Tracking the tracker:

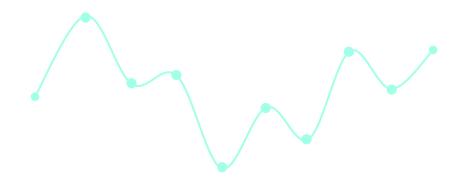
Time Series Analysis in Python From First Principles

Kenneth Emeka Odoh

Data Science & Machine Learning Meetups

Unbounce, Suite #400 - 401 W. Georgia Street Vancouver, BC

March **20th**, 2018



Talk Plan

- Bio
- Motivations & Goals
- Theory
- Application: Forecasting
- Application: Anomaly Detection
- Deployment
- Lessons
- Conclusion

Bio

Education

Masters in Computer Science (University of Regina)
 2013 - 2016

Work

- Research assistant in the field of Visual Analytics 2013 2016
 - (<u>https://scholar.google.com/citations?user=3G2ncgwAAAAJ&hl=en</u>)
- Software engineer working in an IOT startup (Current position)

This talk focuses on two applications in time series analysis:

Forecasting

• Can I use the past to predict the future?

Anomaly Detection

• Is a given point, normal or abnormal?

"Simplicity is the ultimate sophistication" --- Leonardo da Vinci

Motivation

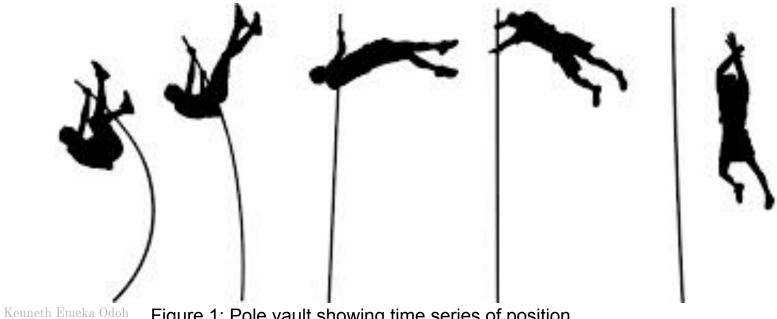
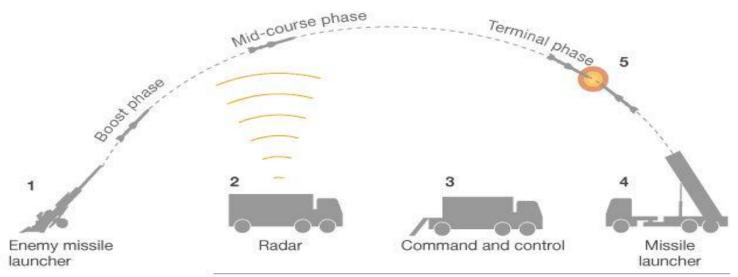


Figure 1: Pole vault showing time series of position

[http://vaultermagazine.com/middle-school-womens-pole-vault-rankings-from-athletic-net-5/]

How the Terminal High Altitude Area Defense System (Thaad) works



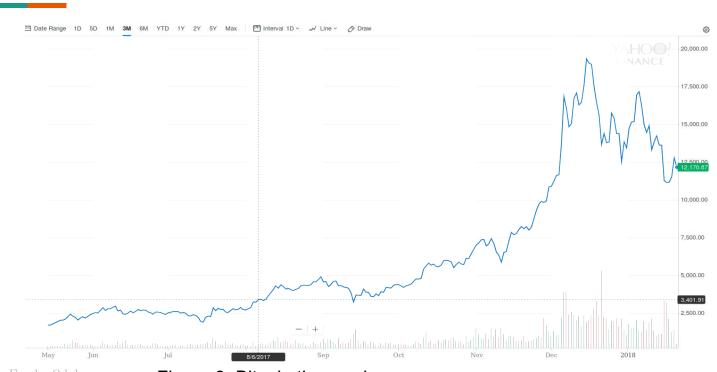
Thaad system

Source: Federation of American Scientists, US Department of Defense

Kenneth Emeka Odoh

Figure 2: Missile Defence System

[http://www.bbc.com/news/world-asia-39214525]



Kenneth Emeka Odoh

Figure 3: Bitcoin time series

[yahoo finance]

Goals

- A real-time model for time series analysis.
- A less computational intensive model thereby resulting in scalability.
- A support for multivariate / univariate time series data.

introducing

pySmooth

(https://github.com/kenluck2001/pySmooth) and

Anomaly

(https://github.com/kenluck2001/anomaly)

Application: Forecasting

Let's dive into the maths for some minutes



[http://www.boom-studios.com/2017/12/01/20th-century-fox-buys-girl-detective-adventure-movie-goldie-vance/]

