Trend Following – Expected Returns

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Abstract

We derive and calculate an ex-ante expectation for generalized trend-following rules, both on a single market as well as for a portfolio of trend strategies. Furthermore, the efficiency of trend-following rules applied to synthetic market data with varying degrees of drift and autocorrelation is discussed. Finding indicate that there is a positive relationship between drift, autocorrelation and the theoretically extractable Sharpe ratio for a trend following strategy. Drift is more important, since it is theoretically unbounded, but strong auto-correlation can create positive returns in the absence of long term drift.

The expected Sharpe ratio of a trend strategy is proportional to the absolute drift and auto-correlation of a market above a threshold. The expected return of a portfolio of trend strategies is also sensitive to the average correlation between the individual trading systems. Anyone engaging in trend following strategies, should expect to generate positive returns if the drift is strong enough or if there is enough autocorrelation, but should seldom expect to beat a market.

Keywords: CTA, Trend Following, Managed Futures

Trend Following – Expected Returns

Trend-following is an established trading and investment strategy that has shown success of being able to adapt to new and changing market conditions. There is an established research body that discusses actual applications of trend following strategies, mostly for Futures market. Commonly, it is an investment thesis based on a singular view of markets behavior, i.e. Identifiable, slowly moving information diffusion that creates trends when markets move from one equilibrium to another. For most trading applications, there is a large degree of variation around the theme, where most applications mix different behavioral theories.

This paper estimates, using normally distributed random data with controlled drift and autocorrelation, what the requirements are for a public domain trend following strategy to achieve profitability. Furthermore, we create expectation for a portfolio of trend strategies. One of the technical rules, due the specification is not dependent on auto-correlation, but that is less of a problem for real world applications, since auto-correlation in markets is close to zero on a daily basis.

Estimates are presented as point estimate, in reality, there is a wide confidence interval around most of the estimates in this report. The reader is assumed to be able to adjust for the statistical boundaries around the presented figures.

The conclusion in this paper forms an important basis for the estimation of the expected return for a broad range of momentum based strategies.

Literature Overview

Most of the papers published on this subject have been written to describe or explain the rationale of why trend following rules have been able to deliver positive returns from markets, as an ex-post phenomenon. Here, a different approach is used. Evaluating under which conditions a trend exploiting strategy is expected to work and how much you should have made from a particular market, given generalized characteristics of a market. A manager that is trading a specific set of markets, should have generated returns that are of the same magnitude and efficiency as our returns are indicating. Thus, the results developed in this report does not only allow someone to create expectations for a strategy, but also to develop benchmarking strategies for existing strategies.

Trend-following is also one of the more popular investment strategies within the hedge funds industry that is classified as "Managed Futures"[1]. (Hurst, Yao, & Pedersen, 2012), (Erb & Harvey, 2006) and (Burghardt & Walls, Managed Futures for institutional investors, 2011) represent a sample of books and articles discussing the value, construction and implementation of trend-following strategies.

The theoretical foundation of trend following under various drift and autocorrelation assumptions can be found in (Acar, 1996) and further analyzed in (Lequeux, 2003). This is a framework that this paper analysis and develops.

Synthetic market data

We make use of simulated market data, while not real, it allows us to isolate component under which trend following strategies are profitable. Since real market data does not exist in isolation, it is problematic making strong assumption about the underlying structure and profitable of the strategies that we seek to evaluate.

Synthetic markets are simulated with varying degrees of drift and/or autocorrelation. Both positive and negative values for drift and autocorrelation are tested, under the assumption that momentum strategies are symmetrical. Simulated markets are not real market data, but interesting results can still be expected to be found. A prior article, (Wojtów, 2012) find outperformance of real markets over random markets with trend elements, which could hint that markets are not well described by random data.

The data is also different from other perspectives; it does not have leverage effects, long-term memory and more importantly no real life interaction from traders and investors. This makes the experiment only a partial solution.

For simplicity, for the first experiments, an assumption is a world without transaction costs. A world with transaction costs reduce the profitability for the trading strategies, especially so for strategies with frequent transactions.

Data Generator

As part of our experiment, long term synthetic market data stream is simulated, using a normal distribution as the random return generator with a known fixed drift (μ) as well as a with a fixed variance (σ^2):

$$X \sim N(\mu, \sigma^2)$$

Adding predictable auto-correlation to the time series requires a few more steps. The normal distribution is independent and identically distributed and does by definition lacks auto-correlation. Auto-correlation is controlled by adding dependency on prior values. For this experiment, only the first auto-correlation effect is controlled. Our random data generator then turns into the following:

$$B_t = \rho B_{t-1} + A$$

Where A, B are $N(0, \sigma^2)$ and ρ is the first order autocorrelation. Furthermore, any unintentional drift in B, is removed and then add back our intended drift component μ_t. Unintentional drift is an issue for non-zero values of ρ.

$$R_t = B_t - \mu_b + \mu_t$$

Throughout this paper, the variance is set to 10% per annum (p.a.) and the drift is allowed to vary between negative 25% to positive 25% p.a. using a hypothetical trading year of 260 days. The auto-correlation is allowed to vary between negative 1 and positive 1. Thus, the simulated random market has a controlled volatility, drift and first order auto-correlation. Compared to real markets, only values around zero are found and the range is wider than what can be observed.

