34.0966	20.5026
240.4378	7.1162	-10.2063	-10.2063

Table 7: SPY/VOO Results – 42 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
5011.3600	1.8741	165.8112	29.9607
74.2074	1.0354	-1.5060	-1.5060
716.8370	2.3794	18.6532	37.1595
48.2661	1.6018	6.5427	12.2496
38.9164	1.3937	-2.8876	0.8325

Table 8: SPY/IVV Results – 42 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
1590.8253	1.2190	53.5640	24.0320
32.4506	1.0453	35.0266	13.2000

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
947.5063	1.8786	161.1508	40.0378
71.3227	1.2201	16.5171	13.6596
24.2234	1.4429	3.7948	2.8422

Table 9: VOO/IVV Results – 42 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
3601.0849	2.2708	11.4795	13.3879
35.0916	1.0976	88.5970	5.4620
8968.9280	3.7847	367.2042	109.1223
1490.3089	1.9166	41.5809	40.7911
24.1494	1.4533	3.1653	2.8613

Table 10: IWM/SLY Results – 42 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
453.0948	25.5839	297.6076	24.3897
109.5298	15.1713	176.4985	26.5694
69.1808	14.3634	33.2011	40.5763
207.1944	16.2788	77.2026	44.2579
96.0204	25.5232	-1.4232	-1.4232

Table 11: VIOO/VTWO Results – 42 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
115.4667	23.8881	17.0488	30.3674
131.0821	14.4536	-7.5889	-7.5889
224.8380	11.1635	15.3820	28.0120
204.2568	12.1856	1.2009	1.2009
97.0636	18.0437	-27.4710	16.7396

Table 12: USO/OIL Results – 42 Day Period

Total Ret. (%) Non-Roll CO	Total Ret. (%) Non-Roll NC	Total Ret. (%) Rolling CO	Total Ret. (%) Rolling NC
27.7059	16.7665	190.5921	100.2309
524.3192	26.1971	-33.5078	6.6263
71.8502	6.7645	44.0904	44.0904
94.6806	9.3719	20.5386	8.0573
378.5742	15.1188	70.9296	70.9296

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## [1] 0
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Based on total returns for each period shown in the tables above, the optimized results ('Non-Roll') are *significantly* different between the two strategies whereas the total returns for the tested periods are significantly closer. The above results show that the total returns per trading period can vary significantly from one period to the next. However, Tables 13-17 below show that the overall sample standard deviation for returns for each portfolio is very low, regardless of the trading period, and so this would indicate that the returns during the entire 12-month sample are more stable than the period-to-period fluctuation in total returns would indicate. This can be seen in the box-plots above as well. There can be significant outliers for certain portfolios in different periods, but generally the returns have a small spread around the median.

Table 13: 12-month Results for 5 trading day window

	t Stat	MSE	TR CO	TR NC	SR CO	SR NC	Avg Ret CO	Avg Ret NC	Ret Std Dev CO	Ret Std Dev NC
SPY/VOO	0.905	19229625	3.387745e+12	6.422102e+11	5.720	5.225	0.0014754	0.0013721	0.0122650	0.0097637
SPY/IVV	1.167	21575680	3.543178e+11	1.439271e+11	6.306	6.209	0.0013810	0.0012965	0.0142309	0.0096580
VOO/IVV	0.964	26139141	2.322039e+16	2.771348e+15	12.338	11.168	0.0019698	0.0017834	0.0174643	0.0107610
IWM/SLY	0.179	79957895	-2.777375e+01	5.921352e+01	0.075	0.052	0.0002992	0.0002124	0.0146373	0.0053038
VIOO/VTWO	0.147	30042125	-3.648767e+00	1.248275e+02	0.052	0.071	0.0001895	0.0002473	0.0105563	0.0050014
USO/OIL	0.241	155082962	3.243471e+03	9.714998e+02	0.108	0.080	0.0005032	0.0003686	0.0160827	0.0082458

Table 14: 12-month Results for 11 trading day window

	t Stat	MSE	TR CO	TR NC	SR CO	SR NC	Avg Ret CO	Avg Ret NC	Ret Std Dev CO	Ret Std Dev NC
SPY/VOO	1.143	52911251	3.378970e+05	5.082253e+05	7.440	7.402	0.0006540	0.0006660	0.0083585	0.0075084
SPY/IVV	2.269	69425977	4.843308e+05	5.314546e+04	8.890	6.504	0.0007034	0.0005563	0.0131245	0.0074378
VOO/IVV	1.131	45192473	2.100703e+07	5.030370e+06	17.130	14.658	0.0008732	0.0007842	0.0113505	0.0081303
IWM/SLY	0.147	71063617	-1.503781e+01	1.391029e+01	0.136	0.075	0.0002678	0.0001430	0.0210902	0.0049431
VIOO/VTWO	0.133	32489837	1.698316e+01	1.492000e+01	0.101	0.090	0.0001560	0.0001432	0.0053647	0.0040837
USO/OIL	0.262	185408918	6.004772e+01	1.295912e+01	0.117	0.070	0.0002658	0.0001567	0.0122149	0.0075723

Table 15: 12-month Results for 21 trading day window

	t Stat	MSE	TR CO	TR NC	SR CO	SR NC	Avg Ret CO	Avg Ret NC	Ret Std Dev CO	Ret Std Dev NC
SPY/VOO	1.421	64000147	19.747638	32.367618	2.019	2.620	0.0001722	0.0001915	0.0063462	0.0054485
SPY/IVV	2.029	78844984	25.560796	29.467518	2.582	2.810	0.0001861	0.0001867	0.0065606	0.0054818
VOO/IVV	1.584	62999674	8661.867394	1007.531504	11.915	6.281	0.0005387	0.0003978	0.0130549	0.0092801
IWM/SLY	0.134	54666266	3.360343	8.729868	0.148	0.132	0.0001068	0.0001211	0.0092498	0.0037138
VIOO/VTWO	0.173	66407009	5.514254	4.414772	0.127	0.101	0.0001036	0.0000851	0.0047657	0.0038107
USO/OIL	0.357	641697219	8.714563	7.407480	0.125	0.100	0.0001612	0.0001272	0.0101105	0.0072192

Table 16: 12-month Results for 31 trading day window

	t Stat	MSE	TR CO	TR NC	SR CO	SR NC	Avg Ret CO	Avg Ret NC	Ret Std Dev CO	Ret Std Dev NC
SPY/VOO	1.266	74746604	7.8380188	6.103136	2.408	2.093	0.0001243	0.0001019	0.0062649	0.0043479
SPY/IVV	1.812	91866348	6.1321119	2.929071	2.062	1.328	0.0001090	0.0000642	0.0060782	0.0045569
VOO/IVV	2.064	79259205	15.1731497	12.231819	3.247	2.902	0.0001843	0.0001709	0.0075577	0.0072189
IWM/SLY	0.097	29451205	23.3879826	6.229902	0.438	0.175	0.0002122	0.0001033	0.0136175	0.0030235
VIOO/VTWO	0.214	96755956	4.3764567	3.235458	0.161	0.118	0.0001040	0.0000716	0.0054186	0.0033501
USO/OIL	0.140	35364444	0.6082134	2.135444	-0.012	0.058	0.0000260	0.0000613	0.0095413	0.0067165

Table 17: 12-month Results for 42 trading day window

	t Stat	MSE	TR CO	TR NC	SR CO	SR NC	Avg Ret CO	Avg Ret NC	Ret Std Dev CO	Ret Std Dev NC
SPY/VOO	0.887	25360720	3.2141112	1.987161	1.987	0.846	0.0000951	0.0000446	0.0085706	0.0023018
SPY/IVV	1.430	43114206	6.5488550	2.298280	3.122	1.087	0.0001264	0.0000529	0.0052174	0.0020626
VOO/IVV	2.506	65593265	14.3474541	3.621515	5.571	1.873	0.0001799	0.0000841	0.0062820	0.0033287
IWM/SLY	0.077	16992790	25.5799725	3.147310	0.691	0.163	0.0002894	0.0000755	0.0185576	0.0033124
VIOO/VTWO	0.252	105669153	0.9160628	1.821992	-0.002	0.100	0.0000103	0.0000417	0.0052187	0.0031739
USO/OIL	0.222	44947423	5.7363147	5.682008	0.291	0.232	0.0001465	0.0001352	0.0089144	0.0076350

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## [1] 0
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Considering how much initial investment was made in the first asset, $100,000, the MSE is misleading since even small weight adjustments in each asset can lead to substantially different initial values. Thus, the MSE of all estimated long-run means $\hat{\theta}$ is considerably exaggerated by the actual portfolio value. The t-statistic, however, is a better measure of the fit of the optimal θ_{t-1}^i as a predictor for θ_t^i. The three portfolios composed of ETFs tracking the S&P 500 have (relatively) very high t-statistics, indicating that the hypothesis that the optimized θ_{t-1}^i is a good predictor of the actual θ_t^i is not well supported. However, the t-statistic is considerably lower for the other three portfolios, two of which also track equity indexes.

Additionally, there seems to be a slight correlation between smaller t-statistic and lower returns/Sharpe Ratios. Higher t-statistics tends to correlate with higher overall returns and Sharpe Ratio. Table 18 below shows that, generally, shorter testing periods have a higher positive correlation between t-statistic and total returns, for both strategies, with the exception of the 31 day testing period. The correlation for the 42-day period is close to 0, and slightly negative for the non-closeout strategy. This seems to indicate that the more accurate the long-run mean is as a predictor of the future mean, the lower the correlation is with overall returns. This is a somewhat paradoxical result in that the more inaccurate the parameter θ is as a predictor of future long-run mean, then the more positively correlated it is with total returns. However, the graphical and results tables appear to support this as well, where we see significantly higher returns, generally, for portfolios who t-statistic is high. So, the more 'stable' the parameter θ is, the higher overall returns we can predict albeit with significantly higher risk-exposure due to the buildup of long or short positions.

Table 18: Pearson Correlation between t-statistic and total returns for each strategy and trading period test

	Total Ret CO	Total Ret NC
5 Days	0.3871328	0.3871922
11 Days	0.1885304	0.1948037
21 Days	0.3785784	0.4016496
31 Days	0.0545757	0.5546301
42 Days	0.0274624	-0.0786470

The final conjecture, that the closeout strategy trades less optimally per trade but that higher number of total trades leads to higher overall returns, is not supported by the results. Tables 19-23 show that, almost always, the closeout strategy trades more frequently but less optimally (lower RPT). However, the total returns for the closeout strategy don't consistently dominate those of the non-closeout, and there is no correlation between the periods/portfolios where the non-closeout has higher total returns but also trades more optimally. (I'm not certain why the 5-day period returns infinite RPT for the VOO/IVV portfolio, but we assume that the RPT are similar to those of IWM/SLY based on observation of their similar results elsewhere.)

