2015
The Graduate School Entrance Examination
Physics
9:00 am－11:00 am

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

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Problem 1

Consider a conical surface with a half apex angle of 45° and a bottom radius $2R_0$. As shown in Figure 1.1, the conical surface is placed with the vertex down and with its axis vertical. The vertex has a small hole. There is a string placed through the hole with mass points 1 and 2 (mass m) attached at both ends. Diameter of the hole and thickness of the string are sufficiently small. Mass and stretch of the string and friction are negligible. The acceleration of gravity is denoted by g. Answer the following questions.

I. Mass point 1 is in horizontal uniform circular motion on the conical surface with a radius R_0 and a velocity v_0. Obtain the velocity v_0 of mass point 1.

II. The system is in motion stated in Question I. When the string breaks suddenly, mass point 1 moves upward on the conical surface.

1. Obtain the angular velocity of mass point 1 about the axis of the conical surface and the vertical component of the velocity of mass point 1 at the moment when mass point 1 reaches a height H from the vertex. Use v_0 in both equations.
2. Answer with reason whether mass point 1 flies out of the top edge of the conical surface or not.

III. The motion of mass point 2, stated in Question I, was vertically perturbed. Then, mass point 2 started vertical small-amplitude oscillation.

1. Let the position of mass point 1 be expressed in cylindrical coordinate (h, r, θ). The origin of the coordinate system is the vertex of the conical surface, and the h-axis directs vertically upward. The following differential equation describes the motion of mass point 1. Obtain the coefficients a and b.

$$\frac{d^2r}{dt^2}+ar\left(\frac{d\theta}{dt}\right)^2+bg=0 \qquad (1)$$

2. Let the r-coordinate of mass point 1 be $r = R_0 + \varepsilon$ and derive a differential equation about the infinitesimal displacement ε, then obtain the period of the small-amplitude oscillation.

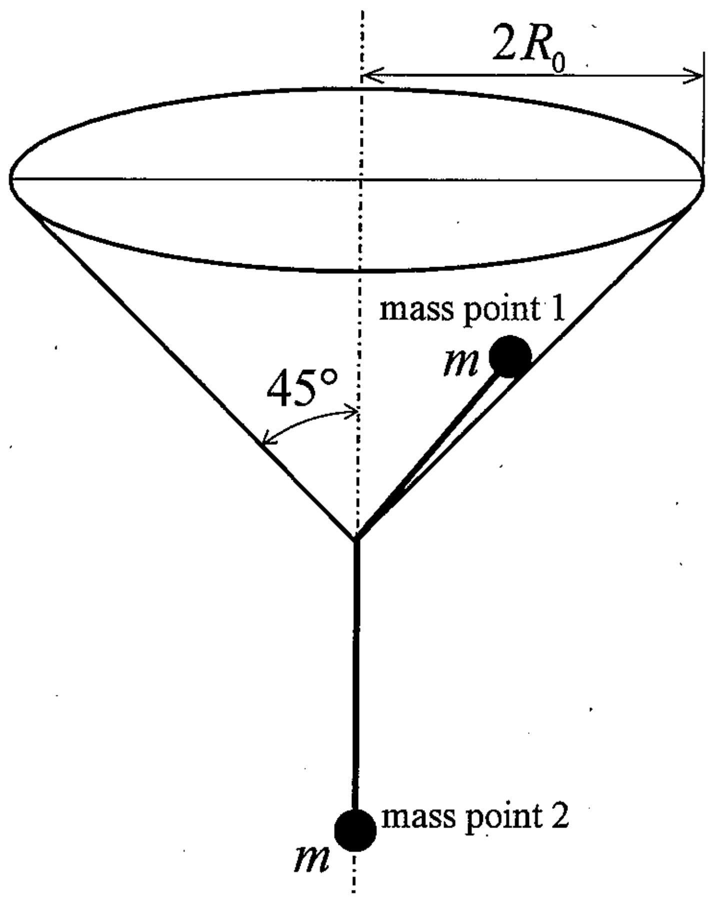

Figure 1.1

Problem 2

Suppose that a straight conductive rod (OP) with length a is rotating in the x-y plane with an angular velocity ω around its end point O in vacuum under a uniform magnetic flux density B_z in the $+z$ direction, as shown in Figure 2.1. The moment of inertia of the conductive rod around the rotating axis is denoted by J. Assuming that the thickness and electric resistance of the conductive rod and frictions can be ignored, answer the following questions.

I. Suppose that the conductive rod is rotating with a constant angular velocity $\omega = \omega_c$.

1. Obtain the area the conductive rod sweeps per unit time.
2. Using the answer above, obtain the voltage V between the end points O and P of the conductive rod.
3. List up all the electro-magnetic forces acting on electrons inside the conductive rod, and discuss the balance among them.

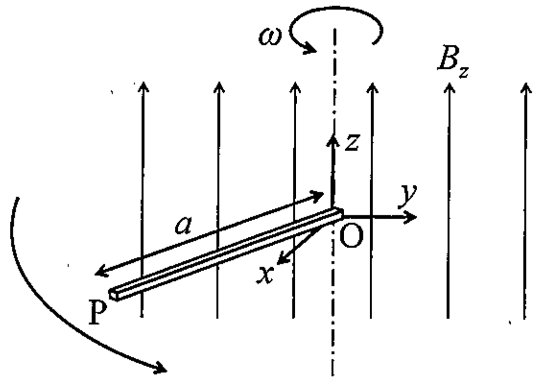

Figure 2.1

II. As shown in Figure 2.2, a circular conductive ring (radius a, centered at O) with no electric resistance is placed so that it always contacts with the conductive rod at P. Then, a resistor (electric resistance R) is connected between O and the ring. Here, ignore any magnetic fields generated by currents in the circuit other than the conductive rod.

1. The conductive rod rotates with a constant angular velocity ω_c, when a torque T_c is externally applied around the rotating axis. Obtain the torque T_c.

2. The rotation of the conductive rod distorts the uniform magnetic field. Draw schematically the magnetic flux lines around the conductive rod observed from point P toward O.
3. The angular velocity of the conductive rod at time $t = 0$ is ω_0, and no external torque is applied to it. Derive the equation that describes the temporal change of the angular velocity ω of the conductive rod, and plot ω against time t.

III. As shown in Figure 2.3, an inductor (inductance L) is connected in series with the circuit discussed in Question II.

1. Derive the differential equation that describes the angular velocity ω of the conductive rod.
2. The angular velocity ω shows an oscillatory behavior under a certain condition. In this situation, obtain the relation that the inductance L should satisfy.

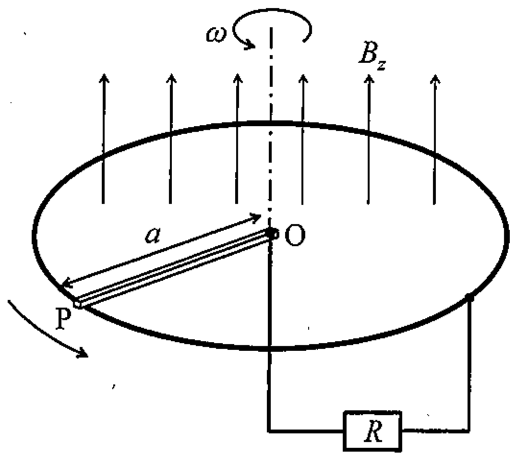

Figure 2.2

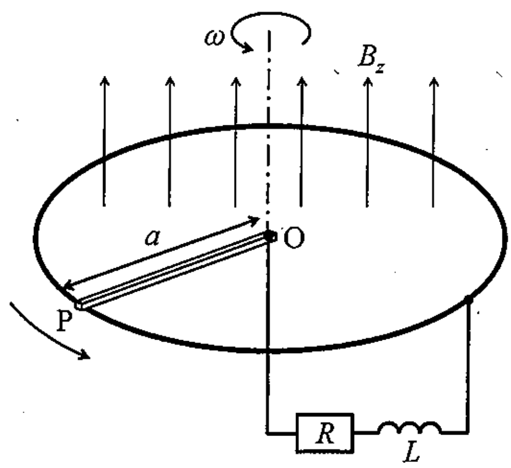

Figure 2.3

Problem 3

Consider various expansion processes of gases. Here, internal energy and volume per unit mole, absolute temperature, pressure and gas constant are denoted as U, V, T, p, and R, respectively. Molar specific heat at constant volume C_V and molar specific heat at constant pressure C_p are both constant regardless of the conditions.

I. Show that the following relation holds true for an ideal gas,

$$C_p - C_V = R . \tag{1}$$

II. Consider a quasi-static adiabatic expansion process or a quasi-static isothermal expansion process of an ideal gas of pressure p_0 and volume V_0. Draw schematically pressure-volume relations (i.e., adiabatic and isothermal lines) in a graph that clarifies the difference between the two processes.

Note that the following Poissons's equation is satisfied for a quasi-static adiabatic expansion process,

$$pV^{(C_p/C_V)} = \text{constant}. \tag{2}$$

III. Consider the process where an ideal gas undergoes adiabatic free expansion into vacuum. Show that the temperature of the gas after the expansion is the same as that before the expansion. Then, show that the process is irreversible.

IV. An ideal gas of volume V_1 at a temperature T_1 undergoes following two expansion processes A or B.

Process A: The gas undergoes an adiabatic free expansion to volume V_2. Then, after the gas is equilibrated, the gas undergoes a quasi-static adiabatic expansion to volume V_3.

Process B: The gas undergoes a quasi-static adiabatic expansion to volume V_2. Then, the gas undergoes an adiabatic free expansion to volume V_3.

The final temperatures of the two processes were found to be the same. Derive the relation among V_1, V_2, and V_3.

V. Consider an adiabatic free expansion process of a van der Waals gas that obeys the equation of state, $\left(p+a/V^2\right)\left(V-b\right)=RT$. Here, a and b are positive constants peculiar to the gas. The gas of volume V_4 undergoes adiabatic free expansion to volume V_5. Express the temperature difference ΔT, using all or part of V_4, V_5, R, a, b, and C_V. Then, explain the origin of the temperature change from the property of van der Waals gas.

The following relation may be used if necessary.

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p \tag{3}$$

Problem 4

I. Consider the refraction of light (a plane wave) as shown in Figure 4.1. Light propagating in vacuum enters a homogeneous medium with a refractive index n at an angle of incidence θ_i and refracts at an angle of refraction θ_r. Here, the speeds of light in vacuum and in the medium are denoted by c and v, respectively. Answer the following questions.

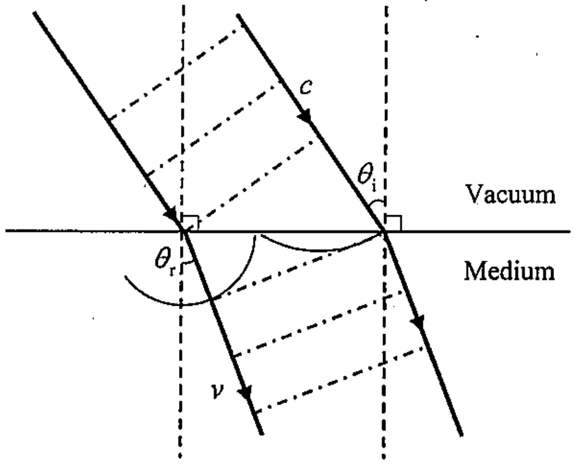

Figure 4.1

1. Derive the law of refraction (Snell's law) in terms of θ_i, θ_r, and n, using Huygens' principle.
2. In the quantum mechanical view, light energy is carried by photons. Using angular frequency ω and wavenumber k, the energy and momentum of a photon are denoted by $\hbar\omega$ and $\hbar k$, respectively (Here, $\hbar = h/2\pi$, h: Planck's constant). Express the relationship between the momentum $\hbar k_0$ in vacuum and the momentum $\hbar k_1$ in the medium, using all or part of θ_i, θ_r, c, v, and n.

II. Consider the refraction by a prism and the diffraction by an optical grating in vacuum. As shown in Figure 4.2, light (wavelength λ) enters the prism (refractive index n, apex angle α) at an angle of incidence θ_i and emerges at an angle of emergence θ_o. Next, as shown in Figure 4.3, light is normally incident to an optical grating composed of a large number of small prisms (refractive index n, apex angle β, width d) which are periodically arranged without any space. Answer the following questions.

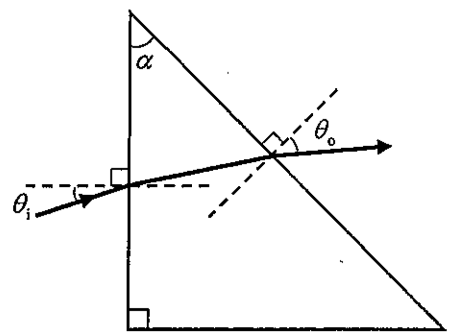

Figure 4.2

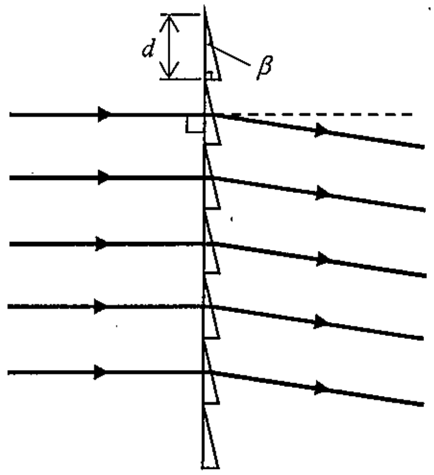

Figure 4.3

1. To determine the refractive index n of the prism, the angle of incidence θ_i and the angle of emergence θ_o are measured (see Figure 4.2). Express the refractive index n of the prism in terms of θ_i, θ_o, and α.
2. When the angle of incidence θ_i in Figure 4.2 is changed in the range of $0° \leq \theta_i < 90°$, light in a certain range of θ_i is not transmitted from the slant face but emerges from the bottom face. Express the range of θ_i in terms of α and n.
3. In Figure 4.3, when the first-order diffracted light from the optical grating and the refracted light by the small prisms propagate in the same direction, express the apex angle β of the small prisms in terms of d, n, and λ. Assume that the width d of the prisms is large enough compared to the wavelength λ of light and the apex angle β of the small prisms is sufficiently small.

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Problem 1

Consider a sphere S1 of radius r and mass m having a uniform volume density, a sphere S2 of radius r and mass $3m$ having a uniform volume density, and a spherical shell S3 of radius r and mass m having infinitesimally thin thickness with the uniform area density. As shown in Figure 1.1, when one of the above spheres (the sphere or the spherical shell) is first placed at Point A on a rough slope which is inclined to the horizontal plane by an angle θ, and then gently released, the sphere starts rolling down along the slope. Denoting the acceleration due to gravity as g, the gravitational force acts at the center of the sphere O, and in addition the normal force N and the friction force F act at the contact point P between the sphere and the slope. Angular velocity around O is denoted as ω, where the counter-clock-wise direction is defined as positive. Answer the following questions.

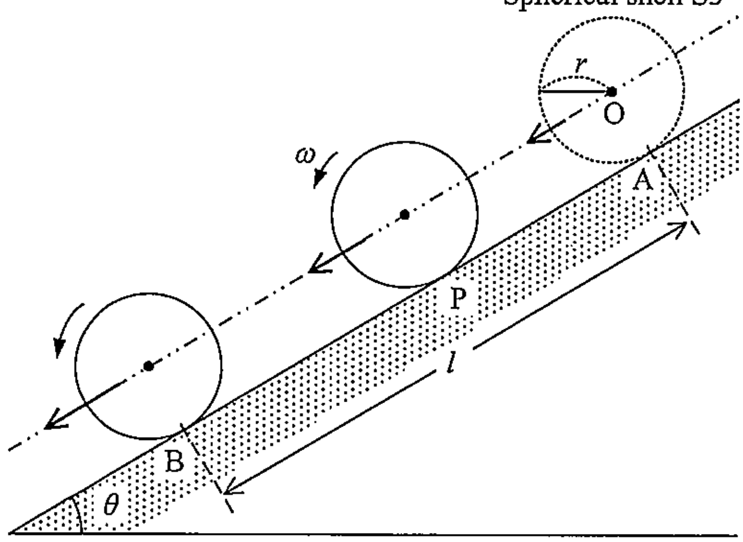

Figure 1.1

I. Derive the moment of inertia I_1 and I_3 about the axis through the center of sphere, respectively for Sphere S1, and for Spherical shell S3. Describe not only the answer but also the process by which you arrive at the answer.

II. Consider the case where Sphere S1 rolls down without slipping. The static friction coefficient between the sphere and the slope is hereafter denoted as μ_0.

1. Write down the following equations: the equation of motion for the center of gravity of the sphere, the equation of force balance in the direction perpendicular to the slope, and the equation of rotational motion around the center of gravity.
2. Obtain the acceleration of the center of gravity when Sphere S1 is rolling down along the slope. In addition, find the relation between μ_0 and θ when Sphere S1 rotates without slipping.

III. Consider the case where slippage occurs while Sphere S1 rolls down the slope. When Sphere S1 is gently released from Point A on the slope, it starts rolling down with slipping. In this case, denoting the coefficient of kinetic friction between Sphere S1 and the slope as μ ($<\mu_0$), the relation $F = \mu N$ holds. Taking the moment of release as time $t = 0$, write down the relative slipping velocity $q(t)$ of Sphere S1 with respect to the slope at the contact point P, as a function of time.

IV. Under conditions where no slippage occurs, Spheres S1, S2 and S3 are released gently from Point A separately, so as to begin rolling down along the slope. Consider Point B located at a distance l down the slope from Point A. Denoting the time required for each of the spheres S1, S2, and S3 to travel from Point A to Point B as T_1, T_2, and T_3 respectively, calculate T_2 / T_1 and T_3 / T_1.

Problem 2

As shown in Figure 2.1, a rod of radius a and height h ($h \gg a$), made of a material having electrical conductivity σ and dielectric constant ε, is placed in vacuum, and is equipped with a pair of electrodes on the top and the bottom. Straight conductors are extended from the top and the bottom electrodes, in line with the center axis of the rod, and wired to a current source and a switch at a far distance; hence the electric or the magnetic field produced by the current source, the switch, and the wiring to them can be neglected. The resistance of the electrodes and the wirings are also negligible. We ignore the electric field concentration on the electrode edges, so that the electric field can be assumed to be uniform, confined inside the rod, in the direction parallel to the rod axis. The dielectric constant of vacuum is denoted as ε_0, and the rod's magnetic permeability is equal to that of vacuum μ_0.

