Understanding ETNs on VIX Futures

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ABSTRACT

This paper aims to improve transparency in the market for direct, leveraged and inverse exchange-traded notes (ETNs) on VIX futures. The first VIX futures ETNs were issued in 2009. Now there are about 30 of them, with a market cap of about \$3 billion and trading volume on some of these products can reach \$5 billion per day. Yet volatility trading is highly complex and regulators should be concerned that many market participants lack sufficient understanding of the risks they are taking. We recommend that exchanges, market-makers, issuers and potential investors, as well as regulators, read this paper to improve their understanding of these ETNs.

We provide a detailed explanation of the roll yield and convexity effects that drive the returns on VIX futures ETNs, and we track their volatility and assess their performance over an eight-year period starting in March 2004, by replicating their values using daily close VIX futures prices. We explain how ETN issuers can construct almost perfect hedges of their suite of ETNs and control their issue (most ETNs are callable) to make very significant profits under all bootstrapped scenarios. However, market knowledge has precipitated front-running of the issuer's hedging activities, making profits more difficult to control. Moreover, for hedging the ETNs such large positions must be taken on VIX futures that the ETN market now leads the VIX futures that they are supposed to track. The result has been an evident increase in the volatility of VIX futures since 2009. If this increase in statistical volatility induces an increase in VIX futures implied volatility, a knock-on effect would be higher prices of VIX options whilst S&P options are unaffected.

A previous discussion paper, Alexander and Korovilas (2012), provided incontrovertible evidence that single positions on direct VIX futures ETNs of any maturity – including mid-term and longer-term trackers – could only provide a diversification/hedge of equity exposure during the first few months of a great crisis of similar magnitude to the banking collapse in late 2008. By contrast, the present discussion paper shows that some highly attractive long-term investment vehicles can be simply constructed by holding certain portfolios of VIX futures ETNs. In particular, we introduce a new class of 'roll-yield arbitrage' ETN portfolios which we call ETN^2 (because they allocate between direct and inverse VIX futures tracker ETNs) and ETN^3 portfolios (that allocate between static and dynamic ETN^2). These portfolios have positive exposure to mid-term direct-tracker ETNs and (typically) negative exposure to short-term direct-tracker ETNs (equivalently, positive exposure to short-term inverse-tracker ETNs). Their unique risk and return characteristics make them highly attractive long-term investments, as well as superb diversifiers of stocks, bonds and commodities.

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1. Introduction

Futures contracts on the Standard and Poor's (S&P) 500 volatility index (VIX) began trading on the CBOE[1] futures exchange in March 2004. Because the VIX index is not tradable there is no unique closed-form, arbitrage free, cost-of-carry relationship connecting the VIX index with the price of a VIX futures. In fact, there is often a sizeable difference between the index and its futures prices.[2] Still, the futures price represents the risk-neutral expectation of VIX at maturity, and as such VIX futures offer a volatility exposure that is still very highly correlated with the VIX index and with the over-the-counter (OTC) S&P 500 variance swaps brokered by investment banks. Moreover, unlike variance swaps, the futures have no credit risk. Hence, the investment side of a commercial bank no longer needs to rely solely on risky OTC trades to gain exposure to volatility.

Since 2004, VIX futures have been actively promoted as having diversification benefits and other unique characteristics. Sophisticated market players now trade VIX futures for speculation, directional exposure, arbitrage, diversification and vega hedging. And now any type of investor – e.g., a pension funds or an individual – has easy access to volatility trades on the New York stock exchange (NYSE) through the exchange-traded notes (ETNs) that track constant-maturity VIX futures. These products have some adverse features not shared by futures: (i) They retain the credit risk of the issuer, which has been relatively high since the banking crisis; (ii) A small investor may be trapped into an illiquid investment because the issuer will only redeem the shares early in large lots; (iii) Many ETNs have a callable feature whereby the issuer can call back the shares at any time, with a short call notice period. On the other hand, ETN issuers charge only a small early redemption fee and an annual service fee related to their hedging costs.

In 2009, Barclays Bank PLC issued VXX and VXZ, their 1-month and 5-month constant-maturity VIX futures trackers. Their performance is directly linked to that of the S&P 500 VIX Short-Term Futures Index and the S&P 500 VIX Mid-Term Futures Index respectively.[3] More brokers, notably ETRACS of UBS AG and VelocityShares of Credit Suisse, quickly followed suit with other tracker ETNs, 2 $\times$ leveraged products and inverse exposures to the S&P constant-maturity VIX futures indices. By December 2011 about 30 VIX-linked ETNs were trading in very high volumes on secondary markets,[4] the primary market being the NYSE Arca. About \$875 million was traded per day, on average, during the first two months of 2012 (not a particularly volatile period) on two of these ETNs; VXX, the Barclays iPath 1-month constant maturity tracker and TVIX, its supra-speculative, twice leveraged extension issued by Credit Suisse in November 2010. The U.S. Securities and Exchange Commission (SEC) are currently scrutinizing the dangers of ever-more complex leveraged exchange-traded instruments. [5] After many adverse press reports about TVIX, Bloomberg reported that the SEC investigators will review the product.[6]

[1] Chicago Board Options Exchange.

[2] Another (minor) difference is that VIX futures are settled on the special opening quotation price, which is based on traded prices of S&P 500 options fed in the VIX formula, whereas the VIX itself is based on the options mid price.

[3] See Standard & Poor's (2011b) for details of the methodology underpinning their calculation.

[4] Including four ProShares volatility-based products that trade as exchange-traded funds.

[5] See Reuters, February 23, 2012: "Analysis: Out of control? Volatility ETN triggers risk concerns."

[6] See: "SEC Said to Review Credit Suisse VIX Note". Bloomberg, March 29, 2012.

Issuers of ETNs, the exchanges that list volatility products, and S&P (which calculates the indices used for indicative values) are all promoting VIX futures and their ETNs as products that are suitable for investors seeking diversification, such as pension funds and mutual funds. However, the frenetic ETN market activity has spilled over to the VIX futures market, increasing the volatility of VIX futures so that they have now become some of the most risky of all exchange-traded instruments.[7] Besides, Alexander and Korovilas (2012) provide a detailed and thorough demonstration that individual position in VIX futures, or their ETNs, offer no opportunities for diversification of equity exposure, except during the onset of a major crisis. In short, they should be entered only as speculative trades.

On the other hand, one of the most important conclusions of this paper is that certain *portfolios* of VIX futures, or their ETNs, which typically take a short position on short-term VIX futures and a long position on longer-term VIX futures, can offer unique risk and return characteristics that should be attractive to many long-term investors, as well as providing superb opportunities for diversification of stocks, bonds and/or commodities. These portfolios might be referred to as a new class of 'roll-yield arbitrage' ETN portfolios, although they are not riskless. The idea is to trade on the differential roll costs at different points along the VIX futures term structure. We call these portfolios ETN^2 when they allocate between direct and inverse VIX futures tracker ETNs, and ETN^3 portfolios when they allocate between static and dynamic ETN^2.

We explain the mechanics under-pinning the design of two recently-launched volatility ETNs – the XVIX, issued by UBS AG in November 2010, and the XVZ, issued by Barclays Bank PLC in August 2011 – that fall into the category of ETN^2. Different ETN^2 products can have quite diverse risk and return characteristics. For instance, by replicating the indicative values of the XVIX and VXZ daily, from inception of VIX futures in March 2004 until December 2011, we show that: (i) the XVIX has almost zero correlation with the VIX, a relatively low volatility and it performs best during tranquil, trending market conditions; and (ii) the XVZ has a strong positive correlation with VIX, a relatively high volatility, and is highly profitable only during volatile periods.

This finding motivates a new class of ETN^3 portfolios that allocate between ETN^2s.[8] These ETN^3 have superior performance and greater diversification potential than the corresponding static and dynamic ETN^2s, according to a wide variety of performance criteria. We label these static and dynamic ETN^3 portfolios CVIX and CVZ, respectively.

ETN issuers can hedge their exposure to early redemption of ETNs perfectly, provided they can trade VIX futures at the daily closing price, because it is this price that determines the redemption value for a VIX futures ETN. Given the service fee charged we explain how issuers should guarantee significant profits, net of hedging costs, provided they issue a controlled portfolio of ETNs. However, there has recently been large-scale front-running activity on the very high volumes that must be traded near the market close on prompt and second to mature VIX futures for the purpose of

[7]For instance, during the onset of the Eurozone debt crisis in August 2011 the statistical (GARCH) volatility of the prompt VIX futures contract exceeded 200%, at a time when the average value traded on this contract alone exceeded one billion USD per day.

[8]Like all other volatility ETN innovations, ETN^3 can be replicated by trading a term structure of VIX futures.

hedging the two largest ETNs, VXX and TVIX. As a result, Credit Suisse stopped the issue of TVIX (sending its traded price to a 90% premium over indicative value at one point, such is the speculative demand on this product) and re-opened it only on the condition that the hedging risk be passed on to the market makers, i.e. they guarantee VIX futures closing prices to Credit-Suisse.

The outline of this discussion paper is as follows: Section 2 begins with explaining the fundamental building-blocks for constructing volatility ETNs, viz. indices of investable, constant-maturity VIX futures which are computed by the S&P. These indices determine the indicative value of VIX futures ETNs. Section 3 analyses the statistical features of constant-maturity VIX futures tracker products, and their implications for new volatility ETNs that are based on trading differential roll-yield effects along the term structure. Section 4 describes how the ETN^2 and ETN^3 products that we consider could be replicated using the term structure of VIX futures, and then examines their performance using a variety of criteria over the eight-year sample period. In Section 5 we explore the risk and return characteristics of one static and one dynamic ETN^3, comparing their performance with the ETNs already available, and demonstrating their great potential for diversification of exposures to other traditional asset classes: equities, commodities and bonds. Section 6 explains how the issuers of ETNs can structure their products and the size of their issue so that they can hedge their liabilities almost perfectly, and moreover make significant profits, provided the hedge on VIX futures is traded at the closing price on the CBOE. Section 7 concludes.

2. Constant-Maturity VIX Futures

For the period March 26, 2004 to December 31, 2011 we obtained Bloomberg data on the daily close (last traded price or index value) for: (i) all VIX futures contracts; (ii) VIX and VXV, i.e. the 30-day and 93-day implied volatility indices calculated by CBOE; (iii) the S&P constant-maturity VIX futures indices; (iv) the two most well-established volatility ETNs, i.e. the VXX and VXZ, Barclays' 1-month and 5-month constant-maturity VIX futures trackers; (v) XVIX and XVZ, the two recently-launched ETN^2.[9]

2.1. Constant-Maturity Prices

From the VIX futures prices we construct two sets of six synthetic time series, each set representing constant maturities of $m = 30, 60, \ldots, 180$ days. The first set is a non-investable futures price time series and the second is a futures return time series, which is investable. The constant-maturity futures price on day t is derived as:[10]

$$p_t^m = \alpha_t p_t^s + (1 - \alpha_t) p_t^l, \quad \alpha_t = \frac{l - m}{l - s} \tag{1}$$

[9](i) are only available from the inception of VIX futures on March 26, 2004; (iii) start only in December 2005 and prior to this we construct the indices by applying the S&P methodology to traded VIX futures; (iv) are available only since their launch in January 2009, but we replicate their indicative values using actual and reconstructed S&P indices; (v) have very few traded prices, starting only in November 2010 and August 2011 respectively, but again we replicate their indicative values as for (iv).

[10]Note that in Eq. (1) and also in Eq. (2) below, we exclude observations with maturity less than five business days. This is to avoid irregularities caused by near-to-maturity trading, as well as the special settlement process in the VIX futures market.

where p^m is the synthetic futures price for a specific maturity m, p^s and p^l are the prices of the shorter and longer maturity exchange-traded futures contracts that straddle the maturity m, with $s < m < l$, and each maturity is measured in calendar days to expiry. These price series provide a visualization of the futures term structure and its properties, based on market traded prices but, returns based on these synthetic prices are not realizable.

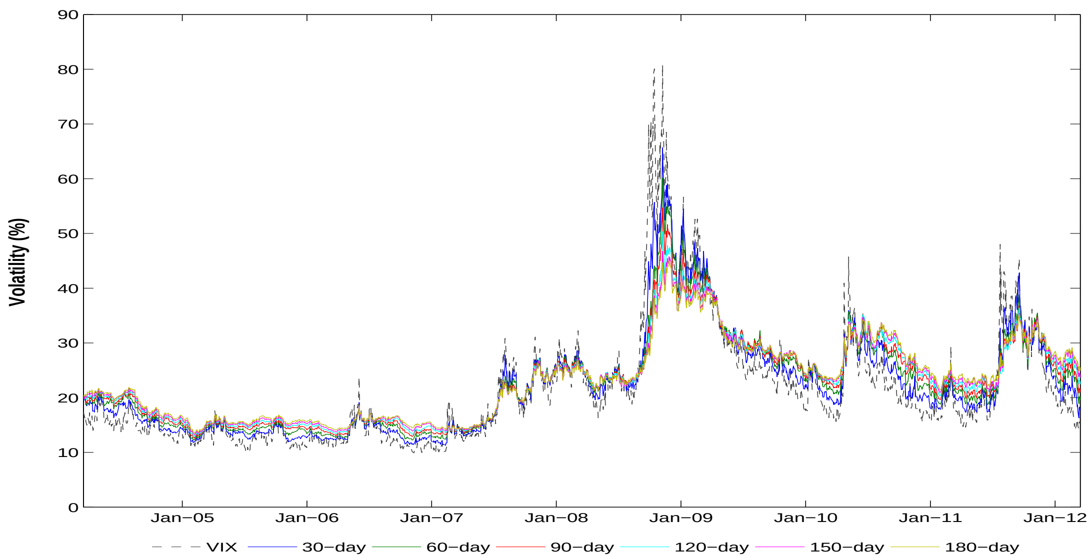

Fig. 1 Term structure evolution of the six constant-maturity VIX futures series, and the VIX 30-day implied volatility index. March 2004 – December 2011.

Fig. 1 depicts the VIX futures price series for the six different maturities under consideration, and the VIX 30-day implied volatility index that underlies the futures. Since the financial crisis starting in mid 2007 volatility has never returned to its previous levels. In September 2008, precipitated by the Lehman Brothers collapse, VIX futures were trading at around 50%. Since then volatility was especially high during the Greek crisis of May 2010 and the wider Eurozone debt crisis beginning in August 2011.

The term structure of VIX futures is typically in contango. Brief periods of backwardation only accompany excessively volatile periods. For our later analysis (in section 4.3) it is interesting to note that, over the entire sample, the 30-day futures price was greater than (less than) the 150-day futures price about 24% (76%) of the time. In some periods, particularly from Q2-2007 to Q2-2008, the long-term contracts at 150 or 180 days to expiry are trading at very similar levels to the short-term contracts at 30 or 60 days to expiry, which implies that the market believes the VIX is at its long-term level.

When the term structure is in backwardation its slope is typically inconsistent with the time-series properties of VIX. For instance, on January 2, 2009 the (synthetic) 180-day futures closed at 36%; but this overestimated the implied value of VIX on July 2, indeed on that date the VIX was below 30%. In other words, the VIX mean-reverts more quickly than the term-structure of

VIX futures would imply. Even if the slope is underestimated during periods of backwardation, the slope of the term structure is still very steep. It has been particularly pronounced since the onset of the banking crisis in 2008. This feature makes VIX futures particularly suitable for the type of differential roll-yield trades that we shall propose in this paper.

