Cryptocurrencies: Stylized Facts, & Risk Based Momentum Investing

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1 Introduction

The motivation of this research is in two folds, to understand the distributional characteristics of cryptocurrencies by means of stylized facts, and also to assess the feasibility of risk based and trend following approaches to investing in this asset class. Cryptocurrencies are more of a recent phenomenon, unlike the traditional asset classes. This poses an explicit constraint on the availability of longer history and also reliability of investment performance. Acknowledging such constraint, I focus my analysis based on the few years of data that is available.

To select the list of currencies for my analysis, I follow a simple method to find a subset of the universe. My analysis period starts from October 2017 till present. For each year in my sample, I find the top 10 currencies which were ranked by market cap at the beginning of that year. The final list consists of the union of all the top 10 lists by year. There is an implicit look ahead bias, since the top 10 currencies in the most recent periods may not have been in the list in prior years. There is also survivorship bias, since I only constrain my analysis to a subset of the universe. However, the purpose of my analysis is not to investigate the efficacy of individual security selection, rather to understand the underlying distribution of these assets, which brings in the interesting question regarding the feasibility of risk based investing and also the existence of momentum in this particular asset class. Thus I only focus on these 22 currencies which have the longest available sample (data starting from October 2017 till present) and also contributed to overall market liquidity at any point in time during my analysis. The reason behind starting 2 months prior 2018 is to have some burn in period for my risk and signal estimation. I pull all the data from CoinMarketCap API. All my calculations are based on daily data, the strategies I show are based on daily rebalancing. I also assume zero cash rate and absence of transaction cost when showing the results.

I divide my analysis in two sections. First, I demonstrate a few stylized facts in cryptocurrency returns. I try to answer two key questions, whether there are distributional characteristics in the time series of crypto returns which helps us predict risk ex-ante and also, whether there is evidence of trend or persistence in returns which may justify momentum. In the next section, I conduct performance analysis of three different sets of strategies, first incorporating risk based approaches to long-only portfolio construction, second, incorporating risk based approaches with trend following signals as a position sizing overlay for long only portfolios and the third one incorporating risk based approaches plus trend for long-short

portfolios. The benchmark to compare in this case is simply a equal weighted basket of the currencies.

2 Stylized Facts

Figure 1 outlines the annualized summary statistics for the currency pairs. Compared to traditional risky assets, cryptocurrencies are highly volatile with annualized ranges between 60% to 158%. The highly volatile nature also potentially explains the large deviations between its' arithmetic and compound returns. The high variance drag is also the reason why there is big differences between the simple Sharpe Ratios and The maximum drawdowns are very high, all greater than 90%. Another interesting view, unlike traditional risky assets like equities, cryptocurrencies demonstrate positive skewness.

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IDs	Arith. Mean	Compound Mean	Volatility	SR	t-Stat	Geo SR	Max DD	Ulcer	Calmar	Skewness	Kurtosis
ETH	122.6%	70.0%	99.3%	1.23	3.85	0.71	94.0%	2.10	1.30	0.14	14.67
EOS	84.5%	6.4%	130.4%	0.65	1.71	0.05	91.9%	1.09	0.92	5.61	105.57
BTC	65.6%	54.3%	66.5%	0.99	3.56	0.82	84.5%	1.28	0.78	0.25	10.09
XMR	98.9%	45.9%	108.0%	0.92	3.11	0.42	95.5%	1.42	1.04	1.40	13.68
TRX	135.2%	30.4%	140.3%	0.96	2.50	0.22	96.0%	1.70	1.41	4.56	52.22
XRP	103.9%	42.7%	125.4%	0.83	2.95	0.34	95.9%	1.29	1.08	5.55	98.14
LINK	133.7%	37.4%	121.3%	1.10	2.85	0.31	87.9%	2.58	1.52	0.92	7.10
всн	58.7%	-2.6%	115.1%	0.51	1.34	-0.02	98.0%	0.70	0.60	1.47	12.52
BNB	183.5%	87.8%	118.6%	1.55	4.07	0.74	80.1%	4.17	2.29	2.82	28.59
ZEC	39.9%	-19.1%	128.4%	0.31	0.88	-0.15	98.5%	0.44	0.41	5.74	135.58
REP	99.1%	18.0%	127.5%	0.78	2.39	0.14	94.8%	1.37	1.05	2.26	24.95
LTC	74.6%	27.2%	106.0%	0.70	2.54	0.26	97.4%	0.99	0.77	4.09	64.01
STEEM	98.4%	-4.7%	158.3%	0.62	1.84	-0.03	98.6%	1.12	1.00	4.71	65.60
ADA	118.9%	30.5%	124.6%	0.95	2.46	0.24	97.8%	1.54	1.21	4.99	71.25
MIOTA	63.3%	-0.2%	113.6%	0.56	1.48	0.00	97.9%	0.76	0.65	0.65	6.59
MAID	82.8%	23.0%	110.4%	0.75	2.55	0.21	96.2%	1.30	0.86	0.93	8.31
DCR	106.7%	36.7%	113.6%	0.94	2.82	0.32	92.4%	1.70	1.15	1.43	8.27
DOGE	146.6%	59.8%	173.3%	0.85	2.95	0.35	95.3%	1.87	1.54	15.04	427.04
ETC	119.6%	30.2%	155.4%	0.77	2.23	0.19	92.1%	1.72	1.30	17.45	568.33
XLM	106.3%	38.9%	124.6%	0.85	2.85	0.31	96.3%	1.51	1.10	3.83	38.97
XEM	133.7%	57.3%	132.5%	1.01	3.23	0.43	98.3%	1.74	1.36	4.64	75.61
DASH	119.6%	52.5%	137.9%	0.87	2.99	0.38	97.4%	1.57	1.23	10.06	262.48

