

# UNKNOWN UNKNOWN: VOL-OF-VOL AND THE CROSS SECTION OF STOCK RETURNS

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## Abstract

This paper investigates how uncertainty about expected stock returns is priced in the cross-section of stocks. Uncertainty is proxied by the volatility of option-implied volatility (vol-of-vol), with higher vol-of-vol signaling more uncertainty among investors about expected stock returns. We find that high vol-of-vol stocks *underperform* low vol-of-vol stocks by circa 0.85 percent over the next month, or about 10 percent per year. This negative vol-of-vol effect cannot be explained by exposures to many previously documented factors, persists for more than 18 months, and also holds in a sample of ADRs. Statistical tests cannot confirm that the vol-of-vol effect is driven by arbitrage frictions and optimism bias, or by exposures to jump risk or stochastic volatility risk. Moreover, we do not find vol-of-vol to be a priced risk factor in traditional asset pricing models, or to reflect higher-order risk. Our results seem inconsistent with rational pricing of uncertainty by a representative agent, and indicate strong information linkages between option and stock markets.

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*“There are known knowns.  
 There are things we know we know.  
 And we also know there are known unknowns.  
 That is to say, we know there is some things we do not know.  
 But there are also unknown unknowns.  
 The ones we don’t know we don’t know.”*

– Donald Rumsfeld, former U.S. Secretary of Defense

## 1 Introduction

Uncertainty can have strong effects on behavior after controlling for risk (Ellsberg, 1961). Following Knight (1921), recent research in finance recognizes that asset prices are not only risky in that objective probabilities can be assigned to potential outcomes. Asset prices are also uncertain to the extent that investors need time to learn about the probabilities (e.g., Pastor and Veronesi, 2003; Cremers and Yan, 2009; Korteweg and Polson, 2009; Ozoguz, 2009), or do not observe probabilities altogether (see, among many others, Dow and Werlang, 1992; Epstein and Wang, 1994; Cao et al., 2005; Hansen et al., 1999; Leippold et al., 2008; Bossaerts et al., 2010; see Epstein and Schneider (2010) for a recent overview).

Notwithstanding the rapidly evolving theory on the implications of uncertainty for asset prices, existing empirical research has been confined to aggregate uncertainty (e.g., Cao et al., 2005; Anderson et al., 2009), uncertainty in an experimental setting (e.g., Ahn et al., 2007; Bossaerts et al., 2010), or forms of uncertainty that only indirectly relate to asset prices (e.g., Zhang, 2006). Yet, the question of how uncertainty *about expected returns on the stock itself* affects stock prices remains an open empirical question that has yet to be explored. We take a first step in this direction by proposing a measure for stock-level uncertainty that is intuitive and theoretically motivated. Specifically, we argue that the volatility of option-implied volatility (vol-of-vol) can be viewed as a proxy for such uncertainty, and study its effect in the cross-section of stock returns.

Although uncertainty about expected stock returns is difficult to observe, vol-of-vol is a natural candidate for the following reasons. First, options are written on the stock itself, traded by a large number of agents, and observed on a daily frequency. Second, option prices are forward-looking by nature, making them an

appealing basis to measure investors' uncertainty *ex ante*. Third, options' implied volatilities (IV) measure the risk-neutral expectation of a stock's future realized return volatility (Carr and Wu, 2009) and are primarily driven by expected stock price volatility (e.g., Christensen and Prabhala, 1998). Since IV gauges the perceived risks to investors regarding expected stock returns, such risks are "known unknowns," i.e., dynamics that investors know they don't know. Vol-of-vol captures the variation in investors' expectations about return volatility, thereby representing stochastic, second-order beliefs about stock returns. Second-order beliefs have theoretical underpinnings to account for the effect of uncertainty, formalized by, among others, Segal (1987), Klibanoff et al. (2005), and Nau (2006). Hence, vol-of-vol measures "stochastic unknowns", with a higher vol-of-vol indicating greater uncertainty among investors about expected stock returns.<sup>1</sup> Therefore, vol-of-vol reflects the extent to which investors don't know what they don't know, a notion we think of as "unknown unknowns."

Our results reveal that, compared to otherwise similar stocks in our sample from 1996 to 2009, stocks with a higher vol-of-vol earn significantly *lower* future returns. When we sort stocks by vol-of-vol into value-weighted quintile portfolios, stocks in the highest vol-of-vol quintile underperform stocks in the lowest vol-of-vol quintile by 0.85 percent in the month following portfolio formation, equivalent to about 10 percent per year. This negative vol-of-vol effect is not explained by loadings on the market, the Fama and French (1993) size and book-to-market factors, or the Carhart (1997) momentum factor, witnessing a four-factor alpha of -0.69 percent a month for the high-minus-low vol-of-vol portfolio. Assuming that vol-of-vol captures uncertainty about expected stock returns, our results strongly suggest that uncertainty has a negative effect on future stock returns in the cross-section.

Portfolio sorts and Fama-MacBeth regressions indicate that this vol-of-vol effect is distinct from more than twenty previously documented return drivers including size (Banz, 1981), beta (Black et al., 1972; Fama and MacBeth, 1973), book-to-market (Fama and French, 1992), momentum (Jegadeesh and Titman, 1993) and short-term reversal (Jegadeesh, 1990; Lehmann, 1990); idiosyncratic volatility (Ang et al., 2006b, 2009), past month's maximum return (Bali et al., 2011), skewness (Harvey and Siddique, 2000), and kurtosis; stock turnover (Datar et al., 1998) and Amihud's stock liquidity (Amihud, 2002); option liquidity, changes in call and put implied volatilities (Ang et al., 2010), the implied-minus-realized volatility spread (Goyal and Saretto, 2009; Bali and Hovakimian, 2009), call-minus-put implied volatilities (Cremers et al., 2011; Bali and Hovakimian, 2009), and the volatility skew (Xing et al., 2010); heterogeneity in beliefs (Diether et al., 2002) and proxies for information uncertainty (Zhang, 2006); private information (Easley et al., 2002; Durnev et al., 2003); leverage (Bhandari, 1988); stock price response delay (Hou and Moskowitz, 2005); and short-sale constraints (Nagel, 2005). Further, the vol-of-vol effect persists for more than 18 months, is also found in a separate sample of U.S.-listed American deposit receipts (ADRs) on non-U.S. firms, is not driven by specific events during our sample period, holds for a variety of vol-of-vol definitions, and is found in value-weighted, equal-weighted, quintile and decile portfolios.

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<sup>1</sup>A comparable approach is taken by Izhakian (2012) who measures risk by the variance of price realizations and uncertainty by the variance of probability of gain or loss.

Economic theory offers several alternative explanations for the negative vol-of-vol effect that are not based on uncertainty. We examine four explanations in more detail, but find that none offer a satisfactory explanation for the vol-of-vol effect. First, the effect may be caused by optimism bias or high vol-of-vol stocks deviating further from fundamental value than low vol-of-vol stocks. However, while this could explain underperformance, it does not explain the significantly positive alpha's that we find for low vol-of-vol stocks. More generally, the vol-of-vol effect persists after controlling for short-sale constraints, and is not stronger when arbitrage risk is higher. Second, the vol-of-vol effect could indicate the presence of a negative premium on volatility risk (Bakshi and Kapadia, 2003) or jump risk (Bali and Hovakimian, 2009; Cremers et al., 2011), with high vol-of-vol stocks having different exposures to aggregate volatility or jump risk than low vol-of-vol stocks. However, we find that the vol-of-vol effect remains economically and statistically significant after controlling for stock-level exposures to aggregate volatility or jump risk. Third, exposures to vol-of-vol might be priced in a factor model as in Ross (1976)'s arbitrage pricing theory. Similarly, vol-of-vol could be a negatively priced risk factor if it provides a hedge against deteriorating investment opportunities (Campbell, 1993, 1996; Ang et al., 2006b). We formally test whether the lower returns on higher vol-of-vol stocks reflect a priced risk factor in traditional asset pricing models, but cannot confirm this econometrically. Fourth, vol-of-vol reflects second-order beliefs and hence may capture some form of higher-order or asymmetric risk not captured in measures like idiosyncratic volatility, beta, skewness, or kurtosis. However, after studying the future returns distribution of the various vol-of-vol portfolios, we find no evidence for this explanation. Hence, the vol-of-vol effect can be reconciled with only little existing research in asset pricing.

To the best of our knowledge we are the first to examine how firm-level uncertainty about expected stock returns is priced in the cross-section of stocks. Previous work has examined uncertainty about various important dimensions other than expected returns on the stock itself, which describe the behavior of stock returns only indirectly. For instance, Zhang (2006) studies uncertainty about the quality of information, and finds that information uncertainty enhances other return anomalies. Our paper is different in that vol-of-vol focuses on the uncertainty investors have about future movements of a stock itself. Investors incorporate this aspect, we find, directly into the pricing of a stock. Also, Cremers and Yan (2009) and Pastor and Veronesi (2003) study uncertainty about the future (accounting) profitability of a firm and find that it affects asset valuations, but leave effects on stock returns unexplored. Yet another strand of research measures uncertainty by the dispersion of beliefs among investment professionals.<sup>2</sup> However, vol-of-vol is

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<sup>2</sup>For instance, Morgan (2002) shows that when risks are opaque, rating agencies disagree more often on the initial ratings on newly issued bonds. In the cross-section, Diether et al. (2002) show that dispersion in analysts' earnings forecasts leads to lower subsequent returns, while Qu et al. (2004) find a positive effect on expected returns. Anderson et al. (2005) construct a factor specification for short-term and long-term forecast dispersion and also find a positive price on analyst dispersion. Using *aggregate* survey forecasts, Anderson et al. (2009) find a positive price of dispersion, and argue that a variant of this measure among professional forecasters reflects model uncertainty (for a similar point, see Drechsler (2008)). Harris and Raviv (1993), Shalen (1993), and Graham and Harvey (1996) find that dispersion among newsletter "forecasts" is positively related to historical volatility, implied volatility, and volume. Also, Bessembinder et al. (1996) examine the open interest on the Standard & Poor's (S&P) 500 Index futures as a measure for disagreement in opinions. However, the open interest measure relates primarily to trading activity rather than stock returns.

calculated from (option) market prices and measures *time-series* variation of *volatility* forecasts, whereas dispersion statistics are calculated from analysts forecasts and capture *cross-sectional* variation in *earnings* forecasts.<sup>3</sup> Our empirical analysis indeed shows that vol-of-vol is different from, and the vol-of-vol effect is hardly affected by, any of the proxies for uncertainty used in these studies, indicating that vol-of-vol reflects a distinct form of uncertainty. Further, Drechsler and Yaron (2011) argue that the expected variance premium at the index level captures attitudes toward uncertainty about economic fundamentals. By contrast, our vol-of-vol measure is related directly to uncertainty about expected stock returns and, unlike the expected variance premium, is not likely to also reflect rational compensations for volatility risk. Empirically, we find that the vol-of-vol effect is distinctly different from the volatility risk premium at the stock level or exposures to the volatility risk premium at the index level. Finally, investors may see kurtosis, which focuses on fat tails in the return distribution, as being representative of uncertainty. We show that vol-of-vol is fundamentally different from, and robust to, kurtosis measures.

Our findings indicate that the equity option market contains information that is reflected later in stock prices. This adds to previous research arguing and showing that information diffuses slowly into and across markets (Hong and Stein, 1999; Hong et al., 2007), and that the option market contributes to price discovery in the stock market (Chakravarty et al., 2004). Similarly, Bali, Ang and Cakici (2010), Goyal and Saretto (2009), Xing, Zhang, and Zhao (2010), Bali and Hovakimian (2009), Cremers and Weinbaum (2010) and Yan (2011) document a significant relation between various measures extracted from option prices and future stock returns. However, the vol-of-vol effect is different from, and robust to, each of these measures. Our findings suggest that understanding the joint dynamics and pricing of option and stock markets requires the modeling of such information spillovers.

This paper proceeds as follows. Section 2 introduces a very simple model that describes the link between vol-of-vol and uncertainty, and explains how we construct the vol-of-vol measure. Section 3 describes our dataset and introduces an array of control variables. Section 4 presents the vol-of-vol effect and demonstrates how vol-of-vol affects future stock returns by means of portfolio sorts and Fama and MacBeth (1973) regressions. Section 5 explores several alternative explanations for our results. Finally, Section 6 concludes. The Appendix defines the explanatory variables used in this study in full detail.

## 2 Vol-of-vol and stock-level uncertainty

This section starts by a simple model that clarifies our choice for vol-of-vol as a measure for uncertainty about expected stock returns. We then discuss how we empirically measure vol-of-vol.

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<sup>3</sup>Moreover, since it is derived directly from market prices, vol-of-vol has an advantage over analyst-based measures by circumventing self-selection problems and optimism bias in analyst forecasts (McNichols and O'Brien, 1997), and not being distorted by incentive-related effects.

## 2.1 Theoretical motivation for vol-of-vol

We theoretically motivate vol-of-vol based on Klibanoff et al. (2005), who model preferences in the case of uncertainty as a functional of the following double-expectational form:

$$V(f) = \int_P \phi \left( \int_S u(f) d\pi \right) d\psi. \quad (1)$$

Here,  $u(f)$  characterizes attitude towards risk in a standard way, and Klibanoff et al. (2005) show that  $\phi(\cdot)$  captures the investor's attitude towards uncertainty. Following the standard approach in the literature, we assume that the outcomes in each possible state of the world are known but that probabilities are uncertain. Our motivation for vol-of-vol as a measure of uncertainty is best understood by first considering the inner expectation, and then the outer expectation.

We start with the utility function  $u(f)$ , where  $f(\cdot)$  is a real-valued function defined on a state space  $S$ , and  $u$  is a Von Neumann-Morgenstern utility function. The inner integral reflects the expected utility in case of known (or estimated) expected probabilities. In our case, the function  $f(\cdot)$  represents the investor's return  $r$  in state of the world  $s$ . Hence, the investor orders her preferences over random return paths by:

$$V(f) = \int_P \phi \left( \int_S u(r) d\pi \right) d\psi. \quad (2)$$

We assume a standard mean-variance utility function for the inner integral,  $u(r) = \mu_S - \frac{\gamma}{2}\sigma_S^2$ , with  $\mu_S$  and  $\sigma_S$  denoting the mean and standard deviation of  $r$  over states  $S$ , and  $\gamma$  capturing investor's risk aversion.

Next, the outer integral resembles subjective uncertainty about the probabilities on each state of the world, and hence about expected utility (i.e., the inner integral). In the case of mean-variance utility, this implies uncertainty about  $\mu_S$  and  $\sigma_S$ . That is,  $P = \{\Pi_1 = (\mu_{S,1}, \sigma_{S,1}), \dots, \Pi_K = (\mu_{S,K}, \sigma_{S,K})\}$  represents the set of  $K$  possible probability distributions over  $S$ , and  $\Psi = (\psi_1, \dots, \psi_K)$  represents the agent's subjective beliefs about  $P$ . The central limit theorem states that, as  $K$  tends to infinity, the beliefs about the potential probability distributions will be distributed normally. Thus, we can define the probability measure  $\Psi$  by a normal distribution over  $P$ :

$$P \sim \mathcal{N}(\mu_P(\mu_S), \mu_P(\sigma_S), \sigma_P(\mu_S), \sigma_P(\sigma_S)). \quad (3)$$

Each of these terms has an intuitive interpretation, with  $\mu_P(\mu_S)$  representing consensus beliefs about expected returns and  $\mu_P(\sigma_S)$  representing consensus beliefs about risk (i.e., volatility). The term  $\sigma_P(\mu_S)$  can be interpreted as the dispersion in beliefs about expected returns, akin to dispersion in point predictions of earnings estimates (Diether et al., 2002), credit ratings (Morgan, 2002), and economic forecasts (Anderson et al., 2009; see also footnote 2). Vol-of-vol represents investor assessments of the last term,  $\sigma_P(\sigma_S)$ , which captures the uncertainty about the "right prior" with regard to the expected riskiness of stock returns,  $\sigma_S$ . This is analogous to the idea of "unknown unknowns."<sup>4</sup>

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<sup>4</sup>We note that interpreting uncertainty about the volatility parameter as structural uncertainty is not new. See, for example, Weitzman (2007).

## 2.2 Measuring vol-of-vol

We operationalize the model above by employing the standardized volatility of option implied volatility (vol-of-vol) measure as a proxy for uncertainty about expected stock returns. Implied volatility (IV) generally measures the risks perceived by investors that surround future stock prices. The volatility of IV, then, captures the variation in investors' assessments of these risks, or second-order beliefs about future stock prices. Since the absolute changes in IV tend to be larger for high-volatility stocks than for low-volatility stocks, we filter the effect of expected risk from our uncertainty measure by scaling the standard deviation of IV of stock  $i$  on day  $t$  ( $\sigma_{i,t}^{IV}$ ) with the average IV over the past month ( $\bar{\sigma}_{i,t}^{IV}$ ). Thus, we calculate the vol-of-vol for stock  $i$  on day  $t$  as follows:

$$VoV_{i,t}^{1M} = \frac{\sqrt{\frac{1}{20} \sum_{j=t-19}^t (\sigma_{i,j}^{IV} - \bar{\sigma}_{i,t}^{IV})^2}}{\bar{\sigma}_{i,t}^{IV}}, \quad (4)$$

where

$$\bar{\sigma}_{i,t}^{IV} = \frac{1}{20} \sum_{j=t-19}^t \sigma_{i,j}^{IV},$$

and implied volatility  $\sigma_{i,j}^{IV}$  is calculated as the average implied volatility of the ATM call option and ATM put option. We require at least 12 non-missing observations in order to compute vol-of-vol. We also delete vol-of-vol values that are not available for more than ten days. In our main analysis, we use a one-month window with daily data to balance time-variation in vol-of-vol against the precision of the vol-of-vol estimates.<sup>5</sup>

We use data of U.S.-listed options that are written on individual stocks trading on the NYSE, AMEX, and NASDAQ exchanges. We use the standard OptionMetrics database to obtain daily implied volatilities (IVs), closing bid and ask prices, option strikes and maturities, as well as information on options' volume and open interest. For individual equity options (all of which are American), OptionMetrics provides IVs from Cox et al. (1979)'s binomial tree-based algorithm, which incorporates discrete dividend payments and early exercise. We use these IVs to calculate vol-of-vol.<sup>6</sup> The option data run from January 1st, 1996 (the first date in the OptionMetrics database), until September 30th, 2009, with which we analyze future returns from February 1996 until October 2009 (for monthly returns) or December 2009 (for longer horizons).