The generated returns, are similar to existing markets values. Several realization of the experiment looks realistic but some does not look realistic. Both effects are due to randomness, but a sample path is presented in *Figure 1*.

As a fixed volatility is used for the random market, it is easy to compute the Sharpe ratio (an efficiency measure):

$$S = \frac{E\left[R - R_f\right]}{\sigma^2}$$

The Sharpe ratio is used is a measure of signal to noise, where a higher measure means higher risk adjusted profitability and is preferable. As the simulator is generating excess return, R_f is defined as zero which simplifies the equation.

As our trading systems do not transform volatility or scales signals inversely proportional to the volatility, only the sign of the returns. This implies that the trading system and the simulated market data have the same sum of squared returns for zero-drift markets.

For the purpose of this experiment, simulation of 100,000 random market return for each value of μ_t,ρ and lookback is generated for each trial. The experiment is repeated five times. This amount of data represents close to 2000 years of simulated market data for each parameter combinations, creating statistically reliable data upon which we can perform our calculations.

The random data generator is based on the Wichmann-Hill algorithm (Wichmann & Hill, 1982).

Theoretical Framework

A framework for determining if the expected return, for a linear Gaussian process with drift was developed in (Acar, 1996) which is compatible with our random market data generator. While one can have different views on if markets are Gaussian enough, the equation below gives a relationship between expected return, drift and autocorrelation for a given market.

$$E(R_{t+1}) = \sqrt{\frac{2}{\pi}}\sigma\rho e^{-\mu_f^2/2\sigma_f^2} + \mu(1 - 2\Phi(-\mu_f\ /\sigma_f\ ))$$

Where ρ is the first order autocorrelation, Φ is the cumulative function of $N(0,1)$, μ is the expected value and σ is the standard deviation. As noted in (Lequeux, 2003), in the special case of pure random walk with drift, the return is negative function of the volatility. That is, the higher volatility and the lower the drift is, the worse is the expected Sharpe ratio. In the case of no-drift, but varying degrees of auto-correlation, the returns are positive function of the auto-correlation. The returns are expressed without transaction costs.

We plot the function, for varying degrees on ρ and μ, (σ is constant of 10% p.a.) (*Figure 2*). The strategy has positive expected Sharpe for most levels of drift and auto-correlation. The except is a range of values where the drift is low combined with negative auto-correlation.

The formula does not offer a framework for the length of the lookback itself and as observed later, this has importance. The results are limited by the Sharpe of the market itself as that would represent a perfect prediction. One reason for why the trend-strategy does not manage to do better is by dynamic choice. It will at some point be against the long-term drift. At this point, the trade has a negative expected return, by definition.

From the equation, we also note that auto-correlation is more important when drift is low and for a range of values, trend following strategies are predicted to have positive expected returns despite no long-term market drift.

Under these specifications of the model, the highest Sharpe a binary trend strategy can deliver, in a single market, is the Sharpe of the market itself.

Trend-following strategies

A set of previously known and described trend following strategies are applied to the synthetic market data. While there are a large number of variations of technical rules or indicators, the chosen strategies only contain one parameter and has been described in the technical literature. They are thus established, with long term track-records but also robust. The parameter, being the same for both indicators, the lookback of a strategy. This is the window over which the strategy collects information, i.e. what the strategy knows.

The rule predicts the next period (day) returns and is updated every time there is a new data point arrives. The results of the systems are yesterday signal times the market return today[2]. Admittedly, the systems presented are strikingly simple, but have been used in the literature to replicate and explain behavior of trend following strategies.

Moving Average

A moving average is a filter strategy, that filters out noise and smooths out a time series. There are multiple variations of moving averages and for a good exposition see (Kaufman, 1998) which covers variations from the standard moving average, double smoothed, average-modified, weighted, triangular, pivot-point weighted, geometric and exponential variations of the same calculations. The strategy used in this specification, is based on the simple moving average:

$$MA_t = \frac{r_t + r_{t-1} + r_{t-2} + \cdots + r_n}{n} = \frac{1}{n}\sum_{t=1}^{n} r_t, \qquad n \leq t$$

This analysis use the trading systems referred in (Kaufman, 1998), the "simple moving average" system:

$$\begin{cases} MA_t > 0 \; buy \; or \; stay \; long \\ MA_t \leq 0 \; sell \; or \; stay \; short \end{cases}$$

Letting t vary from five days to 200 days we execute the trading strategy on the simulated data. The strategy is always in the market and does not scale positions up or down, depending on trend strength. This is a binary strategy, both based on position taking and logic.