Answer the questions about the electric and magnetic fields created inside and outside of the rod when an electrical current flows from the top electrode to the bottom electrode.

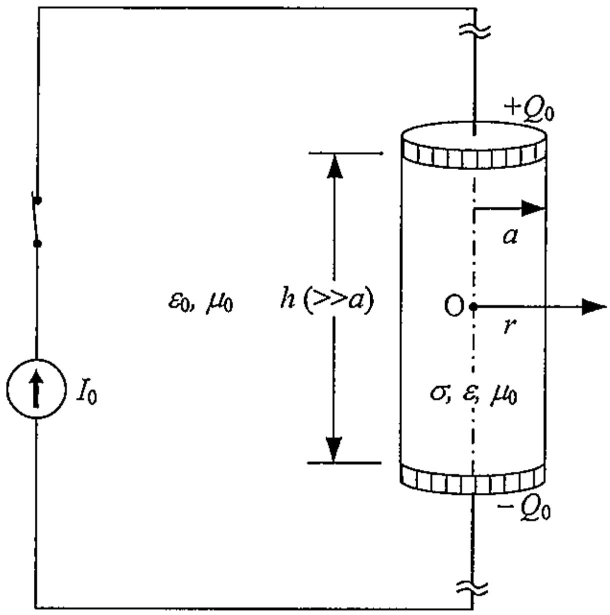

Figure 2.1

I. When a constant current I_0 flows through the rod, the top and the bottom electrodes develop electrical charges $+Q_0$ and $-Q_0$, respectively. You can assume that the electrical charge distribution is uniform on the electrodes.

1. Find the electrical resistance R between the top and the bottom electrodes.
2. Express the relationship between the electrical current density j and the electric field E in the rod.
3. Derive the relationship between the current I_0 and the electrical charge Q_0 using the Gauss's law.
4. Find the electrostatic capacitance C between the electrodes when the constant current I_0 flows.
5. Express the magnetic flux density as a function of the distance r measured from the center of the rod O and perpendicular to the axis of the rod. Consider both inside ($r \leq a$) and outside ($r > a$) the rod separately.

II. When the switch is opened at time $t = 0$, the conduction current inside the rod decays as a function of time, which can be expressed as follows.

$$I(t) = I_0 \exp\left(-\frac{t}{RC}\right) \tag{1}$$

Assuming that the current distribution in the rod is uniform, answer the following questions.

1. Express the displacement current in the rod, as a function of time.
2. Which way (upward or downward) is the direction of the displacement current? Give reasons.
3. Express the magnetic flux density as a function of the distance r measured from the center of the rod O and perpendicular to the axis of the rod. Consider both inside ($r \leq a$) and outside ($r > a$) the rod separately.

Problem 3

Consider heat processes where the working fluid is an ideal gas (gas constant is R). The molar specific heat at constant volume C_V, as well as the molar specific heat at constant pressure C_P, are assumed to be constant regardless of conditions. Assuming all state changes are quasi-static, answer the following questions. In all cases, describe the process with which you arrive at your answers.

I. We take the working fluid of one mole. We shall denote the pressure as P, the absolute temperature T, the volume V, the internal energy U, and the entropy S.

1. Prove the following formula (1).

$$dU = C_V\, dT \tag{1}$$

2. Find the entropy change dS caused by the temperature change dT and the pressure change dP.
3. Let S_0 be the entropy at the absolute temperature T_0 and the pressure P_0. Find the entropy S at the absolute temperature T and the pressure P.

II. As shown in Figure 3.1, gasses with different pressure, each in its equilibrium state, are separated by a diaphragm and contained in a thermally insulated container. Let this be State 'a'. In this state, the volume, the pressure, and the mole number of the gases (gas 1 and gas 2) in each compartment separated by the diaphragm are denoted as V_1, V_2, P_1, P_2, n_1 and n_2, respectively. It is also assumed that both compartments are maintained at the same temperature T_a. When the diaphragm is ruptured, the two gasses mix without chemical reactions and finally reach another equilibrium, which we shall call State 'b'.

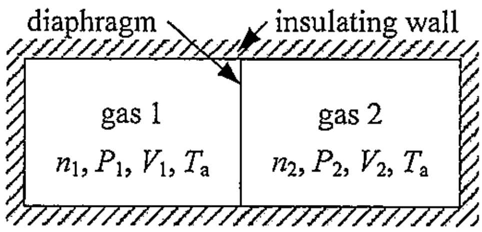

Figure 3.1

1. Express the pressure of the gas P_b in terms of P_1, P_2, n_1 and n_2, after the equilibrium of State 'b' is reached.
2. Find the entropy difference of the gas between State 'a' and State 'b', and show that the process from State 'a' to State 'b' is irreversible.

III. Answer the following questions about the Carnot cycle whose *P-V* (pressure-volume) diagram is depicted in Figure 3.2.

1. Does the entropy increase or decrease, during the process from State B to State C ? Answer the question with reasons.
2. Draw the *T-S* (temperature-entropy) diagram.
3. Derive the efficiency of the Carnot cycle using the *T-S* diagram.

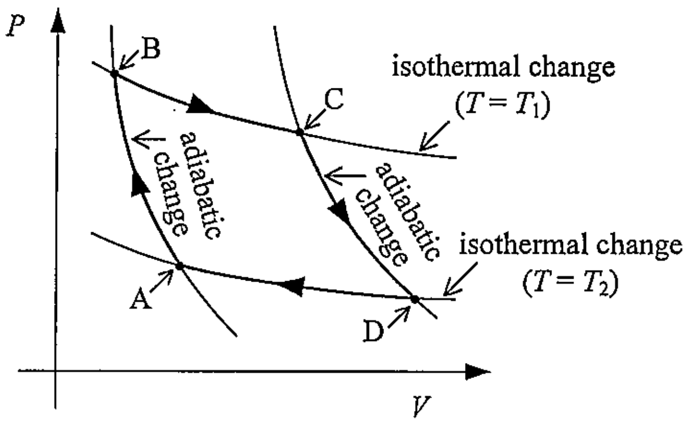

Figure 3.2

Problem 4

Consider a particle of mass m that vibrates along the x axis under a periodic external force $mf\cos\omega t$ (f is a positive constant, ω denotes angular frequency and t time). The equation of motion of this particle can be written as $m\ddot{x}+2m\mu\dot{x}+m\omega_0^2 x = mf\cos\omega t$ ($\ddot{x}=d^2x/dt^2$). Here, $-m\omega_0^2 x$ is the restoring force (the force proportional to the distance x from the origin O, ω_0 is the characteristic angular frequency) and $-2m\mu\dot{x}$ is the resisting force (the force proportional to $\dot{x}=dx/dt$, μ is a positive constant). The position of the particle $x_S(t)$ in the steady state, i.e., after a sufficient time has elapsed, can be written using its initial phase α as follows:

$$x_S(t)=\frac{f}{\sqrt{(\omega_0^2-\omega^2)^2+4\mu^2\omega^2}}\cos(\omega t-\alpha)\,,\quad \tan\alpha=\frac{2\mu\omega}{\omega_0^2-\omega^2}. \tag{1}$$

$x_S(t)$ can also be written using the elastic amplitude A_e and the absorption amplitude A_a as

$$x_S(t)=A_e\cos\omega t+A_a\sin\omega t. \tag{2}$$

In solving the problems below, the following formulae about the integrals over one period (from time τ to $\tau+T$) may be used, where T ($T=2\pi/\omega$) is the period of oscillation.

$$\frac{1}{T}\int_\tau^{\tau+T}\cos^2\omega t\,dt=\frac{1}{2},\quad \frac{1}{T}\int_\tau^{\tau+T}\sin^2\omega t\,dt=\frac{1}{2},\quad \frac{1}{T}\int_\tau^{\tau+T}\sin\omega t\cdot\cos\omega t\,dt=0. \tag{3}$$

I. Answer the following questions concerning the steady state oscillation. Here, $p(t)$ is the power supplied by the external force, and P is the time average of $p(t)$ over one period. In addition, $w(t)$ is the total mechanical energy of the particle, and W is the time average of $w(t)$ over one period.

1. Write A_e and A_a using ω, ω_0, μ, and f.
2. Making use of the definition that $p(t)$ is given by a product of the external force and $\dot{x}$, write P using all or part of m, f, ω, and A_a. In addition, briefly explain its physical interpretation in terms of the energy supplied and the energy consumed.

3. Write W using all or part of m, f, ω, ω_0, and μ.
4. P takes the maximum value P_{max} at $\omega = \omega_0$. Derive the full width at half maximum $\Delta\omega$ ($\Delta\omega = \omega_+ - \omega_-$) from the two points of ω (ω_+ and ω_-, $\omega_+ > \omega_-$) at which P becomes $P_{max}/2$ in the ω dependence of P.

II. Consider how a transient, caused by the external force starting to act at t = 0, finally reaches the steady state. The position of this particle $x_{\mathrm{T}}(t)$ is given by

$$x_{\mathrm{T}}(t) = \exp(-\mu t)(B_1 \cos\omega_1 t + B_2 \sin\omega_1 t) + x_{\mathrm{S}}(t), \tag{4}$$

where B_1 and B_2 are constants, and $\omega_1 = (\omega_0^2 - \mu^2)^{1/2}$. Answer the following questions when the resisting force is sufficiently small ($\omega_0 > \omega_1 >> \mu$). Here, the initial conditions are $x_{\mathrm{T}}(0) = 0$ and $\dot{x}_{\mathrm{T}}(0) = 0$.

1. Write B_1 and B_2 using all or part of ω, ω_1, μ, A_e, and A_a.
2. Derive an approximate solution for $x_{\mathrm{T}}(t)$ in the resonant condition ($\omega = \omega_1$).
3. Derive an approximate solution for $x_{\mathrm{T}}(t)$ when $\omega \approx 0.1\omega_0$. Draw a sketch of $x_{\mathrm{T}}(t)$ with x_{T} as the vertical axis and t as the horizontal axis.

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As depicted in Figure 1.1, a slender uniform rigid rod AB of negligible thickness, length l and mass M, rests vertically on a horizontal plane. The initial position of A is defined as the origin. Axes x and y are defined as shown. When a negligibly small velocity is given to the upper end B in the positive x direction, the rod starts to tilt. Let the center of gravity of the rod be G, and denote the acceleration of gravity as g. Answer the following questions. Neglect friction from the horizontal plane and air. For questions I to IV, you can assume that the rod end A maintains contact with the horizontal plane.

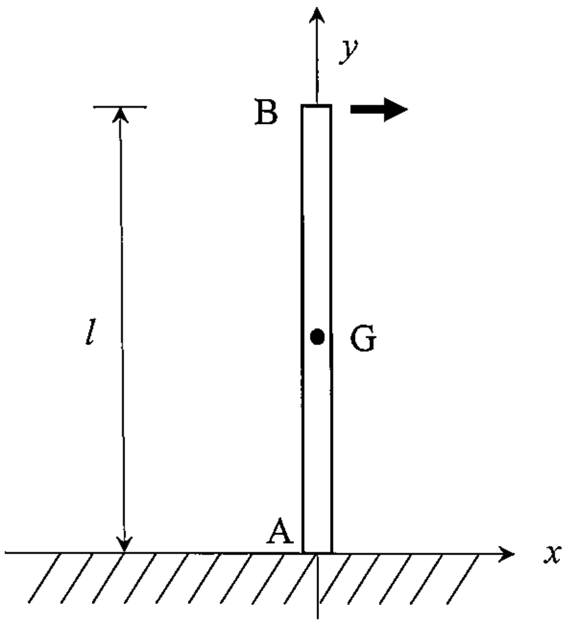


Figure 1.1

I. Show that the moment of inertia I of the rod about the axis normal to the xy-plane going through G can be expressed as $I = \frac{1}{12}Ml^2$.

II. Let θ be the angle between rod AB and the y axis. Angle θ increases with time from $\theta = 0$. Derive the equations of motion of the rod for translational and rotational motions about G. Here, normal force R acts on the rod from the horizontal plane at A.

III. Derive the differential equation for the angle θ. The equation should only include g and l.

IV. Obtain the angular velocity of the rod about G and the velocity of B just before B touches the horizontal plane.

V. Prove that the rod end A maintains contact with the horizontal plane until B touches the horizontal plane.

Problem 2

As shown in Figure 2.1, two semicircular conductive plates having radius r are held parallel to each other with distance z_0 in a vacuum environment. Permittivity of vacuum is ε_0. Let the upper and lower conductive plates be Electrode A and Electrode B, respectively. The center of the straight edges of Electrode A and Electrode B are named O_A and O_B. Electrode A can rotate around Point O_A. Let the angle formed by the two straight edges of the electrodes be θ, where $\theta = 0$ when there is no overlap between the electrodes. In this problem, consider only the electric field perpendicular to the electrode planes, and ignore the edge effects. Answer the following questions.

I. Let the angle of electrodes θ be $\pi/2$. True charges Q and $-Q$ $(Q > 0)$ are put on Electrode A and Electrode B, respectively.

1. Obtain the electric field intensity E in the area where the electrodes are overlapping.
2. Obtain the electric potential V_1 of Electrode A to Electrode B.
3. Obtain the capacitance C between Electrode A and Electrode B.
4. Angle θ is increased slowly from $\theta = \pi/10$ to $\theta = 19\pi/10$. Derive the potential V of Electrode A to Electrode B as a function of angle θ, and draw a graph of V versus θ.

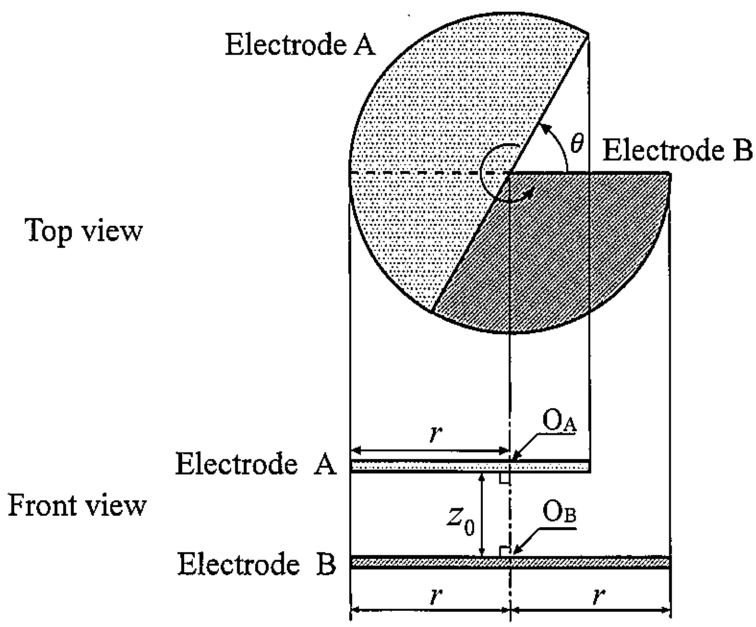


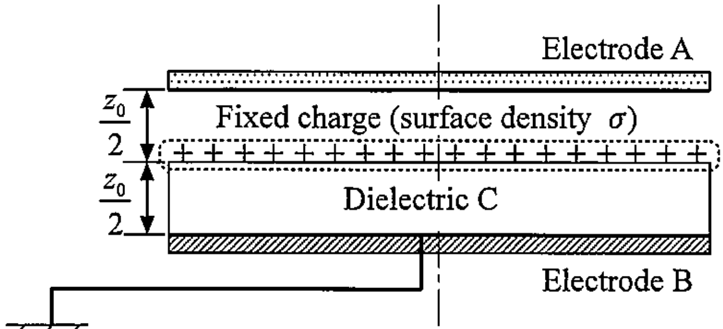
Figure 2.1

II. Charges on Electrode A and Electrode B are set to 0. Angle θ is set to $\theta = \pi$. As shown in Figure 2.2, a semicircular Dielectric C having radius r, thickness $z_0/2$ and relative permittivity k is placed directly over Electrode B. While grounding only Electrode B, true charge is fixed on the upper surface of Dielectric C with a uniform surface density $\sigma\ (\sigma > 0)$. Polarization occurs in Dielectric C and the electric field within the dielectric becomes smaller.

1. Draw electric force lines and electric flux lines in two separate front views. When drawing, assume $k = 2$ and let density of the lines reflect the values of $\varepsilon_0 E$ and electric flux density D.
2. Obtain true charges Q_A and Q_B on Electrode A and Electrode B.

III. Following the previous question, Electrode A is also connected to the ground as shown in Figure 2.3.

1. Let the induced true charges on Electrodes A and B be Q'_A and Q'_B, and the intensity of electric field between the upper surface of Dielectric C and Electrode A be E'_A and that in Dielectric C be E'_B. Express E'_A and E'_B, respectively, as functions of Q'_A and Q'_B.
2. Derive the relationship between E'_A and E'_B, and obtain Q'_A and Q'_B.
3. Draw electric force lines and electric flux lines in two separate front views. When drawing, assume $k = 2$ and let density of the lines reflect the values of $\varepsilon_0 E$ and electric flux density D.
4. Next, θ is set to $\theta = 0$. Electrode A is slowly rotated with a constant angular velocity $\omega\ (\omega > 0)$. Derive the current flowing into Electrode A as a function of time t, $I(t)$, while $0 < \theta < 2\pi$.



Front view

Figure 2.2

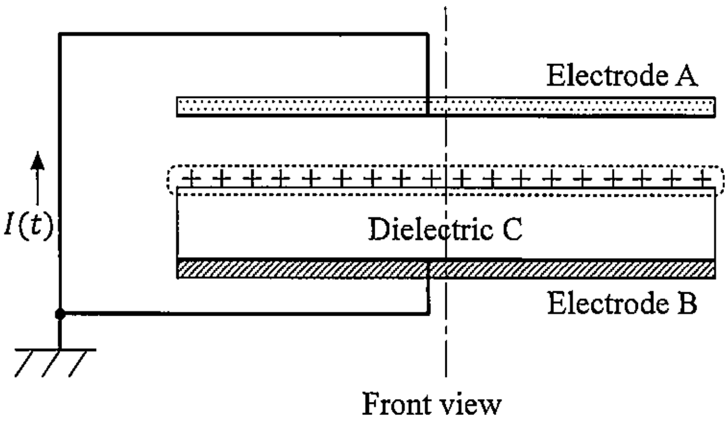


Figure 2.3

Problem 3

I. The following equation of state is applicable for an ideal gas,

$$pv = RT\,. \tag{1}$$

Here, p is the pressure, v is the volume per unit mass or specific volume, T is the temperature, and R is a constant determined by the gas species. Answer the following questions.