We apply the following screening criteria on all options to ensure that we select well-traded and well-priced options that contain reliable information. First, we exclude 'special' options that do not expire on

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<sup>5</sup>Our results are similar if we require less or more non-missing observations, do not correct for unavailable observations, or measure vol-of-vol over longer time windows.

<sup>6</sup>IVs are also available through OptionMetrics' Volatility Surface file, which contains interpolated IVs for constant levels of maturity and moneyness. However, in preliminary analyses, we find that these IVs sometimes vary because of arbitrary changes in the options used to calculate the Surface. For example, the OptionMetrics 30-day at-the-money put IV is interpolated from four put options, with strike prices straddling the stock price and maturities straddling 30 days. As the included options approach expiration, one or more of the four options will be replaced by other options, often causing a spurious change in the estimated implied volatility. Since vol-of-vol relies on the time-series properties of IV, we choose not to use the Volatility Surface, but instead rely on the individual options' database containing all actual implied volatility quotes.

the third Friday of a month. This filters out non-standard option series that are only partially available in the sample and generally have lower liquidity. Second, we retain only those options that have positive open interest, positive best bid price, and non-missing implied volatility values between 3 percent and 200 percent. Third, we eliminate all options that have bid-ask spreads exceeding 25 percent of the average between the bid and ask price.

Since most activity for options is concentrated at the short end, we require a maturity between 10 and 52 trading days for our main option measures, thereby selecting options with a remaining time to maturity (TTM) of approximately one month.<sup>7</sup> We separate call and put options into at-the-money (ATM), out-of-the-money (OTM), and in-the-money (ITM) options following Ofek et al. (2004) and Xing et al. (2010). An option is defined as ATM when the ratio of the strike price to the stock price (strike-to-spot) is between 0.95 and 1.05. Similarly, an option is defined as OTM when the ratio is lower than 0.95 (but higher than 0.80), and ITM when the ratio is higher than 1.05 (but lower than 1.20). Options with ratios below 0.80 and above 1.20 are dropped from the sample. When multiple options fall into the same group, we select the option with moneyness closest to 1.00 (ATM), 0.95 (OTM) or 1.05 (ITM).<sup>8</sup>

### 3 Data and empirical measures

The central result of this paper is that high vol-of-vol stocks earn lower average future returns than low vol-of-vol stocks. In Section 3.1, we proceed by matching the vol-of-vol measure to stock market, accounting, and other data before describing the final sample. In Section 3.2, we introduce several control variables that predict cross-sectional returns in previous research and could potentially explain the relation between vol-of-vol and future stock returns.

#### 3.1 Other data

Stock prices and returns data are obtained from the Center for Research in Security Prices (CRSP). We select all data for ordinary common shares (CRSP share codes 10 and 11) listed on the NYSE, AMEX and NASDAQ, and exclude closed-end funds and REITs (SIC codes 6720-6730 and 6798). We exclude “penny stocks” with prices below \$5 (Amihud, 2002; Zhang, 2006), and “micro caps” by requiring a market capitalization of at least \$225mln (e.g., D’Avolio, 2002) at the end of 2009 (discounted at the risk-free rate). This threshold roughly corresponds to the smallest 10% of NYSE stocks and is to eliminate stocks with difficult-to-measure prices and fundamentals, and illiquid stocks with potential market microstructure problems. Furthermore, D’Avolio (2002) shows that about one-third of these excluded stocks are difficult to short since institutional lenders generally do not have a position in them, and have high shorting costs. Hence, these criteria imply that we select stocks with relatively low short-sale constraints. We adjust the

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<sup>7</sup>We have verified that results are robust to changing the options’ TTM to three, six or twelve months.

<sup>8</sup>Unreported analysis reveals that the results are qualitatively similar when weighting all ATM options by their volume or open interest.



data for delisting returns (obtained from the CRSP delisting file) as suggested by Shumway (1997) and Shumway and Warther (1999), assuming a delisting return of -30 percent (for NYSE and AMEX stocks) or -55 percent (for NASDAQ stocks) if the corresponding delisting code is performance-related.

We match OptionMetrics data to monthly CRSP data using the procedure outlined by Duarte et al. (2005), and select option data on the one-but-last trading day of a month to match to stock returns over the next month(s). This one-day implementation lag avoids spurious findings caused by non-synchronous trading between options and stocks due to slightly different closing times of the exchanges (Battalio and Schultz, 2006), and takes into account the time needed for less technologically advanced investors to process the option information. Following Fama and French (1992), we match Compustat accounting data to CRSP after six months following fiscal year end. Accounting data are required to have a 3-year history to prevent survivorship bias. Realized earnings are obtained from Compustat’s quarterly item 8 (income before extraordinary items) and matched to CRSP after the earnings announcement date. Analyst forecasts, dispersion, and revision data are from Thomson Financial’s Institutional Brokers Estimate System (I/B/E/S). For I/B/E/S, the U.S. unadjusted file is used to mitigate the problem of imprecise forecasts (Diether et al., 2002). Data on institutional ownership are from the Thomson Financial 13f database, and we use Kenneth French’s online data library to obtain the risk-free rate, market, size, value, and momentum factors.

Table 1, panel (a) provides an overview of our sample relative to the CRSP universe. A substantial number of stocks satisfy our screening criteria, and firms with sufficient OptionMetrics data tend to have stocks with larger market capitalization. In the first year that OptionMetrics data became available, 26 percent of the firms in the CRSP universe have sufficient listed option data available, and this increases to almost 50 percent at the last years of our sample. The stocks represent 69 percent to 88 percent of U.S. market capitalization, indicating that larger firms tend to have well-traded options listed on their stocks. Hence, our sample is tilted towards larger stocks that are generally better tracked and better investable.

Table 1, panel (b) presents summary statistics on vol-of-vol for each year in our sample. The statistics are computed by first value-weighting vol-of-vol per month for each firm, and then averaging per year. The average vol-of-vol level tends to increase during turbulent market years, and a similar pattern emerges from the median vol-of-vol. Moreover, vol-of-vol varies substantially across firms with an average standard deviation of about 6 percent.

### 3.2 Control variables

In this section, we discuss a range of variables that will be controlled for in the empirical analysis, classified into six categories. The variables and their abbreviations are described in full detail in the Appendix, and discussed as they are used in the analysis. Throughout the text, we mention additional (but untabulated) control variables in the footnotes to supplement our results.

**Canonical characteristics** Our first set of control variables are the canonical characteristics based on firm size (Banz, 1981), beta (Black et al., 1972; Fama and MacBeth, 1973), book-to-market (Fama and French, 1992), momentum (Jegadeesh and Titman, 1993), and short-term reversal (Jegadeesh, 1990; Lehmann, 1990). Since each of these characteristics have been associated with future returns, the cross-sectional dispersion in expected returns might be less for stocks within portfolios based on these characteristics.

**Return distribution characteristics** Vol-of-vol is related to return dynamics or distribution characteristics by definition, which have a well-known impact on stock returns. In addition to the systematic risk as measured by a stock’s beta, stock returns might also be affected by non-systematic risk (Ang et al., 2006b, 2009), and by skewness and fat-tails in the return distribution. For instance, Bali et al. (2011) show that average return difference between stocks in the lowest decile and the highest decile based on the previous month’s maximum return exceeds 1 percent per month. Also, Barberis and Huang (2008) develop a behavioral setting in which positively skewed securities become overpriced and earn negative average excess returns. Finally, vol-of-vol bears resemblance to kurtosis, which describes the degree to which a stocks’ return distribution is weighted towards its tails. The reason is that the tails of the stock return distribution thicken when the variance of the normal can take several values. Since each of these factors have power in explaining stock returns, we examine whether idiosyncratic volatility, the maximum return, skewness, and kurtosis can explain the negative vol-of-vol effect.<sup>9</sup>

**Liquidity characteristics** The negative impact of vol-of-vol on future stock returns might relate to a liquidity effect. In general, stocks with relatively high liquidity require lower expected returns (Amihud and Mendelson, 1986). We control for stock liquidity in terms of the Amihud illiquidity measure (Amihud, 2002) and a stock’s turnover (Datar et al., 1998). We also examine the vol-of-vol effect for the quintile of stocks with the largest market capitalization, and stocks that are traded on the New York Stock Exchange (NYSE).

**Option-based characteristics** The negative vol-of-vol effect might also be explained by several option-based characteristics that predict future equity returns. Specifically, higher levels of vol-of-vol might be the result of stronger bid-ask bounces in option prices, and high vol-of-vol stocks might be subject to more measurement error in their vol-of-vol statistic than low vol-of-vol stocks. To control for this possibility we control for option liquidity, proxied by the average ATM option bid-ask spread. We also control for option measures from the following studies. Bali and Hovakimian (2009) and Cremers et al. (2011) find that stocks with a low spread between the IVs of ATM put and call options (i.e., ATM volatility skew) outperform stocks with a high spread. Next, measuring investor concerns about negative price movements, Xing et al. (2010) find that stocks with the largest spread between OTM put IV and ATM call IV (i.e., OTM volatility skew)

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<sup>9</sup>In addition to individual skewness and kurtosis, systematic skewness (Harvey and Siddique, 2000) and systematic kurtosis (Dittmar, 2002) could affect future stock returns. Therefore, we repeat our analysis when controlling for these characteristics, as discussed in the Results section. The results are comparable to what is reported in the main tables, but are omitted to conserve space.

underperform stocks with the smallest OTM skew (see also Yan (2011)). Furthermore, Bali and Hovakimian (2009) and Goyal and Saretto (2009) find that a strategy that buys (sells) stocks with the lowest (highest) spread between IV and the past month’s realized volatility (RV) yields positive returns. This measure is of particular interest for our purposes since it proxies for the difference in risk-neutral expectation and physical expectation of the return volatility, which (for the IV of options struck as ATM) is commonly interpreted as a volatility risk premium that might partly reflect compensations for uncertainty (see Bollerslev et al., 2009; Drechsler and Yaron, 2011). Finally, Ang et al. (2010) find that large monthly increases in call IV precede positive stock returns over the following month, and increases in put IV precede negative returns.<sup>10</sup>

**Uncertainty-related characteristics** Vol-of-vol directly measures firm-level uncertainty about expected stock returns. While we focus on uncertainty surrounding returns on the stock itself, we should control for several other forms of uncertainty that might impact stock returns. For instance, the literature on parameter uncertainty (e.g., Pastor and Veronesi, 2003; Cremers and Yan, 2009) proxies uncertainty by size and firm age. The idea is that smaller firms have higher parameter uncertainty, and that parameter uncertainty declines over a firm’s lifetime due to learning. Zhang (2006) uses size and age as proxies for uncertainty about the value of information and a stock, together with for example analyst coverage, forecast dispersion, and return volatility.<sup>11</sup> Related, Diether et al. (2002) find that a smaller degree of consensus among analysts, or more dispersion in the expected earnings of a firm, negatively predicts stock returns. Moreover, calling it “information risk,” Easley et al. (2002) find that the existence of private information that cannot be inferred from prices (either about a common component of asset returns or about a single asset) should affect asset prices. Considering the lack of consensus in this literature and the potential relation with vol-of-vol, we control for all of these uncertainty measures.

**Other characteristics** In addition to the above, we control for leverage, information delay and short-sale constraints. Since the equityholders’ claim on firm value is limited in levered firms, higher debt levels increasingly transmit variations in total firm value to the equity holders (Black and Scholes, 1973). As a consequence, stock prices might be more uncertain for highly levered firms. Since more levered firms have been shown to earn higher returns (Bhandari, 1988), we control for leverage. Further, Hou and Moskowitz (2005) show that the delay with which information is reflected in a stock’s price affects future stock returns. Since stocks surrounded with more uncertainty might incorporate information more slowly into their prices, we also control for price delay. Finally, Miller (1977) argues that, in the presence of short-sale constraints, bearish investors may not be able to price highly short-sale constrained stocks. This leads to overpricing and

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<sup>10</sup>In addition, one might argue that vol-of-vol is related to IV, which reflects the expected volatility of a stock, and which might also relate to uncertainty about stock returns. For example, Drechsler and Yaron (2011) argue that the volatility risk premium, which is also reflected in IV, captures attitudes towards uncertainty about economic fundamentals. However, unreported results show that the vol-of-vol effect persists after controlling for IV. Results are qualitatively similar when we include IV as a control variable or when vol-of-vol is orthogonalized with respect to RV or IV. We economize on space by omitting this result because we already control for RV and IV-RV and because Bali and Hovakimian (2009) find that IV has no reliable predictive power for future stock returns.

<sup>11</sup>Blitz and Van Vliet (2007) find that higher volatility coincides with *lower* future returns in the cross-section of stocks.

more negative future returns on these stocks compared to stocks that are less short-sale constrained. For example, Nagel (2005) shows that short-sale constraints affect future stock returns. To investigate whether the vol-of-vol effect is driven by the effects of short-sale constraints, we control for this variable as well.

## 4 The vol-of-vol effect

This section describes the “vol-of-vol effect” of high vol-of-vol stocks underperforming low vol-of-vol stocks. In Section 4.1, we discuss how the various stock characteristics vary across quintile portfolios based on vol-of-vol. In Section 4.2, we analyze the effect of vol-of-vol on future stock returns using single portfolio sorts. We then re-examine the vol-of-vol effect after controlling for the stock characteristics using double sorts in Section 4.3, and Fama-MacBeth regressions in Section 4.4. A selection of robustness checks is presented in Section 4.5.

Our starting point is the ranking of all stocks in ascending order based on vol-of-vol at the end of month  $t$  (taking into account a one-day implementation lag). We then sort the stocks into quintile portfolios. The first portfolio (“Low”) contains the stocks with the lowest vol-of-vol values, the fifth portfolio (“High”) contains the stocks with the highest vol-of-vol values. The result is illustrated in Figure 1 that shows the value-weighted average of vol-of-vol for each of the five quintiles. We plot these averages from eleven months before until eleven months after portfolio formation. The cross-sectional dispersion in vol-of-vol is highest around portfolio formation. Moreover, the relative ranking of the portfolios is persistent over time with clear differences between the vol-of-vol portfolios, both during the months before portfolio formation and during the months thereafter.

### 4.1 Characteristics of vol-of-vol portfolios

For each month and each quintile portfolio, we compute value-weighted averages for the stock characteristics at portfolio formation (except for size, which is computed on an equal-weighted basis). Next, we compute the time-series average and  $t$ -statistic over the months in our sample. The results are presented in Table 2, which shows how the stock characteristics vary across the vol-of-vol quintiles. The top row reports the average vol-of-vol around portfolio formation, as also illustrated in Figure 1 around portfolio formation.

For the sake of brevity, we do not discuss each of the stock characteristics reported in Table 2. We do note that, as vol-of-vol increases across the quintiles, so do many variables that capture some dimension of risk or uncertainty. For instance, high vol-of-vol stocks have higher beta (“Beta”) and are characterized by higher idiosyncratic volatility (“Idiosync. volatility”), higher past month maximum returns (“Maximum return”), and a more positively skewed (“Skewness”) and leptokurtic (“Kurtosis”) return distribution. Furthermore, the volatility skew (“OTM Skew”) increases with vol-of-vol suggesting that stocks surrounded with more uncertainty raise concerns about downside risk. Vol-of-vol relates negatively to the spread between implied volatility and historical, realized volatility (“IV-RV spread”). This suggests that firms surrounded with more

uncertainty about expected stock returns are less exposed to negatively priced volatility risk premia. Turning to uncertainty-related characteristics, high vol-of-vol stocks have higher forecast dispersion (“Forecast dispersion”) and higher total volatility (“Volatility”), which are indicative of higher information uncertainty (Zhang, 2006) and analyst disagreement (Diether et al., 2002). By contrast, high vol-of-vol stocks have higher analyst coverage, which indicates less information uncertainty in Zhang (2006). Also, high vol-of-vol stocks tend to belong to younger firms (“Age”), although the relation with age is not monotonic. This again suggests that vol-of-vol relates weakly (but significantly) to parameter uncertainty (Pastor and Veronesi, 2003) or information uncertainty (Zhang, 2006). Collectively, the results in this section show that vol-of-vol is significantly related to many previously documented forms of uncertainty, although the directions are mixed.<sup>12</sup>

In correlation analyses and regressions of vol-of-vol on these characteristics (unreported), we confirm the relationship between vol-of-vol and the previously documented proxies for uncertainty. However, the  $R^2$  averages to a maximum of about 15% indicating that vol-of-vol also captures a distinct part of uncertainty that is not reflected in previously proposed measures.

**Portfolio characteristics** We also compute the average number of stocks in each portfolio (“Avg. number of stocks/month”) and the percentage of stocks that stay in the portfolio from one month to the next (“Fraction in portfolio next month”). The one-but-last row of Table 2 indicates that each portfolio-month combination consists of a substantial number of stocks, with more than 230 stocks on average. The same holds when studying the number of stocks in each vol-of-vol portfolio over time (unreported). The number of stocks is smallest at the start of our sample in 1996, but the portfolios always contain more than 130 stocks. Finally, the last row of Table 2 shows that extreme levels of vol-of-vol tend to persist from one month to another. On average, 33 percent of the stocks in the lowest and highest vol-of-vol quintiles stay in their respective quintile during the next period, a percentage significantly larger than the 20 percent expected under random allocation.