Theory

- Time Series Data
 - \circ Y, where t=1,2,3,...,n where t is index and n is number of points.
- Time Series Decomposition

$$Y_t = V_t + S_t + H_t + \xi_t, \quad t = 1, ..., n$$

Sometimes the relation may be in multiplicative form

$$Y_t = V_t * S_t * H_t * \xi_t, \quad t = 1, ..., n$$

Where Y_t is predicted time series, V_t is trend, S_t is cyclic component, H_t is holiday component, and ξ_t is residual.

pth-Order Difference Equation, Autoregressive, AR(p)

$$\mathsf{AR}(\mathsf{p}) = \boldsymbol{\Theta}_1 \, \mathsf{Y}_{\mathsf{t-1}} + \boldsymbol{\Theta}_2 \, \mathsf{Y}_{\mathsf{t-2}} + ... + \boldsymbol{\Theta}_{\mathsf{p}} \, \mathsf{Y}_{\mathsf{t-p}} + \, \boldsymbol{\varepsilon}_{\mathsf{t}} \,, \, \mathsf{for} \, \mathsf{i} = 1, ..., \, \mathsf{p}$$

Where Θ is parameter, and ε_t is noise respectively. See **EWMA** for anomaly detection as an example of AR.

Moving Average Process, MA(q)

$$\mathsf{MA}(\mathsf{q}) = \boldsymbol{\Phi}_1 \, \boldsymbol{\varepsilon}_{\mathsf{t-1}} + \boldsymbol{\Phi}_2 \, \boldsymbol{\varepsilon}_{\mathsf{t-2}} \, + \ldots + \boldsymbol{\Phi}_{\mathsf{q}} \, \boldsymbol{\varepsilon}_{\mathsf{t-q}} \; , \mathsf{for} \, \mathsf{i} = 1, \ldots, \mathsf{q}$$

Where Φ is parameter, and $\varepsilon_{_{t}}$ is noise respectively.

Autoregressive Moving Average, ARMA

$$\mathsf{ARMA}(\mathsf{p},\mathsf{q}) = \Theta_1 \, \mathsf{Y}_{\mathsf{t}-1} + \Theta_2 \, \mathsf{Y}_{\mathsf{t}-2} \, + \ldots + \Theta_{\mathsf{p}} \, \mathsf{Y}_{\mathsf{t}-\mathsf{p}} + \, \boldsymbol{\varepsilon}_{\mathsf{t}} + \boldsymbol{\Phi}_1 \, \boldsymbol{\varepsilon}_{\mathsf{t}-1} + \boldsymbol{\Phi}_2 \, \boldsymbol{\varepsilon}_{\mathsf{t}-2} \, + \ldots + \boldsymbol{\Phi}_{\mathsf{q}} \, \boldsymbol{\varepsilon}_{\mathsf{t}-\mathsf{q}}$$

Where p, q, Θ , and Φ are parameters respectively.

Autoregressive Integrated Moving Average, ARIMA

ARIMA(p, d, q) = ARMA(p+d,q)

See Box-Jenkins methodology for choosing good parameters.

- ARCH
- GARCH (related to exponential weighted moving average). See section on anomaly detection.
- NGARCH
- IGARCH
- EGARCH

• • •

For more information,

see link (https://en.wikipedia.org/wiki/Autoregressive conditional heteroskedasticity)

Kenneth Emeka Odoh

Why Kalman Filters?

- Allows for using measurement signals to augment the prediction of your next states. State is the target variable.
- Allows for correction using the correct state from the recent past step.
- Allows for online prediction in real time.

It is good to note the following:

- How to identify the appropriate measurement signals?
- Measurement captures the trend of the state.
- Modeling can include domain knowledge e.g physics.

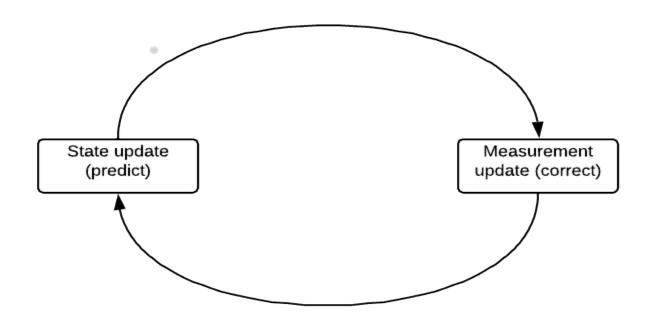


Figure 4: How kalman filter works

Kalman filter

$$\mathbf{x}_{k} = F(\mathbf{x}_{k-1}, \mathbf{w}) + B \cdot v_{k-1}$$

$$\begin{bmatrix} x_{k} \\ x_{k-1} \\ \vdots \\ x_{k-M+1} \end{bmatrix} = \begin{bmatrix} f(x_{k-1}, \dots, x_{k-M}, \mathbf{w}) \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} x_{k-1} \\ \vdots \\ x_{k-M} \end{bmatrix} \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdot v_{k-1}$$

$$y_k = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \cdot \mathbf{x}_k + n_k$$

Where x, y, F,n, v are states, measurement, function, measurement noise, and state noise respectively.