1. The coefficient of volumetric expansion α and the isothermal compressibility k_T are given by,

$$\alpha = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_p, \tag{2}$$

and

$$k_T = -\frac{1}{v}\left(\frac{\partial v}{\partial p}\right)_T. \tag{3}$$

For the case of an ideal gas, express α and k_T using any one of the state quantities of Equation (1). Here, subscripts p and T mean that the pressure p and the temperature T are kept constant.

2. Show that the difference between the specific heat at constant pressure c_p and the specific heat at constant volume c_v is expressed as,

$$c_p - c_v = \frac{vT\alpha^2}{k_T}, \tag{4}$$

for gases in general, including an ideal gas. Here, c_p and c_v are given by,

$$c_p = T\left(\frac{\partial s}{\partial T}\right)_p, \tag{5}$$

and

$$c_v = T\left(\frac{\partial s}{\partial T}\right)_v, \tag{6}$$

where s is the entropy per unit mass. Subscript v denotes that specific volume v is kept constant. Use the following Maxwell's equations,

$$\left(\frac{\partial v}{\partial T}\right)_p = -\left(\frac{\partial s}{\partial p}\right)_T, \tag{7}$$

$$\left(\frac{\partial p}{\partial T}\right)_v = \left(\frac{\partial s}{\partial v}\right)_T, \tag{8}$$

and the chain rule,

$$\left(\frac{\partial p}{\partial v}\right)_T \left(\frac{\partial v}{\partial T}\right)_p \left(\frac{\partial T}{\partial p}\right)_v = -1. \tag{9}$$

3. Consider a system composed of an ideal gas, where the state of the system changes quasi-statically in a reversible process from the thermodynamic equilibrium State 1 (p_1, v_1, T_1) to another thermodynamic equilibrium State 2 (p_2, v_2, T_2). Express the change in s using c_v, v_1, v_2, R, T_1, and T_2. Assume c_v to be constant.
4. Assuming that the reversible change from State 1 to State 2 is adiabatic in the previous question I.3, express T_2/T_1, using v_1, v_2, and the ratio of specific heats $\kappa = c_p/c_v$.

II. Consider the thermal efficiency of a quasi-static cycle for an ideal gas. Here, q_A, q_B, q_C, q_D are heat supplied per unit mass, and q_E, q_F, q_G are heat exhausted per unit mass. Assume that the specific heat at constant pressure c_p and the specific heat at constant volume c_v are constant. Answer the following questions.

1. Cycle A depicted as a $p-v$ diagram in Figure 3.1, consists of four reversible processes; adiabatic change 1→2, isovolumetric change

$2 \to 2'$, adiabatic change $2' \to 4$, and isovolumetric change $4 \to 1$. Express the thermal efficiency $(q_A - q_E)/q_A$ of this cycle, using the compression ratio $\varepsilon = v_1/v_2$ and the ratio of specific heats $\kappa = c_p/c_v$.

2. Cycle B depicted in Figure 3.2, consists of four reversible processes; adiabatic change $1 \to 2$, isobaric change $2 \to 3$, adiabatic change $3 \to 4$, and isovolumetric change $4 \to 1$. Express the thermal efficiency $(q_B - q_F)/q_B$ of this cycle, using ε, κ, and the cut-off ratio $\sigma = v_3/v_2$.
3. Cycle C depicted in Figure 3.3, consists of five reversible processes; adiabatic change $1 \to 2$, isovolumetric change $2 \to 2'$, isobaric change $2' \to 3$, adiabatic change $3 \to 4$, and isovolumetric change $4 \to 1$. Express the thermal efficiency $(q_C + q_D - q_G)/(q_C + q_D)$ of this cycle, using ε, κ, σ, and the pressure rise ratio $\rho = p_3/p_2$.
4. Among the three cycles A, B and C mentioned above, name the cycles with the largest and the smallest thermal efficiency under the same compression ratio ε. Give reasons. Let $\varepsilon > 2$, $\kappa = 4/3$, $\rho > 1$, $\sigma = 2$, and $2^{1/3} = 1.26$.

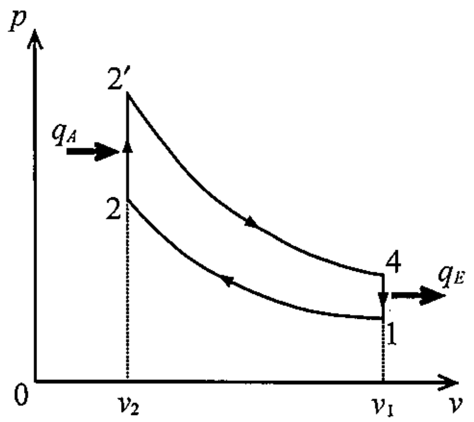


Figure 3.1

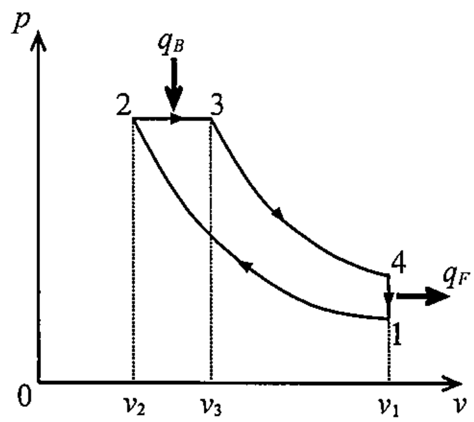


Figure 3.2

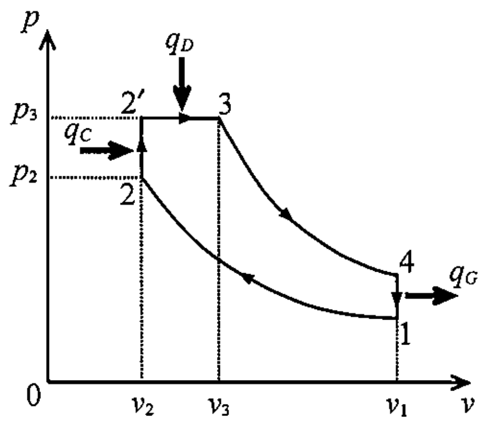


Figure 3.3

Problem 4

I. Consider the vibration in a one dimensional ring lattice consisting of N atoms with mass M_0, interconnected by springs with spring constant K_s. N is sufficiently large, and neighboring atoms can locally be modeled to be in a straight line as shown in Figure 4.1. The distance between the neighboring atoms at equilibrium is given as a. Answer the following questions.

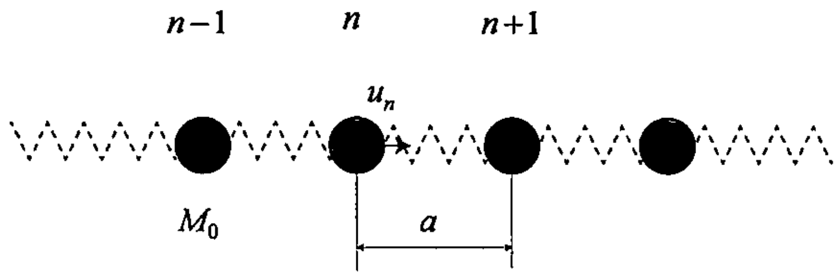


Figure 4.1

1. Displacement of the n^{th} atom from its equilibrium is denoted as u_n. Express the equation of motion of the atom using M_0, K_s, u_n, u_{n+1}, and u_{n-1}. Force and displacement are defined to be positive in the rightward direction in Figure 4.1.
2. The general solution of the equation of motion in question I.1 is given as,

$$u_n = u \exp\{-i(\omega t - kna)\}, \tag{1}$$

where u is the amplitude of vibration of each atom, ω is the angular frequency, and k is the wavenumber. Using this general solution, derive the equation giving the relation between ω and ka.

Next, consider the vibration in a one dimensional ring lattice consisting of two species of atoms with masses M_1 and M_2, alternately interconnected with springs of spring constant K_s as shown in Figure 4.2. The atoms can locally be modeled to be in a straight line. The distance between the neighboring atoms at equilibrium is given as a. Answer the following questions.

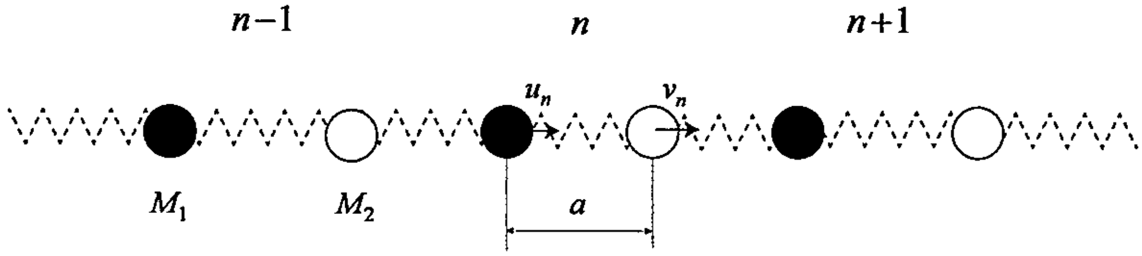


Figure 4.2

3. Displacement of the n^{th} atom with mass M_1 from its equilibrium is denoted as u_n, and that with mass M_2 as v_n. Express the two equations of motion for the two atoms using M_1, M_2, K_s, u_n, u_{n+1}, u_{n-1}, v_n, v_{n+1}, and v_{n-1}. Force and displacement are defined to be positive in the rightward direction in Figure 4.2.
4. Derive the general solution for the equation obtained in the previous question. Here, let the amplitudes of u_n and v_n be u and v, respectively.
5. Derive the equation giving the relation between ω^2 and ka.

II. In quantum mechanics, the harmonic oscillation for a one dimensional lattice as introduced in question I is given as the solution of the following Schrödinger equation,

$$\left\{-\frac{\hbar^2}{2M_0}\frac{d^2}{dx^2}+\frac{1}{2}M_0\omega^2x^2\right\}\varphi(x)=E\varphi(x), \tag{2}$$

where $\varphi(x)$ is the wavefunction, x is the atomic position, E is the eigenvalue, and $\hbar = h/2\pi$ where h is the Planck's constant. The wavefunctions of the ground state and the first excited state of this Schrödinger equation are given by,

$$\varphi_0(x)=C_0\exp\left(-\frac{M_0\omega}{2\hbar}x^2\right), \tag{3}$$

$$\varphi_1(x)=C_1\sqrt{\frac{M_0\omega}{\hbar}}\,x\exp\left(-\frac{M_0\omega}{2\hbar}x^2\right), \tag{4}$$

where C_0 and C_1 are the normalizing constants.

1. Derive the eigenvalue for the ground state E_0 and that for the first excited state E_1 using $\varphi_0(x)$ and $\varphi_1(x)$.
2. Show that the expectation values of the atomic position $\langle x \rangle$ and momentum $\langle p \rangle$ become 0 both in the ground state and the first excited state.

2018
The Graduate School Entrance Examination
Physics
1:00 pm－3:00 pm

GENERAL INSTRUCTIONS

Answers should be written in Japanese or English.

1. Do not open the problem booklets, whether in English or Japanese, until the start of the examination is announced.

2. Notify your proctor if you find any printing or production errors.

3. Answer two problems out of the four problems in the problem booklet.

4. You are given two answer sheets. Use one answer sheet for each problem. You may use the reverse side if necessary.

5. Fill in your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of each answer sheet represent the problem number that you answer (P 1, P 2, P 3, P 4) and also the class of the course (master M, doctor D) that you are applying. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks per sheet.

6. You may use the blank sheets of the problem booklet for rough papers without detaching them.

7. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.

8. You may not take the booklet or answer sheets with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

I. Figure 1.1 shows a rocket flying upward in the vertical $+z$ direction. The rocket is propelled by expulsion of a gas in the opposite direction. The gas is expelled with constant velocity u relative to the rocket in the direction opposite to the advancing direction of the rocket. Except for the propulsive force, assume that all external forces such as air resistance and gravitation do not apply to the rocket. Answer the following questions.

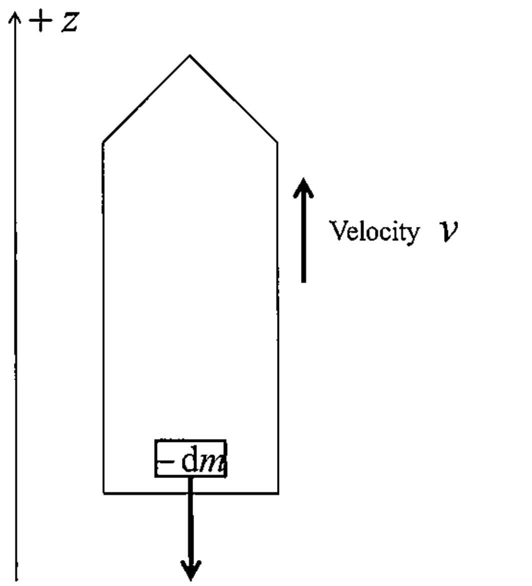


Figure 1.1

1. During a short interval $\mathrm{d}t$, from t to $t+\mathrm{d}t$, the rocket ejects some gas, and the velocity of the rocket increases from v to $v+\mathrm{d}v$. During this time, the mass of the rocket changes from m to $m+\mathrm{d}m$ ($\mathrm{d}m<0$). The momentum of the rocket at time t is equal to the sum of the momentum of the rocket at time $t+\mathrm{d}t$, and the momentum of the gas (with the mass of $-\mathrm{d}m\,(>0)$) expelled during $\mathrm{d}t$. Under these conditions, find the relational expression between $\mathrm{d}v$, $\mathrm{d}m$, m and u. You may neglect the second-order terms of small values, such as $\mathrm{d}v\cdot\mathrm{d}m$.
2. The rocket continues to expel the gas for a long time, and the mass of the rocket decreases from m_i to m_f. Find the total increase in the velocity of the rocket.

II. Now assume that gravitational acceleration g applies to the rocket downward in the vertical $-z$ direction in Question I. Answer the following questions.

1. Find the time-rate-of-change in the velocity of the rocket v as a function of t on the basis of change in the momentum at short interval $\mathrm{d}t$. Here, the gas is expelled at a constant rate, such that m can be described by the following equation

$$m = m_i(1-kt), \tag{1}$$

where k is a positive constant.

2. The rocket continues to expel the gas for a long time, and then the velocity of the rocket increases from v_i to v_f as the mass of the rocket decreases from m_i to m_f. Find an expression for v_f.

III. Figure 1.2 shows the rocket flying in the upward vertical $+z$ direction. Assume that the rocket moves in the zx-plane. The axis of the rocket is tilted at an angle θ against the upward vertical $+z$ direction. Forces $L = K_L\theta$ and $D = K_D\theta$ are applied to the rocket along the horizontal ($+x$) and the vertical ($-z$) directions, respectively, at a position which is at distance ℓ_1 on the axis in front of the center of gravity G of the rocket. K_L and K_D are positive constants, respectively. θ is initially very small but non-zero. Additionally, the rocket engine is set at the bottom of the rocket on the axis (at a distance ℓ_2 from G), and provides a force of constant magnitude F to the terminal end of the rocket. The angle of this force can be changed to stabilize the rocket. The angle between F and the axis of the rocket is δ, as shown in Figure 1.2. Herein, the moment of inertia around the center of gravity G of the rocket along the axis perpendicular to the zx-plane can be expressed by I, which is assumed to be independent of time. Providing that θ and δ are very small, $\sin\theta$ and $\sin\delta$ can be approximated to $\sin\theta \cong \theta$ and $\sin\delta \cong \delta$, respectively, while $\cos\theta$ and $\cos\delta$ can be approximated to $\cos\theta \cong 1$ and $\cos\delta \cong 1$, respectively. Answer the followings.

1. Derive an equation of rotational motion for the rocket, and find a second-order differential equation of θ as a function of time t. Neglect terms of θ^2.

2. Show that $|\theta|$ increases with time when $\delta = 0$.
3. When δ is controlled such that $\delta = \alpha\theta$, find the condition on the constant α such that θ converges to zero as $t \to \infty$.
4. When δ is controlled such that $\delta = \alpha\theta + \beta(\mathrm{d}\theta / \mathrm{d}t)$, find the condition on the constants α and β such that θ converges to zero as $t \to \infty$.

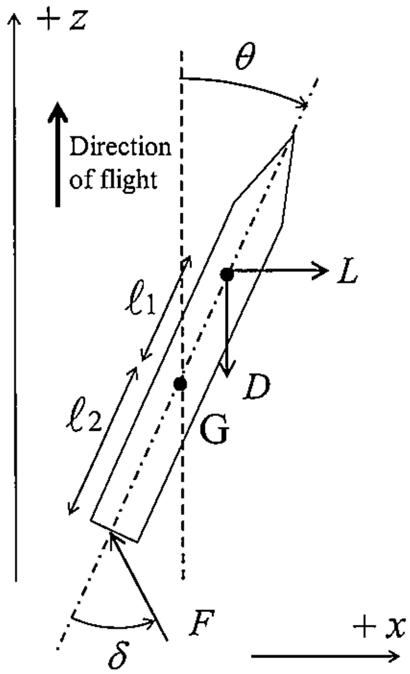


Figure 1.2

Problem 2

As shown in Figure 2.1, a parallel plate capacitor is placed in vacuum. The length and width of the capacitor are a and b, respectively. The gap between the two metal electrodes is d, which is much smaller than both a and b $(d \ll a, b)$. Let the permittivity of the vacuum be ε_0. A dielectric of permittivity $\varepsilon\ (> \varepsilon_0)$ is inserted in the gap. The length, width, and thickness of the dielectric are $a/2, b$, and d, respectively. The position of the left edge of the dielectric is labeled by x $(0 \le x < a)$, as shown in Figure 2.1. This dielectric can be moved in the range of $0 \le x < a$ without friction. When $0 \le x \le a/2$, the dielectric fits completely between the electrodes. On the other hand, when $a/2 < x < a$, the dielectric partly lies outside of the electrodes.

As shown in Figure 2.2, one can switch the connection between this capacitor and terminals A, B, or C for selecting one of three types of circuit. Answer the following questions. Neglect the capacitance, inductance, and resistance except for those shown in Figure 2.2.