## 4.2 Single portfolio sorts

We continue by computing, for each of the five vol-of-vol portfolios, the value-weighted and equal-weighted return over the following month. We then form a high-minus-low (“High-Low”) vol-of-vol portfolio that buys the high vol-of-vol portfolio and sells the low vol-of-vol portfolio. This position is held for one month. For each of the quintile and High-Low portfolios, Table 3 reports time-series averages of the cross-sectional average vol-of-vol (“Vol-of-vol”), average excess returns (“Excess return”), and the intercepts from the regression of excess portfolio returns on: i) a constant and the excess market return (“CAPM alpha”), ii) the previous

<sup>12</sup>Moreover, unreported analyses reveal that vol-of-vol tend to relate to firm-related properties that are generally associated with higher uncertainty about the future. For example, high vol-of-vol firms tend to have higher R&D (as a proportion of assets), more intangible capital (measured by property, plant and equipment over total assets), and higher expected long-term growth than low vol-of-vol firms. By contrast, vol-of-vol does not reliably relate to profitability (as measured by return-on-equity), the ratio of external financing to assets, or past one or five years sales growth. None of these measures affect subsequent conclusions, but are omitted from the paper to conserve space.

model augmented by the size and value factors as in Fama and French (1993) (“3F alpha”), and iii) the previous model augmented by the momentum factor following Carhart (1997) (“4F alpha”).

**Value-weighted portfolios** Panel (a) of Table 3 contains the results after value-weighting stocks within each portfolio. During our sample period, low vol-of-vol stocks earn on average 0.59 percent per month in excess of the risk free rate, whereas high vol-of-vol stocks earn -0.26 percent. The difference as implemented in the High-Low portfolio equals an economically large -0.85 percent per month, with a highly significant  $t$ -statistic of -2.83. A similar significantly negative performance is observed for the alphas in the CAPM and three-factor Fama and French (1993) model, indicating that the market, value, and size factors do not drive the return spread on the High-Low vol-of-vol portfolio. Similarly, the alpha in the four-factor regression is economically substantial, with a return differential of -0.69 percent per month and a significant  $t$ -value of -2.39. This indicates that vol-of-vol is also distinct from exposures associated with momentum.

Figure 2 graphically illustrates the “vol-of-vol effect”. Portfolio returns decrease almost monotonically from quintile 1 (Low) to quintile 5 (High). We note that the most dramatic drop in monthly returns occurs when moving from quintile 4 to quintile 5 (0.40 percent in returns vs. 0.41 percent in 4F alpha). Strikingly, this pattern is similar to the dramatic increase in average vol-of-vol from quintile 1 to quintile 5 (i.e., 0.06 percent).

**Equal-weighted portfolios** The results for equal-weighted portfolios, presented in panel (b) of Table 3, also reveal an economically important and statistically significant negative effect of vol-of-vol on future stock returns. The average excess return (4F alpha) difference between the Low vol-of-vol and High vol-of-vol portfolio is -0.50 percent (-0.44 percent) per month with a  $t$ -statistic of -3.07 (-2.72). Hence, compared to the value-weighted portfolios, excess returns and alphas are economically smaller (in absolute terms), but still substantial, with higher statistical significance. Unreported results reveal that the effect of vol-of-vol on future stock returns tends to be stronger for stocks with higher market capitalization (unlike most other anomalies), thereby causing the economically smaller performances when applying equal weighting. We attribute this pattern to larger cap stocks having more option series and more frequently traded option series on them, which contributes to a better-quality of IV and, hence, our vol-of-vol measure. Furthermore, we expect IV to be a better measure of expected volatility, and hence vol-of-vol being a better measure of uncertainty about stock price movements, for larger cap stocks. Indeed, unreported results show that IV is a better predictor of realized volatility over the subsequent month for larger cap stocks, consistent with this conjecture.

We focus on value-weighted returns in the remainder of this paper, since equal-weighted portfolios are tilted towards the smaller stocks, which (besides having a lower-quality vol-of-vol measure) are economically less important and generally more difficult and costly to trade.<sup>13</sup> An argument against our decision to

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<sup>13</sup>Although it does not seem to impact our results, another argument for value weighting is that equal-weighted returns might be biased upwards in the presence of bid-ask bounces (Blume and Stambaugh, 1983) and other forms of microstructure

present value-weighted returns is given by Bali and Cakici (2008) who find that equal- or value-weighted weighting schemes lead to different results on the link between idiosyncratic volatility and stock returns. We reiterate that the vol-of-vol effect is robust to equal-weighting each stock in our sample.

### 4.3 Double sorts

To verify that the vol-of-vol effect is not explained by any of the characteristics discussed above, we next examine the performance of vol-of-vol sorted portfolios after controlling for each of them. To this end, we form quintile portfolios at the end of each month by sorting on the variables that potentially explain the negative vol-of-vol effect. Next, we further sort each quintile portfolio into five additional vol-of-vol portfolios, which results in a total of 25 portfolios. Subsequently, we average each of the vol-of-vol portfolios across the five quintiles that could explain the vol-of-vol effect, in order to produce portfolios with dispersion in vol-of-vol but are similar in terms of the explanatory variables. This procedure allows us to control for each characteristic without assuming a parametric form on the relationship between vol-of-vol and future stock returns. In addition, we form a high-minus-low (“High-Low”) vol-of-vol portfolio that buys the resulting high vol-of-vol portfolio and sells the resulting low vol-of-vol portfolio. For each of these portfolios, we compute average value-weighted excess returns and alphas over the following month. The results of these double sorts are presented in Table 4. Each panel represent one class of characteristics.

Panel (a) of Table 4 demonstrates that the negative relation between vol-of-vol and future stock returns is not affected by canonical characteristics. That is, the negative vol-of-vol effect remains statistically significant with  $t$ -statistics if 2.12 and higher. The average excess return (4F alpha) of the High-Low portfolio equals -0.48 percent (-0.47 percent) per month when controlling for beta, -0.65 percent (-0.61 percent) when controlling for book-to-market, -0.37 percent (-0.32 percent) when controlling for size, -0.90 percent (-0.83 percent) when controlling for momentum, and -0.80 percent (-0.75 percent) when controlling for short-term reversal. Hence, the vol-of-vol effect remains economically important and is not explained by return effects related to the common factor characteristics.

Panel (b) of Table 4 presents results on whether idiosyncratic volatility, maximum return, skewness, and kurtosis explain the vol-of-vol effect. Despite the strong linkages between vol-of-vol and each of these characteristics, the negative relation between vol-of-vol and future stock returns persists after controlling for each of them. More specifically, the excess return (4F alpha) of the High-Low vol-of-vol portfolio ranges from -0.76 percent to -0.86 percent (-0.66 percent to -0.75 percent) per month, with  $t$ -statistics ranging between -2.73 and -3.59 (-2.31 and -3.07). Hence, none of these characteristics are able to explain the vol-of-vol effect. This finding is of particular interest for kurtosis that focuses on fat tails in the return distribution, and

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noise (Asparouhova et al., 2010). Specifically, Asparouhova et al. (2010, 2012) show that if security-level explanatory variables are positively correlated with the amount of microstructure noise, portfolio-sorted returns and coefficients in empirical asset pricing tests may be overstated, with equally weighted portfolios being especially sensitive to this bias. By inducing a negative correlation between returns and microstructure noise, the value-weighting of stocks generally offsets the bias (Asparouhova et al., 2012).

therefore seems related to vol-of-vol.<sup>14</sup>

In panel (c) of Table 4, we can see whether stock liquidity explains the lower returns for high vol-of-vol stocks relative to low vol-of-vol stocks. Since the most liquid stocks tend to be the most relevant ones from an investment perspective, we examine the vol-of-vol effect for the largest and most liquid stocks more closely. For the largest quintile of stocks (“Largest stocks (Top size quintile)”) and NYSE stocks (“Largest stocks (NYSE stocks only)”), the average excess return (4F alpha) of the High-Low portfolio is -0.73 and -0.84 percent (-0.62 and -0.69 percent) per month, respectively, with  $t$ -statistics of -2.18 and -2.98 (-2.00 and -2.51). Unlike many other anomalies, the vol-of-vol effect is quite pronounced for the largest stocks. Furthermore, the negative vol-of-vol effect remains significant for the most liquid stocks as measured by the Amihud illiquidity measure (“Most liquid (Amihud)”) and the highest turnover (“Most liquid (turnover)”), with a High-Low excess return (4F alpha) of -0.76 and -1.62 percent (-0.63 and -1.42 percent) per month, respectively, and corresponding  $t$ -statistics of -2.33 and -3.62 (-2.10 and -3.17). These results indicate that the vol-of-vol effect is not explained by differences in stock liquidity.

Panel (d) of Table 4 shows that the vol-of-vol effect is not strongly affected by the option bid-ask spread, ATM volatility skew, OTM volatility skew, IV-RV spread, first monthly differences in call IVs, and first monthly differences in put IVs. The average High-Low excess returns (4F alphas) range between -0.61 and -0.86 percent (-0.56 and -0.75 percent) per month, with  $t$ -statistics ranging between -2.51 and -2.87 (-2.26 and -2.65). This indicates that the vol-of-vol effect is not explained by bid-ask noise in option prices or previously documented, option-related characteristics.<sup>15</sup>

Panel (e) of Table 4 presents results on whether the lower returns on high vol-of-vol stocks can be explained by previously proposed measures for uncertainty. Controlling for age, analyst coverage, forecast dispersion, or volatility does not mitigate the vol-of-vol effect. Negative average excess returns (4F alphas) in the High-Low portfolio range between -0.72 and -0.84 percent (-0.60 and -0.73 percent) per month, and  $t$ -statistics range between -2.95 and -3.41 (-2.28 and -2.99). Similarly, controlling for private information does not explain the High-Low difference either, with excess returns (4F alphas) of -0.45 percent (-0.42 percent) per month and  $t$ -statistics of -2.37 (-2.26). Hence, none of these previously documented other aspects of uncertainty are able to explain the negative vol-of-vol effect, in line with our conjecture that vol-of-vol captures a distinct form of uncertainty.

Panel (f) of Table 4 present similar results after controlling for leverage, stock price delay, and short sale constraints. Average excess returns (4F alphas) range from -0.44 to -0.87 percent (-0.41 to -0.76 percent) per month, with  $t$ -statistics ranging between -2.25 and -3.14 (-2.17 and -2.92). Hence, these characteristics do not explain the vol-of-vol effect either.<sup>16</sup>

<sup>14</sup>In unreported double-sorts, we also find a persistent negative and significant vol-of-vol effect after controlling for co-skewness (Harvey and Siddique, 2000), co-kurtosis (Dittmar, 2002), and downside beta (Ang et al., 2006a).

<sup>15</sup>In unreported double-sorts, we also find a persistent negative and significant vol-of-vol effect after controlling for changes in ATM skew (Cremers and Weinbaum, 2010). Moreover, one could argue that vol-of-vol is related to IV, which reflects the expected volatility of a stock, and which may also relate to a stock’s uncertainty (see footnote 10). However, the vol-of-vol effect also persists after including IV as a control variable.

<sup>16</sup>In unreported double-sorts, we also find a persistent negative and significant vol-of-vol effect after controlling for various



## 4.4 Regression tests

The previous section indicates that portfolios formed by sorting on vol-of-vol generate substantial profits that are not explained by any single control variable. In this section, we estimate Fama and MacBeth (1973) regressions to simultaneously control for a range of control variables, to avoid the specification of breakpoints, and to further take advantage of the cross-sectional variation in vol-of-vol and the control variables. Each month, we conduct cross-sectional regressions of stock returns on vol-of-vol and one or more control variables, each of which is winsorized at the 1st and 99th percentile to limit the effect of outliers. The regressions are estimated using OLS and take the following form:

$$r_{i,t+1} - r_{t+1}^f = \alpha + \beta X_{i,t} + \varepsilon_{i,t+1}, \quad (5)$$

where  $r_{i,t+1}$  is the realized return on stock  $i$  in month  $t + 1$ ,  $r_{t+1}^f$  is the risk free rate over month  $t + 1$ ,  $X_{i,t}$  is a collection of predictor variables at time  $t$  for stock  $i$ , and  $\varepsilon_{i,t+1}$  is the prediction error which is assumed to be normally distributed with mean zero.

Next, we conduct tests on the time-series averages of the slope coefficients from the regressions. To account for potential autocorrelation and heteroskedasticity in the coefficients, we compute Newey and West (1987)-adjusted  $t$ -statistics based on the time-series of the coefficient estimates. Table 5 shows the results, classified in the same categories as before.

In panel (a) of Table 5, model (1) regresses the next month's return against current vol-of-vol. The coefficient on vol-of-vol is 0.035 with a  $t$ -statistic of -3.20. With a sample-wide standard deviation of vol-of-vol of about 6 percent (see panel (b) in Table 1), a two-standard deviation increase in vol-of-vol is associated with economically substantial lower returns of about -0.42 percent over the following month. Regressions (2)-(6) add one of the five canonical characteristics (beta, book-to-market, size, momentum, and short-term reversal) to regression (1). The coefficients on vol-of-vol effect remain economically large and highly significant, whereas most of the coefficients on the canonical cross-sectional stock return predictors are not significantly different from zero. Regression (7) adds all risk factors jointly with similar results. This is in line with recent evidence that beta does not pay off (Frazzini and Pedersen, 2010) and that most canonical return anomalies are substantially reduced after 1993 (Chordia et al., 2010) and for the large cap stocks (Fama and French, 2008). In panels (b)-(f), we use model (7) as the base specification.

In panel (b) of Table 5, the coefficients on vol-of-vol range from -0.022 to -0.026 and are slightly smaller than those in panel (a). Still, the vol-of-vol effect remains economically substantial and highly significant in all models, with  $t$ -statistics ranging between -2.72 and -3.24. When all return distribution characteristics are added jointly, the coefficient on vol-of-vol remain economically important and statistically strong with a value of -0.026 and a  $t$ -statistic of -3.53. Of the returns distribution characteristics only idiosyncratic

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other previously documented cross-sectional return drivers including the return on equity, the expected earnings-reporting month effect, I/B/E/S forecast revisions over the previous month, the Altman distress score, I/B/E/S long-term growth expectations, historical sales growth, R&D/total assets, growth in capital expenditures, property, plant and equipment (PPE) to total assets, net payout yield, and change in institutional ownership.

volatility seems to reliably predict returns, indicating that the vol-of-vol effect is not explained by these characteristics either.

In panel (c) of Table 5, the coefficients on vol-of-vol are of similar magnitude and significance in all specifications, varying between -0.024 and -0.025 with  $t$ -statistics between -2.88 and -3.00. The coefficients on Amihud illiquidity and stock turnover are not significant in any specification. This confirms that stock liquidity characteristics cannot explain the vol-of-vol effect.

In panel (d) of Table 5, the coefficient on vol-of-vol ranges from -0.029 to -0.037, with  $t$ -statistics of -2.39 and -2.70. Noteworthy is also that most of the coefficients on option-based characteristics are not significant when estimated individually or collectively. This lack of statistical significance on the other option characteristic variables might be caused by a sharply reduced sample size, as some of the option characteristics require the availability of multiple option contracts. Since it is especially the OTM skew variable that decreases the number of observations, we have re-run the regressions after omitting the OTM skew variable (unreported). The resulting vol-of-vol coefficients have similar size and significance levels. Hence, bid-ask noise and option characteristics from previous studies are not able to explain the vol-of-vol effect either.

Finally, in panel (e) of Table 5, the coefficients on vol-of-vol, again, remain economically large, ranging between -0.022 and -0.026, and highly significant, with  $t$ -statistics ranging between -2.94 and -3.25. Of the uncertainty-related and other characteristics, analyst coverage, analyst dispersion, private information, and short-sale constraints seem to reliably predict returns. However, in line with the results of the previous subsection that vol-of-vol captures a distinct form of uncertainty, none of these previously documented other aspects of uncertainty or other characteristics are able to explain the negative vol-of-vol effect.

In summary, Table 5 strongly indicates that vol-of-vol has predictive power for future stock returns with a highly significant, negative coefficient in each specification, consistent with the patterns observed in Tables 3 and 4.

## 4.5 Robustness

In the previous two subsections, we have shown that the negative cross-sectional relation between vol-of-vol and future monthly stock returns is not explained by a range of control variables. We conclude this section by checking the sensitivity of our results to a series of robustness checks.

First, one may wonder how persistent the vol-of-vol effect is. To evaluate return persistence, we proceed by tracking the average excess returns and four-factor alphas of the High-Low vol-of-vol quintile portfolio up to twenty-four months after portfolio formation. The results, depicted in Figure 3, show that (negative) returns continue to accumulate at a slowly decreasing rate. For more than 18 months after portfolio formation, average excess returns and four-factor alphas of the High-Low portfolio remain significantly negative at the 95% confidence level. Hence, the vol-of-vol effect is quite persistent.

Second, to verify that our results are not specific to our sample, we test the cross-sectional relationship

between vol-of-vol and subsequent stock returns for a sample of U.S.-listed American depository receipts (ADRs) on non-U.S. firms. Since ADRs are not included in previous analyses and tend to have returns that are similar to the locally listed shares, similar results for ADRs provide some confidence that the vol-of-vol effect also holds out-of-sample. We calculate returns and alphas based on an equal-weighting scheme,<sup>17</sup> and construct tercile (not quintile) portfolios to have a substantial number of ADRs in each portfolio. Panel (a) of Table 6 presents the average excess returns and one, three, and four-factor alphas of each tercile and the High-Low portfolio after single-sorting ADRs on vol-of-vol. We observe that returns decrease from tercile 1 (Low) to tercile 3 (High), resulting in an average excess return (4F alpha) for the High-Low portfolio of -1.20 percent (-1.14 percent). Besides economically important, the vol-of-vol effect is also highly significant statistically with a  $t$ -statistic of -4.66 (-4.24). To summarize, the vol-of-vol effect is present not only among U.S.-listed stocks but also in ADRs, suggesting that our results are not driven by data-snooping.

Third, the negative vol-of-vol effect could be clustered in specific time periods. In Figure 4, we plot the average excess return of the High-Low portfolio for each month in calendar time from January 1996 to October 2009. The negative vol-of-vol effect is present in around 60 percent of the months, and negative returns tend to be larger (in absolute terms) than positive returns for any given year throughout the sample period. Overall, it seems unlikely that the vol-of-vol effect is driven by any specific time period. To more formally test this, we calculate a sup- $F$  test statistic that tests for one or more structural breaks in the value-weighted portfolio returns of the High-Low portfolio (Andrews, 1993; Bai and Perron, 1998). The null hypothesis of no break cannot be rejected: the test statistic is 5.29 with a  $p$ -value of 0.223. Moreover, the sup- $F$  test yields similar results when we use equal-weighted portfolio returns, suggesting that the vol-of-vol effect does not change in magnitude or direction over our sample period. As an additional check, we also run a regression of monthly vol-of-vol returns on a time trend, which yields a significantly negative average vol-of-vol effect, but an insignificant effect of the time trend. Hence, the vol-of-vol effect seems not clustered in specific time periods.