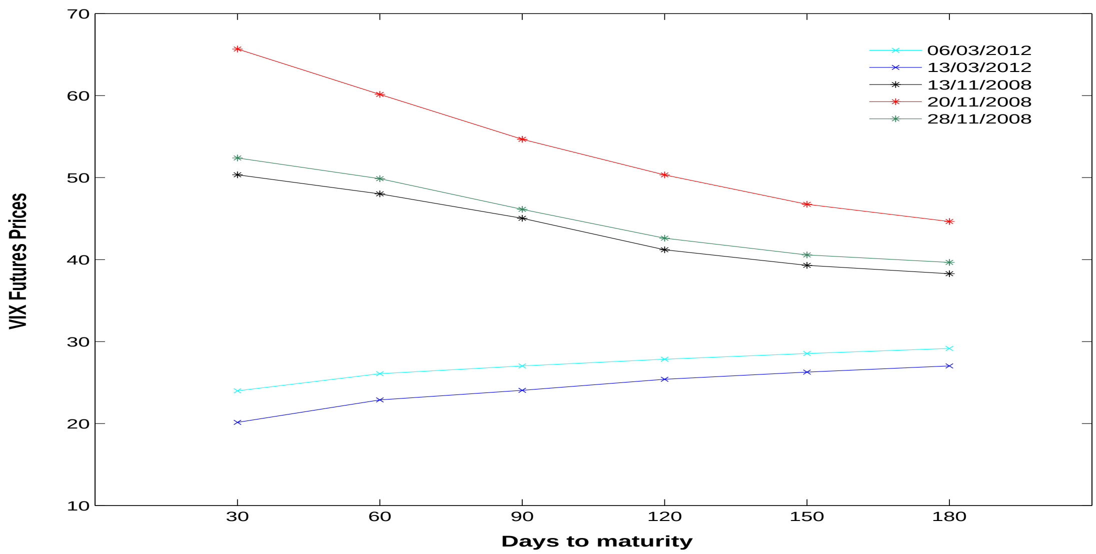

Fig. 2 Term structure of VIX futures while in contango (2010) and backwardation (2008).

Fig. 2 depicts how the typical shift and slope of the VIX futures term structure varies, according to whether the market is in contango or backwardation. In November 2010 the term structure exhibited contango, being steeper at the short-end than at the longer maturities. Also, the shifts in the term-structure are relatively small – each curve is depicted at weekly intervals. By contrast, during November 2008 the market was in distinct backwardation. At such times the (now negative) slope of the term structure is very much steeper at the short-end, and the shifts are of greater magnitude. For instance, the term structure shifted upward by 17 volatility points at the short-end during a single week, and then returned almost to the previous level by the end of the next week.

2.2. Investable Returns and the S&P Indices

The indicative values of VIX futures ETNs are based on the S&P indices of investable constant-maturity (ICM) VIX futures derived from their daily closing prices. As explained by Galai (1979), an ICM discretely compounded return on VIX futures may be obtained via linear interpolation between the returns on the two futures with maturity dates either side of the constant maturity. To construct such a series we set

$$r^m_{t+1} = \beta_t r^s_t + (1-\beta_t) r^l_{t+1}, \quad \beta_t = \alpha_t p^s_t / p^m_t, \qquad r^i_{t+1} = \frac{p^i_{t+1} - p^i_t}{p^i_t}, \tag{2}$$

where r^i_{t+1} are the individual discretely-compounded returns on the tradable futures contracts with maturities $i = s$ (short) and $i = l$ (long).[11] Basing an ETN on the ICM returns given by Eq. (2) retains the one-to-one correspondence between the weights derived for the ETN and the weights on the futures that are actually traded. In other words, the ETN issuer may replicate the product using the exchange-listed VIX futures, because at any point in time there is a unique interpolation constant β_t for each ICM futures return, which is used via Eq. (2) to distribute into weights on traded VIX futures.[12]

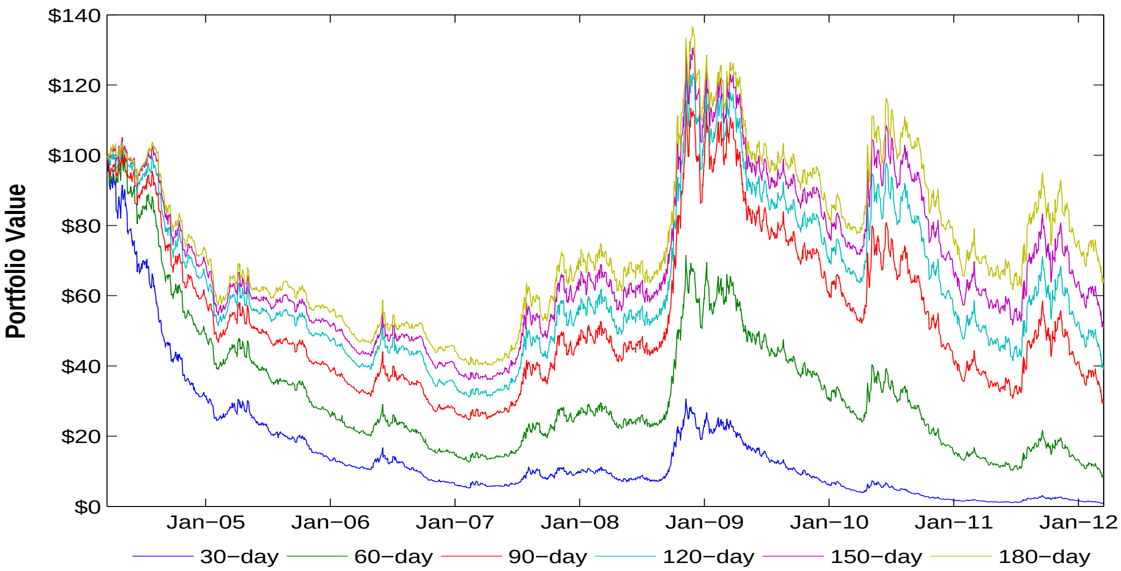

Fig. 3 Evolution of the value of \$100 invested in the VIX futures with constant-maturity returns, constructed using Eq. (2), in March 2004. These represent the S&P indices of maturity T, with $T = 30, 60, \ldots, 180$ days.

To illustrate the difference between the constant-maturity price series Eq. (1), which are displayed in Fig. 1 and the attainable returns for an investor following the constant-maturity methodology as in Eq. (2), Fig. 3 depicts the performance of a theoretical \$100 investment in each of the six constant maturities. The series represent the S&P indices of maturity $30, 60, \ldots, 180$ days.

To attain the ICM return one must rebalance the portfolio of the two straddling VIX futures daily to keep the required constant-maturity exposure. Since the VIX futures term structure is almost always in contango, at each rebalancing there is a small but almost always negative roll cost created by selling the lower price shorter-term futures and buying the higher price longer-term futures. This small daily roll cost creates highly negative long-run returns for the investor in a short-term VIX futures tracker ETN. But because of the convexity in the VIX futures term structure, replicating a long-maturity ETN performs relatively well, because the difference in roll

[11] Whenever there are not two contracts that straddle the desired maturity, linear extrapolation rather than linear interpolation is performed using the two closest contracts to that maturity. In that case a short position in one of the two contracts used in the calculation is necessary.

[12] Note that returns on VIX futures, as for any other futures contract, directly represent an excess over the current risk-free rate return – see Bodie and Rosansky (1980) and Fortenbery and Hauser (1990) for further explanation.

costs between the two straddling VIX futures contracts is smaller than it is at the short end of the term stucture. That is, the size of the roll yield depends on the slope of the term structure, which is steepest at the short end. Hence, the negative roll yield when the term structure is in contango is much greater at the short end of the VIX futures term structure. This is the main reason why the 1-month tracker VXX has performed so much worse than the 5-month tracker VXZ. For instance, \$100 invested in the VXZ on January 29, 2010 was worth a little over \$60 by December 30, 2011, but the same invested in VXX was worth less than \$9!

2.3. Roll Cost and Convexity Effects

The discretely-compounded daily return derived from the price index p_t^m is not realisable, and not equal to r_t^m. The difference between them is called the roll yield of maturity m and its negative, the roll cost, is denoted c_t^m. The roll cost captures the loss (or gain) from rolling a futures position from a shorter maturity contract to one of a longer maturity when the futures market is in contango (or backwardation) on day t.

If we ignore transactions costs so that all prices are mid prices, we can simply define the roll cost to be Eq. (2) minus the one-period return based on Eq. (1). On applying a little algebra it can be shown that:[13]

$$c_{t+1}^m = r_{t+1}^m - \left[(p_{t+1}^m - p_t^m)/p_t^m\right] = \frac{p_{t+1}^s - p_{t+1}^l}{p_t^m}\,(\alpha_{t+1} - \alpha_t). \tag{3}$$

Typically (i.e. unless a new pair of contracts is rolled into) we have $\alpha_t > \alpha_{t+1}$. For instance, when rebalancing inter-week rather than over a weekend, $\alpha_t - \alpha_{t+1} = (l-s)^{-1}$.[14] When the market is in contango, $p_t^l > p_t^s$ so the roll cost (3) is positive. The roll cost is only negative (i.e. roll yield positive) when the term structure is in backwardation. The VIX futures term structure is very often in contango, as we have seen from Fig. 1, so a very large and negative roll yield has substantially eroded the returns realised on all the standard tracker ETNs products since their launch.

Convexity of the term structure induces a different sensitivity of ETNs returns during periods of contango and backwardation. To see this, note that the short end of the VIX futures term structure is much more volatile, i.e. r_t^s is usually of much greater magnitude than r_t^l. The sensitivity of r_t^m to r_t^s and r_t^l depends on whether term structure is in backwardation or contango. For instance, set $\alpha_t = 0.5$, so that $\beta_t = (2p_t^m)^{-1}p_t^s$ and $1-\beta_t = (2p_t^m)^{-1}p_t^l$. In contango, $\beta_t < 1-\beta_t$ and the constant-maturity return r_{t+1}^m given by (2) has a relatively low sensitivity to r_{t+1}^s. But in backwardation, where returns typically have much greater magnitude (cf. Fig. 2) we have $\beta_t > 1-\beta_t$ so r_{t+1}^m has a higher sensitivity to r_{t+1}^s. This convexity effect means that, as the market swings from contango into steep backwardation at the beginning of a crisis, very rapid gains are made on the synthetic short-term VIX futures; but thereafter equally rapid losses are made, as the backwardation declines and the term structure returns to contango.

[13]The S&P uses another definition but without apparent justification, e.g. Standard & Poor's (2011a).

[14]The level of $l-s$ depends on the distance between the two contracts used in the constant-maturity calculation, which is most often either 28 or 35 days in our sample.

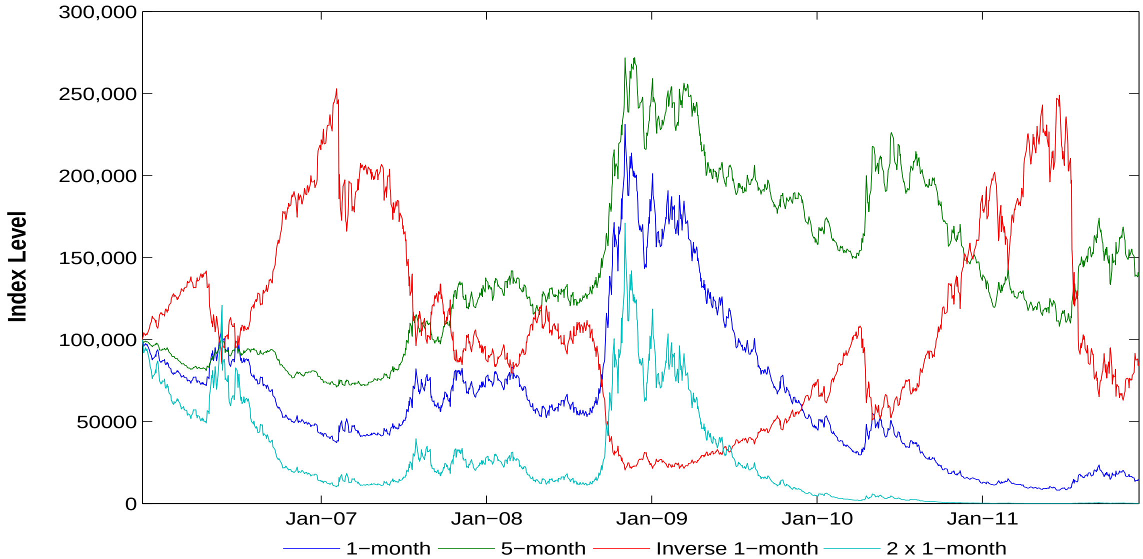

Fig. 4 S&P 30-day and 150-day constant maturity indices, the inverse 30-day index and 2 $\times$ leverage on the 30-day index, December 2005 – December 2011.

To witness this convexity effect in action, Fig. 4 displays the S&P 500 constant-maturity indices underlying four major ETNs: the VXX. VXZ, XIV and TVIX.[15] These indices were first quoted in December 2005, starting at a value of 100,000.[16] The convexity effect induces a heightened sensitivity to the short-term return during a period of strong backwardation such as the banking crisis at the end of 2008. Thus, as the market swung from contango to backwardation at the beginning of the crisis, the direct tracker indices jumped upward. For instance, during the two months following the collapse of Lehman Bros on September 17, 2008 the 30-day index rose by more than 200%, the inverse 30-day index fell by 73%, the 2 $\times$ leveraged 30-day index rose by 670%, and the 150-day index rose by 73%. But as the market swung back to contango the indices lost the value they gained and the higher the gain the greater and more rapid the subsequent losses.

The convexity effect is positive for purchasers of direct tracker ETNs, but leads to large negative drawdowns for the inverse tracker products. For instance, the XIV, whose indicative value is represented by the red line in Fig. 4, lost about 50% of its value during the first few days of the Eurozone crisis in August 2011.

We conclude that the direct, inverse and leveraged tracker ETNs are likely to be amongst the most risky of all exchange-traded products. We now move to a statistical analysis of the returns on investable constant-maturity VIX futures, which will verify this claim.

[15] Daily returns on VXX and VXZ are returns on the S&P 1-month and 5-month constant-maturity total return indices, respectively, less the investor fee. Credit Suisse issued the 2 $\times$ leveraged product TVIX (and the XIV inverse ETN whose returns are minus the returns on the S&P 30-day index, less a 1.35% annual fee) in November 2010.

[16] Prior to this date their evolution is proportional to the series displayed in Fig. 3, which we started at 100. It is interesting to note that between March 2004 and December 2005 the 30-day S&P index lost over 80% of its value.

3. Statistical Analysis of S&P Indices

Panel A of Table 1 reports summary statistics on the six ICM VIX futures returns that determines the value of the S&P indices, computed using Eq. (2) with maturities, $30, 60, \ldots, 180$ calendar days.[17] As we move to longer maturities the returns represent longer term expectations of volatility and are consequently less variable over time. This is consistent with the well-known Samuelson effect in all futures markets – see Samuelson (1965). However, the decrease in volatility as we move along the term structure of VIX futures is much more pronounced than it is in most other financial and commodity futures. Another unique and prominent feature is that, due to the roll-cost effects explained in the previous section, there are much larger negative returns at the short end of the term structure than at the long end. Panel B presents the correlation matrix of these returns over the entire sample. Clearly the series form a very highly correlated system, with correlations ranging between 0.86 and 0.98.

Table 1
Descriptive statistics for returns on investable, constant-maturity VIX futures.

Panel A: Summary statistics

Contract	Annualized Mean	Volatility	Sharpe Ratio	Skewness	Kurtosis	Total Return
30-day	-33.57%	57.20%	-0.5869	1.16	5.38	-97.94%
60-day	-16.92%	43.35%	-0.3902	0.88	4.46	-87.12%
90-day	-4.79%	36.73%	-0.1305	0.77	4.22	-59.24%
120-day	-2.15%	32.84%	-0.0655	0.72	4.40	-44.37%
150-day	-0.36%	30.29%	-0.0119	0.67	4.52	-31.90%
180-day	0.62%	28.69%	0.0215	0.68	4.75	-23.76%

Panel B: Correlations

	30-day	60-day	90-day	120-day	150-day	180-day
30-day	1.00					
60-day	0.97	1.00				
90-day	0.93	0.98	1.00			
120-day	0.91	0.95	0.98	1.00		
150-day	0.88	0.92	0.95	0.98	1.00	
180-day	0.86	0.90	0.93	0.95	0.98	1.00

Panel A reports the summary statistics for investable, constant-maturity VIX futures daily returns. Sample means, standard deviations and corresponding Sharpe ratios are annualized based on 250 trading days per year. We present figures for excess kurtosis not kurtosis here and later, in Tables 6 and 9. Panel B is the correlation matrix for the returns under consideration. All figures are significant at the 1% level. The sample is between March 26, 2004 and December 30, 2011.