Momentum

Momentum is a simpler strategy, in terms of what it is looking to extract from the markets. The most basic implementation of the strategy will be studied, which is essentially looking at the slope between two points, n-days apart. The indicator is defined as:

$$Mom_t = close_t - close_{t-n}$$

In line with the above, define the trading rule as the following:

$$\begin{cases} Mom_t > 0 \; buy \; or \; stay \; long \\ Mom_t \leq 0 \; sell \; or \; stay \; short \end{cases}$$

Momentum is an information poor strategy; it ignores much of what is happening between the two measurement points and is thus more dependent on the actual behavior of a market. Compared to the moving average, the information density is similar for small n, but different for large n. The strategy can thus also be perceived as a strategy that seek to identify and exploit acceleration rather than pure drift.

$$\frac{Mom_t}{n+1} = SMA_{t,n} - SMA_{t-1.n}$$

From the definition of the drift equation, there is no explicit acceleration of price moves, beyond the first day. Due to lack of explicit acceleration in our time series, we cannot expect that the momentum strategy should have a higher efficiency than the moving average strategy. On top of that, the way the random market data is generated suits the moving average strategy better. Other ways to generate random market data are outside the scope of this paper.

Results

In the section, an exposition and discussion about the results of each set of rules take place. With regards to the moving averages, the same general shape as for the theoretical result are observed. It has positive expected returns for all values except for combinations of low drift and negative autocorrelation. High Sharpe ratios are observed for high values of autocorrelation (*Figure 3*).

Our results for the momentum rule is slightly more complicated (*Figure 4*). The strategy is unprofitable for all negative values of auto-correlations and insensitive to drift for positive auto-correlation values. Like with the moving average, the worst results are observed when negative auto-correlation is combined with low drift. In *Figure 5*, the impact of the auto-correlation is isolated, by removing all results with drift (i.e. the solution with zero drift). The solution is broadly speaking similar for both systems. For the momentum strategy, there is strong dependence on the auto-correlation combined with a short time horizon. This is perhaps not unexpected when reflecting upon the definition of the market data generator. For short-term trend-strategies, there is an absolute requirement on positive auto-correlation for profitability. Imposing further drift, as before, reduces the hurdle and the strategy can remain profitable despite negative auto-correlation (before transaction costs).

Looking across all drift values, there is a tendency for the moving average strategy to be more profitable, the longer the time frame is under compatible large drift. For short term strategy, the dependency is different, highlighted in red above. That is, there is a tradeoff between potential returns under zero drift combined with lower returns under stronger drift. For the momentum strategy, there is no easily discernable consistent pattern.

The short-term behavior of these trading strategies are different. Behavior and implications under transaction costs assumptions is explored in the next section. Short-term trading has a potential to deliver remarkable Sharpe ratios as displayed *Figure 5*.

Transaction costs

Before transaction costs, there are only minor drift and/or auto-correlation requirements for trend strategies to be profitable. As part of the simulation, the number of trades are tracked. Transaction frequency is a function of the lookback. Shorter term strategies change position more frequently as a function of volatility in the market (*Figure 6*).

Estimating real transaction costs is difficult due to differentiated market structure. There are multiple markets, multiple volatility levels and market specific restrictions that create differentiated transaction costs for each specific market. For the sake of simplicity, for the simulated market, transaction can be executed at approximately the average bid-ask spread for liquid markets. This is true for small volume, controlled and well-hidden trading activities. For larger orders, there are generally higher transaction costs.

Imposing transaction costs for two strategies result in break-even requirements that are pushed higher. For the zero-drift momentum strategy, adding transaction costs has managed to increase the breakeven auto-correlation from 0 to 0.3 for a modest transaction cost 2.5 basis points (bp) for each trade. Higher transaction costs (measured as transaction cost, slippage, commissions etc.) will reduce the profitability even more.

With regards to the moving average strategy, imposing transaction cost require markets to have a higher drift than before. For short-term strategy, there is little chance of being profitable unless the underlying market has a Sharpe ratio widely exceeding long term expectations. For long term strategies, we note profitability in line with long term market data. This is across the range of the previously discussed auto-correlation values.

The results are not surprising, increasing friction in a system, will result in higher break-even Sharpe ratios. That said, from the charts presented in this paper, the requirements for profitability are lower when using long term strategies. In fact, generating profits from short-term strategies using a trend-following strategy might be impossible, at least for the limited set of strategies tested in this paper.

The choice of which strategies to use, should be as independent of the underlying market characteristics as possible. Therefore, optimal ex-ante Sharpe ratios might only be achievable as a long term trader if opting for a portfolio that consists of longer term strategies. Short-term trend-following strategies are restricted by requirements on market drift, auto-correlation and transaction costs. Short term portfolio construction requires a different approach as the expectations on market characteristics are high.

Portfolio efficiency

To estimate the expected efficiency of a portfolio of the analyzed strategies, the starting point is the estimation the Sharpe of a single market. We observe that markets exhibit long-term auto-correlation close to zero and we can therefore reduce the complexity of our model:

$$E(R_{t+1}) = \mu(1 - 2\Phi(-\frac{\mu_f}{\sigma_f}))$$

Thereafter, estimate expected correlation between systems. Under the assumption that volatility can be managed properly, the expected return of a portfolio can be calculated, given a correlation level. Table 1 shows the expected efficiency for different market levels.

Mirroring our observations, (Burghardt, Kirk, & Liu, Capacity of the managed futures industry, 2013) estimates that a trend-following has single market Sharpe of 0.1. Given our external and the values presented herein, a reasonable estimate for a single market Sharpe is between 0.1 and 0.2. Expected Sharpe of the portfolio for various degrees of system correlation[3] can be calculated using standard formulas for portfolio risk (*Figure 7*). From the above, a higher correlation is not beneficial to a portfolio of trend-based strategies using similar markets.