Fourth, in the model from Section 2.1, we derive a vol-of-vol definition that is not scaled by the average IV over the past month. To verify that the scaling does not drive our results, we create portfolios based on unscaled vol-of-vol. Panel (b) of Table 6 present the results. The High-Low difference remains economically and statistically strong, and shows that the effect of unscaled vol-of-vol is even stronger than the effect of scaled vol-of-vol.

Fifth, by forming vol-of-vol portfolios that follow procedures that are standard in the literature, we ignore possible industry clustering within the portfolios. However, one could argue that high and low vol-of-vol stocks cluster in certain industries at various points in time, so that the vol-of-vol effect is partly driven by industry effects. Therefore, we next form industry-neutral vol-of-vol portfolios by constructing vol-of-vol

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<sup>17</sup>We do not use value weights since the market value of ADRs would include only those shares trading in the US. Moreover, even if we would also include the market value of shares traded abroad, value weights are likely to be biased. The reason is that many of these foreign companies are controlled by family groups, which can have a significant effect on the market value of a firm that is not captured by shares outstanding times market price, and which influences the economic importance of an ADR for investors.

quintile portfolios within each two-digit SIC code industry and averaging each vol-of-vol portfolio over the various industries. The bottom row of panel (b) in Table 6 (“Industry neutral”) present the results. The vol-of-vol effect is roughly of similar magnitude as before witnessing an average excess return (4F alpha) of the High-Low portfolio of -0.63 percent (-0.60 percent). However, removing between-industry variation within each of the vol-of-vol portfolios increases the  $t$ -statistic to -4.26 (-4.06), while four-factor alphas in the “Low” (“High”) portfolio are now significantly larger (smaller) than zero. This suggest that the vol-of-vol effect becomes even stronger after controlling for industry effects.<sup>18</sup>

Finally, we perform a series of additional checks, of which we do not report results to conserve space. Specifically, the negative vol-of-vol effect also persists when we (i) construct decile portfolios instead of quintile portfolios; (ii) include previously excluded stocks with prices below \$5 or with end-of-2009 market capitalization smaller than \$225mln; (iii) compute vol-of-vol exclusively from OTM put IVs, ATM put IVs, or ATM call IVs; (iv) use equal weighted, open-interest weighted or volume-weighted average IVs within the ATM call and/or put category; (v) standardize vol-of-vol by the IV measured at the beginning or end of the month; (vi) require three, ten, or twenty non-missing IV observations when calculating vol-of-vol; (vii) use IVs of options with three, six, or twelve months to maturity.

## 5 Potential explanations

The previous sections have shown that high vol-of-vol stocks robustly achieve lower future returns than low vol-of-vol stocks. This indicates a negative relationship between uncertainty about expected stock returns and future stock returns in the cross-section. In this section, we discuss several alternative explanations for the negative vol-of-vol effect. We first consider optimism bias and, more generally, deviations from fundamental value. Next, we examine whether vol-of-vol relates to stochastic volatility and jump risk premia. Subsequently, we investigate whether vol-of-vol reflects a negatively priced risk factor. Finally, we examine to what extent the vol-of-vol effect is explained by higher-order or asymmetric risk.

### 5.1 Does vol-of-vol relate to deviations from fundamental value?

The lower future returns on stocks with higher vol-of-vol could indicate that the prices of high vol-of-vol stocks are higher than justified by their fundamental value. For example, the negative vol-of-vol effect could reflect an optimism bias. Miller (1977) and Chen et al. (2001) argue that prices reflect a more optimistic valuation if short-sale constraints prevent pessimistic investors from holding a short position in a stock. As a consequence, they argue that when disagreement about the profitability of a stock increases, market prices rise relative to the true value of a stock and expected returns are negative. Since vol-of-vol relates to disagreement about the risk of a stock, the negative vol-of-vol effect might also be explained by overoptimism in the presence of short sales constraints.

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<sup>18</sup>Furthermore, unreported analyses reveal that the vol-of-vol effect is present in almost all industries at the two-digit SIC code level.

To investigate this explanation, we repeat the double-sorting procedure from Section 4.3 by sorting stocks into quintile portfolios based on short-sale constraints (proxied by residual institutional ownership). Next, we further sort each portfolio into quintile portfolios based on vol-of-vol. Panel (a) of Table 7 presents the average excess returns of the resulting 25 portfolios and the High-Low portfolios (expressed in returns and 4F alphas) for the short-sale constraints and vol-of-vol quintiles.

The magnitude of the vol-of-vol effect, presented in the “High-Low” and “High-Low (4F alpha)” columns, decreases monotonically with short-sale constraints. When moving from “Low short-sale constraints” to “High short-sale constraints”, the vol-of-vol effect decreases from -0.38 to -1.68 percent (-0.40 to -1.51 percent) per month in terms of (absolute) average excess returns (4F alphas). The decrease is significant with a  $t$ -statistic of -2.75 (-2.36). In the “Low short-sale constraints” quintile, the vol-of-vol effect is smallest in magnitude (about -0.40 percent per month) and statistically not distinguishable from zero.<sup>19</sup> These results suggest that the vol-of-vol effect is more pronounced among stocks held less by professional investors.

Although this seems in line with an optimism bias and a short-sale constraints based explanation, several other results suggest that this explanation is unlikely to fully explain the vol-of-vol effect. First, in the presence of higher short-sale constraints, the vol-of-vol effect is substantially driven by significantly *positive* abnormal returns in the Low vol-of-vol portfolio. The portfolios in the “Low” column labeled “3”, “4” and “High short-sale constraints” have average excess returns of 0.66 percent, 1.05 percent and 0.85 percent, respectively. Second, the effect of short-sale constraints on low vol-of-vol stocks is directly opposite to that on high vol-of-vol stocks. In the final row labeled “High-Low”, the positive effect of short sale constraints in the Low vol-of-vol portfolio mirrors the negative impact on the High vol-of-vol portfolio (0.71 percent versus -0.58 percent). This pattern cannot be explained by an optimism bias in the presence of short-sale constraints, which only has implications for the high vol-of-vol portfolios.

Several bits of evidence throughout our paper seem to corroborate this view. For instance, the vol-of-vol effect is present among the largest and most liquid stocks, for which disagreement and short-sale constraints tend to be smaller. Furthermore, our sample excludes penny stocks and micro-caps suggesting that short-sale constraints are already relatively low (D’Avolio, 2002). In light of these differences, we conclude that the vol-of-vol effect is at best only partially explained by optimism bias.

More generally, if vol-of-vol is driven by pricing errors of *any* kind, we expect such errors to be larger when arbitrage is more risky. For instance, De Long et al. (1990) and Shleifer and Vishny (1997) argue that financial markets might not always be informationally efficient when, among others, arbitrage is risky, since arbitrage risk deters arbitrageurs from exploiting pricing errors. In fact, previous studies argue that arbitrage risk amplifies anomalies such as book-to-market, post-earnings announcement drift, accounting accruals, and momentum (Ali et al., 2003; Mendenhall, 2004; Mashruwala et al., 2006). We explore this mispricing-based explanation by first sorting stocks into quintile portfolios based on arbitrage risk, followed by sorting stocks into quintile portfolios based on vol-of-vol. We proxy arbitrage risk by idiosyncratic

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<sup>19</sup> As an additional check (unreported) we repeated the double sort using the percentage of institutional ownership instead of residual institutional ownership, and find similar results.

volatility, since an increase in idiosyncratic volatility generally makes arbitrage more risky, and results in smaller optimal positions by arbitrageurs (Shleifer and Vishny, 1997; Pontiff, 2006).

Panel (b) of Table 7 shows the performance of the 25 resulting portfolios and the High-Low portfolios (in terms of excess returns and 4F alphas) for the arbitrage risk and vol-of-vol quintiles. The effect of vol-of-vol is significant for stocks with low or moderate arbitrage risk. By contrast, the vol-of-vol effect is statistically indistinguishable from zero when arbitrage risk is the highest. Moreover, the leftmost columns of the row labeled “High-Low” do not show a significantly different vol-of-vol effect for stocks with low arbitrage risk and high arbitrage risk. Hence, the vol-of-vol effect is not more pronounced for stocks surrounded by high arbitrage risk, which seems inconsistent with any mispricing-based explanation.

## 5.2 Does vol-of-vol relate to exposures to stochastic volatility and jump risk?

The negative vol-of-vol effect potentially reflects a compensation for systematic stochastic volatility risk or jump risk. Higher vol-of-vol indicates increased IV dynamics, so that the vol-of-vol statistic might simply capture systematic volatility risk. Alternatively, vol-of-vol might reflect higher risk of jumps in prices. For example, Andersen et al. (2002) find that only those specifications that incorporate stochastic volatility *and* jump diffusion deliver acceptable option prices. Their result is consistent with option pricing models in which implied volatilities reflect volatility or jump risk (e.g., Heston, 1993; Pan, 2002).

To evaluate whether stochastic volatility and jump risk explain our results we take the following approach. First, we estimate jump and volatility risk factor loadings at the individual stock level following Cremers et al. (2011). Specifically, we regress daily excess returns over a one year rolling window on the excess market return and either an aggregate stochastic volatility factor or an aggregate jump risk factor. In the regressions we require at least 12 degrees of freedom. To control for potential issues of infrequent trading (Dimson, 1979) we also include the first lags of the regressors and compute stochastic volatility or jump risk exposures as the sum of the current and the lagged coefficients. We use the return of at-the-money, market-neutral straddles on S&P 500 index options and the first differences in the CBOE volatility index (VIX) as stochastic volatility factors, and the slope of the implied volatility skew on S&P 500 index options and the returns of out-of-the-money index puts on the S&P 500 as jump risk factors. While several other measures could be used, Cremers et al. (2011) show that these measures yield the highest priced stochastic volatility and jump risk premia in the cross-section of stocks. Subsequently, we double-sort stocks into 25 portfolios based on the estimated volatility or jump risk loadings over the current month and vol-of-vol at the beginning of the current month to construct vol-of-vol portfolios with similar stochastic volatility or jump risk exposure.

Table 8 presents the results for each vol-of-vol quintile and the High-Low portfolio, expressed in average excess returns and 4F alphas. In panel (a), individual stocks’ exposure to market-neutral S&P index straddles (“S&P500 straddle betas”) fail to explain the vol-of-vol effect, which remains significant with a High-Low average excess return (4F alpha) of -0.67 percent (-0.56 percent) per month and a  $t$ -statistic of -2.82 (-

2.46). Similarly, High-Low differences in returns (4F alphas) remain economically important and statistically significant after controlling for exposures to first differences in the CBOE volatility index (“ $\Delta$ VIX betas”), with an average value of -0.64 percent (-0.53 percent) per month and a  $t$ -statistic of -2.69 (-2.28).

In panel (b), controlling for exposures to the change in the slope of the implied volatility skew (“ $\Delta$ option skew betas”) still yields a strong vol-of-vol effect, witnessing an average High-Low excess return (4F alpha) of -0.86 percent (0.73 percent) per month with a  $t$ -statistic of -3.04 (-2.66). Similarly, High-Low differences in excess returns (4F alphas) after controlling for exposures to returns on out-of-the-money puts on S&P 500 (“OTM put betas”) are -0.94 percent (-0.82 percent) per month with a  $t$ -statistic of -3.37 (-2.95). These results reject an explanation of the negative vol-of-vol effect based on individual stocks’ systematic volatility risk or jump risk exposures.<sup>20</sup>

### 5.3 Is vol-of-vol a priced risk factor?

Finding lower returns on stocks with higher vol-of-vol could also reflect a risk factor that is systematically priced in a factor model as in Ross (1976)’s arbitrage pricing theory. Similarly, vol-of-vol can be a negatively priced risk factor if it provides a hedge against an aggregate decrease in investment opportunities (Campbell, 1993, 1996; Ang et al., 2006b). We investigate this explanation as follows.

Following common procedures, we first construct a vol-of-vol factor by taking the value-weighted High-Low vol-of-vol quintile portfolio. Then, requiring at least 12 degrees of freedom and using a one-year estimation window, we regress each stock’s daily excess returns against the daily returns on this vol-of-vol factor, the market factor, and their first lags to control for infrequent trading (Dimson, 1979). Next, we take the sum of the coefficients on the vol-of-vol factor and its lag, and use it as an instrument for the future expected factor loadings (i.e., vol-of-vol betas). If our vol-of-vol result reflects exposures to a systematically priced risk factor, then a stock with a high vol-of-vol factor loading should have a lower average return than a stock with a low vol-of-vol factor loading.

Panel (a) of Table 9 presents results from single sorts. To facilitate comparison, the top row labeled “Vol-of-vol characteristic” re-states the single-sort result on the vol-of-vol characteristic from Table 3. The row labeled “Vol-of-vol beta” reports average excess returns of five portfolios formed after sorting stocks each month on the vol-of-vol factor loading. While the decrease in excess returns over the vol-of-vol beta quintiles is slightly larger than for the vol-of-vol characteristic, the High-Low differences in excess returns and four-factor alphas are statistically insignificant with  $t$ -statistics of -1.40 and -1.31, respectively. Hence, single sorts fail to indicate a significant factor explanation of the vol-of-vol effect.<sup>21</sup>

The sort on the vol-of-vol beta may correlate with the vol-of-vol characteristic, which may increase noise in the resulting portfolio returns. Therefore, we proceed by following the approach used by Daniel and Titman (1997), Daniel et al. (2001), and Davis et al. (2000). We form 25 portfolios by equally dividing

<sup>20</sup>These results are robust to calculating ex ante factor loadings or using a one-month estimation window.

<sup>21</sup>When we use a monthly (instead of annual) window to estimate the vol-of-vol factor loading we obtain even weaker results, with High-Low returns (4F alphas) of -0.08 percent (0.11 percent) and a  $t$ -statistic as low as -0.12 (0.14).

each of the vol-of-vol quintiles into five value-weighted portfolios based on the estimated vol-of-vol factor loadings. Next, within each vol-of-vol factor loading quintile we average across the vol-of-vol characteristic portfolios. This results in sets of portfolios that consist of stocks with similar levels of vol-of-vol, but with different loadings on the vol-of-vol factor. If the vol-of-vol result reflects exposures to a systematically priced risk factor, then a stock with a high vol-of-vol factor loading should have a lower average return than a stock with a low vol-of-vol factor loading but a similar vol-of-vol characteristic.

Panel (b) of Table 9 reports the results. As we move from portfolio “Low” to “High”, we are moving from portfolios with low average loadings on the vol-of-vol factor to portfolios with high loadings. Excess returns decrease in this direction, as do the 4F alphas. However, High-Low differences in excess returns and 4F alphas are not significant with  $t$ -statistics of -1.01 and -0.67, respectively, consistent with the result of the single portfolio sorts reported in panel (a). Hence, we find no significant relation between vol-of-vol factor loadings and average returns.<sup>22</sup>

The insignificant link between the vol-of-vol loadings and returns potentially reflects the fact that *ex ante* loadings are weak predictors of *ex post* loadings. Demonstrating that this is not the case, the bottom half of panel (b) shows that both average *ex ante* vol-of-vol factor loadings (“*Ex ante* vol-of-vol beta”) and average *ex post* factor loadings (“*Ex post* vol-of-vol beta”) increase monotonically across the portfolios. This indicates that the above method does achieve considerable dispersion in the *ex post* factor loadings. Furthermore, the last row (“Vol-of-vol characteristic”) verifies that this procedure causes little to no variation in the vol-of-vol characteristic. Hence, our sorting procedure produces substantial variation in the vol-of-vol factor loadings that is independent of the vol-of-vol characteristic.<sup>23</sup> Collectively, we cannot confirm econometrically that the vol-of-vol effect is explained by factor exposures.

## 5.4 Does vol-of-vol reflect higher-order risk or return asymmetries?

Vol-of-vol might also reflect higher-order or asymmetric risk patterns not captured by idiosyncratic volatility, (downside) beta, skewness or kurtosis. For example, low vol-of-vol stocks or portfolios may carry substantial downside or lower-tail risk to which investors are generally averse and that is compensated for with higher future returns. Alternatively, high vol-of-vol stocks or portfolios may have more upside potential (e.g., one of them might be the ‘new Google’) and a more attractive upper tail in expectation. This is a return property that investors generally prefer and requires a lower future return. To explore this possibility, we examine the distribution of future monthly returns in more detail for each of the five quintile portfolios.

Table 10 contains the results. In panels (a) and (b), the first data column (“Avg”) re-states the average excess returns on the five vol-of-vol portfolios from Table 3. Panel (a) contains the time-series averaged statistics of the cross-sectional, value-weighted future returns distribution of the stocks included within each

<sup>22</sup>The results are similar when we use contemporaneous vol-of-vol exposures, monthly estimation windows, or an equal-weighted vol-of-vol factor.

<sup>23</sup>In unreported results, we double-check the predictive power of the *ex ante* vol-of-vol betas by regressing excess returns on the High-Low characteristic-balanced vol-of-vol beta portfolios on the market and vol-of-vol factor. This yields a vol-of-vol factor loading of 0.91 with a  $t$ -statistic of 6.42.



quintile portfolio, as well as the difference in statistics between the High vol-of-vol portfolio and the Low vol-of-vol portfolio (“High-Low”). Panel (b) presents statistics that describe the time-series returns distribution of each of the quintile portfolios themselves, the difference in statistics between the High vol-of-vol portfolio and the Low vol-of-vol portfolio (“High-Low”), and the returns distribution of the High-Low quintile portfolio described in previous Sections (“Hedge”).