3.1. Volatility Analysis

We examine the statistical volatility of the indicative values for VIX futures ETNs by applying a generalized autoregressive conditional heteroscedasticity (GARCH) framework to daily returns on S&P indices of different maturities. To capture the possibility of an asymmetric volatility response to shocks we employ the well-known Glosten, Jagannathan, and Runkle (1993) (GJR) model. That

[17] The results here omit transaction costs and the investment fee for VIX futures ETNs (these are analysed later, in Sections 6 and 5).

is, the return r_t is assumed to follow the conditional mean and variance equations:

$$r_t = c + \sigma_t z_t, \quad z_t \sim \text{NID}(0,1), \tag{4}$$

$$\sigma_t^2 = \omega + \alpha\epsilon_{t-1}^2 + \phi 1_{\{\epsilon_{t-1}<0\}}\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2. \tag{5}$$

Here α determines the reaction of conditional volatility to market shocks, β determines the persistence of volatility, ϕ determines the asymmetric response of volatility, the indicator function $1_{\{\epsilon_t<0\}}$ being 1 if $\epsilon_t < 0$ and 0 otherwise, and ω determines the level of the long-term (steady state) variance $\bar{\sigma}^2 = \omega/(1-\alpha-\beta-0.5\gamma)$.

Table 2 reports the coefficient estimates and their standard errors when the GJR model is fitted to each of the S&P index daily returns. The 30-day index, which underlies the VXX, is the most sensitive to market shocks, since its conditional volatility has the greatest value for $\hat{\alpha}$. But with all $\hat{\alpha}$'s in the range 0.14 – 0.17, even the returns on longer maturity indices are highly sensitive to market movements. By contrast, their volatilities mean-revert fairly rapidly, given that $\hat{\beta}$ takes fairly low values (in the range 0.868 – 0.884). The values of $\hat{\phi}$ are clearly differentiated across maturities, with the asymmetry in the volatility response becoming less pronounced in the contracts of longer maturities. All the estimates of ϕ are negative and significant, meaning that a negative return on volatility is good news, since it implies positive equity returns, and thus the volatility of volatility decreases in response.[18]

Table 2
Maximum likelihood estimation of GJR models on constant-maturity VIX futures returns.

	30-day	60-day	90-day	120-day	150-day	180-day
ω	3.02×10^{-5}	1.20×10^{-5}	6.52×10^{-6}	4.49×10^{-6}	3.76×10^{-6}	3.58×10^{-6}
	(9.53)	(6.38)	(6.03)	(4.68)	(5.55)	(6.29)
α	0.1692	0.1637	0.1472	0.1566	0.1406	0.1424
	(12.70)	(11.68)	(9.99)	(10.29)	(10.59)	(11.43)
β	0.8683	0.8750	0.8831	0.8733	0.8841	0.8790
	(97.12)	(83.83)	(83.81)	(79.29)	(87.68)	(95.14)
ϕ	-0.1227	-0.1075	-0.0799	-0.0700	-0.0623	-0.0523
	(-7.51)	(-6.55)	(-4.87)	(-4.09)	(-4.08)	(-3.48)
$\bar{\sigma}$	56.27%	44.66%	41.09%	46.89%	38.16%	43.09%
Min. Vol.	25.95%	17.21%	13.29%	10.92%	10.43%	9.80%
Max. Vol.	204.56%	155.24%	121.96%	103.60%	89.19%	87.72%

In-sample maximum likelihood estimation of GJR-GARCH coefficients for each VIX futures series. Numbers in parentheses denote t–statistics under the null hypothesis that coefficients are equal to zero. All coefficients are significant at the 1% level. $\bar{\sigma}$ denotes the long term average for volatility, i.e. for the GJR model the annualized squared root of $\bar{\sigma}^2 = \omega/(1-\alpha-\beta-0.5\phi)$. Min. Vol. and Max. Vol. denote the minimum and the maximum conditional in-sample volatility for each VIX futures, respectively.

Thus, the VIX is expected to revert towards an average value by the time the longer-term futures contracts mature. The long-term volatility of the long-term futures is lower than it is for the short-term futures. Even the 60-day VIX futures have a long-term volatility of 44.66%, much less than

[18]This finding is consistent with the strong evidence of a negative leverage effect (i.e. a positive ϕ coefficient in the GJR model) when such a model is applied to a stock return – see Black (1976) and Engle and Ng (1993).

Table 3
Maximum likelihood estimation of GJR models on constant-maturity VIX futures returns.

	30-day	60-day	90-day	120-day	150-day	180-day
Panel A: March 26, 2004 - December 31, 2010						
ω	2.94×10^{-5}	1.58×10^{-5}	6.47×10^{-6}	4.12×10^{-6}	3.24×10^{-6}	3.61×10^{-6}
	(8.09)	(6.52)	(6.57)	(4.81)	(5.65)	(7.02)
α	0.1878	0.1459	0.1360	0.1463	0.1377	0.1343
	(11.16)	(9.49)	(9.46)	(9.80)	(10.51)	(10.60)
β	0.8602	0.8854	0.8932	0.8842	0.8896	0.8877
	(68.39)	(82.82)	(89.57)	(84.39)	(95.76)	(99.78)
ϕ	-0.1510	-0.1187	-0.0886	-0.0795	-0.0698	-0.0661
	(-8.49)	(-6.89)	(-5.90)	(-5.09)	(-4.85)	(-4.83)
$\alpha + 0.5\phi$	0.1123	0.0866	0.0917	0.1065	0.1028	0.1013
$\bar{\sigma}$	51.67%	37.62%	32.66%	33.37%	32.67%	28.62%
Panel B: January 1, 2011 - March 31, 2012						
ω	1.21×10^{-4}	7.57×10^{-5}	3.98×10^{-5}	3.00×10^{-5}	2.44×10^{-5}	1.96×10^{-5}
	(2.25)	(3.09)	(2.52)	(2.27)	(2.34)	(2.08)
α	0.1365	0.2044	0.2144	0.1906	0.1762	0.1653
	(3.41)	(3.61)	(3.29)	(3.01)	(2.73)	(2.50)
β	0.8393	0.7721	0.7666	0.7768	0.7805	0.7888
	(17.04)	(15.27)	(16.55)	(16.54)	(15.69)	(14.84)
ϕ	-0.0544	-0.0417	-0.0083	0.0127	0.0370	0.0453
	(-1.09)	(-0.54)	(-0.10)	(0.15)	(0.41)	(0.48)
$\alpha + 0.5\phi$	0.1093	0.1835	0.2102	0.1969	0.1947	0.1880
$\bar{\sigma}$	76.70%	65.33%	65.55%	53.51%	49.54%	46.00%

In-sample maximum likelihood estimation of GJR-GARCH coefficients for each VIX futures series. Numbers in parentheses denote t–statistics under the null hypothesis that coefficients are equal to zero. All coefficients are significant at the 1% level. $\bar{\sigma}$ denotes the long term average for volatility, i.e. for the GJR model the annualized squared root of $\bar{\sigma}^2 = \omega/(1 - \alpha - \beta - 0.5\phi)$.

the long-term volatility of 30-day futures, at 56.27%. Moreover, the long-term volatility does not fall much further for the 90-day and longer maturity contracts: all longer-maturity contracts have similar $\bar{\sigma}$. The implication here is that equity investors expect VIX to display a very rapid mean-reversion, typically taking more than 1 month but less than 2 months to revert to normal levels after a shock. This is also evident from historical volatilities presented in Table 1.

Fig. 5 depicts the GJR conditional volatility estimates over the entire sample. The conditional volatility is much higher and more variable for the 30-day futures compared with other maturities. A maximum volatility of more than 200% for the 30-day VIX futures occurred during the onset of the Eurozone debt crisis of August 2011. The TVIX was launched one month later, but had it existed in August 2011 its volatility would have been over 400%! The volatility of VIX futures also peaked at the time of the Lehmann Brothers collapse in September 2008, and many times after this. Indeed all VIX futures, even those of longer maturities, have displayed numerous jumps in their volatilities along with a generally increasing level of volatility.

As the trading volume on VIX futures has increased, due to large-scale hedging activity of ETNs issuers, they have become more volatile relative to the VIX index itself. For instance, between January 2, 2007 and March 31, 2010, a period when the average VIX index value was 27%, the 30-day VIX futures had an average volatility of 60%. But between April 1, 2010 and

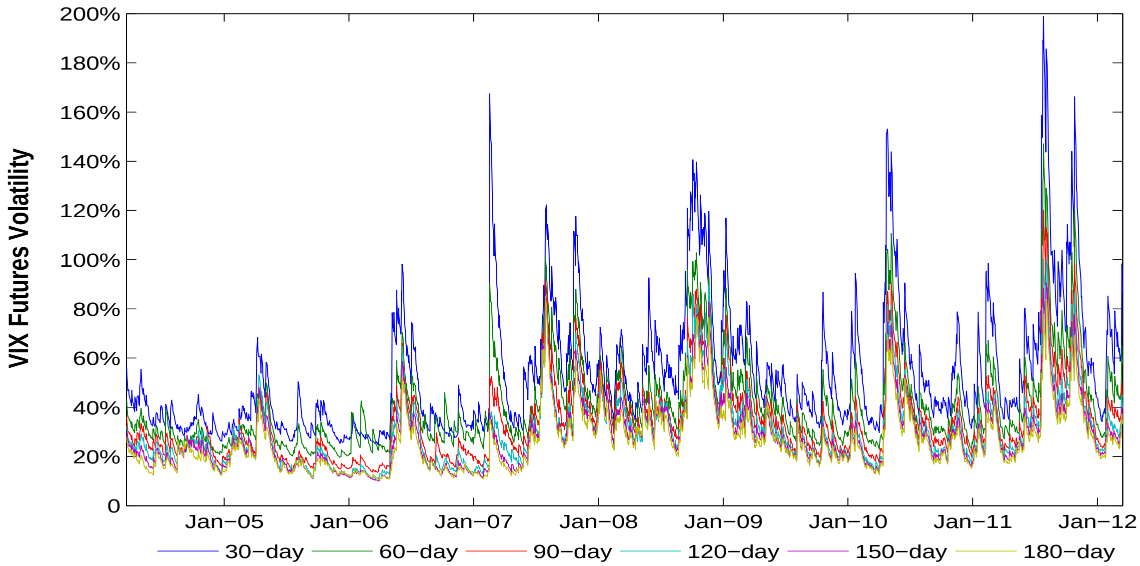

Fig. 5 GJR-GARCH volatilities for the six VIX futures series based on the maximum likelihood estimation of the model for conditional variance: $\hat{\sigma}_t^2 = \hat{\omega} + \hat{\alpha}\epsilon_{t-1}^2 + \hat{\phi}1_{\{\epsilon_{t-1}<0\}}\epsilon_{t-1}^2 + \hat{\beta}\hat{\sigma}_{t-1}^2$ where $\epsilon_t|I_t = R_t - \bar{R} \sim N(0, \sigma_t^2)$ with $I_t = \{R_t, R_{t-1}, \ldots\}$, quoted as an annualized standard deviation. Estimates for ω, α, ϕ and β for each VIX futures series are given in Table 2.

March 31, 2012, when the average VIX index value was 23%, the annualized volatility of daily returns on 30-day VIX futures was 73%. Assuming this higher statistical volatility influences the expectations of players in options markets, we should expect a spillover effect whereby VIX option prices increase relative to S&P 500 option prices.

3.2. Correlation Analysis

The S&P indices returns form a very highly correlated system, as is evident from the correlation matrix displayed in Panel B of Table 1. Hence, their returns are an ideal input to principal component analysis (PCA), a statistical methodology which identifies the key factors that drive a highly correlated the system. To fix ideas, consider a given set of $T \times n$ return series $\mathbf{X}$ with correlation matrix $\mathbf{C}$. Then the principal components of $\mathbf{C}$ are the columns of the $T \times n$ matrix $\mathbf{P}$ such that $\mathbf{P} = \mathbf{XW}$, where $\mathbf{W}$ is the $n \times n$ orthogonal matrix of eigenvectors of $\mathbf{C}$. The total variation in system $\mathbf{X}$ is the sum of the eigenvalues of $\mathbf{C}$, and we label these eigenvalues λ_i with $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n > 0$. The mth component is determined by the eigenvector belonging to the m^{th} largest eigenvalue, so that the proportion of total variation explained by the m^{th} principal component is $\lambda_m/(\lambda_1+\cdots+\lambda_n)$. This way we identify how many key factors explain most of the total risk in the system. Note that principal components are, by construction, mutually uncorrelated because $\mathbf{W}$ is an orthogonal matrix.

Table 4 reports the PCA of the correlation matrix of S&P index returns, estimated over the entire sample. Panel A presents the proportion of total variation in the system explained by each component. The first component alone explains almost 95% of the movements in the system and

Table 4
Principal components analysis on the correlation matrix of VIX futures constant maturity returns.

Contract	PC1	PC2	PC3	PC4	PC5	PC6
Panel A: Proportion of variation explained						
	94.73%	3.44%	1.04%	0.49%	0.19%	0.11%
Panel B: Eigenvectors						
30-day	0.3980	-0.6185	0.5062	-0.3890	0.2166	0.0674
60-day	0.4101	-0.3886	-0.1205	0.5116	-0.5848	-0.2500
90-day	0.4143	-0.0962	-0.5043	0.2797	0.4912	0.4953
120-day	0.4138	0.1774	-0.4293	-0.4517	0.1330	-0.6256
150-day	0.4105	0.4019	0.0443	-0.3661	-0.5256	0.5077
180-day	0.4026	0.5140	0.5373	0.4130	0.2754	-0.1963

Panel A reports the proportion of total risk explained by each factor driving the term structure of the S&P index returns, based on the sample from March 2004 to December 2011. Panel B reports the eigenvectors (i.e. factor weights for the principal component representation) for each return series.

with three factors representing the system we are able to explain over 99% of the total variation. Panel B of Table 4 reports the eigenvectors, i.e. the factor weights for each S&P index return series. Reading across the first row in panel B and using only three components, we have with over 99% accuracy, the following representation of the standardized returns $\tilde{r}_t^{30}$ on 30-day S&P index:[19]

$$\tilde{r}_t^{30} \approx 0.3980\mathrm{PC}_{1,t} - 0.6185\mathrm{PC}_{2,t} + 0.5062\mathrm{PC}_{3,t}. \tag{6}$$

Similar three-factor representations for futures of other maturities show that the first component captures an almost exact parallel shift in the term structure, in that if PC_1 moves by 1% and the other components remain fixed, then all standardized futures shift upwards by about 0.4%. The first eigenvalue tells us that 94.73% of the movements in standardized VIX futures over the sample consist of parallel shifts of this type. The second component represents an almost exact linear tilt in the standardized term structure, with shorter maturities increasing whilst longer maturities decrease. Only 3.44% of the variation in the standardized VIX futures term structure can be attributed to movements of this type.[20]

3.3. Implications for Roll-Yield Arbitrage Trades

The typical movements of the S&P indices may be decomposed into changes arising from: (a) the roll cost c_t^m, representing a slide along the VIX term structure from a maturity of i days to maturity of $i-1$ days,[21] for each contract $i = s, l$ straddling the maturity m; and (b) a movement of the entire VIX futures term structure from day $t-1$ to day t. The roll cost is relatively small and highly predictable, compared with the large and unpredictable movements of type (b). A

[19] The standardized return $\tilde{r}_t^{30}$ is the return at time t minus its sample mean divided by its sample standard deviation.

[20] PCA is not merely a statistical tool for gaining further insight to the statistical behaviour of the term structure of VIX futures. When applied to the VIX futures covariance matrix it becomes a tool for generating GARCH covariance matrices of VIX futures. In the appendix these covariance matrices will be used to examine how effectively the traders in VIX futures can diversify their risk across the term structure.

[21] Or subtract more than 1 day over a weekend, when using business rather than calendar day counts. And here 'typical' excludes the effect when moving to a new pair of contracts.

roll-yield 'arbitrage' seeks trades that are immunized against movements of type (b) so that their main exposure is purely to the roll cost.[22]

The PCA analysis in the preceding sub-section shows that about 95% of the observed moves in standardized returns are parallel shifts of the entire term structure. Hence, long-short trades in standardized returns of different maturities should hedge about 95% of the large and unpredictable daily movements of the term structure. That is, the amount gained or lost through the movement at the short end of the term structure would be almost exactly offset by an equal opposite loss or gain at the long end of the term structure.