When allocating risk to trading systems, market correlation is transformed into a reactive function that can be long or short the underlying asset and would thus not be fully correlated to the underlying market. Thus, there are reasons to expect that correlation between trading strategies is lower than what the market correlation above would imply[4]. This is somewhat harder to estimate since it not possible to accurately account for all variations of trading systems that there might be. The results are summarized in the Table 2 for varying degrees of correlation and underlying markets.

Conclusion

Based on the established framework, return generated from medium to long-term trend following strategies are due to market drift and auto-correlation. Based on reasonable value of drift and autocorrelation available from liquid markets, trend-following, on a stand-alone basis, is a low Sharpe strategy, with an expected per-market Sharpe of approximately 0.15. This is a function of the market drift. We also estimate that based on available market data, a broadly diversified portfolio of trend strategies has an expected Sharpe ratio between 0.4 and 0.6.

Trend following portfolio returns, depends, not only on having a low market correlation, but is more dependent on having a low correlation between trading-systems. Trend-following can thus be referred to as a portfolio effect and would require diversification across markets to be successful, at least so for longer term momentum strategies.

For traders that are using trend-following strategies, it would seem prudent to continue to look for uncorrelated markets/trading systems while trying to explicitly understand why a market would have a persistent auto-correlation and/or a high risk adjusted drift.

Arguing the correlation makes it difficult to extract trend-following returns may be a fact, but if the correlation is expected to remain higher than the historical norm, a systematic strategy may also need to search for different strategies.

References

Acar, E. (1996). *Expected returns of directional forecasters.*

Burghardt, G., & Walls, B. (2011). *Managed Futures for institutional investors.* Bloomberg Press.

Burghardt, G., Kirk, E., & Liu, L. (2013). *Capacity of the managed futures industry.* Newedge.

Erb, C. B., & Harvey, C. R. (2006). The Tactical and Strategic Value of Commodity. *62*.

Hurst, B. K., Yao, O. H., & Pedersen, L. H. (2012). A Century of Evidence on TrendFollowing. AQR.

IMF. (October 2015). *Global Financial Stability Report.* New York: IMF.

Kaufman, P. (1998). *Trading Systems and Methods.*

Lequeux, P. (2003). Optimal Allocation of Trend-Following Rules: An Application Case of Theoritical Results. In C. Dunnis, J. Laws, & P. Naim, *Applied quantitative methods for trading and investments.* Wiley Finance.

Nilsson, L. (2015). *Continuous Risk Adjustments - Does the Half-Life Have Implications for the Distribution.* Retrieved from http://ssrn.com/abstract=2667156

Wichmann, B. A., & Hill, I. D. (1982). Algorithm AS 183: An efficient and portable pseudo-random number generator. *Applied Statistics, 31*(2).

Wojtów, Ł. (2012). *Theoretical basis and a practical example of trend following.*

Tables

Table 1

Calculated relationship between market Sharpe and Trend following Sharpe for a single market. We note that markets need an expected Sharpe (positive or negative) above 0.2 for trend strategies to be profitable.

Expected Market Sharpe	Expected Trend following Sharpe
0.2	0.03
0.3	0.07
0.4	0.12
0.5	0.19

Calculated values are based on long-term ranges for a set of diversified markets.

Table 2

Expected Portfolio Sharpe for varying degrees of market-system efficiency for portfolios of trend following strategies. Table shows the expected portfolio Sharpe for a set of trend strategies with a pair-wise correlation ranging from 1.0 to 0.1 where the underlying strategy has a Sharpe ratio of 0.15.

For portfolio of strategies we note that the efficiency converges to the square root of the number of component as the correlation between return stream goes to zero.

Number of assets	1.0	0.9	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1	0.0
2	0.15	0.15	0.16	0.16	0.17	0.17	0.18	0.19	0.19	0.20	0.21
4	0.15	0.16	0.16	0.17	0.18	0.19	0.20	0.22	0.24	0.26	0.30
8	0.15	0.16	0.17	0.17	0.19	0.20	0.22	0.24	0.27	0.33	0.42
16	0.15	0.16	0.17	0.18	0.19	0.21	0.23	0.26	0.30	0.38	0.60
32	0.15	0.16	0.17	0.18	0.19	0.21	0.23	0.26	0.32	0.42	0.85
64	0.15	0.16	0.17	0.18	0.19	0.21	0.23	0.27	0.33	0.44	1.20

As the IMF noted in *(IMF, October 2015)*, correlations has remained high after the financial crisis of 2008. This would generally reduce the expected Sharpe ratio of any portfolio.

Figures

Figure 1 First 1000 data points for the simulation using a drift of 10% p.a. and an autocorrelation of 0.1 on an index basis. The path resembles an actual market, with the exception of time varying volatility.