Table 19: Returns Per Trade and total number of returns for 5-day sample

	Ret Per Trade (%) CO	No. Trades CO	Ret Per Trade (%) NC	No. Trades NC
SPY/VOO	0.3820075	600.7547	0.4181849	568.5283
SPY/IVV	0.4047642	588.4906	0.4492698	561.2830
VOO/IVV	0.6637415	577.4717	0.7233151	537.2830
IWM/SLY	0.1862887	639.0943	0.2053472	589.3585
VIOO/VTWO	Inf	544.2642	Inf	513.8491
USO/OIL	0.2444528	601.8113	0.2630264	551.6226

Table 20: Returns Per Trade and total number of returns for 11-day sample

	Ret Per Trade (%) CO	No. Trades CO	Ret Per Trade (%) NC	No. Trades NC
SPY/VOO	0.2010875	1361.500	0.2427667	1239.667

	Ret Per Trade (%) CO	No. Trades CO	Ret Per Trade (%) NC	No. Trades NC
SPY/IVV	0.1994542	1368.750	0.2143500	1227.167
VOO/IVV	0.3600042	1368.250	0.4169792	1211.667
IWM/SLY	0.1039792	1348.917	0.1139500	1212.500
VIOO/VTWO	0.1090417	1232.875	0.1231708	1133.708
USO/OIL	0.0955292	1432.458	0.1045167	1273.625

Table 21: Returns Per Trade and total number of returns for 21-day sample

	Ret Per Trade (%) CO	No. Trades CO	Ret Per Trade (%) NC	No. Trades NC
SPY/VOO	0.0517750	2766.750	0.0672833	2534.250
SPY/IVV	0.0483000	2954.833	0.0627833	2570.500
VOO/IVV	0.1146167	2836.417	0.0988833	2482.750
IWM/SLY	0.0477167	2677.167	0.0628917	2362.833
VIOO/VTWO	0.0495417	2557.667	0.0595083	2207.667
USO/OIL	0.0548833	2543.583	0.0658167	2275.083

Table 22: Returns Per Trade and total number of returns for 31-day sample

	Ret Per Trade (%) CO	No. Trades CO	Ret Per Trade (%) NC	No. Trades NC
SPY/VOO	0.0373250	3950.500	0.0406500	3562.250
SPY/IVV	0.0570875	3631.375	0.0583750	3114.375
VOO/IVV	0.0432250	3711.625	0.0520250	3220.125
IWM/SLY	0.0659750	3447.375	0.0535875	2878.875
VIOO/VTWO	0.0358500	3710.125	0.0378500	3415.375
USO/OIL	0.0281875	3830.750	0.0384250	3520.000

Table 23: Returns Per Trade and total number of returns for 42-day sample

	Ret Per Trade (%) CO	No. Trades CO	Ret Per Trade (%) NC	No. Trades NC
SPY/VOO	0.07000	3495.0	0.08274	3214.6
SPY/IVV	0.05402	4445.0	0.06422	3955.0
VOO/IVV	0.04018	5176.4	0.03468	4589.6
IWM/SLY	0.04588	5125.2	0.03106	4402.0
VIOO/VTWO	0.01956	5508.6	0.03126	4619.8
USO/OIL	0.69190	4824.8	0.92764	3880.4

Overfitting

The strategy is a not quite a 'rolling' strategy, but rather a step-shift where each shift is however many days is being tested in the strategy (i.e. 5, 11, etc). Considering that there are 78 observations per day, then the number of observation associated with the strategy is either $n = 390, 858, 1638, 2418$, or 3276 per trading period. So rather than continuously removing and then adding 78 degrees of freedom, the whole window is adding and removing n observations simultaneously. When optimizing parameters and deriving indicators and signals, we rely on n observations during $t-1$ to trade using n observations during t. However, each observation in t is independent of $t-1$, so it can be argued that there are n degrees of freedom in my strategy. The success of short term strategies is likely an anomaly and not a reliable result (i.e. too good to be true, so the methodology should be reevaluated). However, with increasing degrees of freedom for the longer trading/testing periods, the results stabilize and tend to become more realistic in terms of total returns and Sharpe Ratio.

One way to increase degrees of freedom would be to use one of the longer optimization periods, such as 31 or 42 days, and then trade based on the parameters, indicators, and signals derived for a longer period of time – that is, extend the market data used for out-of-sample testing. The results from this will be more robust since any short term anomolies will likely be smoothed out/diluted.

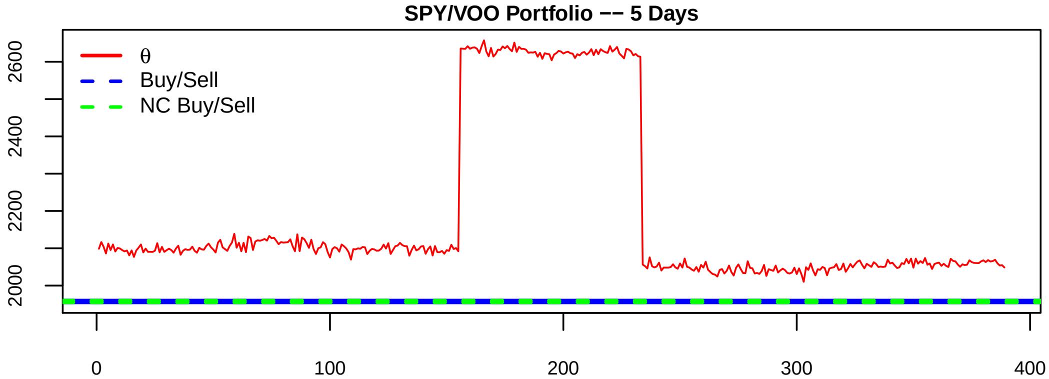
Conclusion

From the results above, we can conclude that pairs traded portfolios consisting of ETFs tracking the same index or commodity can be modelled well as an Ornstein-Uhlenbeck process. The portfolios exhibit strong mean-reversion properties, and this allows for the possibility of statistical arbitrage. The technique of optimizing parameters, indicators, and signals during period $t-1$ and using these to trade during period t have produced mixed results. High total returns tend to be associated with the shorter testing/trading windows, but these are likely the result on an anomoly and also expose the trader to high risk due to significant buildup of long or short positions. Longer periods have more stable returns and lower risk, but are lower in absolute value.

The conjecture that the closeout strategy would consistently trade less optimally but with higher frequency, leading to higher total returns, appears to be incorrect, at least in this instance. The closeout strategy does have consistently more trades and lower returns per trade in any given period, but this does no correlate to higher total returns. Additionally, it seems that the parameter estimate θ_{t-1} and the optimal asset weights derived based on maximum likelihood estimation generally do not serve as good parameters to use during period t. By association, the indicators and signals derived in $t-1$ are not optimal in period t.

Appendix

Below are additional graphical results from the testing conducted not discussed above, both portfolio and returns resutls along with boxplots of returns.


SPY/VOO Portfolio -- 5 Days
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Buy/Sell
NC Buy/Sell
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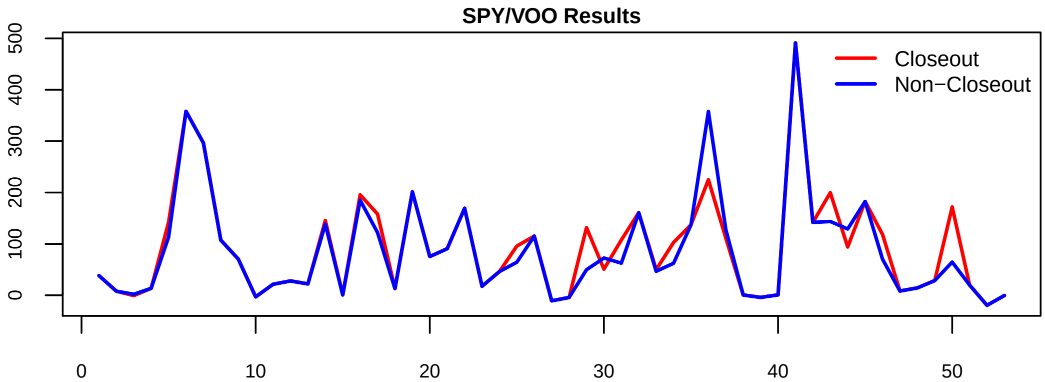
2200
2400
2600
0
100
200
300
400


SPY/VOO Results
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Non−Closeout
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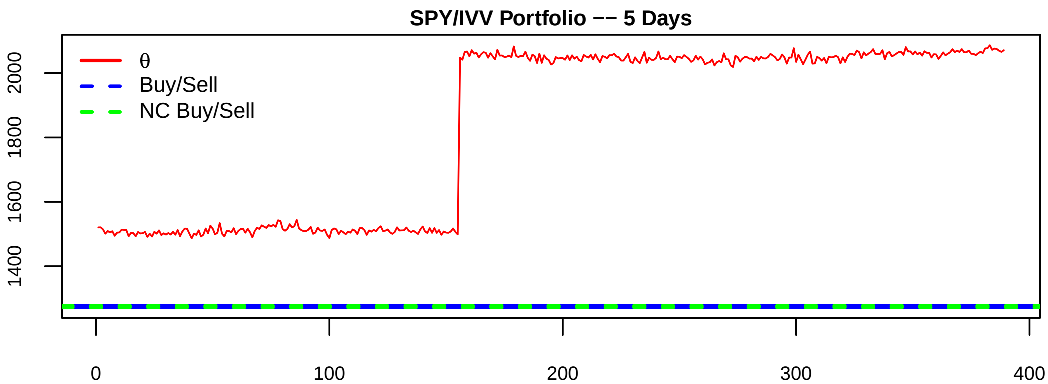
200
300
400
500
0
10
20
30
40
50


