Does Realized Skewness Predict the Cross-Section of Equity Returns?*

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Abstract

We use intraday data to compute weekly realized variance, skewness, and kurtosis for equity returns and study the realized moments' time-series and cross-sectional properties. We investigate if this week's realized moments are informative for the cross-section of next week's stock returns. We find a very strong negative relationship between realized skewness and next week's stock returns. A trading strategy that buys stocks in the lowest realized skewness decile and sells stocks in the highest realized skewness decile generates an average weekly return of 19 basis points with a t-statistic of 3.70. Our results on realized skewness are robust across a wide variety of implementations, sample periods, portfolio weightings, and firm characteristics, and are not captured by the Fama-French and Carhart factors. We find some evidence that the relationship between realized kurtosis and next week's stock returns is positive, but the evidence is not always robust and statistically significant. We do not find a strong relationship between realized volatility and next week's stock returns.

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Keywords: Realized volatility; skewness; kurtosis; equity markets; cross-section of stock returns.

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1 Introduction

We examine the properties of higher moments computed from intraday returns. Merton (1980) first noted that volatility can be measured arbitrarily precisely as the sampling frequency increases. This insight was later applied to the measurement of time-varying volatility in the realized volatility literature, which constructs daily measures of realized volatility computed from intraday squared returns. Extending the now well-established concept of realized volatility, we compute realized skewness and kurtosis from intraday cubed and quartic returns. We show that under realistic assumptions, based on a continuous-time specification of equity price dynamics that includes stochastic volatility and jumps, the realized moments converge to well-defined limits. The limits of the higher realized moments are determined by the jump parameters of the continuous-time price process. Using Monte Carlo techniques, we verify that the measurement of the realized higher moments is reliable in finite samples, and that it is robust to the presence of market microstructure noise as well as to quote discontinuities in existence prior to decimalization.

Our empirical strategy uses an extensive sample of weekly data. We aggregate daily realized moments to obtain weekly realized volatility, skewness, and kurtosis measures for over three million firm-week observations. We find considerable time-variation in the cross-sectional percentiles. While the cross-sectional dispersion in realized volatility has decreased during our sample period, the cross-sectional dispersion in realized skewness and kurtosis has increased. The median cross-sectional kurtosis has also increased significantly.

Next we examine the relationship between higher moments computed from intraday returns and future stock returns. We sort stocks into deciles based on the current-week realized moment and compute the subsequent one-week return of the trading strategy that buys the portfolio of stocks with a high realized moment (volatility, skewness, or kurtosis) and sells the portfolio of stocks with a low realized moment.

When sorting on realized volatility, the resulting portfolio return differences are not statistically significant. However, when sorting by realized skewness, the long-short value-weighted portfolio produces an average weekly return of -19 basis points with a t-statistic of -3.70. The resulting four factor Carhart risk adjusted alpha for the long-short skewness portfolio is also close to -19 basis points per week. We find a positive relation between realized kurtosis and subsequent stock returns, but the economic magnitude is smaller and the results are less significant.

We confirm the negative relation between realized skewness and future returns using Fama-MacBeth regressions. We also investigate the robustness of these findings when controlling for a number of well-documented determinants of returns: firm size (Fama and French (1993)), the book-to-market ratio (Fama and French (1993)), market beta, lagged return (Jegadeesh (1990), Lehmann (1990) and Gutierrez and Kelley (2008)), historical skewness, idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), coskewness (Harvey and Siddique (2000)), realized volatility and kurtosis, maximum return (Bali, Cakici, and Whitelaw (2009)), the number of analysts that follow the firm

(Arbel and Strebel (1982)), illiquidity (Amihud (2002)), and the number of intraday transactions. Realized skewness continues to be highly significant in explaining the cross section of returns after controlling for these factors. Finally, results for realized skewness are robust to the January effect and are significant when considering only NYSE stocks. We also show that the cross-sectional results obtain for alternative holding periods.

The positive relation between realized kurtosis and future returns is also confirmed using Fama-MacBeth regressions. However, robustness exercises indicate that the results for realized kurtosis are not as economically and statistically significant as the results for realized skewness.

To verify that our measures of higher moments are not contaminated by microstructure noise, and to make sure that we are effectively measuring asymmetry and fat tails, we investigate four additional measures of skewness and kurtosis using high frequency data. In the first measure, the return drift is removed from the realized moments. A second measure uses jump-robust estimates of realized volatility to compute higher moments. The third measure is an enhanced version of the realized moment that uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005) to compute realized volatility. This subsampling methodology ensures that useful data are not ignored and provides a more robust estimator of the realized moment. The fourth approach uses percentiles of the high-frequency return distribution as alternative measures to capture skewness and kurtosis.

We find that the relation between realized kurtosis and stock returns is not always robust to different implementations. However, the negative relation between realized skewness and future stock returns is robust. We show that our realized skewness measure captures jumps in returns, and therefore has a different information content than historical skewness measures based on daily returns, which also capture the diffusive part of returns.

Finally we further investigate the relation between realized skewness and other documented determinants of returns. Two-way sorts on realized skewness and firm characteristics also confirm that the relationship between realized skewness and returns is significant. We pay particular attention to the relationship between realized skewness computed from intraday returns during the week and the total stock return for the week. The short-term return reversal effect is well-documented in the literature,[1] and while we find that realized skewness and weekly return are related, their effects on the subsequent week's return are different. Furthermore, when we compute realized skewness adjusting for the weekly return drift, the subsequent week's cross-sectional pattern in returns is preserved. When constructing a portfolio that combines the reversal strategy and the realized skewness strategy, the Sharpe ratio is generally larger than for the two individual strategies and the tail risk for the combined portfolio is much lower than for the reversal strategy alone.

Ang, Hodrick, Xing, and Zhang (2006) find that stocks with high idiosyncratic volatility earn low returns. Motivated by their findings, we also explore the relationship between realized skewness,

[1] Return reversals at the daily, weekly and monthly frequencies have been studied by Lehmann (1990), Jegadeesh (1990), Cox and Peterson (1994), Avramov, Chordia, and Goyal (2006), and Gutierrez and Kelley (2008) among others.

idiosyncratic volatility and subsequent stock returns. We find that when idiosyncratic volatility increases, low skewness stocks are compensated with higher returns while high skewness stocks are compensated with lower returns. This pattern is stronger for small stocks. Therefore, skewness provides a partial explanation of the idiosyncratic volatility puzzle. Similar findings obtain when using realized volatility instead of idiosyncratic volatility.

Our results build on the voluminous econometric literature on realized volatility, which is succinctly surveyed in Andersen and Benzoni (2009). Schwert (1989) and Paye (2012) use daily returns to construct monthly and quarterly realized volatility and investigate economic drivers of the secular variation in market volatility. Hsieh (1991) contains perhaps the first application of realized volatility in finance using intraday data, and Andersen, Bollerslev, Diebold, and Ebens (2001) are among the first to the study realized volatility of individual equities. Fleming, Kirby, and Ostdiek (2003) are the first to apply realized volatility and correlation from intraday data in portfolio allocation. Recently, Bollerslev, Osterrieder, Sizova, and Tauchen (2013) have used realized volatility to assess the risk-return relationship and to forecast future market returns, and Corsi, Fusari, and La Vecchia (2013) use realized volatility to develop a new class of option valuation models. Our paper is the first to exploit the information in high-frequency data to compute firm-level realized skewness and kurtosis as well as realized volatility. We also use these measures in a large-scale analysis of the cross-section of equity returns, using a cross-section of more than two thousand firms each week during our 21-year sample.

Our work also builds on existing studies of the theoretical relationship between higher moments and stock returns. Kraus and Litzenberger (1976) show theoretically that coskewness is a determinant of the cross-section of stock returns. Different theoretical arguments suggest that assets' (idiosyncratic) skewness may explain asset returns. Barberis and Huang (2008) demonstrate that assets with greater skewness have lower returns when investors make decisions according to cumulative prospect theory.[2]

The remainder of the paper is organized as follows. Section 2 estimates the weekly realized higher moments from intraday returns and constructs portfolios based on these moments. It also investigates the limiting properties of the realized higher moments. Section 3 computes raw and risk-adjusted returns on portfolios sorted on realized volatility, skewness, and kurtosis, and estimates Fama-MacBeth regressions including various control variables. Section 4 contains a series of robustness checks. Section 5 further explores the results by investigating double sorts on higher moments and other firm characteristics, and by documenting the interaction of volatility, skewness, and returns as well as the interaction of lagged returns, skewness, and returns. Section 6 concludes.

[2] See also Mitton and Vorkink (2007), Brunnermeier, Gollier, and Parker (2007), and Dittmar (2002).

2 Constructing Moment-Based Portfolios

In this section we first describe the data. We then show how the realized higher moments are computed and we investigate their limiting properties. Finally, we form portfolios by sorting stocks into deciles based on the weekly realized moments, and then report on the characteristics of these portfolios.

2.1 Data

We analyze every listed stock in the Trade and Quote (TAQ) database from January 4, 1993 to December 31, 2013. TAQ provides historical tick by tick data for all stocks listed on the New York Stock Exchange, American Stock Exchange, Nasdaq National Market System, and SmallCap issues. Stocks with prices below $5 are excluded from the analysis. We record prices every five minutes starting at 9:30 EST and construct five-minute log-returns for the period 9:30 EST to 16:00 EST for a total of 78 daily returns. We construct the five minute grid by using the last recorded price within the preceding five-minute period. If there is no price in a period, the return for that period is set to zero.

To ensure sufficient liquidity, we require that a stock has at least 80 daily transactions to construct a daily measure of realized moments.[3] The average number of intraday transactions per day for a stock is over two thousand. The weekly realized moment estimator is the average of the available daily estimators (Wednesday through Tuesday). Only one valid day of the realized moment is required to have a weekly estimator.

We use data from three additional databases. From the Center for Research and Security Prices (CRSP) database, we use daily returns of each firm to calculate weekly returns (from Tuesday close to Tuesday close), historical equity skewness, market beta, lagged return, idiosyncratic volatility, maximum return over the previous week and month, and illiquidity; we use monthly returns to compute coskewness as in Harvey and Siddique (2000);[4] we use daily volume to compute illiquidity; and we use outstanding shares and stock prices to compute market capitalization. COMPUSTAT is used to extract the Standard and Poor's issuer credit ratings and book values to calculate book-to-market ratios of individual firms. From Thomson Returns Institutional Brokers Estimate System (I/B/E/S), we obtain the number of analysts that follow each individual firm. These variables are discussed in more detail in Appendix A.

2.2 Computing Realized Higher Moments

We first define the intraday log returns for each firm. On day t, the ith intraday return is given by

$$r_{t,i} = p_{t,\frac{i}{N}} - p_{t,\frac{(i-1)}{N}}, \tag{1}$$

[3] We repeated the analysis using a minimum of 100, 250, and 500 transactions instead. The results are similar.

[4] Computing co-moments with high-frequency data is not straightforward due to synchronicity problems between stock and index returns.

where p is the natural logarithm of the price and N is the number of return observations in a trading day. The opening log price on day t is $p_{t,0}$ and the closing log price on day t is $p_{t,1}$. We use five-minute returns so that in 6.5 trading hours we have $N = 78$.

The well-known daily realized variance (Andersen and Bollerslev (1998) and Andersen, Bollerslev, Diebold and Labys (2003)) is obtained by summing squares of intraday high-frequency returns

$$RDVar_t = \sum\nolimits_{i=1}^{N} r_{t,i}^2. \tag{2}$$

As is standard, we do not estimate the mean of the high-frequency return because it is dominated by the variance at this frequency. See Section 4.1 for a robustness check.

Its model-free nature is an appealing characteristic of this volatility measure compared to other estimation methods (see Andersen, Bollerslev, Diebold, and Labys (2001) and Barndorff-Nielsen and Shephard (2002) for details). Moreover, as we will discuss below, realized variance converges to a well-defined quadratic variation limit as the sampling frequency N increases.

Given that we are interested in measuring the asymmetry of the daily return's distribution, we construct a measure of ex-post realized daily skewness based on intraday returns standardized by the realized variance as follows

$$RDSkew_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RDVar_t^{3/2}}. \tag{3}$$

The interpretation of this measure is straightforward: negative values indicate that the stock's return distribution has a left tail that is fatter than the right tail, and positive values indicate the opposite.

We are interested in extremes of the return distribution more generally, and so we also construct a measure of realized daily kurtosis defined by

$$RDKurt_t = \frac{N\sum_{i=1}^{N} r_{t,i}^4}{RDVar_t^2}. \tag{4}$$

The limits of the third and fourth moment when the sampling frequency N increases will be analyzed below as well. The scaling of $RDskew_t$ and $RDKurt_t$ by $\sqrt{N}$ and N ensures that their magnitudes correspond to daily skewness and kurtosis.

Our cross-sectional asset pricing analysis below is conducted at the weekly frequency. We therefore construct weekly realized moments from their daily counterparts as follows. If t is a Tuesday then we compute

$$RVol_t = \left(\frac{252}{5}\sum\nolimits_{i=0}^{4} RDVar_{t-i}\right)^{1/2}, \tag{5}$$

$$RSkew_t = \frac{1}{5}\sum\nolimits_{i=0}^{4} RDSkew_{t-i}, \tag{6}$$

$$RKurt_t = \frac{1}{5}\sum\nolimits_{i=0}^{4} RDKurt_{t-i}. \tag{7}$$

Our cross-sectional analysis below is conducted at the weekly frequency and t will therefore denote a week from this point on. Each trading day has 78 five-minute intervals so that the weekly realized moments for each firm are based on 390 observations. Note that, as is standard, we have annualized the realized volatility measure to facilitate the interpretation of results.

We compute $RVol_t$, $RSkew_t$, and $RKurt_t$ for more than three million firm-week observations during our January 1993 to December 2013 sample period. Figure 1 summarizes the realized moments. The top-left panel of Figure 1 displays a histogram of the realized volatility measure pooled across firms and weeks. As often found in the realized volatility literature, the unconditional distribution of realized equity volatility is right-skewed. The top-right panel in Figure 1 shows the time-variation in the cross-sectional percentiles using three-month moving averages. The cross-sectional dispersion in realized equity volatility is clearly not constant over time and seems to have decreased through our sample period.

The middle-left panel of Figure 1 shows the histogram of realized equity skewness. The skewness distribution is very fat-tailed and strongly peaked around zero. The middle-right panel of Figure 1 shows the time-variation in the cross-sectional skewness percentiles. The cross-sectional dispersion in realized equity skewness has increased through our sample.

The bottom-left panel of Figure 1 shows the histogram of realized equity kurtosis. Similar to realized volatility, realized kurtosis appears to be approximately log-normally distributed. The vast majority of our sample has a kurtosis above 3, strongly suggesting fat-tailed returns. The bottom-right panel of Figure 1 shows that the cross-sectional distribution of realized equity kurtosis has become more dispersed over time, matching the result found for realized skewness.

Figure 1 reveals interesting patterns in the cross-sectional distribution of higher moments over time. We attempt to exploit these patterns for cross-sectional equity pricing in Section 3. First, in order to anchor our analysis, we investigate the limiting properties of higher order moments computed from high-frequency data.

2.3 Limiting Properties of Realized Moments

The limiting properties of realized variance have been studied in detail in the econometrics literature, however, much less is known about realized skewness and kurtosis. In this section, we therefore investigate the realized moments under the assumption that the underlying continuous time price process follows a jump-diffusion process with stochastic volatility.

Assume that the log-price p of a security evolves according to the stochastic equation

$$p_T = \int_0^T \mu_s ds + \int_0^T \sigma_s dW_s + J_T, \tag{8}$$

where μ is a locally bounded predictable drift process, σ is a positive càdlàg process, and J is a pure jump process, and where without loss of generality $p_0 = 0$.

Andersen, Bollerslev, Diebold and Labys (2003) show the following general limit result as the sampling frequency N goes to infinity for realized variance from time 0 to T

$$RM(2) \equiv \sum_{i=1}^{N} \left(p_{T\frac{i}{N}} - p_{T\frac{(i-1)}{N}}\right)^2 \xrightarrow{\mathcal{P}} \int_0^T \sigma_s^2 ds + \sum_{0<s\leq T} (\Delta p_s)^2 \,, \text{ as } N \to \infty. \tag{9}$$

where $\Delta p_s = p_s - p_{s-}$ denotes the size of the jump at time s. This limit as the sampling frequency goes to infinity contains two elements: integrated variance as well as the sum of squared jumps. We see that the simple realized variance based on five minute returns in equation (2) contains both volatility and jumps in the limit. We will consider jump-robust measures of volatility in Section 4.2.

For the realized third moment, the results in Barndorff-Nielsen, Kinnebrock, and Shephard (2010) and Jacod (2012) can be used to show that

$$RM(3) \equiv \sum_{i=1}^{N} \left(p_{T\frac{i}{N}} - p_{T\frac{(i-1)}{N}}\right)^3 \xrightarrow{\mathcal{P}} \sum_{0<s\leq T} (\Delta p_s)^3 \,, \text{ as } N \to \infty. \tag{10}$$

This result is important in two respects: First, it shows that the realized third moment in the limit separates the jump contribution from the continuous contribution to cubic variation and it captures just the jump part. It does not for example capture skewness arising from correlation between return and variance innovations (the "leverage" effect). Second, it shows that the sign of the average jump size is captured: Firms with (on average) positive jumps will have a positive realized third moment and vice versa.

For the realized fourth moment, Barndorff-Nielsen and Shephard (2004) and Jacod (2012) show that

$$RM(4) \equiv \sum_{i=1}^{N} \left(p_{T\frac{i}{N}} - p_{T\frac{i-1}{N}}\right)^4 \xrightarrow{\mathcal{P}} \sum_{0<s\leq T} (\Delta p_s)^4 \,, \text{ as } N \to \infty. \tag{11}$$

We see again that the fourth realized moment in the limit captures the jump component but not the continuous component of quartic variation. The continuous component could for example arise from volatility of volatility in the price process. The fourth power in the limit expression of course implies that positive and negative jumps are treated equally so that only the magnitude of jumps is captured, and not their direction.

We conclude from these general results for the third and fourth realized moments that we may expect very different estimates of skewness and kurtosis depending on the frequency of data used to estimate these moments. Skewness estimates from moving windows of daily or weekly data are likely to have different averages than skewness measures constructed from intraday data. This is in contrast to the second moment, where using higher-frequency returns yields more and more efficient estimates of quadratic variation.

To fix ideas, consider an example where the log price of a security evolves according to the particular affine stochastic differential equation

$$dp_t = \left(\mu - \tfrac{1}{2}V_t - \bar{\mu}_J \lambda\right) dt + \sqrt{V_t} dW_t^{(1)} + J dN_t, \tag{12}$$
$$dV_t = \kappa\left(\theta - V_t\right) dt + \sigma\sqrt{V_t} dW_t^{(2)}, \tag{13}$$

where μ is the drift parameter, κ is the mean reversion speed to the long-term volatility mean θ, and σ is the diffusion coefficient of the volatility process V_t. $W_t^{(1)}$ and $W_t^{(2)}$ denote two standard Brownian motions with correlation ρ, and N_t is an independent Poisson process with arrival rate λ. The jump size J is distributed $N\left(\mu_J, \sigma_J^2\right)$. The value for $\bar{\mu}_J$ is set to $\exp\left(\mu_J + \sigma_J^2/2\right) - 1$.

For this process, the limits of the expected value of the realized moments are given by

$$IM(2) \equiv \lim_{N\to\infty} E_0\left[RM(2)\right] = \theta T - \left(\theta - V_0\right)\frac{\left(1 - e^{-\kappa T}\right)}{\kappa} + \lambda\left(\mu_J^2 + \sigma_J^2\right) T \tag{14}$$
$$IM(3) \equiv \lim_{N\to\infty} E_0\left[RM(3)\right] = \lambda\left(\mu_J^3 + 3\mu_J\sigma_J^2\right) T, \tag{15}$$
$$IM(4) \equiv \lim_{N\to\infty} E_0\left[RM(4)\right] = \lambda\left(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4\right) T. \tag{16}$$

This result can be derived using the transform analysis in Duffie, Pan, and Singleton (2000). Appendix B provides the details.