Figure 1: Summary Statistics of Cryptocurrencies: 2017/10-Present

Figure 2 plots the unconditional rank correlation matrix of my selected universe. I also overlay the matrix with a dendogram based on hierarchial clustering, with similar currency pairs being adjacent to each other. The average rank correlation across these 22 pairs is 0.56. This stylized fact is very in align with traditional assets, where the average correlations within an asset class clusters tends to be quite high and statistically significant.

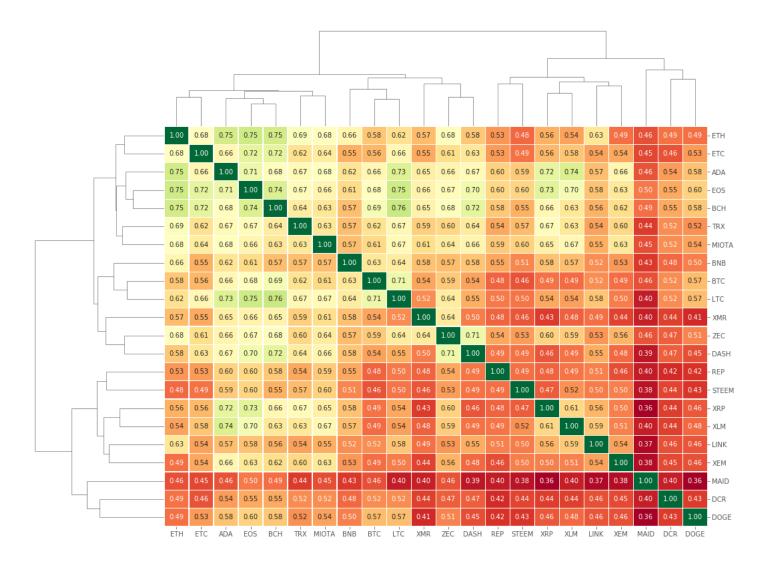


Figure 2: Rank Correlation Matrix with Clustered Dendogram Rank Correlation Matrix - Hierarchial Clustered Points

Figure 3 plots the autocorrelation of returns by four different lags, 1 day, 3 days, 5 days and 10 days. It's very interesting to see that even under lag 1, the currency pairs demonstrate some autocorrelation and higher than traditional markets. However, the signs are mixed, some demonstrate positive autocorrelation while some demonstrate negative autocorrelation. This implies over the very short period, the selected assets can either demonstrate trend or mean reversion. Autocorrelations become positive as for almost all currencies as we increase the number of lags. One thing to note here, that this level of autocorrelation is quite high compared to traditional markets (i.e. S&P 500) where autocorrelations even upto 100 day lags don't show any discernible pattern.

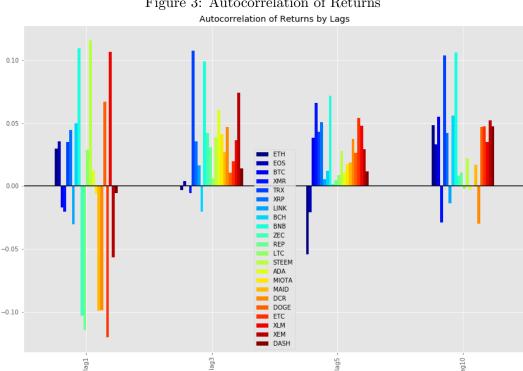


Figure 3: Autocorrelation of Returns

Figure 4 plots a similar bar chart as before, but this time using squared returns or instantaneous variance. In this case, we see a very different picture. At lag 1, the autocorrelations are positive and statistically significant for almost all pairs. The autocorrelations stays significantly positive for some of the pairs up to 5 day lags, although show a declining pattern as we increase the lag length. The high autocorrelation for squared returns has important implication, since this gives us some confidence in predicting risk measures more accurately than predicting returns in the shorter horizon.

Let's run a more formal analysis by designing a volatility targeting strategy for each asset. For each coin, I compute ex-ante volatility based on EWMA with 63 day halflife. Next, using I construct a daily rebalancing volatility targeting strategy for each asset. Figure 5 plots the Geometric Sharpe Ratios for each asset based on the volatility targeting strategy and compares it against a simple long only strategy (buy and hold). Almost all the currency pairs show improved risk adjusted performance for their volatility targeting strategies.