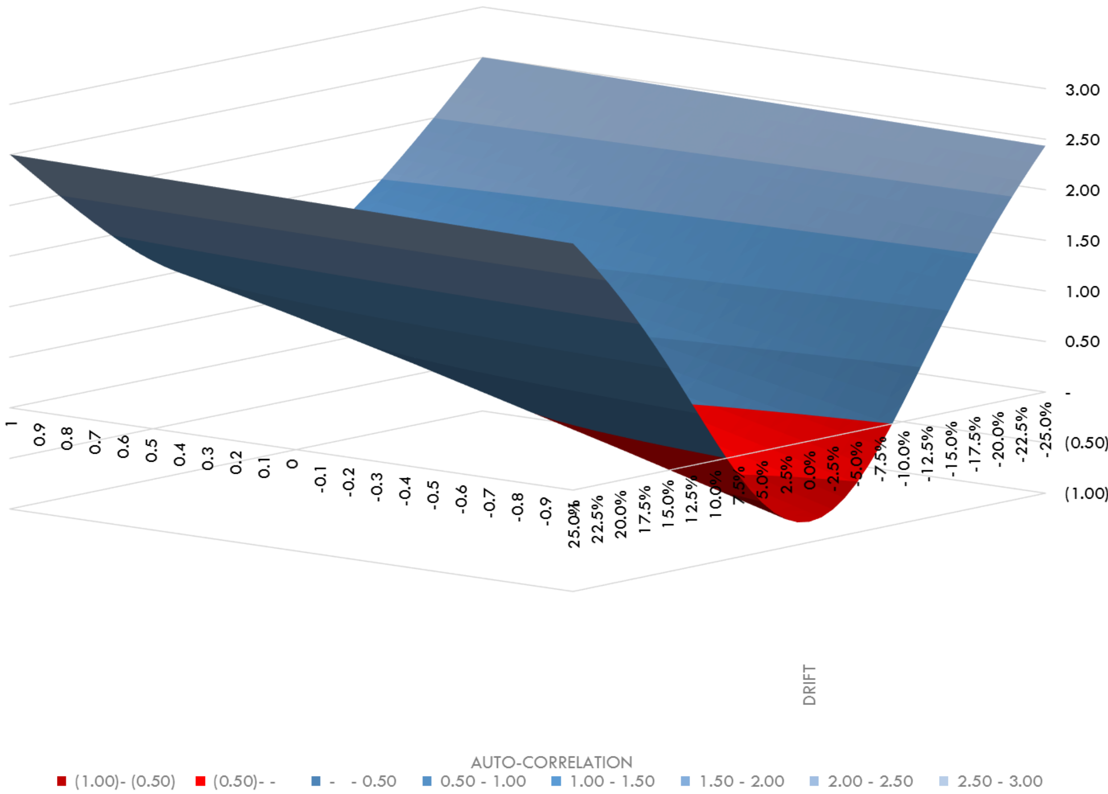

Figure 2 Theoretical profitability as a function of auto-correlation and drift (x-axis: market drift, y-axis: auto-correlation and z-axis Sharpe). We note that the strategy deployed here shows a uniform pattern of profitability.

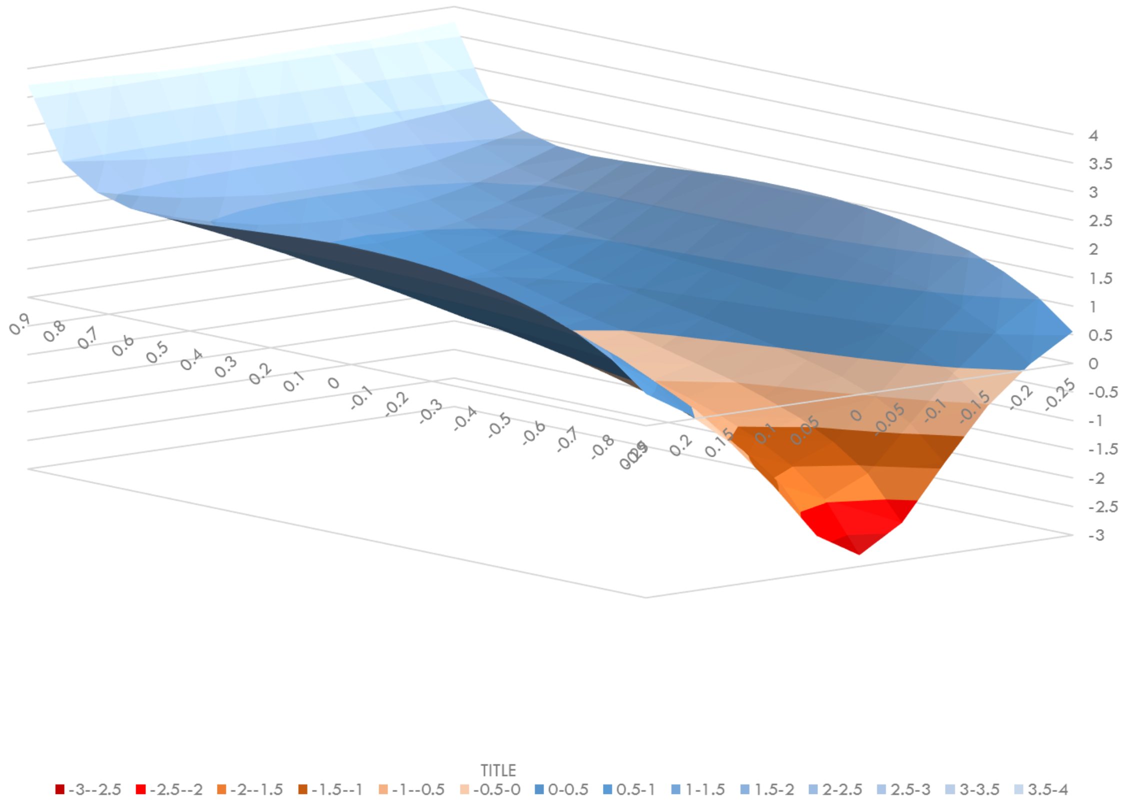

Figure 3 Moving Average rules (x-axis: market drift, y-axis: auto-correlation and z-axis Sharpe). The moving average rules is sensitive to negative auto-correlation and will not produce positive expected return under such conditions (unless the drift is high).