These results confirm the intuition from the general case. Consistent with equations (9), (10), and (11), while the second moment contains both jump and diffusion parameters, the third and fourth moments depend exclusively on jump parameters. It is important once again to note that whereas realized variance converges to the total quadratic variation of the process, the realized skewness and kurtosis estimates do not converge to the total cubic and quartic variation. Total cubic variation includes diffusive skewness, arising for example from the correlation ρ between the two Brownian innovations. Total quartic variation includes diffusive kurtosis arising from volatility of volatility, which is captured by the parameter σ. In the limit, our measures pick up the contribution to skewness and kurtosis from jumps only. Total cubic and quartic variation can only be estimated from long samples of low-frequency returns or by using options to retrieve risk-neutral moments as in Neuberger (2012) and Conrad, Dittmar, and Ghysels (2013) who rely on the methodology in Bakshi, Kapadia, and Madan (2003). The conclusion is once again that skewness (and kurtosis) measures computed from high-frequency data are likely to contain different information from those computed from daily data or from options. We discuss these differences in detail in Section 5.5.

In Appendix C we provide Monte Carlo evidence on the finite-sample properties of $RM(j)$ in the presence of market microstructure noise and discontinuities in quoted prices. The results show that the finite-sample moment estimates are well behaved.

2.4 Portfolio Sort Characteristics

Each Tuesday, we form portfolios by sorting stocks into deciles based on the weekly realized moments. Table 1 reports the time-series sample averages for the moments and different firm characteristics, by decile. Panel A reports the time-series averages for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Column 1 contains the portfolio of stocks with the smallest average realized moment, and column 10 contains the portfolio of stocks with the highest realized moment. The characteristics include firm size, book-to-market ratio, historical skewness using daily returns from the previous month, market beta from the market model regression, lagged return, idiosyncratic volatility as in Ang, Hodrick, Xing, and Zhang (2006), coskewness as in Harvey and Siddique (2000), maximum return of the previous month, illiquidity as in Amihud (2002), number of analysts from I/B/E/S, credit rating, stock price, number of intraday transactions, and number of stocks per decile. On average there are just below 300 companies per decile each week.

Table 1, Panel A displays results for the ten decile portfolios based on realized volatility. Realized volatility increases from 17.3% for the first decile to 128.4% for the highest decile. Interestingly, realized skewness has a negative relation with realized volatility, and realized kurtosis shows an increasing pattern through the volatility deciles. Furthermore, companies with high realized volatility tend to be small, followed by fewer analysts, less coskewed with the market, and they have a lower stock price. A positive relation exists between realized volatility and historical skewness, market beta, lagged return, idiosyncratic volatility and maximum monthly return. Finally, no pattern is observed between realized volatility and book-to-market, number of intraday transactions, and credit rating.

Panel B of Table 1 shows that realized skewness equals -1.16 for the first decile portfolio and 1.13 for the tenth decile. Firms with a high degree of asymmetry, either positive or negative, are small, highly illiquid, followed by fewer analysts, and the number of intraday transactions for these firms is lower.

Panel C of Table 1 reports on the decile portfolios based on realized kurtosis. The average kurtosis ranges from 4.0 to 17.5 across the deciles. Firm characteristics that are positively related to realized kurtosis include realized volatility, historical skewness, lagged return, idiosyncratic volatility, illiquidity and maximum monthly return. Variables that have a negative relation with realized kurtosis include size, market beta, coskewness, number of I/B/E/S analysts, stock price, and number of intraday transactions.

Figure 2 complements Table 1 by reporting the time series of realized higher moments by firm size (Panel A), book-to-market (Panel B), and market beta (Panel C). For each moment we plot the average value for each of the terciles using 3-month moving averages as in Figure 1. Figure 2 clearly shows that the variation in realized volatility (left column) and realized kurtosis (right column) is robustly related to the cross-sectional variation in size, book-to-market, and market beta. Over time, realized volatility tends to be consistently high for firms with small market caps, low book-to-market and high market betas. Realized kurtosis tends to be consistently high for small caps, high book-to-market firms, and low beta firms. While these general patterns may not generally be surprising, the high and increasing realized kurtosis for low beta firms is perhaps not expected.

It may partly explain the high rewards for holding low beta stocks as recently documented by Frazzini and Pedersen (2014). The change in the cross-sectional distribution of realized volatility by market capitalization in the top left panel is interesting as well. In the early part of the sample the volatility of small caps is roughly five times that of large caps, whereas at the end of the sample their volatility levels are almost identical.

In sharp contrast to volatility and kurtosis, the variation in skewness seems largely unrelated to the three well-known determinants of returns studied in Figure 2. These results suggest that skewness may have the most potential of the three moments to serve as a new independent driver of variation in expected return in the cross section. This is the topic to which we now turn.

3 Realized Moments and the Cross-Section of Stock Returns

In this section, we first analyze the relationship between the current week's returns and the previous week's realized volatility, realized skewness, and realized kurtosis. Subsequently, we use the Fama and MacBeth (1973) methodology to conduct cross-sectional regressions, and to determine the significance of each higher realized moment individually and simultaneously. We also use these cross-sectional regressions to control for other firm-specific determinants of returns.

3.1 Sorting Stock Returns on Realized Volatility

Every Tuesday, stocks are ranked into deciles according to their realized volatility. Then, using returns over the following week, we construct value- and equal-weighted portfolios. Table 2, Panel A reports the time series average of weekly returns for decile portfolios based on the level of realized volatility.

The value-weighted returns increase from 22 basis points for decile 1 to 33 basis points for decile 10. Equal-weighted returns increase from 21 basis points to 27 basis points. Thus, the returns of the long-short portfolio, namely one that buys stocks in decile 10 and sells stocks in decile 1, are positive for both value-weighted and equal-weighted portfolios. The positive relation between individual volatility and stock returns is not consistent with Ang, Hodrick, Xing, and Zhang (2006) who use daily data and longer samples to estimate volatility. However, neither our value-weighted nor the equal-weighted long-short portfolio returns nor their alphas are statistically significant. The four factor model employs the three Fama and French (1993) factors (excess market-return, size, and book-to-market) and the Carhart (1997) momentum factor.

We conclude that in our sample, realized volatility and next week's stock returns are not robustly related.

3.2 Sorting Stock Returns on Realized Skewness

Table 2, Panel B reports the time-series average of weekly returns for decile portfolios grouped by realized skewness.

The value-weighted and equal-weighted returns both show a decreasing pattern between realized skewness and the average stock returns over the subsequent week. The return for the portfolio of stocks with the lowest level of skewness is 43 basis points for value-weighted portfolios and 50 basis points for equal-weighted portfolios, while the return for stocks with the highest level of realized skewness is 24 basis points for value-weighted and 16 for equal-weighted portfolios. The weekly return difference between portfolios 10 and 1 is −19 basis points for value-weighted returns and −34 for equal-weighted returns. Both differences are statistically significant at the one percent level. The equal-weighted return difference is larger than the value-weighted return difference, suggesting that the relationship between skewness and subsequent returns is larger for small firms.

We also assess the empirical relationship between realized skewness and stock returns by adjusting for standard measures of risk. The table presents, for each decile, alphas relative to the Carhart four factor model. Note that alphas are large and statistically significant for value- and equal-weighted portfolios across deciles. In addition, the difference between the alphas of the tenth and first deciles is −19 and −34 basis points for value- and equal-weighted portfolios, respectively. Most alphas are estimated to be positive, which may reflect transaction costs (Lesmond, Ogden, and Trzcinka (1999)) or the construction of the factors (Cremers, Petajisto, and Zitzewitz (2012)).

Our long-short skewness returns are also large when compared with the standard four factor returns. In our sample the weekly return on the market factor is 14 basis points per week, the size factor return is 4 basis points, the value factor return is 6 basis points, and momentum yields 13 basis points per week on average (not reported in the tables).

Finally, the economic and statistical significance of the relation between return and realized skewness is not limited to the top and bottom deciles. The table also reports on the economic and statistical significance of the return difference between the ninth and second decile, as well as between the eighth and third decile. In both cases these differences are economically and statistically significant, for returns as well as alphas, and regardless of the weighting scheme. We also construct the Patton and Timmermann (2010) monotonicity tests. We test the null hypothesis of no monotonicity against the alternative hypothesis of returns that monotonically decrease with realized skewness. The p-value associated with this test is essentially zero for both value-weighted and equal-weighted returns, strongly rejecting the null of no monotonicity.

Our findings are consistent with the empirical results in Boyer, Mitton, and Vorkink (2010). They construct a measure of expected idiosyncratic skewness that controls for firm characteristics and find that stocks with higher expected idiosyncratic skewness yield lower future returns. Zhang (2006) measures the skewness for a given firm by the cross-sectional skewness of the firms in that industry, and documents a negative relation between skewness and stock returns. Our four factor Carhart risk adjusted alpha of −19 basis points per week. Boyer, Mitton, and Vorkink (2010) and Zhang (2006) report premiums of −67 and −36 basis points per month, respectively.

Skewness measures extracted from options yield contradictory results on the relation between option implied skewness and future returns in the cross-section. While Xing, Zhang, and Zhao

(2010) and Rehman and Vilkov (2010) document a positive relation, Conrad, Dittmar, and Ghysels (2013) find a negative one. In other related work, Kelly and Jiang (2014) study the relationship between tail estimates and returns. Cremers, Halling, and Weinbaum (2015) study the relationship between returns and aggregate jump risk.

In conclusion, we find strong evidence of a negative cross-sectional relationship between realized skewness and future stock returns. Realized skewness is an important determinant of the cross-sectional variation in subsequent one-week returns, and its effect is not captured by standard measures of risk.

3.3 Sorting Stock Returns on Realized Kurtosis

Panel C of Table 2 documents the average next-week stock returns for decile portfolios based on realized kurtosis. Value-weighted and equal-weighted portfolio returns both increase with the level of realized kurtosis. For value-weighted portfolios, decile 1 has an average weekly return of 22 basis points, compared to 32 basis points for decile 10. Thus, the long-short portfolio generates a return of 10 basis points with a t-statistic of 2.01. A similar result is found for the equal-weighted portfolio, where the long-short realized kurtosis premium equals 12 basis points with a t-statistic of 2.75.

The estimates for the Carhart four-factor alpha are smaller and less statistically significant compared to those for the raw returns. The value-weighted alpha for the long-short portfolio is 9 basis points and the equal-weighted alpha is 11 basis points. The equal-weighted alpha is significant at the 5% level, while the value-weighted alpha is significant at the 10% level.

Comparing Panels A, B, and C of Table 2, we conclude that, while the evidence suggests a positive relation between realized kurtosis and returns, realized skewness appears to be the most reliable moment-based indicator of subsequent one-week equity returns in the cross section.

3.4 Fama-MacBeth Regressions

To further assess the relationship between future returns and realized volatility, realized skewness, and realized kurtosis, we carry out various cross-sectional regressions using the method proposed in Fama and MacBeth (1973). Each week t, we compute the realized moments for firm i and estimate the following cross-sectional regression:

$$r_{i,t+1} = \gamma_{0,t} + \gamma_{1,t} RVol_{i,t} + \gamma_{2,t} RSkew_{i,t} + \gamma_{3,t} RKurt_{i,t} + \phi_t' Z_{i,t} + \varepsilon_{i,t+1}, \tag{17}$$

where $r_{i,t+1}$ is the weekly return (in bps) of the ith stock for week $t+1$, and where $Z_{i,t}$ represents a vector of characteristics and controls for the ith firm observed at the end of week t.

Table 3 reports the time-series average of the γ and ϕ coefficients for five cross-sectional regressions. The first column presents the results of the regression of the stock return on lagged realized volatility. The coefficient associated with realized volatility is 5.1 with a Newey-West t-statistic of 0.41. This confirms that there does not seem to be a significant relationship between realized volatility and stock returns. The second and third columns confirm the relation between the stock return and lagged realized skewness and realized kurtosis respectively. In column 2, the coefficient associated with realized skewness is -20.5 with a Newey-West t-statistic of -8.07. Similarly, in column 3, the coefficient on realized kurtosis is 0.92 with a t-statistic of 2.82. In the fourth column, we report regression results using all higher moments simultaneously. The coefficients on lagged realized skewness and realized kurtosis remain statistically significant, and are again negative and positive respectively. The third and fourth realized moments appear to explain different aspects of stock returns.

In the last column, we add control variables to ensure that realized skewness and realized kurtosis are not a manifestation of previously documented relationships between firm characteristics and stock returns. As is standard, we control for size, book-to-market, and market beta computed in a regression using daily returns on the market during the previous 12 months. We also include realized Value-at-Risk.[5] Furthermore, we control for other previously documented relationships between stock returns and firm characteristics, such as lagged weekly return (Lehmann (1990)), historical skewness, idiosyncratic volatility (Ang, Hodrick, Xing, and Zhang (2006)), and coskewness measured by the variability of the stock's return with respect to changes in the level of volatility following Harvey and Siddique (2000), the maximum (and minimum) daily return during the previous month (Bali, Cakici, and Whitelaw (2009)). Because our data are weekly, we also include the maximum (and minimum) daily return over the previous week as a robustness check. Finally, some control variables are related to the illiquidity and visibility of individual stocks. This includes the number of analysts following a stock (see Arbel and Strebel (1982)), the measure of illiquidity proposed in Amihud (2002), and the number of intraday transactions.

We find that the coefficients of realized skewness and realized kurtosis are still significant when including the control variables in the last column of Table 3, with Newey-West t-statistics of -6.59 and 2.45, and preserve their signs with coefficients of -11.2 and 0.70, respectively. The absolute value of the point estimates is smaller for both skewness and kurtosis compared to columns (2)-(4), suggesting that some of the cross-sectional patterns documented in Table 2 overlap with some of the control variables in Table 3. We analyze the relation between realized skewness and the various control variables in Section 5.1. Lagged return turns out to be the control variable most correlated with realized skewness, and we investigate it in detail in Section 5.2.

Table 3 indicates that control variables such as realized volatility, idiosyncratic volatility, and coskewness do not play a significant role in the cross-section of returns at a weekly level, while variables such as size, lagged return, realized Value-at-Risk, maximum and minimum return, and illiquidity are relevant. The negative sign on the coefficients related to size and lagged return and the positive sign of the coefficient related to book-to-market confirm existing results in the literature.

[5] Realized Value-at-Risk is computed at the 1% level using 5-minute returns during the week. Alternatively, we used expected shortfall at the 1% level. The results for expected shortfall are similar to those for Value-at-Risk and they are therefore not reported.

To provide more perspective on the relation between realized moments and the characteristics and controls, Table 4 provides the correlation matrix for all regressors used in Table 3. Realized skewness is most highly correlated with the minimum weekly return (16.2%) and the lagged return (22.3%). Note that in our sample these correlations are lower than those between other well-known determinants of the cross-section of returns, such as idiosyncratic volatility and the maximum return during the month (88.8%), or between lagged return and the maximum return during the week (47.0%). Also note that while lagged return is correlated with maximum monthly return (24.3%), and idiosyncratic volatility (15.7%), realized skewness seems to be nearly uncorrelated with these and other firm characteristics, confirming and strengthening the results from Table 2.

In summary, Table 3 demonstrates that the economic and statistical significance of realized skewness for the cross-section of weekly returns is robust to the inclusion of various control variables. Table 3 is also supportive of the relation between realized kurtosis and subsequent returns. The estimated sign on realized kurtosis is consistent with the evidence in Table 2, but the results in Table 3 are stronger. This is not necessarily surprising: sorts as in Table 2 are more intuitive, but the more formal regressions in Table 3 capture linear relationships better and control for confounding effects.

4 Robustness Analysis

In this section, we further explore the relation between realized moments and stock returns. First, we investigate the relation between current-week realized moments and next-week returns when using drift-adjusted realized moments. Second, we investigate if our findings on skewness and kurtosis continue to hold up when using jump-robust realized volatility estimators. Third, we look at long-short returns for different subsets of our data. Fourth, we investigate alternative measures of realized skewness. Fifth, we use different holding periods for returns.

4.1 Drift-Adjusted Realized Moments

The computation of daily realized volatility, realized skewness, and realized kurtosis in equations (2), (3) and (4) assumes that the 5-minute mean return is zero. This is a standard assumption for such short time intervals. However, because the resulting skewness measure may capture a weekly return drift and thus the return reversal effect, it is imperative to check if our results are robust to changing this assumption. To ensure that our measures are not contaminated by the weekly total return, we define the following daily realized measures adjusted for the drift:

$$DriftRDVar_t = \sum_{i=1}^{N} (r_{t,i} - \mu_{w(t),i})^2, \tag{18}$$

$$DriftRDSkew_t = \frac{\sqrt{N} \sum_{i=1}^{N} (r_{t,i} - \mu_{w(t),i})^3}{DriftRDVar_t^{3/2}}, \tag{19}$$

$$DriftRDKurt_t = \frac{N \sum_{i=1}^{N} (r_{t,i} - \mu_{w(t),i})^4}{DriftRDVar_t^2}, \tag{20}$$

The notation $\mu_{w(t),i}$ reflects that for any given day t in a given week and for any stock i, the average return for that day (scaled to 5 minutes) is computed using all days in that week, to obtain more precise estimates. The weekly realized moments, $RVol_{drift}$, $RSkew_{drift}$, $RKurt_{drift}$, are then computed using equations (5), (6) and (7) with the drift-adjusted daily moments.

Panel A of Table 5 reports the time-series average of weekly returns for decile portfolios grouped by $RSkew_{drift}$. The equal- and value- weighted long-short returns are negative and statistically significant. The value-weighted long-short return is -11.10 basis points with a t-statistic of -2.42. Equal-weighted long-short returns are larger. The Carhart four factor long-short alphas are very similar to the long-short raw return.

Panel B of Table 5 reports the time-series average of weekly returns for decile portfolios grouped by $RKurt_{drift}$. The equal- and value- weighted long-short returns are positive and significant. The Carhart four factor long-short alpha is significant for value-weighted returns at the 10% level. We do not report on $RVol_{drift}$ because the results are very similar to Table 2 and therefore inconclusive.

We also run the two-step Fama-MacBeth regressions using the drift-adjusted moments. Table 6 repeats the five regressions of Table 3 but now using drift-adjusted realized moments. In columns (1)-(3), we run the univariate regressions for the new realized volatility, skewness, and kurtosis measures. In column (4), we use the three moments jointly. Finally, column (5) includes all the control variables used in Table 3. The coefficients on the drift-adjusted realized skewness are negative and highly statistically significant in all regressions. Similarly, the coefficients for realized kurtosis adjusted by drift are positive and statistically significant in all regressions. These results show that our realized skewness measure is not merely capturing a weekly drift in return that reverses in the subsequent week. We provide additional evidence on the reversal effect in Section 5.2 below.

4.2 Higher Moments with Jump-Robust Realized Volatility Estimators

For the class of affine jump-diffusion models often considered in the volatility literature, the limit of the sum of intraday squared returns in (2) can be written as the sum of jump variation and integrated variance. The realized variance estimator used in the empirics so far therefore captures both jumps and diffusive volatility in the limit. This does not invalidate it as an ex-post measure for the total daily quadratic variation, but it does suggest a robustness exercise using measures that separate jump variation and integrated variance.

Several volatility estimators that are robust to the presence of jumps have been developed in the literature. They are designed to only capture integrated variance in the limit. The so-called bipower variation estimator of Barndorff-Nielsen and Shephard (2004) is defined by

$$BPV_t = \frac{\pi}{2}\frac{N}{N-1}\sum_{i=1}^{N-1}|r_{t,i+1}|\,|r_{t,i}|$$

which converges in the limit to integrated variance, even in the presence of jumps.

Motivated by the presence of large jumps that may bias upward the bipower variation measure in realistic settings when N is finite, Andersen, Dobrev and Schaumburg (2010) have recently developed two alternative jump-robust estimators, defined by

$$\begin{aligned} MinRV_t &= \frac{\pi}{\pi-2}\left(\frac{N}{N-1}\right)\sum_{i=1}^{N-1}\min\left\{|r_{t,i}|,|r_{t,i+1}|\right\}^2, \text{ and} \\ MedRV_t &= \frac{\pi}{6-4\sqrt{3}+\pi}\left(\frac{N}{N-2}\right)\sum_{i=2}^{N-1}\text{median}\left\{|r_{t,i-1}|,|r_{t,i}|,|r_{t,i+1}|\right\}^2. \end{aligned}$$

These estimators will also both converge to the integrated variance when N goes to infinity and in the presence of large jumps they typically have better finite sample properties than BPV_t.