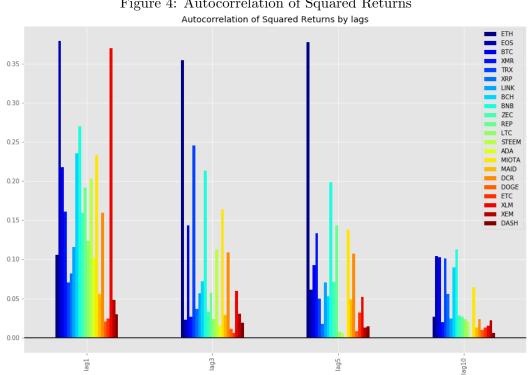
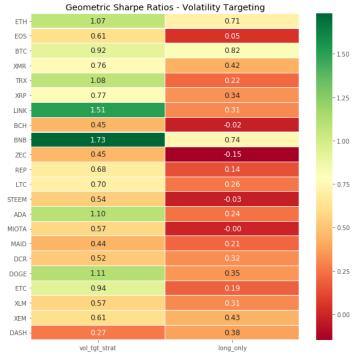


Figure 4: Autocorrelation of Squared Returns

Figure 5: Autocorrelation of Squared Returns



The next question I wanted to ask whether like other risky assets, if the risk is asymmetric. In equity markets, it's commonly known as leverage effect, where daily returns are negatively correlation to changes in volatility. This is important since, alongside higher autocorrelation of squared returns, a risk targeting approach to an asset with asymmetric risk profile can provide an extra risk adjusted performance. To assess the existence of this phenomenon, I compute daily volatility based on an exponentially weighted process with 63 day halflife. Next for each asset, I compute the unconditional rank correlation of daily returns against the changes in volatility. Figure 5 plots the correlations. All the currency pairs demonstrate very low correlation to changes in volatility.

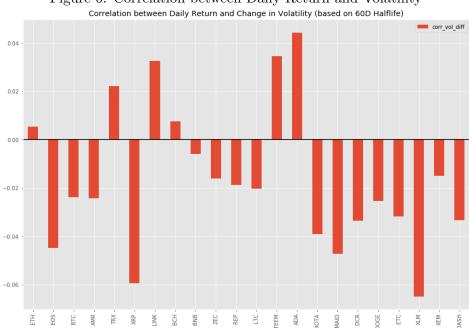


Figure 6: Correlation between Daily Return and Volatility

In order to further assess this finding, I also compute the upside and downside deviation for each pair. Upside (downside) deviation is simply the standard deviation of positive (negative) demeaned returns. Figure 6 plots both measures side by side. All the cryptocurrencies in our sample demonstrates a higher upside deviation compared to their downside deviation. A particular one that stands out is the meme coin 'DOGE', whose upside deviation is almost 3x its downside deviation. The asymmetric risk to the upside maybe related to the wide speculative nature of some of these coins, where the overall variation of the distribution is mostly explained by the wide retail speculation driven by "FOMO" or "Fear of Missing out", causing their prices to rise. This also brings in another interesting point, absent of fundamentals if the underlying dynamics of these coins are mostly driven by wide speculative over/under reaction, this can give rise of potential trends in the price process, as opposed to mean reversion.

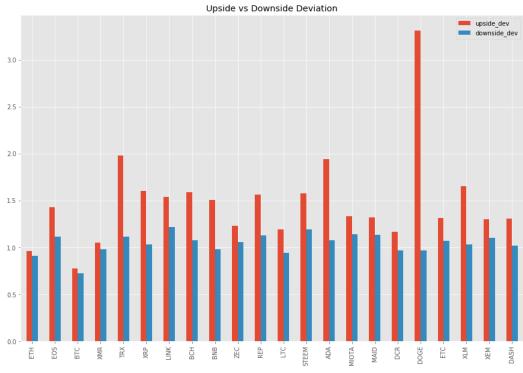


Figure 7: Upside & Downside Deviation

To evaluate the overall "trendiness" of these assets, I compute two different measures:

Hurst Exponent: The Hurst exponent is a measure of long term memory of financial time series. This method measures how much a time series' speed of diffusion is different from the geometric brownian motion assumption. The calculation of Hurst exponent is influenced by rescaled range analysis, which assesses how much the variation of time series changes with length of period being considered. This is a nutshell measure of the intensity of randomness of a price process. A random walk process has a hurst exponent of 0.50. Hurst exponent greater than 0.50 suggests trend and less than 0.50 suggest mean reversion in the price process.

Volatility Ratio: This measure is simply the ratio between annualized volatility computed over quarterly sample returns and annualized volatility computed over daily sample returns. A ratio greater than one suggests higher volatility over lower frequency relative to a higher one, due to positive auto correlation in the price process.

Figure 7 shows the two measures for each currency pair. The volatility ratio is greater than 1 across all assets. Also, the hurst exponents are greater than 0.50 for the majority except for a few currency pairs. The rank correlation between the two measures is 0.47.