A long-short ETN portfolio with positions weighted by the inverse of their respective volatilities will capture this offsetting effect. For instance, since the 30-day futures have an average volatility approximately twice that of the 150-day futures (cf. Table 1) a short position on 30-day VIX futures (long position on XIV) offset by a long position of twice the size on 150-day futures (VXZ, VIIZ or VXEE) should immunize against 95% the risk arising from movements of type (b). This type of reasoning may well have been used by UBS to derive the XVIX, a roll-yield arbitrage ETN that the bank issued in November 2010. The XVIX was the first example of an ETN^2, having a static allocation to the inverse 30-day VIX futures tracker and twice this allocation to the direct 150-day VIX futures tracker.

4. Taxonomy of VIX Futures ETNs

Table 5 displays a chronology of VIX futures ETNs issued by Barclays (in red), Credit Suisse (in blue) and UBS (in black). The first two columns give the constant maturity of the respective indicative S&P index,[23] and the maturity of the notes. The third column gives the leverage, i.e. 1 for a direct tracker, −1 for an inverse tracker, and 2 for a direct leveraged tracker. Then follows: the annual fee, higher for the inverse products because the issuer charges a significant fee for hedging costs; the market cap of each note on February 29, 2012 and the average volume traded on each note between January 2 and February 29, 2012.

The market cap on VXX is still almost double that of TVIX. Together their cap is 64% of the total cap of all ETNs. Having been issued later, the UBS products have the lowest market cap (5% of the total) and the Barclays products constitute 56% of the total market cap. The average daily trading volume during the first two months of 2012 was 44.59 million shares over all ETNs, with 36.81 million shares traded on VXX and TVIX alone (amounting to about $875 million dollars per day) and 23.87 million shares traded on average, per day, on just the Barclays products. On August 8, 2011 a total of $4,973,659,365 was traded on the VXX alone. Clearly, Barclays still dominates the market although shares on the TVIX and XIV products of Credit Suisse are being traded in increasingly high volumes.

The remainder of this section explores the potential benefits from roll-yield arbitrage trades

[22] We use the term 'arbitrage' here because it has slipped into market usage. But, as mentioned in the introduction, it is not really an arbitrage because it is not riskless.

[23] And for the two ETN^2, XVIX and XVZ, which attempt to exploit differential roll yields, the maturities of the two indices they allocate between.

Table 5
ETN chronology.

Ticker[*§]	Inception	Maturity VIX futures (m)	Maturity Note (y)	Leverage[†]	Service fee[‡]	Market Cap. (million $)[⁎]	Average Volume[⁎]
VXX[a,b]	Jan-2009	1	10	1	0.89%	1,218.36	23,499,211
VXZ[a]	Jan-2009	5	10	1	0.89%	233.26	329,478
VIIX[a]	Nov-2010	1	20	1	0.89%	15.82	32,605
VIIZ	Nov-2010	5	20	1	0.89%	7.63	829
XVIX	Nov-2010	1 vs. 5	30	1	0.85%	23.04	7,539
XVZ	Aug-2011	1 vs. 5	10	1	0.95%	187.33	25,816
VXAA	Sep-2011	1	30	1	0.85%	5.94	1,210
VXBB	Sep-2011	2	30	1	0.85%	7.19	370
VXCC	Sep-2011	3	30	1	0.85%	8.14	660
VXDD	Sep-2011	4	30	1	0.85%	8.37	192
VXEE	Sep-2011	5	30	1	0.85%	8.58	371
VXFF	Sep-2011	6	30	1	0.85%	9.20	182
XXV	Jul-2010	1	10	-1	0.89%	15.66	13,586
XIV[b]	Nov-2010	1	20	-1	1.35%	427.55	7,321,582
ZIV[b]	Nov-2010	5	20	-1	1.35%	9.09	15,229
AAVX	Sep-2011	1	30	-1	5.35%	11.79	3,484
BBVX	Sep-2011	2	30	-1	5.35%	11.43	1,623
CCVX	Sep-2011	3	30	-1	5.35%	10.82	765
DDVX	Sep-2011	4	30	-1	5.35%	10.96	132
EEVX	Sep-2011	5	30	-1	5.35%	10.50	891
FFVX	Sep-2011	6	30	-1	5.35%	10.59	100
IVOP	Sep-2011	1	10	-1	0.89%	6.95	4,602
VZZB	Nov-2010	5	10	2	1.78%[c]	2.79	12,049
TVIX	Nov-2010	1	20	2	1.65%	676.04	13,315,361
TVIZ	Nov-2010	5	20	2	1.65%	7.32	4,464

[*] We colour ETNs according to their issuer: Barclays Bank PLC (red), Credit Suisse AG (blue) and UBS AG (black).
[§] Except VXX, VXZ and XVZ, all other products have automatic termination clauses and all products except Barclays' are callable at any time by the issuer.
[†] For products with leverage other than 1, actual leverage depends on day and time of transaction.
[‡] All ETNs carry an extra redemption fee: 0.05% (red and blue) and 0.125% (black).
[⁎] Market cap as of February 29, 2012; average daily volume computed between January 2, 2012 and February 29, 2012.
[a] Options on these ETNs are available to trade on CBOE.
[b] Split executed by the issuer: 1 for 4 (VXX), 10 for 1 (XIV), 8 for 1 (ZIV).
[c] The 3m LIBOR is added on top of the service fee.

between inverse short-term and direct long-term VIX futures trackers. We begin by replicating and analysing the performance of XVIX and XVZ, two such trading strategies that have recently been issued as volatility ETNs. Then we introduce a new class of ETN^3 products which trade on the different performance of the ETN^2 when the VIX futures term structure is in backwardation and contango. This is followed by an analysis of more general diversifying trades along the entire term structure, based on minimum variance portfolios.

4.1. The XVIX and XVZ Exchange-Traded Notes

The ETRACS Daily Long-Short VIX ETN (XVIX) is one of the thirteen VIX-based ETNs brokered by UBS AG. It first traded on November 30, 2010 and matures on November 30, 2040.

It is a static exposure across the VIX futures term structure capturing 50% of the performance of an inverse position in the VXX (the 30-day VIX futures tracker ETN) and 100% the performance of a long position in the VXZ (the 150-day VIX futures tracker ETN). This strategy exploits the roll yield differences between short-term and long-term contracts which are evident when the VIX futures term structure is in contango, i.e. most of the time. Letting r_t^{xvix} denote the excess return on the static carry strategy at time t we have:

$$r_t^{xvix} = -\frac{r_t^{30}}{2} + r_t^{150} \tag{7}$$

where r_t^{30} and r_t^{150} are the excess returns earned on constant-maturity 30-day and 150-day VIX futures contracts respectively (i.e. the returns captured by the two respective S&P constant-maturity indices as these are already described). Based on our previous comments in Section 3.3, we see that this relationship hedges about 95% of the term structure movements so that it earns the differential roll yield across the two maturities, which is typically positive because the market is in contango.

Barclays launched the dynamic VIX ETN (XVZ) in August 2011 and full details of this product are given in Barclays Bank PLC (2011). Letting x_{30} and x_{150} denote the allocations on day $t-1$ to the 30-day and 150-day VIX futures contracts, respectively, the excess return on the XVZ strategy at time t is given by:

$$r_t^{xvz} = x_{t-1}^{30} r_t^{30} + x_{t-1}^{150} r_t^{150} \tag{8}$$

The allocations x_{30} and x_{150} are derived relative to target allocations y_{30} and y_{150} using the following process. Let the ratio of the 30-day implied volatility index (VIX) to the 93-day implied volatility index (VXV) at time t be denoted Y_t. Thus, when $Y_t < 1$ ($Y_t \geq 1$) the short end of the VIX term structure is in contango (backwardation). Now define target allocations $[y_t^{30},\ y_t^{150}]$ on 30-day and 150-day VIX futures as:

$$[y_t^{30},\ y_t^{150}] = \begin{cases} [-0.3,\ 0.7] & \text{if} \quad Y_{t-1} < 0.9, \\ [-0.2,\ 0.8] & \text{if} \quad 0.9 \leq Y_{t-1} < 1, \\ [0,\ 1] & \text{if} \quad 1 \leq Y_{t-1} < 1.05, \\ [0.25,\ 0.75] & \text{if} \quad 1.05 \leq Y_{t-1} < 1.15, \\ [0.5,\ 0.5] & \text{if} \quad Y_{t-1} \geq 1.15. \end{cases} \tag{9}$$

The actual allocation applied on the strategy at time t is then:

$$x_t^{30} = \begin{cases} x_{t-1}^{30} & \text{if} \quad x_{t-1}^{30} = y_t^{30} \\ \min\left\{x_{t-1}^{30} + 12.5\%, y_t^{30}\right\} & \text{if} \quad x_{t-1}^{30} < y_t^{30} \\ \max\left\{x_{t-1}^{30} - 12.5\%, y_t^{30}\right\} & \text{if} \quad x_{t-1}^{30} > y_t^{30} \end{cases} \tag{10}$$

and

$$x_t^{150} = \begin{cases} x_{t-1}^{150} & \text{if } x_{t-1}^{150} = y_t^{150} \\ \min\left\{x_{t-1}^{150} + 12.5\%, y_t^{150}\right\} & \text{if } x_{t-1}^{150} < y_t^{150} \\ \max\left\{x_{t-1}^{150} - 12.5\%, y_t^{150}\right\} & \text{if } x_{150,t-1} > y_t^{150} \end{cases} \quad (11)$$

The XVZ is a relatively complex dynamic strategy that attempts to allocate between VXX and VXZ to gain full advantage of the roll-yield differences along the VIX futures term structure, by switching the positions in the short-term and long-term end of the term structure depending on whether the VIX term structure is in contango or backwardation.

4.2. Replication Results for Volatility ETN^2

We replicate the values of the XVIX and XVZ back to March 2004 using all available data on VIX futures and S&P's constant-maturity indices.[24] Note that XVIX and XVZ are total return products, i.e. the 91-day U.S. T-bill is added to their underlying index return. However, here we present the excess returns on the products since, assuming they can borrow money at close to the T-bill rate, it is excess returns that are of interest to large investors. We also deduct the annual service fee (which is 0.85% and 0.95% per annum, for XVIX and XVZ respectively) so that the results correspond to the actual excess return received by the investor. We also present results for the two underlying ETNs, VXX and VXZ, after accounting for their service fee of 0.89% p.a..

Table 6
Descriptive statistics of ETNs and volatility roll-yield trades, and correlation matrix with VIX and VXV.

Panel A: Summary statistics

Contract	Annualized Mean	Volatility	Sharpe Ratio	Skewness	Kurtosis	Total Return
VXX	-35.68%	56.38%	-0.6328	1.08	5.03	-98.19%
VXZ	-1.50%	30.06%	-0.0498	0.71	4.57	-37.36%
XVIX	15.94%	13.42%	1.1875	-0.33	5.02	224.22%
XVZ	17.75%	24.00%	0.7394	1.56	17.55	220.93%

Panel B: Correlations

	VXX	VXZ	XVIX	XVZ	VIX	VXV
VXX	1.00					
VXZ	0.90	1.00				
XVIX	-0.09	0.36	1.00			
XVZ	0.71	0.85	0.42	1.00		
VIX	0.87	0.79	-0.05	0.62	1.00	
VXV	0.89	0.82	-0.02	0.67	0.94	1.00

Panel A reports the summary statistics for the two volatility roll-yield trades under consideration (ETN^2). Sample means, standard deviations and corresponding Sharpe ratios are annualized based on 250 trading days per year. Panel B is the correlation matrix for the ETN^2 returns and the implied volatility indices, VIX and VXV. The sample is between March 26, 2004 and December 30, 2011.

Table 6 presents some descriptive statistics based on our replicated sample. Compared with

[24] Since the S&P started quoting constant-maturity indices only in December 2005, the first 21 months of data are computed using our own computation of the constant-maturity futures returns based on Eq. (2). Note that between December 2005 and December 2011, the 30-day and 150-day returns based on Eq. (2) had correlation of 99.82% and 99.32% with the Short Term and Mid Term S&P VIX futures indices respectively - not an exact 1 because of minor differences of very few basis points arising from the difference in calendar/business days interpolation.

the tracker ETNs, the XVIX and XVZ are much less volatile. Moreover, they successfully turn the negative volatility risk premium and negative roll-yield effect into positive returns. The simple, static XVIX outperformed the more complex, dynamic XVZ on a risk-adjusted basis (as measured by the Sharpe ratio) and although its average annual mean return was lower, its total return was slightly greater than that of the XVZ. The higher volatility, large positive skew and very high kurtosis of the XVZ returns shows that its performance is characterised by a few very large positive returns. Timing an investment in XVZ would therefore be crucial, assuming one could always find a buyer in the secondary market when wishing to offload the investment. The simpler static strategy seems a better bet for a longer-term investment, being much less exposed to extreme daily shocks. Nevertheless, even the XVIX has a higher kurtosis than the tracker ETNs, so potential investors should be aware that both products are exposed to significant daily fluctuations.

Panel B of Table 6 shows the correlation matrix of daily returns on the ETN^2 with the daily returns on the VIX and the VXV. The XVZ has a significant positive correlation with both the VIX and the VXV (not surprizingly, since they are used as indicators for changing allocations) but the static XVIX strategy has virtually zero correlation with the VIX and the VXV, indeed the correlations are even slightly negative. This shows that the XVIX could be regarded as an investment instrument which lies outside the equity volatility asset class.

4.3. Introducing Volatility ETN^3

The XVIX and XVZ indicative returns between March 2004 and December 2011 are depicted in the middle section of Fig. 6 below, along with some summary statistics. Later on, a more detailed analysis will show that the XVIX and XVZ have a complementary performance. That is, the XVIX performs best when the market is in contango and the XVZ only performs well during market crashes, i.e. when the VIX futures term structure swings into steep backwardation, at which time it performs very well indeed, exactly when the XVIX makes a significant loss. Hence, we ask: it is possible to enhance returns further by holding a combination of the XVIX and XVZ? To illustrate the possible benefits of such an ETN based on ETN^2 (called an ETN^3, for obvious reasons) we now consider one static and one dynamic allocation to the two already-issued ETN^2:

- The static ETN^3, CVIX, allocates 75% of capital to XVIX and 25% to XVZ. This allocation is chosen because it corresponds almost exactly to the proportion of days in our sample that the relationship between the 30-day contract and 150-day contract was in contango (75%) and backwardation (25%), as previously noted;

- The dynamic ETN^3, CVZ, holds XVIX when the VIX term structure is in contango (i.e. $Y_t < 1$) and XVZ when VIX term structure is in backwardation ($Y_t \geq 1$).

Any prospective issuer of these ETN^3s need not replicate the product by actually holding shares of the XVIX and XVZ – indeed, the ETN redemption constraints would preclude this. Instead, direct trading along the term structure of VIX futures, with daily rebalancing to create synthetic ICM futures as described in Section 2, allows their performance to be replicated exactly. Since replication is so easy there are numerous other rules for allocating between the two ETN[2] that might be considered. Indeed, we expect major volatility ETN issuers to be actively developing such products at the time of writing. The allocations specified above were merely the simplest possible static and dynamic rules based on the observation that the XVIX tends to performs best when the VIX term structure is in contango, and the VXZ performs better when the VIX term structure is in backwardation.

Fig. 6 shows the daily returns to the VXX and VXZ (above), the XVIX and XVZ (middle) and CVIX and CVZ (below) plus some descriptive statistics: the annualized mean daily return, the volatility, Sharpe ratio and total return computed over the entire sample period, net of fees. Clearly, the two VIX futures trackers VXX and VXZ are high-risk, low-return ETNs. They are clearly undesirable as stand-alone investments and should only be used for short-term speculation. By contrast, the differential roll-yield trades, XVIX and XVZ, have a much better risk-adjusted performance: the XVIX has a lower risk than the XVZ, and a higher total return over the period. However, its average daily return is lower, at only 15.94% in annual terms compared with 17.75% for the XVZ.