To assess the robustness of our cross-sectional return results, Table 7 reports the value-weighted and equal-weighted weekly returns of the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) when using the three alternative realized volatility estimators, BPV_t, $MinRV_t$, and $MedRV_t$. To facilitate comparisons, the first column of Table 7 uses the standard $RVol_t$ from (5) and thus reproduces the last column of Table 2. Each panel in Table 7 reports the value-weighted and the equal-weighted long-short returns. Alpha is again computed using the Carhart four factor model.

Panel A in Table 7 reports the long-short results for the three alternative realized volatility estimators. We find that the insignificant relationship between return and volatility remains when alternative estimators of realized volatility are used.

Panel B in Table 7 shows the long-short results for realized skewness when scaling by the three alternative realized volatility estimators. We see that the strong negative relationship between realized skewness and returns found in Table 2 is robust to changing the denominator in

$$RDSkew_t = \frac{\sqrt{N}\sum_{i=1}^{N} r_{t,i}^3}{RDVar_t^{3/2}}. \tag{21}$$

to be any of the three jump-robust volatility estimators defined in this section.

Panel C in Table 7 presents results for realized kurtosis when scaling by the three alternative realized volatility estimators. Point estimates for the long-short returns and alphas are consistently positive and fairly large, but results for value-weighted alphas are only marginally significant for the bipower variation estimator.

We conclude that the strong negative relationship between realized skewness and subsequent returns in the cross-section is not an artefact of a particular measure of realized volatility. The results hold up when we use estimators of realized volatility that are jump-robust. The long-short returns for realized kurtosis are also consistently large and mostly significant.

4.3 Subsamples

Panel A of Table 8 reports value- and equal-weighted returns of portfolios sorted on realized skewness across different subsamples. Keim (1983) documents calendar-related anomalies for the month of January, in which stocks have higher returns than in the rest of the year. Panel A of Table 8 presents the average weekly returns for the month of January and for the rest of the year for both value- and equal-weighted portfolios. As expected, returns for the month of January are consistently higher than returns for the rest of the year.

The difference between the returns of portfolios with high-skewness stocks and portfolios with low-skewness stocks is negative and significant for both January and non-January periods. This is the case for value-weighted as well as equal-weighted portfolios.

We previously documented that stocks with high and low levels of skewness tend to be small. Hence, we examine if the effect of skewness is exclusively driven by small NASDAQ stocks. By only including stocks from the New York Stock Exchange (NYSE), row 3 of Table 8 shows that the effect of realized skewness is present among NYSE stocks. Hence, small NASDAQ stocks are not driving our results.

In Table 8, Panel B, we analyze the value-weighted and equal-weighted returns of portfolios sorted on realized kurtosis for different subsamples. The long-short portfolio returns are positive for all subsamples. For value-weighted portfolios the realized kurtosis premium is positive, but often not statistically significant, again indicating that the cross-sectional relationship between realized kurtosis and future returns is less robust than the one documented for realized skewness. We omit results for realized volatility across subsamples because they confirm the results of Table 2, and we do not find a robust pattern. These results for volatility and kurtosis are representative of other robustness analyses we conducted: for volatility we usually do not find a significant effect, whereas for kurtosis the results are not always significant. In the remainder of the paper we will therefore focus on realized skewness.

4.4 Alternative Measures of Realized Skewness

We now investigate the robustness with respect to the implementation of realized skewness by analyzing two alternative estimators. The first estimator, $SubRSkew$, uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005) for realized volatility, which provides measures robust to microstructure noise. This method consists of constructing overlapping low-frequency subsamples on a high-frequency grid. Instead of one realized measure based on a single five-minute return grid, we now have six estimators of realized skewness using subsamples of 30-minute returns for the period 9:30 EST to 16:00 EST. Subsamples start every five minutes (at 9:00, 9:05, 9:10, 9:15, 9:20 and 9:25), but employ 30-minute returns. Subsequently, the realized skewness estimator is computed as the average of the six (overlapping) estimators obtained from the subsamples. Additionally, we compute $SubRSkew_{drift}$ using the subsampling methodology on the drift adjusted realized measures described in the previous section.

The second alternative estimator of intraday skewness depends solely on quartiles from the intraday return distribution. As proposed in Bowley (1920), a measure of skewness that is based on quartiles can be defined as

$$SK2_t = (Q_3 + Q_1 - 2Q_2)/(Q_3 - Q_1), \tag{22}$$

where Q_i is the i^{th} quartile of the five-minute return distribution.

Panel A of Table 9 includes our results for $RSkew$ from Table 2, as well as results for the estimators $SubRSkew$ and $SubRSkew_{drift}$ which are based on the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005), and for the $SK2$ measure based on quartiles in (22).

Panel A of Table 9 documents negative value-weighted and equal-weighted long-short returns for the alternative measures $SubRSkew$, $SubRSkew_{drift}$, and $SK2$. The long-short return is smaller for the $SK2$ and $SubRSkewdrift$ measures (for value-weighted returns), but it is much larger for $SubRSkew$ compared with the $RSkew$ measure. The subsampling skewness measure, $SubRSkew$, has a long-short raw value-weighted return of -42 basis points and a Carhart alpha of -43 basis points with t-statistics of -6.22 and -6.38. The subsampling skewness adjusted by drift has a premium of -18 basis points with a t-statistic of -3.27 and a Carhart alpha of -20 basis points with a t-statistic of -3.73. We conclude that all these alternative measures of realized skewness, except for the quantile-based $SK2$, yield statistically significant negative long-short alphas.

Panel B of Table 9 contains results for another robustness analysis, using realized skewness calculated over different intervals: one day, three days, one week, two weeks, and one month. The results for $RSkew_{1w}$ in the middle column are the benchmark results from Table 2. The long-short returns are small and statistically insignificant in the value-weighted case when only one day of high frequency data is used. All other results are statistically significant, and the long-short returns are of a roughly similar magnitude.

4.5 Alternative Forecast Horizons

Thus far our empirics have been based on weekly returns and weekly realized moments. In this section we keep the weekly frequency when computing realized moments but we increase the return holding period from one week to up to one month.

Table 10 contains the results for (overlapping) two-week, three-week, and one-month returns. We report the returns of decile portfolios formed from realized moments, and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment). Each panel reports on both value-weighted and equal-weighted portfolios and include t-statistics computed from robust standard errors. Alpha is again computed using the Carhart four factor model.

The strong negative relationship between realized skewness and returns in Table 2 is confirmed for all three alternative horizons. This relationship is significant for raw returns as well as alphas, and for value-weighted and equal-weighted portfolios.

We have repeated this analysis for realized volatility and kurtosis. Volatility most often yields a negative sign and kurtosis a positive sign. The results for volatility are usually not statistically significant, and kurtosis yields significant results for returns but not always for alphas. This confirms our other findings, and we therefore do not report the results to save space. They are available on request.

5 Exploring the Results

In this section, we further analyze the interaction between realized moments and other firm characteristics using double sorts. Subsequently we pay special attention to the interaction between realized skewness, lagged returns, and the cross-section of returns, as well as the relation between realized skewness, realized volatility, and the cross-section of returns. Finally, we compare the performance of high-frequency and low-frequency skewness measures.

5.1 Realized Skewness and Firm Characteristics

This section further analyzes the interaction between realized moments and other firm characteristics. Consider size as an example. While anomalies for small firms may be interesting, Fama and French (2008) point out that to ensure the general validity of an anomaly, small (microcaps), medium, and large firms ought to all exhibit the anomaly. We use an independent double sort methodology to analyze the realized skewness premium for five different size portfolios. We sort stocks into quintiles by size and by realized skewness. Using the intersection of the two characteristics, we compute value- and equal-weighted returns for the resulting twenty-five portfolios. Row 1 of Table 11 reports the return from being long the highest skew portfolio and short the lowest skew portfolio for each of the five size quintiles. This methodology yields a realized skewness premium conditional on size, and allows us to assess if the realized skewness premium is economically significant for all size levels. We also provide double sorting results for the other control variables used in Tables 3 and 6 above.

Again using size in row 1 as an example, the realized skewness value-weighted premium of −55.5 basis points for quintile 1 can be earned by buying small stocks (microcaps) with high realized skewness and selling small stocks with low realized skewness. For big firms in quintile 5, the corresponding premium is −17.8 basis points. All five size groups exhibit the realized skewness anomaly, but the premium is larger for small stocks. This finding explains why the effect of realized skewness is weaker for value-weighted portfolios when compared to equal-weighted portfolios, as evident in Table 2. The stronger negative effect of skewness for small firms is also consistent with Chan, Chen, and Hsieh (1985), who show that there are risk differences between small and large firms. The realized skewness premium and t-statistics are of similar magnitude for equal-weighted returns across size quintiles.

The evidence in favor of the skewness premium is overwhelming. Table 11 indicates that the realized skewness premiums are negative and statistically significant at the 5% level for almost all cases, for value-weighted and equal-weighted portfolios. The relationship between realized skewness and subsequent returns appears to be robust to all firm characteristics and does not seem to be a proxy for any of them. The strongest interaction is found between realized skewness and lagged weekly returns and we will therefore investigate this relationship in detail below.

We also performed a double sort on realized kurtosis and firm characteristics (not reported). The results indicate that while the long-short return is positive in the large majority of cases, the results are not as strong as for skewness in Table 11. Again we do not report these results to save space; they are available on request.

5.2 Realized Skewness and the Reversal Effect

The correlation matrix in Table 4 includes a wide variety of firm characteristics and existing determinants of the cross-section of returns. Realized skewness is most highly correlated to lagged return (22.3%). In the double sorting exercise in Table 11, the long-short return based on realized skewness is negative in four of the five quintiles for lagged weekly return, but it is not always significant. We therefore provide a more detailed analysis of the relation between realized skewness and lagged return.

5.2.1 Double Sorts on Realized Skewness and Lagged Returns

In Panel A of Table 12, we provide detailed results for double sorts on lagged return and realized skewness. All results are for one-week returns. Using the one-week lagged return in the second row, the long-short realized skewness return is negative in eight of ten cases, but the negative returns are not always statistically significant. When we use a different window to compute the lagged return, the results for realized skewness are stronger, because the two measures now contain more independent information.

In Panel B of Table 12, we repeat the double sorts from Panel A, but we now use the $SubRSkew$ measure from Table 9, which uses the subsampling methodology suggested by Zhang, Mykland, and Ait-Sahalia (2005). This measure is more robust to microstructure noise, and Table 9 indicates that it yields stronger results. Panel B of Table 12 shows that the long-short realized skewness return is always negative using this measure, and the statistical significance is higher than in Panel A, but overall the results confirm that there is some overlap between realized skewness and the reversal effect.

Panels C and D of Table 12 further analyze this relation. We report the overlap between the realized skewness and lagged return quintiles used in Panels A and B, on average across time. For instance, in Panel C, 36.0% of the firms in the lowest realized skew quintile are also in the lowest lagged return quintile, whereas 9.8% of the firm in the lowest realized skewness quintile are in the highest lagged return quintile. Panels C and D confirm the message from the simple correlation matrix in Table 4: there is some overlap between the two phenomena but also independent variation.

5.2.2 A Direct Comparison of Realized Skewness and the Reversal Effect

Tables 13 and 14 provide a direct comparison of the returns based on realized skewness and reversal (lagged returns). We report results for the one-week (Table 13) and one-month (Table 14) horizons. Panel A reports results for value-weighted returns and Panel B for equal-weighted returns. We present the average returns and alphas for the short-long portfolios that generate a positive return by selling negative skewness and buying positive skewness, using $RSkew$ as well as $SubRSkew$. For value-weighted returns, the short-long returns based on lagged returns are roughly similar to $SubRSkew$ and larger than $RSkew$. For equal-weighted returns, the differences are larger.

Figure 3 documents the pattern over time for short-long returns obtained by sorting on $RSkew$ and lagged return, for the one-week horizon and value-weighted returns. We use three-month moving averages of returns to smoothen the time series. The black and grey lines depict the short-long returns associated with realized skewness and lagged return respectively. Both returns vary substantially over time. It is of course to be expected that the return on a risk factor is at times positive and negative at other times. Most importantly, Figure 3 shows that the time-series of short-long returns differ substantially. They move together in some brief subperiods of the sample, but are very different in others.

The partial overlap between realized skewness and the reversal effect may be useful for interpreting our results. For a risk-based interpretation, persistence is critically important. However, most measures of skewness are not very persistent, and this also holds for the realized skewness in our sample. An alternative explanation for our findings is provided by Nagel (2012),[6] who interprets the returns of short-term reversal strategies as a proxy for the returns from liquidity provision. A similar explanation may apply to our findings given the correlation between the realized skewness and lagged return variables.

Tables 13 and 14 document several other aspects of the distribution of short-long returns. In addition to average short-long returns and alphas, we report on standard deviation, skewness, kurtosis, and the Sharpe ratio. We also report value-at-risk and expected shortfall at the 1 percent level computed using historical simulation, and the correlation between the short-long returns from

[6]We thank our WFA discussant, Bryan Kelly, for suggesting this interpretation.

skewness and lagged returns.

First consider the one-week returns in Table 13. The short-long returns based on lagged returns are higher than those based on the *SubRSkew* measure, but this is outweighed by a larger standard deviation, leading to a smaller Sharpe ratio. Skewness and kurtosis for the returns associated with the *SubRSkew* measure do not suggest higher tail-risk than in the case of the strategy based on lagged return. This is in turn reflected in substantially lower value-at-risk and expected shortfall for the *SubRSkew* strategy. The returns associated with the *RSkew* measure have the lowest value-at-risk and expected shortfall, but the Sharpe ratio is substantially lower as well.[7] The results for monthly returns in Table 14 confirm these findings.

5.2.3 Combining Realized Skewness and the Reversal Effect

The right-hand side of Table 13 reports on short-long portfolios that combine lagged returns and realized skewness. We report on two strategies that combine information from lagged returns and realized skewness. The first strategy puts equal weights in the two portfolios, while the second strategy has optimal weights that maximize the Sharpe ratio. We obtain several interesting conclusions. First, it is relatively straightforward to improve on the average returns, Sharpe ratios, and tail risk measures for the lagged return strategy by adding information from realized skewness. Second, the information based on *SubRSkew* is much more valuable than the information based on *RSkew*. Finally, and perhaps most surprisingly, adding information from lagged returns to the information from *SubRSkew* does not lead to significant improvements, even when the weights are optimized. In some cases a slightly higher Sharpe ratio obtains, but only at the expense of higher tail risk. The results for monthly returns in Table 14 confirm the conclusions from Table 13.

Overall, Tables 13 and 14 suggest that there is a substantial amount of information in realized skewness that is independent of the information in lagged returns. This independent information is much better captured by the *SubRSkew* measure, which is less contaminated by microstructure noise.

Figure 4 further analyzes the differences between realized skewness and lagged returns. We depict average characteristics in each decile, sorted on realized skewness (black) and lagged return (grey) respectively. The deciles are similar with respect to the size of the firms and illiquidity, but different in every other respect, notably so with respect to market beta, maximum and minimum return, and the number of transactions.

These differences are perhaps not surprising given the structure of the correlation matrix in Table 4. In the double sorts in Table 11, none of the indicators that intuitively ought to be related to lagged returns seems to be impacting significantly on the negative sign for realized skewness. The correlation matrix in Table 4 indicates that this is to be expected, because while realized skewness

[7] Note that the skewness associated with both strategies is positive, indicating an additional benefit besides the attractive Sharpe ratios. To the best of our knowledge this important fact has not yet been reported in the reversal literature.

has a correlation of 16.2% with the minimum weekly return and 22.3% with the lagged return, its correlation with other indicators is nearly zero. Contrast this with lagged return, which has a correlation of 47.0% with the maximum weekly return and 34.5% with the minimum weekly return. Notice also the high correlations between idiosyncratic volatility and some of these measures, while the correlation between realized skewness and idiosyncratic volatility is essentially zero.

Figure 4 confirms that while there is some relation between the long-short returns from realized skewness and lagged return, realized skewness contains a component that is independent of existing indicators. We attribute this finding to the intuition from Section 2.3 that realized skewness captures the jump component of returns. Lagged return is partly composed of these jumps, but it also captures the diffusive component. Minimum returns are more correlated with realized skewness than maximum returns, presumably because realized skewness mainly captures large negative jumps.

5.3 Realized Skewness and Realized Volatility

We now further examine the interaction between the effects of realized skewness and realized volatility on returns. We construct portfolios using a double sort on realized skewness and realized volatility and then examine subsequent stock returns. First, we form five quintile portfolios with different levels of realized skewness. Within each of these portfolios, we form five portfolios that have different levels of realized volatility.[8] Panel A of Figure 5 shows the value- and equal-weighted returns for the 25 portfolios double sorted on realized skewness and realized volatility. The variation in the moments across portfolios is large. Realized skewness increases from -0.844 to $+0.825$ and realized volatility increases from 20% to about 110% across portfolios. Equal-weighted portfolios with low realized volatility of 20% have very similar returns for all five levels of realized skewness, between 20 and 30 basis points. However, as realized volatility increases, the return of low and high realized skewness portfolios strongly diverges. Portfolios with high realized volatility of 110% report the highest and the lowest return of all 25 portfolios. Stocks with the lowest realized skewness earn the highest average equal-weighted return of 66 basis points, and stocks with the highest realized skewness earn the lowest average return of -2 basis points. For value-weighted portfolios, similar conclusions obtain, but Panel A of Figure 5 indicates that the differences between portfolios are smaller. The nature of the relation between realized skewness, realized volatility, and returns is therefore clearly size-dependent.

Our findings are consistent with the mechanics of the equilibrium model in Colacito, Ghysels, and Meng (2012), which contains an interaction between second and third moments. As positive asymmetry increases, volatility is welfare increasing as it implies a larger probability of an extremely good state of the economy. The opposite is true for the case of negative skewness, since higher volatility increases the likelihood of a left tail event.

We conclude that it is important to account for skewness when analyzing the return/volatility

[8] Sorting on realized volatility first, and subsequently on realized skewness, does not change the results.

relationship. Highly volatile stocks may earn low returns, which seems counterintuitive, but the reason is that their skewness is high, especially for small stocks.

5.4 Realized Skewness and Idiosyncratic Volatility

Building on our findings regarding volatility and skewness, we now investigate whether realized skewness can explain the idiosyncratic volatility puzzle uncovered by Ang, Hodrick, Xing, and Zhang (2006). They find that stocks with high idiosyncratic volatility earn lower returns than stocks with low idiosyncratic volatility, contradicting the implications of mean-variance models.

To study the interaction between realized skewness and idiosyncratic volatility on stock returns we employ double sorting. Panel B of Figure 5 shows the returns of the 25 equal-weighted portfolios for different levels of idiosyncratic volatility. Just as with realized volatility in Panel A of Figure 5, high idiosyncratic volatility is compensated with high returns only if skewness is low. Investors are willing to accept low returns and high idiosyncratic volatility in exchange for high positive skewness. For value-weighted portfolios, the pattern is less prominent.

These results suggest that investors may trade-off high idiosyncratic volatility and low returns for high skewness, because they like positive skewness. Preference for skewness may explain some aspects of the idiosyncratic volatility puzzle, but this explanation is clearly related to firm size.

5.5 Low Frequency Measures of Skewness

In this section we investigate the cross-sectional pricing of historical skewness computed using daily returns over different horizons. We begin by computing six different measures of historical skewness using close-to-close returns from daily data: $HSkew_{5D}$ uses a moving window of five trading days to compute skewness, $HSkew_{1M}$ uses one month of daily data, etc. The longest window is 60 months of daily data in $HSkew_{60M}$. The six measures of historical skewness in Panel A of Table 15 yield very different results from the realized skewness measures in Table 9: Using 5-day, 6-month, 12-month, and 24-month windows, long-short returns and alphas are positive and statistically significant, consistent with the coefficient on historical skewness in Table 3, which is obtained using one month of daily data. For the other two measures, the 1-month and the 60-month windows, the results are usually not statistically significant. In summary, results for historical skewness critically depend on the estimation window used.