Figure 8: Trend Testing Measures

	Vol Ratio	Hurst Exponent				
ETH	4.35	0.54				
EOS	3.89	0.54				
BTC	2.54	0.55				
XMR	1.52	0.49				
TRX	5.06	0.52				
XRP	2.98	0.45				
LINK	1.97	0.44				
BCH	1.70	0.48				
BNB	2.66	0.59				
ZEC	1.14	0.46				
REP	1.45	0.49				
LTC	2.97	0.51				
STEEM	2.36	0.53				
ADA	1.88	0.59				
MIOTA	2.22	0.51				
MAID	1.66	0.46				
DCR	7.85	0.59				
DOGE	2.37	0.56				
ETC	1.54	0.50				
XLM	3.99	0.55				
XEM	3.89	0.54				
DASH	3.02	0.48				

Now, let's run a more formal test. For each asset, I examine the time series predictability of daily returns conditional on past returns over a given horizon. The regression specification is as follows,

$$r_{t+1} = \alpha + \beta_h \mathbf{sign} (r_{t:t-h}) + \epsilon_{t+1}$$
(1)

 $r_{t:t-h}$ is the past cumulative return over a horizon h in days. For each coin, I run 9 different regression of the same specification with 9 different lags. Next, I compute the t-Statistic of the regression coefficient. A statistically significant positive t-Stats will denote time series predictability of future returns based on past returns. Figure 8 shows the results in a heatmap. Unlike traditional assets, where momentum is more evident over half to a full year, cryptocurrencies demonstrate a much different picture. Short term momentum predict future returns better than long term momentum. For all the coins, the t-Stats starting from 5 days to a month (20 days) is highly significant. Also, the longer lookbacks like 190 days and 252 days bear statistically insignificant predictability.

t-Stats: Momentum Betas 11.54 8.30 6.30 2.33 1.78 1.15 1.52 FTH 11.17 1.30 0.43 EOS 8.20 5.63 12 12.35 8.50 5.53 1.51 0.90 BTC 11.62 7.18 5.86 1.45 0.64 1.21 0.88 XMR 1.74 9.23 1.52 6.12 5.35 0.78 0.28 TRX - 10 10.26 7.75 5.54 2.24 1.58 2.22 XRP 11.93 8.32 5.04 2.27 1.41 1.39 LINK 0.74 7.64 5.31 1.84 0.93 0.84 BCH 9.95 0.55 7.19 4.92 1.68 1.61 0.56 BNB - 8 12.88 9.05 5.71 ZEC 9.07 6.13 5.32 0.83 REP 11.59 7.57 1.69 1.24 0.92 LTC 5.64 - 6 8.09 5.58 1.45 0.61 STEEM 10.07 7.77 6.22 4.06 2.10 1.55 ADA 11.08 MIOTA 7.54 5.83 2.12 8.08 4.77 1.44 1.72 1.56 MAID 11.43 7.14 6.29 DCR 1.74 1.25 DOGE 6.85 5.47 4.47 1.51 11.75 8.42 5.90 1.37 ETC 10.07 8.73 6.75 XLM 8.08 6.31 XEM 11.07 8.05 5.98 DASH 1.17 2.15 2.17 5 10 20 60 120 160 190 252 lags in days

Figure 9: Momentum t-Stats: Daily Returns

How about monthly returns? I rerun the same analysis, but this time resample the returns monthly. Figure 9 outlines the t-Stats for each asset by lags. The shorter lookbacks (5 days) are not significant anymore, with on average negative signs, suggesting mild mean reversions. Some of the coins show statistically significant mean reversions for 2 week lookbacks. Mid term lookbacks (3-6 months) now become statistically significant on average, with positive t-Stats greater than 3 on some of the key coins (i.e. BTC, ETH and LTC etc.). Longer lookbacks still show weak predictability, similar conclusion as our previous analysis. This suggests that the length of lookbacks can vary in terms of predictability depending on the rebalancing scheme of our portfolio.

t-Stats: Momentum Betas 0.69 0.19 1.02 4.50 2.08 2.61 2.17 1.48 FTH -3.49 0.33 -0.26-0.321.24 2.53 1.14 0.23 EOS 0.31 5.12 2.84 2.44 1.96 2.09 1.15 BTC -0.18 -2.40 -0.27 1.80 0.67 0.47 1.60 XMR 1.39 2.97 TRX 1.74 1.69 0.72 1.23 1.18 1.59 1.70 0.58 XRP - 3 -3.13 -1.82 2.57 1.70 0.72 1.36 1.36 LINK -0.97 -0.75 0.08 3.06 2.49 BCH 1.18 0.55 1.45 -0.35 -0.311.24 3.06 2.37 1.48 1.43 1.01 BNB - 2 -0.39 -0.53 5.13 2.92 2.19 ZEC 1.20 1.10 -0.17 -0.36 -0.22 4.78 2.42 1.53 1.56 1.83 REP - 1 -0.68 -2.13 0.20 4.18 1.91 1.55 LTC 1.51 -0.54 -0.64 -0.57 2.91 1.76 STEEM 2.39 1.77 1.79 0.22 -0.32 3.42 1.63 2.61 1.65 1.33 ADA - 0 0.57 -0.83 0.57 1.71 1.37 1.59 1.42 MIOTA 0.44 -0.15 -0.21 2.88 1.69 0.82 1.31 MAID 0.82 1.45 2.75 5.32 3.51 2.19 DCR 2.22 - -1 DOGE -0.04 2.39 1.86 1.60 1.45 1.45 2.26 2.01 1.97 0.59 ETC -2.77 1.49 -0.14 2.86 2.06 1.99 XLM 3.10 1.64 2.32 2.74 -0.91 -0.663.02 1.80 1.89 XEM -0.58 -0.85 3.17 DASH -2.14 1.07 1.10 0.78 1.02 5 10 60 20 120 160 190 252 lags in days