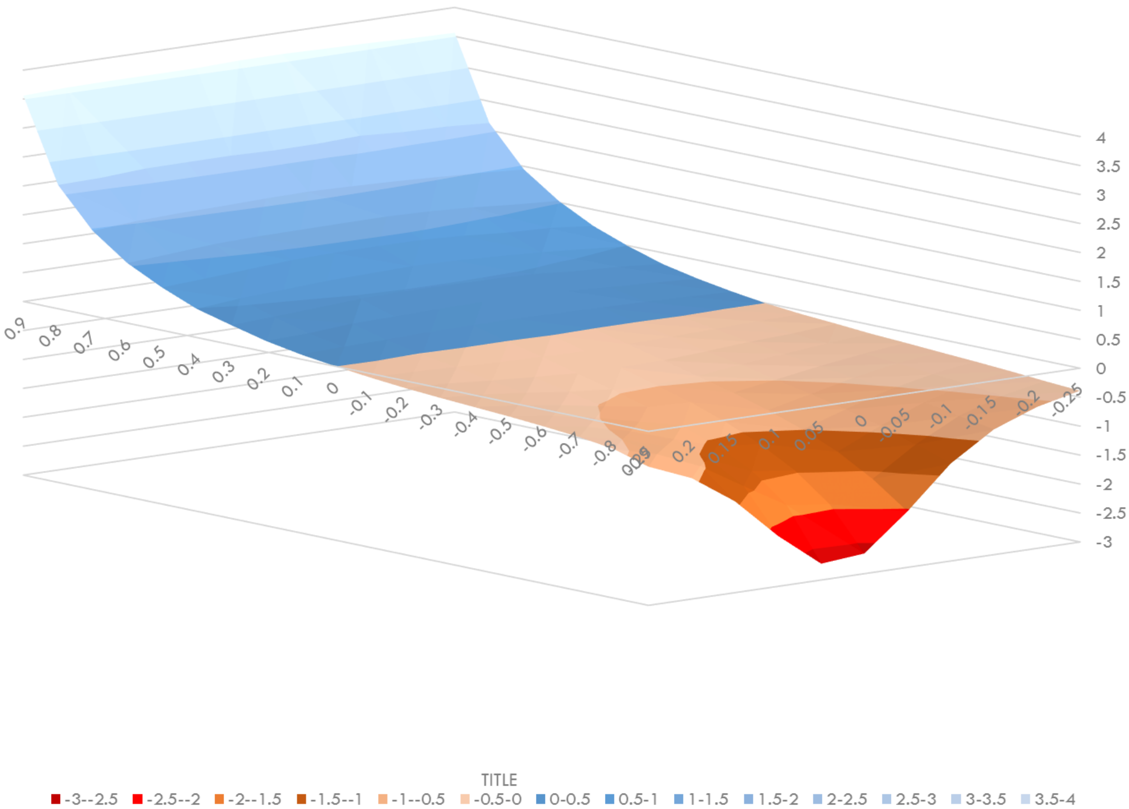

Figure 4 Momentum Results (x-axis: market drift, y-axis: auto-correlation and z-axis Sharpe). The momentum rule suffers from negative performance under negative auto-correlation, but does not depend on auto-correlation for larger values of the drift. The momentum rule is less sensitive to auto-correlation due to the construction of the strategy.