These findings highlight the importance of the limiting results from Section 2.3: Skewness measures computed from high-frequency data capture jumps and contain different information from skewness measures computed from daily data. It is therefore not surprising that the return premia from sorting stocks on low-frequency $HSkew$ measures are different from the return premia from sorting stocks on high-frequency $RSkew$ measures. These results suggest a need for asset pricing models with differential risk premia on the diffusive and jump components of skewness.

To try and reconcile the results on historical skewness with the results for realized skewness, Panel A of Table 15 includes results for one more measure of historical skewness and four measures of realized skewness. The first measure of realized skewness is $RSkew$ computed using five-minute returns, which is the benchmark result from Table 2. The three other measures of $RSkew$ are computed using 30-minute, 60-minute, and half-day (195-minute) returns, respectively. We emphasize that we do not advocate the use of these measures of realized moments. Their only purpose is to illustrate the relation between realized and historical skewness. The same remark applies to the measure of historical skewness computed using a five-day window and daily open-to-close returns. We conclude that when realized skewness is computed using lower-frequency returns, the resulting estimate is more similar to the historical measure obtained using a five-day window. Moreover, when we compute the historical skewness using a five-day window and using daily open-to-close returns, which is more in line with the information used by the realized volatility measure, the resulting point estimates become closer to those for $RSkew_{1/2D}$. Comparing the 5-day open-to-close and the 5-day close-to-close historical skewness results suggests that the positive return spread from sorting on historical skewness is largely driven by the contribution to historical skewness from overnight price moves.

To provide more perspective on the relation between historical and realized skewness measures, it is instructive to compare their time paths. We do not report these figures because of space constraints, but the high correlation between most of the realized skewness measures on the one hand and most of the historical skewness measures on the other hand is striking. The correlation matrix between the different measures is sufficient to make this point. It is presented in Panel B of Table 15. Notice how the high-frequency $RSkew$ measures are quite highly correlated with each other, and the low-frequency $HSkew$ measures are highly correlated with each other, but note also that the cross-correlation between $RSkew$ and $HSkew$ measures is low. They clearly capture different aspects of the return-generating process.

6 Conclusion

We document the cross-section of realized higher moments. We introduce model-free estimates of higher moments based on the methodology used by Hsieh (1991) and Andersen, Bollerslev, Diebold, and Ebens (2001) to estimate realized volatility. We use five-minute returns to obtain daily measures of higher moments, and subsequently aggregate this measure up to the weekly frequency. We find considerable time-variation in the cross-sectional distribution.

We then investigate the cross-sectional relationship between realized higher moments of individual stocks and future stock returns. We find a reliable and significant negative relationship between realized skewness and next week's stock returns in the cross-section. We find little evidence of a reliable relation between realized volatility and the cross-section of next week's stock returns. For realized kurtosis, overall the evidence indicates a positive relationship, but the result is not always robust to variations in the empirical setup. Fama-MacBeth regressions and double sorting confirm that realized skewness is not a proxy for firm characteristics such as size, book-to-market, realized volatility, market beta, historical skewness, idiosyncratic volatility, coskewness, maximum return over the previous month or week, analysts coverage, illiquidity or number of intraday transactions. Among determinants of the cross-section of returns documented in the existing literature, realized skewness is most closely related with the reversal effect.

We analyze the relationship between realized skewness and realized volatility in more detail. When double sorting on realized skewness and volatility, we find that stocks with negative skewness are compensated with high future returns for higher volatility. However, as skewness increases and becomes positive, the positive relation between volatility and returns turns into a negative relation. We conclude that investors may accept low returns and high volatility because they are attracted to high positive skewness. We perform a similar analysis for realized skewness and idiosyncratic volatility, and find that skewness may help explain the idiosyncratic volatility puzzle in Ang, Hodrick, Xing, and Zhang (2006), who document that stocks with high idiosyncratic volatility earn low returns.

Several interesting topics are left for future research. We have analyzed the cross-section of equity returns, but we have not investigated if realized higher moments are useful for time-series forecasting of future returns. A direct comparison of realized skewness and realized semivariance, proposed by Barndorff-Nielsen, Kinnebrock, and Shephard (2010) and related to realized skewness, may also be of interest. Patton and Sheppard (2011) demonstrate that realized semivariance leads to improved forecasts of future volatility, which also suggests that realized skewness may be of interest for forecasting future volatility. Finally, it may prove interesting to repeat our exercise using jump estimates constructed from high-frequency data, as in Li (2013).

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Appendix A: Data

- Following Fama and French (1993), size (in billions of dollars) is computed each June as the stock price times the number of outstanding shares. The market cap is held constant for a year.

- Following Fama and French (1993), book-to-market is computed as the ratio of book common equity over market capitalization (size). Book common equity is defined using COMPUSTAT's book value of stockholders' equity plus balance-sheet deferred taxes and investment tax credit minus the book value of preferred stock. The ratio is then computed as the book common equity at the end of the fiscal year over size at the end of December.

- Historical skewness for stock i on day t is defined as

$$HSkew_{i,t} = \frac{1}{N} \sum_{s=0}^{N} \left(\frac{r_{i,t-s} - \mu_i}{\sigma_i} \right)^3, \tag{23}$$

  where N is the number of trading days, $r_{i,t-s}$ is the daily log-return of stock i on day $t-s$, μ_i is the mean for stock i and σ_i is the standard deviation of stock i. We estimate historical skewness using 5 day, 1, 6, 12, 24, and 60 months of daily returns.

- Market beta is computed at the end of each month using a regression of daily returns over the past 12 months.

- Following Ang, Hodrick, Xing, and Zhang (2006), idiosyncratic volatility is defined as

$$idvol_{i,t} = \sqrt{var(\varepsilon_{i,t})}, \tag{24}$$

  where $\varepsilon_{i,t}$ is the error term of the three-factor Fama and French (1993) regression. The regression is estimated with daily returns over the previous 20 trading days.

- Following Harvey and Siddique (2000), coskewness is defined as

$$CoSkew_{i,t} = \frac{E\left[\varepsilon_{i,t}\varepsilon_{m,t}^2\right]}{\sqrt{E\left[\varepsilon_{i,t}^2\right]} E\left[\varepsilon_{m,t}^2\right]}, \tag{25}$$

  where $\varepsilon_{i,t}$ is obtained from $\varepsilon_{i,t} = r_{i,t} - \alpha_i - \beta_i r_{m,t}$, where $r_{i,t}$ is the monthly return of stock i on month t, $r_{m,t}$ is the market monthly return on month t. This regression is estimated at the end of each month using monthly returns for the past 24 months.[9]

- Maximum return is defined as the maximum daily return over the previous month or week.

[9] Harvey and Siddique (2000) use data for the past 60 months to estimate coskewness. This approach considerably reduces our sample of firms. We therefore estimate coskewness using only 24 months, which Harvey and Siddique (2000) also consider.

- Following Avramov, Chordia, and Goyal (2006), Jegadeesh (1990), and Lehmann (1990), lagged return is defined as the weekly return over the previous week from Tuesday to Monday.

- Following Amihud (2002), stock illiquidity on day t is measured as the average of the ratio of the absolute value of the return over the dollar value of the trading volume over the previous year
$$illiquidity_{i,t} = \frac{1}{N}\sum_{s=0}^{N}\left(\frac{|r_{i,t-s}|}{|volume_{i,t-s} * price_{i,t-s}|}\right), \tag{26}$$
where N is the number of trading days, $r_{i,t-s}$ is the daily log-return of stock i on day $t-s$, $volume_{i,t-s}$ is the daily volume of stock i on day $t-s$, and $price_{i,t-s}$ is the price of stock i on day $t-s$. We use 252 trading days to estimate illiquidity.

- The credit rating is retrieved from COMPUSTAT and is then assigned a numerical value as follows: AAA=1, AA+=2, AA=3, AA-=4, A+=5, A=6, A-=7, BBB+=8, BBB=9, BBB-=10, BB+=11, BB=12, BB-=13, B+=14, B=15, B-=16, CCC+=17, CCC=18, CCC-=19, CC=20, C=21 and D=22. When no rating is available, the default credit rating value is 8.

Appendix B: Expected Values of Realized Moment Limits

To derive the expected value of the realized moment limits for the model in (12)-(13), we employ the fact that the log-price p_t in equation (12) is an affine model, so there exists a closed-from solution for its moment generating function (MGF). In this appendix, we find an explicit representation of the forward MGF of p_t, which is then used to derive the expected value of the realized moment limits. From the MGF of p_t, see Duffie, Pan, and Singleton (2000), and using the law of iterated expectations, we have the forward MGF

$$\begin{aligned}
\varphi_t(u,\tau) &\equiv E\left[\exp\left(u\left(p_{t+\tau}-p_t\right)\right)\middle|\mathcal{F}_0\right] \\
&= E\left[E\left[\exp\left(u\left(p_{t+\tau}-p_t\right)\right)\middle|\mathcal{F}_t\right]\middle|\mathcal{F}_0\right] \\
&= E\left[\exp\left(\alpha(u,\tau)+\beta(u,\tau)V_t\right)\middle|\mathcal{F}_0\right] \\
&= \exp\left(\alpha(u,\tau)\right)E\left[\exp\left(\beta(u,\tau)V_t\right)\middle|\mathcal{F}_0\right] \\
&= \exp\left(\alpha(u,\tau)\right)\exp\left(-A\left(-\beta(u,\tau),t\right)-B\left(-\beta(u,\tau),t\right)V_0\right),
\end{aligned}$$

where

$$\begin{aligned}
A(\varsigma,t) &= \frac{\varsigma e^{-\kappa t}}{1+\varsigma\frac{\sigma^2}{2\kappa}\left(1-e^{-\kappa t}\right)},\ B(\varsigma,t)=\frac{2\kappa\theta}{\sigma^2}\log\left(1+\varsigma\frac{\sigma^2}{2\kappa}\left(1-e^{-\kappa t}\right)\right), \\
\alpha(u,t) &= \left(\mu-\lambda\bar{\mu}_J\right)ut-\frac{\kappa\theta}{\sigma^2}\left(\left(\gamma+b\right)t+2\log\left(1-\frac{\gamma+b}{2\gamma}\left(1-e^{-\gamma t}\right)\right)\right)+\lambda t\left(\phi(u)-1\right)-\mu t, \\
\beta(u,t) &= -\frac{a\left(1-e^{-\gamma t}\right)}{2\gamma-\left(\gamma+b\right)\left(1-e^{-\gamma t}\right)},
\end{aligned}$$

with $a = u - u^2$, $b = \sigma\rho u - \kappa$, $\gamma = \sqrt{b^2 + a\sigma^2}$, $\phi(u) = \exp\left(\mu_J u + \frac{1}{2}\sigma_J^2 u^2\right)$, and $\tau = 1/N$.

We now need to find the expected value of the limits of the realized moments defined in (14),(15) and (16). We start by rewriting

$$\begin{aligned}
\lim_{N\to\infty} E\left[\sum_{i=1}^{N}\left(p_{T\frac{i}{N}} - p_{T\frac{i-1}{N}}\right)^j\right] &= \lim_{N\to\infty}\sum_{i=1}^{N} E\left[\left(p_{T\frac{i}{N}} - p_{T\frac{i-1}{N}}\right)^j\right] \\
&= \lim_{N\to\infty}\sum_{i=1}^{N} \left.\frac{\partial^j \varphi_{T\frac{i-1}{N}}}{\partial u^j}(u,\tau)\right|_{u=0}.
\end{aligned}$$

The Taylor series expansion of $\left.\frac{\partial^j \varphi_t}{\partial u^j}(u,\tau)\right|_{u=0}$ around $\tau = 0$ yields

$$\left.\frac{\partial^2 \varphi_t}{\partial u^2}(u,\tau)\right|_{u=0} = \left(\theta + \lambda\left(\mu_J^2 + \sigma_J^2\right) - e^{-\kappa t}\left(\theta - V_0\right)\right)\tau + O\left(\tau^2\right), \tag{27}$$

$$\left.\frac{\partial^3 \varphi_t}{\partial u^3}(u,\tau)\right|_{u=0} = \lambda\left(\mu_J^3 + 3\mu_J\sigma_J^2\right)\tau + O\left(\tau^2\right), \tag{28}$$

$$\left.\frac{\partial^4 \varphi_t}{\partial u^4}(u,\tau)\right|_{u=0} = \lambda\left(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4\right)\tau + O\left(\tau^2\right), \tag{29}$$

where $O\left(\tau^2\right)$ denotes all terms of order 2 and above. As N tends to infinity, τ converges to zero, so that the only remaining terms in equations (27), (28), and (29) are those of order 1. The limit of the sum of these terms coincides with the definition of the Riemann-Stieltjes integral, so that integrating these terms with respect to t over the sampling interval $[0, T]$ gives (14), (15), and (16).

Appendix C: Monte Carlo Evidence

In practice, microstructure noise is present in high frequency prices. To simulate market microstructure noise, we define the observed log price p_t^* as

$$p_t^* = p_t + u_t, \tag{30}$$

where u_t is i.i.d. Gaussian noise with mean zero and variance σ_u^2. Hence, the observed log price p_t^* is a noisy observation of the non-observable true price p_t.

Several studies use Monte Carlo simulations to investigate the properties of realized variance estimators when allowing for market microstructure noise, see for instance Andersen, Bollerslev, and Meddahi (2011), Gonçalves and Meddahi (2009), and Ait-Sahalia and Yu (2009). Based on their work, we conduct the following Monte Carlo study.

We simulate $100,000$ paths of the log price process p_t using the Euler scheme at a time interval $\tau = 1$ second. The parameters for the continuous part of the process are set to $\mu = 0.05$, $\kappa = 5$, $\theta = 0.04$, $\sigma = 0.5$, $V_0 = 0.09$, and $\rho = -0.5$. The microstructure noise, u_t, is modeled with a normal distribution of mean zero and standard deviation of 0.05%. These parameter values are similar to those employed by Ait-Sahalia and Yu (2009). The parameters for the jump component are set at $\lambda = 100$, $\mu_J = 0.01$, and $\sigma_J = 0.05$.

To assess the impact of the microstructure noise at different sampling frequencies, we use signature plots as proposed in Andersen, Bollerslev, Diebold, and Labys (2000). The signature plots provide the sample mean of a daily realized moment based on returns sampled at different intraday frequencies. We take as an observation period $T = 1$ day, that is $T = 1/252$, and we assume a day has 6.5 trading hours.

Panel A of Figure A.1 shows the signature plots of $RM(j)$ as defined in (9), (10), and (11). This figure includes 99% confidence bands around the Monte Carlo estimates. For the second moments in the first row of panels the confidence intervals are very tight around the Monte Carlo estimate making them barely noticeable in the plot. For the third and fourth moments (the second and third row of panels), the 99% confidence intervals contain the Monte Carlo estimate as well as the theoretical limit.

The signature plot for the second moment $RM(2)$ depicts the well-known effect that microstructure noise has on realized volatility: as the sampling frequency increases (moving from right to left in the figure), the variance of the noise dominates that of the price process; but for lower frequencies, this effect attenuates. In contrast, the microstructure noise does not affect the signature plots of $RM(3)$ and $RM(4)$ in the same way. There is a small and insignificant bias in $RM(3)$ and $RM(4)$ relative to $IM(3)$ and $IM(4)$ but the bias does not increase with the intraday frequency.

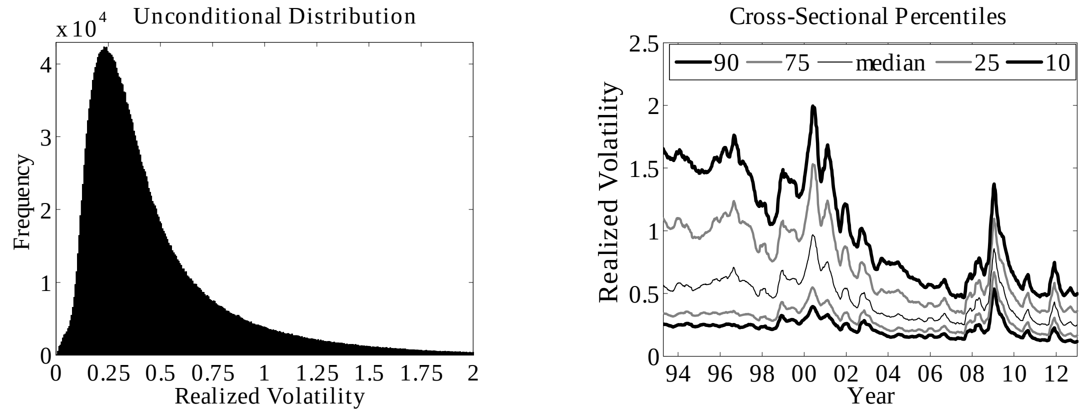
Chakravarty, Wood, and Van Ness (2004) document that bid-ask spreads declined significantly following the decimalization of NYSE-listed companies in 2001. This indicates that pre-decimalization prices exhibit an additional bid-ask spread generated by fractional minimum increments. To gauge the effect of this discontinuity on the realized moment measures, we conduct a Monte Carlo study similar to the one above, with the exception that observed prices are now measured in sixteenths of a dollar. To isolate the effect of fractional minimum increments, we assume here that observed prices are not affected by microstructure noise.

Panel B of Figure A.1 shows the signature plots for the realized moments. The plots reveal that realized volatility is the only moment affected by fractional changes in observed prices. As the frequency increases, the discontinuity of observed prices creates noise that is picked up by the volatility measure. However, the noise does not affect the third and fourth moments as shown by the 99% confidence intervals, which contain the Monte Carlo estimate as well as the theoretical limit.

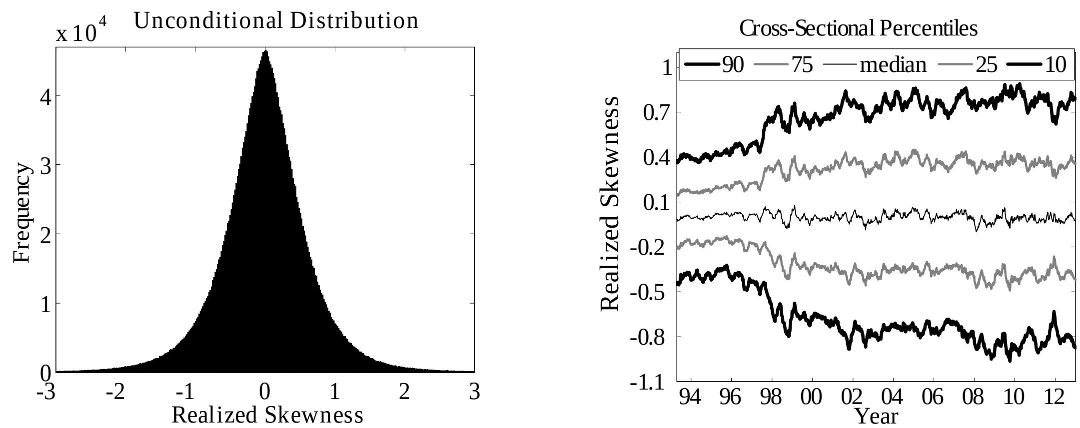
Figure 1
Histogram and Percentiles of Realized Moments

We display histograms and 3-month moving averages of percentiles of realized moments for the cross-section of stocks during the period January 1993 to December 2013. Figures for realized volatility, realized skewness, and realized kurtosis are reported in Panel A, Panel B, and Panel C, respectively. The sample contains 3,696,592 firm-week observations.