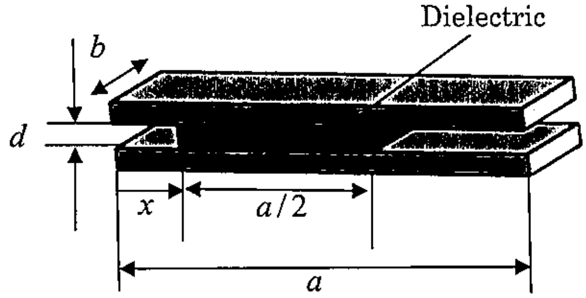


Figure 2.1

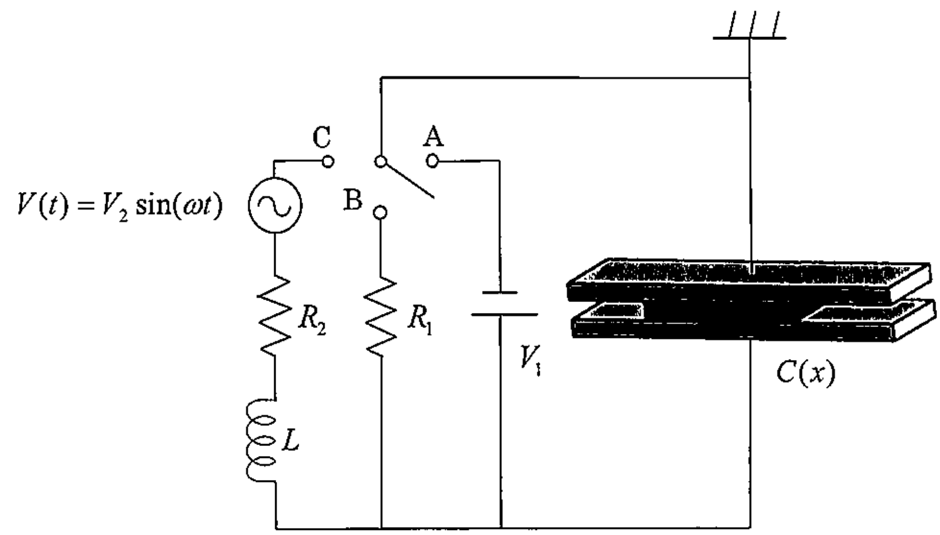


Figure 2.2

I. The switch is connected to terminal A and left for a long time. The constant DC voltage across the capacitor is V_1.

1. Derive the capacitance of the capacitor $C(x)$ as a function of x within the range of $0 \le x < a$. Use what you need from $x, \varepsilon_0, \varepsilon, a, b, d, V_1$.
2. Derive the energy stored in the capacitor $U(x)$ as a function of x within the range of $0 \le x < a$. Use what you need from $x, \varepsilon_0, \varepsilon, a, b, d, V_1$.
3. Derive the change of charge $\Delta Q(x)$ in the capacitor when the dielectric is slowly moved over a distance Δx within the range of $a/2 < x < a$. Use what you need from $\Delta x, x, \varepsilon_0, \varepsilon, a, b, d, V_1$.
4. Derive the force $F_1(x)$ acting on the dielectric along the horizontal direction as a function of x. Use what you need from $x, \varepsilon_0, \varepsilon, a, b, d, V_1$. Let the range of x be $0 \le x < a$. The direction of the force is defined as positive when the force acts on the dielectric such that the value of x increases (such that the dielectric moves towards the right in Figure 2.1).

II. The dielectric is set at $x = 3a/4$ and the switch is connected to terminal A. After a long time, the switch is switched from terminal A to terminal B. Let the time be $t = 0$ when the switch is switched. The resistance in the connected circuit is R_1. Derive the force acting on the dielectric $F_2(t)$ as a function of t. Use what you need from $t, x, \varepsilon_0, \varepsilon, a, b, d, V_1, R_1$. The direction of positive force is defined as in Question I.4. Let R_1 be large enough such that the magnetic field induced by the displacement current in the capacitor can be neglected.

III. The switch is connected to terminal C. The resistance in the connected circuit is R_2 and the inductance of the coil is L. The AC voltage of the electric power source is given by $V(t) = V_2 \sin(\omega t)$, where ω is the angular frequency.

1. The effective value of the power consumption, $P_2(x)$, in the resistor R_2 can be expressed as a function of x. Derive the condition of L for $P_2(x)$ to have a local maximum in the range of $0 \le x < a$. Use what you need from $\varepsilon_0, \varepsilon, a, b, d, V_2, R_2, \omega$.
2. Sketch a graph of the voltage waveform applied on the capacitor when $P_2(x)$ becomes maximum under the condition derived in Question III.1. Also sketch the voltage waveform of the electric power source on the same graph. Explain the physics behind the relation between the two waveforms.

Problem 3

I. Consider two solid objects, a and b, at temperatures T_1 and T_2, respectively, which have the same heat capacity C and are isolated from the outer environment. When object a is brought into contact with object b, the temperatures of these objects become the same, and the system reaches a thermal equilibrium state. Note that the volume of these objects does not change after the contact. Calculate the temperature T_f in the thermal equilibrium state and the change in entropy of this system, ΔS, after the objects are brought into contact. Prove that the entropy increases $(\Delta S > 0)$.

II. Consider an irreversible heat engine A and a reversible heat pump B, which operate between two heat reservoirs with constant temperatures as in Figure 3.1. The irreversible heat engine A absorbs heat Q_2^A from the high-temperature heat reservoir R_2, performs work W, and releases heat Q_1^A to the low-temperature heat reservoir R_1 in one cycle. The reversible heat pump B absorbs heat Q_1^B from the low-temperature heat reservoir R_1 through the work W generated by the irreversible heat engine A, and provides heat Q_2^B to the high-temperature heat reservoir R_2. Here, the efficiency of the irreversible heat engine and the reversible heat pump are denoted as $\eta_A(=W/Q_2^A)$ and $\eta_B(=W/Q_2^B)$, respectively. From the following relationships (a)-(c), choose all which do not satisfy the second law of thermodynamics, and explain why.

(a) $\eta_A < \eta_B$, (b) $\eta_A = \eta_B$, (c) $\eta_A > \eta_B$.

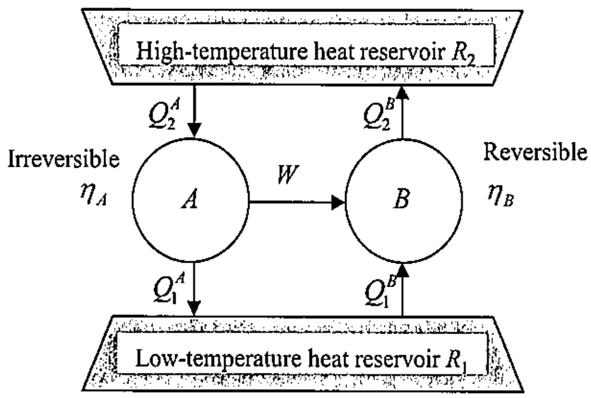


Figure 3.1

III. Consider a closed vessel filled with a photon gas at temperature T in an isolated environment. In a thermal equilibrium state, the pressure p of the photon gas, which is independent of volume V, is described with the following equation

$$p = \frac{1}{3} a T^4 , \tag{1}$$

where a is a positive constant. Answer the following questions.

1. Explain why this photon gas does not have a heat capacity at constant pressure.
2. When the volume of the vessel V is quasi-statically changed at a constant temperature, according to the first law of thermodynamics, Equation (2) is true, where U is the internal energy of the photon gas and S is its entropy. The change in the internal energy is described with Equation (3). Prove Equation (3) using the first law of thermodynamics. You may use the Maxwell relation as in Equation (4).

$$dU = TdS - pdV . \tag{2}$$

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p . \tag{3}$$

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V . \tag{4}$$

3. Calculate the internal energy $U(T,V)$ and entropy $S(T,V)$ of the photon gas in the thermal equilibrium state. Here, the internal energy and the entropy are both zero at absolute zero $(T = 0)$.
4. Figure 3.2 shows a Carnot cycle in which the working medium is a photon gas. This cycle consists of four processes: isothermal expansion (A→B), adiabatic expansion (B→C), isothermal compression (C→D), and adiabatic compression (D→A). The amount of heat that the system absorbs in the isothermal expansion process at temperature T_2 is denoted as Q_2, and the amount of heat that the

system releases in the isothermal compression process at temperature $T_1(<T_2)$ is denoted as Q_1. Calculate Q_2 and Q_1. Here, the quantity of heat is defined as positive when the system absorbs heat.

5. Show that the Clausius equality (5) is true in this case.

$$\frac{Q_2}{T_2}+\frac{Q_1}{T_1}=0 \tag{5}$$

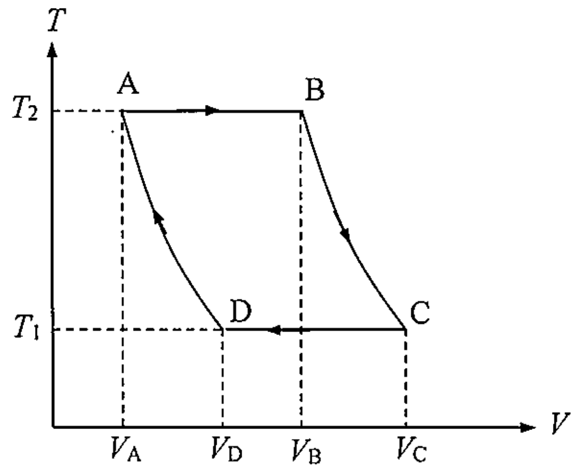


Figure 3.2

Problem 4

Answer the following questions about the wave nature of light and particles and also relativistic effects.

I. As shown in Figure 4.1, light passing through a single slit and then double slits propagates towards a screen. Interference fringes are generated on the screen. Answer the following questions. Here, the widths of all slits are much smaller than the wavelength of the light.

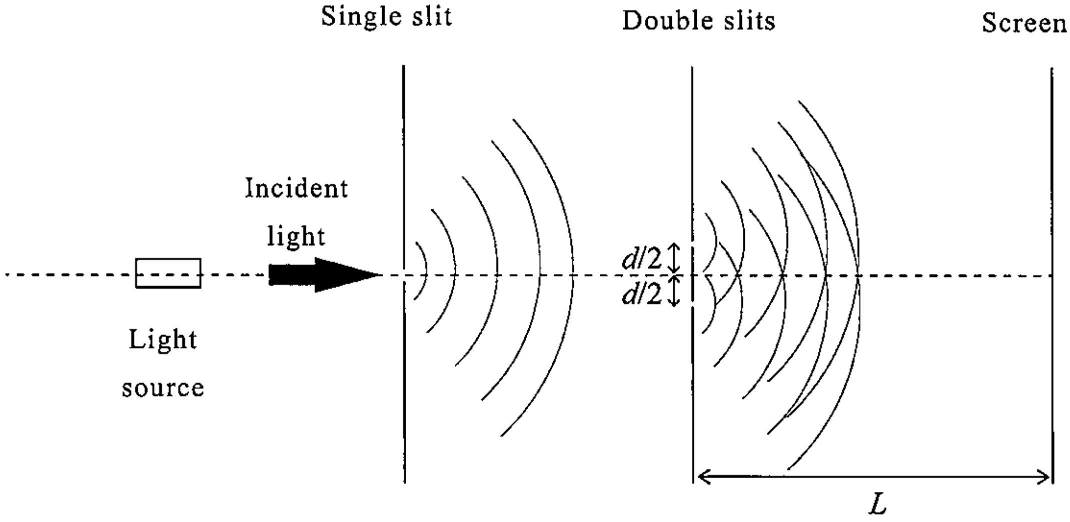


Figure 4.1

1. The incident light source produces monochromatic light with a visible wavelength of λ. The separation of the double slits is d, as shown in Figure 4.1. The distance between the double slits and the screen is L. Under these conditions, express the interval x of the interference fringes on the screen, using λ, d, and L. Here, you can assume that L is much larger than d and λ.
2. Next, the incident light source is changed from the monochromatic one to a white one. Explain the differences between the pattern on the screen in the case of monochromatic light and white light. You may draw figures if needed.

II. Instead of light, now consider that an electron beam passes through the double slits and propagates towards the screen shown in Figure 4.1. In quantum mechanics, electrons have wave-particle duality. Therefore, as with light, interference fringes of electrons appear on the screen. The wavelength λ of electrons is given by $\lambda = h/p$, where h is Planck's constant and p is the momentum of the electrons. Answer the following questions.

1. The mass and the charge of an electron are m and e, respectively. Express the wavelength λ of the electron accelerated by an electric potential difference V in vacuum, using m, e, h, and V. Here, you can ignore any relativistic effects.
2. Calculate the value of the electric potential difference V necessary for generating an electron with a wavelength of 2.0 Å, using the equation that you derived in Question II.1. Use the following values for your calculation.

Planck's constant $h = 6.6 \times 10^{-34}$ [J s]
Charge of electron $e = 1.6 \times 10^{-19}$ [C]
Mass of electron $m = 9.1 \times 10^{-31}$ [kg]

III. As the velocity of accelerated electrons approaches the velocity of light, relativistic effects cannot be ignored. Answer the following questions. Here, Planck's constant and charge of electron is h and e, respectively.

1. (x, y, z) and (x', y', z') are defined as coordinate systems in two inertial frames, S and S', respectively. t and t' are times in S and S', respectively. Now, the two coordinate systems move in parallel. At times $t = t' = 0$, the coordinate origin and clock of S' exactly coincide with those of S. S' is moving at a constant velocity v in the positive direction along the x-axis when seen from S. The coordinate transformations between S and S' are defined with the following equations

$$x' = \alpha(x - vt), \quad y' = y, \quad z' = z. \tag{1}$$

$$x = \alpha(x' + vt'), \quad y = y', \quad z = z'. \tag{2}$$

A pulse of light is emitted from the origin in the positive direction along the x-axis at times $t = t' = 0$. The speed of light c is invariant in both inertial frames. By considering the positions of the light at time t in S and at time t' in S', express the positive coefficient α in terms of v and c.

2. Considering relativistic effects, the momentum p of the electron with uniform motion at a velocity v is expressed as

$$p = \alpha m_0 v, \tag{3}$$

where the coefficient α is that which was obtained in Question III.1, and m_0 is the static mass of the electron. The total energy E of this electron is expressed as

$$E^2 = m_0^2 c^4 + p^2 c^2. \tag{4}$$

Here, the rest energy is defined as $m_0 c^2$.

Consider an electron with a constant velocity after being accelerated by an electric potential difference V in vacuum. Express the wavelength λ and the velocity v of this electron, using all or some of c, m_0, e, V, and h. Here, you must take relativistic effects into account.

2019
The Graduate School Entrance Examination
Physics
1:00 pm－3:00 pm

GENERAL INSTRUCTIONS

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3. Answer two problems out of the four problems in the problem booklet.

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5. Fill in your examinee number and the problem number in the designated places at the top of each answer sheet. The wedge-shaped marks on the top edge of each answer sheet represent the problem number that you answer (P 1, P 2, P 3, P 4) and also the class of the course (master M, doctor D) that you are applying. At the end of the examination, follow your proctor's instructions and cut out carefully the two corresponding wedge marks per sheet.

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Examinee Number	No.

Write your examinee number in the space provided above.

Problem 1

I. Consider a pulley of radius a and mass M having a uniform volume density, which can rotate freely without friction around the fixed point O, as shown in Fig. 1.1. Weight 1 of mass m_1 and weight 2 of mass m_2 are suspended from the pulley by a string, and then gently released. Assume that the pulley and weights are stationary at the initial state. g is the acceleration due to gravity, and we assume that $m_2 > m_1$. Mass, thickness, and elongation of the string, as well as the dimension of weights are negligible. There is no slipping between the pulley and the string. Answer the following questions.

1. Derive the moment of inertia of the pulley around the fixed point O. Also write the derivation process.
2. When the weight is released at time $t=0$, derive the velocity of weight 1 along the y direction, as a function of time t.

II. A cylinder of mass M, outer diameter $2a$, and inner diameter a rotates with angular velocity ω_0 around the central axis, as shown in Fig. 1.2. Consider the motion when the cylinder is gently put on a horizontal plane with rough surface. Answer the following questions. Here, the cylinder is a rigid body having a uniform volume density, the coefficient of kinetic friction between the cylinder and the horizontal plane is μ, and the acceleration due to gravity is g. Also, the energy loss of the cylinder when rolling on the plane without slipping is negligible.

1. Derive the moment of inertia of the cylinder around the central axis. Also write the derivation process.
2. When the cylinder is put on the plane at time $t=0$, it starts rolling with slipping. Obtain the velocity of the cylinder's center of gravity and the angular velocity around the cylinder's center of gravity, as functions of time t, when the cylinder is rolling with slipping.
3. Derive the time t_1 and distance x_1 which are required for the cylinder to start rolling without slipping.
4. A rough slope forming an angle θ with the horizontal plane is located far from the distance x_1. Calculate the maximum height to which the cylinder can rise by rolling without slipping. Assume that

θ is small enough that the cylinder can move from the horizontal plane to the slope while maintaining contact.

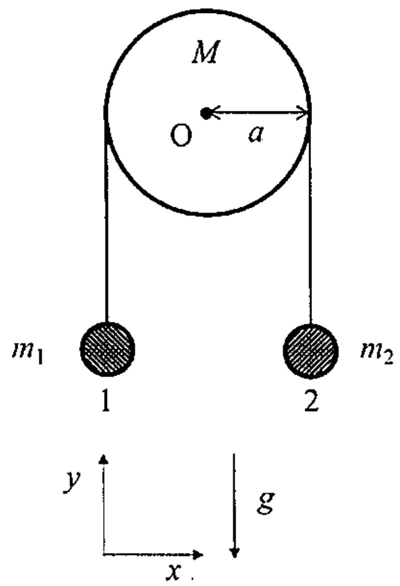

Figure 1.1

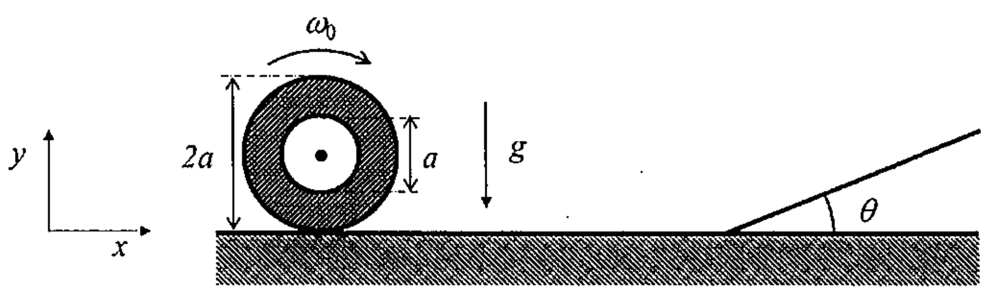

Figure 1.2

Problem 2

Consider electric and magnetic fields induced by electric charges, magnetization, and currents in vacuum. Let the dielectric constant and magnetic permeability of vacuum be ε_0 and μ_0, respectively. You can neglect gravity. Answer the following questions.