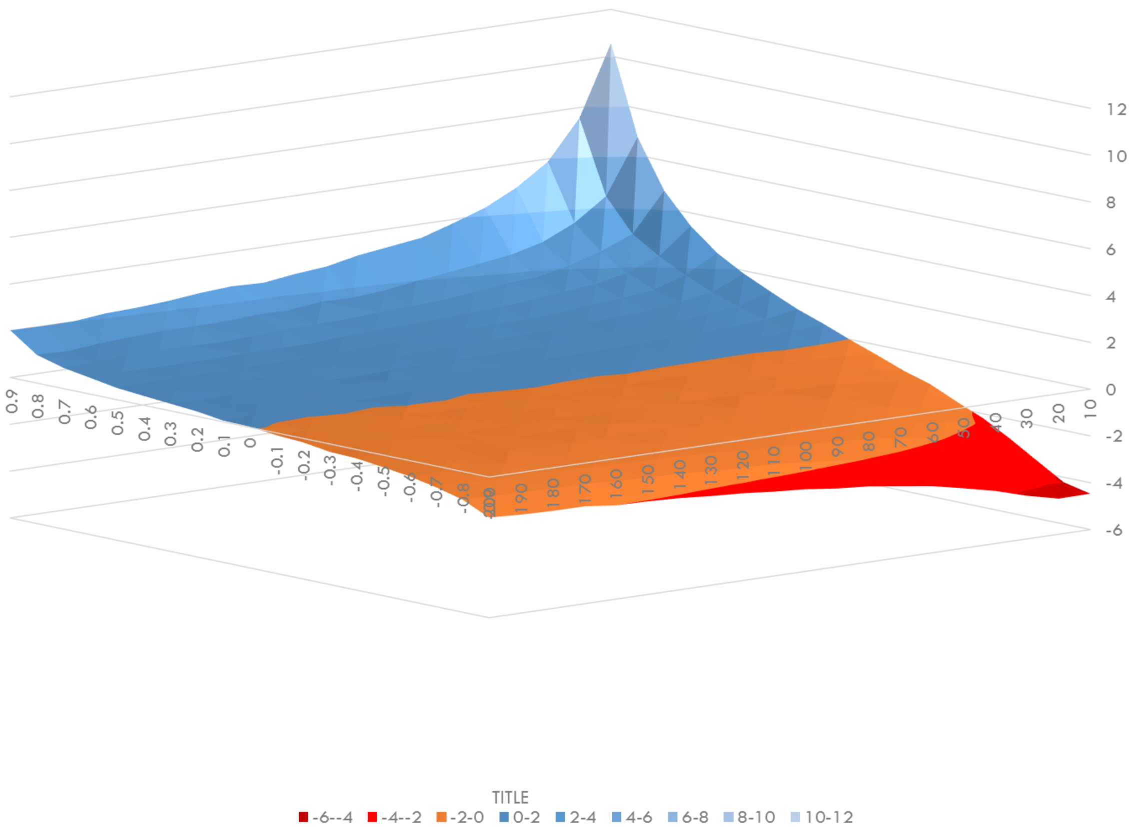

Figure 5 Zero-drift solutions momentum (x-axis: market drift, y-axis: auto-correlation and z-axis Sharpe). This isolate the behavior of the momentum strategy, showing that the strategy can potentially be profitable for low values of drift, but that it requires high values of auto-correlation.