Figure 10: Momentum t-Stats: Monthly Returns

Finally, I conduct factor analysis to understand the common risk drivers across these assets. I run a principal component analysis on the unconditional covariance matrix of the 22 coins. Next, I extract the eigenvalues and the corresponding eigenvectors for each principal component. Figure 10 plots the explained variance ratios and also the factor loadings for the first three principal components. The first three principal components explain around 50% of the total variation, with the individual explained variance being 38.4%, 6.5% and 5.1% respectively. I also show the factor loadings heatmap for the first three components. The first principal component has positive loadings across all coins, suggesting the presence of a market risk factor. The next principal component looks unique in the sense it captures the variation explained by DOGE, granting its unique presence as a "meme coin" and capture the risk unexplained by the market factor due to its unique characteristic compared to the other coins.

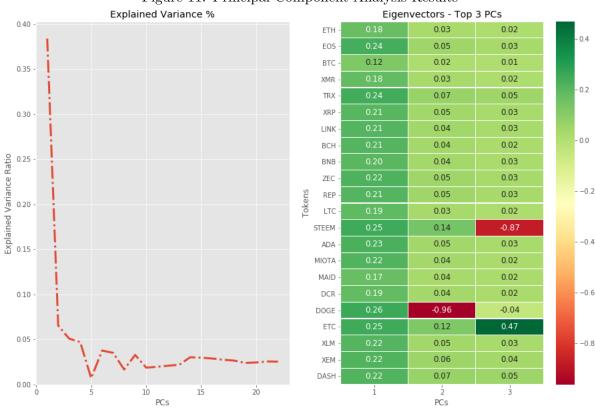


Figure 11: Principal Component Analysis Results

3 A Trend Following System for Cryptocurrencies

In order to broadly test the efficacy of trends in cryptocurrencies, I design a trend following system. For each CTA asset i, the signal strength is calculated based on the median of four separate models.

$$X_{i,t}^{CTA} = med\left(S_{i,t}^{k}\right) \; ; \; X_{i,t}^{CTA} \in [-1,1] \; or \; [0,1] \tag{2}$$

Each model k is processed from 4 different trend estimators, denoted as S^k ,

1. Risk Adjusted Momentum:

For each lookback $n \in [10, 20, 30, 40, 50]$, the risk adjusted momentum signal is calculated as,

$$x_n = \frac{\Pi_t^n (1 + rx_t)}{\sigma_n \sqrt{n}} \tag{3}$$

The numerator of the formula is simply the compound returns over the period. $\sigma_n \sqrt{n}$ is the daily volatility scaled over the lookback period.

2. EMA Crossover:

For each lookback set, $s \in [5, 10, 20] \& n \in [20, 40, 80]$, the EMA Crossover signal is calculated as,

$$x_n = \frac{EWMA[P; hl[s]] - EWMA[P; hl[p]]}{EWSD[P; hl[p]]}$$

$$(4)$$

$$hl[s] = \log 0.5 / \log (1 - 1/s)$$
 (5)

3. EMA Breakout:

For each lookback $n \in [10, 20, 30, 40, 50]$, the EMA Breakout signal is calculated as,

$$x_n = \frac{P - EWMA[P; span = n]}{EWSD[P; span = n]}.$$
(6)

4. Max Breakout:

For each lookback $n \in [10, 20, 30, 40, 50]$, the max breakout signal is calculated as,

$$x_n = \frac{P - max [P; n]}{EWSD [P; ; span = n]}.$$
 (7)

max[P;n] is maximum of price over n period lookback.