Panel (a) reveals that except for the 95th percentile (“P95”) and the maximum returns (“Max”), the returns distribution of the “High” portfolio is more negative than the “Low” portfolio. Hence, high vol-of-vol stocks have more negative average returns than low vol-of-vol stocks for the grand majority of stocks in the sample. Additionally, the below-median High-Low return differences are larger (in absolute value) than above-median differences, which indicates that downside risk exceeds upside potential. By contrast, stocks with the highest returns (“Max”) have on average a positive High-Low difference, with average maximum returns increasing monotonically in vol-of-vol. However, the difference in maximum returns is limited to 6.35 percent between the best performing individual stocks in the High and Low vol-of-vol portfolios. Hence, upside potential does not seem to justify the negative vol-of-vol effect.

In panel (b), we confirm the findings from panel (a) by examining the returns distribution of vol-of-vol portfolios. Higher vol-of-vol portfolios have lower returns over time up to the 75th percentile (columns “Min” to “P75”), which is hard to align with a risk-based explanation. As before, below-median return differences (“High-Low”) are generally larger in absolute value than above-median differences, indicating downside risk that exceeds upside potential. By contrast, for the 95th percentile and up, the higher vol-of-vol portfolios generally achieve higher returns. When focusing on the maximum monthly return for each quintile portfolios (“Max”), the High-Low return differential (“High-Low”) is positive, but limited with a value of 7.07 percent.

Furthermore, when we look at the future returns distribution of the High vol-of-vol minus Low vol-of-vol portfolio (“Hedge”), negative returns again tend to be larger (in absolute values) than positive returns, even when we would correct for the average return of the High-Low portfolio of 0.85 percent. For example, the Min, P1, P5, and P25 equal -19.50 percent, -16.22 percent, -6.21 percent, and -2.57 percent, respectively, well below (in absolute value) the Maximum (14.85 percent), P99 (10.63 percent), P95 (3.62 percent), and P75 (0.94 percent), respectively. Hence, the future stock or portfolio returns distributions of various vol-of-vol portfolios provide no evidence for an higher-order or asymmetric risk based explanation.

## 6 Conclusion

A large stream of academic literature shows different and strong behavioral responses to risky problems where objective probabilities are given, and to (fundamentally) uncertain problems where the odds are unobserved (Knight, 1921; Ellsberg, 1961). This extends directly to the asset management industry after the market crash of 2008, where a well-heard remark was that “markets look attractively priced, but volatility of volatility, or uncertainty, is too high.” Following the above quote, we postulate that the volatility of

option-implied volatility (vol-of-vol) can be viewed as a proxy for uncertainty about expected stock returns, and document how vol-of-vol predicts the cross section of future stock returns. In fact, since it measures the “variability of variability,” we argue that vol-of-vol reflects the extent to which investors don’t know what they don’t know, a notion we think of as “unknown unknowns”.

Our results indicate that, compared to otherwise similar stocks, stocks earn significantly *lower* returns if our proxy for uncertainty, the vol-of-vol, is higher. In terms of excess returns (four-factor alphas), a value-weighted portfolio of stocks in the highest vol-of-vol quintile underperforms a portfolio of stocks in the lowest vol-of-vol quintile by about 10 percent (8 percent) per year. This is substantial, especially given the fact that our sample is biased towards larger stocks. For comparison, the canonical book-to-market and momentum effects range between zero and four percent per year in our sample.

We demonstrate that the negative relationship between vol-of-vol and future stock returns is of a distinctly different nature than over 20 previously documented return drivers. The negative effect persists after controlling for size, beta, value, momentum and short-term reversal factors; idiosyncratic volatility, skewness, kurtosis, and the past month’s maximum return; stock liquidity and option liquidity; previously documented option-based return predictors; information uncertainty, parameter uncertainty or heterogeneity in beliefs proxied by size, age, analyst coverage, forecast dispersion, total volatility, and private information; leverage, stock price response delay, and short-sale constraints. Furthermore, we find that the vol-of-vol effect persists for more than 18 months, also holds in a separate sample of U.S.-listed ADRs on non-U.S. firms, cannot be ascribed to subperiods within our sample period, is robust to various changes in the definition of vol-of-vol or the construction of vol-of-vol portfolios, and is found in value-weighted and equal-weighted portfolios as well as in Fama-MacBeth regressions.

We examine several possible explanations for the negative vol-of-vol effect that are not related to uncertainty about stock returns. First, the low returns on high vol-of-vol stocks might be due to prices of high vol-of-vol stocks deviating more from fundamental value. However, significantly positive returns on low vol-of-vol stocks are inconsistent with overpricing due to short-sale constraints. Furthermore, we find evidence against mispricing *per se* in that the vol-of-vol effect is not significantly stronger when arbitrage risk is higher. Second, stocks with high vol-of-vol might have different exposures to aggregate volatility or jump risk than stocks with low vol-of-vol. However, the exposures account for very little evidence of the lower returns for higher vol-of-vol stocks. Third, exposures to vol-of-vol may be priced in a factor model as in Ross (1976)’s arbitrage pricing theory, or vol-of-vol can be a negatively priced risk factor if it provides a hedge against deteriorating investment opportunities (Campbell, 1993, 1996; Ang et al., 2006b). To test the validity of this explanation, we construct a factor to mimic vol-of-vol, and sort stocks into subportfolios on the basis of their sensitivity to the vol-of-vol factor while controlling for the vol-of-vol characteristic. While we find strong patterns in pre-formation and post-formation loadings, we do not find evidence that vol-of-vol underperformance is explained by vol-of-vol being a priced risk factor. Finally, vol-of-vol might capture some form of higher-order or asymmetric risk. However, after studying the future returns distributions of various

vol-of-vol portfolios, we find little evidence for such an explanation either. Hence, the documented vol-of-vol effect does not fit any of these alternative explanations.

Our empirical findings are consistent with models where information diffuses gradually from option into stocks markets, but seemingly contradict the mainstream literature on uncertainty and asset pricing. Assuming that vol-of-vol captures uncertainty, our results strongly suggest that uncertainty has a negative effect on expected stock returns in the cross-section. This result is opposite to the more intuitive idea that a cross-sectional uncertainty factor is compensated for by higher stock returns (assuming investors are uncertainty-averse). From finance theory we might expect that, when agents are uncertainty-averse, stocks trade at a discount when returns are surrounded with more uncertainty, leading to a positive relationship between uncertainty and expected stock returns. In fact, our results seemingly contrasts with previous findings that uncertainty is positively priced at the index level (Anderson et al., 2009).

Nevertheless, it is possible to reconcile the negative vol-of-vol effect with recent models in which a negative uncertainty premium occurs if investors differ in their uncertainty preferences (e.g., Cao et al., 2005; Easley and O'Hara, 2009; Bossaerts et al., 2010). This literature follows Dow and Werlang (1992) who demonstrate that when an investor's uncertainty aversion is sufficiently high, she will not hold a risky asset. Cao et al. (2005) extend this result by arguing that such limited participation can lead to a lower equity premium when investors are heterogeneous in their uncertainty preferences. If multiple agents differ sufficiently in their aversion against uncertainty, the more uncertainty-averse investors may shy away from stocks surrounded with high uncertainty. As a consequence, the risky asset is held – and priced – only by investors who are sufficiently less uncertainty-averse and who require low uncertainty premiums.<sup>24</sup> In fact, Bossaerts et al. (2010) use this line of reasoning to explain the low return on growth stocks, which are likely to be the more uncertain securities. Experimental evidence has confirmed heterogeneous uncertainty preferences (Ahn et al., 2007), as well as the impact of uncertainty on participation in asset markets (Bossaerts et al., 2010). Furthermore, Uppal and Wang (2003) show that uncertainty-averse agents hold less diversified portfolios since investors shy away from assets they feel more uncertain about.

Yet another explanation for the negative vol-of-vol effect is that, when controlling for risk, investors may actually be willing to pay a premium for betting on highly uncertain events (such as stock market investments). Experimental studies argue that people tend to have such 'uncertainty-loving' preferences if they feel more knowledgeable or competent about, or if they feel more familiar or experienced with, the decision at hand (Heath and Tversky, 1991). Brenner and Izhakian (2012) make a case for this explanation by using an empirical measure for uncertainty in an aggregate time series context. Comparable with our results, their empirical evidence suggests that uncertainty has a negative effect on future index-level returns.

Future research is needed to identify the precise mechanism that underlies the vol-of-vol effect, or effects of uncertainty on asset prices in general. This is a challenging task, since any empirical investigation is

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<sup>24</sup>The idea is comparable to Amihud and Mendelson (1986) and Beber et al. (2011) who study a "cliente effect" of having different types of investors with different expected holding periods, and find that each type trades assets with different relative spreads.

hampered by the unobservable nature of uncertainty. Therefore, it would be interesting for future research to develop other theoretically motivated measures for various forms of uncertainty at the stock-level, and compare them with vol-of-vol. Clearly, the implications of uncertainty on financial markets deserve further study.

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## 7 Appendix: Variable Definitions

This section describes the variables used in the text, their respective sources, and their abbreviations in the tables.

- **Beta** (Black et al., 1972; Fama and MacBeth, 1973; “Beta”) is estimated for each individual stock  $i$  at the end of month  $t$  using a CAPM regression over one year of weekly returns. Specifically, we estimate  $r_{i,\tau} - r_\tau^f = \alpha_i + \beta_{i,t} r_\tau^M + \varepsilon_{i,\tau}$ , where  $r_{i,\tau}$  is the return on stock  $i$  over week  $\tau$ ,  $r_\tau^M$  is the market return in week  $\tau$ , and  $r_\tau^f$  is the risk-free rate in week  $\tau$ . We proxy  $r_\tau^M$  by the CRSP daily value-weighted index and  $r_\tau^f$  by the Ibbotson risk-free rate. Beta equals the coefficient  $\beta_{i,t}$ .
- **Book-to-market** (Polk et al., 2006; “Book-to-market”) is book equity divided by market capitalization at the end of the previous fiscal year, and is updated every 12 months beginning in July. Book equity is for the fiscal year ending in the preceding calendar year and equals the sum of stockholders’ equity plus deferred taxes, investment tax credit, post-retirement benefit assets net of liabilities, minus preferred stock.
- **Size** (Banz, 1981; “Size”) is the equity market capitalization (price times shares outstanding) at the end of the previous month. In our regressions we take the natural logarithm of size in order to remove the extreme skewness in this variable.
- **NYSE Only** (“NYSE Only”) is a dummy variable equal to one for stocks traded on the NYSE, and zero otherwise
- **Momentum** (Jegadeesh and Titman, 1993; “Momentum”) is the cumulative stock return over the previous 11 months, i.e., starting at time  $t - 12$  and ending at time  $t - 1$  to isolate momentum from the short-term reversal effect.
- **Short-term reversal** (Jegadeesh, 1990; Lehmann, 1990; “Short-term reversal”) is last month’s stock return (i.e., from time  $t - 1$  to  $t$ ).
- **Idiosyncratic volatility** (Ang et al., 2006b; Bali et al., 2011; “Idiosync. volatility”) for each stock  $i$  is computed as the standard deviation of the daily residuals from the Fama and French (1993) three-factor model. Specifically, we estimate  $r_{i,\tau} - r_\tau^f = \alpha + \beta_{i,t} r_\tau^M + h_{i,t} HML_\tau + s_{i,t} SMB_\tau + \varepsilon_{i,\tau}$ , where  $r_\tau^M$  is the market return over period  $\tau$ ,  $HML_\tau$  is the return of the Value-minus-Growth portfolio over period  $\tau$ ,  $SMB_\tau$  is the return on the Small-minus-Big portfolio over period  $\tau$ , and  $\varepsilon_{i,\tau}$  is the idiosyncratic return on stock  $i$  over period  $\tau$  using daily returns over rolling annual periods. Subsequently, we compute idiosyncratic volatility as the standard deviation of  $\varepsilon_{i,\tau}$  over the past year .
- **Maximum return** (Bali et al., 2011; “Maximum return”) of each stock is the maximum daily return over the past month (i.e., from time  $t - 1$  to  $t$ ).

- **Skewness** (Xu, 2007; “Skewness”) is the historical third-order centralized moment calculated as  $E(x - \mu)^m / \sigma^m = 1/N \sum_{t=1}^n (x_i - \bar{x})^m / \hat{\sigma}_x^m$ , where  $\bar{x}$  and  $\hat{\sigma}_x$  are the sample mean and standard deviation of daily returns on stock  $i$  over the past year, and  $m = 3$ .
- **Kurtosis** (“Kurtosis”) is the fourth-order centralized moment calculated similarly with  $m = 4$ .
- **Amihud illiquidity** (Amihud, 2002; “Amihud illiquidity”) is computed as the absolute value of daily returns divided by daily dollar volume (in millions) measured annually. For NASDAQ firms, volume is divided by two to account for inter-dealer double-counting.
- **Turnover** (Datar et al., 1998; “Turnover”) equals last month’s number of shares traded in stock  $i$  as a percentage of total shares outstanding.
- **Option bid-ask spread** (“Option bid-ask spread”) is computed as the previous month’s average of the bid-ask spreads on at-the-money options.
- **At-the-money skew** (Bali and Hovakimian; Cremers and Weinbaum; “ATM Skew”) is the difference between implied volatilities of the ATM call and put options at time  $t$ .
- **Out-of-the-money skew** (Xing et al., 2010; “OTM Skew”) is the difference between the implied volatility of the OTM put options and the average of the implied volatilities of the ATM call and put options at time  $t$ .
- **Implied volatility - realized volatility spread** (Bali and Hovakimian, 2009; Goyal and Saretto, 2009; “IV-RV spread”) is the difference between the average of the implied volatilities of the ATM call and put options at time  $t$ , and last month’s realized volatility computed from daily returns.
- **Change in the ATM call IV** or **Change in ATM put IV** (Ang et al., 2010; “Change in the call IV,” “Change in put IV”) equal the monthly first difference between time  $t$  and  $t - 1$  in the implied volatilities of the ATM call or put options.
- **Age** (Pastor and Veronesi, 2003; Zhang, 2006; “Age”) equals the number of years up to time  $t$  since a firm first appeared on the CRSP tapes. In our regressions we take the natural logarithm of age in order to remove the extreme skewness in this variable.
- **Analyst coverage** (Zhang, 2006; “Analyst coverage”) is the number of analysts following the firm over the last month.
- **Forecast dispersion** (Diether et al., 2002; Zhang, 2006; “Forecast dispersion”) is the standard deviation in analysts’ next fiscal year’s I/B/E/S earnings forecasts, scaled by price, all measured at time  $t$ .
- **Volatility** (Zhang, 2006; “Volatility”) is the standard deviation of weekly returns of each stock  $i$  over the past year ending at the end of month  $t$ .

- **Leverage** (Bhandari, 1988; “Leverage”) is defined as 1 minus book equity (see the variable definition of Book-to-market) divided by total assets (COMPUSTAT item code: AT), updated every twelve months beginning in July.
- **Private information** (Durnev et al., 2003; “Private information”) is calculated after running a regression of each stock’s excess return on the excess returns of the market index and the index for industry  $j$  to which stock  $i$  belongs;  $r_{i,t} - r_t^f = \alpha_i + \beta_{i,t}(r_\tau^M - r_\tau^f) + \gamma_{i,t}(r_{j,\tau} - r_\tau^f) + \varepsilon_{i,t}$ . Private information is measured as  $1 - R^2$  obtained from this regression. The regression are run on weekly data over the past year up to time  $t$  using the the CRSP value-weighted market index, the value-weighted industry index based on a firm’s two-digit SIC industry classification, and  $r_t^f$  from Ibbotson.
- **Stock price delay** (Hou and Moskowitz, 2005; “Stock price delay”) is calculated after running a regression of the weekly excess returns of stock  $i$  on contemporaneous and four weeks of lagged returns on the market portfolio over the past year up to time  $t$ ,  $r_{i,t} - r_t^f = \alpha_i + \beta_{i,t}(r_\tau^M - r_\tau^f) + \sum_{n=1}^4 \delta_i^{(-n)}(r_{\tau-n}^M - r_\tau^f) + \varepsilon_{i,t}$ . Price delay equals one minus the ratio of the  $R^2$  from the regression restricting  $\delta_i^{(-n)} = 0, \forall n \in [1, 4]$  to the  $R^2$  from the regression without restrictions,  $1 - \frac{R_{R,it}^2}{R_{U,it}^2}$ .
- **Short-sale constraints** (Nagel, 2005; “Short-sale constraints”) are measured by “residual” institutional ownership (low institutional ownership indicates high short sale constraints), calculated as institutional ownership corrected for size effects. Following (Nagel, 2005), we use the residual from cross-sectional regressions of institutional ownership against firm sizeduring each quarter. Institutional ownership is measured as the fraction of shares of stock  $i$  held by institutional investors during the quarter prior to the latest earnings announcement, as reported on Thomson Financial’s CDA/Spectrum Institutional (13f) Holdings. We set institutional ownership to zero if no ownership data are available for a firm-quarter during the 180 days prior to the earnings announcement.
- **Stochastic volatility** (“S&P500 straddle betas” (Cremers et al., 2011) or “ $\Delta$ VIX betas” (Ang et al., 2006b)) **and jump risk** (Cremers et al., 2011; “ $\Delta$ option skew betas” or “OTM put betas”) are the factor loadings of stock  $i$  at time  $t$  ( $f_{it}$ ) that are estimated from the following regression:

$$r_{i,\tau} - r_\tau^f = \alpha + \beta_{i,t}(r_\tau^M - r_\tau^f) + f_{i,t}\Delta F_\tau + \varepsilon_{i,\tau},$$

where  $r_\tau^M$  is the equity market return (proxied by the CRSP value-weighted index),  $r_\tau^f$  is the Ibotson risk-free rate, and  $f_{i,t}$  captures firm  $i$ ’s exposure to the risk factor  $F$ , which is either the returns of at-the-money, market-neutral S&P index straddles (Cremers et al., 2011) or daily changes in the VIX (Ang et al. (2006b); Cremers et al. (2011)) as proxies for systematic stochastic volatility risk, or the change in the slope of the implied volatility skew (Yan, 2011; Cremers et al., 2011) or returns on out-the-money puts on S&P 500 options (Cremers et al., 2011) as proxies for systematic jump risk. More precisely:

- **Market-neutral straddle returns** (Cremers et al., 2011; “S&P 500 Straddle”) are computed by constructing zero-beta straddles using nearest to ATM and one month maturity index options on the S&P 500 by solving the problem

$$\begin{aligned} r_{MN} &= \theta r_c + (1 - \theta) r_p \\ \theta \beta_c + (1 - \theta) \beta_p &= 0, \end{aligned}$$

where  $r_{MN}$  is the market-neutral straddle return,  $r_c$  ( $r_p$ ) is the return on the call (put),  $\beta_c$  ( $\beta_p$ ) is the market beta of the call (put), and  $\theta$  is the weight invested in the call. To implement this, we solve for  $\theta$  using Black-Scholes option betas, following Coval and Shumway (2001); Cremers et al. (2011).