I. As shown in Fig. 2.1, electric charges are distributed over a disk with a hole (inner radius: a, outer radius: b) with a uniform surface density $-\sigma\,(\sigma > 0)$ in the xy plane. Let the center of the disk be O.

1. Find the electric potential and field at the point $\mathrm{P}(0, 0, z)$.
2. Express the electric field obtained in I.1 in the case where $z \gg b$ and briefly explain the physical meaning of this expression.
3. Show that the electric field obtained in I.1 is equal to that induced by the infinite plane sheet of uniform electric charge when $a \to 0$ and $b \to \infty$.
4. A particle with negative electric charge $-q\,(q > 0)$ and mass m is put at the point $\mathrm{C}(0, 0, c)\,(c > 0)$ which is very far from O and given an initial velocity of $\overrightarrow{v_0} = (0, 0, -v_0)\,(v_0 > 0)$. Find the z-coordinate of the particle when it is closest to O. Here, v_0 is small enough that the z-coordinate of the particle z_c is always much larger than b $(z_c \gg b)$ during the motion of the particle. After a sufficient period of time has elapsed (i.e. time $t \to \infty$), the velocity of the particle converges towards a certain $\overrightarrow{v_c}$. Find $\overrightarrow{v_c}$.
5. A particle with positive electric charge $q\,(q > 0)$ and mass m is put at the point $\mathrm{D}(0, 0, d)\,(d \approx 0)$ near O and then gently released. Explain the motion of this particle and find its period. You can neglect electromagnetic wave emission from the particle during the motion.

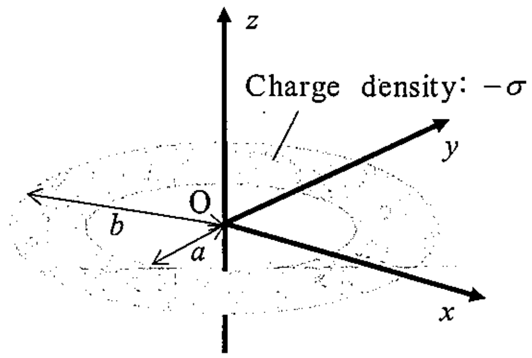

Figure 2.1

II. Consider a magnetic cylinder with an inner radius a, outer radius b, thickness t and rectangular cross section in vacuum.

1. As shown in Fig. 2.2, a magnetic cylinder is magnetized along the central axis of the cylinder with constant magnitude M. Explain in what part of the cylinder the magnetization currents (the currents inducing magnetization) flow and in what directions.
2. As shown in Fig. 2.3, a magnetic cylinder is magnetized along the circumferential direction of the cylinder with constant magnitude M. Find the magnetic field and magnetic flux density in and outside the cylinder.
3. As shown in Fig. 2.4, a circular solenoid is formed by winding a conductive wire uniformly around this magnetic cylinder (total number of coils: N). Find the total magnetic flux penetrating the cross section of the magnetic cylinder when the current I flows in the solenoid. We assume that the cylinder is not magnetized before the current flows and neglect leakage of the magnetic flux. Let the magnetic permeability of the cylinder be μ.
4. Assume the magnetic flux density and magnetization are uniform in the circular solenoid. Find the magnetic flux density and magnetization in the cylinder using the average circumference of $2\pi(a+b)/2$. In addition, show that this magnetic flux density is equal to that calculated from the magnetic flux in II.3, when the inner radius is much larger than the difference between inner and outer radii (i.e. $a \gg b-a$) by considering the assumption that $a \gg b-a$ and the

magnetic flux density is uniform in the cylinder. If necessary, you may use the approximation $\ln(1+x) \approx x$ when $|x| \ll 1$.

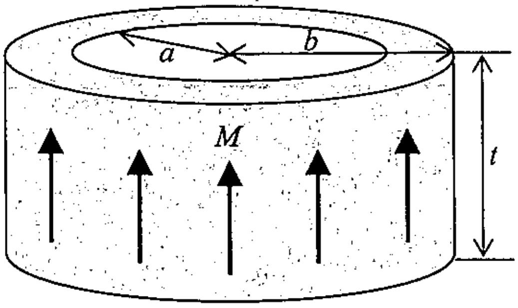

Figure 2.2

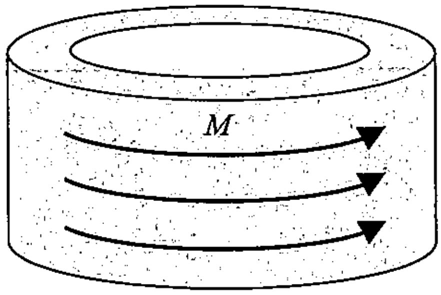

Figure 2.3

Total number of coils: N

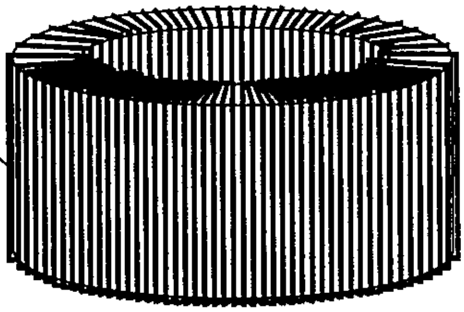

Figure 2.4

Problem 3

I. The following equation shows the first law of thermodynamics.

$$d'Q + d'W = dU. \tag{1}$$

Here Q is the amount of heat supplied to the system, W is the amount of work done on the system, and U is the internal energy. The specific heat at constant volume, C_V, is expressed as,

$$C_V = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V. \tag{2}$$

Here, n, V, and T are the number of moles, the volume, and the temperature, respectively. Let P and C_P be the pressure and the specific heat at constant pressure, respectively. Answer the following questions.

1. Show that the following relation holds true for an ideal gas in a quasi-static adiabatic process.

$$nC_V dT + PdV = 0. \tag{3}$$

2. Show that the following relation holds true for an ideal gas in a quasi-static adiabatic process using equation (3).

$$TV^{\gamma-1} = \text{constant}, \tag{4}$$

where γ is the ratio of specific heat ($\gamma = C_P / C_V$). You may use the fact that the relationship between C_P and C_V can be expressed as $C_P - C_V = R$ (R: the gas constant) for an ideal gas.

II. An ideal gas of n moles is contained in a cylinder with a constant cross-section Z and the gas is manipulated according to the quasi-static thermal cycle, A→B→C→D→A, depicted in Fig. 3.1. Here, P, V, and T are the pressure, the volume, and the temperature, respectively. The processes A→B and C→D are isothermal and the processes B→C and D→A are adiabatic.

(Volume, Temperature) of the gas at state A, B, C, and D, are (V_1, T_1), (V_2, T_1), (V_3, T_2), and (V_4, T_2), respectively. Assume that the friction between the cylinder and the piston is negligible during the adiabatic processes and isothermal expansion process, whereas a constant friction force f is applied between the cylinder and the piston as shown in Fig. 3.2 during the isothermal compression process (C→D). All the work done by the friction force is absorbed by the contained ideal gas. Here R is the gas constant. Answer the following questions.

1. Find the amount of heat absorbed by the gas, Q_1 (>0), during the process A→B.
2. Find the amount of heat absorbed by the gas, Q_2 (<0), during the process C→D.
3. Find the efficiency η ($\eta=(Q_1+Q_2)/Q_1$) of the cycle A→B→C→D→A, using n, R, f, Z, T_1, T_2, V_3, and V_4.

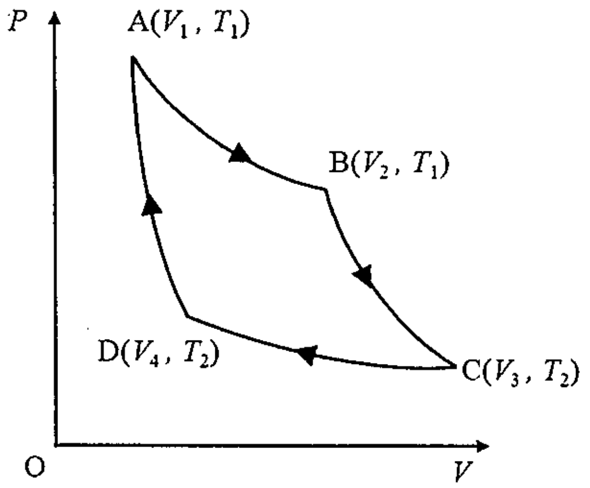

Figure 3.1

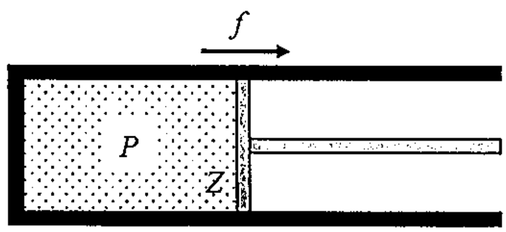

Figure 3.2

III. Consider a reversible cell, where the volume change is negligible during the reaction and the work is done only by the charges. The amount of work done by the cell is expressed as Edq, with electromotive force E and charge q. In the reversible cell, the first law of thermodynamics is expressed as,

$$dU = TdS - Edq, \tag{5}$$

where U is the internal energy, T is the temperature, and S is the entropy. The Helmholtz free energy is defined by

$$F = U - TS. \tag{6}$$

Answer the following questions.

1. Show that the following relations between Helmholtz free energy F, electromotive force E, and entropy S hold true. You may consider that F can be expressed with independent variables q and T.

$$E = -\left(\frac{\partial F}{\partial q}\right)_T, \tag{7}$$

$$S = -\left(\frac{\partial F}{\partial T}\right)_q. \tag{8}$$

2. Show that the following relation about the internal energy change holds true.

$$\left(\frac{\partial U}{\partial q}\right)_T = -E + T\left(\frac{\partial E}{\partial T}\right)_q. \tag{9}$$

3. Assume the electromotive force of the reversible cell is expressed as,

$$E = 0.49 + 0.0002T \text{ [V]}. \tag{10}$$

Calculate the amount of heat absorbed by the cell when the cell is discharged at 200 mA for 10 seconds at 300 K. Note that charge movement dq during the discharge is expressed as $dq = It$ with current I and time t. Here the temperature variation and the electromotive force variation of the cell are negligibly small during the discharge.

Problem 4

Consider the principle of the acceleration sensor. Here, a hollow rigid box containing a body (regarded as a point mass) of mass m is connected to the box by a spring (an element generating a restoring force proportional to the displacement from the natural length: linear coefficient k) and a damper (an element generating a resisting force proportional to the velocity of the body: linear coefficient c) as shown in Fig. 4.1. The box and the body do not rotate, and move only vertically. The coordinate of the box in space and the coordinate of the body relative to the box from the natural length of the spring are denoted x and y, respectively. Note that the time is t and no gravity is considered.

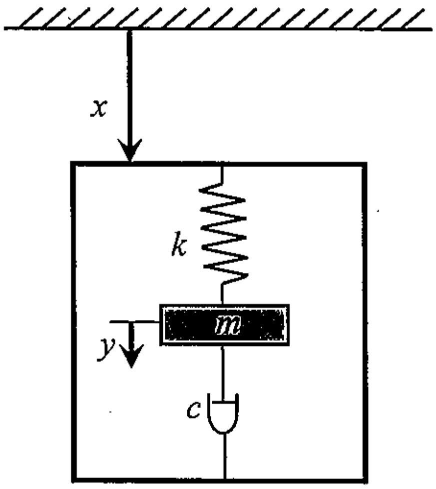

Figure 4.1

I. Express the inertial force acting on the body using m, x, y, and t.

II. The restoring force on the body by the spring can be expressed as $-ky$ and the resisting force acting on the body by the damper can be expressed as $-c\dfrac{dy}{dt}$. Derive the equation of motion of the body.

III. Consider an oscillation of the box with amplitude a and angular frequency ω, expressed as $x = a\cos\omega t$. When only the steady-state response is considered, answer the following questions.

1. Express the response y of the body in the form $y = y_0 \cos(\omega t - \beta)$. Here, express y_0 and β using γ and μ as defined below, and a. Note that $\omega_0 = \sqrt{\frac{k}{m}}$.

$$\gamma = \frac{c}{2\sqrt{mk}}, \tag{1}$$

$$\mu = \frac{\omega}{\omega_0}. \tag{2}$$

2. Show the relationship between y_0 and μ by a diagram, in the case when $c = 0$ and in the case when $c = 2\sqrt{mk}$.

3. When ω is large enough compared to ω_0, approximate y_0 and β. Based on this result, deduce a method to estimate the response x of the box using the response y of the body.

4. When ω is small enough compared to ω_0, approximate y_0 and β. Based on this result, deduce a method to estimate the acceleration $\frac{d^2x}{dt^2}$ of the box using the response y of the body.

2020
The Graduate School Entrance Examination
Physics
1:00 pm－3:00 pm

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Problem 1

Consider an object A, consisting of a tube of outer radius $2r$ and inner radius r, and a solid cylinder of radius r that fits inside the tube. The tube and the solid cylinder share the same central axis as shown in Fig. 1.1. The tube and the solid cylinder are rigid bodies of uniform identical material. The tube has mass $3m$ and the solid cylinder has mass m. The amount of clearance between the inner surface of the tube and the outer surface of the solid cylinder can be ignored, and the solid cylinder can rotate inside the tube.

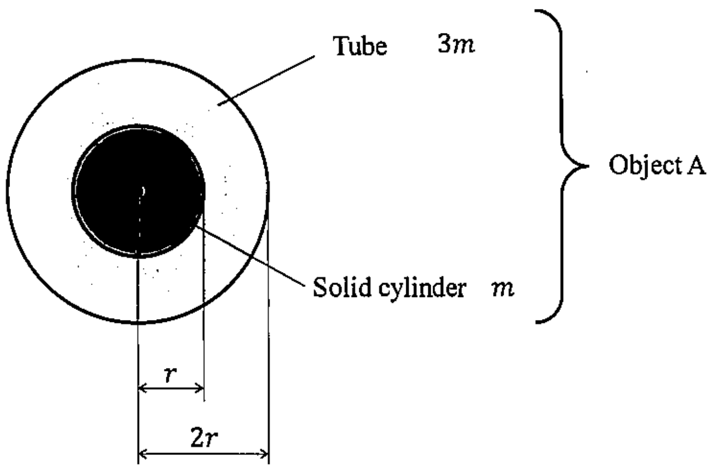

Figure 1.1

I. Derive the moment of inertia of the tube I_T and the solid cylinder I_C around the central axis, respectively. Also write the derivation process.

II. Consider a horizontal plane QR and an inclined plane (a slope) PQ at angle θ to the horizontal plane as shown in Fig. 1.2. The slope PQ has friction, and the horizontal plane QR has no friction. It is assumed that the object A can move between the horizontal plane and the slope while maintaining contact, and that there is no energy loss in transition from movement between the slope and the horizontal plane. The acceleration due to gravity is g.

In all of the following questions, the height of the central axis of the object A when on the horizontal plane is set as 0. You may use I_T and I_C as the moment of inertia of the tube and the solid cylinder around the central axis, respectively.

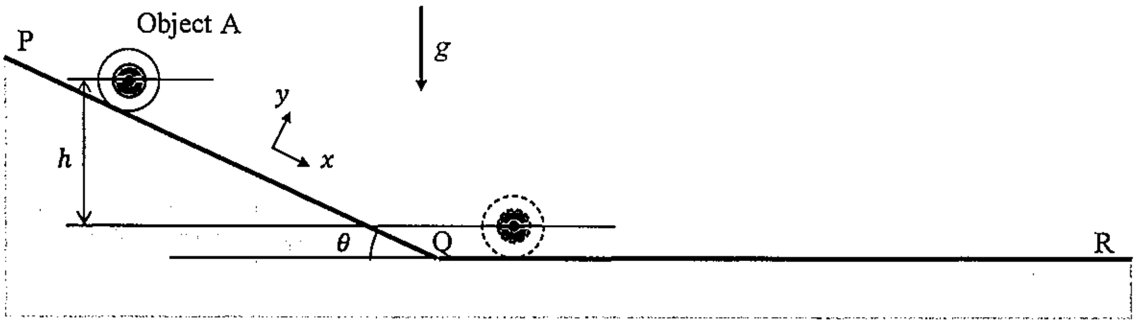

Figure 1.2

1. Consider the case where the friction between the inner surface of the tube and the outer surface of the solid cylinder can be ignored and the solid cylinder can rotate smoothly inside the tube around the same central axis. The object A was gently placed on the slope PQ so that the height of the central axis of the object A was h as shown in Fig. 1.2. Both the tube and the solid cylinder were not rotating. The tube then started rolling down the slope PQ smoothly without sliding.

   Obtain the translational velocity of the center of gravity of the object A in the horizontal direction v_1, the angular velocity of the tube around its central axis ω_{T1}, and the angular velocity of the solid cylinder around its central axis ω_{C1}, immediately after reaching the horizontal plane.

2. Next, consider the case where the solid cylinder can rotate inside the tube around the same central axis, but friction acts between the inner surface of the tube and the outer surface of the solid cylinder. The magnitude of the dynamic friction force between the tube and the solid cylinder is f. As shown in Fig. 1.2, when the object A was gently placed on the slope PQ so that the central axis of the object A was set at the height of h while neither the solid cylinder nor the tube was rotating, the object A started rolling smoothly down the slope PQ, without sliding. At this time, it was observed that the inner solid cylinder was slidingly rotating with a different angular velocity to the outer tube. Answer the following questions.