Next, each x_n is processed into a signal strength as follows,

- For each model and each lookback n, I normalize each x_n by their exponential weighted volatility.

$$z_n = \frac{x_n}{EWSD[x_n, span = 252]} \tag{8}$$

Next, each z_n is transformed into a position size based on a sigmoid function capped at 2 sigma, denoted as s_n ,

$$s_n = 2 * \left[\min \left(\max \left(\frac{1}{1 + e^{-z_n}}, 1 - \frac{1}{1 + e^{-2}} \right), \frac{1}{1 + e^{-2}} \right) \right] - 1$$
 (9)

For the long only case,

$$s_n = \left[\min \left(\max \left(\frac{1}{1 + e^{-z_n}}, 1 - \frac{1}{1 + e^{-2}} \right), \frac{1}{1 + e^{-2}} \right) \right]$$
 (10)

Figure 3 below shows an illustration of the formula above,

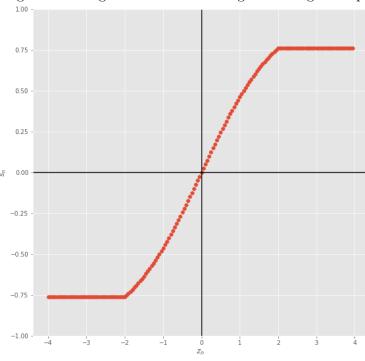


Figure 12: Long Short Position Sizing with 2 Sigma Cap

- The final signal strength from each model k is computed as,

$$S^k = \sum_n w_n s_n \tag{11}$$

$$w_n = \frac{2^{-n/hl}}{\sum_n 2^{-n/hl}} \tag{12}$$

Here, the raw weights, denoted as $2^{-n/1}$, is chosen based on a halflife hl of 1 month.

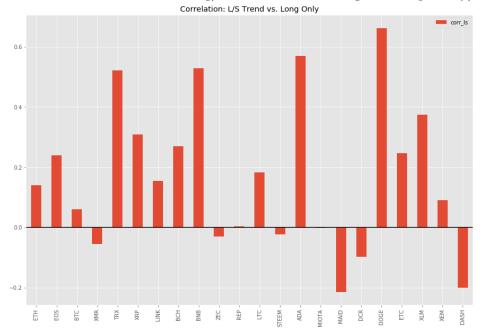
Next, I compute the position sizes for each asset, both under long-short and long only cases and design a daily rebalanced strategy. Figure 13 shows the geometric sharpe ratios for each asset. The first column 'long out' is a long only position sizing strategy, the second column 'long short' is the long short strategy and the final column is simply the buy and hold strategy. Compared to the buy and hold strategy, both trend following based overlays demonstrate higher risk adjusted returns for most coins. Absent of position sizing based on volatility targeting, position sizing based on trend following signals do provide improved performance. The relative outperformance in sharpe ratios compared to a buy and hold strategy indicates that the trend following system manages to increase position/exposure during periods of favorable trends while cutting risk or moving towards more short position during periods of unfavorable trends. I also plot the correlation between long/short strategy and the simple buy/hold. With the exception of a few coins (TRX, ADA and DOGE), most of them demonstrate low correlation to the buy and hold strategy, suggesting lesser long bias in trends. The higher correlations can also be a by product of the sample period, since we are only looking at roughly four and half years of data and conditional correlations of a trend following strategy against its underlying asset may vary over time, specially looking at other

traditional markets with longer history.

Geometric Sharpe Ratios - Trend Following Overlay 1.31 ETH 2.0 0.61 0.05 0.68 EOS 1.20 1.02 0.82 BTC 0.42 0.68 XMR 1.08 0.59 TRX XRP 0.65 0.47 - 1.5 1.38 0.44 BCH BNB 1.21 0.74 ZEC 0.14 REP - 1.0 0.79 1.00 LTC STEEM ADA 0.73 MIOTA 0.52 MAID 0.5 0.94 0.81 DCR 1.29 1.30 DOGE ETC 0.76 0.66 0.92 0.79 XLM 0.61 1.01 0.43 0.0 XEM DASH long_out long_short long_only

Figure 13: Geometric Sharpe Ratios - Trend Following

Figure 14: Correlation between Long/Short Trend Following and Simple Buy/Hold



4 Risk Based Portfolio Construction

In this section I describe the methodologies behind the risk based allocation framework. I considered four different allocation frameworks. For each of the four allocation framework, I considered three variations, allocation with no trend overlay, allocation with long only trend sizing overlay and allocation with long-short trend overlay. Below I outline the allocation rule for each cases. I drop the time subscript t to generalize the equations.

1) Equal Weighting (ew):

The allocation rules to equal weighted portfolio, denoted as \mathbf{w} , a $N \times 1$ vector of weights, can be computed as one of the three below forms,

$$\mathbf{w} = \begin{cases} \frac{30\%}{N*\sigma_p} \mathbf{1}_N; & Long \ Only \\ \frac{30\%}{N*\sigma_p} \mathbf{X}_{long}; & Long \ with \ Trend \\ \frac{30\%}{N*\sigma_p} \mathbf{X}_{long/short}; & Long/Short \ with \ Trend \end{cases}$$
(13)

 $\frac{30\%}{\sigma_p}$ is the target gearing for an annualized volatility of 30%, $\mathbf{1}_N$ is a vector of ones, and \mathbf{X} is a vector of signals.