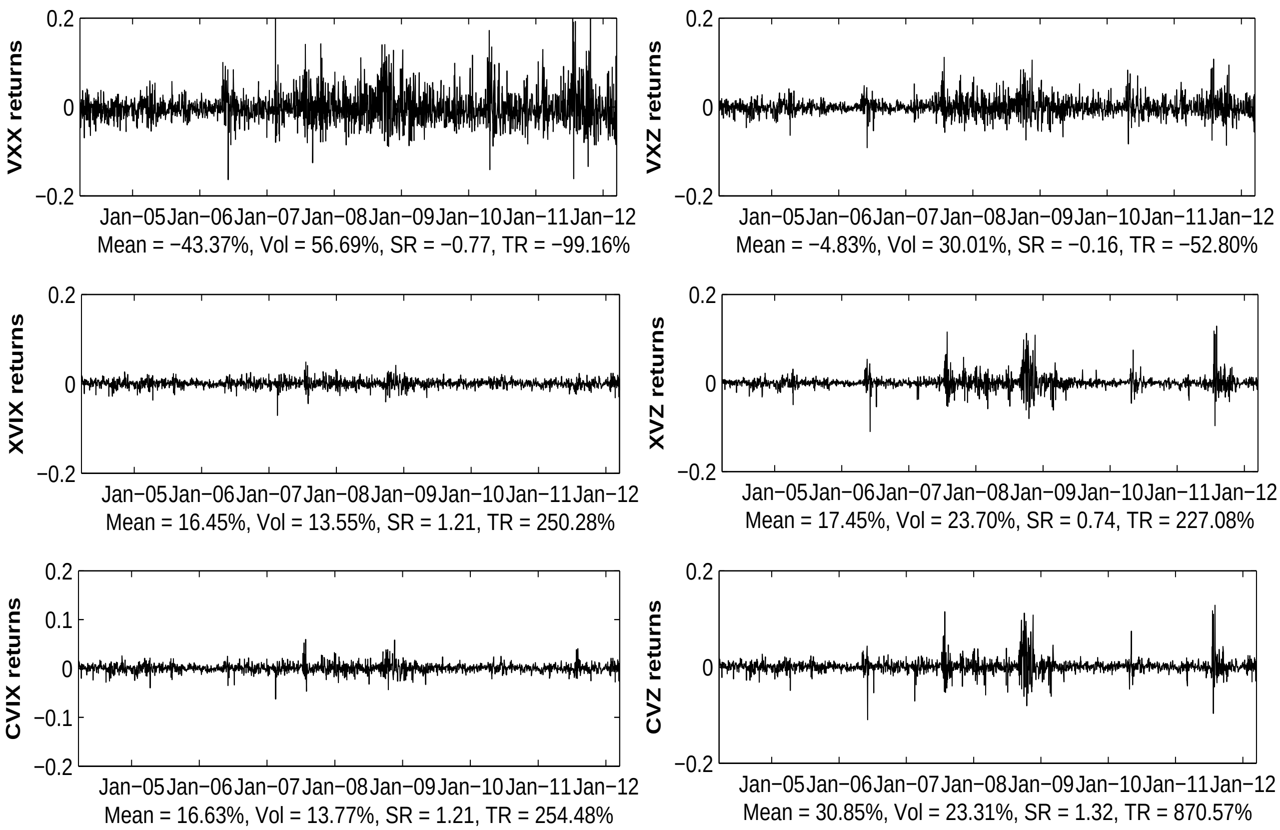

Fig. 6 Daily returns on volatility ETNs (VXX, VXZ, XVIX, XVZ, CVIX and CVZ), annualized mean daily return, volatility, Sharpe ratio and total return between March 26, 2004 and December 30, 2011.

The proposed ETN^3 products, the CVIX and CVZ, are highly correlated with the two ETN^2s: the static products (XVIX and CVIX) have a sample correlation of 91.78% and the dynamic products (XVZ and CVZ) have a sample correlation of 91.68%. The CVIX has a performance very similar to the XVIX but the CVZ performs very much better than the XVZ. In fact, since 2004 CVZ has had an average daily return of over 30% (annualized), with a Sharpe ratio of 1.31 and a total return of almost 800%.

4.4. Diversifying Risk along the VIX Futures Term Structure

Although the main objective when structuring an ETN product is to maximize return, there is also a diversification benefit arising from any long-short position on VIX futures of two different maturities, since their returns are highly correlated. This diversification effect could be further enhanced by trading more than two different constant-maturity tracker ETNs. In this section we consider how an ETN^2 product might be structured by replicating a portfolio containing VIX futures of all available maturities. Such portfolios should also be of internal interest to the issuing bank. This is because the investment arm of a large commercial bank could be actively participating in the VIX futures market in several ways other than issuing ETNs: (1) it may act as a market maker or a broker executing clients' orders;[25] (2) other departments may use VIX futures to hedge their volatility exposure; and (3) a separate proprietary trading desk might speculate in the market. Since the bank may be actively engaged in all these operations simultaneously, open positions in the VIX futures book of the bank overnight may be unavoidable. Thus, the risk management department of the bank as a whole and its Chief Risk Officer in particular should be interested in minimizing the risk exposure by effectively diversifying its positions along the VIX term structure.

Suppose that the bank has trades along the whole VIX futures term structure and is net long, with weights vector $\mathbf{w}_t = (w_{1t}, w_{2t}, \ldots, w_{nt})$ where $w_{1t} + w_{2t} + \cdots + w_{nt} = 1$. Denote by $\hat{\mathbf{\Sigma}}_t$ the estimate of the $n \times n$ covariance matrix of the returns on the contracts at time t. Then the global minimum-variance portfolio, i.e. the exposure along the term structure that will give the lowest variance at time t, is given by the weights vector:

$$\mathbf{w}_t^* = \frac{\hat{\mathbf{\Sigma}}_t^{-1}\mathbf{1}}{\mathbf{1}'\hat{\mathbf{\Sigma}}_t^{-1}\mathbf{1}} \tag{12}$$

where $\mathbf{1}$ is the $n \times 1$ vector of 1's. The minimum variance attainable is

$$\sigma_t^{2*} = \mathbf{w}_t^{*\prime}\hat{\mathbf{\Sigma}}_t\mathbf{w}_t^*. \tag{13}$$

Many banks would seek to be net long on VIX futures, not only to hedge the short volatility exposure of writing options and variance swaps but also to accommodate the long volatility positions of institutional clients that seek to diversify their equity exposures. However, a minimum-variance portfolio of VIX futures can also be net short, with weights $-\mathbf{w}_t^*$.

[25] A list of authorised market makers and brokerage firms in VIX futures market can be found in `http://cfe.cboe.com/tradecfe/brokers.aspx`.

We shall include the ETN derived from out-of-sample minimum variance allocations in our empirical results below, and label this potential ETN^2 product the MVX, for obvious reasons. For this the estimate $\hat{\boldsymbol{\Sigma}}_t$ of the covariance matrix in (13) is computed daily, using a simple rolling equally-weighted historical covariance matrix calculated from the last N returns.[26]

5. Performance of ETNs: The Investors Perspective

This section reports detailed results on the comparative performance of VIX futures ETNs, based on their indicative values between March 2004 and December 2011. First we give a brief description of the performance criteria that we apply.

5.1. Performance Measures

The most widely used risk-adjusted performance measure, introduced by Sharpe (1966, 1975), is the ratio of the expected excess annual return on a portfolio to its standard deviation. We follow standard practice by estimating the Sharpe ratio as the mean of daily excess returns divided by its standard deviation, annualizing this ratio through multiplying by $\sqrt{250}$. The Sharpe ratio adjusts for risk as measured by the portfolio volatility whereas the similar performance measure introduced by Sortino and Van Der Meer (1991) adjusts only for downside risk, as measured by the square root of semi-variance introduced by Markowitz (1959). In addition to the Sharpe and Sortino ratios, we examine the "Omega" ratio introduced by Keating and Shadwick (2002), which is the ratio of the expectation of the positive excess returns to the expectation of the negative excess returns. A positive Omega ratio indicates that positive returns tend to outweigh the negative returns, on average. As another measure of downside risk which quantifies the extent of possible margin calls on the issuer of the ETN, we calculate the maximum daily drawdown over the entire sample from March 2004 to September 2011.

Finally we compute the manipulation-proof performance measure Theta derived by Goetzmann, Ingersoll, Spiegel, and Welch (2007). Manipulation of a performance measure is due to the fund manager's actions, like window dressing or closet indexing, which increase the value of a performance measure, but add no real value to the investor. The properties of such a manipulation-proof performance measure are similar to a power utility pay-off. In particular, the measure depends on the relative risk aversion of the investor, γ. We set $\gamma = 3$, but our empirical results are qualitatively robust to the choice of this parameter. For a portfolio with daily excess returns r_t measured over a sample size of size T it is defined as

$$\Theta = \frac{250}{(1-\gamma)} \ln \left(T^{-1} \sum_{t=1}^{T} (1 + r_t)^{1-\gamma} \right). \tag{14}$$

Similar to Ederington (1979) the effectiveness of the diversification is measured by the per-

[26] But the disadvantages of this approach are well known - see Alexander (2009) for example. Baba, Engle, Kraft, and Kroner (1991) and many others advocate instead the multivariate GARCH model, where a conditional covariance matrix is estimated at every time t using maximum likelihood methods. The results of using a GARCH covariance matrix are given in the appendix.

centage reduction in variance when the minimum-variance portfolio is held, relative to holding the undiversified benchmark position over a period of one day. That is,

$$E_t = \frac{\hat{\sigma}^2_{\theta,t} - \hat{\sigma}^2_{\pi,t}}{\hat{\sigma}^2_{\theta,t}}, \quad \text{where} \tag{15}$$

$$\hat{\sigma}^2_{\theta,t} = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1}\left(r^{\theta}_{t-i}\right)^2, \quad \text{and} \quad \hat{\sigma}^2_{\pi,t} = (1-\lambda)\sum_{i=1}^{\infty} \lambda^{i-1}\left(r^{\pi}_{t-i}\right)^2, \tag{16}$$

where r^{θ}_t and r^{π}_t are the daily returns on the benchmark portfolio, and on the minimum-variance portfolio, respectively. Here $0 \leq \lambda \leq 1$, with higher values resulting in smoother time series for $\hat{\sigma}^2_{\theta,t}$ and $\hat{\sigma}^2_{\pi,t}$. When $\lambda = 1$ we have the standard equally-weighted sample variances, but we prefer to use $\lambda < 1$ for exponentially smoothed sample variances as these change over time according to market conditions.

5.2. Empirical Performance of ETNs

Table 7 reports the range of performance measures described above, applied to the already-issued ETNs in question (i.e. the VXX, VXZ, XVIX, and XVZ) over the entire sample period and over three sub-periods: March 26, 2004 – October 31, 2006 (a period of stable trending equity markets); November 1, 2006 – May 31, 2009 (which covers the credit and banking crises); and June 1, 2009 – December 30, 2011 (a period that includes the Eurozone debt crisis). In each period we compare their performance to the performance of the ETN^3 products defined above, i.e. the CVIX and CVZ. Panel A results for the tracker ETNs show that long-term investments in these products are unwise. Although the VXZ performs better than the VXX, the VXZ could still incur a daily loss of around -10%, and the expected return is negative. The only sub-period when the tracker ETNs performed reasonably well was the central period covering the credit and banking crises, and over this period the long-term VIX futures tracker VXZ outperformed the short-term tracker VXX according to each criterion.

Turning now to the ETN^{2}s: both XVIX and XVZ outperform the tracker ETNs over the entire period, and over the first and last sub-samples in Panel B and Panel D (when the XVIX also outperforms the XVZ). This is according to each criterion, except for the large maximum daily loss observed for the XVZ. During the credit and banking crisis period (Panel C) their performance was comparable to that of the tracker ETNs, with the XVZ clearly a better investment than the XVIX. Still, the large daily drawdowns highlight that the VIX futures market can swing very quickly from contango into strong backwardation, with inverse contracts rapidly appreciating in value, especially the short-term ones. In this case the potential profits from differential roll-yield strategies can easily turn into large losses.

The performance of the ETNs and ETN^{2}s is clearly regime-dependant, and the aim of our proposed ETN^{3}s is to build portfolios that will perform relatively well irrespective of the market regime – or at least be more immune to the regime than the already-issued ETNs. Examining the performance of the ETN^{3}s over the whole period, both the static mix of XVIX and XVZ (i.e. the

Table 7
Performance measures for ETN^n, $n = 1, 2, 3$.

	ETN		ETN^2		ETN^3	
	VXX	VXZ	XVIX	XVZ	CVIX	CVZ
Panel A: Full Sample						
Sharpe ratio	-0.6328	-0.0498	1.1875	0.7394	1.1896	1.3111
Sortino ratio	-0.9789	-0.0761	1.7322	1.2106	1.7893	2.1649
Omega ratio	0.8924	0.9909	1.2315	1.1858	1.2430	1.3424
Maximum daily loss	-16.35%	-9.16%	-7.09%	-10.96%	-6.25%	-10.96%
Theta	-81.69%	-14.85%	13.22%	9.33%	13.50%	22.71%
Panel B: March 26, 2004 - October 31, 2006						
Sharpe ratio	-2.6751	-1.5779	1.2081	-0.3174	0.8276	0.7343
Sortino ratio	-3.3698	-2.0236	1.7334	-0.4047	1.1418	0.9635
Omega ratio	0.6249	0.7537	1.2320	0.9341	1.1576	1.1563
Maximum daily loss	-16.35%	-9.16%	-3.65%	-10.96%	-3.96%	-10.96%
Theta	-112.47%	-37.89%	12.45%	-8.06%	7.66%	7.64%
Panel C: November 1, 2006 - May 31, 2009						
Sharpe ratio	0.7697	1.0588	0.7664	1.3681	1.1579	1.5526
Sortino ratio	1.2706	1.7141	1.1110	2.3175	1.7958	2.6325
Omega ratio	1.1416	1.2109	1.1451	1.3279	1.2368	1.3851
Maximum daily loss	-12.58%	-7.43%	-7.09%	-8.05%	-6.25%	-8.05%
Theta	-8.83%	19.04%	8.59%	28.79%	15.77%	34.08%
Panel D: June 1, 2009 - December 30, 2011						
Sharpe ratio	-0.9295	-0.3008	1.8812	0.6555	1.7468	1.5454
Sortino ratio	-1.4563	-0.4668	2.8551	1.1612	2.7270	2.8449
Omega ratio	0.8483	0.9476	1.3662	1.1762	1.3516	1.4558
Maximum daily loss	-16.12%	-8.64%	-2.32%	-9.65%	-2.52%	-9.65%
Theta	-122.63%	-25.24%	18.58%	7.51%	17.09%	26.56%

Table reports various performance measures for the excess returns, net of fees, on the two tracker ETNs (VXX and VXZ) the two ETN^2 differential roll-yield strategies (XVIX and XVZ) and their static and dynamic ETN^3 combinations (CVIX and CVZ). The full sample is between March 26, 2004 and December 30, 2011.

CVIX) and the dynamic, regime-dependant mix (i.e. the CVZ) have higher Sharpe ratios, Sortino ratios, Omega ratios and Theta than any of the existing ETNs. However, the XVIX outperforms both ETN^3s during the first sub-sample and XVZ outperforms CVIX during the second sub-sample. CVZ experiences the same large drawdowns as the XVZ but they are still much smaller than those experienced on the VXX. According to the ultimate test of the manipulation-proof measure Theta, the CVZ is the preferred amongst all the ETNs except during the first sub-sample, when the XVIX performed best. As with the two ETN^2s, the static CVIX outperforms the dynamic CVZ only during the stable trending market of the first sub-sample.

5.3. The Minimum-Variance ETN

Fig. 7 depicts the minimum-variance weights derived using Eq. (12), starting in March 28, 2005. They are fairly stable over time, generally in the range of $[-1, 1]$, except for the long-term contracts that were trading on very low volumes until mid 2007. As expected, the minimum-variance portfolio is balanced, with long and short positions evenly distributed.