Panel A: Realized Volatility



Panel B: Realized Skewness



Panel C: Realized Kurtosis

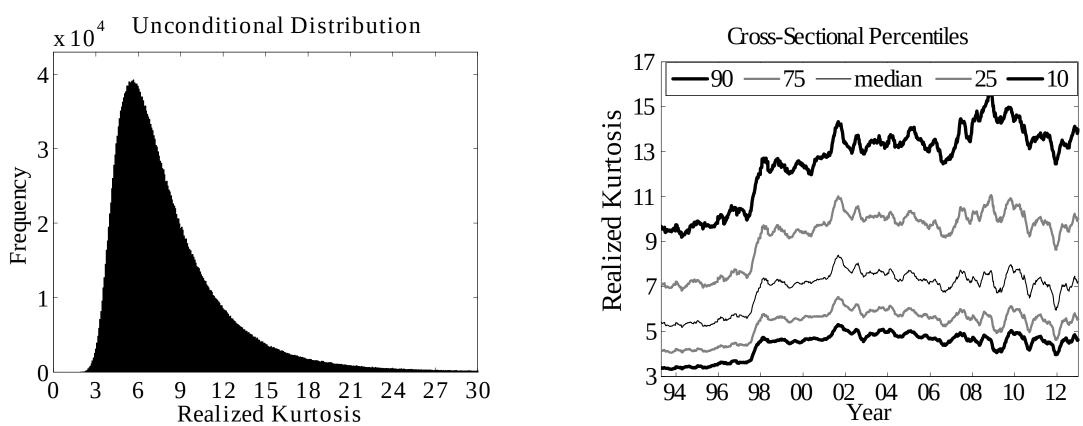
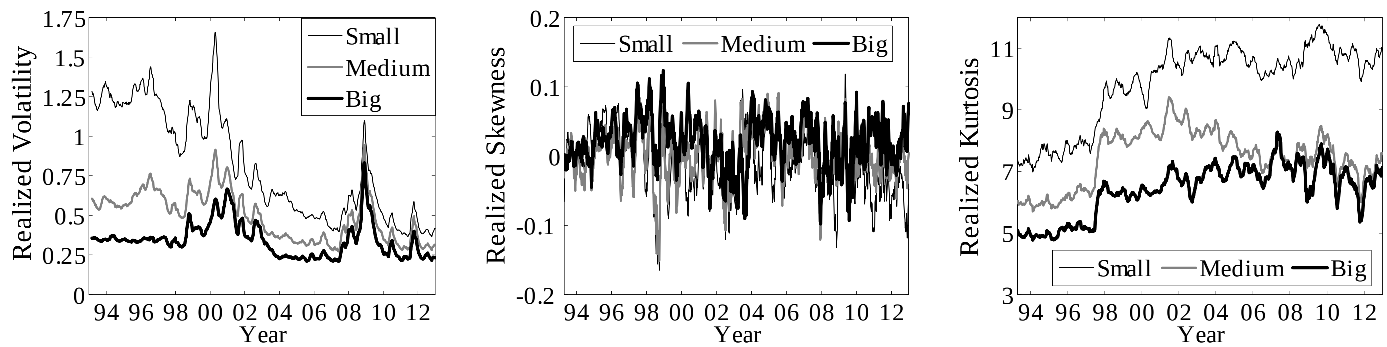


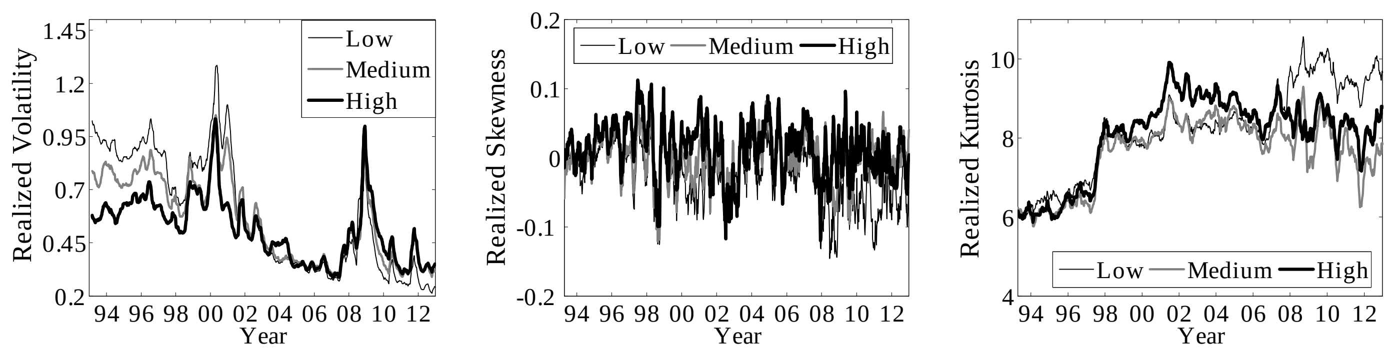
Figure 2
Realized Moments by Firm Characteristic

We display 3-month moving averages of the terciles of realized moments by size, book-to-market, and market beta. Panel A displays results for three size groups, Panel B for three book-to-market groups, and Panel C for three market beta groups. The sample begins January 1993 and ends December 2013.

Panel A: by Size



Panel B: by Book to Market



Panel C: by Market Beta

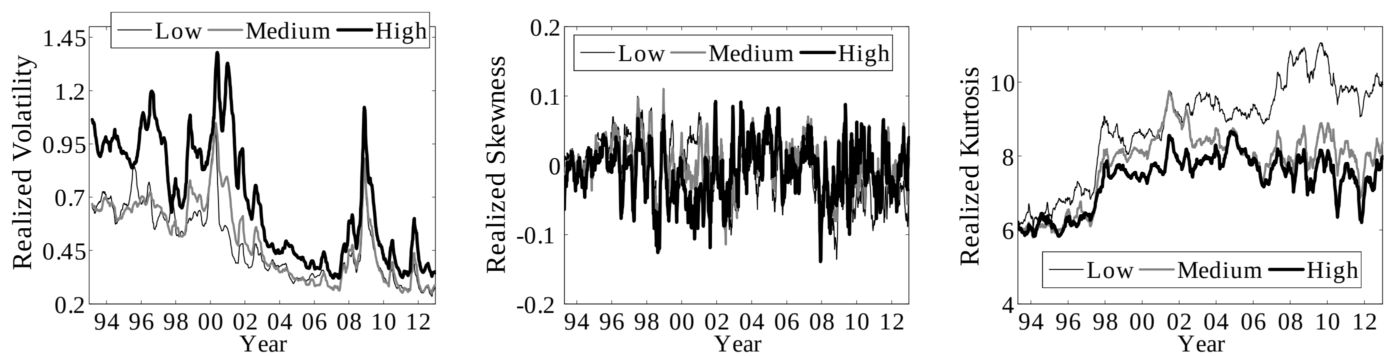


Figure 3

Short-Long Returns based on Realized Skewness and Lagged Return

Each week, we rank stocks based on realized skewness and lagged return, and compute the value-weighted weekly returns of the difference between portfolio 1 and portfolio 10 (short-long return) from January 1993 to December 2013. The plot shows the 3-month moving average of the short-long return.

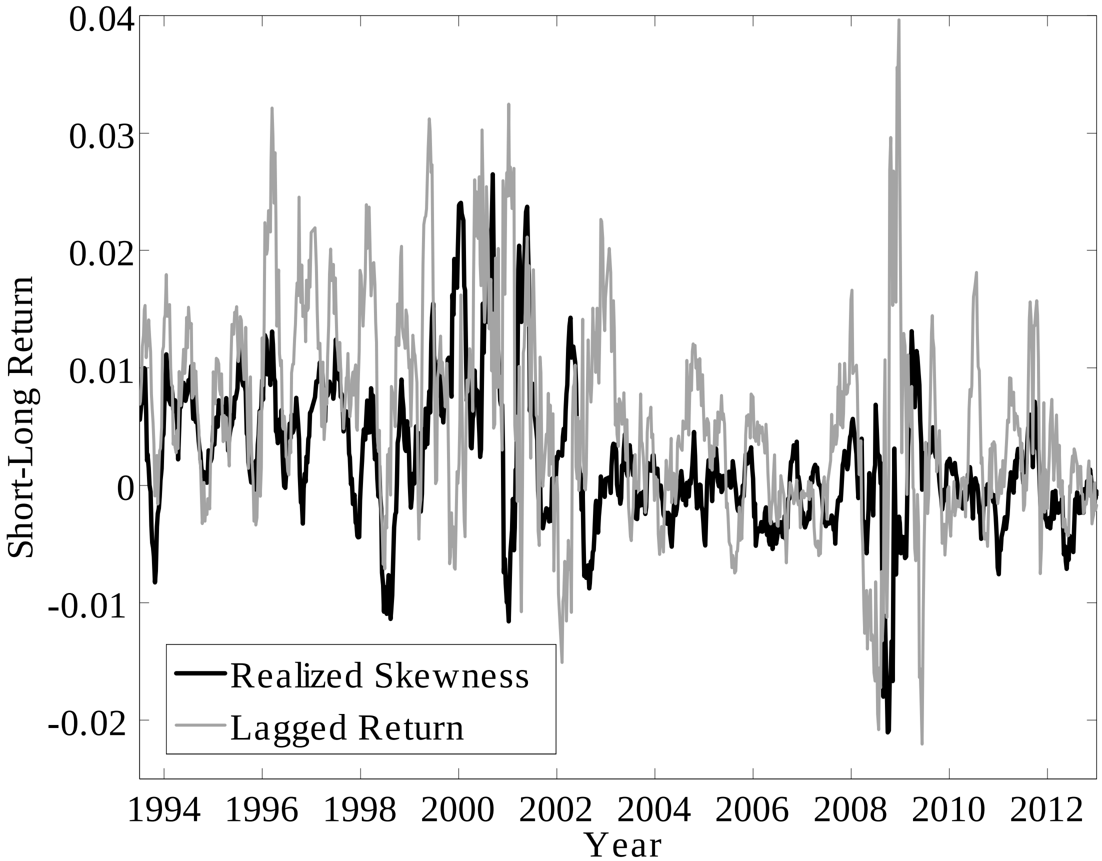


Figure 4
Characteristics of Portfolios Sorted by Realized Skewness and Lagged Return

Each week, stocks are ranked by either realized skewness or lagged return, and then sorted into deciles. We report the average firm characteristic for each decile portfolio. Firm characteristics are realized volatility, realized kurtosis, size, market beta, maximum return, minimum return, illiquidity and number of intraday transactions. The sample begins January 1993 and ends December 2013.

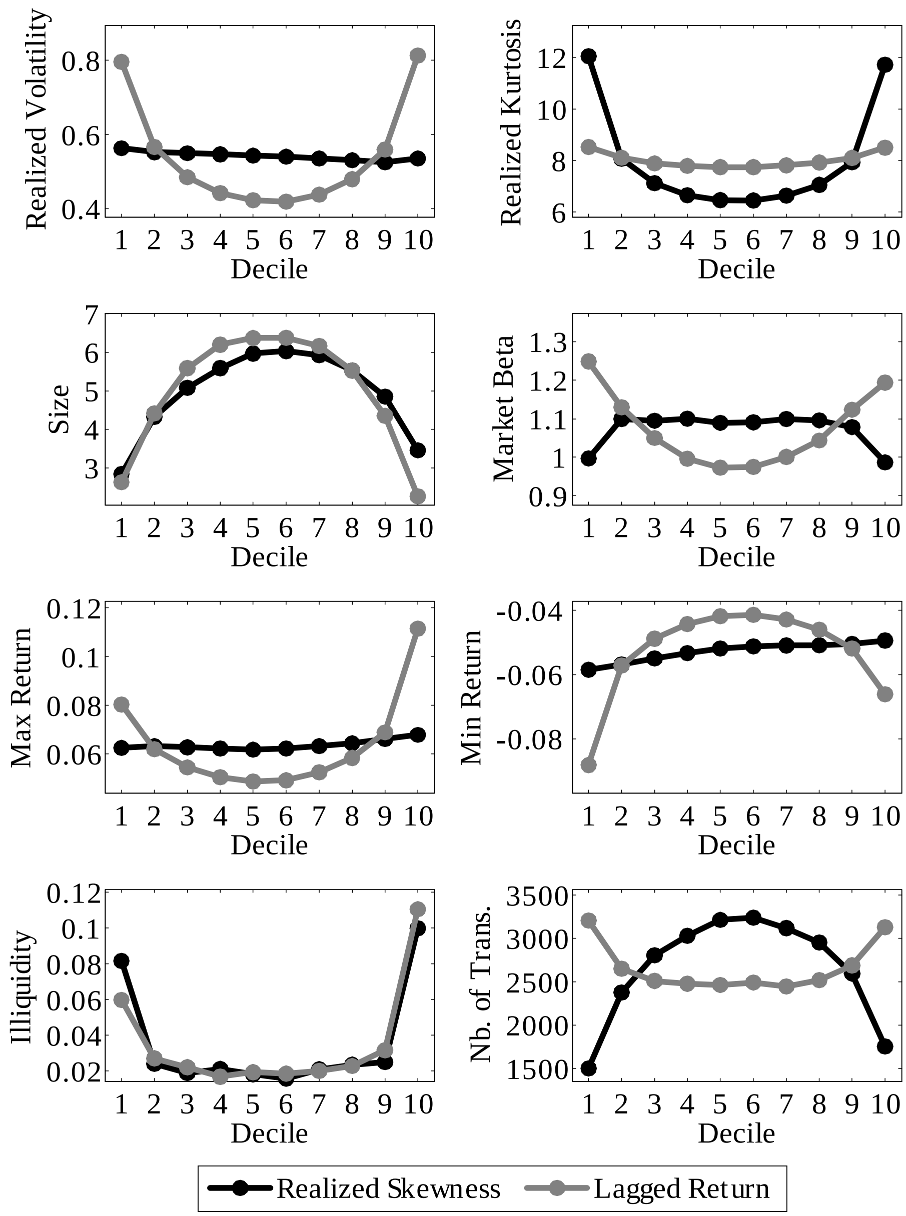
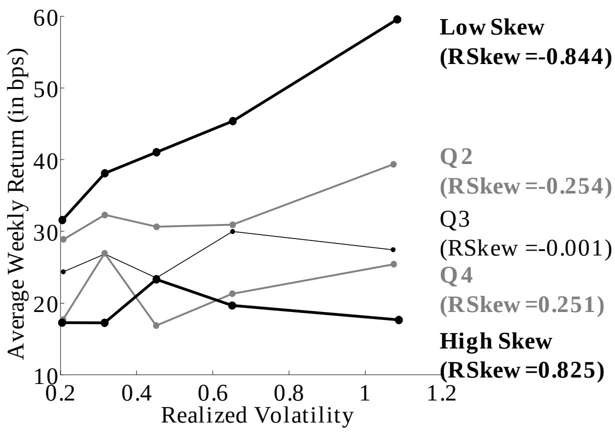


Figure 5
Interaction between Realized Volatility, Realized Skewness, and Stock Returns

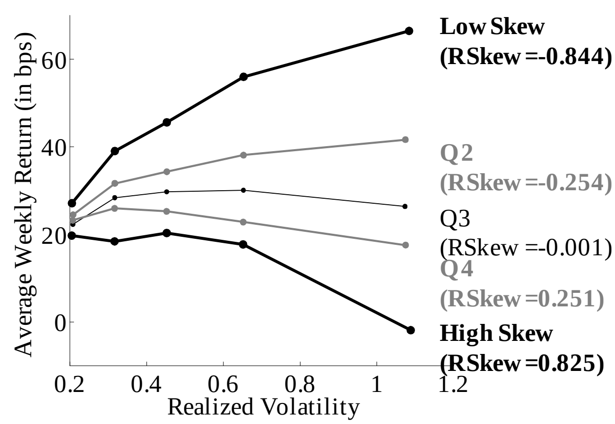
Each week, stocks are first ranked by realized skewness into five quintiles and then, within each quintile, stocks are sorted once again into five quintiles by realized volatility (Panel A) and idiosyncratic volatility (Panel B). We report value- and equal-weighted returns (in bps) for different levels of volatility and realized skewness. Each line represents different quintiles of realized skewness. The average realized skewness in each quintile is reported in parentheses. The sample begins January 1993 and ends December 2013.

Panel A: Interaction between Realized Volatility, Realized Skewness, and Stock Returns

Value-Weighted

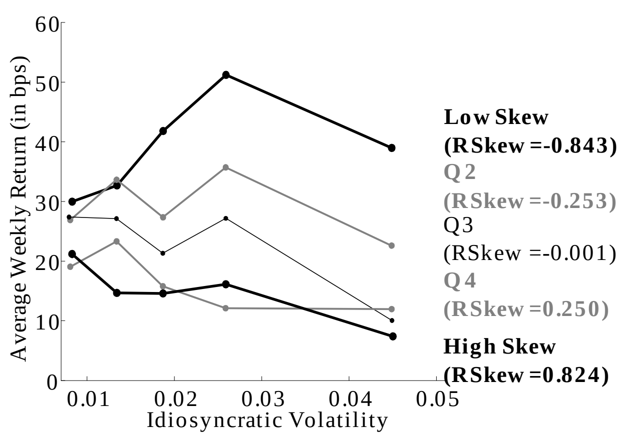


Equal-Weighted



Panel B: Interaction between Idiosyncratic Volatility, Realized Skewness, and Stock Returns

Value-Weighted



Equal-Weighted

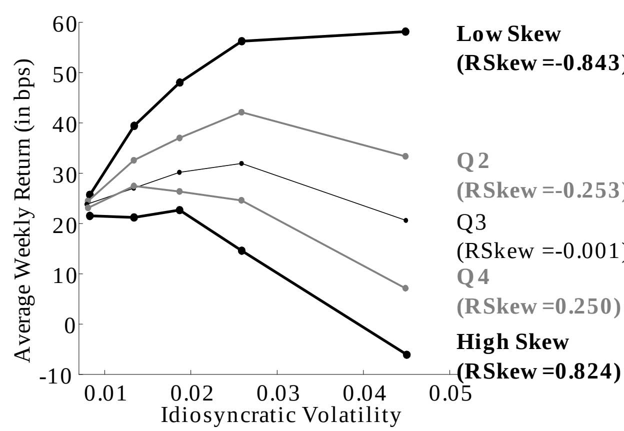


Figure A.1
Signature Plots for Realized Moments

We show signature plots for the three daily realized moments, $RM(2)$, $RM(3)$, and $RM(4)$ computed using equations (9), (10), and (11). The intraday sampling frequency on the horizontal axis is in seconds. The dotted lines represent the theoretical limit of realized moments as given by equations (14), (15), and (16). Monte Carlo estimates are plotted in continuous dark lines. Confidence intervals at 99% are shown in grey lines. In Panel A, observed prices have a microstructure noise component, which is simulated from a mean-zero normal distribution with a standard deviation of 0.05%. In Panel B, prices are only observed in sixteenths of a dollar.