2) Inverse Volatility (ivol): The allocation rules to inverse volatility portfolio is computed as,

$$\mathbf{w} = \begin{cases} \frac{30\%}{\sigma_p} * \frac{\frac{1}{\operatorname{diag}(\sqrt{\Sigma})}}{\sum_{i}^{N} \frac{1}{\sigma_i}}; & Long \ Only \\ \frac{30\%}{\sigma_p} * \frac{\frac{\mathbf{x}_{long}}{\operatorname{diag}(\sqrt{\Sigma})}}{\sum_{i}^{N} \frac{\mathbf{X}_{i,long}}{\sigma_i}}; & Long \ with \ Trend \\ \frac{30\%}{\sigma_p} * \frac{\frac{\mathbf{x}_{long/short}}{\operatorname{diag}(\sqrt{\Sigma})}}{\sum_{i}^{N} \frac{|\mathbf{X}_{i,long/short}|}{\sigma_i}}; & Long/Short \ with \ Trend \end{cases}$$

$$(14)$$

3) Inverse Variance (ivar): The allocation rules to inverse variance portfolios is computed as,

$$\mathbf{w} = \begin{cases} \frac{30\%}{\sigma_p} * \frac{\frac{1}{\operatorname{diag}(\Sigma)}}{\sum_{i}^{N} \frac{1}{\sigma_i^2}}; & Long \ Only \\ \frac{30\%}{\sigma_p} * \frac{\frac{\mathbf{X}_{long}}{\operatorname{diag}(\Sigma)}}{\sum_{i}^{N} \frac{\mathbf{X}_{i,long}}{\sigma_i^2}}; & Long \ with \ Trend \\ \frac{30\%}{\sigma_p} * \frac{\frac{\mathbf{X}_{long/short}}{\operatorname{diag}(\Sigma)}}{\sum_{i}^{N} \frac{|\mathbf{X}_{i,long/short}|}{\sigma_i^2}}; & Long/Short \ with \ Trend \end{cases}$$

$$(15)$$

4) Equal Risk Contribution (erc):

The objective functions for the equal risk contribution portfolios are as follows,

- Long Only:

$$\max_{\mathbf{w}} \sum_{i}^{N} \log(w_{i})$$
s.t. $\sqrt{\mathbf{w}' \Sigma \mathbf{w}} \le 30\%$ (16)

- Long Only with Trend:

$$\max_{\mathbf{w}} \sum_{i}^{N} X_{i,long} \log (w_i)$$
s.t. $\sqrt{\mathbf{w}' \Sigma \mathbf{w}} \le 30\%$

- Long Short with Trend:

$$\max_{\mathbf{w}} \sum_{i}^{N} |X_{i,long/short}| \log(|w_{i}|)$$

$$\text{s.t.} \sqrt{\mathbf{w}' \Sigma \mathbf{w}} \leq 30\%$$

$$w_{i} * X_{i,long/short} > 0$$
(18)

The risk model used for all the estimations is based on a conditional covariance matrix with exponentially weighted volatilities with 63 day half life, and exponentially weighted correlation matrix with 252 day halflife. Figure 15 outlines the performance statistics for each allocation rule. The last row, 'ew long' is a simple benchmark based on a equal weighted portfolio of all the 22 coins, rebalanced daily.

		Arith. Mean	Compound Mean	Volatility	SR	t-Stat	Geo SR	Max DD	Ulcer	Calmar	Skewness	Kurtosis
	ew	22.1%	19.0%	30.7%	0.72	1.53	0.62	46.6%	0.99	0.48	-0.84	7.86
Risk Based	ivol	23.4%	20.1%	31.6%	0.74	1.57	0.64	48.0%	0.99	0.49	-0.86	7.58
nisk baseu	ivar	24.4%	21.1%	32.2%	0.76	1.60	0.65	49.1%	1.00	0.50	-0.85	7.54
	erc	23.3%	20.1%	31.5%	0.74	1.57	0.64	48.1%	0.99	0.48	-0.87	7.70
Risk Based	ew	23.6%	20.7%	30.6%	0.77	1.63	0.68	45.5%	1.13	0.52	-0.83	8.13
+ Trend	ivol	28.2%	26.0%	31.5%	0.89	1.89	0.83	45.8%	1.37	0.62	-0.79	7.46
Long Only	ivar	29.9%	27.9%	32.2%	0.93	1.97	0.87	46.3%	1.45	0.64	-0.79	7.40
Long Only	erc	28.0%	25.9%	31.4%	0.89	1.89	0.83	46.0%	1.35	0.61	-0.80	7.57
Risk Based	ew	38.9%	40.4%	31.8%	1.22	2.59	1.27	23.2%	4.10	1.68	1.06	8.24
+ Trend	ivol	42.3%	44.9%	32.5%	1.30	2.75	1.38	22.7%	4.50	1.86	0.99	7.56
Long Short	ivar	45.4%	49.2%	32.9%	1.38	2.92	1.50	22.3%	4.86	2.03	0.90	7.07
Long Short	erc	33.5%	32.6%	32.4%	1.03	2.19	1.01	35.4%	2.26	0.94	0.47	4.96
	ew_long	59.0%	21.4%	87.4%	0.67	1.43	0.24	89.6%	0.90	0.66	-0.68	5.18