VIX futures trading is very strongly concentrated on contracts that are relatively close to

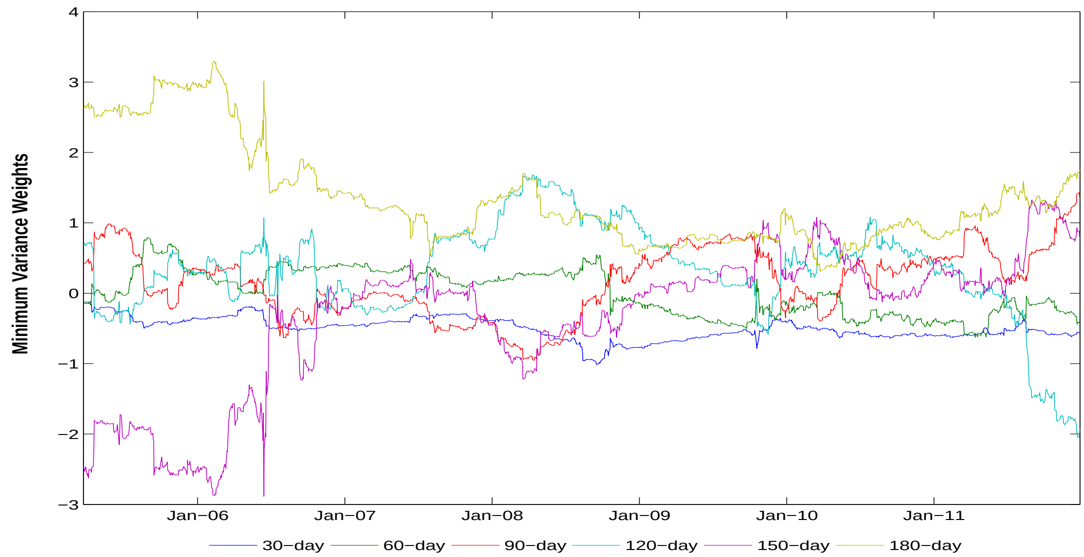

Fig. 7 Minimum variance weights on the constant-maturity VIX futures based on equally weighted covariance matrices computed using a rolling-window methodology.

maturity. For instance, during the first two weeks of August 2011 the average daily volume of VIX futures traded was about 40,000 on the August contract, 34,000 on the September contract, 10,000 on the October contract and then decreasing from about 7,500 on the November contract to less than a 1,000 contracts per day for the February and March 2012 expires. This was a volatile period and overnight risks might be considerably reduced by taking a more balanced exposure across the whole term structure, as prescribed by the minimum-variance portfolio, MVX.

On each day between March 28, 2005 and December 30, 2011 we compute the equally weighted covariance matrix $\hat{\mathbf{\Sigma}}_t$ on the ICM VIX futures contracts, based on the last 250 returns.[27] The minimum-variance positions are kept for one day and the realized out-of-sample portfolio return is recorded. Then we roll the sample forward one day and repeat to obtain a series $\{r_t^{\pi}\}_{t=N+1}^{T}$ of out-of-sample returns on the minimum-variance portfolio.

We label the ETN based on minimum variance portfolio weights the MVX. Indicative returns on the MVX can only be computed from March 2005, since we have (arbitrarily) employed a 1-year sample to estimate the required covariance matrix. Over the period March 2005 – December 2011 the MVX had a Sharpe ratio of 0.89 with a volatility of 23.58%, not including investor fees. Over the same period the volatility of the CVZ was only marginally higher (24.61%) and the CVIX had a much lower volatility (13.90%). This is possible because the minimum-variance allocation is only in-sample, and the returns are recorded out-of-sample.

We now consider the diversification effectiveness of the MVX relative to a benchmark portfolio consisting of the VXX. That is, we set $\theta = 30$ days in Eqs. (15) and (16) and use the RiskMetrics

[27] We also used several other values for N, but the variance reduction was fairly robust to this choice. Results are available from the authors on request.

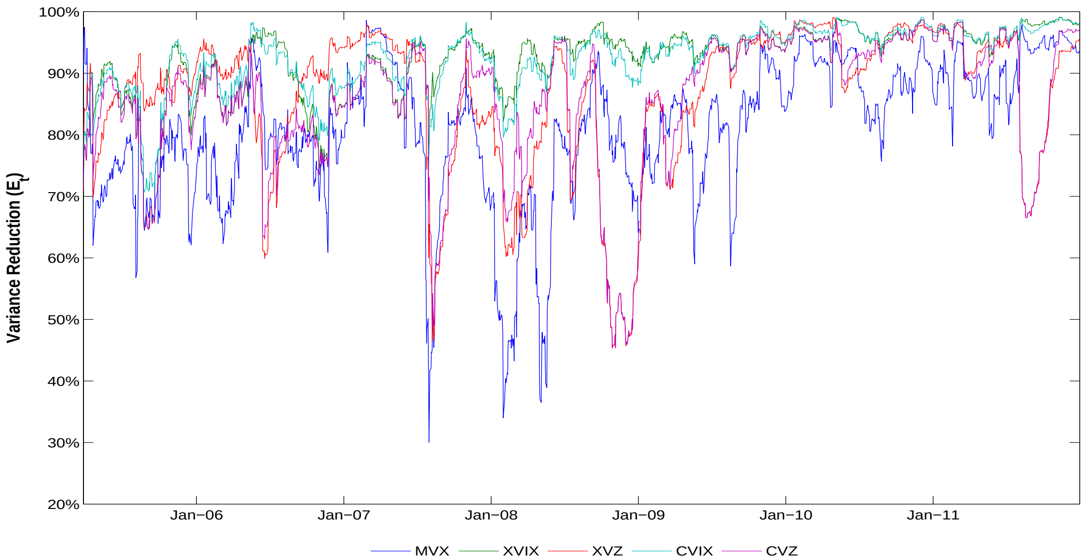

Fig. 8 The variance reduction from exposure to MVX, or one of the new static or dynamic differential roll-yield ETNs, relative to an exposure to VXX. A figure of 90% indicates that the variance of the MVX (or other ETN) is 10% of the variance of the VXX.

value $\lambda = 0.94$ in Eq. (16) and hence compute the time series E_t of variance reducation. For comparison, we also replace the variance of MVX by the variance of XVIX, of XVZ, CVIX and CVZ, yielding the five different time series of variance reduction effectiveness displayed in Fig. 8. Surprizingly, a the greatest variance reduction is achieved with the static ETN^2 and ETN^3, i.e. the XVIX and CVIX. These ETNs typically have variance between 5% and 20% of the VXX, in other words the variance reduction is between 95% and 80%. During the last two years, their variance has never exceeded 10% of the variance of VXX. This shows that it is difficult to construct allocations along the VIX term structure that have a lower volatility than the existing (and potential) static differential roll-yield ETNs.

From the issuer's perspective, the MVX has some advantages over single tracker products, which are hedged using only two VIX futures contracts. Concentrated exposures often incur margin calls, especially during volatile periods, but margin costs for a diversified portfolio should be very small because the exposure is balanced: brokers typically charge only a few basis points on the difference between interest earned on the margin when the futures price rises, and interest paid on the margin when the futures price falls. On the other hand, diversifying a concentrated exposure along the term structure incurs a hedging cost all along the curve. Apart from considering the internal activities of the bank, the attraction of issuing a diversified ETN such as MVX will depend on the relative size of fees and hedging costs plus costs of maintaining a margin account.

5.4. Risk and Return on ETN^2 and ETN^3

Here we present a detailed analysis of the risk-returns profile of all the ETN^2 and ETN^3 products considered so far. Fig. 9 presents a rolling picture of the Sharpe ratio, mean daily return (annualized) and the annual volatility of the ETN^2 and ETN^3 products, including the minimum-variance ETN (MVX). Each statistic is computed using a sample of the previous 250 days, then the sample is rolled over daily and the statistics are re-calculated.

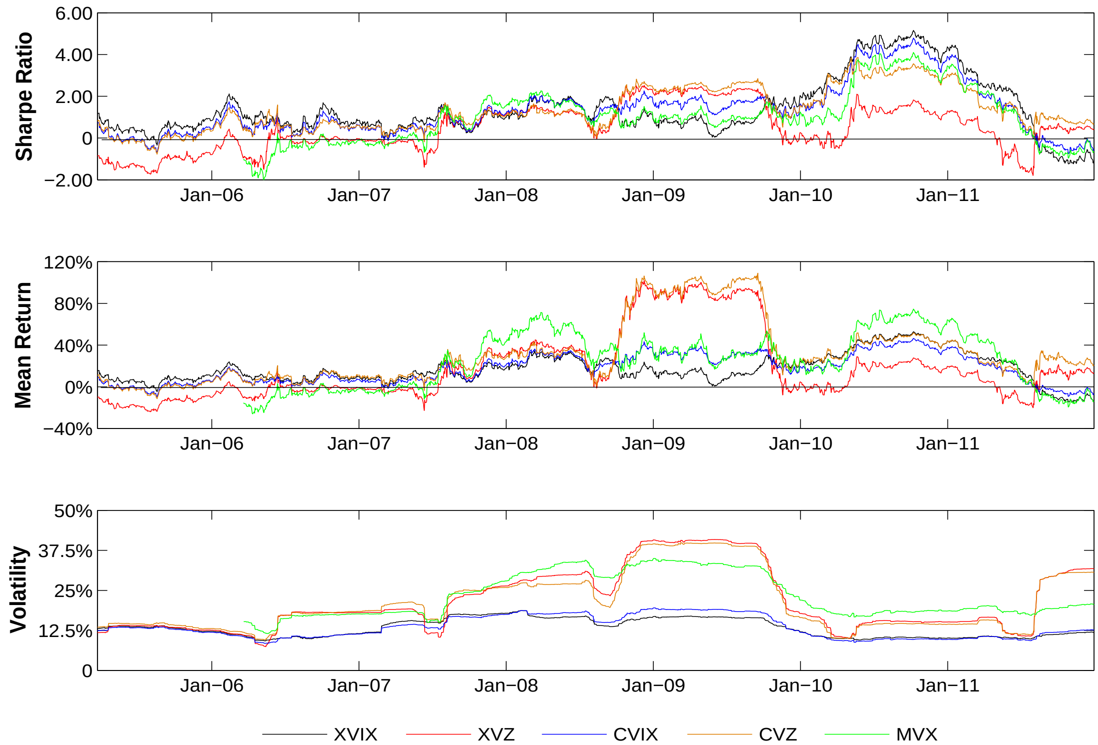

Fig. 9 Sharpe ratio, mean and volatility computed ex-post using the previous 250 days of returns on the ETN^2 and ETN^3 and on the minimum-variance ETN (MVX), March 2005 – December 2011.

Whilst the dynamic products XVZ and CVZ have a consistently higher volatility, they also have an extremely large return in Q4 2008, at the onset of the banking crisis. Surprizingly, the MVX also has a very good return – indeed it produced the best performance of all at the end of 2007 and in Q2 2010. The two static ETNs, the XVIX and CVIX, have the lowest volatility throughout and a consistently positive mean return until the end of the sample. Unfortunately, during 2011 these static strategies performed very badly indeed, with a Sharpe ratio rapidly falling from an unprecedented high of almost 5 (!) into negative territory within the space of 12 months. The Sharpe ratio of all ETNs fell during this period, and the CVZ it is the only one of the seven ETNs that consistently held a positive mean return, when computed over the previous 250 days, over the entire sample period.

5.5. Comparison with Standard Asset Classes

Next we compare the performance of the proposed ETN^3 products with the performance of standard asset classes (US equities, commodities and bonds) and assess the potential for diversification of investments in standard assets by allocating a portion of capital to the CVIX or the

CVZ. To proxy the US equity market we use the S&P 500 stock index; to proxy the US commodity market we employ the S&P GSCI commodity index (SPGSCI) which allocates between the major physical commodities that have active futures traded on US exchanges; and as a proxy for the US fixed income market we use the Barclays Aggregate Bond Fund (AGG) which corresponds to the performance of the total US investment grade bond market. We subtract a proxy for the risk-free rate (91-day U.S. T-bill) from the returns on all variables to construct excess over the risk-free rate returns. Again, we use the longest possible historical period (almost 8 years) for this analysis.

Table 8
Descriptive statistics and correlation matrix of ETN^3s and traditional asset classes.

Panel A: Summary statistics

	Annualized Mean	Volatility	Sharpe Ratio	Skewness	Kurtosis	Total Return
CVIX	16.32%	13.72%	1.1896	0.15	6.68	232.95%
CVZ	30.80%	23.49%	1.3111	1.48	18.92	798.35%
S&P 500	2.75%	22.17%	0.1239	-0.04	9.93	2.28%
SPGSCI	3.68%	5.77%	0.6383	-2.60	73.50	31.67%
AGG	13.05%	26.90%	0.4850	-0.14	2.03	109.02%

Panel B: Correlations

	CVIX	CVZ	S&P 500	SPGSCI	AGG	VIX
CVIX	1.00					
CVZ	0.80	1.00				
S&P 500	-0.35	-0.53	1.00			
SPGSCI	-0.11	-0.17	0.33	1.00		
AGG	0.11	0.04	-0.12	0.01	1.00	
VIX	0.24	0.40	-0.75	-0.24	0.11	1.00

Panel A reports the summary statistics for daily excess returns on the two volatility ETN^3 products that we propose, and compares them with those for the S&P 500, the SPGSCI commodity index and the AGG bond index. Sample means, standard deviations and corresponding Sharpe ratios are annualized based on 250 trading days per year. Panel B is the correlation matrix for these returns and the VIX. The sample is March 26, 2004 to December 30, 2011.

Table 8 reports the results. Panel A shows that the ETN^3 products clearly outperformed the three standard asset classes. Investing in commodities was profitable but highly risky and the Sharpe ratio for commodities is much worse than those of the ETN^3s. Results are even worse for the other two asset classes; investing in equity would have been only marginally profitable with an annualized mean return of just 2.75%. Moreover, if a fund manager had employed a buy-and-hold strategy over the whole period he would have experienced a return of just 2.28%. Investment in the bond market would have yielded the highest total return of the three conventional asset classes, but it is still far less than the return that could have been realized had the CVIX and CVZ been available to investors. Total returns from buy-and-hold from March 2004 to December 2011 were almost 235% for the CVIX and almost 800% for the CVZ. The CVIX has negligible skewness and relatively low kurtosis. Whilst the CVZ has a high kurtosis (almost 19) it also has a strong positive skewness, so extreme positive returns have been more common than extreme negative returns.

A considerable potential for diversification with standard asset classes is also offered by the CVIX, and even more so by the CVZ. Panel B of Table 8 shows that both ETNs have a weak positive correlation with the VIX index and a strong negative correlation with the S&P 500 equity index. They also display a small negative correlation with the SPGSCI commodity index and the CVIX has a small positive correlation with the AGG bond index. By comparison, the correlation between the S&P 500 and the SPGSCI indices is, at 0.33, relatively strong. Moreover, the CVZ has a correlation with the AGG bond index that is insignificantly different from zero.

6. Hedging ETNs: Concerns for Issuers and Regulators

The holder of an ETN has the right to redeem early (typically in lots of 25,000 shares) at the closing indicative value one business day after the holder gives notice (through his broker) of the redemption. During volatile periods early redemptions can be extremely large. For instance, during the first few days of the Euro crisis in August 2011 the number of VXX shares outstanding more or less halved, from 42 million to 21 million. The early redemption features of ETNs require the issuer to secure a profit through hedging his exposure daily, to earn at least the annual service fee after accounting for hedging costs.

In this section we derive expressions for net assets and liabilities on the hedging account; we illustrate the extent of profits that can be made by banks that issue ETNs, using some straightforward examples; we perform a scenario analysis, which issuers should be employing for risk management; and explain to regulators the mechanics (including various avenues for front-running) of the hedging activities that tie the NYSE with the CBOE.