(i) Consider the motion of the object A on the slope PQ. As shown in the Fig. 1.2, consider a coordinate system in which the x axis is parallel to the slope PQ and the y axis is perpendicular. The translational velocity of the center of gravity of the object A along the slope is v, the angular velocity of the tube around its central axis is ω_{T}, the angular velocity of the solid cylinder around its central axis is ω_{C}, the x component and the y component of resultant force acting between the tube and the solid cylinder are N_{x} and N_{y} respectively, and normal force and friction force acting on the tube from the slope are N_{PQ} and F_{PQ} respectively.

① Show equations of motion of the tube's center of gravity motion in the x direction and the y direction and the tube's rotational equation of motion around the central axis of the tube. Also show the relation between v and ω_{T}.

② Show equations of motion of the solid cylinder's center of gravity motion in the x direction and the y direction and the solid cylinder's rotational equation of motion around the central axis of the solid cylinder.

(ii) Consider the motion of the object A on the horizontal plane QR. Answer the following questions by assuming that the translational velocity of the center of gravity of the object A along the horizontal plane, the angular velocity of the tube around its central axis, and the angular velocity of the solid cylinder around its central axis immediately after reaching the horizontal plane are v_{Q}, ω_{TQ}, and ω_{CQ}, respectively. While the object A traveled on the horizontal plane QR, the angular velocity of the tube around its central axis and that of the solid cylinder around its central axis eventually became equal.

① Derive the translational velocity of the center of gravity of the object A along the horizontal plane v_{R} and the angular velocity of the object A around its central axis ω_{R}.

② Derive the energy loss due to the dynamic friction between the inner surface of the tube and the outer surface of the solid cylinder during the movement of object A on the horizontal plane QR.

Problem 2

When an electric current flows through a conductor, the electric field and current density inside the conductor have spatial distributions depending on the frequency. Consider this phenomenon in the following questions. There is a conductor with conductivity σ filling the region $-h \leq y \leq h$ $(h \neq 0)$ in vacuum, as shown in Fig. 2.1. The conductor has infinite lengths in the x and z directions. An electric field E_z applied in the z direction generates electric current density $j_z = \sigma E_z$ in the conductor. The permittivity and magnetic permeability of the conductor are equal to the vacuum permittivity ε_0 and vacuum permeability μ_0, respectively. Due to symmetry, the magnetic flux density generated inside and outside the conductor by j_z has only an x component, $B_x(y)$. Positive E_z denotes an electric field in the $+z$ direction, and positive B_x denotes a magnetic flux density in the $+x$ direction.

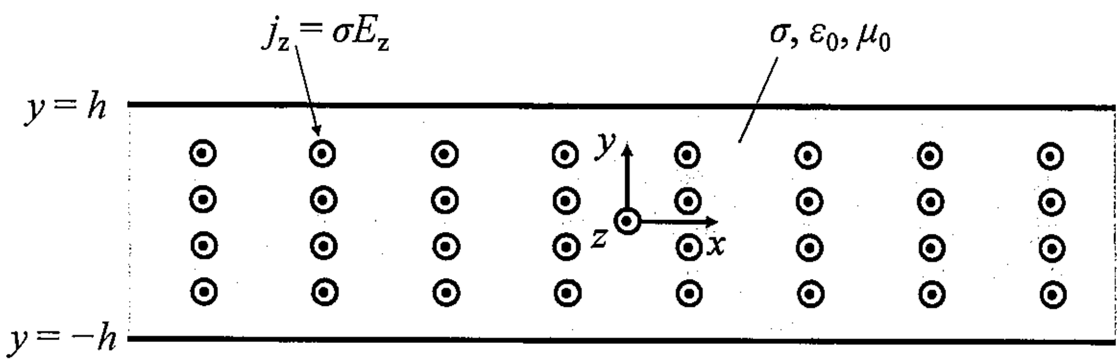

Figure 2.1

I. Consider the case where the electric field E_z is uniform and static.
  1. Express the amount of heat generation per unit volume and unit time inside the conductor, using σ and j_z.
  2. Express the magnetic flux density B_x as a function of y, for the region outside the conductor $y < -h$, inside the conductor $-h \leq y \leq h$, and outside the conductor $y > h$. Assume that $B_x(-y) = -B_x(y)$ because of its symmetry.

II. Consider the case where the electric field varies with time. In this case, the electric field and current density inside the conductor are not necessarily uniform. Both the electric field and the magnetic flux density oscillate with an angular frequency ω, and these are given by $E_z = \mathrm{Re}\{\tilde{E}\exp(i\omega t)\}$ and $B_x = \mathrm{Re}\{\tilde{B}\exp(i\omega t)\}$ with complex parameters $\tilde{E}$ and $\tilde{B}$. i is the imaginary unit.

1. Given the above electromagnetic properties of the conductor, Maxwell's equations inside the conductor contain the following two equations:

$$\frac{1}{\mu_0}\nabla \times \boldsymbol{B} - \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} = \boldsymbol{j}, \tag{1}$$
$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = \boldsymbol{0}, \tag{2}$$

where $\boldsymbol{E}$, $\boldsymbol{B}$, and $\boldsymbol{j}$ are the electric field, the magnetic flux density, and the current density, respectively. Prove that the complex electric field $\tilde{E}$ satisfies the following equation (3), based on equations (1) and (2). Assume that the x and y components of the electric field are zero, the y and z components of the magnetic flux density are zero, $\partial E_z/\partial x = \partial E_z/\partial z = 0$,
and $\partial B_x/\partial x = \partial B_x/\partial z = 0$.

$$\frac{d^2\tilde{E}}{dy^2} + (\varepsilon_0\mu_0\omega^2 - i\omega\mu_0\sigma)\tilde{E} = 0. \tag{3}$$

2. When the conductor is a metal and the conductivity is sufficiently high ($\sigma \gg \varepsilon\omega$), equation (3) can be approximated as follows:

$$\frac{d^2\tilde{E}}{dy^2} - i\alpha^2\tilde{E} = 0, \tag{4}$$

where $\alpha = (\omega\mu_0\sigma)^{\frac{1}{2}}$. Find a general solution to this differential equation.

3. A current of $2h\mathrm{Re}\{j_c \exp(i\omega t)\}$ per unit length in the x direction flows in the conductor, where j_c is a real constant. The complex electric field satisfies the following equation.

$$\int_{-h}^{h} \sigma\tilde{E}\,dy = 2hj_c. \tag{5}$$

Assume that $\tilde{E}(-y) = \tilde{E}(y)$ because of its symmetry. Find the solution to equation (4) inside the conductor for these conditions. You can simplify the solution, using $\cosh(\beta) = (\exp(\beta) + \exp(-\beta))/2$ and $\sinh(\beta) = (\exp(\beta) - \exp(-\beta))/2$, where β is a complex number.

4. Inside the conductor, the squares of the electric field amplitudes at the middle ($y = 0$) and at the upper surface ($y \to h$) are denoted by $|\tilde{E}(0)|^2$ and $|\tilde{E}(h)|^2$, respectively.
   (i) The angular frequency ω of the electric field is low ($\omega \to 0$). Choose the correct one from the following. Explain the reason.
      a. $|\tilde{E}(0)|^2 \ll |\tilde{E}(h)|^2$
      b. $|\tilde{E}(0)|^2 \approx |\tilde{E}(h)|^2$
      c. $|\tilde{E}(0)|^2 \gg |\tilde{E}(h)|^2$
   (ii) The angular frequency ω of the electric field is high ($\omega \gg \frac{1}{\mu_0 \sigma h^2}$). Choose the correct one from the following. Explain the reason.
      a. $|\tilde{E}(0)|^2 \ll |\tilde{E}(h)|^2$
      b. $|\tilde{E}(0)|^2 \approx |\tilde{E}(h)|^2$
      c. $|\tilde{E}(0)|^2 \gg |\tilde{E}(h)|^2$

Problem 3

Consider a perfect gas defined by the equation of state (1) and a Van der Waals gas defined by the equation of state (2).

$$PV = nRT, \tag{1}$$

$$\left\{P + a\left(\frac{n}{V}\right)^2\right\}(V - nb) = nRT. \tag{2}$$

Here, P, V, n, R and T are the pressure, the volume, the amount of substance (number of moles), a gas constant and the thermodynamic temperature, respectively. Also, a and b are assumed to be constants. For a quasi-static process of these gases, the first law of thermodynamics is expressed as

$$dU = TdS - PdV, \tag{3}$$

where U and S are the internal energy and the entropy, respectively. Answer the following questions. Note that the specific heat at constant volume C_V is assumed to be constant and is expressed as

$$C_\mathrm{V} = \frac{1}{n}\left(\frac{\partial U}{\partial T}\right)_V. \tag{4}$$

I. Consider the following relations for both a perfect gas and a Van der Waals gas. Using equation (3), $\left(\frac{\partial U}{\partial V}\right)_T$ is expressed as equation (5).

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial S}{\partial V}\right)_T - P. \tag{5}$$

Using equation (5) and a Maxwell relation of $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$, a thermodynamic equation of state can be derived as expressed in equation (6) without using the entropy S, which we cannot measure directly.

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P. \tag{6}$$

Find $\left(\frac{\partial U}{\partial V}\right)_T$ for both a perfect gas and a Van der Waals gas.

II. Consider real gas effects during quasi-static expansions. A gas of a unit amount of substance (1 mol), pressure P_0, volume V_0, and thermodynamic temperature T_0 is inside a cylinder fitted with a piston as the initial condition. Assume that the system is thermally isolated from the external environment. Answer the following questions.

1. The gas undergoes an adiabatic reversible expansion to a volume of $2V_0$ from V_0 by displacement of the piston as shown in Fig. 3.1. Find the thermodynamic temperatures T and the entropy changes ΔS for both a perfect gas and a Van der Waals gas after such an expansion.

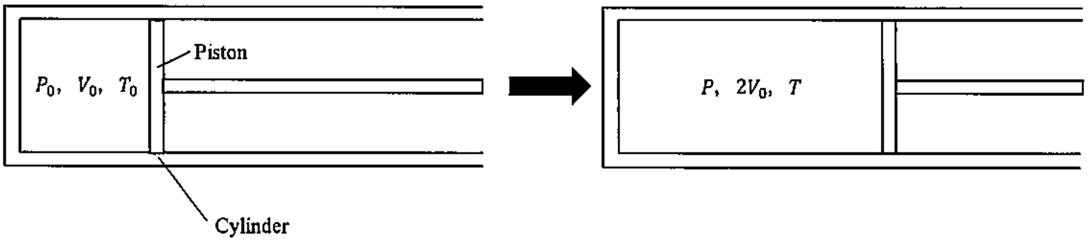

Figure 3.1

2. Next, consider that the gas in the cylinder is heated by a heater and is reversibly expanded to a volume of $2\,V_0$ from V_0 at the constant thermodynamic temperature of T_0 as shown in Fig. 3.2. Find the changes in the internal energy and the entropy, ΔU and ΔS, for both a perfect gas and a Van der Waals gas after such an expansion. Also, explain the reason for the difference of the change in the internal energy ΔU between a perfect gas and a Van der Waals gas.

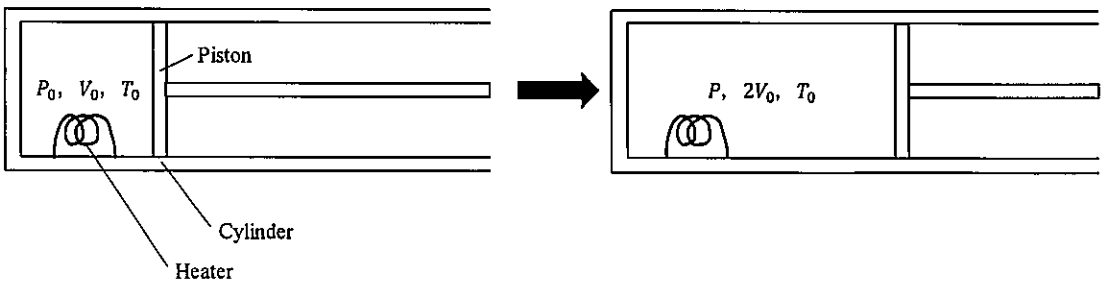

Figure 3.2

III. Chamber A of volume V_0 and Chamber B of volume V_0 are connected by a valve as shown in Fig. 3.3. At the initial condition, a perfect gas with pressure P_0, volume V_0, and thermodynamic temperature T_0 is in Chamber A. The gas consists of a unit amount of substance (1 mol). Chamber B is evacuated. Consider expansion of the gas in Chamber A by opening the valve. Answer the following questions, assuming that the system is thermally isolated from the external environment.

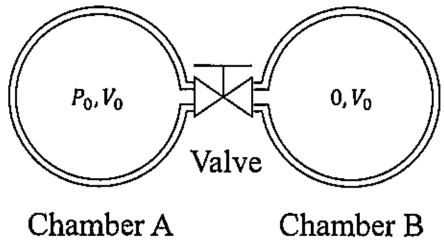

Figure 3.3

1. Find the thermodynamic function (quantity of state) which remains constant during this process.
2. Find the thermodynamic temperature T and the entropy change ΔS after such an expansion.
3. Explain about the irreversibility of this process, with reasoning.

Problem 4

A rod of length L is fixed at both ends by support A and support B as shown in Fig. 4.1. Assume that moments are not applied at both ends of the rods.

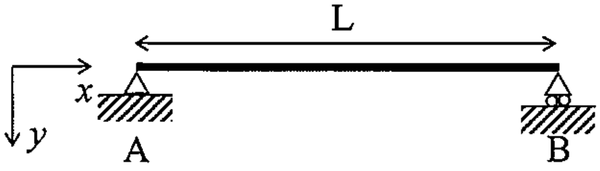

Figure 4.1

I. Consider the bending deformation of the rod when a distributed load $q(x)$ per unit length is applied to the rod in the y direction. Assume that the deformation of the rod is infinitesimally small, and consider only displacement in the y direction (the downward direction is positive as shown in Fig. 4.2). The mass of the rod is negligible. Answer the following questions.

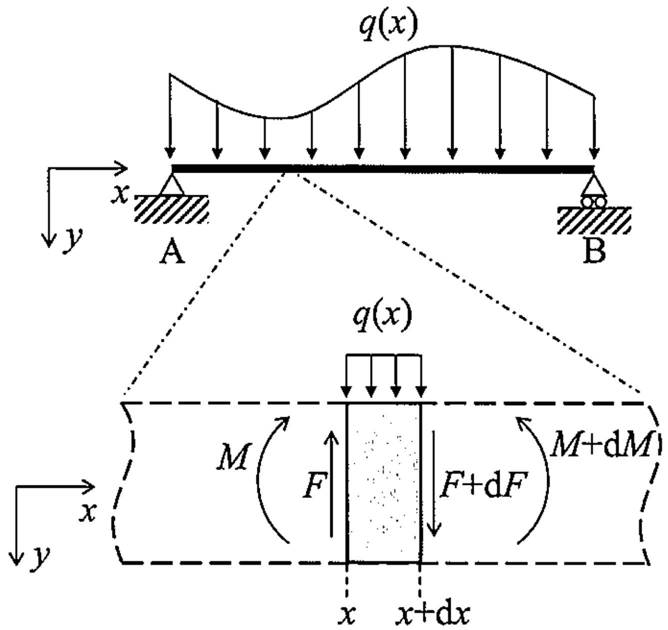

Figure 4.2

1. Consider an element of the rod at any position x. Assume that the force F and the moment M act on the cross section of the rod to maintain static equilibrium as shown in Fig. 4.2.

   (i) Show that equation (1) holds true between the force F and the distributed load $q(x)$ in this case.

   $$\frac{dF}{dx} = -q(x). \tag{1}$$

   (ii) Show that equation (2) holds true between the moment M and the force F in this case.

   $$\frac{dM}{dx} = F. \tag{2}$$

2. When the deformation of the rod is infinitesimally small, it is assumed that the rod is deformed only by the moment M. In this case, equation (3) holds true between the displacement y, and the moment M. Here, R is assumed to be a constant. When the distributed load $q(x) = \mathrm{k}$ is applied (k is a constant), derive the maximum displacement of the rod. Here, assume that $y = 0$ at both ends of the rod.

   $$\mathrm{R}\frac{d^2y}{dx^2} = -M. \tag{3}$$

II. Next, consider the bending vibration of the rod shown in Fig. 4.1. Here, consider about free vibration. Assume that the rod is uniform, the density of the rod is ρ, and the cross sectional area is S. Also, assume that the deformation of the rod is infinitesimally small, and consider only motion in the y direction. Gravitational forces are negligible. Answer the following questions.

1. Consider the motion of an element of the rod as shown in Fig. 4.3. By considering the equation of motion of the element in the figure, and by using equations (2) and (3), show that the equation of motion of the bending vibration of the rod in the y direction is expressed as equation (4). Assume that the element moves only in the y direction, and that the force in the x direction and the rotation by the moment are negligible.

$$\mathrm{R}\frac{\partial^4 y}{\partial x^4} + \rho \mathrm{S}\frac{\partial^2 y}{\partial t^2} = 0. \tag{4}$$

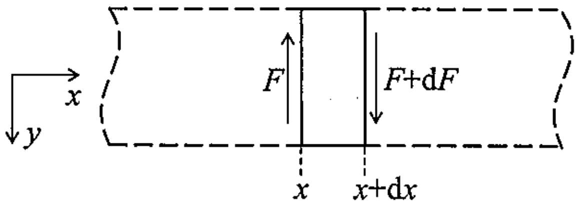

Figure 4.3

2. Assume that the solution of equation (4) obtained in Question II. 1 is $y(x,t) = X(x)\exp(i\omega t)$, where i is the imaginary unit. Show that the general solution $X(x)$ of equation (4) is expressed by equation (5). Here, $\mu^4 = \frac{\rho S \omega^2}{R}$, and $C_1 \sim C_4$ are constants. Also, $\exp(ix) = \cos x + i\sin x$, $\sinh x = (\exp(x) - \exp(-x))/2$, and $\cosh x = (\exp(x) + \exp(-x))/2$.

$$X(x) = C_1 \sin \mu x + C_2 \cos \mu x + C_3 \sinh \mu x + C_4 \cosh \mu x. \tag{5}$$

3. In the case when the rod is fixed at both ends similar to Fig. 4.1, $y = 0$ at both ends of the rod.
(i) Obtain the possible values of μ using equation (5).
(ii) Explain the behavior of the bending vibration based on the result of Question II. 3. (i).

問 題 訂 正

科目名：物理学

第２問　II. 1.　５行目（５ページ）

（誤）… それぞれ電場，磁場，電流密度である。

（正）… それぞれ電場，磁束密度，電流密度である。

No correction in the English version.