SPY/IVV Portfolio -- 5 Days
θ
Buy/Sell
NC Buy/Sell
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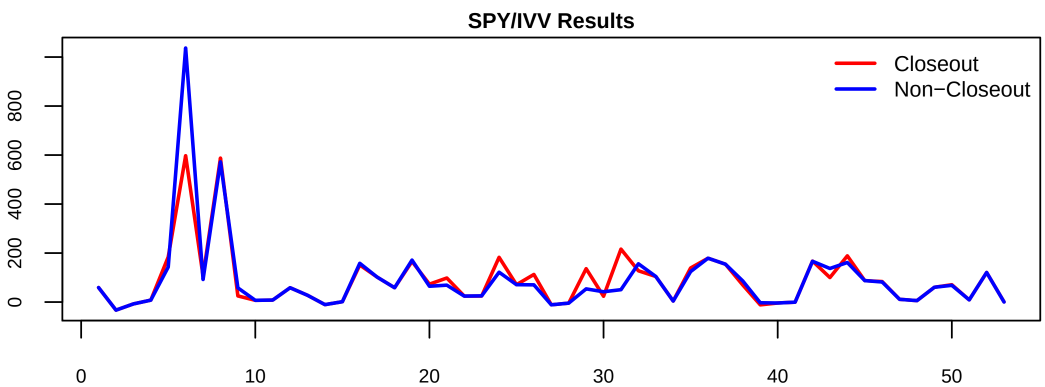
1600
1800
2000
0
100
200
300
400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Closeout
Non−Closeout
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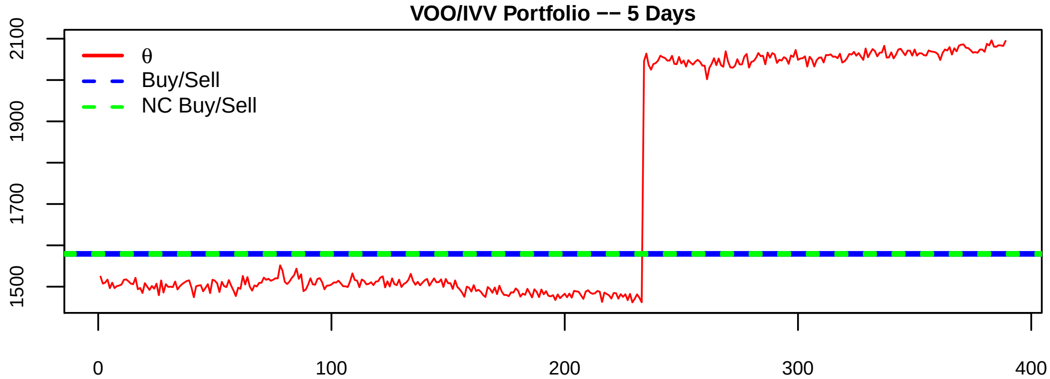
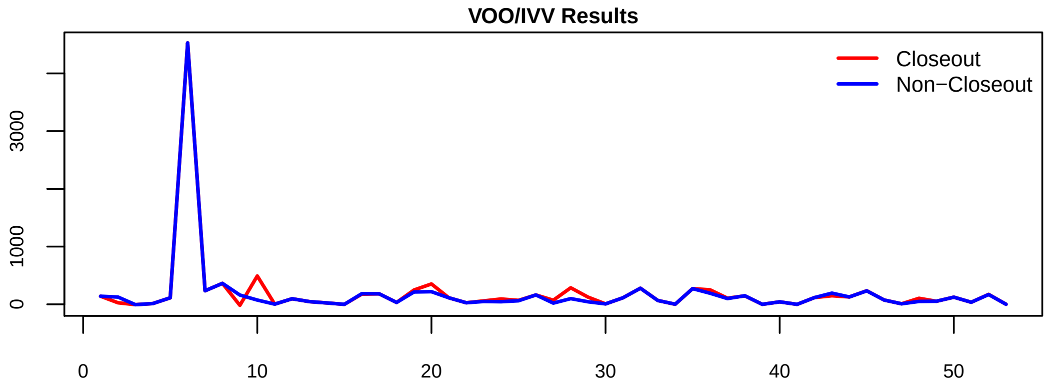
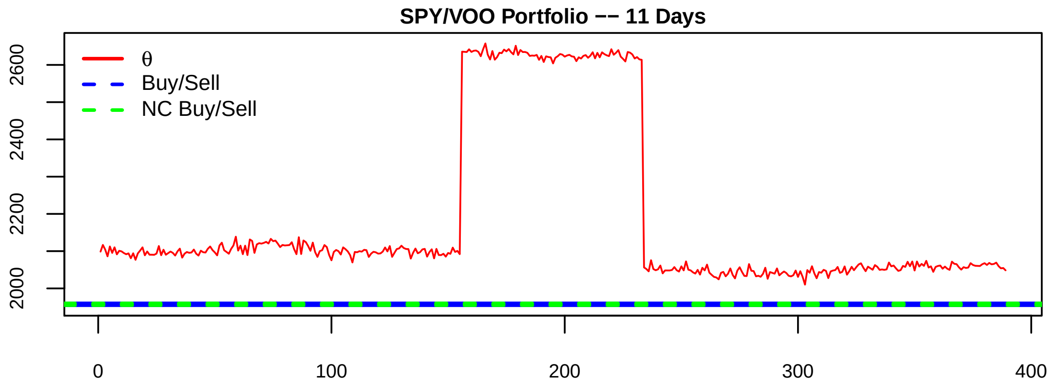
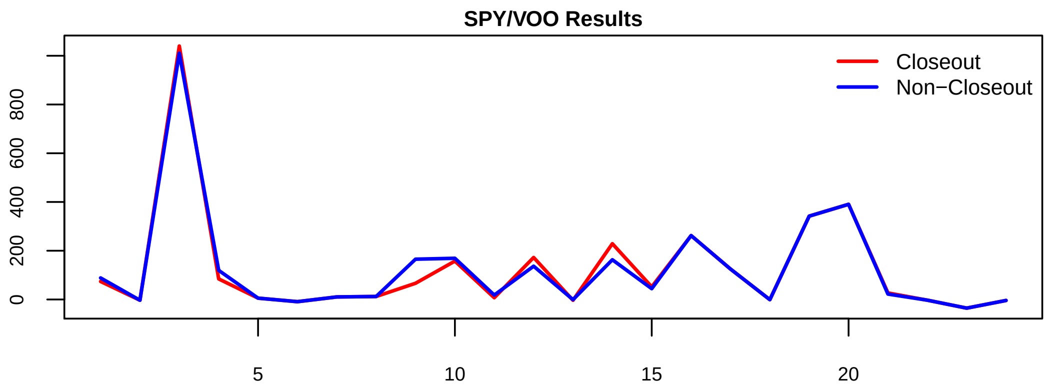
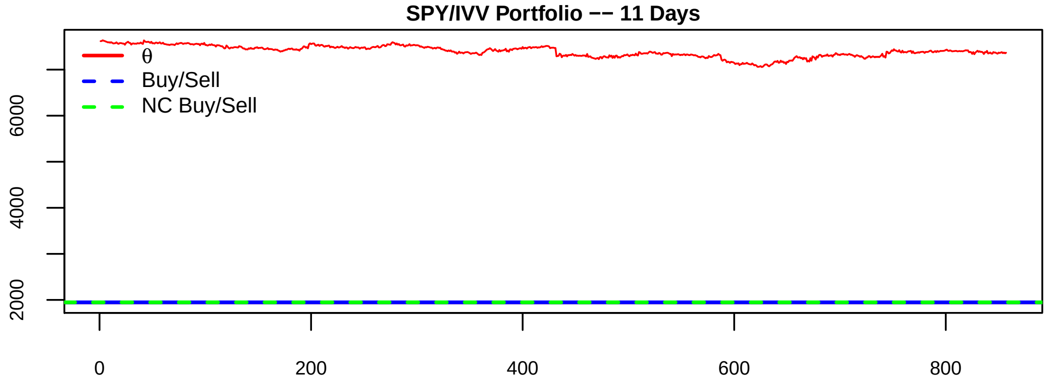
400
600
800
0
10
20
30
40
50

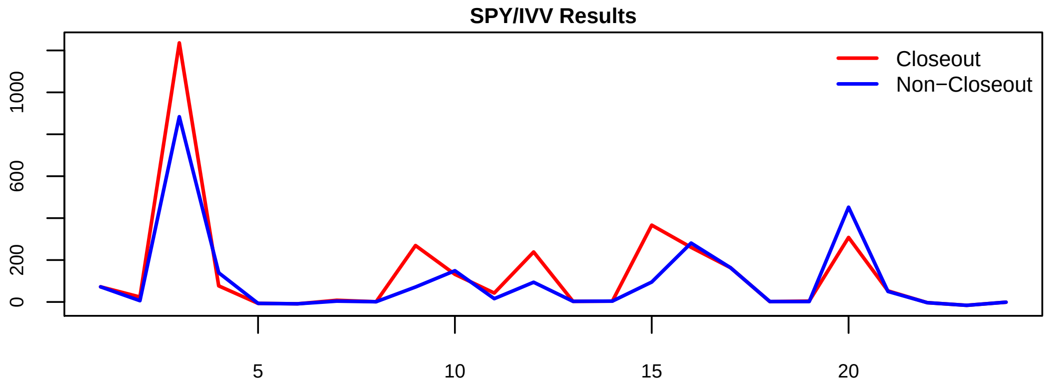

VOO/IVV Portfolio –– 5 Days
θ
Buy/Sell
NC Buy/Sell
1500
1700
1900
2100
0
100
200
300
400

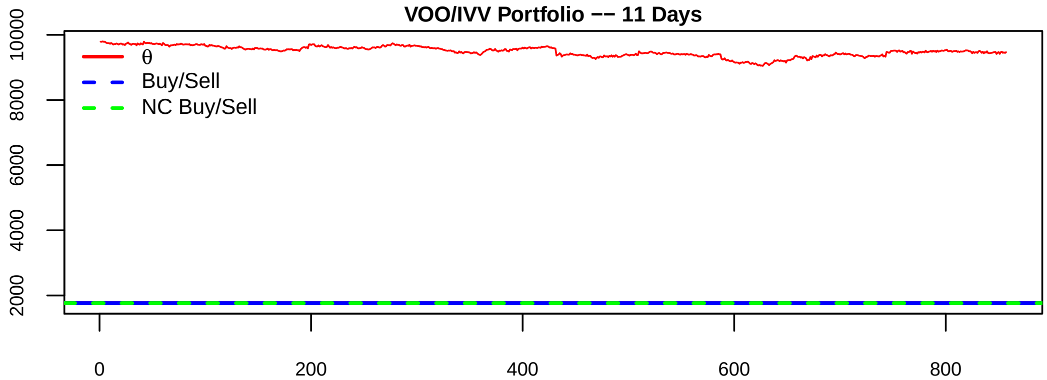

VOO/IVV Results
Closeout
Non−Closeout
0
1000
3000
0
10
20
30
40
50

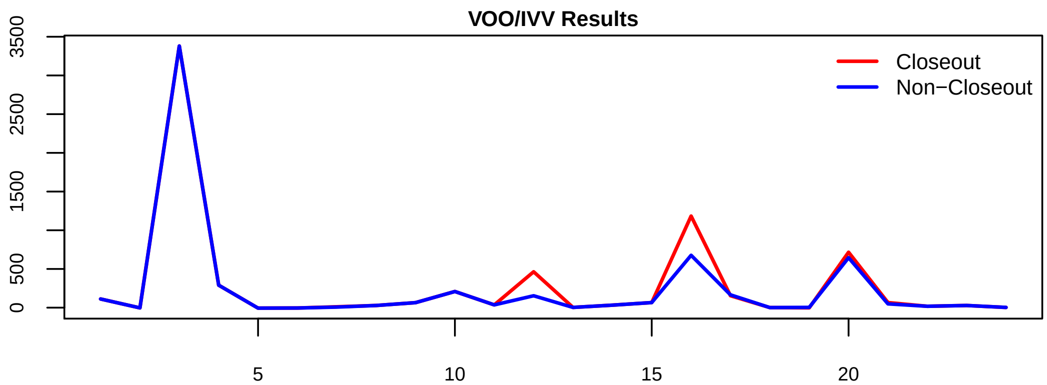

SPY/VOO Portfolio –– 11 Days
θ
Buy/Sell
NC Buy/Sell
2000
2200
2400
2600
0
100
200
300
400

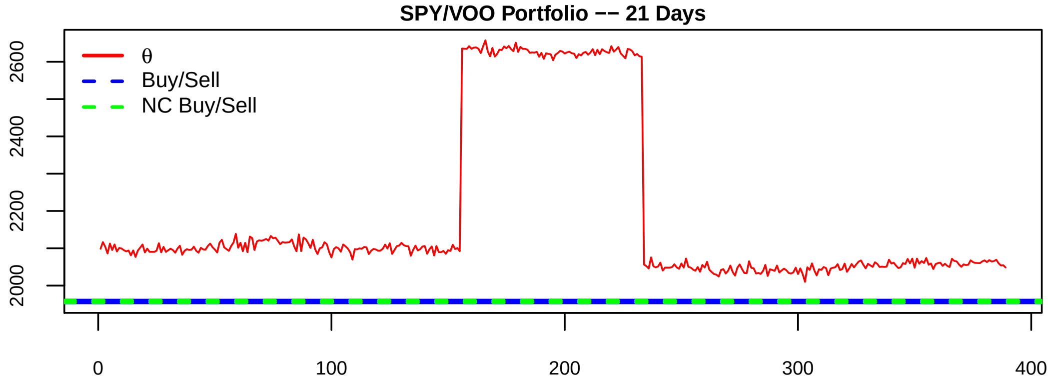

SPY/VOO Results
Closeout
Non−Closeout
0
200
400
600
800
5
10
15
20

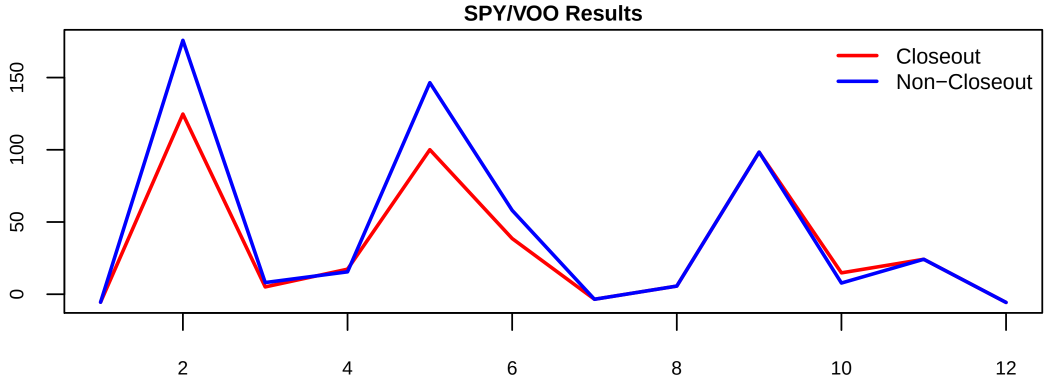

SPY/IVV Portfolio −− 11 Days
θ
Buy/Sell
NC Buy/Sell
2000
4000
6000
0
200
400
600
800