Figure 15: Performance Statistics: 2018-Present

The risk based methods show sizeable improvement compared to the simple long only benchmark. The geometric sharpe ratios are higher compared to the simple benchmark, which has a Geo Sharpe Ratio of 0.24. However across the four allocation framework, there is not much difference with regards to risk adjusted performance. The story changes a bit when I add the trend overlay. All the risk based methods not only outperform the simple benchmark, but also equal weighted (ew) trend approach, suggesting synergy in performance when incorporating trend following with risk based allocation rules. The inverse

variance (ivar) portfolio stands out to be the best out of all the models, with a Geo SR of 0.87 versus the equal weighted trend (ew) with a Geo SR of 0.68. Next, I also look at long-short portfolios. Fully aware that during the periods in the backtest, cryptocurrencies weren't shortable at scale, thus one can potentially expect outsized risk adjusted performance. The champion model in this case, again the inverse variance portfolio, has a sharpe ratio of 1.50. All the other models also show decent performance compared to the simple benchmark.



Figure 16: Cumulative Performance - Scaled to 15% Volatility: 2018-Present

5 Simulation Analysis

Given the short period of data used for this analysis, it can be quite misleading to simply rely on the point estimates for each of performance metrics from the backtests. For this reason, I run a simulation based test to generate distribution for the performance metrics. In particular, I use the champion risk based method, the ivar portfolios as candidates for this analysis. Also, I am mostly interested in a long only crypto portfolio, so I disregard the long/short models for this analysis.

I first sample 5000 paths from historical data using a stationary block bootstrap method. I assume an average block length of 21 days. Next, for each path I compute three separate portfolios, 1) the long equal weighted portfolio 2) the inverse variance portfolio and also 3) the inverse variance portfolio with trend overlay. Next, for each path and portfolio I compute the necessary performance statistics and compute their distribution quantiles. Figure 17 outlines the quantiles for some of the key measures in question. Figure 18 plots the overlapping density plots of the portfolios for each given metric.

The median results are consistent with the historical backtests. The inverse variance with trend outperforms the other three cases, with Geo SR of 1.02. The distributions for Geo SRs are much tighter for the risk based methods versus the simple benchmark, where the 10th and 90th quantiles are -0.17 and 2.30 versus 0.17 and 2.09 for ivar plus trend. Looking at the distributions of compound mean, the worst case 10th percentile for long only benchmark is much lower (-15.3%) compared to the other two portfolios (2.4% and 5.4%), suggesting greater return deterioration due to the variance drag. Furthermore, the maximum drawdown distributions are much skewed to the right compared to the risk based portfolios, with a probability of 10% of a drawdown event greater than or equal to 95.1% in the benchmark versus 57.2% for the ivar plus trend portfolio.

Compound Quantile **Portfolio** Geo SR Max DD Ulcer Calmar Mean long equal -15.3% -0.1768.8% 0.39 0.28 10.0% 2.4% 0.07 ivar 31.1% 0.26 0.14ivar + trend 5.4% 0.17 30.1% 0.38 0.20 long equal 60.4% 0.68 83.5% 1.90 1.06 50.0% 28.5% 0.89 42.4% 1.63 0.71 ivar ivar + trend 31.8% 40.8% 1.02 1.87 0.80 2.30 long equal 206.2% 95.1% 4.63 2.09 90.0% 4.25 ivar 61.8% 1.93 59.4% 1.57 ivar + trend 66.4% 2.09 57.2% 4.65 1.70

Figure 17: Simulation Results - Quantile of Performance Metrics

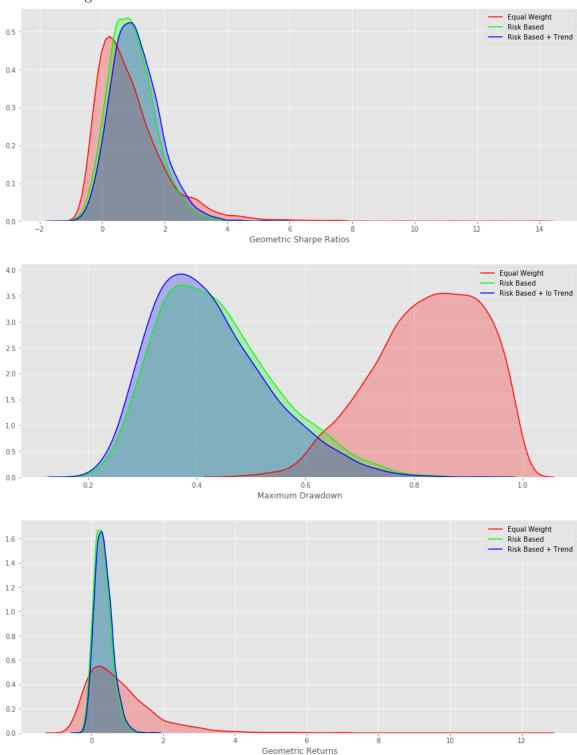


Figure 18: Simulation Results - Distributions of Performance Metrics