Panel A:
Market Microstructure Noise

Panel B:
Discrete Price Changes

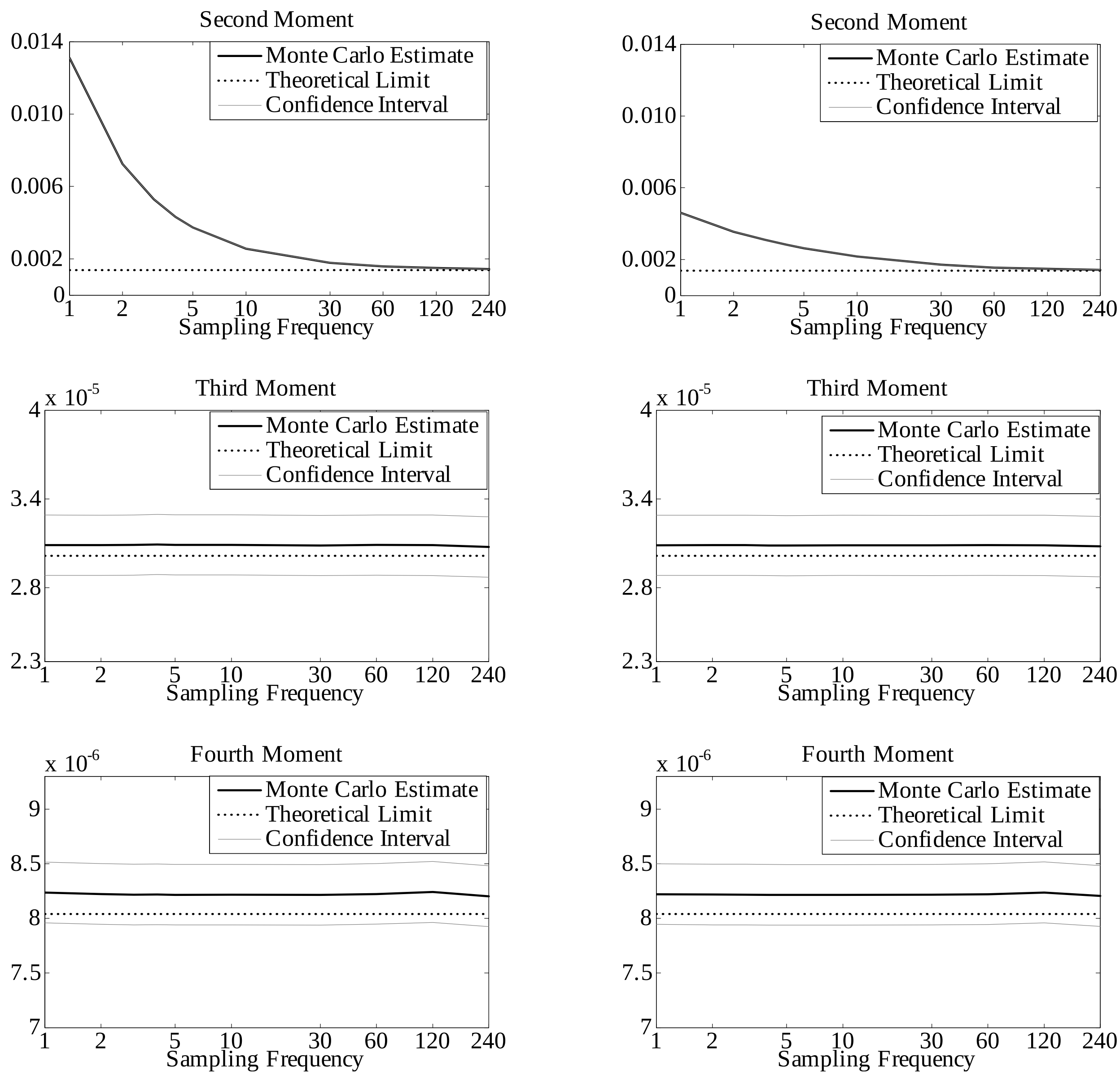


Table 1
Characteristics of Portfolios Sorted by Realized Moments

Each week, stocks are ranked by their realized moment and sorted into deciles. The equal-weighted characteristics of each decile are computed over the same week. This procedure is repeated for every week from January 1993 through December 2013. Panel A displays the average results for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Average characteristics of the portfolios are reported for the realized moments, Size (market capitalization in $billions), BE/ME (book-to-market equity ratio), Historical Skewness (one month historical skewness from daily returns), Market Beta, Lagged Return, Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000)), Maximum Return (of the previous month), Illiquidity (monthly average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), Number of Analysts (from I/B/E/S), Credit Rating (1= AAA, 8= BBB+, 17= CCC+, 22=D), Price (stock price), Intraday Transactions (intraday transactions per day) and Number of Stocks.

Panel A: Characteristics of Portfolios Sorted by Realized Volatility

Deciles	1	2	3	4	5	6	7	8	9	10
Realized Volatility	0.173	0.239	0.289	0.344	0.411	0.493	0.592	0.713	0.880	1.284
Realized Skewness	0.016	0.011	0.006	-0.001	-0.003	-0.006	-0.011	-0.016	-0.024	-0.022
Realized Kurtosis	7.5	7.2	7.3	7.4	7.6	7.8	8.1	8.4	8.9	9.9
Size	13.17	11.95	9.01	6.33	4.42	3.01	2.02	1.29	0.85	0.49
BE/ME	0.251	0.461	0.537	0.603	0.579	0.447	0.392	0.364	0.368	0.326
Historical Skewness	0.129	0.106	0.115	0.129	0.149	0.168	0.198	0.219	0.250	0.327
Market Beta	0.68	0.81	0.89	0.97	1.06	1.16	1.25	1.32	1.33	1.26
Lagged Return	0.004	0.004	0.004	0.004	0.004	0.005	0.005	0.006	0.008	0.022
Idiosyncratic Volatility	0.010	0.012	0.014	0.016	0.019	0.021	0.025	0.028	0.033	0.044
Coskewness	-0.003	-0.007	-0.011	-0.016	-0.017	-0.018	-0.022	-0.025	-0.029	-0.037
Maximum Return	0.030	0.035	0.040	0.046	0.053	0.061	0.070	0.080	0.093	0.128
Illiquidity	0.010	0.007	0.007	0.008	0.010	0.012	0.017	0.022	0.036	0.219
Number of Analysts	8.4	9.5	9.7	9.5	8.9	8.2	7.3	6.2	5.2	3.5
Credit Rating	7.3	7.6	8.0	8.4	8.7	8.9	8.9	8.9	8.8	8.5
Price	87.6	75.4	46.9	35.7	31.7	28.2	25.3	22.0	18.4	15.5
Intraday Transactions	2,287	2,812	3,029	3,102	3,038	2,899	2,715	2,461	2,228	2,015
Number of Stocks	298	299	299	299	298	299	299	299	299	298

Table 1 (Continued)

Panel B: Characteristics of Portfolios Sorted by Realized Skewness

Deciles	1	2	3	4	5	6	7	8	9	10
Realized Skewness	-1.16	-0.54	-0.33	-0.18	-0.06	0.06	0.18	0.32	0.53	1.13
Realized Volatility	0.562	0.552	0.549	0.546	0.543	0.540	0.535	0.530	0.525	0.535
Realized Kurtosis	12.0	8.1	7.1	6.7	6.5	6.5	6.6	7.1	7.9	11.7
Size	2.84	4.33	5.08	5.59	5.96	6.03	5.92	5.53	4.85	3.45
BE/ME	0.345	0.355	0.420	0.482	0.488	0.529	0.482	0.461	0.393	0.370
Historical Skewness	0.155	0.164	0.165	0.168	0.169	0.174	0.178	0.188	0.204	0.224
Market Beta	1.00	1.10	1.10	1.10	1.09	1.09	1.10	1.10	1.08	0.99
Lagged Return	-0.028	-0.017	-0.009	-0.003	0.003	0.009	0.015	0.022	0.031	0.042
Idiosyncratic Volatility	0.023	0.023	0.022	0.022	0.021	0.021	0.022	0.022	0.022	0.023
Coskewness	-0.024	-0.020	-0.017	-0.017	-0.016	-0.015	-0.014	-0.014	-0.014	-0.018
Maximum Return	0.062	0.063	0.063	0.062	0.062	0.062	0.063	0.064	0.066	0.068
Illiquidity	0.082	0.024	0.019	0.021	0.018	0.016	0.021	0.023	0.025	0.100
Number of Analysts	6.0	7.5	7.8	8.0	8.1	8.1	8.2	8.2	7.9	6.6
Credit Rating	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4	8.4
Price	30.5	37.8	40.4	40.9	40.3	39.2	42.8	40.9	39.3	34.6
Intraday Transactions	1,502	2,378	2,805	3,029	3,211	3,239	3,118	2,953	2,595	1,753
Number of Stocks	298	299	299	299	298	299	299	299	299	298

Panel C: Characteristics of Portfolios Sorted by Realized Kurtosis

Deciles	1	2	3	4	5	6	7	8	9	10
Realized Kurtosis	4.0	4.8	5.4	5.9	6.6	7.3	8.2	9.4	11.3	17.5
Realized Volatility	0.436	0.481	0.509	0.526	0.539	0.550	0.561	0.575	0.594	0.647
Realized Skewness	0.006	0.006	0.005	0.005	0.005	0.001	-0.002	-0.009	-0.016	-0.049
Size	13.88	8.33	6.17	5.01	4.10	3.49	2.95	2.47	1.99	1.27
BE/ME	0.781	0.592	0.499	0.389	0.403	0.381	0.322	0.308	0.314	0.337
Hskew	0.134	0.157	0.165	0.177	0.184	0.182	0.190	0.191	0.201	0.208
Beta	1.09	1.15	1.15	1.14	1.12	1.11	1.08	1.04	0.99	0.86
Lagged Return	0.004	0.006	0.006	0.007	0.007	0.007	0.007	0.007	0.007	0.007
Idiosyncratic Volatility	0.018	0.020	0.021	0.022	0.023	0.023	0.023	0.024	0.024	0.025
Coskewness	-0.006	-0.007	-0.011	-0.013	-0.016	-0.018	-0.020	-0.023	-0.026	-0.032
Maximum Return	0.053	0.059	0.062	0.064	0.065	0.065	0.066	0.066	0.066	0.069
Illiquidity	0.005	0.006	0.007	0.013	0.016	0.018	0.020	0.030	0.050	0.185
Number of Analysts	10.8	10.0	9.2	8.6	7.9	7.4	6.8	6.2	5.4	3.9
Credit Rating	8.0	8.3	8.4	8.4	8.5	8.5	8.5	8.5	8.4	8.4
Price	43.7	49.8	47.1	44.5	42.4	38.2	35.7	31.0	29.1	25.1
Intraday Transactions	5,906	4,204	3,511	3,007	2,578	2,225	1,858	1,526	1,144	629
Number of Stocks	298	299	299	299	298	299	299	299	299	298

Table 2
Realized Moments and the Cross-Section of Stock Returns

We report value- and equal-weighted weekly returns (in bps) of decile portfolios formed from realized moments, the corresponding t-statistics (in parentheses), and the return difference between portfolio 10 and portfolio 1, 9 and 2, and 8 and 3, over the period January 1993 to December 2013. Panel A displays the results for realized volatility, Panel B for realized skewness, and Panel C for realized kurtosis. Each panel reports the value-weighted portfolios and the equal-weighted portfolios. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized moment measure. Alpha is the intercept from time-series regressions of the returns of the portfolio using the Carhart four factor model.

Panel A: Realized Volatility and the Cross-Section of Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low	9-2	8-3
						Value weighted							
Raw Returns	21.81	24.83	29.05	27.07	25.12	27.82	30.03	26.04	33.61	32.87	11.05	8.78	-3.01
	(4.15)	(4.00)	(4.06)	(3.24)	(2.62)	(2.50)	(2.38)	(1.81)	(2.17)	(1.93)	(0.72)	(0.69)	(-0.30)
Alpha, C4	8.95	9.07	11.09	8.67	5.85	8.19	9.74	4.91	12.51	12.16	1.97	2.19	-7.43
	(3.36)	(3.86)	(4.87)	(3.17)	(1.68)	(1.85)	(1.92)	(0.83)	(1.79)	(1.42)	(0.21)	(0.27)	(-1.12)
						Equal weighted							
Raw Returns	21.30	24.58	26.88	30.04	29.26	32.67	32.33	34.33	35.40	27.43	6.13	10.82	7.45
	(4.51)	(4.12)	(4.10)	(4.14)	(3.58)	(3.48)	(3.05)	(2.90)	(2.69)	(1.92)	(0.50)	(1.10)	(0.97)
Alpha, C4	8.41	8.00	8.88	10.44	8.06	10.94	9.66	11.76	12.17	4.43	-5.23	2.92	1.63
	(3.79)	(3.85)	(4.30)	(5.20)	(3.89)	(4.45)	(3.30)	(3.45)	(2.78)	(0.75)	(-0.79)	(0.57)	(0.41)

Panel B: Realized Skewness and the Cross-Section of Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low	9-2	8-3
						Value weighted							
Raw Returns	43.34	34.36	35.21	25.57	29.50	22.36	24.23	15.12	17.06	24.09	-19.26	-17.31	-20.09
	(5.08)	(4.02)	(4.08)	(2.92)	(3.52)	(2.80)	(3.02)	(1.90)	(2.21)	(3.27)	(-3.70)	(-3.72)	(-4.53)
Alpha, C4	26.02	17.25	18.34	8.94	14.91	6.83	7.87	-0.55	-0.21	8.08	-19.19	-18.70	-20.13
	(6.43)	(5.19)	(5.61)	(2.66)	(5.07)	(2.66)	(2.74)	(-0.19)	(-0.08)	(2.58)	(-3.70)	(-4.01)	(-4.52)
						Equal weighted							
Raw Returns	50.23	42.85	36.45	30.99	30.00	24.20	25.48	21.04	17.27	15.83	-34.41	-25.58	-15.41
	(5.55)	(4.53)	(3.87)	(3.35)	(3.33)	(2.73)	(2.91)	(2.45)	(2.07)	(2.07)	(-8.86)	(-7.49)	(-5.44)
Alpha, C4	30.41	22.06	15.69	10.56	9.56	3.93	5.21	0.74	-2.54	-2.75	-34.42	-25.85	-16.19
	(8.62)	(7.09)	(5.70)	(4.17)	(4.21)	(1.91)	(2.55)	(0.35)	(-1.16)	(-1.11)	(-9.26)	(-7.82)	(-5.86)

Panel C: Realized Kurtosis and the Cross-Section of Stock Returns

	Low	2	3	4	5	6	7	8	9	High	High-Low	9-2	8-3
						Value weighted							
Raw Returns	21.69	27.60	25.97	30.56	26.30	22.58	28.71	31.75	23.04	32.15	10.46	-4.57	5.77
	(2.58)	(3.42)	(3.21)	(3.82)	(3.32)	(2.80)	(3.67)	(3.90)	(3.00)	(4.35)	(2.01)	(-1.06)	(1.35)
Alpha, C4	5.94	11.23	8.47	13.91	9.66	6.15	11.28	14.49	5.93	15.86	8.68	-6.55	4.78
	(2.18)	(4.95)	(3.26)	(5.45)	(3.39)	(2.00)	(3.85)	(4.21)	(1.71)	(4.31)	(1.80)	(-1.61)	(1.11)
						Equal weighted							
Raw Returns	23.31	27.41	27.90	27.24	29.35	29.07	29.58	31.23	33.56	35.62	12.31	6.16	3.33
	(2.64)	(3.01)	(3.02)	(2.92)	(3.19)	(3.20)	(3.33)	(3.55)	(3.99)	(4.69)	(2.75)	(1.93)	(1.25)
Alpha, C4	5.50	6.95	6.85	5.80	7.80	8.00	8.73	10.84	14.18	18.16	11.41	5.98	2.74
	(2.24)	(3.18)	(3.01)	(2.45)	(3.21)	(3.39)	(3.63)	(4.17)	(5.05)	(6.09)	(3.09)	(2.12)	(1.08)

Table 3
Fama-MacBeth Cross-Sectional Regressions

We report results from Fama-MacBeth cross-sectional regressions of weekly stock returns (in bps) on firm characteristics for the period January 1993 to December 2013. Firm characteristics are Realized Volatility, Realized Skewness, Realized Kurtosis, Size (market capitalization in $billions), BE/ME (book-to-market equity ratio), Market Beta, Realized Value-at-Risk - 1% from 5-minute returns over one week, Lagged Return (in bps), Historical Skewness (one month historical skewness from daily returns), Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000) with 24 months of data), Maximum (minimum) Return of previous month and week in bps, Number of Analysts (from I/B/E/S), Illiquidity (5-day average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), and Number of Intraday Transactions. We report the average of the coefficient estimates for the weekly regressions along with the Newey-West t-statistic (in parentheses).

	(1)	(2)	(3)	(4)	(5)
Intercept	29.0	33.7	25.7	24.0	110.6
	(4.02)	(3.45)	(2.44)	(2.95)	(3.93)
Realized Volatility	5.1			2.8	-1.1
	(0.41)			(0.21)	(-0.13)
Realized Skewness		-20.5		-22.2	-11.2
		(-8.07)		(-9.35)	(-6.59)
Realized Kurtosis			0.92	0.92	0.7
			(2.82)	(2.31)	(2.45)
log (Size)					-5.9
					(-2.34)
log (BE/ME)					2.6
					(1.75)
Market Beta					-3.7
					(-0.57)
Realized Value-at-Risk - 1%					0.05
					(2.38)
Lagged Return					-0.026
					(-9.36)
Historical Skewness					0.4
					(0.29)
Idiosyncratic Volatility					-280
					(-1.02)
Coskewness					21.6
					(1.49)
Maximum Monthly Return					0.001
					(0.11)
Minimum Monthly Return					0.0114
					(2.06)
Maximum Weekly Return					-0.014
					(-3.02)
Minimum Weekly Return					-0.014
					(-2.59)
log (Number of Analysts+1)					0.2
					(0.08)
Illiquidity					-130
					(-3.31)
log (Number of Intraday transactions)					7.2
					(2.97)
Adjusted Rsquare	0.022	0.002	0.002	0.028	0.088

Table 4

Correlations of Firm Characteristics

We report unconditional correlations (in %) of firm characteristics for the period January 1993 to December 2013. Firm characteristics are Realized Volatility, Realized Skewness, Realized Kurtosis, Size (market capitalization in $billions), BE/ME (book-to-market equity ratio), Market Beta, 1% Realized Value-at-Risk computed from 5-minute returns over one week, Lagged Return (in bps), Historical Skewness (one month historical skewness from daily returns), Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000) with 24 months of data), Maximum (minimum) Return of previous month and week in bps, Number of Analysts (from I/B/E/S), Illiquidity (5-day average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), and Number of Intraday Transactions.

	Realized Volatility																	
Realized Skewness	-1.4	Realized Skewness																
Realized Kurtosis	9.0	-3.0	Realized Kurtosis															
log (Size)	-42.7	2.1	-37.8	log (Size)														
log (BE/ME)	-20.1	0.1	0.5	-2.5	log (BE/ME)													
Market Beta	8.5	0.5	-7.1	4.4	0.8	Market Beta												
Realized Value-at-Risk - 1%	79.4	-8.1	12.5	-39.6	-16.5	5.7	Realized Value-at-Risk - 1%											
Lagged Return	5.8	22.3	0.7	-5.4	-0.2	-0.1	0.6	Lagged Return										
Historical Skewness	10.1	2.5	1.0	-5.5	-3.4	3.1	7.5	8.0	Historical Skewness									
Idiosyncratic Volatility	67.9	0.0	4.5	-32.0	-16.7	12.6	54.7	15.7	15.3	Idiosyncratic Volatility								
Coskewness	-7.1	0.2	-1.7	5.6	7.0	2.0	-5.7	0.2	-1.4	-6.1	Coskewness							
Maximum Monthly Return	53.6	2.4	2.0	-24.6	-7.8	7.0	42.4	24.3	30.0	88.8	-4.1	Maximum Monthly Return						
Minimum Monthly Return	-59.7	4.0	-0.8	21.7	8.7	-9.6	-48.9	9.0	16.4	-69.6	4.8	-45.5	Minimum Monthly Return					
Maximum Weekly Return	50.8	12.0	3.3	-19.2	-6.2	5.4	38.4	47.0	12.5	59.0	-3.3	62.4	-35.2	Maximum Weekly Return				
Minimum Weekly Return	-55.0	16.2	-1.6	16.5	6.9	-7.4	-47.6	34.5	1.0	-48.0	4.3	-33.2	64.1	-27.1	Minimum Weekly Return			
log (Nb Analysts+1)	-10.1	3.1	-25.9	64.8	0.2	7.8	-12.9	-1.6	1.3	-0.8	4.7	-2.8	-1.0	-2.5	0.3	log (Nb Analysts+1)		
Illiquidity	1.1	0.0	1.4	-1.4	0.7	-0.2	0.8	0.2	0.2	1.0	-0.2	0.9	-0.7	0.8	-0.9	-0.7	Illiquidity	
log (Nb of Intraday Trans.)	-17.3	2.0	-31.9	59.6	32.4	10.4	-18.0	-0.8	-1.6	-3.9	5.0	0.9	-3.8	3.1	-5.4	42.4	-0.6	log(Nb Intraday Trans.)

Table 5
Drift Adjusted Realized Moments and the Cross-Section of Stock Returns

We report value- and equal-weighted weekly returns (in bps) of decile portfolios formed from drift adjusted realized moments, the corresponding t-statistics (in parentheses), and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) over the period January 1993 to December 2013. Panel A displays results for the drift adjusted realized skewness and Panel B for the drift adjusted realized kurtosis. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized moment measure. Alpha is the intercept from time-series regressions of the returns of the portfolio using the Carhart four factor model.