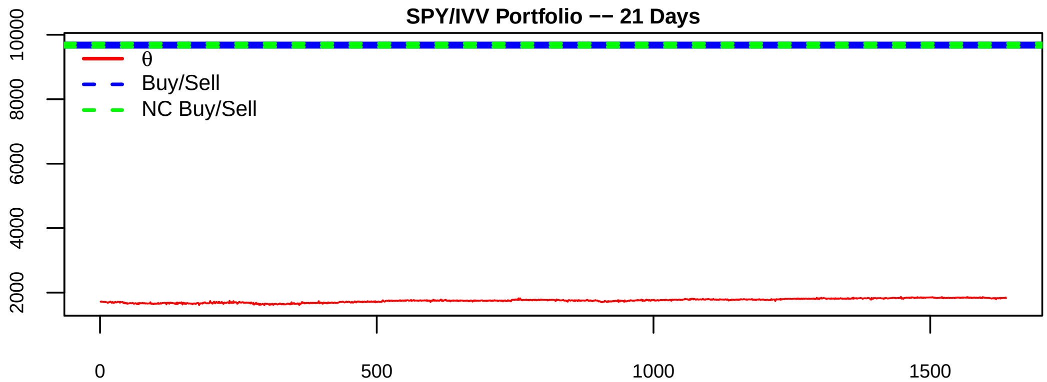

SPY/IVV Results
Closeout
Non−Closeout
0
200
600
1000
5
10
15
20

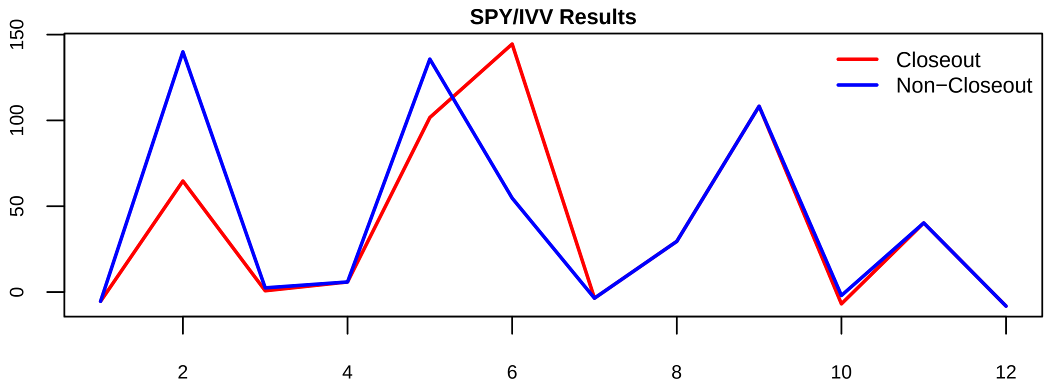

VOO/IVV Portfolio −− 11 Days
θ
Buy/Sell
NC Buy/Sell
2000
4000
6000
8000
10000
0
200
400
600
800

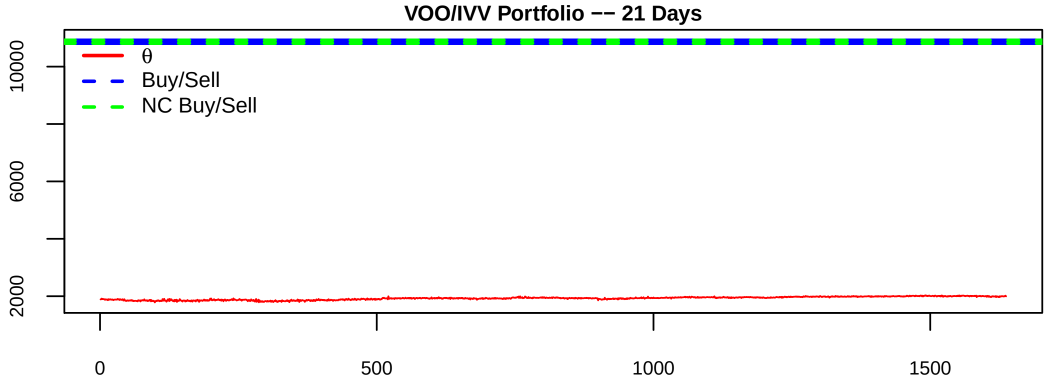

VOO/IVV Results
Closeout
Non−Closeout
0
500
1500
2500
3500
5
10
15
20

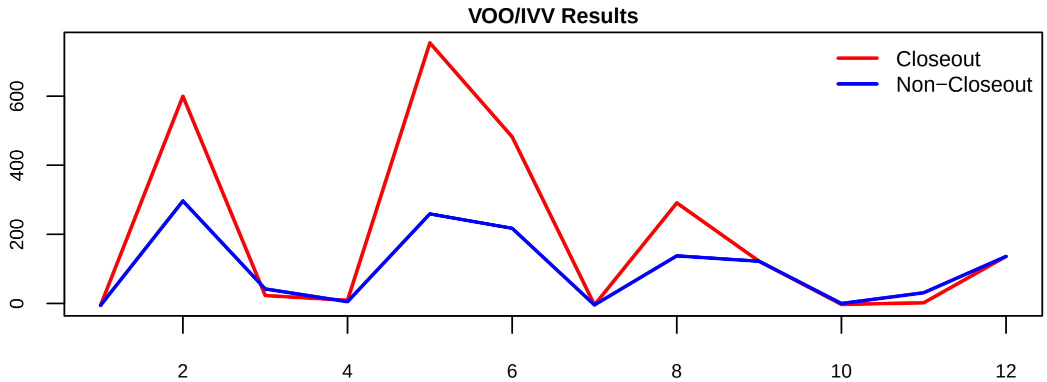

SPY/VOO Portfolio −− 21 Days
θ
Buy/Sell
NC Buy/Sell
2000
2200
2400
2600
0
100
200
300
400

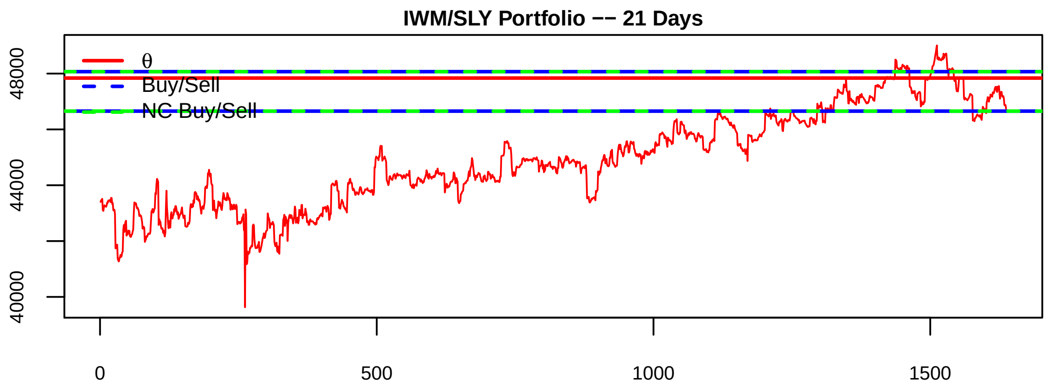

SPY/VOO Results
Closeout
Non−Closeout
0
50
100
150
2
4
6
8
10
12

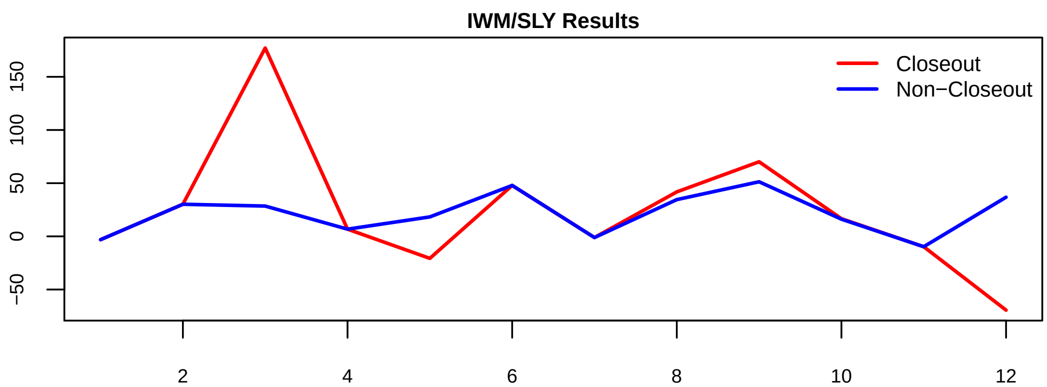

SPY/IVV Portfolio −− 21 Days
θ
Buy/Sell
NC Buy/Sell
2000
4000
6000
8000
10000
0
500
1000
1500