6.1. Perfect Hedges Based on Indicative Values

Let x_t^m denote the net liability facing the issuer of a single constant-maturity tracker ETN, with maturity m, on day t. Let r_{t+1}^m denote the return on the m-maturity S&P index from day t to day $t+1$, which is given by Eq. (2). Let f^m denote the (daily equivalent) constant investment fee and let $\tilde{r}$ denote the daily return on the risk-free asset. Then from day t to day $t+1$ the liability x_t^m changes to

$$x_{t+1}^m = x_t^m(1 + r_{t+1}^m + \tilde{r} - f^m).$$

The assets held by the issuer in the risk-free asset are x_0^m on the day of issue and thereafter the assets y_t^m on day t grow at the risk-free rate $\tilde{r}$, plus the P&L from the hedge of his liabilities on day t. Suppose the issuer hedges his liabilities x_t^m at time t using the synthetic m-maturity futures. Ignoring any margin costs, taking an exposure of x_t^m in the futures with return r_{t+1}^m adds the P&L $x_t^m r_{t+1}^m$ to the issuer's assets, so that in the absence of transaction costs his total assets at time $t+1$ would be given by $y_t^m(1+\tilde{r}) + x_t^m r_{t+1}^m$. However, there is a small daily transaction cost of

$$h_{t+1}^m = \frac{s_{t+1}^m}{2}|x_t^m + 1 - x_t^m(1 + r_{t+1}^m)|$$

arising from rebalancing the positions in the two futures that straddle the target maturity.[28] Here

[28] Recall that during a weekday the amount $(l-s)^{-1}$, which is usually 1/28 or 1/32 of the exposure, is transferred from the shorter to the longer maturity futures. Over a weekend it is 3 times this amount.

s_t^m represents the bid-ask spread for an m-maturity futures exposure on day t.[29] Hence,

$$y_{t+1}^m = y_t^m(1+\tilde{r}) + x_t^m r_{t+1}^m - h_{t+1}^m.$$

Thus, the net assets (i.e. assets – liabilities) at time $t+1$ are given by

$$z_{t+1}^m = y_{t+1}^m - x_{t+1}^m = (y_t^m - x_t^m)(1+\tilde{r}) + x_t^m f^m - h_{t+1}^m. \tag{17}$$

6.2. Empirical Examples

Fig. 10 depicts three empirical examples showing, on the left, the daily average flow of fees and hedging costs, averaged over the previous month. The right-hand graphs depict the cumulative net assets accruing to the issuer of ETNs, as a proportion of the current liability. The examples are: (i) an initial issue of equal amounts in the 30-day tracker VXX and its inverse tracker, the AAVX; (ii) an initial issue of XVIX, plus two-thirds of this exposure in the inverse 150-day tracker and one-third in the direct 30-day tracker; and (iii) an initial issue of equal amounts on the XVIX, and the four direct and inverse trackers at 30-day and 150-day maturities, i.e. five ETNs in all. We suppose all shares in issue were sold on the primary market on March 30, 2004 and that the number of shares outstanding is assumed constant, i.e. there are number issues of shares less the shares redeemed early on any day is assumed constant.

The net asset path (assets minus liabilities) is not shown, but it grows monotonically over the entire period in all three cases. Net assets as a proportion of the current liabilities also remains positive over the entire period even though the fees earned cover the hedging costs only in case (i) and (iii). In case (ii) the excess of hedging costs over fees earned erodes the asset side of the balance sheet so that they are much less than in cases (i) and (iii).

Of course, the initial exposures change over time. In particular, the liabilities for direct and inverse trackers vary inversely, so that the liability to the inverse tracker grows when the liability to the direct tracker falls. For this reason the net assets can fall, as a proportion of the liabilities. For instance, by March 20, 2009 the net assets in case (i) – i.e. an initial issue of equal dollar amounts in VXX and AAVX – had risen to nearly 85% of the current liability, net of fees and hedging costs. But one year later, by March 20, 2010 it had fallen steadily too less than 30% of the current liability. This is because the liability to the inverse tracker was gradually rising during the long contango market following the banking crisis. Clearly the issuer of a suite of products may find it advantageous to issue further notes in direct tracker products at such times, if possible. This way, the basket of ETNs can be kept in balance so that not only the net assets are always growing, but also net assets are always growing as a proportion of the current liability.

[29]In the next section we report an empirical analysis of net assets accruing to the issuer of ETNs over time, reporting fees and hedging costs separately. Here we assume constant spreads of 25bps for the 30-day and 30bps for the 150-day trackers. It is true that spreads have decreased dramatically over time, but our purpose is to assess the profitability of a potential future issuance of ETNs, so it is appropriate to assume hedging costs are fixed at their current size.

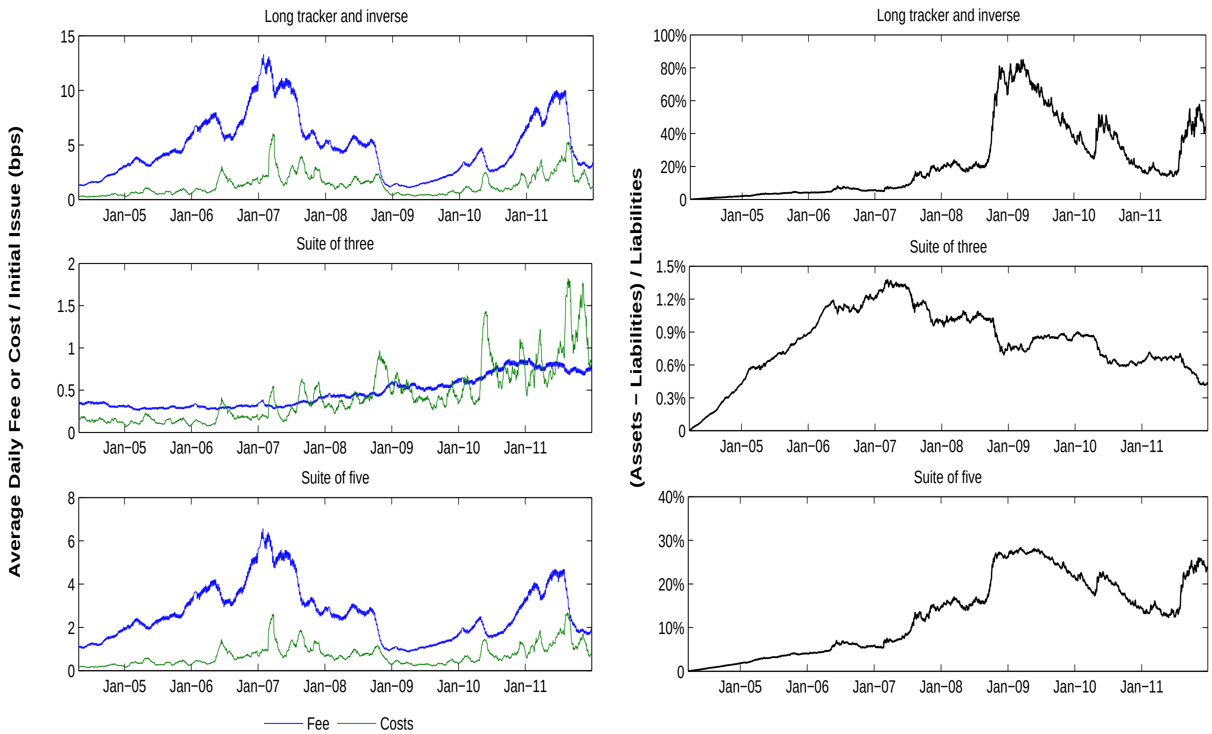

Fig. 10 Daily average fees and transaction costs, averaged over the previous month (on the left) and the cumulative net assets accruing to the issuer of ETNs, as a proportion of the current liability (on the right).

6.3. Scenario Analysis

The examples selected in Fig. 10 consider only three from a very large number of possible strategies for issuing ETNs. Moreover, we considered just one (historical) scenario for the evolution of the S&P indices that determine the indicative values of the ETNs. We now apply a statistical bootstrap methodology to simulate a distribution for net assets accruing to the issuer, as a proportion of liabilities, at different time horizons in the future. For each distribution we use the bootstrap to simulate 10,000 possible paths such as those shown in Fig. 10, each with a different, randomly-selected sequence of historical returns on the S&P indices.

Continuing to assume that issuance is equal to early redemptions, we use the block bootstrap to simulate a distribution for net assets and liabilities at some future point in time.[30] More precisely, we start with an initial number of notes sold on the primary market in our three cases: (i) equal initial dollar amounts in VXX and AAVX; (ii) XVIX with balanced exposures to inverse 150-day and direct 30-day; and (iii) XVIX with equal amounts in direct and inverse 30-day and 150-day trackers. We re-sample with replacement 1250 observations from our historical data on m-maturity indices in such a way that the cross-correlation and autocorrelation properties are retained. So we select a date at random and then sample 10 consecutive daily returns on all the constant maturity indices, starting from this date. Repeating this process 125 times gives a bootstrapped

[30]Assuming the initial issue is left unchanged and there are no early redemptions. More complete simulations would adjust for further issues and early redemptions, but for brevity we report only the base case in this paper.

Table 9
Bootstrapped Sample Statistics: (Assets – Liabilities)/Liabilities.

Year	1	2	3	4	5
Case (i): 30-day direct and inverse trackers, VXX and AAVX					
Mean	2.41%	5.12%	8.10%	11.41%	14.97%
Stdev	0.43%	1.30%	2.82%	4.98%	7.86%
Skew	-0.16	0.57	1.05	1.33	1.55
Kurt	1.18	1.36	3.26	4.11	4.64
Max	4.57%	12.87%	37.90%	60.13%	79.65%
Min	0.49%	0.80%	1.38%	1.14%	2.22%
Case (ii): XVIX with 1/3rd in 30-day and 2/3rd in inverse 150-day trackers					
Mean	1.81%	3.37%	4.62%	5.56%	6.21%
Stdev	0.24%	0.65%	1.17%	1.74%	2.31%
Skew	-0.39	-0.08	0.14	0.32	0.51
Kurt	0.31	-0.06	-0.08	-0.08	0.16
Max	2.64%	5.95%	10.82%	14.39%	18.19%
Min	0.77%	1.06%	1.11%	1.15%	1.03%
Case (iii): XVIX Suite of Five					
Mean	2.06%	4.05%	5.90%	7.59%	9.10%
Stdev	0.29%	0.77%	1.45%	2.25%	3.22%
Skew	-0.34	0.27	0.74	0.92	1.30
Kurt	0.88	0.93	2.15	2.03	3.36
Max	3.34%	7.68%	19.30%	25.22%	32.77%
Min	0.74%	1.04%	1.53%	1.46%	1.72%

The mean, standard deviation, skewness and excess kurtosis of the distribution of 10,000 block-bootstrapped net asset values, as proportion of liabilities, after n years, with $n = 1, 2, ..., 5$. We also report the minimum and maximum values obtained in all 10,000 simulations.

multivariate time series with 1250 observations. From this simulated time series we compute the net assets as a proportion of liabilities, exactly as we did for the real historical data in Fig. 10, but we retain only five points viz. the net assets one year after the issue, as a proportion of the liabilities at that time, and the same again for n years after the issue, with $n = 2, ..., 5$, assuming 250 trading days per year. Then we repeat the block bootstrap and again compute the net assets (as a proportion of liabilities) after n years, with $n = 1, 2, ..., 5$. Repeating this procedure 10,000 times simulates a distribution of possible values for net assets as a proportion of liabilities, based on the block-bootstrap methodology. Table 9 reports the summary statistics.
Comparing the results for the three different issuance strategies we note:

1. The means increase over time in all cases. Case (i) produces the greatest values on average, followed by case (iii) and finally case (ii). The uncertainty (as measured by the standard deviation) also increases over time and has similar ordering to the mean;

2. The t-ratio of mean to standard deviation decreases over time but still, the mean is significantly greater than zero even after five years; this t-ratio is at most 7.5 [case (ii) after one year] and at least 1.9 [case (i) after five years];

3. The distributions are near normal after one year, but thereafter skewness is positive and growing over time in all three cases. Excess kurtosis does not necessarily grow over time – see case (ii) for instance, which gives the lightest-tailed distributions of all;

4. The minimum value obtained over all 10,000 simulations is always positive. For instance, after five years net assets accruing from the XVIX suite of five would be at least 1.72% and at most 32.77% of liabilities; and net assets from the VXX – AAVX combination would be at least 2.22% and at most 79.65% of liabilities – assuming no change to the initial issue of equal amounts in each ETN.

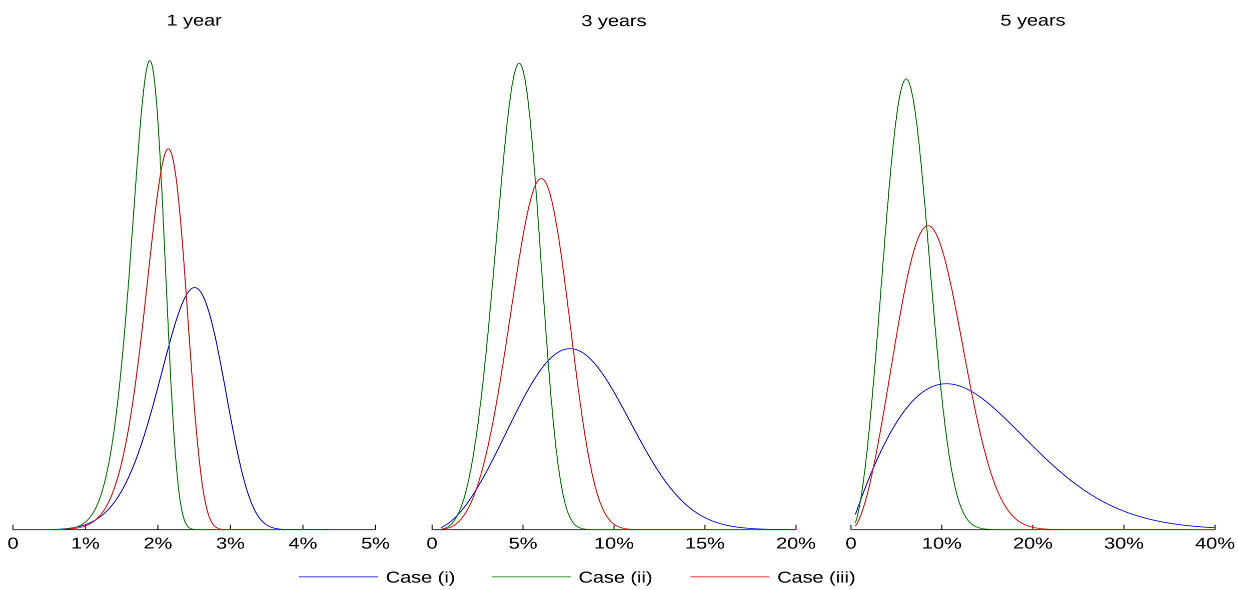

Fig. 11 Densities of current assets minus current liabilities of the ETN issuer as a proportion of current liabilities: one year after issue (left); three years after issue (middle); and five years after issue (right). Case (i) is equal initial dollar amounts in VXX and AAVX, shown in blue; case (ii) is XVIX with balanced exposures to inverse 150-day and direct 30-day shown in red; and case (iii) is XVIX with equal amounts in direct and inverse 30-day and 150-day trackers, shown in green. No re-issue or early redemptions included in the scenarios. Assets include accrual of the principal raised at the risk free rate, investment fees, profit and loss on the VIX futures positions and hedging costs. Densities are based on 10,000 block bootstraps of 1250 observations from the historical data between March 2004 and December 2011.

Fig. 11 displays the entire density of net asset value as a proportion of liabilities for each issuance strategy and at one, three and five years after issue. The graph on the left depicts one-year densities for the three different issuance strategies; in the middle we compare the three-year densities and on the right we compare the five-year densities.

In each case the near-normal one-year density with low mean and variance evolves over time into densities with progressively higher means and variances, positive skewness and [except in case (ii)] excess kurtosis. The most uncertain of the relative net asset values arises in case (i), where only the direct and inverse 30-day trackers are issued, and the most certain relative net asset values are for case (ii), the XVIX with balanced exposures to inverse 150-day and direct 30-day trackers. However, the expected relative net asset value is also largest in case (i) and lowest in case (ii). Relative net asset values exceeding 3% after one year, 12% after three years or 20% after five years are only ever realized in case (i).