第２問　II. 2.　１行目（５ページ）

（誤）… 導電率が十分に高い場合($\sigma \gg \underline{\varepsilon}\omega$)，

（正）… 導電率が十分に高い場合($\sigma \gg \underline{\varepsilon_0}\omega$)，

Problem 2　II. 2.　Line 1-2　(Page 6)

(incorrect)... the conductivity is sufficiently high ($\sigma \gg \underline{\varepsilon}\omega$),

(correct)... the conductivity is sufficiently high ($\sigma \gg \underline{\varepsilon_0}\omega$),

2019 年 8 月 27 日

工学系研究科

大学院入試委員会本部

2021

The Graduate School Entrance Examination

Physics 1 (Mechanics)

Problem Number P1

Answer Time 60 minutes

GENERAL INSTRUCTIONS

1. Do not look at the Problems until the start of the examination has been announced.
2. Use 6 Answer Sheets and 3 Draft Sheets.
3. Do not use the back faces of the Answer Sheets or the Draft Sheets.
4. Fill in your examinee number in the designated places at the top of all the Answer Sheets and the Draft Sheets.
5. Answers must be written in Japanese or English.
6. Answers must be marked within the solid frame on the Answer Sheets.
7. Any Answer Sheet with marks or symbols irrelevant to your answers is considered to be invalid.
8. The Problems are described in Japanese on pages 5-6 and in English on pages 7-8.
9. Scrolling, expansion and reduction of the Problems are permitted. Keyboard operation is prohibited.

- Show the derivation processes as well as the results.
- Continue the answer even if network trouble occurs.

Physics 1 (Mechanics)

Answer all Questions I, II and III.

For the system shown in Fig. 1.1, consider objects A (mass: m) and B (mass: M) which move along x axis. Object A moves with a constant velocity $v\,(> 0)$. Object B connected to an end of a spring (spring constant: k) is at rest. The other end of the spring is fixed to the wall. When object A collides with object B, answer the following questions. Define time $t = 0$ at the moment of collision, and assume that object B will not collide with the wall. No force is exerted from the outside. Ignore the sizes of objects A and B. The origin of x axis is fixed at the initial position of object B.

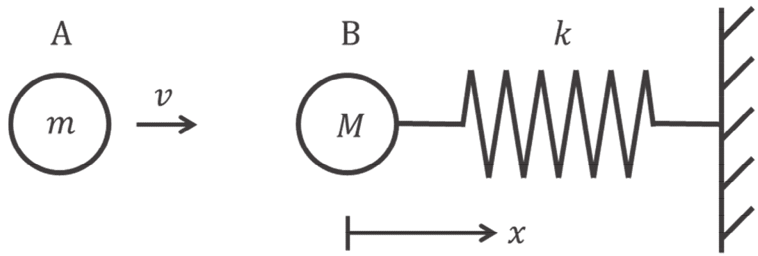

Figure 1.1

I. When objects A and B are united at the moment of collision and then move together, answer the following questions.

1. Determine the position x of object B as a function of t.
2. Determine the time when object B first comes back to the initial position after the collision. In addition, determine the velocity of object B at this moment.

Continued on the next page.

II. When the collision between objects A and B is completely elastic, answer the following questions.

1. Determine the velocities of objects A and B just after the collision.
2. Assuming $M = m$, plot the positions of objects A and B after the collision on a graph as functions of time.

III. When the coefficient of restitution between objects A and B is e $(0 < e < 1)$ and $\dfrac{M}{m} = 2$, the second collision occurs at time $t = \dfrac{7\sqrt{2}\,\pi}{6}\sqrt{\dfrac{m}{k}}$.

Answer the following questions.

1. Determine the velocities of objects A and B just before the second collision using the coefficient of restitution e.
2. In this case, determine the coefficient of restitution e.

2022

The Graduate School Entrance Examination

Physics 2

Electromagnetism

Answer Time 60 minutes

GENERAL INSTRUCTIONS

1. Do not open the problem booklet until the start of the examination is announced.
2. Notify your proctor if you find any printing or production errors.
3. The problems are described in Japanese on pages 2-4 and in English on pages 8-10.
4. Answer all questions.
5. 1 answer sheet is given. You may use the reverse side if necessary.
6. Do not write anything in the box at the upper left of the answer sheet.
7. Fill in your examinee number in the designated place at the top of the answer sheet.
8. Answers must be written in Japanese or English.
9. You may use the blank pages of the problem booklet for drafts without detaching them.
10. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.
11. You may not take the booklet or answer sheet with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

日本語の注意事項はおもて表紙にある。

Physics 2 (Electromagnetism)

Answer all Questions I, II, III, IV, and V.

As shown in Fig. 2.1, two concentric cylindrical homogeneous conductors A and B with the length L and negligible thickness are located in vacuum. Let radii of the conductors A and B be a and b $(a < b)$, respectively. Assume that the length L is long enough $(L \gg a, b)$. Let a dielectric constant of vacuum be ε_0.

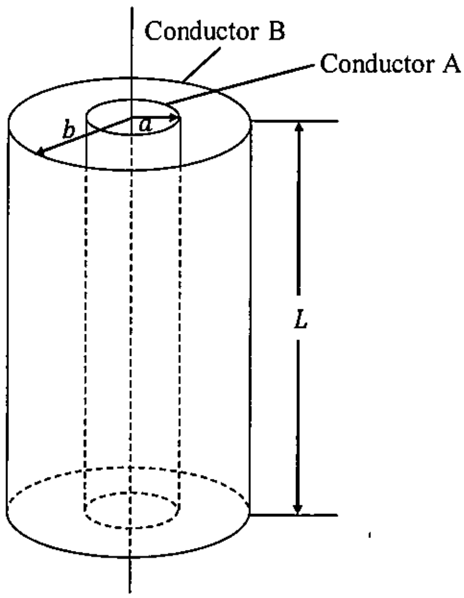

Figure 2.1

I. Direct electric currents with magnitude I flow upward and downward in the conductors A and B, respectively along the central axis direction. Derive the magnitude and direction of the magnetic field at the point with distance r from the central axis of the conductors.

II. Assume that the conductors A and B are uncharged and ungrounded at the initial state. Secondly, electric charge Q_1 $(Q_1 > 0)$ is given to the conductor A. Thirdly, the conductor B is grounded. For each case of before and after the conductor B is grounded, derive the magnitude of the electric field E at the point with distance r from the central axis, respectively. Additionally, draw the graphs of E versus r, respectively.

Continued on a later page.

III. The region between the conductors A and B is completely filled with a material with a dielectric constant $\varepsilon(\varepsilon > \varepsilon_0)$ and an electrical conductivity σ. Assume that the conductor A is ungrounded and the conductor B is grounded.

1. Derive the electrical resistance between the conductors A and B.
2. At the time $t = 0$, a charge $Q_2(Q_2 > 0)$ is given to the conductor A. Explain how a charge in the conductor A changes with time t by drawing a graph of its time variation.

Continued on a later page.

IV. As shown in Fig. 2.2 (a view of the conductors from the direction of their central axis), the space between the conductors A and B is filled with a medium 1 (dielectric constant: ε_1, electrical conductivity: σ_1) for the inner region and a medium 2 (dielectric constant: ε_2, electrical conductivity: σ_2) for the outer region. The regions have a coaxial cylindrical interface with a radius $x(a < x < b)$. Then, a voltage is applied between the conductors A and B. After sufficient time passes, let $I(I > 0)$ be the magnitude of a steady current that flows between the conductors A and B. Derive the surface charge density accumulated at the interface of the media 1 and 2.

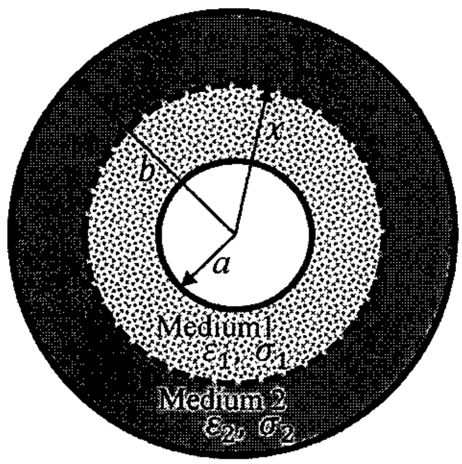

Figure 2.2

V. Consider the case where the electrical resistivities of the media 1 and 2 are high enough that they can be regarded as insulators. Electric charges $+Q_3$ and $-Q_3$ $(Q_3 > 0)$ are given to the conductors A and B, respectively.

1. Derive the electrostatic energy of this system.
2. Derive the magnitude of the force per unit area at the cylindrical interface of the media. Assume $\varepsilon_1 > \varepsilon_2$.

Problem 1

Consider a simple pendulum consisting of a weight of mass m, whose size is negligible, attached to the bottom end of a rigid rod of length l, whose mass and thickness are negligible. Take the x-axis horizontally and the y-axis vertically downward. The top end of the rigid rod is attached to a hinge, and the simple pendulum can rotate freely in the xy-plane using the hinge as a frictionless fulcrum. Let the angle between the rigid rod and the y-axis be θ, and denote the time as t. Define $\dot{\theta} \equiv \frac{d\theta}{dt}$ and $\ddot{\theta} \equiv \frac{d^2\theta}{dt^2}$. The gravitational acceleration is denoted by g. Assume air resistance to be negligible.

I. As shown in Fig. 1.1, the hinge is fixed to the origin O.

1. From the equations of motion of the weight in the x- and y-directions, derive the differential equation satisfied by θ.
2. Assuming that θ always satisfies $|\theta| \ll 1$, neglect the second order and higher order terms of θ. Then, $\sin\theta \approx \theta$ and $\cos\theta \approx 1$. The weight is initially at rest at $(x, y) = (0, l)$ and then launched at time $t = 0$ with the initial speed u_1 in the positive direction of the x-axis. Determine θ at time t (≥ 0).

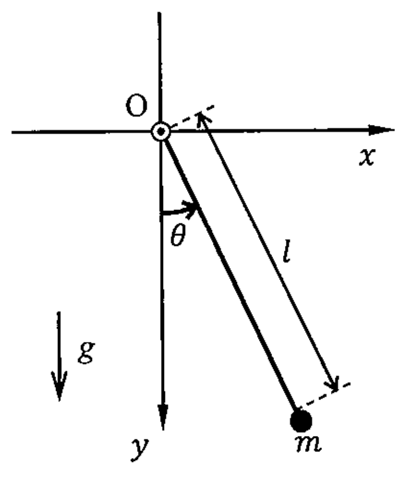

Figure 1.1

II. As shown in Fig. 1.2, the hinge is fixed to the tip of a spring that can only extend and contract in the horizontal direction. The spring constant is k, and the masses of the spring and hinge are negligible. Let δ be the x coordinate of the hinge, and $\delta = 0$ when the spring is at its natural length. Assume that θ always satisfies $|\theta| << 1$. Then, $\left|\sqrt{\frac{l}{g}}\dot{\theta}\right| << 1$ and $\left|\frac{l}{g}\ddot{\theta}\right| << 1$ are also satisfied, and we can neglect the second order and higher order terms of the dimensionless quantities θ, $\sqrt{\frac{l}{g}}\dot{\theta}$, and $\frac{l}{g}\ddot{\theta}$, as well as the second order and higher order terms of their combinations.

1. From the equation of motion of the weight in the y-direction, determine the tension in the rigid rod.
2. Assuming that the forces acting on the hinge in the x-direction are balanced, derive the equation that holds between δ and θ.
3. In the case of Question II.2, derive the differential equation satisfied by θ from the equation of motion of the weight in the x-direction.
4. Initially, the hinge is at rest at the origin O, and the weight is at rest at $(x,y) = (0, l)$. At time $t = 0$, the weight is launched with the initial speed u_2 in the positive direction of the x-axis. Determine θ at time t (≥ 0).

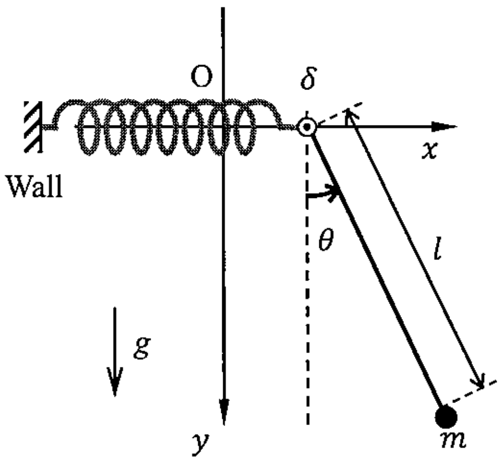

Figure 1.2

III. As shown in Fig. 1.3, from $t=0$, the hinge is forced to move horizontally with the displacement $\xi = A\sin(\omega t)$ with respect to the origin O. A and ω are real constants, and $\omega \neq \sqrt{\frac{g}{l}}$. For $t<0$, the hinge and the weight were at rest at the origin O and $(x, y) = (0, l)$, respectively. Assuming that the approximation in Question I.2 is valid, determine θ at time t (≥ 0).

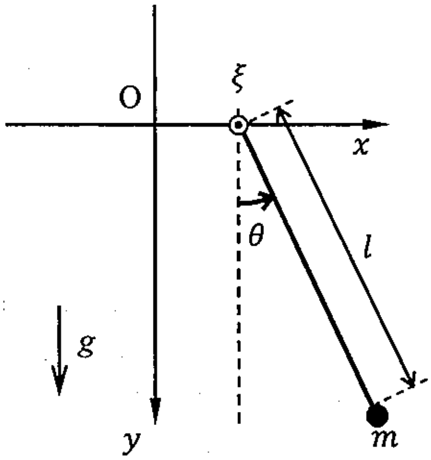

Figure 1.3

Problem 2

Answer both Questions I and II.

I. As shown in Fig. 2.1, consider the cylindrical region with a radius of a and an infinite length in vacuum. In this cylindrical region, electric charges are uniformly distributed with a volume charge density of ρ (> 0). The electric permittivity of the vacuum is ε_0.

1. Derive the magnitude and direction of the electric field vector $\boldsymbol{E}(r)$ at the distance r from the central axis of the cylindrical region for both the inside and outside of the cylindrical region.
2. Next, the cylindrical region is surrounded by a concentric metallic hollow cylinder with infinite length, whose inner and outer radii are $2a$ and $3a$, respectively. Fig. 2.2 shows the cross section perpendicular to the central axis of the cylindrical region in this case. The metallic hollow cylinder does not affect the charge distribution in the cylindrical region. The metallic hollow cylinder remains electrically neutral as a whole. Derive the areal charge density σ induced on the inner surface of the metallic hollow cylinder.
3. In the case of Question I.2, determine the magnitude of the electric field vector at the distance $5a$ from the central axis.
4. Let the electric potentials at the distance $5a$ from the central axis be V_1 and V_2 in the cases of Questions I.1 and I.2, respectively. Derive $\Delta V = V_1 - V_2$. Here, assume that the electric potential on the central axis is zero.

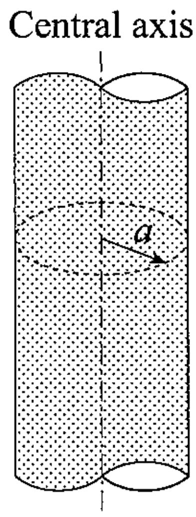

Figure 2.1

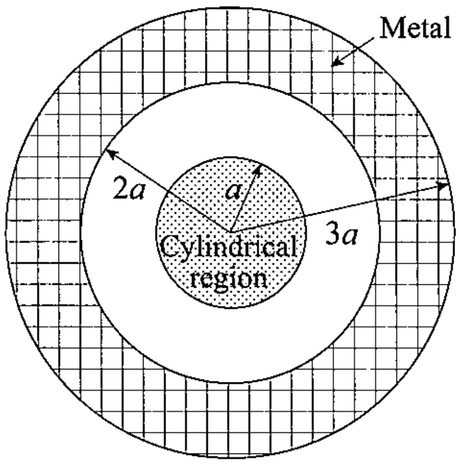

Figure 2.2

II. As shown in Fig. 2.3, consider the case where a plane electromagnetic wave with the wave vector $\boldsymbol{k}_0$ ($|\boldsymbol{k}_0| = k_0$) propagating in vacuum (electric permittivity of ε_0 and magnetic permeability of μ_0) along the z-axis with speed c_0 is incident normally on a uniform insulating medium with electric permittivity $4\varepsilon_0$ and magnetic permeability μ_0. The interface between vacuum and the insulating medium lies in the xy-plane that includes the origin O, and the region $z \geq 0$ is filled with the insulating medium.

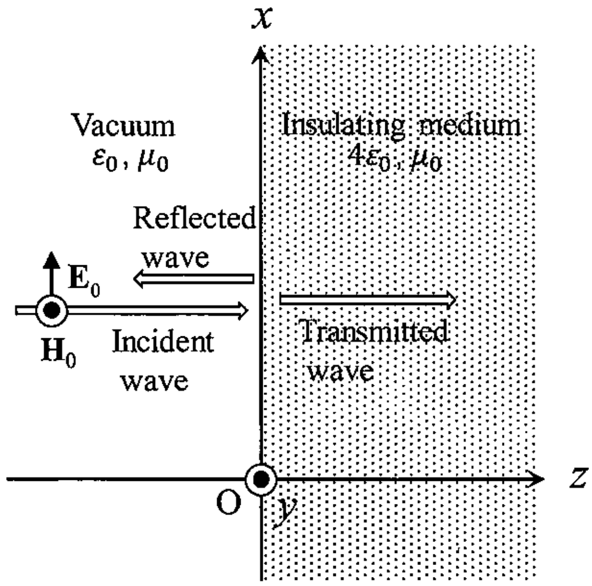

Figure 2.3

Let the electric field vector of the incident wave at time t be $\boldsymbol{E}_0 = (E_0 \cos[k_0(z - c_0 t)],\ 0,\ 0)$, where E_0 is a real constant. Let the magnetic field vector of the incident wave be $\boldsymbol{H}_0$.

1. Using Maxwell's equations, show that $\boldsymbol{H}_0 = \left(0,\ \dfrac{E_0}{\mu_0 c_0} \cos[k_0(z - c_0 t)],\ 0\right)$.
2. By applying Maxwell's equations to $\boldsymbol{E}_0$ and $\boldsymbol{H}_0$, show that the relation $c_0 = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}}$ holds.