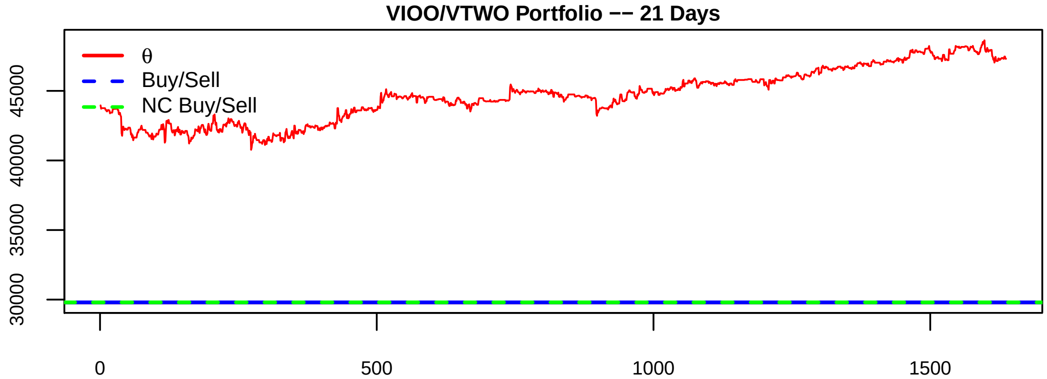

SPY/IVV Results
Closeout
Non−Closeout
0
50
100
150
2
4
6
8
10
12

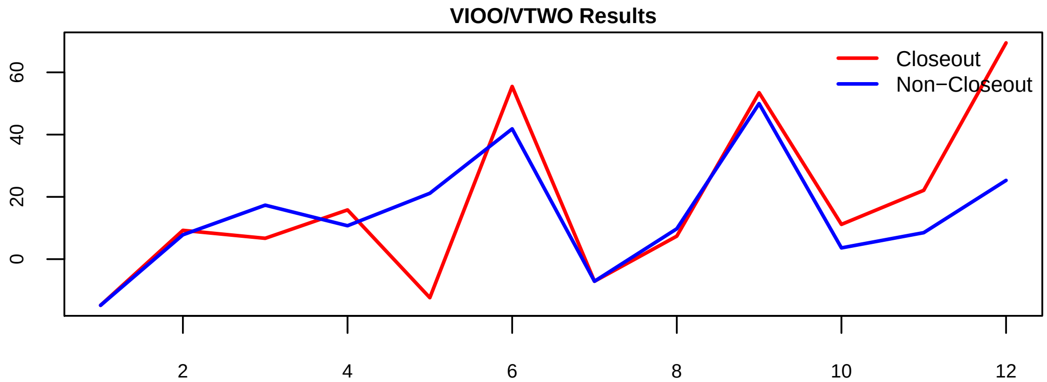

VOO/IVV Portfolio −− 21 Days
θ
Buy/Sell
NC Buy/Sell
2000
6000
10000
0
500
1000
1500

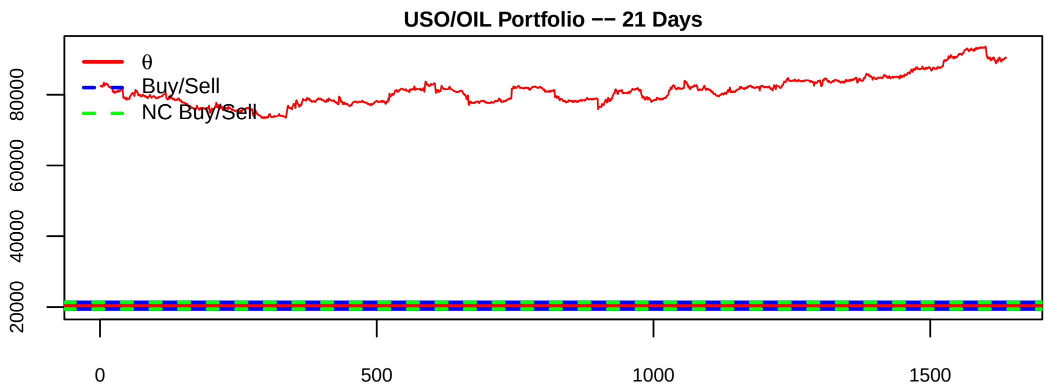

VOO/IVV Results
Closeout
Non−Closeout
0
200
400
600
2
4
6
8
10
12

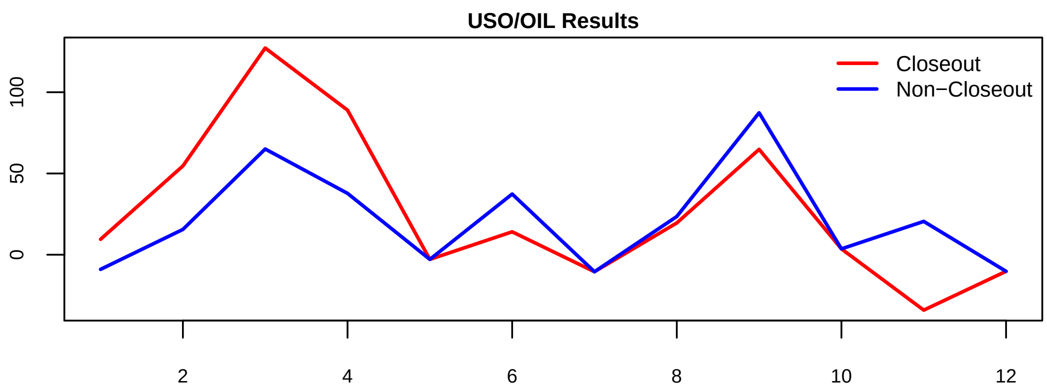

IWM/SLY Portfolio −− 21 Days
θ
Buy/Sell
NC Buy/Sell
40000
44000
48000
0
500
1000
1500

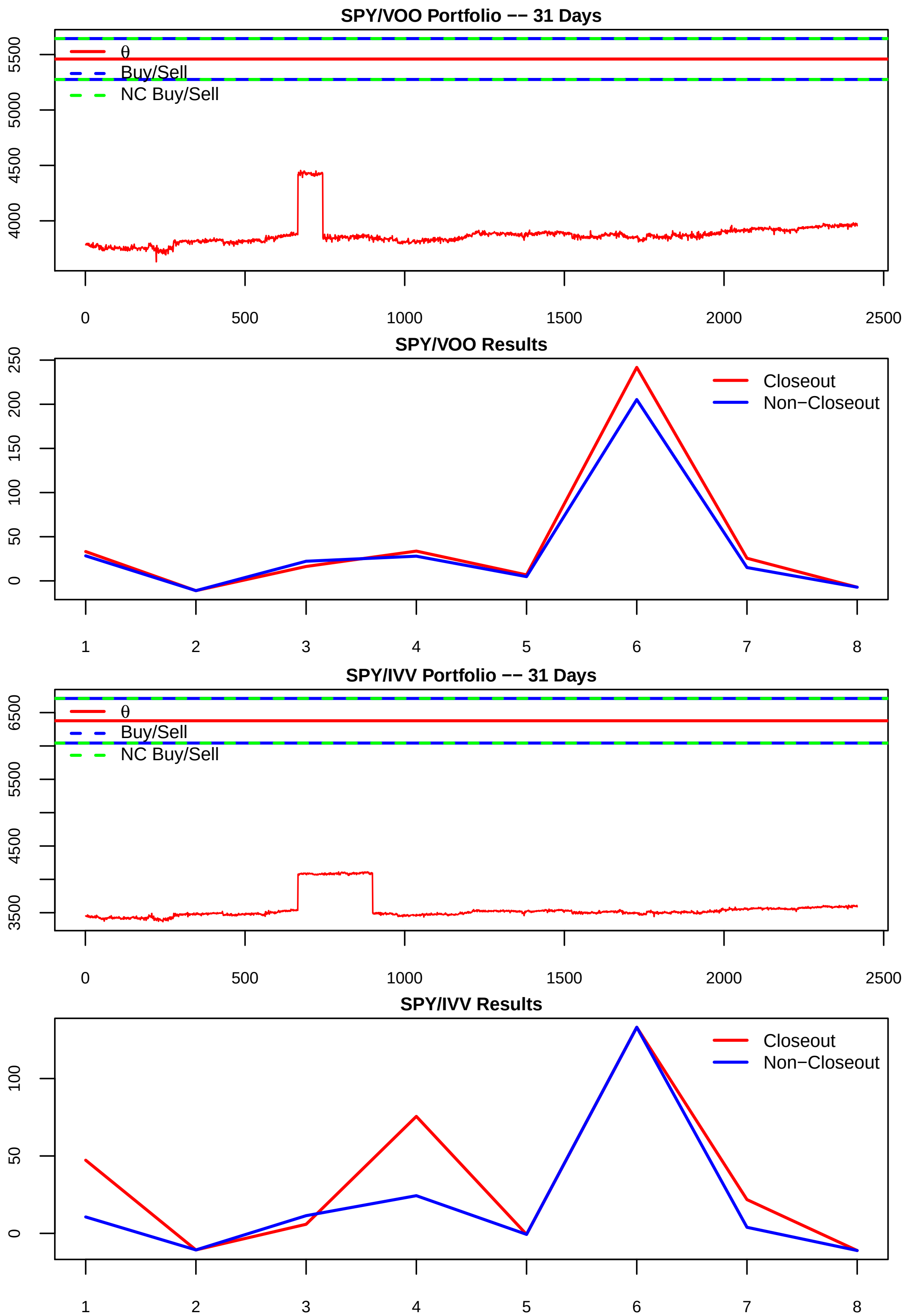

IWM/SLY Results
Closeout
Non−Closeout
−50
0
50
100
150
2
4
6
8
10
12

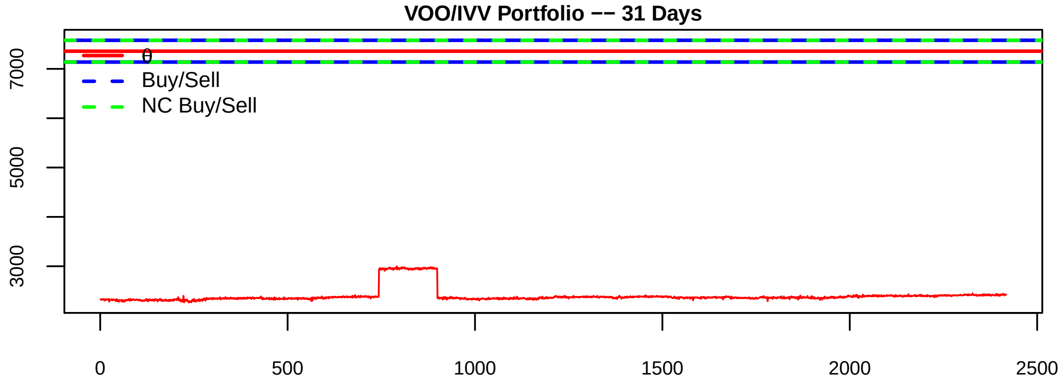

VIOO/VTWO Portfolio -- 21 Days
θ
Buy/Sell
NC Buy/Sell
30000
35000
40000
45000
0
500
1000
1500

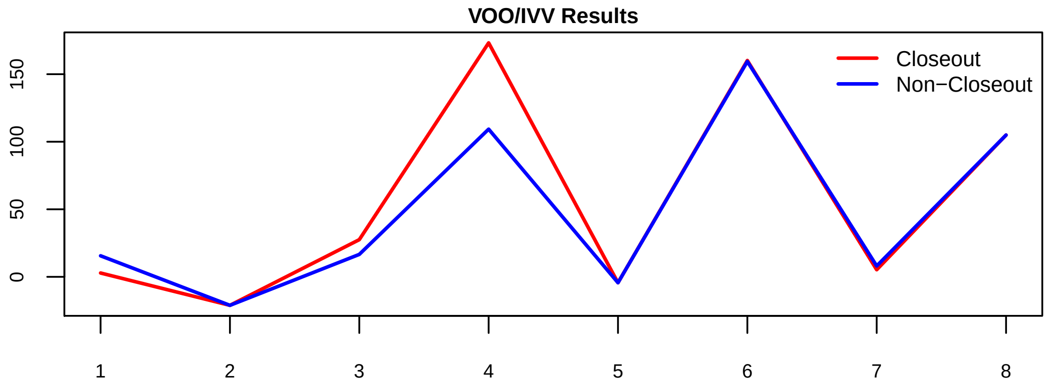

VIOO/VTWO Results
Closeout
Non−Closeout
0
20
40
60
2
4
6
8
10
12

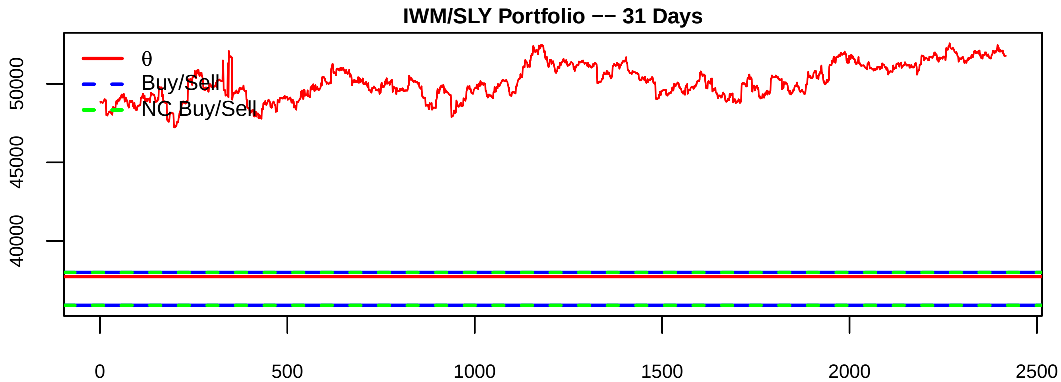

USO/OIL Portfolio -- 21 Days
θ
Buy/Sell
NC Buy/Sell
20000
40000
60000
80000
0
500
1000
1500