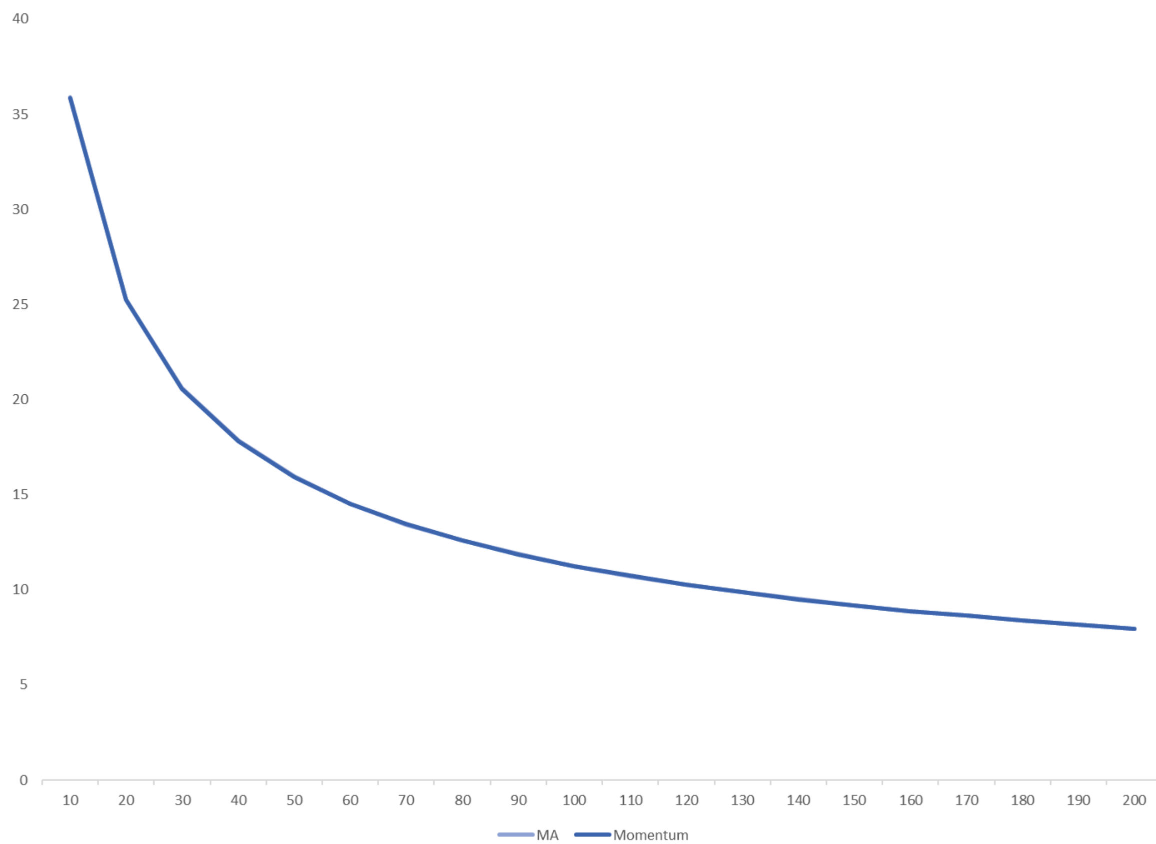

Figure 6 Number of trades per trading system, as a function of the lookback, 10% volatility p.a. for the system. The shorter the lookback, the more transactions are generated. A larger amount of transaction leads to a larger amount of transaction costs. The expected number of transactions per year, follows the framework developed in (Lequeux, 2003) and is independent of the trading system that was select. Conditional upon the lookback, the two explored trading systems have approximately the same number of expected trades per year. More interesting, is that market structure (drift and autocorrelation) does only have a minor impact on trading frequency. These results are stylized and does not hold for all trend strategy implementations.

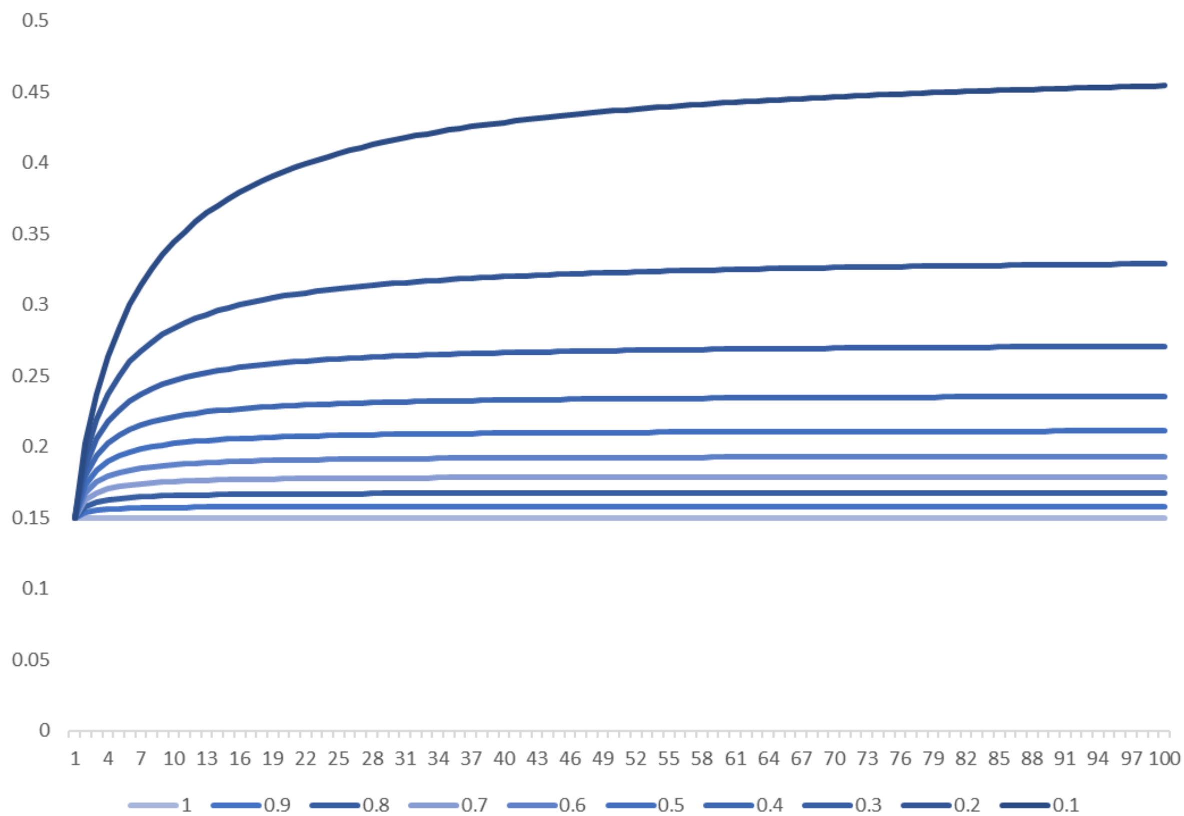

Figure 7 Expected portfolio Sharpe for a portfolio of n assets with correlation x, for an assumed efficiency of 0.15 for a single market trend strategy. As seen in *Figure 7*, the lower the correlation, the more efficient the portfolio is. This has been calculated for portfolios with up to 100 assets with the same correlation, although the number of liquid futures markets is somewhere between 30 and 50. Furthermore, futures market does have differentiated liquidity and the weighting by liquidity would reduce the actual number of true tradeable markets if liquidity is an issue for portfolio construction.

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1 The alternative classification Commodity Trading Advisor (CTA) is also used. CTA is a regulatory classification which requires registration with the National Futures Associations (NFA) whereas Managed Futures does not, in itself, require regulatory oversight. The terms are used interchangeably.

2 For practical purpose, there should be a short delay between the signal generation and the trade due to computation and implementation delays. For electronic trading and signal generation, this is generally small and close to infinitesimal compared to the expected holding period. We have not implemented this delay in the strategies.

3 Please note, that this is not market correlation, but rather the market-system correlations.

4 This is more pronounced the more times the system shifts from long to short, that is for shorter-term strategies and the realized correlations will fully show the benefit.