Let $\boldsymbol{E}_{\mathrm{T}}$, $\boldsymbol{H}_{\mathrm{T}}$, $\boldsymbol{k}_{\mathrm{T}}$, and c_{T} be the electric field vector, magnetic field vector, wave vector, and speed of the electromagnetic wave transmitted through the interface, respectively. Let $\boldsymbol{E}_{\mathrm{R}}$, $\boldsymbol{H}_{\mathrm{R}}$, $\boldsymbol{k}_{\mathrm{R}}$, and c_{R} be the electric field vector, magnetic field vector, wave vector, and speed of the electromagnetic wave reflected by the interface, respectively. Here, $\boldsymbol{k}_{\mathrm{R}} = -\boldsymbol{k}_0$, $c_{\mathrm{R}} = c_0$, and $c_{\mathrm{T}} = \dfrac{1}{\sqrt{4\varepsilon_0\mu_0}} = \dfrac{1}{2}c_0$. Use the continuity of the tangential component of the electric field vector $\boldsymbol{E}$ and the continuity of the tangential component of the magnetic field vector $\boldsymbol{H}$ at the interface to answer the following Questions II.3 and II.4.

3. Express $\boldsymbol{k}_{\mathrm{T}}$ in terms of $\boldsymbol{k}_0$.
4. Obtain $\boldsymbol{E}_{\mathrm{T}}$, $\boldsymbol{E}_{\mathrm{R}}$, $\boldsymbol{H}_{\mathrm{T}}$, and $\boldsymbol{H}_{\mathrm{R}}$.

Let $\boldsymbol{S}_0$, $\boldsymbol{S}_{\mathrm{T}}$, and $\boldsymbol{S}_{\mathrm{R}}$ be the Poynting vectors of the incident, transmitted, and reflected waves, respectively. The Poynting vector $\boldsymbol{S}$ is defined by $\boldsymbol{S} \equiv \boldsymbol{E} \times \boldsymbol{H}$.

5. Obtain $\boldsymbol{S}_0$, $\boldsymbol{S}_{\mathrm{T}}$, and $\boldsymbol{S}_{\mathrm{R}}$.
6. Obtain the transmission coefficient T and reflection coefficient R of the incident wave. T and R are given by $T = \dfrac{|\boldsymbol{S}_{\mathrm{T}}|_{\mathrm{ave}}}{|\boldsymbol{S}_0|_{\mathrm{ave}}}$ and $R = \dfrac{|\boldsymbol{S}_{\mathrm{R}}|_{\mathrm{ave}}}{|\boldsymbol{S}_0|_{\mathrm{ave}}}$, respectively. Here, $|\boldsymbol{A}|_{\mathrm{ave}}$ is the time average of the magnitude of a vector $\boldsymbol{A}$.

2024

The Graduate School Entrance Examination

Physics

13:00 – 15:00

GENERAL INSTRUCTIONS

1. Do not open the problem booklet until the start of the examination is announced.
2. Notify your proctor if you find any printing or production errors.
3. Answers must be written in Japanese or English. The problems are described in Japanese on pages 2–9 and in English on pages 12–19.
4. Answer all problems.
5. Two answer sheets are given. Use one answer sheet for each Problem (1 and 2). You may use the reverse side if necessary.
6. Write the problem number (1 or 2) that you answer in the upper left box of the answer sheet.
7. Fill in your examinee number in the designated place at the top of the answer sheet.
8. You may use the blank pages of the problem booklet for drafts without detaching them.
9. Any answer sheet with marks or symbols irrelevant to your answers is considered to be invalid.
10. Do not take the answer sheets or the booklet with you after the examination.

Examinee Number	No.

Write your examinee number in the space provided above.

日本語の注意事項はおもて表紙にある。

Problem 1

As shown in Fig. 1.1, a uniformly dense disk of radius R and mass M is put on a 30 degree angle slope. The thickness of the disk is negligible. This disk is attached to a thread at point P on the circumferential sidewall. The other end of this thread is connected to a spring with a spring constant K and this spring is fixed at a distance $2R$ from the slope. The spring does not contact with the disk. The disk moves in-plane and the trajectory of the disk center is maintained parallel to the slope. Air resistance and the masses of the thread and the spring are ignored. The gravitational acceleration is denoted by g.

The x-axis is defined parallel to the slope downward and x is the position of the disk center. As shown in Fig. 1.1, the original x position ($x = 0$) is defined when the spring is at its natural length, the thread is not rolled on the disk, and the line crossing the point P and the disk center is perpendicular to the slope. As indicated in Fig. 1.2, the disk does not slip on the slope and rotates while winding up the thread. In this figure, the disk at the original position ($x = 0$) is indicated with dashed line.

Answer all the following questions from I to IV.

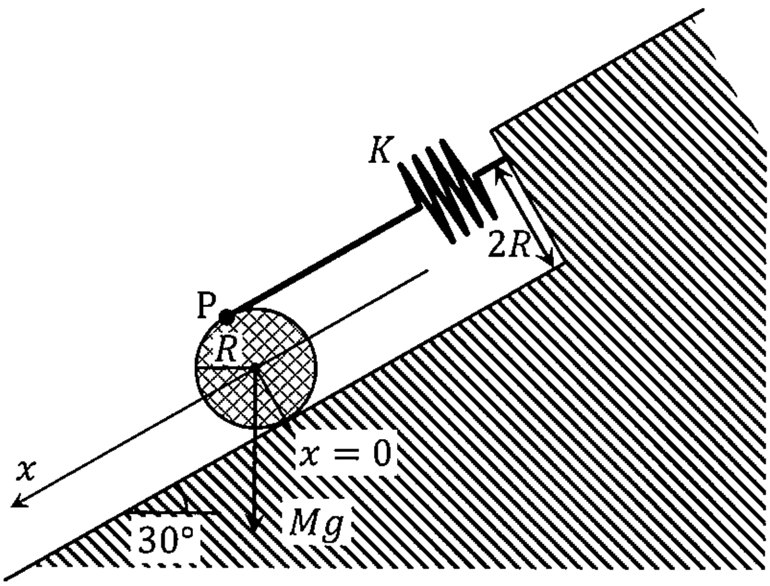

Figure 1.1

I. Derive the moment of inertia I of the disk.

II. From the original position $(x = 0)$, the disk is put down on the slope quasi-statically while it is rotated as shown in Fig. 1.2. Determine the x-coordinate x_0 when the disk reaches the equilibrium position.

III. The disk is set back to the original position $(x = 0)$ and gently released $\left(\frac{dx}{dt} = 0\right)$ at $t = 0$. Answer the following questions.

1. Determine the differential equation expressing the disk center position x as a function of time t.

2. Solve the differential equation in the previous question and draw a graph of the disk center position x as a function of time t.

3. The static friction coefficient between the slope surface and the disk is denoted by μ. Determine the condition that the parameter μ must satisfy in order to keep the non-slippage movement between the slope and the disk.

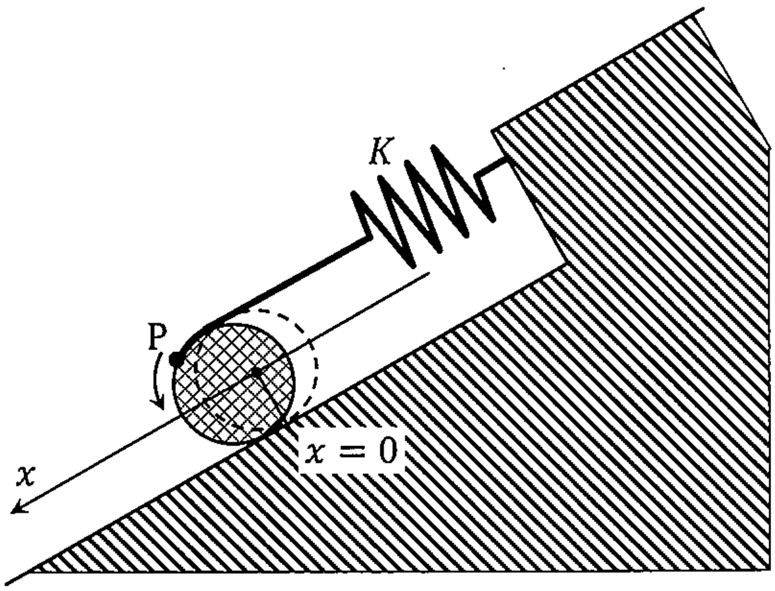

Figure 1.2

IV. Before starting the movement as indicated in Question III, an obstacle is attached on the slope. This obstacle contacts with the disk at the equilibrium position ($x = x_0$) as shown in Fig. 1.3. The contact is with the pointed tip of the obstacle at a distance h from the slope. No slippage happens between the disk and the obstacle and the angular momentum is conserved during the contact.

Now the disk is gently released $\left(\frac{dx}{dt} = 0\right)$ at the original position ($x = 0$) as in Question III. Determine the condition that h must satisfy for the disk to detach from the slope after the contact with the obstacle.

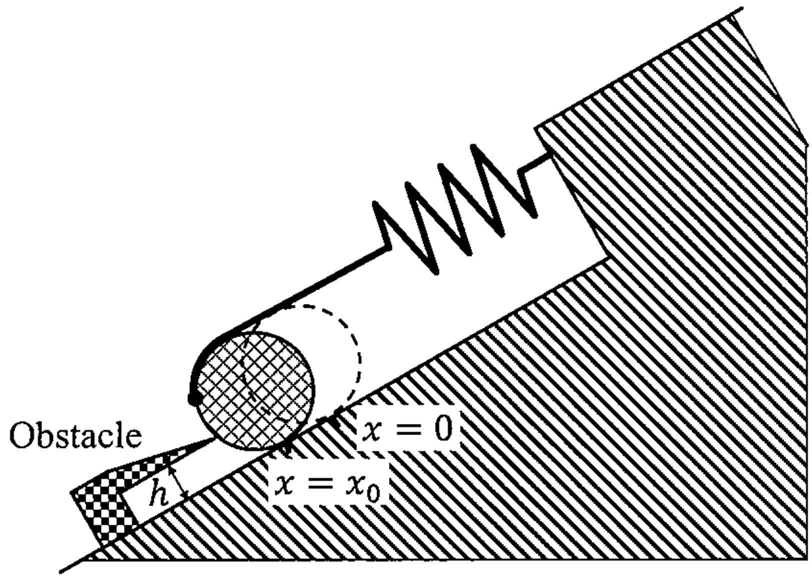

Figure 1.3

Problem 2

Answer both questions I and II.

I. Assume that charges are distributed in a vacuum as shown in Fig. 2.1 and Fig. 2.2. In Fig. 2.1, the charges are uniformly distributed with a charge density ρ ($\rho > 0$) in a sphere of radius a. The distance from the center is denoted by r. Let the electric potential $V = 0$ at the center. In Fig. 2.2, the charges are uniformly distributed with a charge density ρ ($\rho > 0$) in the region $-a < x < a$, $-\infty < y < \infty$, and $-\infty < z < \infty$ in the xyz orthogonal coordinate system. Let the electric potential $V = 0$ at $x = 0$. The permittivity in vacuum is denoted by ε_0. Answer the following questions.

1. Find the formula and draw the graph of the potential V as a function of r in the case of Fig. 2.1.

2. Find the formula and draw the graph of the potential V as a function of x in the case of Fig. 2.2.

Next, consider a point charge P with a mass m and a charge $-q$ ($q > 0$) is put without initial velocity in Fig. 2.1 and Fig. 2.2. Assume that the presence and the motion of the point charge P do not affect the existing charge distributions. Answer the following questions.

3. Assume that the point charge P is put at a certain position r_0 in $0 < r < a$ in Fig. 2.1. Find the equation of motion of the point charge P. Solve the equation and describe the motion of the point charge P.

4. Assume that a point charge P is put at a certain position x_0 in $-a < x < a$ ($x \neq 0$) in Fig. 2.2. Find the equation of motion of the point charge P. Solve the equation and describe the motion of the point charge P.

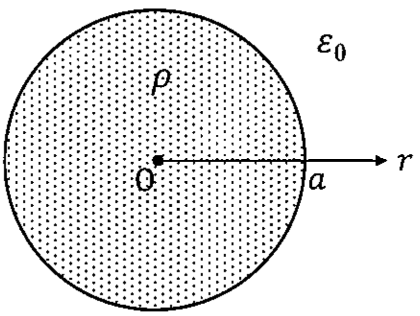

Figure 2.1

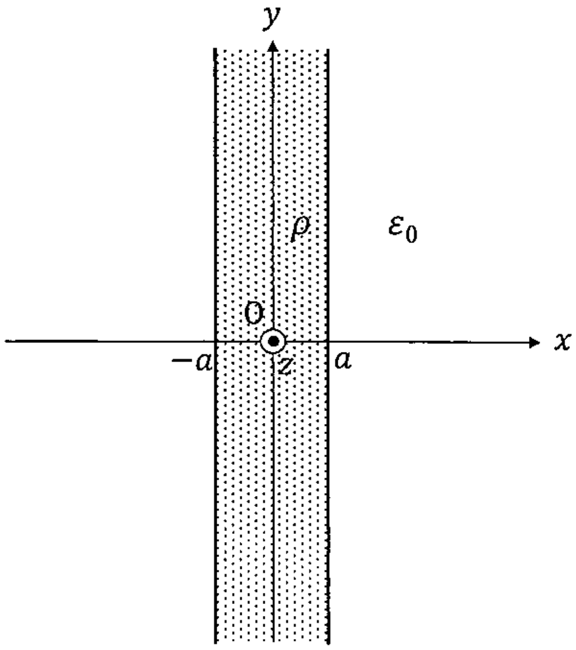

Figure 2.2

II. As shown in Fig. 2.3, a dielectric 1 with permittivity ε_1 and a dielectric 2 with permittivity ε_2 are contacted on the *xy* plane including the origin O in the *xyz* orthogonal coordinate system. The dielectric 1 exists in the region $z \leq 0$ (Region I), and the dielectric 2 exists in the region $z > 0$ (Region II). A point charge with the charge q is placed at the point P $(0,0,a)$ $(a > 0)$. The permittivity in vacuum is denoted by ε_0. First, the method of image charges will be used to obtain the electrostatic fields in Fig. 2.3. Let the electric potential be zero at infinity. Answer the following questions.

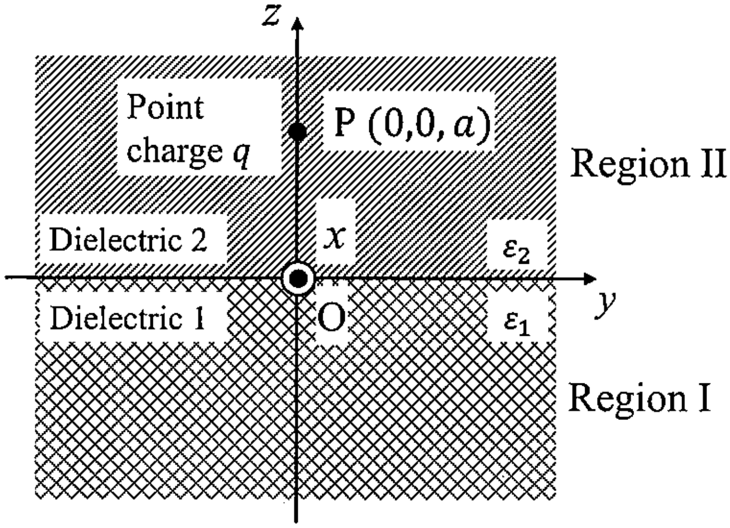

Figure 2.3

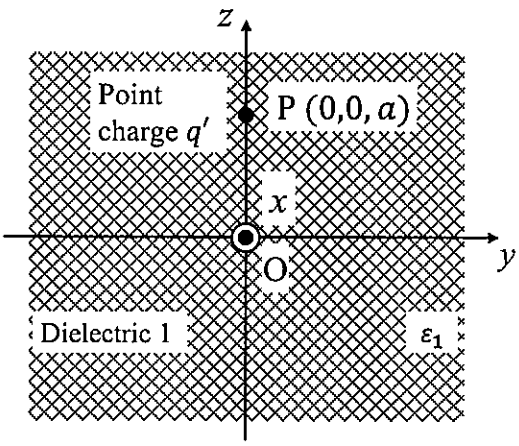

Figure 2.4

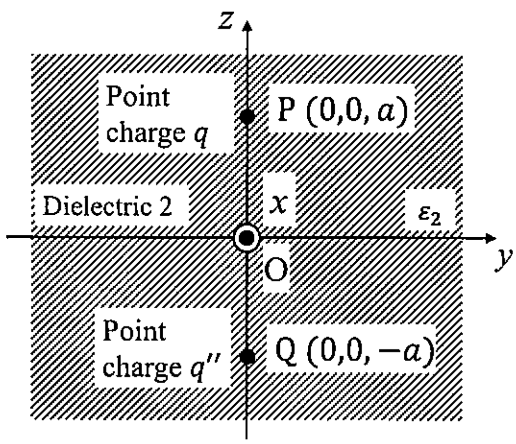

Figure 2.5

1. As shown in Fig. 2.4, assume that the dielectric 1 occupies the entire space and that a point charge with the charge q' is placed at the point P $(0, 0, a)$. Find the electric potential $\phi_1(x, y, z)$ at the point (x, y, z) in Fig. 2.4.

2. As shown in Fig. 2.5, assume that the dielectric 2 occupies the entire space and that a point charge with the charge q'' is placed at the point Q $(0,0,-a)$ in addition to the point charge with the charge q at the point P $(0, 0, a)$. Find the electric potential $\phi_2(x, y, z)$ at the point (x, y, z) in Fig. 2.5.

3. Assume that the electric potentials in Regions I and II in Fig. 2.3 are given by $\phi_1(x, y, z)$ and $\phi_2(x, y, z)$, respectively, obtained in the previous questions. Using $\varepsilon_1, \varepsilon_2$ and q, express q' and q'' defined in Questions II.1 and II.2 so that the boundary conditions at the interface between Regions I and II are satisfied.

4. Find the magnitude of the force on the point charge with the charge q as shown in Fig. 2.3.

5. Consider the force obtained in Question II.4. Assuming $\varepsilon_2 < \varepsilon_1$, find the direction of the force on the point charge with the charge q at the point P $(0, 0, a)$.

6. Find the surface charge density of the polarized charge $\sigma(x, y)$ at the interface of Regions I and II in Fig. 2.3.

7. Find the total polarized charge Σ at the interface in Fig. 2.3.