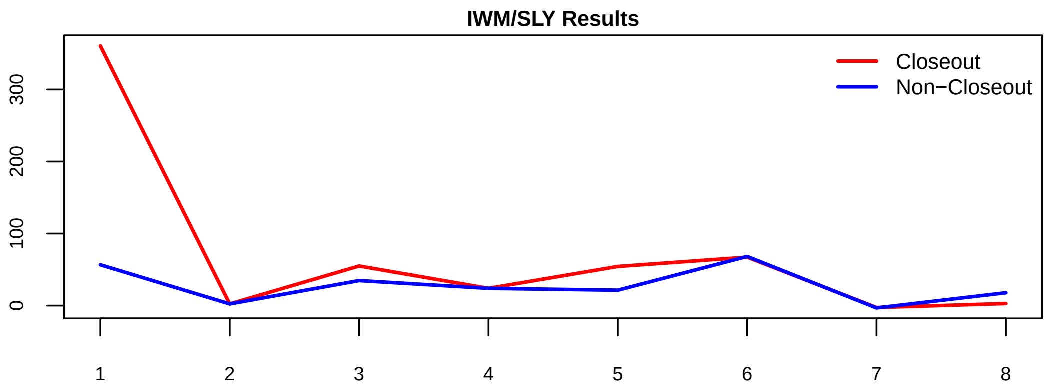

USO/OIL Results
Closeout
Non−Closeout
0
50
100
2
4
6
8
10
12

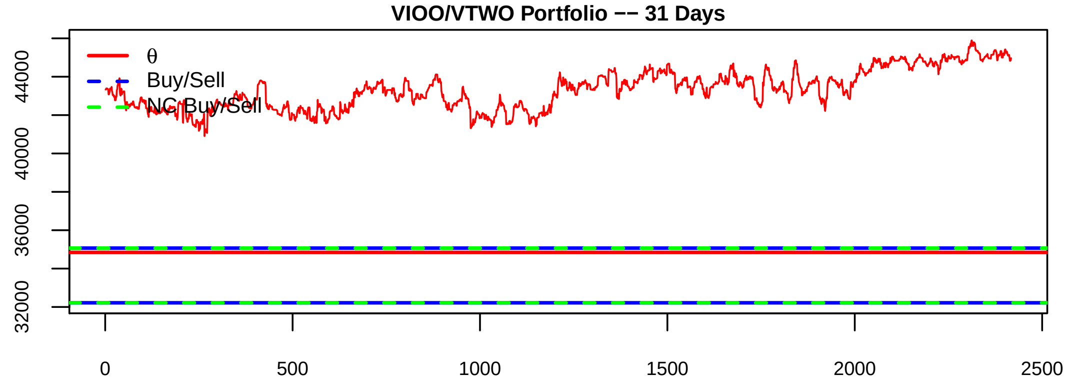

SPY/VOO Portfolio -- 31 Days
θ
Buy/Sell
NC Buy/Sell
5500
5000
4500
4000
0
500
1000
1500
2000
2500
SPY/VOO Results
Closeout
Non−Closeout
250
200
150
100
50
0
1
2
3
4
5
6
7
8
SPY/IVV Portfolio -- 31 Days
θ
Buy/Sell
NC Buy/Sell
6500
5500
4500
3500
0
500
1000
1500
2000
2500
SPY/IVV Results
Closeout
Non−Closeout
100
50
0
1
2
3
4
5
6
7
8

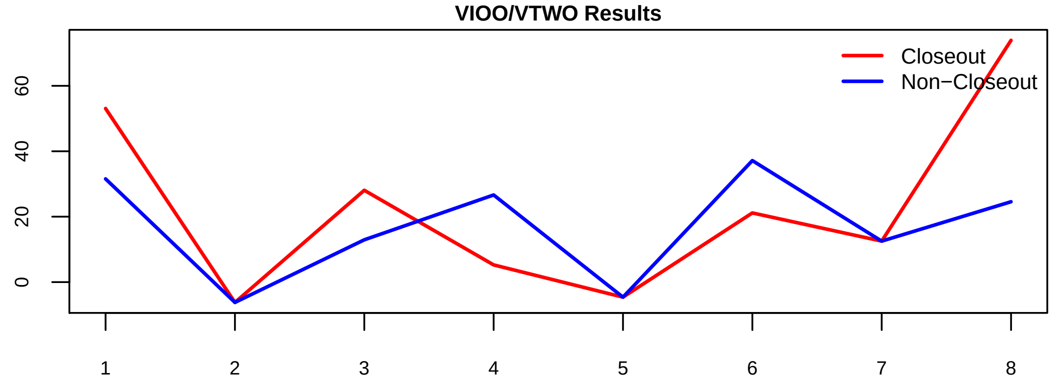

VOO/IVV Portfolio −− 31 Days
θ
Buy/Sell
NC Buy/Sell
3000
5000
7000
0
500
1000
1500
2000
2500

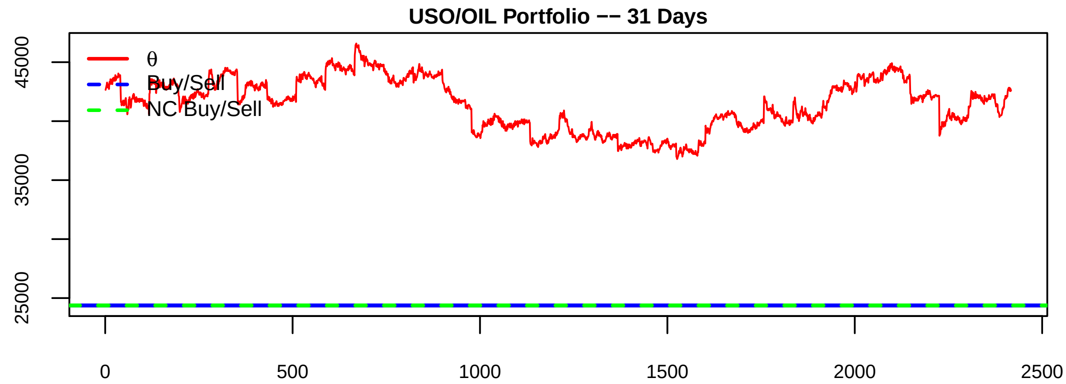

VOO/IVV Results
Closeout
Non−Closeout
0
50
100
150
1
2
3
4
5
6
7
8

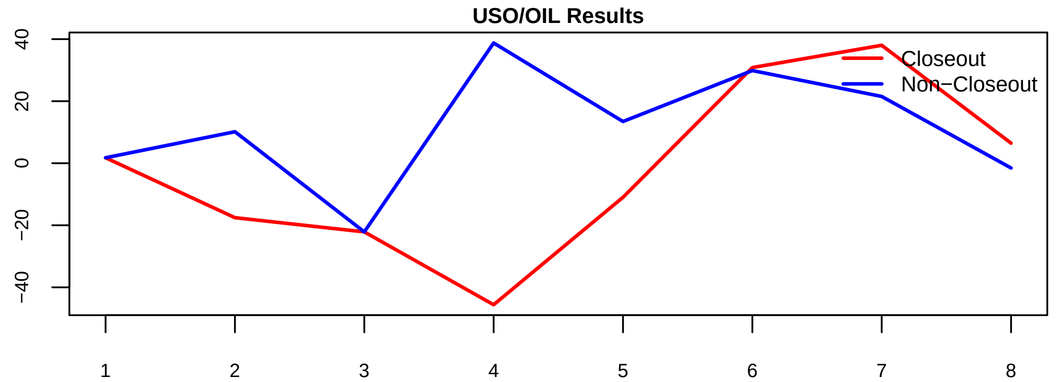

IWM/SLY Portfolio −− 31 Days
θ
Buy/Sell
NC Buy/Sell
40000
45000
50000
0
500
1000
1500
2000
2500

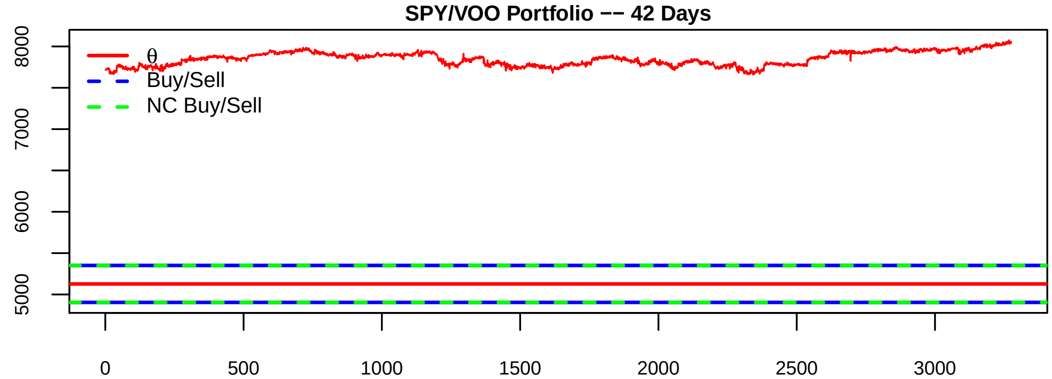

IWM/SLY Results
Closeout
Non−Closeout
0
100
200
300
1
2
3
4
5
6
7
8

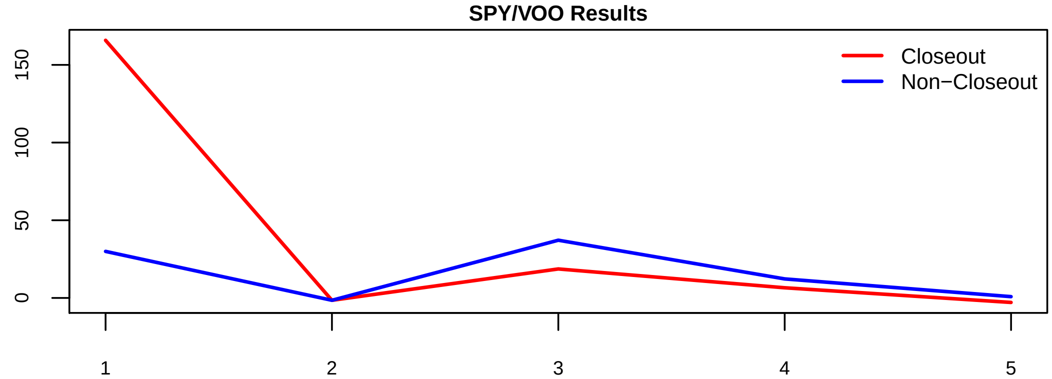

VIOO/VTWO Portfolio −− 31 Days
θ
Buy/Sell
NC Buy/Sell
32000
36000
40000
44000
0
500
1000
1500
2000
2500