- **Changes in VIX** (Ang et al., 2006b; “ $\Delta VIX$ ”) equal first daily differences in the VIX from the Chicago Board Options Exchange.
- **Change in the slope of the implied volatility skew** (Cremers et al., 2011; Yan, 2011; “ $\Delta$ option skew”) is calculated as the daily change in the difference between the implied volatilities of the out-of-the money put option (nearest to 0.95 strike-to-spot ratio) and the average of the implied volatilities of the nearest to at-the-money call and put options, where the options are nearest to one-month maturity S&P500 index options.
- **OTM put return on the S&P 500 options** (Cremers et al., 2011; “ $\Delta$ OTM put”) is the daily return on the out-of-the money put S&P500 index option that is nearest to a 0.95 strike-to-spot ratio and one-month maturity.

We estimate the factor loadings by running the above regressions on daily data over an annual rolling window. In order to control for potential issues of infrequent trading, we also include the factors lagged one day (as proposed by Dimson, 1979) and use the sum of the betas estimated for the contemporaneous and the one period lagged risk factors as the estimated factor loading, following Cremers et al. (2011).

Figure 1: Vol-of-vol portfolios

This figure plots the average vol-of-vol of each vol-of-vol portfolio from 11 months before ( $t-11$ ) till 11 months after ( $t+11$ ) portfolio formation. Vol-of-vol is past month's volatility of option-implied volatility (IV) standardized by average IV (see Section 2.2), and IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one-day implementation lag and value-weight stocks in each portfolio. The sample period runs from January 1996 to October 2009.

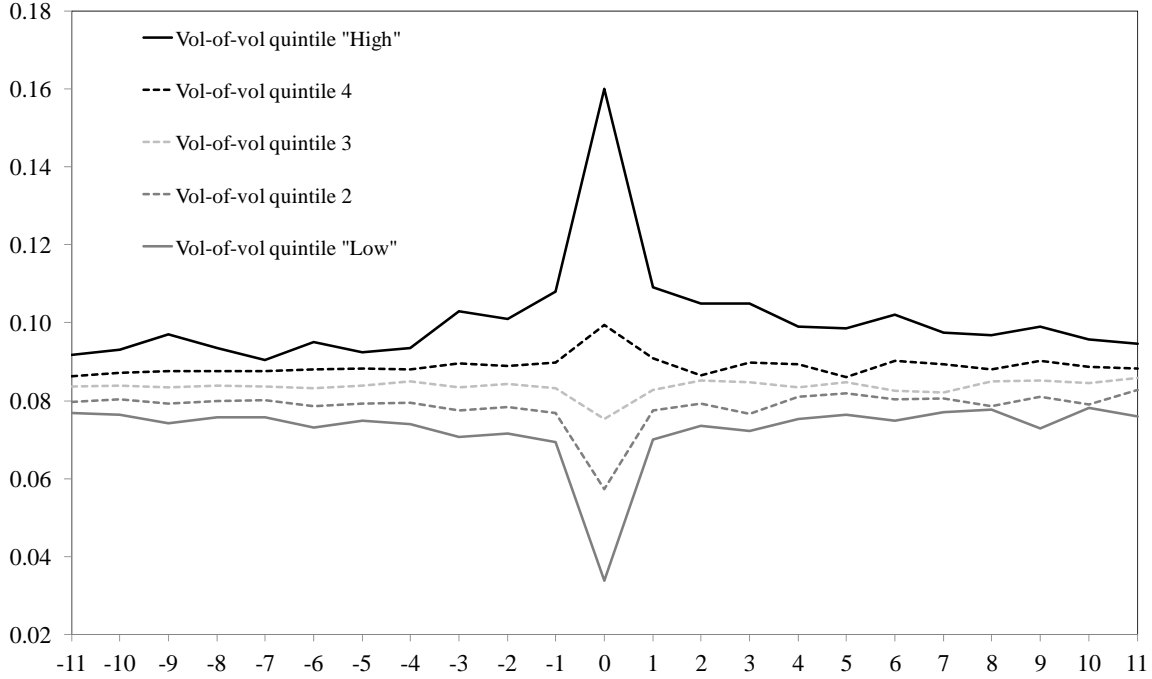


Figure 2: Monthly returns on vol-of-vol portfolios

This figure shows the average monthly excess returns of portfolios sorted on vol-of-vol. Vol-of-vol is past month's volatility of option-implied volatility (IV) standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag and value-weight stocks in each portfolio. We then plot the average excess return (Return; black bars) and four-factor alpha (4F alpha; grey bars) of each portfolio, and the High vol-of-vol minus low vol-of-vol portfolio (High-Low), over the subsequent month. The sample period runs from January 1996 to October 2009.

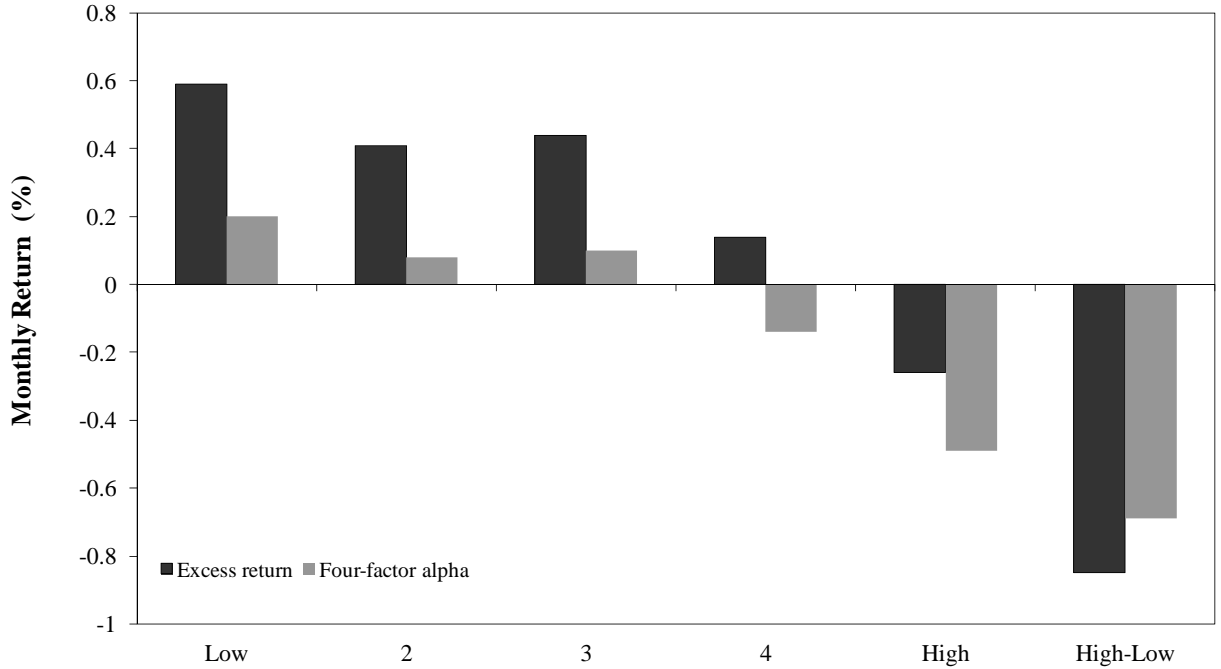


Figure 3: Performance persistence of the High-Low vol-of-vol portfolio

This figure shows the average cumulative returns of the High-Low vol-of-vol portfolio that buys the top quintile portfolio and sells the bottom quintile portfolio. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag and value-weight stocks in each portfolio. We then buy the High portfolio and sell the Low portfolio, and hold this position for the next one to 24 months. The graph plots the average excess returns (black line) and four-factor alphas (grey line) of this strategy, with dotted lines delineating the 95% confidence interval. The sample period runs from January 1996 to October 2009.

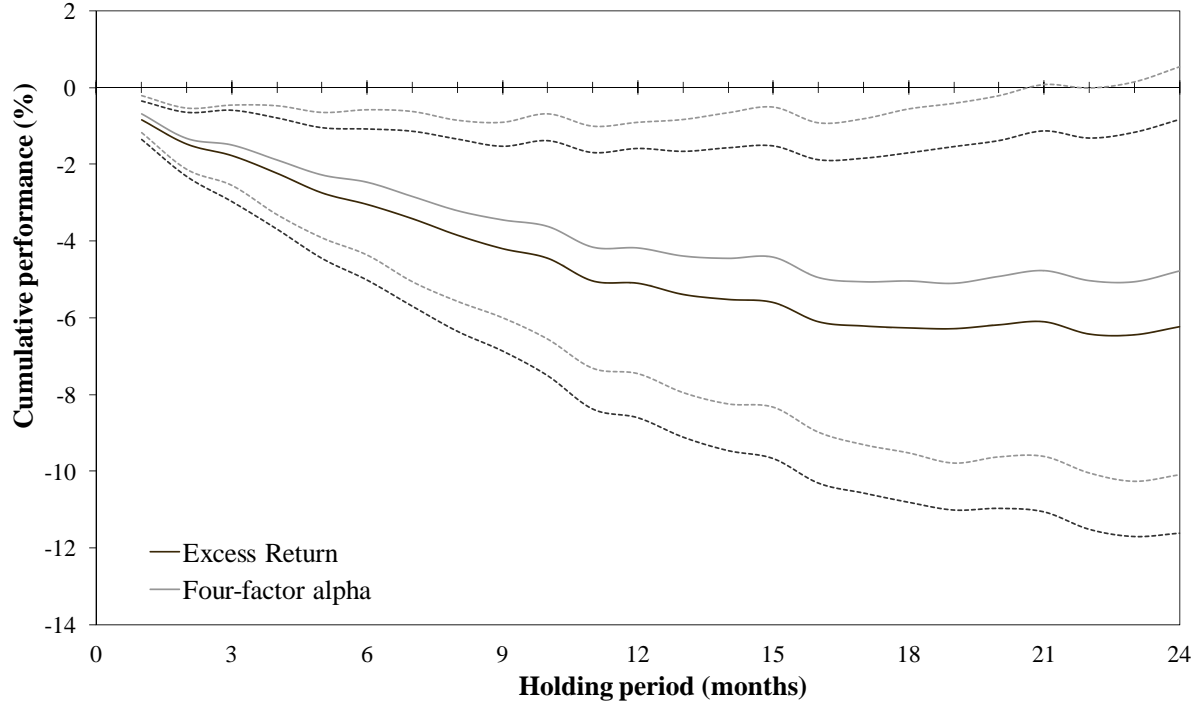




Figure 4: Performance High-Low vol-of-vol portfolio over time

This figure shows the average month-by-month excess returns of the High-Low portfolio that buys the top quintile portfolio and sells the bottom quintile portfolio. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag and value-weight stocks in each portfolio. We then buy the High portfolio and sell the Low portfolio, and hold this position over the subsequent month. The graph plots the average monthly returns on this strategy over our sample period from January 1996 to October 2009.

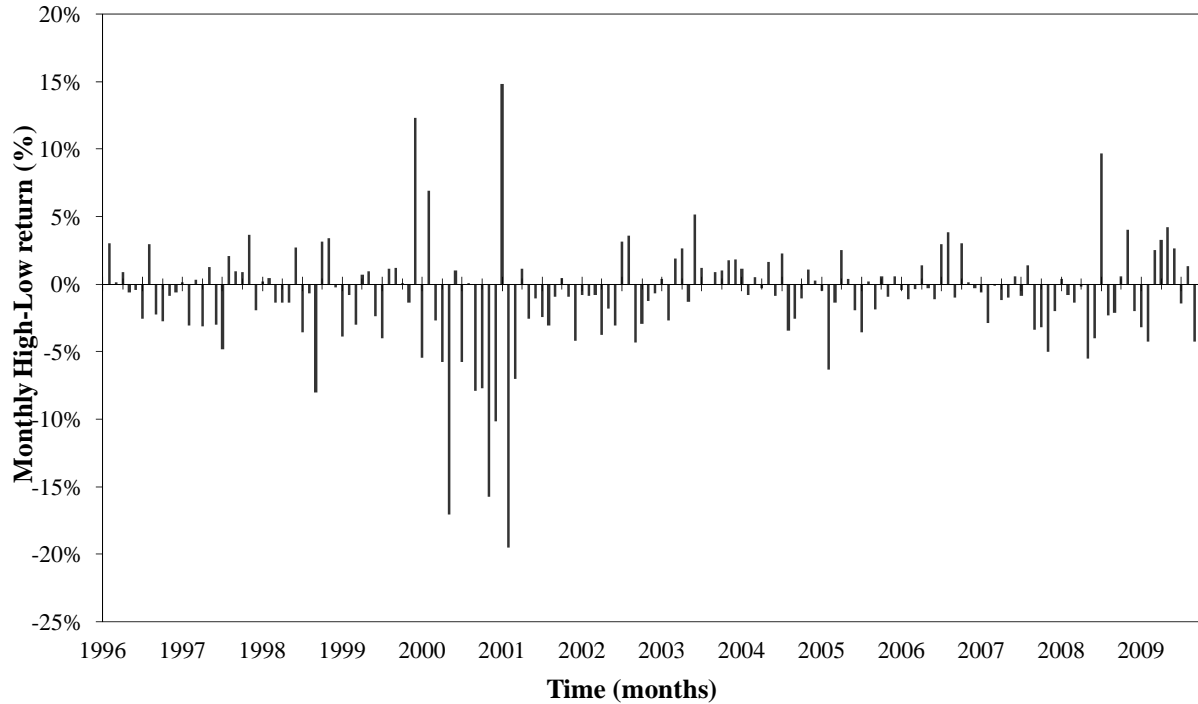


Table 1: Descriptive statistics vol-of-vol sample

This table reports descriptive statistics for vol-of-vol over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). Implied volatility is calculated from at-the-money call and put options with maturity closest to 30 days. Panel (a) reports the coverage statistics of our sample versus the CRSP sample. The first three data columns show the number of CRSP stocks included in our analysis (Nr. of stocks), the number of CRSP stocks for which we could compute our vol-of-vol measure (Nr. of stocks with vol-of-vol), and the number of stocks for which we could compute vol-of-vol as a percentage of the number of CRSP stocks (Pct. of stocks with vol-of-vol). The last three columns show the average market capitalization of CRSP stocks (MV of stocks (\$mln)), the stocks for which we can compute vol-of-vol (MV of stocks with vol-of-vol (\$mln)), and the stocks for which we can compute vol-of-vol as a percentage of the total market capitalization of CRSP stocks (MV of stocks with vol-of-vol (%)). Panel (b) reports year-by-year summary statistics of vol-of-vol. It presents the sample averages of the monthly value-weighted mean, standard deviation, 25th, 50th, and 75th percentiles of vol-of-vol, grouped per annum. The bottom row shows the grand average over our total sample.

(a) Coverage statistics

Year	Nr. of stocks	Nr. of stocks with vol-of-vol	Pct. of stocks with vol-of-vol	MV of stocks (\$mln)	MV of stocks with vol-of-vol (\$mln)	MV of stocks with vol-of-vol (%)
1996	3,414	898	26%	1,975	5,326	69%
1997	3,583	1,241	35%	2,398	5,739	81%
1998	3,498	1,382	39%	3,046	6,689	85%
1999	3,213	1,376	43%	4,047	8,386	88%
2000	3,172	1,341	42%	4,867	10,227	88%
2001	2,700	1,155	43%	4,741	9,741	87%
2002	2,476	1,071	43%	4,441	8,774	84%
2003	2,485	971	39%	4,242	9,062	82%
2004	2,793	1,128	40%	4,600	9,720	84%
2005	2,832	1,167	41%	4,930	10,263	84%
2006	2,889	1,270	44%	5,219	10,323	86%
2007	2,847	1,360	48%	5,853	10,793	87%
2008	2,365	1,147	49%	5,838	10,513	86%
2009	1,969	864	44%	5,101	9,058	76%

Descriptive statistics vol-of-vol sample (continued)

(b) Summary statistics of vol-of-vol

Year	Mean	Std. deviation	25-th percentile	50-th percentile	75-th percentile
1996	9.27%	6.43%	5.27%	7.80%	11.40%
1997	8.95%	6.83%	4.76%	7.22%	10.83%
1998	9.04%	6.43%	5.04%	7.49%	11.15%
1999	8.28%	6.21%	4.09%	6.19%	9.39%
2000	7.90%	6.33%	4.27%	6.54%	9.93%
2001	8.66%	5.55%	5.15%	7.48%	10.68%
2002	9.14%	5.56%	5.48%	7.97%	11.32%
2003	8.45%	4.96%	5.39%	7.46%	10.23%
2004	8.24%	5.56%	5.11%	7.06%	9.84%
2005	8.55%	5.51%	5.21%	7.29%	10.36%
2006	8.84%	6.00%	5.38%	7.42%	10.42%
2007	9.39%	6.15%	6.05%	8.17%	11.31%
2008	9.93%	5.61%	7.19%	9.47%	12.56%
2009	8.59%	4.94%	5.94%	7.73%	10.04%
Average	8.80%	5.86%	5.31%	7.52%	10.67%

Table 2: Stock characteristics of portfolios sorted by vol-of-vol

This table reports average characteristics for portfolios sorted on vol-of-vol over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag and (except for Size which is equal-weighted) value-weight stocks in each portfolio. The table presents average characteristics at the end of month, as well as the difference in means between portfolio High and portfolio Low (High-Low). The top row (Vol-of-vol) shows the average vol-of-vol in each portfolio. Subsequent rows present averages for stock characteristics, each of which is defined in the Appendix. The final two rows present each portfolio's average number of stocks per month, and the fraction of stocks that remain in the same portfolio from one month to the next. We report  $t$ -statistics in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