6.4. The Regulators Perspective

The large-scale hedging activities of ETN issuers on the CBOE market are likely to influence the prices of VIX futures. Moreoever, there can be an indirect speculative activity surrounding this hedging that we now explain. Because the early redemption value of an ETN is determined by the closing prices of the two straddling VIX futures (one day after notice is given) the previous analysis of assets and liabilities in the issuer's ETN account is based on closing prices of VIX futures. Of course, ETN issuers can put their VIX futures hedging trades to the CBOE at any time during the day, but any systematic changes in a VIX futures price after the trade and before market close will bias the hedging account. For this reason, the issuer may see an advantage in timing the rebalance of his hedge as near as possible to the CBOE market close. The number of shares outstanding on each ETN is public knowledge. Therefore, by replicating the analysis described above and using the shares outstanding to compute the net exposure to VIX futures across the suite of ETNs offered by an issuer, speculative traders on the CBOE could predict how much the ETN issuers are required to trade on each VIX futures contract each day for rebalancing the hedge.

This way it is relatively easy for a speculator to predict both the time that an issuer places his hedging trades, and the size and direction of these trades. Hence, a speculator could front-run these trades to create an 'arbitrage' in the CBOE market for their own benefit. The effect of the speculator's activities on the issuer is that the issuer trades VIX futures at a disadvantageous price, because the speculator will exit his position immediately the issuer has transacted the hedge. This type of activity is another channel whereby activities in the ETN market can influence the prices of the very futures they are supposed to track.

A case in point is the TVIX, the $2\times$ leveraged 30-day tracker issued by Credit Suisse. The scale of front-running of their hedging activities recently led to such an increase in hedging costs that the bank suspended further issuance of the ETN, in February 2012. As a result of the excessive speculative demand for TVIX, over the next few days it traded at a significant premium (up to 90%) of its indicative value. Credit Suisse subsequently re-opened the issue only on the condition that the market makers on the NYSE and CBOE agree to transact VIX futures in the required amount for hedging at the daily closing price, thus passing on the hedging risk to the market makers.

Besides the daily rolling behavior that is easily predicted, front-running of the issuer's hedging trades could be based on actual early redemptions. During volatile periods the issuer often needs to off-load a significant number of VIX futures that are no longer required for hedging, as in the VXX example cited in the introduction to this section. The possession of such privileged information by the investor, his broker and the issuer is clearly highly sensitive, because it creates a huge potential for front-running arbitrage.

In order to receive the closing indicative value of the ETN on day $T+1$ the investor is required to give notice of early redemption, through his broker, by 4pm on day T. Since the issuer no longer needs to hold some VIX futures as a hedge against these shares the issuer should sell those futures any time between 4pm on day T and market close on day $T+1$. If the early redemption notice is for a very large number of shares (redemption is in lots of 25,000) and the issuer sells the corresponding VIX futures some time before the market close on day $T+1$, this sell order will have a downward pressure on the redemption value of the ETN. In other words, in the absence of a market shock, the issuer will need to pay the investor less for his ETN shares if he exits his hedge as early as possible. Even if the early redemption order is not very large, because of the implicit roll cost on the ETNs, the expected daily return on the indicative value is negative. Thus, the issuer's expected profit is still positive after any early redemption order. In the absence of a shock that increases volatility this expected profit decreases to zero as the time that the issuer closes out the VIX futures hedge progresses towards market close on day $T+1$.

Both the front-running of ETN hedging and the ability of the issuer to influence the redemption value are consequences of their early redemption conditions. If their terms and conditions were changed so that notice of redemption could be served on the day of redemption instead of the day before, and further, if the redemption value were set at an average price throughout the day, then neither of these issues would arise.

But even without these problems, regulators may be concerned that the direct trading of volatility will increase systemic risk. Changes in volatility act like a catalyst for the herding reactions of financial firms, so volatility markets are propagators of systemic risk. Heavy trading on VIX futures and ETNs has increased the volatility of these volatility markets. In particular, the frequency and amplitude of volatility cycles appears to have increased since the launch of ETNs, making counter-cyclical risk management more difficult and thereby increasing systemic risk.

ETN activities also transmit risk from one market to another. At the time of writing the U.S. Securities and Exchange Commision (SEC) is particularly worried that the large-scale hedging activities by ETN issuers transmits volatility from the NYSE to the VIX futures and options that are traded on the CBOE. If higher VIX option prices subsequently spill over to S&P options, the VIX index itself will increase. Thus, a knock-on effect of hedging VIX futures ETNs could be that a spiral of increasing levels of volatility could ensue.

We have shown, in Section 5, how non-leveraged direct and inverse tracker ETNs can be used to form attractive portfolios for long-term investors. Such portfolios typically take positions in short-term inverse trackers and long-term direct trackers. Typically, short-term speculative traders would be happy to take opposite positions most of the time. So the ETN issuer will have a natural partial hedge which reduces their demand for VIX futures to complete the perfect hedge.

By contrast, the leveraged direct tracker ETNs, such as the TVIX, are purely speculative vehicles which are being traded in increasly large volumes. Consequently, large numbers of VIX futures contracts are required for hedging those ETNs and this could have a major impact on systemic risk. Hence, regulators should be concerned about the existence of such instruments.

7. Summary and Conclusions

Exchanges, regulators, market-makers, speculators, potential investors and the banks that issue VIX futures ETNs may all improve their understanding of these products from reading this paper. This is particularly important given the opaque risks arising from trading VIX futures ETNs on the NYSE Arca, and the spill-over hedging effects on VIX futures contracts traded on the CBOE.

Our chronology of the issues of VIX futures ETNs shows that more than half of them directly track a single constant-maturity VIX futures, including three 2 × leveraged products recently issued by Credit Suisse and Barclays. The market cap of such direct tracker products on February 29, 2012 was nearly $2.5 billion, i.e. 82% of the total market cap of all VIX futures ETNs at that time, the other ETNs being ten inverse trackers plus the two differential roll-yield products, XVIX and XVZ. During the first two months of 2012 about 37 million shares were traded on the direct tracker ETNs every day. But volatility ETNs are amongst the most risky of all exchange-traded products available, especially those such as TVIX that have 2 × leverage on a direct tracker ETN. Had the TVIX be traded during the August 2011 Eurozone crisis its volatility would have exceeded 400%.

Investment returns on individual ETNs are eroded by negative roll-yield effects, except during the (increasingly) brief periods that the VIX futures term structure is in backwardation. However, it is possible to build portfolios of ETNs, which typically have a short exposure to short-term VIX futures trackers and a long exposure to longer-term VIX futures trackers, which offer unique risk and return characteristics through trading the differential roll yield along the VIX futures term structure. To replicate the indicative returns on these portfolios we use the S&P constant-maturity indices from December 2005 and prior to this we replicate the S&P index values ourselves, by linearly interpolating between VIX futures returns. Such indices are investable because they retain a one-to-one correspondence between the synthetic and traded maturities of VIX futures.

We find that XVIX, the ETN^2 issued by UBS AG, offers an almost uncorrelated exposure to VIX but only performs well during stable trending equity markets; whereas XVZ, the ETN^2 issued by Barclays Bank PLC, is highly correlated with VIX but performs well only during a crisis. This motivates us to propose two ETN^3 products which turn out to have risk-return and diversification characteristics that are superior to any of the existing volatility ETNs. We name these the CVIX, which has a static allocation to XVIX and XVZ, and the CVZ, which has a dynamic, regime-dependent allocation to the same two ETN^2. According to a wide variety of risk-adjusted performance measures over almost an eight-year historical period as well as a thorough sub-period analysis, our proposed ETN^3 clearly outperform the volatility ETNs that have already been issued, with two exceptions – UBS's XVIX would have performed best during the period March 2004 – October 2006 and Barclays' XVZ would have outperformed CVIX during the banking crisis.

To illustrate how much risk could be reduced when the ETN has an allocation across the entire term structure of VIX futures we have quantified the variance reduction possible relative to the VXX ETN when the risk is effectively hedged along the term structure of VIX futures. We call this minimum variance portfolio of constant-maturity VIX futures the MVX. As an ETN, the MVX also has excellent return characteristics, but the CVIX is even better at reducing risk and CVZ has almost as low volatility as the MVX. In fact, it would be difficult to construct allocations to the VIX term structure that have a lower volatility than the CVIX. For instance, the CVIX has displayed an average volatility of less than 14% since 2004. Yet, the total buy-and-hold returns from March 2004 to December 2011, net of investment fees, would be about 235% for the CVIX and almost 800% for the CVZ! The Sharpe ratios have averaged at 1.19 for the CVIX and 1.31 for the CVZ, over the eight-year sample period.

The CVIX has negligible skewness and relatively low kurtosis. The CVZ has a strong positive skewness, so extreme positive returns have been more common than extreme negative returns. Whilst both products are positively correlated with VIX (sample correlations are 0.24 for CVIX and 0.40 for CVZ) their correlation with equity is negative (-0.35 for CVIX and -0.53 for CVZ) and correlations with commodities and bonds is either negative or negligible. Hence, their considerable potential for diversification with standard asset classes also makes these products attractive.

To enhance the uptake of ETN^2 and ETN^3 the issuer should make it very clear in the prospectus that VIX futures and the direct tracker ETNs that currently dominate the market may be excellent instruments for short-term bets on the returns to VIX futures of different maturities, but they are far too risky for pension funds or mutual funds to invest in, outside of a very major crisis in equity markets. Alexander and Korovilas (2012) show that this comment also applies to the VXZ and other mid-term VIX futures trackers, although this fact is not yet commonly understood.

We have explained how an ETN issuer can hedge against early redemption of ETNs. Even though the direct tracker ETNs have an expected value of zero at maturity, early redemptions are frequent during volatile periods, so these can pose a significant risk to the issuer. And inverse tracker ETNs do not have a long-term value of zero, so the hedging costs for these products is much higher – indeed hedging the inverse part of the differential roll-yield trades constitutes almost all of the cost. We have demonstrated, algebraically and empirically, how the issuer can hedge an exposure to a suite of ETNs almost exactly, provided their trades on VIX futures are at the daily closing price. Moreover, they should be able to control the issues of their various ETNs and hedge the whole portfolio in such a way that the net assets accruing after hedging costs are very substantial.

The statistical block bootstrap has been applied to build 10,000 scenarios for the evolution of the issuers assets and liabilities over a five-year horizon. Such scenarios capture the cross-correlation and autocorrelation of the ETN returns, as well as accounting for the fees accruing to and the hedging costs paid by the issuer. We only exclude margin costs. Notably the net assets never become negative, under any of the 10,000 bootstrap scenarios, and for all three ETN issuance strategies considered (i.e., the same initial dollar exposure to short-term direct and inverse trackers; the XVIX with offsetting issues in short-term direct and medium-term inverse trackers; and a suite of five ETNs). Under every simulation the net asset value after hedging costs is always positive, even after deducting liabilities arising from the unlikely scenario of full early redemption (or entire issue called). The expected value of net assets relative to liabilities always grows over time, but so does its variance. Nevertheless, even five years after issue the expected net assets relative to liabilities is significantly greater than zero.

In this exercise we have considered only three possible ETN issuance strategies and we bootstrapped scenarios using historical data from March 2004 – December 2011. Although this period includes the credit and banking crisis, even more severe hypothetical stress tests with partial early redemption and/or new issuance scenarios should be performed. The aim should be to construct a diverse array of ETNs (including the CVIX and CVZ which are very attractive stand-alone long-term investments as well as excellent diversification instruments) that makes considerable profits for the fully-hedged issuer under all bootstrapped historical and hypothetically stressed scenarios.

Regulators are watching the recent growth in trading on VIX futures ETNs very closely. We have explained how the predictable, large-scale hedging activities of ETN issuers are a vehicle whereby ETN market trading now influences the prices of the futures contracts that they are supposed to track. There is also a great potential for speculators to front-run the daily hedges of ETN issuers, by trading VIX futures shortly before close on the CBOE. The remedy would be to change the terms and conditions of early redemption to be similar to those of standard passive ETFs, such as the SPDR, and furthermore pass on the hedging responsibility to the market makers.

However, the scale of these hedging activities (especially those on TVIX) has raised the volatility of VIX futures, and this increases the prices of VIX options. If this has the knock-on effect of increasing S&P option prices the VIX index itself will increase, thus raising the level of VIX futures prices. Since volatility acts like a catalyst for the herding reactions of financial firms, regulators may well be concerned that the direct trading of volatility will increase systemic risk.

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Appendix

Here we present a comparison of the variance reduction attainable by using multivariate GARCH rather than equally-weighted covariance matrices for deriving minimum-variance allocations to the VIX futures term structure. Multivariate GARCH models have many parameters. In our case they have at least 72 parameters, since there are 24 distinct elements in a 6×6 covariance matrix and each covariance equation requires at least 3 parameters. Not surprizing, the optimizer fails to converge for some samples. To circumvent this problem we use the fact that VIX futures form a very highly correlated system to apply the orthogonal GARCH (O-GARCH) covariance matrices introduced by Alexander (2001).

We first apply PCA to the unconditional covariance matrix of the ICM VIX futures returns. This allows the returns to be represented as $\mathbf{X} = \mathbf{PW}'$ where $\mathbf{P}$ are the principal components and $\mathbf{W}$ are the eigenvectors of the unconditional covariance matrix. Then, since the columns of

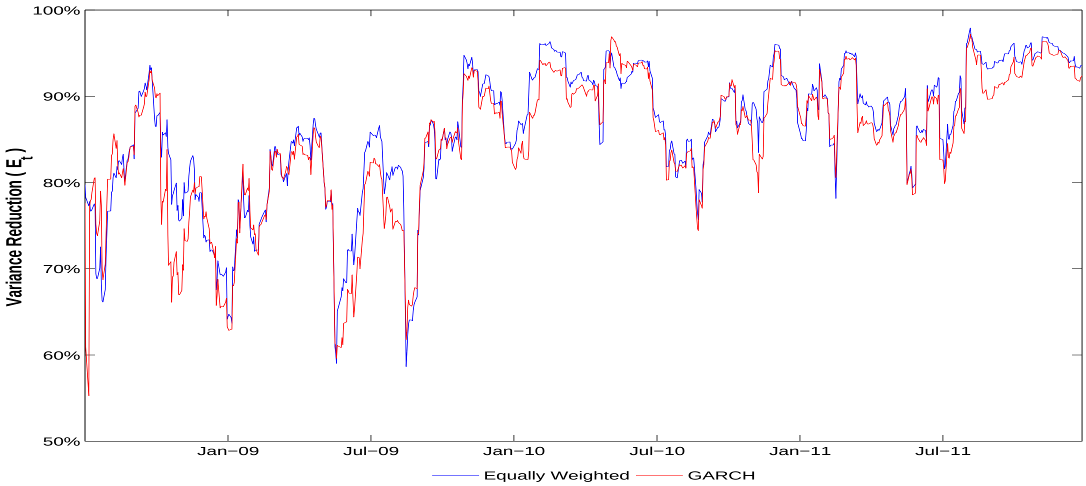

Fig. 12 Effectiveness of holding minimum-variance portfolios of VIX futures overnight. Percentage variance reduction that is possible relative to concentrating positions on 30-day VIX futures. For the variance calculations (16) we set $\lambda = 0.94$.

$\mathbf{P}$ are uncorrelated by construction, we estimate only univariate GARCH models for the principal components. The result is a time-varying an $n \times n$ diagonal matrix of conditional variances $\hat{\mathbf{\Omega}}_t$. Then we use the eigenvector matrix $\mathbf{W}$ to retrieve the conditional covariance matrix estimate of our returns $\mathbf{X}$ as $\hat{\mathbf{\Sigma}}_t^{og} = \mathbf{W}\hat{\mathbf{\Omega}}_t\mathbf{W}'$. Fig. 8 compares the variance reduction that is possible using the equally-weighted $\hat{\mathbf{\Sigma}}_t$ in (12), as in the main text, with using the GARCH $\hat{\mathbf{\Sigma}}_t^{og}$. After all, if there is little improvement from using the more sophisticated GARCH methodology we may as well use the much simpler $\hat{\mathbf{\Sigma}}^{ew}$. We do not report the minimum variance weights that are derived from the GARCH covariance matrix, since these are very unstable over time. Moreover, the variance reduction is similar, and often better, when the weights are based on the equally-weighted covariance matrix. We conclude that one should simply apply equally-weighted covariance matrices to diversify a VIX futures exposure across the term structure.