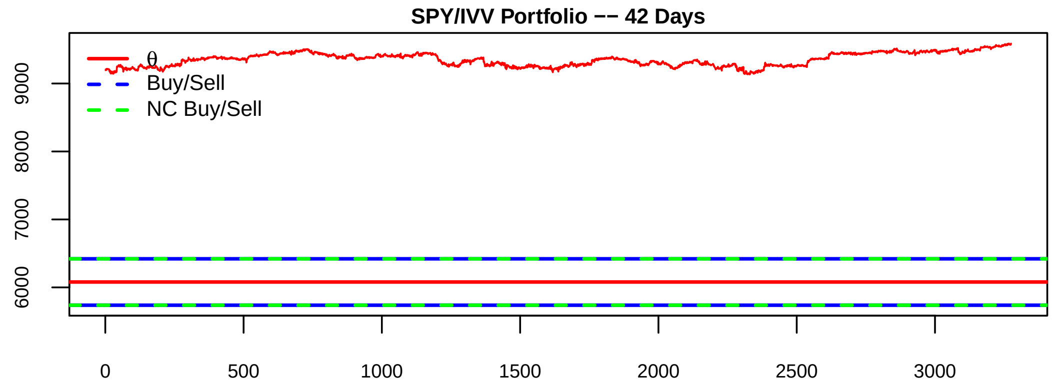

VIOO/VTWO Results
Closeout
Non−Closeout
0
20
40
60
1
2
3
4
5
6
7
8

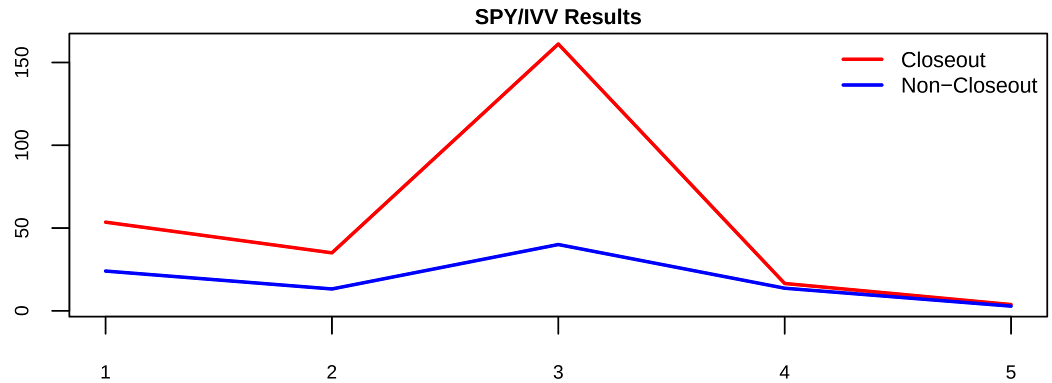

USO/OIL Portfolio −− 31 Days
θ
Buy/Sell
NC Buy/Sell
25000
35000
45000
0
500
1000
1500
2000
2500

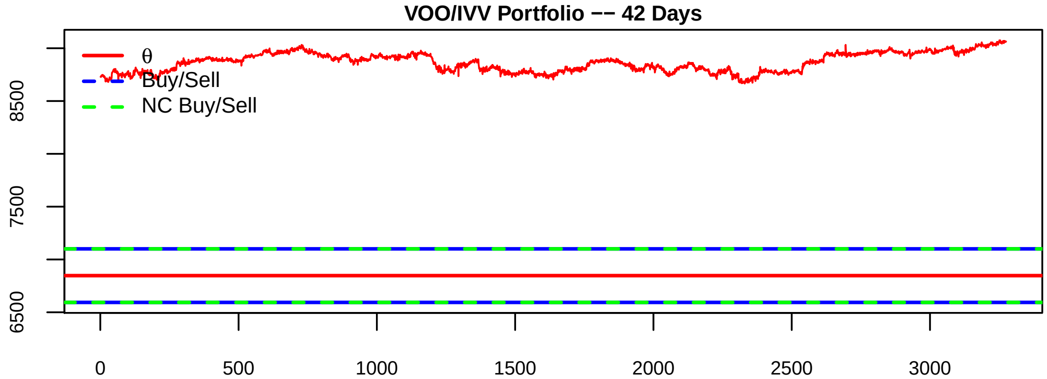

USO/OIL Results
Closeout
Non−Closeout
−40
−20
0
20
40
1
2
3
4
5
6
7
8

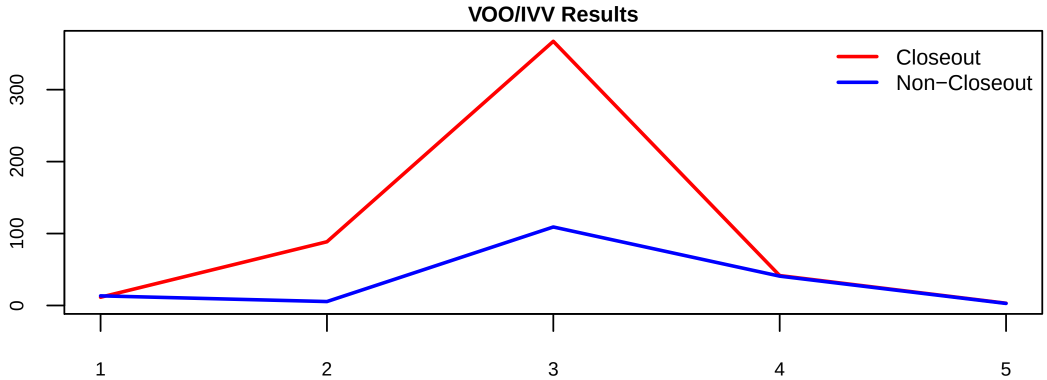

SPY/VOO Portfolio –– 42 Days
θ
Buy/Sell
NC Buy/Sell
5000
6000
7000
8000
0
500
1000
1500
2000
2500
3000

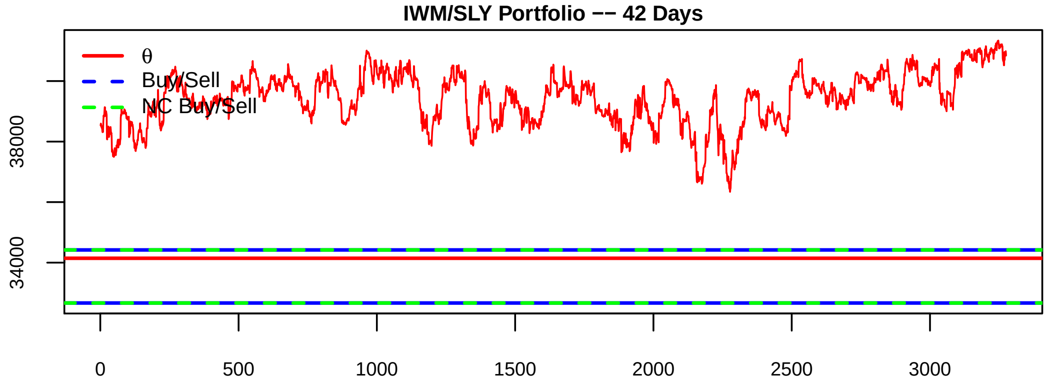

SPY/VOO Results
Closeout
Non−Closeout
0
50
100
150
1
2
3
4
5

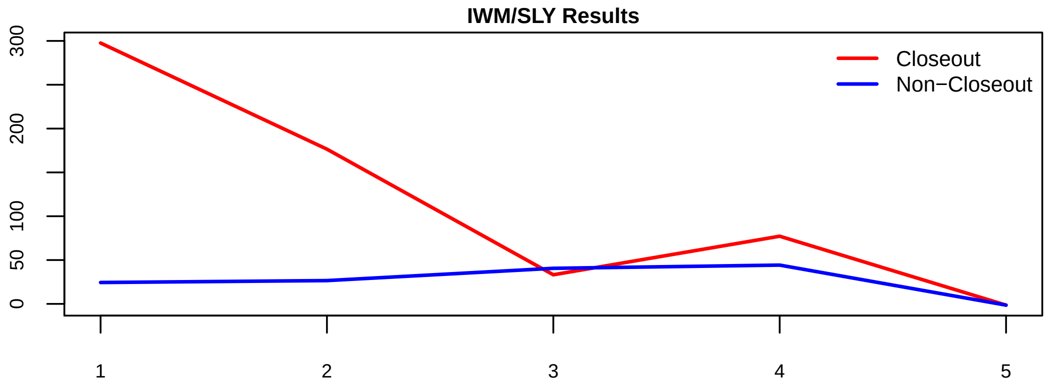

SPY/IVV Portfolio –– 42 Days
θ
Buy/Sell
NC Buy/Sell
6000
7000
8000
9000
0
500
1000
1500
2000
2500
3000