Panel A: Drift Adjusted Realized Skewness

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	36.76	28.11	29.15	23.44	27.93	24.23	25.31	24.81	22.00	25.65	-11.10
	(4.58)	(3.27)	(3.53)	(2.85)	(3.39)	(2.97)	(3.17)	(3.09)	(2.78)	(3.44)	(-2.42)
Alpha, C4	19.47	10.05	12.82	7.39	12.44	8.67	9.32	8.17	6.64	9.32	-11.40
	(5.63)	(2.96)	(4.34)	(2.68)	(4.34)	(3.17)	(3.49)	(3.07)	(2.37)	(3.00)	(-2.49)
						Equal weighted					
Raw Returns	44.40	35.82	33.12	29.26	28.45	26.87	27.40	25.60	21.88	21.41	-22.99
	(5.00)	(3.88)	(3.58)	(3.20)	(3.16)	(3.04)	(3.13)	(2.93)	(2.59)	(2.75)	(-7.16)
Alpha, C4	24.72	15.03	12.63	8.75	8.29	6.38	7.16	5.01	2.09	2.64	-23.33
	(7.55)	(5.22)	(4.85)	(3.61)	(3.67)	(3.07)	(3.53)	(2.41)	(1.00)	(1.12)	(-7.64)

Panel B: Drift Adjusted Realized Kurtosis

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	21.22	27.21	27.07	30.33	26.40	22.01	30.44	31.21	23.48	31.96	10.74
	(2.53)	(3.36)	(3.37)	(3.77)	(3.36)	(2.71)	(3.85)	(3.85)	(3.08)	(4.28)	(2.06)
Alpha, C4	5.45	10.86	9.72	13.60	9.71	5.58	13.18	14.19	6.35	15.50	8.80
	(2.02)	(4.76)	(3.83)	(5.14)	(3.41)	(1.81)	(4.20)	(4.18)	(1.84)	(4.16)	(1.82)
						Equal weighted					
Raw Returns	23.31	26.58	27.85	27.40	29.05	29.35	29.94	30.44	34.68	35.57	12.26
	(2.65)	(2.92)	(3.01)	(2.93)	(3.17)	(3.23)	(3.36)	(3.47)	(4.12)	(4.68)	(2.74)
Alpha, C4	5.55	6.09	6.71	5.90	7.68	8.23	8.95	10.16	15.36	18.05	11.25
	(2.27)	(2.84)	(2.98)	(2.41)	(3.26)	(3.47)	(3.72)	(3.93)	(5.48)	(6.06)	(3.05)

Table 6
Fama-MacBeth Cross-Sectional Regressions for Drift Adjusted Realized Moments

We report results from Fama-MacBeth cross-sectional regressions of weekly stock returns (in bps) on firm characteristics for the period January 1993 to December 2013. Firm characteristics are drift-adjusted Realized Volatility, Realized Skewness, and Realized Kurtosis, Size (market capitalization in \$billions), BE/ME (book-to-market equity ratio), Market Beta, Realized Value-at-Risk - 1% from 5-minute returns over one week, Lagged Return (in bps), Historical Skewness (one month historical skewness from daily returns), Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), Coskewness (computed as in Harvey and Siddique (2000) with 24 months of data), Maximum (and minimum) Return of previous month in bps, Maximum (and minimum) Return of previous week in bps, Number of Analysts (from I/B/E/S), Illiquidity (5-day average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), and Number of Intraday Transactions. We report the average of the coefficient estimates for the weekly regressions along with the Newey-West t-statistic (in parentheses).

	(1)	(2)	(3)	(4)	(5)
Intercept	28.9	33.0	24.5	22.2	110.2
	(4.01)	(3.39)	(2.33)	(2.75)	(3.92)
Realized Volatility - Drift Adjusted	5.3			3.4	-1.5
	(0.42)			(0.26)	(-0.17)
Realized Skewness - Drift Adjusted		-15.8		-16.9	-10.2
		(-6.94)		(-8.08)	(-6.04)
Realized Kurtosis - Drift Adjusted			1.08	1.02	0.6
			(3.23)	(2.54)	(2.28)
log (Size)					-5.9
					(-2.34)
log (BE/ME)					2.6
					(1.72)
Market Beta					-3.7
					(-0.58)
Realized Value-at-Risk - 1%					0.05
					(2.56)
Lagged Return					-0.027
					(-9.56)
Historical Skewness					0.3
					(0.21)
Idiosyncratic Volatility					-281
					(-1.02)
Coskewness					21.5
					(1.49)
Maximum Monthly Return					0.001
					(0.20)
Minimum Monthly Return					0.0119
					(2.14)
Maximum Weekly Return					-0.015
					(-3.39)
Minimum Weekly Return					-0.016
					(-3.01)
log (Number of Analysts+1)					0.2
					(0.10)
Illiquidity					-128
					(-3.26)
log (Number of Intraday transactions)					7.2
					(2.97)
Adjusted Rsquare	0.023	0.001	0.002	0.027	0.087

Table 7
Alternative Realized Volatility Estimators, Higher Moments, and the Cross-Section of Stock Returns

We report the value-weighted and equal-weighted weekly returns (in bps) of the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) during January 1993 to December 2013. Stocks are ranked based on their realized moments using different realized volatility estimators. Panel A displays the results for alternative realized volatility estimators, Panel B for realized skewness scaled by alternative realized volatility estimators, and Panel C for realized kurtosis scaled by alternative realized volatility estimators. Alpha is the intercept from time-series regressions of the returns of the portfolio that buys portfolio 10 and sells portfolio 1 using the Carhart four factor model.

Panel A: Long-Short Returns for Alternative Realized Volatility Estimators

	RV	BPV	minRV	medRV
	Value weighted			
Raw Returns	11.05	10.49	10.74	7.45
	(0.72)	(0.65)	(0.69)	(0.48)
Alpha, C4	1.97	-1.39	-0.37	-3.64
	(0.21)	(-0.14)	(-0.04)	(-0.38)
	Equal weighted			
Raw Returns	6.13	3.45	2.38	2.84
	(0.50)	(0.26)	(0.18)	(0.22)
Alpha, C4	-5.23	-9.92	-10.37	-9.87
	(-0.79)	(-1.39)	(-1.47)	(-1.44)

Panel B: Long-Short Returns for Realized Skewness Scaled by Alternative Realized Volatility Estimators

Realized Skewness scaled by	RV	BPV	minRV	medRV
	Value weighted			
Raw Returns	-19.26	-18.51	-18.07	-20.70
	(-3.70)	(-3.57)	(-4.13)	(-4.31)
Alpha, C4	-19.19	-18.46	-19.07	-21.92
	(-3.70)	(-3.57)	(-4.33)	(-4.53)
	Equal weighted			
Raw Returns	-34.41	-34.25	-20.69	-23.96
	(-8.86)	(-8.70)	(-7.51)	(-8.27)
Alpha, C4	-34.42	-34.07	-21.30	-24.49
	(-9.26)	(-9.05)	(-7.89)	(-8.58)

Panel C: Long-Short Returns for Realized Kurtosis Scaled by Alternative Realized Volatility Estimators

Realized Kurtosis scaled by	RV	BPV	minRV	medRV
	Value weighted			
Raw Returns	10.46	9.34	10.61	10.33
	(2.01)	(1.76)	(2.05)	(2.01)
Alpha, C4	8.68	7.76	9.89	9.21
	(1.80)	(1.59)	(2.24)	(2.03)
	Equal weighted			
Raw Returns	12.31	11.43	12.88	12.56
	(2.75)	(2.52)	(2.72)	(2.77)
Alpha, C4	11.41	10.82	13.35	13.11
	(3.09)	(2.91)	(3.53)	(3.55)

Table 8
Realized Moments and Returns for Different Subsamples

We report weekly returns (in bps) and t-statistics of decile portfolios formed from ranking stocks by their realized moments. We also report the difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment). Panel A displays results for realized skewness and Panel B for realized kurtosis. Each panel reports the value- and equal-weighted decile portfolio returns for the month of January, for all months excluding January, and only for NYSE stocks. The sample begins January 1993 and ends December 2013.

Panel A: Realized Skewness Effects for Different Subsamples

	Value weighted Raw Returns										
	Low	2	3	4	5	6	7	8	9	High	High-Low
January	61.80	30.36	32.07	43.92	32.53	31.50	21.91	23.31	17.56	29.28	-32.51
	(2.12)	(1.07)	(0.96)	(1.54)	(1.18)	(1.22)	(0.79)	(0.86)	(0.72)	(1.14)	(-1.69)
Non-January	41.73	34.72	35.49	23.97	29.24	21.55	24.43	14.41	17.01	23.63	-18.10
	(4.67)	(3.88)	(3.98)	(2.61)	(3.32)	(2.57)	(2.91)	(1.73)	(2.09)	(3.07)	(-3.34)
NYSE	62.47	40.38	38.05	39.61	34.85	28.27	24.76	16.59	21.28	25.36	-37.11
	(5.18)	(3.28)	(3.00)	(3.09)	(2.77)	(2.34)	(2.06)	(1.44)	(1.92)	(2.36)	(-4.38)

	Equal weighted Raw Returns										
	Low	2	3	4	5	6	7	8	9	High	High-Low
January	76.97	64.68	42.23	40.49	47.89	41.61	37.52	40.41	28.60	29.58	-47.39
	(2.43)	(2.03)	(1.35)	(1.33)	(1.61)	(1.39)	(1.28)	(1.38)	(1.04)	(1.22)	(-3.30)
Non-January	47.89	40.94	35.95	30.16	28.44	22.68	24.42	19.35	16.27	14.62	-33.27
	(5.07)	(4.14)	(3.64)	(3.11)	(3.01)	(2.44)	(2.66)	(2.15)	(1.86)	(1.82)	(-8.25)
NYSE	66.75	53.14	44.49	41.11	31.50	30.25	24.34	18.09	14.33	7.33	-59.42
	(5.90)	(4.49)	(3.69)	(3.44)	(2.68)	(2.63)	(2.13)	(1.62)	(1.34)	(0.75)	(-11.48)

Panel B: Realized Kurtosis Effects for Different Subsamples

	Value weighted Raw Returns										
	Low	2	3	4	5	6	7	8	9	High	High-Low
January	23.32	40.71	22.34	43.26	37.59	32.94	41.77	21.22	39.27	32.04	8.72
	(0.89)	(1.54)	(0.82)	(1.48)	(1.41)	(1.14)	(1.77)	(0.80)	(1.49)	(1.20)	(0.45)
Non-January	21.54	26.45	26.29	29.45	25.31	21.67	27.57	32.67	21.61	32.16	10.62
	(2.44)	(3.12)	(3.11)	(3.54)	(3.05)	(2.58)	(3.34)	(3.82)	(2.69)	(4.18)	(1.97)
NYSE	28.88	33.27	27.72	30.57	25.37	29.64	34.93	40.08	35.47	36.66	7.78
	(2.39)	(2.70)	(2.31)	(2.56)	(2.23)	(2.59)	(3.23)	(3.89)	(3.52)	(4.12)	(0.92)

	Equal weighted Raw Returns										
	Low	2	3	4	5	6	7	8	9	High	High-Low
January	28.50	41.63	39.09	48.63	47.28	45.29	48.63	42.92	52.70	55.20	26.70
	(0.98)	(1.36)	(1.27)	(1.55)	(1.52)	(1.52)	(1.60)	(1.53)	(1.84)	(2.17)	(1.73)
Non-January	22.86	26.16	26.92	25.37	27.79	27.65	27.91	30.21	31.89	33.91	11.05
	(2.47)	(2.74)	(2.78)	(2.60)	(2.88)	(2.90)	(3.01)	(3.26)	(3.63)	(4.26)	(2.37)
NYSE	29.87	27.12	31.30	33.47	33.25	29.17	32.33	37.55	38.73	38.41	8.54
	(2.23)	(2.12)	(2.52)	(2.76)	(2.86)	(2.58)	(2.94)	(3.57)	(3.84)	(4.41)	(1.11)

Table 9
Long-Short Returns for Alternative Skewness Measures

We report the long-short weekly returns computed as in Table 2 for alternative realized skewness measures. Panel A uses realized skewness ($RSkew$), $SubRSkew$, the average realized skewness over different subsamples as suggested by Zhang, Mykland, and Ait-Sahalia (2005), $SubRSkew_{drift}$, the average drift adjusted realized skewness over different subsamples as suggested by Zhang, Mykland, and Ait-Sahalia (2005), and the interquartile skewness ($SK2$) defined as $(Q_3 + Q_1 - 2Q_2)/(Q_3 - Q_1)$ where Q_i is the i^{th} quartile of the five-minute return distribution. Panel B uses the 1-day, 3-day, 1-week, 2-week and 1-month average of daily realized skewness computed from 5-minute returns. The sample begins January 1993 and ends December 2013.

Panel A: Realized Skewness

	RSkew	SubRSkew	SubRSkew$_{drift}$	SK2
	Value weighted			
Raw Returns	-19.26	-42.44	-17.53	-5.36
	(-3.70)	(-6.22)	(-3.27)	(-1.29)
Alpha, C4	-19.19	-42.56	-19.87	-6.22
	(-3.70)	(-6.38)	(-3.73)	(-1.52)
	Equal weighted			
Raw Returns	-34.41	-50.14	-8.89	-11.08
	(-8.86)	(-8.66)	(-2.22)	(-3.13)
Alpha, C4	-34.42	-48.64	-8.49	-11.34
	(-9.26)	(-8.96)	(-2.37)	(-3.42)

Panel B: Realized Skewness Calculated over Different Intervals

	RSkew$_{1d}$	RSkew$_{3d}$	RSkew$_{1w}$	RSkew$_{2w}$	RSkew$_{1m}$
	Value weighted				
Raw Returns	-1.00	-19.30	-19.26	-18.81	-19.54
	(-0.20)	(-3.79)	(-3.70)	(-3.36)	(-3.21)
Alpha, C4	-4.95	-21.07	-19.19	-20.69	-22.54
	(-0.98)	(-4.03)	(-3.70)	(-3.64)	(-3.66)
	Equal weighted				
Raw Returns	-5.36	-26.83	-34.41	-33.86	-23.02
	(-1.63)	(-7.44)	(-8.86)	(-7.90)	(-4.94)
Alpha, C4	-7.00	-28.35	-34.42	-36.09	-25.81
	(-2.10)	(-7.74)	(-9.26)	(-8.45)	(-5.62)

Table 10
Weekly Realized Skewness and the Cross-Section of Stock Returns for Different Horizons

We report the value- and equal-weighted returns (in bps) for different horizons of decile portfolios formed from realized skewness, their Newey-West t-statistics (in parentheses) and the return difference between portfolio 10 (highest realized moment) and portfolio 1 (lowest realized moment) over the period January 1993 to December 2013. Raw returns (in bps) are obtained from decile portfolios sorted solely from ranking stocks based on the realized skewness measure. Alpha is the intercept from time-series regressions of the returns of the portfolio that buys portfolio 10 and sells portfolio 1 using the Carhart four factor model. Panel A, Panel B, and Panel C report two-week, three-week, and one-month returns.

Panel A: Two-Week Horizon

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	70.34	61.66	55.33	52.92	53.41	43.03	46.77	34.90	40.33	47.30	-23.03
	(6.16)	(5.34)	(4.82)	(4.71)	(4.78)	(3.98)	(4.20)	(3.23)	(3.97)	(4.74)	(-3.23)
Alpha, C4	37.95	28.08	25.64	21.05	23.43	13.47	14.33	3.63	8.33	16.64	-23.80
	(7.07)	(6.13)	(5.64)	(4.92)	(5.98)	(3.67)	(3.45)	(0.93)	(2.21)	(3.86)	(-3.33)
						Equal weighted					
Raw Returns	77.29	68.39	59.94	55.38	56.80	48.84	52.04	43.47	38.45	39.56	-37.74
	(6.07)	(5.16)	(4.56)	(4.30)	(4.52)	(3.97)	(4.27)	(3.59)	(3.27)	(3.64)	(-6.98)
Alpha, C4	38.89	27.70	19.74	15.41	16.86	8.98	13.09	3.66	-1.04	1.79	-39.58
	(8.19)	(6.46)	(5.12)	(4.39)	(5.32)	(3.11)	(4.63)	(1.23)	(-0.32)	(0.52)	(-7.59)

Panel B: Three-Week Horizon

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	93.87	84.60	83.60	79.18	78.30	63.36	70.96	57.76	59.26	69.19	-24.68
	(6.81)	(6.00)	(6.15)	(5.89)	(5.77)	(4.87)	(5.39)	(4.41)	(4.87)	(5.77)	(-2.88)
Alpha, C4	45.07	34.33	36.89	30.94	32.03	19.35	23.54	9.89	10.22	23.00	-25.77
	(6.89)	(6.04)	(7.27)	(6.01)	(6.24)	(4.26)	(4.84)	(2.02)	(2.24)	(4.19)	(-2.98)
						Equal weighted					
Raw Returns	104.41	95.97	85.50	80.65	85.10	76.14	80.53	69.52	65.14	66.25	-38.16
	(6.70)	(5.89)	(5.32)	(5.15)	(5.49)	(5.05)	(5.38)	(4.67)	(4.54)	(4.91)	(-6.06)
Alpha, C4	45.16	33.46	24.22	19.00	23.52	15.85	20.64	7.81	4.13	8.39	-40.47
	(8.11)	(6.57)	(5.27)	(4.64)	(6.35)	(4.60)	(5.94)	(2.12)	(1.08)	(1.94)	(-6.61)

Panel C: One-Month Horizon

	Low	2	3	4	5	6	7	8	9	High	High-Low
						Value weighted					
Raw Returns	123.99	107.01	109.79	103.37	105.86	90.16	98.24	90.03	90.89	101.15	-22.84
	(7.42)	(6.37)	(6.61)	(6.32)	(6.45)	(5.80)	(6.16)	(5.77)	(6.13)	(7.01)	(-2.25)
Alpha, C4	75.95	57.48	62.35	55.20	59.11	44.80	51.15	43.64	42.60	54.71	-24.94
	(6.60)	(5.37)	(5.93)	(5.28)	(5.52)	(4.64)	(5.03)	(4.45)	(4.46)	(5.65)	(-2.43)
						Equal weighted					
Raw Returns	134.17	124.83	115.81	109.93	114.81	105.35	112.67	101.58	99.87	102.19	-31.98
	(7.13)	(6.35)	(5.97)	(5.79)	(6.09)	(5.74)	(6.22)	(5.63)	(5.73)	(6.20)	(-4.21)
Alpha, C4	74.84	62.05	54.05	48.67	53.09	44.62	52.57	39.39	38.21	42.76	-35.78
	(6.88)	(5.60)	(5.01)	(4.58)	(5.07)	(4.41)	(5.21)	(3.90)	(3.86)	(4.47)	(-4.82)

Table 11
Double Sorting on Firm Characteristics and Realized Skewness

Stocks are sorted into five quintiles each week based on a given firm characteristic as well as on realized skewness, and 25 portfolios are formed based on these two criteria. We then compute value- and equal-weighted average weekly returns of the difference between the highest and lowest skewness portfolio for a given level of the firm characteristic, along with the t-statistic (in parentheses). Firm characteristics are Size (market capitalization in $billions), BE/ME (book-to-market equity ratio), Realized Volatility, Realized Kurtosis, Lagged Return 1-week, Historical Skewness (one month historical skewness from daily returns), Illiquidity (5-day average of the absolute return over daily dollar trading volume times 10^6, as in Amihud (2002)), Number of Intraday Transactions, Maximum (minimum) Return of previous month and week in bps, Number of Analysts (from I/B/E/S), Market Beta, Idiosyncratic Volatility (computed as in Ang, Hodrick, Xing, and Zhang (2006)), 1%-Realized Value-at-Risk (VaR) from 5-minute returns over one week, and Coskewness (computed as in Harvey and Siddique (2000) with 24 months of data). The sample begins January 1993 and ends December 2013.