Category	Variable	Low	2	3	4	High	High-Low	$t$ (High-Low)
	Vol-of-vol	0.03	0.06	0.08	0.10	0.16	0.12	
Canonical characteristics:	Beta	0.96	0.97	1.01	1.03	1.04	0.08***	(3.55)
	Book-to-market	0.37	0.34	0.33	0.33	0.37	0.00	(0.70)
	Size (\$bln)	7.22	9.66	9.86	10.09	7.76	0.54	(1.37)
	Momentum	3.09	2.16	1.81	1.20	-0.29	-3.38***	(-3.89)
	Short-term reversal	2.92	3.40	3.64	3.59	5.14	2.21*	(1.79)
Returns distribution characteristics:	Idiosync. volatility (%)	1.83	1.76	1.78	1.80	1.91	0.08***	(3.87)
	Maximum return (%)	4.31	4.33	4.49	4.72	5.64	1.33***	(10.77)
	Skewness	0.14	0.11	0.15	0.13	0.20	0.06***	(4.28)
	Kurtosis	3.47	3.40	3.41	3.86	5.49	2.02***	(13.09)
Liquidity characteristics:	Amihud illiquidity (%)	0.09	0.05	0.06	0.07	0.12	0.03	(1.43)
	Turnover	1.25	1.19	1.21	1.22	1.36	0.11***	(4.11)

Table 2: Stock characteristics of portfolios sorted by vol-of-vol (Continued)

Category	Variable	Low	2	3	4	High	High-Low	t(High-Low)
	Vol-of-vol	0.03	0.06	0.08	0.10	0.16	0.12	
Options-based characteristics:	Option bid-ask spread	0.19	0.18	0.18	0.18	0.19	0.00	(-0.44)
	ATM skew (%)	-0.57	-0.50	-0.45	-0.44	-0.38	0.19	(1.12)
	OTM skew (%)	3.83	3.94	4.13	4.28	4.59	0.76***	(5.02)
	IV-RV spread (%)	2.22	1.03	0.31	-0.42	-3.12	-5.34***	(-12.19)
	Change in call IV (%)	-0.03	-0.02	-0.02	-0.09	0.25	0.29	(0.40)
	Change in put IV (%)	-0.04	0.02	-0.01	0.15	0.18	0.22	(0.33)
Uncertainty-related characteristics:	Age	34.83	38.98	37.74	36.08	32.81	-2.02***	(-2.64)
	Analyst coverage	19.06	20.54	20.64	20.67	19.49	0.43*	(1.68)
	Forecast dispersion (%)	0.21	0.21	0.21	0.21	0.26	0.04***	(3.53)
	Volatility	15.60	15.52	15.87	16.33	17.67	2.08***	(7.48)
	Private information	0.57	0.53	0.53	0.52	0.56	-0.01	(-0.76)
	Leverage	0.59	0.59	0.59	0.59	0.59	0.00	(-0.26)
Other characteristics:	Stock price delay	0.27	0.26	0.24	0.24	0.26	-0.01	(-0.63)
	Short sale constraints	0.18	0.16	0.25	0.18	-0.02	-0.20***	(-4.09)
	Avg. number of stocks/month	236	237	237	237	237		
Portfolio characteristics:	Fraction in portfolio next month	0.33	0.24	0.22	0.24	0.33		

Table 3: Returns on portfolios sorted by vol-of-vol

This table reports average monthly returns on portfolios sorted on vol-of-vol over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag. The table presents average returns of each portfolio over the subsequent month, as well as the difference in monthly returns between portfolio High and portfolio Low (High-Low). The top row (Vol-of-vol) shows the average vol-of-vol of each portfolio. The remaining rows present excess returns (Excess return) and alphas from the Sharpe-Lintner model (CAPM alpha), from the Fama-French three-factor model (3F alpha), and from the Fama-French-Carhart four-factor model (4F alpha). Panel (a) presents results for value-weighted portfolios, and panel (b) for equal-weighted portfolios. We report  $t$ -values in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Value-Weighted Returns

	Low	2	3	4	High	High-Low
Vol-of-vol	0.03	0.06	0.08	0.10	0.16	0.12
Excess return	0.59 (1.57)	0.41 (1.11)	0.44 (1.09)	0.14 (0.34)	-0.26 (-0.53)	-0.85*** (-2.83)
CAPM alpha	0.28* (1.87)	0.10 (0.77)	0.10 (0.90)	-0.21 (-1.57)	-0.65*** (-3.58)	-0.94*** (-3.17)
3F alpha	0.24* (1.76)	0.11 (0.92)	0.15 (1.39)	-0.14 (-1.02)	-0.55*** (-3.27)	-0.79*** (-2.98)
4F alpha	0.20 (1.36)	0.08 (0.66)	0.10 (0.99)	-0.14 (-1.06)	-0.49*** (-2.76)	-0.69** (-2.39)

(b) Equal-Weighted Returns

	Low	2	3	4	High	High-Low
Vol-of-vol	0.03	0.06	0.08	0.10	0.16	0.12
Excess return	0.40 (0.79)	0.42 (0.83)	0.21 (0.42)	0.09 (0.17)	-0.10 (-0.18)	-0.50*** (-3.07)
CAPM alpha	-0.01 (-0.09)	0.01 (0.05)	-0.2 (-1.29)	-0.34** (-2.06)	-0.52*** (-3.04)	-0.50*** (-2.99)
3F alpha	-0.09 (-0.74)	-0.10 (-0.80)	-0.29*** (-2.64)	-0.41*** (-3.07)	-0.61*** (-4.65)	-0.52*** (-3.26)
4F alpha	-0.10 (-0.77)	-0.06 (-0.50)	-0.26** (-2.52)	-0.36*** (-2.78)	-0.54*** (-4.35)	-0.44*** (-2.72)

Table 4: Returns of portfolios sorted by stock characteristics and vol-of-vol

This table reports average monthly returns of portfolios sorted on stock characteristics and vol-of-vol over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios on the basis of one of the characteristics described in Section 2. Each characteristic is defined in the Appendix. Within each characteristic quintile, we sort stocks into five additional portfolios (Low, 2, 3, 4, and High) based on vol-of-vol and compute the returns on the corresponding portfolios over the subsequent month. We use a one trading day implementation lag and value-weight stocks in each portfolio. For Size, the table presents average excess returns of each of the twenty-five resulting portfolios (Small size, 2, 3, 4, Large size; Low, 2, 3, 4, High), as well as the difference between portfolio High and portfolio Low (High-Low). The column labeled "High-Low (4F alpha)" presents the difference in four-factor alphas between portfolio High and portfolio Low. For the remaining characteristics excluding liquidity characteristics, the table presents the return of each vol-of-vol quintile, averaged over the five characteristic-sorted portfolios. For liquidity characteristics, the table presents returns of the most liquid portfolios. We report  $t$ -statistics in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Canonical characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Beta	0.43 (1.01)	0.38 (0.91)	0.45 (1.03)	0.21 (0.48)	-0.05 (-0.10)	-0.48** (-2.40)	-0.47** (-2.20)
Book-to-market	0.54 (1.38)	0.57 (1.49)	0.43 (1.04)	0.27 (0.65)	-0.12 (-0.26)	-0.65*** (-2.95)	-0.61*** (-2.79)
Size	0.32 (0.61)	0.43 (0.82)	0.16 (0.30)	0.17 (0.31)	-0.05 (-0.09)	-0.37** (-2.31)	-0.32** (-2.12)
Momentum	0.65 (1.60)	0.49 (1.24)	0.50 (1.16)	0.26 (0.55)	-0.25 (-0.52)	-0.90*** (-3.78)	-0.83*** (-3.43)
Short-term reversal	0.55 (1.41)	0.45 (1.09)	0.40 (0.92)	0.10 (0.22)	-0.25 (-0.51)	-0.80*** (-3.29)	-0.75*** (-3.04)

(b) Returns distribution characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Idiosync. volatility	0.29 (0.58)	0.34 (0.62)	0.30 (0.55)	0.04 (0.07)	-0.57 (-0.93)	-0.86*** (-3.10)	-0.75*** (-2.78)
Maximum return	0.49 (1.04)	0.31 (0.70)	0.29 (0.62)	0.17 (0.34)	-0.34 (-0.69)	-0.83*** (-3.59)	-0.75*** (-3.07)
Skewness	0.45 (1.14)	0.52 (1.36)	0.39 (0.92)	0.21 (0.47)	-0.36 (-0.74)	-0.81*** (-2.92)	-0.67** (-2.53)
Kurtosis	0.58 (1.52)	0.47 (1.23)	0.40 (0.99)	0.22 (0.49)	-0.18 (-0.35)	-0.76*** (-2.73)	-0.66** (-2.31)

Table 4: Returns of portfolios sorted by stock characteristics and vol-of-vol (Continued)

## (c) Liquidity characteristics (largest and most liquid stocks)

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Largest stocks	0.51	0.56	0.49	0.26	-0.23	-0.73**	-0.62**
(top size quintile)	(1.48)	(1.50)	(1.32)	(0.62)	(-0.46)	(-2.18)	(-2.00)
Largest stocks	0.57	0.48	0.48	0.20	-0.28	-0.84***	-0.69**
(NYSE stocks only)	(1.58)	(1.36)	(1.35)	(0.50)	(-0.64)	(-2.98)	(-2.51)
Most liquid stocks	0.56	0.53	0.46	0.22	-0.20	-0.76**	-0.63**
(Amihud)	(1.59)	(1.45)	(1.20)	(0.50)	(-0.41)	(-2.33)	(-2.10)
Most liquid stocks	0.67	0.89	0.38	0.02	-0.96	-1.62***	-1.42***
(turnover)	(1.00)	(1.28)	(0.53)	(0.03)	(-1.25)	(-3.62)	(-3.17)

## (d) Option-based characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Option bid-ask spread	0.50	0.52	0.44	0.23	-0.16	-0.66***	-0.56**
	(1.29)	(1.33)	(1.06)	(0.54)	(-0.34)	(-2.74)	(-2.43)
ATM skew	0.51	0.50	0.42	0.28	-0.32	-0.83***	-0.75***
	(1.32)	(1.26)	(0.98)	(0.62)	(-0.62)	(-2.71)	(-2.65)
OTM skew	0.54	0.45	0.39	0.29	-0.32	-0.86***	-0.74**
	(1.35)	(1.13)	(0.97)	(0.64)	(-0.63)	(-2.87)	(-2.57)
IV-RV spread	0.53	0.41	0.33	0.26	-0.20	-0.73***	-0.64**
	(1.29)	(0.99)	(0.78)	(0.56)	(-0.41)	(-2.82)	(-2.37)
Change in call IV	0.46	0.65	0.38	0.29	-0.15	-0.61**	-0.59**
	(1.05)	(1.52)	(0.89)	(0.68)	(-0.33)	(-2.53)	(-2.26)
Change in put IV	0.58	0.63	0.28	0.25	-0.06	-0.64**	-0.60**
	(1.37)	(1.49)	(0.67)	(0.61)	(-0.14)	(-2.51)	(-2.26)



Table 4: Returns of portfolios sorted by stock characteristics and vol-of-vol (Continued)

## (e) Uncertainty-related characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Age	0.46 (1.10)	0.38 (0.78)	0.40 (0.84)	0.16 (0.29)	-0.38 (-0.69)	-0.84*** (-3.12)	-0.68*** (-2.91)
Analyst coverage	0.38 (0.78)	0.29 (0.59)	0.24 (0.43)	-0.27 (-0.50)	-0.44 (-0.83)	-0.81*** (-3.41)	-0.73*** (-2.99)
Forecast dispersion	0.54 (1.35)	0.46 (1.11)	0.37 (0.84)	0.09 (0.18)	-0.29 (-0.56)	-0.83*** (-3.01)	-0.71*** (-2.70)
Volatility	0.42 (0.83)	0.47 (0.93)	0.25 (0.50)	0.09 (0.16)	-0.30 (-0.57)	-0.72*** (-2.95)	-0.60** (-2.28)
Private information	0.45 (1.21)	0.32 (0.87)	0.36 (0.95)	0.10 (0.25)	0.00 (0.01)	-0.45** (-2.37)	-0.42** (-2.26)

## (f) Other characteristics

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Leverage	0.45 (1.14)	0.51 (1.29)	0.34 (0.80)	0.20 (0.45)	-0.20 (-0.42)	-0.65*** (-2.76)	-0.48** (-2.18)
Stock price delay	0.47 (1.22)	0.35 (1.00)	0.46 (1.23)	0.25 (0.63)	0.02 (0.05)	-0.44** (-2.25)	-0.41** (-2.17)
Short sale constraints	0.61 (1.61)	0.50 (1.24)	0.49 (1.16)	0.14 (0.32)	-0.26 (-0.53)	-0.87*** (-3.14)	-0.76*** (-2.92)

Table 5: Fama-MacBeth regression results

This table presents coefficient estimates from monthly Fama-MacBeth (1973) regressions over our sample period from January 1996 to October 2009. We regress excess stock returns over month  $t + 1$  against a constant, vol-of-vol, and a series of stock characteristics, all measured at the end of month  $t$  using a one-day implementation lag. The variable definitions are described in the Appendix. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. All regressions are value-weighted. We report  $t$ -statistics in parentheses that are Newey-West corrected. \*, \*\*, and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Canonical characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.008* (1.80)	0.011*** (3.89)	0.007 (1.31)	0.004 (0.35)	0.006 (1.41)	0.009* (1.92)	0.011 (1.44)
Vol-of-vol	-0.035*** (-3.20)	-0.040*** (-3.46)	-0.036*** (-3.42)	-0.029*** (-2.68)	-0.035*** (-3.38)	-0.033*** (-3.13)	-0.025*** (-3.07)
Beta		-0.003 (-0.79)					-0.003 (-1.04)
Book-to-market			0.005 (1.37)				0.001 (0.67)
ln(Size)				0.001 (0.62)			0.000 (-0.12)
Momentum					0.000 (0.03)		-0.004 (-1.36)
Short-term reversal						-0.003 (-0.63)	-0.107* (-1.82)
Adjusted $R^2$	0.002	0.052	0.014	0.021	0.019	0.015	0.085
Observations	182,632	182,632	182,632	182,632	182,632	182,632	182,632

Table 5: Fama-MacBeth regression results (Continued)

(b) Returns distribution characteristics

	(1)	(2)	(3)	(4)	(5)
Constant	0.028*** (3.75)	0.016** (2.15)	0.012 (1.48)	0.011 (1.46)	0.029*** (3.86)
Vol-of-vol	-0.025*** (-3.13)	-0.022*** (-2.72)	-0.024*** (-3.10)	-0.025*** (-3.24)	-0.026*** (-3.53)
Beta	-0.001 (-0.41)	-0.002 (-0.84)	-0.003 (-1.03)	-0.003 (-1.04)	-0.001 (-0.23)
Book-to-market	>-0.001 (-0.14)	0.001 (0.42)	0.002 (0.71)	0.001 (0.67)	>-0.001 (-0.18)
ln(Size)	>-0.001** (-2.00)	>-0.001 (-0.54)	>-0.001 (-0.17)	>-0.001 (-0.14)	>-0.001** (-2.10)
Momentum	-0.004 (-1.24)	-0.004 (-1.31)	-0.004 (-1.29)	-0.004 (-1.36)	-0.004 (-1.22)
Short-term reversal	-0.007* (-1.71)	-0.006 (-1.26)	-0.007* (-1.81)	-0.007* (-1.80)	-0.007* (-1.62)
Idiosync. volatility	-0.381** (-2.45)				-0.402** (-2.46)
Maximum return		-0.140* (-1.84)			-0.005 (-0.30)
Skewness			-0.101* (-1.68)		-0.001 (-1.28)
Kurtosis				0.000 (0.22)	0.000 (1.48)
Adjusted $R^2$	0.090	0.084	0.082	0.081	0.095
Observations	182,623	182,623	182,623	182,623	182,623

Table 5: Fama-MacBeth regression results (Continued)

(c) Liquidity characteristics

	(1)	(2)	(3)
Constant	0.012 (1.63)	0.012 (1.59)	0.013* (1.76)
Vol-of-vol	-0.024*** (-3.00)	-0.025*** (-2.96)	-0.025*** (-2.88)
Beta	-0.003 (-1.05)	-0.003 (-1.10)	-0.003 (-1.10)
Book-to-market	0.002 (0.71)	0.002 (0.84)	0.002 (0.87)
ln(Size)	>-0.001 (-0.32)	>-0.001 (-0.16)	>-0.001 (-0.33)
Momentum	-0.004 (-1.26)	-0.004 (-1.27)	-0.003 (-1.18)
Short-term reversal	-0.107* (-1.83)	-0.107* (-1.69)	-0.107* (-1.70)
Amihud illiquidity	-0.059 (-0.47)		-0.061 (-0.50)
Turnover		>-0.001 (-0.45)	>-0.001 (-0.42)
Adjusted $R^2$	0.083	0.087	0.089
Observations	182,623	182,623	182,623

Table 5: Fama-MacBeth regression results (Continued)