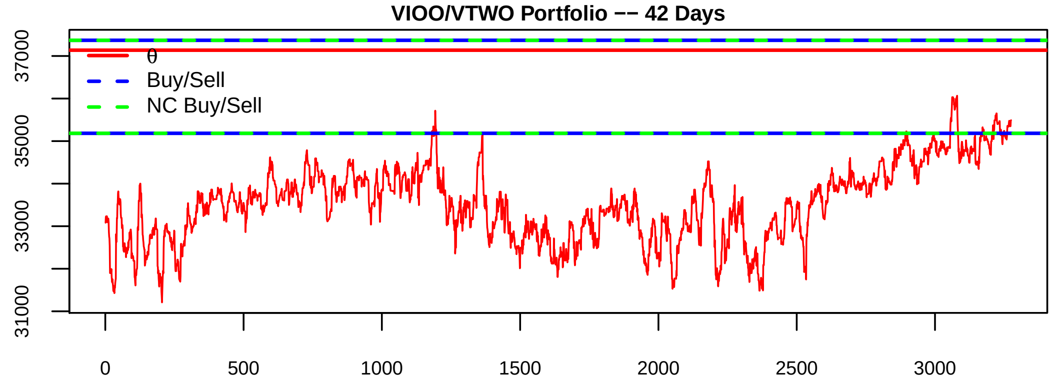
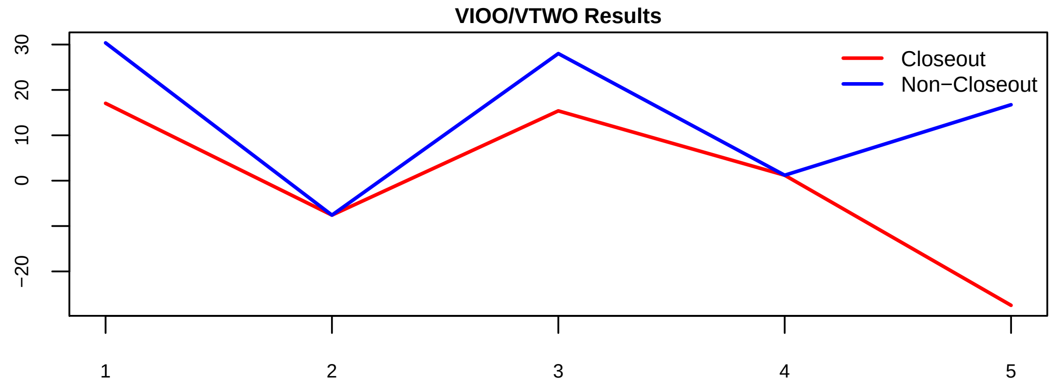
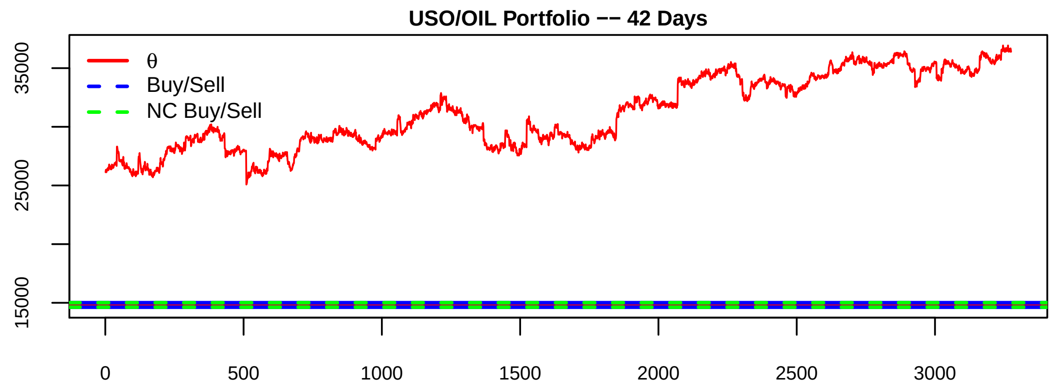
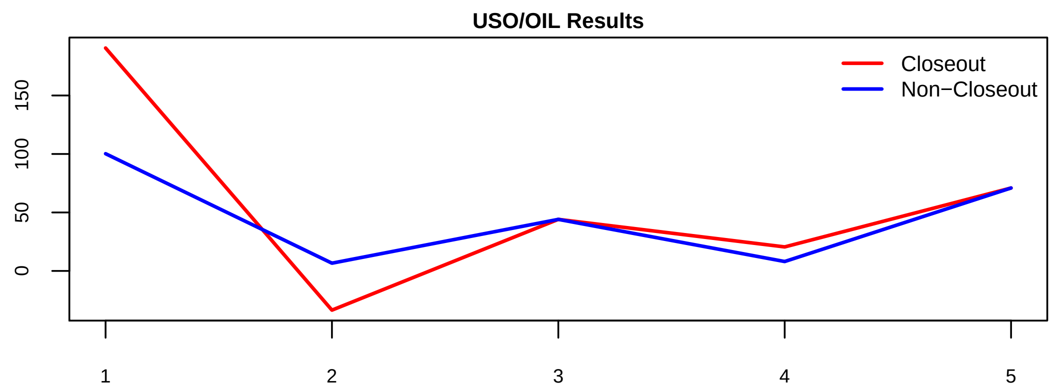

SPY/IVV Results
Closeout
Non−Closeout
0
50
100
150
1
2
3
4
5

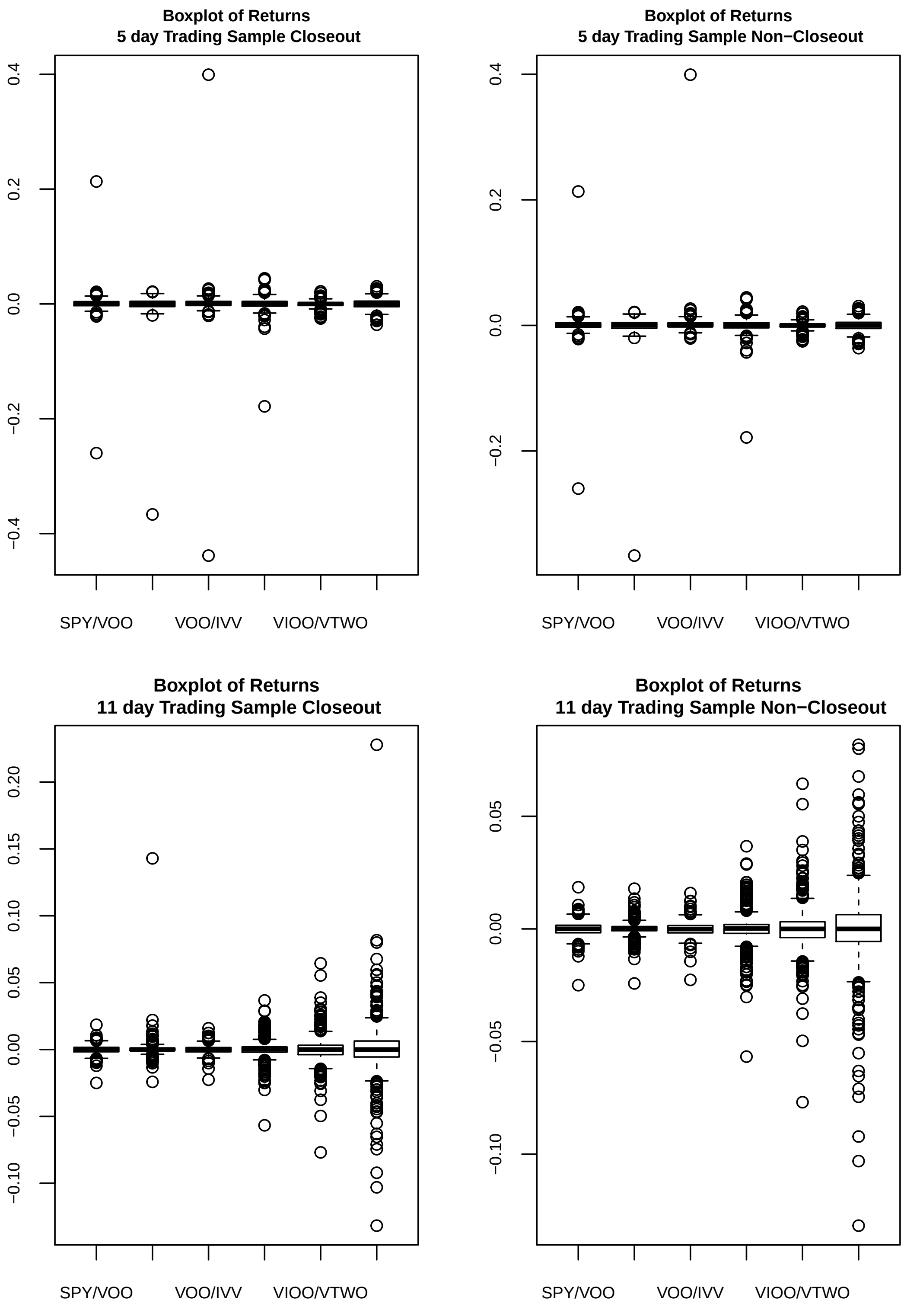

VOO/IVV Portfolio −− 42 Days
θ
Buy/Sell
NC Buy/Sell
6500
7500
8500
0
500
1000
1500
2000
2500
3000

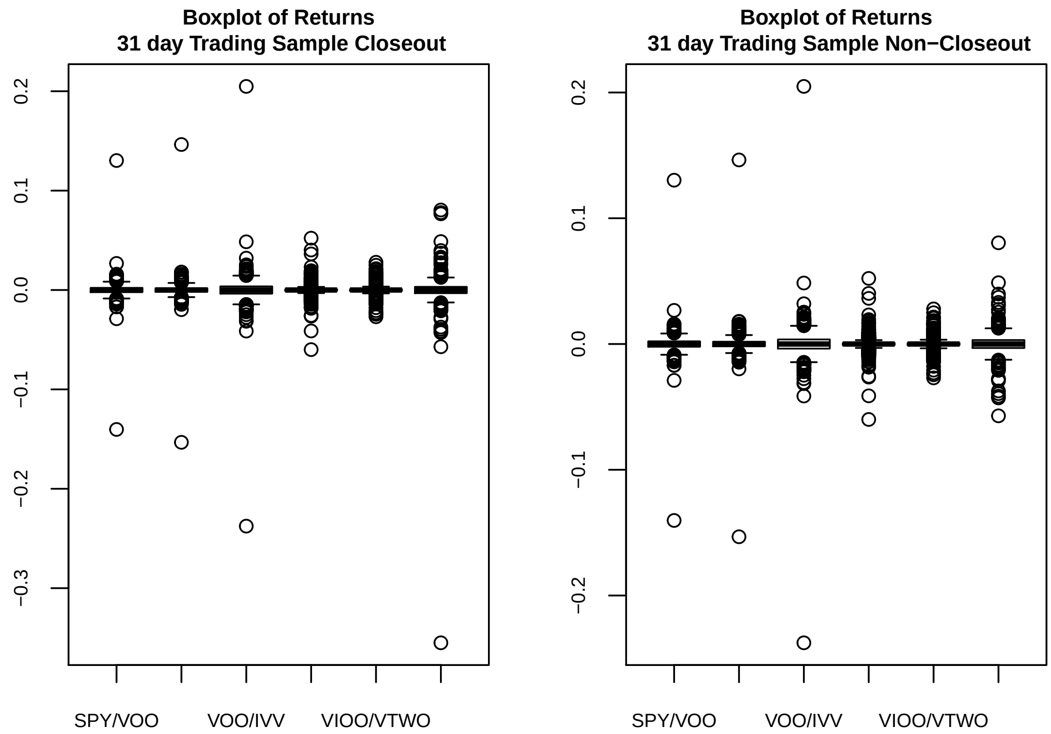

VOO/IVV Results
Closeout
Non−Closeout
0
100
200
300
1
2
3
4
5


IWM/SLY Portfolio −− 42 Days
θ
Buy/Sell
NC Buy/Sell
34000
38000
0
500
1000
1500
2000
2500
3000


IWM/SLY Results
Closeout
Non−Closeout
0
50
100
200
300
1
2
3
4
5


VIOO/VTWO Portfolio −− 42 Days
θ
Buy/Sell
NC Buy/Sell
31000
33000
35000
37000
0
500
1000
1500
2000
2500
3000

VIOO/VTWO Results
Closeout
Non−Closeout
−20
0
10
20
30
1
2
3
4
5

USO/OIL Portfolio −− 42 Days
θ
Buy/Sell
NC Buy/Sell
15000
25000
35000
0
500
1000
1500
2000
2500
3000

USO/OIL Results
Closeout
Non−Closeout
0
50
100
150
1
2
3
4
5


Boxplot of Returns
5 day Trading Sample Closeout
0.4
0.2
0.0
−0.2
−0.4
SPY/VOO
VOO/IVV
VIOO/VTWO
Boxplot of Returns
5 day Trading Sample Non−Closeout
0.4
0.2
0.0
−0.2
SPY/VOO
VOO/IVV
VIOO/VTWO
Boxplot of Returns
11 day Trading Sample Closeout
0.20
0.15
0.10
0.05
0.00
−0.05
−0.10
SPY/VOO
VOO/IVV
VIOO/VTWO
Boxplot of Returns
11 day Trading Sample Non−Closeout
0.05
0.00
−0.05
−0.10
SPY/VOO
VOO/IVV
VIOO/VTWO


Boxplot of Returns
31 day Trading Sample Closeout
0.2
0.1
0.0
−0.1
−0.2
−0.3
SPY/VOO
VOO/IVV
VIOO/VTWO
Boxplot of Returns
31 day Trading Sample Non−Closeout
0.2
0.1
0.0
−0.1
−0.2
SPY/VOO
VOO/IVV
VIOO/VTWO

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