	Value Weighted					Equal Weighted				
Quintiles	Low	2	3	4	High	Low	2	3	4	High
Size	-55.5	-31.2	-25.1	-8.5	-17.8	-58.4	-32.4	-26.8	-7.9	-16.0
RSkew 5-1	(-9.95)	(-6.59)	(-5.92)	(-2.02)	(-3.59)	(-10.19)	(-6.80)	(-6.32)	(-1.92)	(-3.92)
BE/ME	-35.5	-25.5	-15.4	-1.6	-4.0	-44.7	-30.7	-28.6	-25.8	-19.5
RSkew 5-1	(-4.53)	(-4.14)	(-2.79)	(-0.26)	(-0.67)	(-7.41)	(-6.61)	(-6.46)	(-5.79)	(-4.11)
Realized Volatility	-14.3	-20.9	-17.7	-25.7	-41.9	-7.4	-20.6	-25.3	-38.3	-68.3
RSkew 5-1	(-3.45)	(-3.76)	(-2.64)	(-3.09)	(-3.95)	(-3.10)	(-7.01)	(-6.29)	(-8.32)	(-10.30)
Realized Kurtosis	-19.6	-12.1	-14.5	-26.1	-13.3	-30.6	-29.9	-31.8	-32.9	-30.5
RSkew 5-1	(-2.12)	(-1.75)	(-2.54)	(-4.50)	(-2.74)	(-3.58)	(-5.18)	(-7.08)	(-8.38)	(-8.69)
Lagged Return 1w	-12.7	-5.6	-6.8	3.2	-18.8	-36.0	-10.1	0.3	-8.0	-18.9
RSkew 5-1	(-1.38)	(-0.94)	(-1.14)	(0.52)	(-2.26)	(-6.45)	(-2.69)	(0.09)	(-1.95)	(-3.50)
Historical Skewness	-21.9	-22.4	-18.8	-11.4	-21.0	-26.7	-29.2	-25.7	-28.5	-37.7
RSkew 5-1	(-3.39)	(-3.54)	(-2.94)	(-1.66)	(-3.12)	(-6.19)	(-6.79)	(-5.80)	(-6.40)	(-7.85)
Illiquidity	-18.1	-9.8	-18.1	-30.5	-55.4	-16.4	-15.0	-22.9	-32.4	-58.2
RSkew 5-1	(-3.61)	(-2.14)	(-3.89)	(-5.98)	(-8.87)	(-3.60)	(-3.65)	(-5.33)	(-7.39)	(-11.78)
Intra. Transactions	-15.5	-13.9	-16.6	-21.1	-19.6	-26.5	-30.0	-31.4	-32.3	-32.9
RSkew 5-1	(-3.57)	(-3.27)	(-4.19)	(-5.00)	(-3.45)	(-7.87)	(-7.60)	(-7.13)	(-6.15)	(-5.23)
Maximum 1M Return	-17.1	-22.6	-28.0	-15.5	-27.1	-15.7	-24.8	-31.2	-32.1	-54.8
RSkew 5-1	(-3.36)	(-4.12)	(-4.06)	(-2.02)	(-2.81)	(-4.90)	(-6.84)	(-5.39)	(-5.19)	(-7.80)
Minimum 1M Return	-19.6	-31.4	-21.8	-14.7	-16.4	-60.5	-41.3	-29.0	-13.9	-9.8
RSkew 5-1	(-2.07)	(-4.11)	(-3.29)	(-2.88)	(-3.21)	(-9.32)	(-9.15)	(-5.44)	(-2.94)	(-2.14)
Maximum 1W Return	-12.9	-14.8	-19.2	-16.6	-14.4	-29.5	-20.6	-21.0	-24.3	-33.2
RSkew 5-1	(-2.00)	(-2.49)	(-3.02)	(-2.24)	(-1.48)	(-6.37)	(-4.87)	(-3.50)	(-3.96)	(-5.12)
Minimum 1W Return	-16.5	-21.3	-23.3	-7.1	-9.8	-45.0	-23.3	-20.5	-13.2	-16.2
RSkew 5-1	(-1.83)	(-2.85)	(-3.96)	(-1.36)	(-1.67)	(-7.50)	(-5.64)	(-3.85)	(-2.69)	(-3.10)
Nb. of Analysts	-21.9	-21.7	-20.0	-14.5	-21.6	-40.7	-34.9	-30.4	-19.0	-17.8
RSkew 5-1	(-3.55)	(-4.25)	(-4.27)	(-3.12)	(-4.04)	(-8.23)	(-7.85)	(-7.13)	(-4.49)	(-3.81)
Market Beta	-14.6	-16.0	-17.1	-35.7	-22.2	-23.6	-25.2	-26.5	-35.8	-48.9
RSkew 5-1	(-2.74)	(-3.58)	(-3.12)	(-5.40)	(-2.48)	(-5.90)	(-7.52)	(-7.17)	(-8.24)	(-8.63)
Idio. Volatility	-8.8	-18.0	-27.2	-35.1	-31.6	-4.2	-18.2	-25.4	-41.7	-64.3
RSkew 5-1	(-1.99)	(-3.36)	(-3.98)	(-4.27)	(-3.21)	(-1.51)	(-6.23)	(-6.49)	(-8.65)	(-10.53)
Realized VaR - 1%	-8.8	-18.0	-27.2	-35.1	-31.6	-3.0	-16.8	-25.3	-35.1	-66.9
RSkew 5-1	(-1.99)	(-3.36)	(-3.98)	(-4.27)	(-3.21)	(-1.21)	(-5.45)	(-6.42)	(-7.40)	(-10.64)
Coskewness	-14.0	-23.1	-14.1	-16.9	-32.5	-41.2	-29.2	-25.1	-31.3	-30.1
RSkew 5-1	(-2.12)	(-3.08)	(-2.23)	(-2.78)	(-5.40)	(-9.40)	(-6.45)	(-5.92)	(-7.74)	(-7.19)

Table 12
Realized Skewness and Lagged Return

In Panel A (Panel B), we implement independent double sorting between realized skewness (with subsampling) and lagged return for 3 days, 1 week, 2 weeks and 1 month. Panel C (Panel D) reports the overlap between stocks (in %) in the quintile portfolios based on realized skewness (with subsampling) and lagged return. The sample begins January 1993 and ends December 2013.

Panel A: Double Sorting on Lagged Return and Realized Skewness

	Value Weighted					Equal Weighted				
Quintiles	Low	2	3	4	High	Low	2	3	4	High
Lagged Return 3d	-18.9	-8.7	-5.3	-3.6	-11.2	-38.1	-7.3	-7.5	-9.1	-15.2
RSkew 5-1	(-2.04)	(-1.45)	(-0.88)	(-0.61)	(-1.34)	(-6.77)	(-1.94)	(-2.05)	(-2.28)	(-2.70)
Lagged Return 1w	-12.7	-5.6	-6.8	3.2	-18.8	-36.0	-10.1	0.3	-8.0	-18.9
RSkew 5-1	(-1.38)	(-0.94)	(-1.14)	(0.52)	(-2.26)	(-6.45)	(-2.69)	(0.09)	(-1.95)	(-3.50)
Lagged Return 2w	-17.8	-5.9	-6.5	-12.4	-29.3	-49.4	-9.9	-10.9	-10.8	-33.2
RSkew 5-1	(-2.09)	(-1.04)	(-1.16)	(-2.00)	(-3.81)	(-9.38)	(-2.73)	(-3.08)	(-2.67)	(-6.70)
Lagged Return 1m	-32.8	-11.4	-6.7	-13.8	-27.2	-54.5	-19.2	-8.5	-19.1	-37.6
RSkew 5-1	(-3.97)	(-1.93)	(-1.20)	(-2.37)	(-3.94)	(-9.81)	(-5.32)	(-2.53)	(-4.96)	(-7.62)

Panel B: Double Sorting on Lagged Return and Realized Skewness Subsampling

	Value Weighted					Equal Weighted				
Quintiles	Low	2	3	4	High	Low	2	3	4	High
Lagged Return 3d	-42.4	-17.5	-9.1	-15.9	-19.8	-59.9	-10.1	-6.0	-3.6	-14.8
RSkew 5-1	(-4.11)	(-2.49)	(-1.37)	(-1.90)	(-1.83)	(-7.33)	(-2.02)	(-1.24)	(-0.61)	(-1.82)
Lagged Return 1w	-44.4	-13.3	-14.4	-12.3	-41.5	-56.2	-5.9	-3.1	-4.7	-25.5
RSkew 5-1	(-3.69)	(-1.85)	(-2.07)	(-1.65)	(-3.88)	(-6.53)	(-1.35)	(-0.68)	(-0.84)	(-3.19)
Lagged Return 2w	-46.7	-18.3	-14.3	-21.5	-50.0	-65.9	-17.4	-4.2	-17.2	-42.8
RSkew 5-1	(-4.76)	(-2.70)	(-2.26)	(-2.97)	(-5.37)	(-9.45)	(-3.87)	(-0.95)	(-3.30)	(-6.47)
Lagged Return 1m	-65.9	-24.6	-15.3	-29.4	-44.0	-73.5	-21.6	-9.7	-19.6	-53.9
RSkew 5-1	(-7.03)	(-3.71)	(-2.39)	(-4.30)	(-5.55)	(-10.43)	(-4.62)	(-2.24)	(-4.21)	(-8.82)

Panel C: Overlap between stocks (in %) for Realized Skewness and Lagged Return Quintile Portfolios

	Lagged Return Quintile Portfolio					
RSkew Quintile Portfolio	1	2	3	4	5	Total
1	36.0	23.9	17.4	13.0	9.8	100
2	24.7	24.2	20.8	17.2	13.1	100
3	17.5	21.5	22.4	21.1	17.6	100
4	12.7	17.5	21.5	24.1	24.3	100
5	9.1	13.0	18.1	24.7	35.1	100

Panel D: Overlap between stocks (in %) for Realized Skewness with Subsampling and Lagged Return Quintile Portfolios

	Lagged Return Quintile Portfolio					
SubRSkew Quintile Portfolio	1	2	3	4	5	Total
1	44.1	25.7	15.2	9.2	5.7	100
2	25.5	26.5	21.8	15.8	10.5	100
3	16.1	22.3	23.8	21.4	16.4	100
4	9.5	16.4	22.8	26.4	25.0	100
5	4.8	9.2	16.4	27.2	42.5	100

Table 13
Portfolios Based on Realized Skewness and Lagged Returns - One-Week Horizon

We report the performance of short-long portfolios based on measures of realized skewness and lagged return. Panels A and B report performance measures of value-weighted and equal-weighted short-long portfolios based on sorts of lagged return, previous week realized skewness (*RSkew*), and previous week realized skewness using subsampling (*SubRSkew*). Each week, a low-high (P1-P10) spread portfolio is formed and different performance measures are computed with one-week subsequent returns. These measures include the average raw return, alpha, standard deviation, skewness, kurtosis, annual Sharpe ratio, and risk measures associated with tail risk: the 1% value-at-risk over one week computed from the return series, and the expected shortfall which looks at the average of the weekly returns that exceed value-at-risk. Linear correlations between the short-long return strategies based on lagged return and realized skewness are also reported. We also present performance measures for an equal-weighted short-long portfolio strategy based on combining the portfolios based on lagged return and realized skewness, and for a strategy that optimally combines the lagged return and a realized skewness measure by performing a static portfolio maximization of the Sharpe ratio. These strategies are referred to as Combo and Optimal Combo, respectively. For the Optimal Combo strategy, the optimal weight on the realized skewness measure is presented. The sample begins January 1993 and ends December 2013.

Panel A: Short-Long Value-Weighted Returns

	Lagged Return	RSkew	SubRSkew	RSkew		SubRSkew	
				Combo	Optimal Combo	Combo	Optimal Combo
Raw Return (bps)	55.11	19.26	42.44	37.18	37.18	48.78	45.37
t-statistic	(5.09)	(3.70)	(6.22)	(5.46)	(5.46)	(6.19)	(6.50)
Alpha, C4	54.31	19.19	42.56	36.78	36.78	48.36	45.14
t-statistic	(5.12)	(3.70)	(6.38)	(5.51)	(5.51)	(6.31)	(6.66)
Standard Deviation (bps)	353.57	170.28	222.88	222.72	222.72	257.42	228.19
Skewness	0.40	0.22	0.99	0.37	0.37	0.80	1.04
Kurtosis	7.17	9.41	8.74	7.74	7.74	8.65	9.52
Sharpe Ratio	1.10	0.80	1.35	1.18	1.18	1.34	1.41
Value at Risk - 1% (bps)	899.20	422.50	518.00	557.75	557.75	654.75	526.10
Expected Shortfall (bps)	1284.45	620.88	701.29	846.10	846.10	941.55	778.48
Correlation		0.37	0.57				
Optimal Weight					0.50		0.77

Panel B: Short-Long Equal-Weighted Returns

	Lagged Return	RSkew	SubRSkew	RSkew		SubRSkew	
				Combo	Optimal Combo	Combo	Optimal Combo
Raw Return (bps)	83.12	34.41	50.14	58.79	48.08	66.65	63.35
t-statistic	(8.76)	(8.86)	(8.66)	(9.67)	(9.92)	(9.32)	(9.35)
Alpha, C4	80.44	34.42	48.64	57.44	47.32	64.47	61.27
t-statistic	(8.73)	(9.26)	(8.96)	(9.82)	(10.21)	(9.44)	(9.51)
Standard Deviation (bps)	310.01	126.89	189.30	198.69	158.42	233.61	221.37
Skewness	1.01	0.66	0.64	0.97	0.89	1.00	0.97
Kurtosis	8.64	4.36	5.31	6.58	5.33	7.50	7.12
Sharpe Ratio	1.90	1.92	1.87	2.09	2.15	2.02	2.02
Value at Risk - 1% (bps)	756.00	347.58	462.90	462.16	352.97	571.85	546.38
Expected Shortfall (bps)	1033.99	407.93	606.01	598.39	468.10	740.56	689.82
Correlation		0.58	0.74				
Optimal Weight					0.72		0.60

Table 14
Portfolios Based on Realized Skewness and Lagged Returns - One-Month Horizon

We report the performance of short-long portfolios based on measures of realized skewness and lagged return. Panels A and B report performance measures of value-weighted and equal-weighted short-long portfolios based on sorts of lagged return, previous week realized skewness (*RSkew*), and previous week realized skewness using subsampling (*SubRSkew*). Each week, a low-high (P1-P10) spread portfolio is formed and different performance measures are computed with one-week subsequent returns. These measures include the average raw return, alpha, standard deviation, skewness, kurtosis, annual Sharpe ratio, and risk measures associated with tail risk: the 1% value-at-risk over one month computed from the return series, and the expected shortfall which looks at the average of the monthly returns that exceed value-at-risk. Linear correlations between the short-long return strategies based on lagged return and realized skewness are also reported. We also present performance measures for an equal-weighted short-long portfolio strategy based on combining the portfolios based on lagged return and realized skewness, and for a strategy that optimally combines the lagged return and a realized skewness measure by performing a static portfolio maximization of the Sharpe ratio. These strategies are referred to as Combo and Optimal Combo, respectively. For the Optimal Combo strategy, the optimal weight on the realized skewness measure is presented. The sample begins January 1993 and ends December 2013.

Panel A: Short-Long Value-Weighted Returns

	Lagged Return	RSkew	SubRSkew	RSkew		SubRSkew	
				Combo	Optimal Combo	Combo	Optimal Combo
Raw Return (bps)	54.15	22.84	44.95	38.49	34.74	49.55	46.51
t-statistic	(2.51)	(2.25)	(3.37)	(2.81)	(2.84)	(3.20)	(3.46)
Alpha, C4	54.55	24.94	47.87	38.31	34.41	51.21	49.00
t-statistic	(2.55)	(2.43)	(3.62)	(2.81)	(2.82)	(3.35)	(3.69)
Standard Deviation (bps)	702.36	329.39	433.68	445.50	397.73	503.50	436.76
Skewness	0.81	0.13	1.20	0.95	0.90	1.23	1.37
Kurtosis	9.56	7.62	14.99	10.78	10.82	13.41	15.99
Sharpe Ratio	0.27	0.24	0.36	0.30	0.30	0.34	0.37
Value at Risk - 1% (bps)	1809.50	744.40	1142.40	1039.45	923.72	1304.00	1179.47
Expected Shortfall (bps)	2569.39	1157.62	1529.72	1595.24	1434.66	1822.34	1608.81
Correlation		0.42	0.55				
Optimal Weight					0.62		0.83

Panel B: Short-Long Equal-Weighted Returns

	Lagged Return	RSkew	SubRSkew	RSkew		SubRSkew	
				Combo	Optimal Combo	Combo	Optimal Combo
Raw Return (bps)	72.55	31.98	51.01	52.26	42.93	61.78	57.04
t-statistic	(4.04)	(4.21)	(4.40)	(4.54)	(4.69)	(4.50)	(4.59)
Alpha, C4	74.26	35.78	52.26	53.97	44.64	63.26	58.42
t-statistic	(4.14)	(4.82)	(4.55)	(4.71)	(4.91)	(4.63)	(4.73)
Standard Deviation (bps)	583.03	246.99	376.99	373.87	297.43	445.69	404.14
Skewness	0.86	-0.04	0.48	0.77	0.49	0.89	0.77
Kurtosis	8.57	12.22	13.01	10.59	12.38	11.37	12.76
Sharpe Ratio	0.43	0.45	0.47	0.48	0.50	0.48	0.49
Value at Risk - 1% (bps)	1731.10	569.60	931.90	988.55	761.61	1154.50	1120.68
Expected Shortfall (bps)	2128.79	851.57	1375.95	1315.21	1060.26	1583.14	1456.31
Correlation		0.55	0.71				
Optimal Weight					0.73		0.72

Table 15
Realized Skewness and Historical Skewness

Panel A displays long-short weekly returns computed as in Table 2 for realized skewness and historical skewness measures. Realized skewness is the weekly average of daily realized skewness measures computed from 5-minute, 30-minute, 60-minute and half-day returns. Historical skewness is computed with daily returns across different horizons (5 days open/close, 5-days, 1 month, 6 months, 12 months, 24 months and 60 months). Panel B displays the correlation of the realized skewness measures and the historical skewness measures. The sample begins January 1993 and ends December 2013.

Panel A: Realized and Historical Skewness at Different Frequencies

	RSkew				Historical Skewness						
	5min	30min	60min	1/2D	5D (Op./Cl.)	5D	1M	6M	12M	24M	60M
					Value weighted						
Raw Returns	-19.26	-8.32	-1.82	-0.25	3.84	20.25	-4.97	26.28	19.84	15.96	2.22
	(-3.70)	(-1.70)	(-0.38)	(-0.05)	(0.85)	(4.10)	(-1.00)	(4.76)	(3.67)	(2.87)	(0.37)
Alpha, C4	-19.19	-11.23	-6.51	-3.62	0.81	18.18	-6.45	24.45	15.36	12.19	-1.88
	(-3.70)	(-2.23)	(-1.33)	(-0.72)	(0.18)	(3.66)	(-1.31)	(4.43)	(2.93)	(2.29)	(-0.33)
					Equal weighted						
Raw Returns	-34.41	-11.55	-7.03	-5.85	4.64	12.17	-0.82	29.01	20.97	12.70	5.40
	(-8.86)	(-3.66)	(-2.16)	(-1.75)	(1.69)	(3.85)	(-0.19)	(5.27)	(3.62)	(2.08)	(0.86)
Alpha, C4	-34.42	-13.21	-8.82	-8.36	1.12	10.21	-4.96	23.40	14.31	6.87	0.22
	(-9.26)	(-4.12)	(-2.67)	(-2.46)	(0.40)	(3.32)	(-1.38)	(4.90)	(2.99)	(1.41)	(0.04)

Panel B: Correlation Matrix of Realized and Historical Skewness at Different Frequencies (%)

		RSkew				Historical Skewness					
		5min	30min	60min	1/2D	5D Op./Cl.	5D	1M	6M	12M	24M
RSkew	30min	41.41									
	60min	41.06	81.76								
	1/2D	37.74	59.63	70.68							
HSkew	5D (Op./Cl.)	3.34	8.70	10.67	11.89						
	5D	4.20	10.80	13.39	13.98	33.64					
	1M	2.51	3.98	4.07	3.33	8.43	12.45				
	6M	1.14	1.64	1.51	0.89	2.85	4.09	33.77			
	12M	0.88	1.12	0.96	0.35	2.30	3.20	23.95	72.78		
	24M	0.43	0.69	0.49	-0.20	1.94	2.69	17.62	52.48	73.08	
	60M	0.12	0.22	0.09	-0.57	1.34	1.87	10.88	31.11	44.28	62.70