(d) Options-based characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.009 (0.98)	0.008 (0.90)	0.007 (0.86)	-0.006 (0.64)	0.007 (0.76)	0.007 (0.86)	-0.009 (1.03)
Vol-of-vol	-0.034*** (-2.70)	-0.036*** (-2.52)	-0.035*** (-2.54)	-0.035*** (-2.52)	-0.037*** (-2.53)	-0.037*** (-2.68)	-0.029*** (-2.39)
Beta	-0.003 (-0.91)	-0.003 (-0.84)	-0.003 (-0.89)	-0.002 (-0.72)	-0.003 (-0.84)	-0.003 (-0.86)	-0.002 (-0.69)
Book-to-market	0.002 (0.69)	0.002 (0.75)	0.002 (0.76)	0.002 (0.77)	0.002 (0.87)	0.003 (0.89)	0.002 (0.75)
ln(Size)	0.000 (0.16)	0.000 (0.33)	0.000 (0.36)	0.000 (0.53)	0.000 (0.41)	0.000 (0.34)	0.000 (-0.02)
Momentum	0.001 (0.40)	0.001 (0.38)	0.001 (0.30)	0.001 (0.31)	0.001 (0.28)	0.001 (0.24)	0.001 (0.33)
Short-term reversal	>-0.001 (-0.05)	-0.001 (-0.16)	-0.001 (-0.30)	-0.001 (-0.17)	-0.001 (-0.12)	-0.002 (-0.32)	-0.001 (-0.12)
Option bid-ask spread	-0.003 (-0.36)						-0.005 (-0.54)
ATM skew		0.045** (2.33)					0.049* (1.79)
OTM skew			-0.005 (-0.40)				0.005 (0.37)
IV-RV spread				0.003 (0.56)			0.003 (0.54)
Change in call IV					-0.007 (-0.65)		-0.007 (-0.36)
Change in put IV						-0.019** (-2.04)	-0.006 (-0.34)
Adjusted $R^2$	0.096	0.094	0.094	0.095	0.096	0.095	0.106
Observations	80,911	80,911	80,911	80,911	80,911	80,911	80,911

Table 5: Fama-MacBeth regression results (Continued)

(e) Uncertainty characteristics and other characteristics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	0.011 (1.33)	0.015* (1.82)	0.013* (1.66)	0.019*** (2.75)	0.012 (1.53)	0.030*** (3.34)	0.016** (2.32)	0.012 (1.55)	0.036*** (4.20)
Vol-of-vol	-0.026*** (-3.25)	-0.025*** (-3.17)	-0.024*** (-3.08)	-0.023*** (-2.94)	-0.023*** (-3.04)	-0.025*** (-3.18)	-0.026*** (-3.24)	-0.025*** (-3.16)	-0.022*** (-2.99)
Beta	-0.003 (-0.91)	-0.003 (-1.06)	-0.002 (-0.85)	-0.002 (-0.69)	-0.003 (-0.94)	-0.004 (-1.29)	-0.004 (-1.11)	-0.003 (-0.97)	-0.003 (-1.18)
Book-to-market	0.001 (0.47)	0.001 (0.53)	0.002 (0.93)	<0.001 (0.22)	0.001 (0.46)	>0.001 (0.01)	0.001 (0.36)	0.001 (0.55)	<0.001 (0.13)
ln(Size)	>-0.001 (-0.50)	-0.001 (-0.96)	>-0.001 (-0.38)	-0.001 (-1.03)	>-0.001 (-0.22)	-0.001 (-1.44)	<0.001 (-0.50)	<0.001 (-0.27)	-0.052** (-2.56)
Momentum	-0.004 (-1.24)	-0.004 (-1.19)	-0.004 (-1.30)	-0.003 (-1.14)	-0.004 (-1.38)	-0.004 (-1.20)	-0.004 (-1.18)	-0.004 (-1.26)	-0.004 (-1.32)
Short-term reversal	-0.007 (-1.65)	-0.007 (-1.60)	-0.007* (-1.75)	-0.006 (-1.54)	-0.007* (-1.72)	-0.006 (-1.54)	-0.006 (-1.57)	-0.007 (-1.64)	-0.007* (-1.74)
ln(Age)	0.001 (-1.23)								>-0.001 (-0.65)
Analyst coverage		<0.001* (1.77)							<0.001* (1.86)
Forecast dispersion			-0.346* (-1.95)						-0.319** (-2.26)
Volatility				-0.171 (-1.57)					-0.095 (-0.98)
Private information					-0.062** (-2.19)				-0.060** (-2.28)
Leverage						-0.001 (-0.33)			-0.002 (-0.57)
Stock price delay							-0.003 (-1.05)		0.001 (0.33)
Short sale constraints								<0.001*** (3.35)	<0.001*** (3.26)
Adjusted $R^2$	0.084	0.084	0.085	0.090	0.086	0.085	0.084	0.082	0.105
Observations	178,147	178,147	178,147	178,147	178,147	178,147	178,147	178,147	178,147

Table 6: Robustness checks

This table reports robustness results on the vol-of-vol effect. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. We use a one trading day implementation lag and after value-weighting stocks in each portfolio. Panel (a) presents performance measures of tercile portfolios (Low, 2, High) constructed by sorting a sample of American deposit receipts (ADRs) on vol-of-vol following the procedure described in Table 3. Panel (b) presents the results of two additional robustness checks. The top row re-states the results from Table 3. The row labeled "No implied volatility scaling" presents excess returns (Excess return) and four-factor alphas (4F alpha) when vol-of-vol is not scaled by average implied volatility, as in Eq. 4. The row labeled "Industry neutral" presents these performance measures after constructing industry-neutral vol-of-vol portfolios. We report  $t$ -statistics in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Equal-weighted portfolio sorts on ADRs

	Low	2	High	High-Low
Excess return	2.09 (3.53)	1.25 (2.00)	0.89 (1.52)	-1.20 (-4.66)
CAPM alpha	2.01 (3.62)	1.18 (1.99)	0.84 (1.50)	-1.17 (-4.56)
3F alpha	1.80 (3.06)	0.93 (1.50)	0.64 (1.08)	-1.16 (-4.45)
4F alpha	1.85 (3.08)	0.98 (1.53)	0.71 (1.18)	-1.14 (-4.24)

Panel (b): Selection of other robustness checks

	Excess return			4F alpha		
	Low	High	High-Low	Low	High	High-Low
Vol-of-vol effect (Table 3)	0.59 (1.57)	-0.26 (-0.53)	-0.85*** (-2.83)	0.20 (1.36)	-0.49*** (-2.76)	-0.69** (-2.39)
No implied volatility scaling	0.45 (1.50)	-0.53 (-0.74)	-0.98** (-2.24)	0.11 (0.82)	-0.87*** (-3.55)	-0.98*** (-2.83)
Industry neutral portfolios	0.67* (1.79)	0.04 (0.11)	-0.63*** (-4.26)	0.26** (2.21)	-0.34*** (-3.04)	-0.60*** (-4.00)

Table 7: Can deviations from fundamental value explain low returns on high vol-of-vol stocks? This table reports average monthly excess returns and four-factor alphas of portfolios sorted on short sale constraints or arbitrage risk and vol-of-vol, over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Short-sale constraints are proxied by residual institutional ownership and arbitrage risk is proxied by idiosyncratic volatility. Both are defined in the Appendix. Each month we sort stocks in ascending order into quintile portfolios on the basis of short-sale constraints (Low short-sale constraints, 2, 3, 4, High short-sale constraints; see panel (a)) or arbitrage risk (Low arbitrage risk, 2, 3, 4, High arbitrage risk; see panel (b)). Within each quintile, we further sort stocks into five additional portfolios based on vol-of-vol (Low, 2, 3, 4, High). We use a one-trading day implementation lag and value-weight stocks in each portfolio. The table presents average excess returns of the twenty-five resulting portfolios, as well as the difference in monthly returns between portfolio High and portfolio Low (High-Low). The columns labeled "High-Low (4F alpha)" present the High-Low difference in four-factor alphas. We report *t*-statistics in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Short-sale constraints

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Low short-sale constraints	0.13 (0.31)	0.40 (0.83)	0.26 (0.55)	-0.16 (-0.32)	-0.25 (-0.50)	-0.38 (-1.07)	-0.40 (-1.06)
2	0.35 (0.88)	0.58 (1.45)	0.21 (0.44)	0.00 (0.01)	-0.24 (-0.45)	-0.60 (-1.47)	-0.41 (-1.02)
3	0.66* (1.65)	0.44 (1.11)	0.77* (1.70)	0.28 (0.58)	-0.03 (-0.06)	-0.69** (-2.05)	-0.63* (-1.91)
4	1.05** (2.21)	0.86* (1.71)	1.18** (2.41)	0.71 (1.39)	0.06 (0.11)	-0.98** (-2.29)	-0.88** (-2.15)
High short-sale constraints	0.85** (1.99)	0.20 (0.44)	0.05 (0.10)	-0.12 (-0.21)	-0.83 (-1.37)	-1.68*** (-3.66)	-1.51*** (-3.40)
High-Low	0.71** (2.28)	-0.20 (-0.56)	-0.21 (-0.57)	0.04 (0.11)	-0.58 (-1.52)	-1.29*** (-2.75)	-1.11** (-2.36)

(b) Arbitrage risk

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Low arbitrage risk	0.57* (1.82)	0.33 (0.97)	0.82** (2.43)	0.36 (0.98)	-0.23 (-0.64)	-0.80*** (-2.72)	-0.80*** (-2.71)
2	0.46 (1.22)	0.66 (1.52)	0.62 (1.52)	0.10 (0.21)	-0.02 (-0.04)	-0.48 (-1.20)	-0.34 (-0.78)
3	0.80 (1.49)	1.01* (1.83)	0.41 (0.70)	0.19 (0.30)	-0.58 (-0.97)	-1.38*** (-3.24)	-1.14*** (-2.61)
4	0.34 (0.46)	0.36 (0.47)	0.11 (0.14)	0.37 (0.47)	-0.84 (-0.99)	-1.19** (-2.60)	-1.20*** (-2.69)
High arbitrage risk	-0.70 (-0.74)	-0.66 (-0.67)	-0.45 (-0.47)	-0.80 (-0.74)	-1.18 (-1.10)	-0.47 (-0.76)	-0.30 (-0.49)
High-Low	-1.27 (-1.45)	-0.99 (-1.17)	-1.27 (-1.48)	-1.16 (-1.23)	-0.94 (-1.04)	0.33 (0.47)	0.50 (0.77)



Table 8: Can volatility risk exposure or jump risk exposure explain low returns on high vol-of-vol stocks? This table reports average monthly excess returns and four-factor alphas of portfolios sorted on exposures to jump risk or volatility risk and vol-of-vol, over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios on the basis of jump risk exposure or volatility risk exposure. We use two proxies for volatility risk exposure and two proxies for jump risk exposure, each of which is defined in the Appendix. Within each quintile, we sort stocks into five additional portfolios based on vol-of-vol (Low, 2, 3, 4, High). We use a one trading day implementation lag and value-weight stocks in each portfolio. The table presents the excess return of each vol-of-vol quintile over the subsequent month, averaged over the five volatility risk exposure or jump risk exposure portfolios. It also presents the difference between portfolio High and portfolio Low in excess returns (High-Low) and in four-factor alphas (4F alpha)). We report  $t$ -statistics in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Volatility risk exposure

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
S&P500 straddle betas	0.49 (1.19)	0.44 (1.08)	0.37 (0.86)	0.17 (0.36)	-0.19 (-0.38)	-0.67*** (-2.82)	-0.56** (-2.46)
$\Delta$ VIX betas	0.43 (1.07)	0.39 (0.95)	0.38 (0.90)	0.19 (0.41)	-0.21 (-0.41)	-0.64*** (-2.69)	-0.53** (-2.28)

(b) Jump risk exposure

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
$\Delta$ option skew betas	0.51 (1.30)	0.46 (1.18)	0.28 (0.65)	0.22 (0.49)	-0.34 (-0.68)	-0.86*** (-3.04)	-0.73*** (-2.66)
OTM put betas	0.58 (1.45)	0.45 (1.13)	0.38 (0.89)	0.20 (0.44)	-0.36 (-0.70)	-0.94*** (-3.37)	-0.82*** (-2.95)

Table 9: Empirical test of vol-of-vol as a priced risk factor

This table presents test results on whether exposures to a vol-of-vol factor explain stock returns during our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks into quintiles on the basis of vol-of-vol using a one trading day implementation lag and value-weighting stocks in each portfolio. We construct a vol-of-vol factor from the difference between the High and the Low vol-of-vol portfolio. Next, requiring at least 12 degrees of freedom, we measure exposure to the vol-of-vol factor as the sum the coefficients  $\beta_t^V + \beta_{t-1}^V$  from the following regression run over the past year:

$$r_{i,t} - r_t^f = \alpha + \beta_t^V r_t^V + \beta_{t-1}^V r_{t-1}^V + \beta_t^M (r_t^M - r_t^f) + \beta_{t-1}^M (r_{t-1}^M - r_{t-1}^f),$$

where  $r_{i,t}$  is the daily return of stock  $i$ ,  $r_t^f$  is the daily risk-free rate,  $r_t^V$  is the daily return on the vol-of-vol factor, and  $r_t^M$  is the daily return on the market. Panel (a) reports the results of the single-sort portfolio analysis. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, High) on the basis of the vol-of-vol characteristic as in Table 3 (Vol-of-vol characteristic), or on the basis of the estimated vol-of-vol exposure  $\beta_{t-1}^V + \beta_{t-2}^V$  (Vol-of-vol beta). The table reports average excess returns of each portfolio over the subsequent month, as well as the difference in returns between portfolio High and portfolio Low (High-Low). The columns labeled "High-Low (4F alpha)" present the difference in four-factor alphas between portfolio High and portfolio Low. Panel (b) reports the results of the double sorts analysis. Each month we sort stocks in ascending order into quintile portfolios on the basis of the vol-of-vol characteristic. Within each quintile, we sort stocks into five additional portfolios based on the vol-of-vol beta. The row labeled "Average excess returns" presents the monthly excess return of each vol-of-vol beta quintile, averaged over the five vol-of-vol characteristic-sorted portfolios. The rows labeled "Portfolio averages" report the average *ex ante* vol-of-vol beta, the average *ex post* vol-of-vol beta, and the average *vol-of-vol* characteristic of each vol-of-vol beta quintile. We report *t*-statistics in parentheses that are Newey-West corrected. \*, \*\* and \*\*\* indicate significance at the 10%, 5%, and 1% level, respectively.

(a) Single sorts analysis

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Vol-of-vol characteristic	0.59 (1.57)	0.41 (1.11)	0.44 (1.09)	0.14 (0.34)	-0.26 (-0.53)	-0.85*** (-2.83)	-0.69** (-2.39)
Vol-of-vol beta	0.63 (1.70)	0.36 (1.08)	0.43 (1.12)	0.22 (0.46)	-0.27 (-0.38)	-0.90 (-1.40)	-0.68 (-1.31)

(b) Double sorts analysis

	Low	2	3	4	High	High-Low	High-Low (4F alpha)
Portfolio performance							
Vol-of-vol beta	0.62 (1.13)	0.31 (0.80)	0.32 (0.80)	0.10 (0.24)	-0.09 (-0.14)	-0.70 (-1.01)	-0.35 (-0.67)
Portfolio characteristics							
<i>Ex ante</i> vol-of-vol beta	-0.59	-0.19	0.07	0.32	0.84	1.43	
<i>Ex post</i> vol-of-vol beta	-0.26	-0.08	0.00	0.15	0.46	0.72	
Vol-of-vol characteristic	0.08	0.09	0.09	0.09	0.09	0.00	

Table 10: Distribution of monthly stock returns associated with varying levels of vol-of-vol

This table presents distribution characteristics for portfolios sorted on vol-of-vol over our sample period from January 1996 to October 2009. Vol-of-vol is past month's volatility of option-implied volatility (IV), standardized by average IV (see Section 2.2). IV is calculated from at-the-money call and put options with maturity closest to 30 days. Each month we sort stocks in ascending order into quintile portfolios (Low, 2, 3, 4, and High) on the basis of vol-of-vol. We use a one trading day implementation lag and value-weight stocks in each portfolio. Panel (a) presents the time-series average of the value-weighted cross-sectional mean (Avg), standard deviation (Std), minimum (Min), percentiles (P1-P99), and maximum (Max) of individual stock returns within each vol-of-vol portfolio, as well as the difference in returns between portfolio High and portfolio Low (High-Low). Panel (b) presents the mean (Avg), standard deviation (Std), minimum (Min), percentiles (P1-P99), and maximum (Max) of the time-series of returns on the vol-of-vol quintile portfolios, as well as the difference in returns between portfolio High and portfolio Low (High-Low). The rows labeled "Hedge" describe the returns distribution of the High-Low vol-of-vol portfolio (that buys the top quintile and sells the bottom quintile).

Panel (a): Cross-sectional distribution of individual stock returns

	Avg	Std	Min	P1	P5	P10	P25	P50	P75	P90	P95	P99	Max
Low	0.59	12.60	-41.20	-21.04	-12.44	-9.03	-4.21	0.46	5.35	10.52	13.96	23.30	50.76
2	0.41	12.46	-42.21	-21.01	-12.13	-9.11	-4.27	0.31	5.07	9.85	13.46	22.47	51.18
3	0.44	12.57	-43.66	-20.63	-12.47	-9.07	-4.29	0.39	5.20	10.02	13.46	22.38	52.56
4	0.14	12.60	-44.15	-20.63	-12.71	-9.63	-4.82	0.03	5.01	10.23	14.01	22.50	54.70
High	-0.26	12.96	-48.45	-23.70	-13.78	-10.23	-5.45	-0.29	4.96	10.23	14.30	23.26	57.11
High-Low	-0.85	0.36	-7.25	-2.66	-1.34	-1.20	-1.23	-0.75	-0.39	-0.28	0.34	-0.04	6.35

Panel (b): Time-series distribution of portfolio returns

	Avg	Std	Min	P1	P5	P10	P25	P50	P75	P90	P95	P99	Max
Low	0.59	4.63	-18.66	-12.47	-6.41	-4.73	-1.99	1.11	4.51	6.15	7.72	9.22	10.39
2	0.41	4.57	-16.21	-10.92	-7.98	-4.90	-2.17	1.50	3.42	6.42	7.57	9.21	9.86
3	0.44	4.98	-17.96	-15.19	-8.41	-6.19	-1.92	0.66	4.02	6.70	7.99	9.75	11.58
4	0.14	5.20	-16.40	-13.94	-9.25	-6.64	-2.39	0.94	3.71	6.90	7.60	10.29	10.59
High	-0.26	5.89	-22.00	-17.82	-10.85	-7.15	-2.89	0.73	3.53	6.08	8.20	11.95	17.47
High-Low	-0.85	1.26	-3.35	-5.35	-4.44	-2.42	-0.90	-0.38	-0.98	-0.07	0.49	2.73	7.07
Hedge	-0.85	3.93	-19.50	-16.22	-6.21	-4.27	-2.57	-0.82	0.94	2.97	3.62	10.63	14.85