Kalman formulation allows for

- Handling missing data.
- Data fusion

It is a **state space** model.

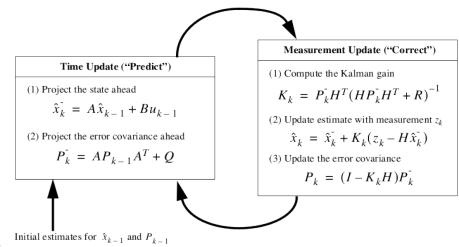
Continuous form of the forward algorithm of an HMM [Andrew Fraser, 2008].

[Julier, 2002]

Discrete Kalman Filter (DKF)

- Capture linear relationship in the data.
- Fluctuates around the means and covariance.
- Eventual convergence as the data increases, thereby improving filter performance (bayesian).

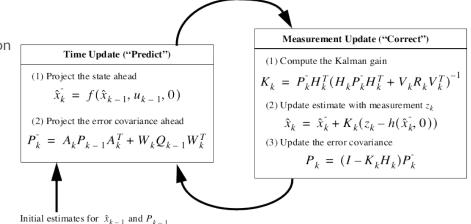
In the presence of nonlinear relationship. The increased **error** in the posterior estimates, thereby leading to suboptimal filter performance (**underfitting**).



Extended Kalman Filter (EKF)

- Makes use of Jacobians and hessians.
 - o Increase computation cost.
 - Need for **differentiable** non-linear function
- Linearizing nonlinear equation using taylor series to 1st order.
- Same computational complexity as Unscented kalman filter.
- Captures nonlinear relationship in the data.

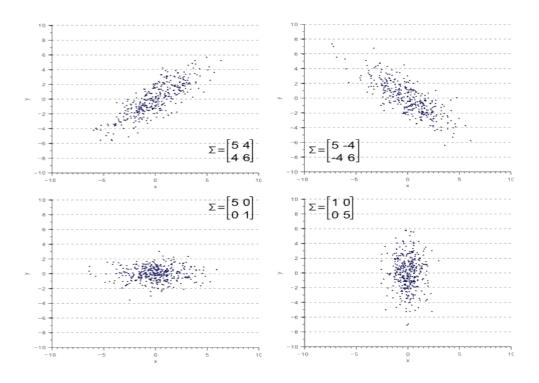
The limited order of taylor series can lead to **error** in the posterior estimates, thereby leading to suboptimal filter performance.



Unscented Kalman Filter (UKF)

- Better sampling to capture more representative characteristics of the model.
 - By using sigma points.
 - Sigma point are extra data points that are chosen within the region surrounding the original data. This helps to
 account for variability by capturing the likely position that data could be given some perturbation. Sampling
 these points provides richer information of the distribution of the data. It is more ergodic.
- Avoid the use of jacobians and hessians.
 - Possible to use any form of **nonlinear** function (not only differentiable function).
- Linearizing nonlinear function using taylor series to 3rd order.
- Same computational efficiency as the EKF.

Kenneth Emeka Odoh [Julier, 2002]



For more info, [http://www.visiondummy.co m/2014/04/geometric-interpre tation-covariance-matrix/]

Figure 5: Geometric interpretation of covariance matrix

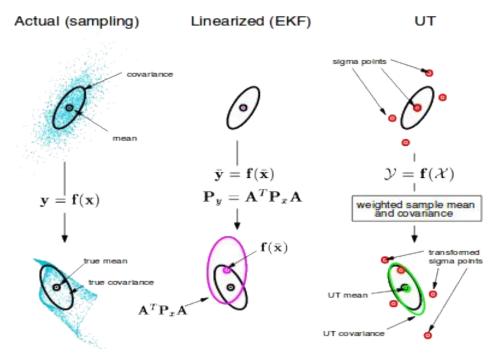


Figure 6: Actual sampling vs extended kalman filter, and unscented kalman filters

[Julier, 2002]

RLS (Recursive Linear Regression)

Initial model at time, t with an update as new data arrives at time t+1.

$$y = \Theta(t)^* x_t$$

At time t+1, we have data, x_{t+1} and y_{t+1} and estimate $\theta(t+1)$ in incremental manner Matrix Inversion Lemma RLS, version 1

At time step t + 1

- 1. Form x(t + 1) using the new data.
- 2. Form $\varepsilon(t+1) = y(t+1) x^T(t+1)\hat{\theta}(t)$
- 3. Form $P(t+1) = P(t) \left[I_m \frac{x(t+1)x^T(t+1)P(t)}{1+x^T(t+1)P(t)x(t+1)} \right]$
- 4. Update $\hat{\theta}(t+1) = \hat{\theta}(t) + P(t+1)x(t+1)\varepsilon(t+1)$
- 5. Loop back to step (1).

Online ARIMA

RLS



'Vanilla' ARIMA

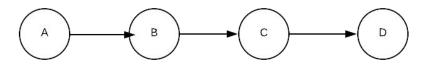
Box-Jenkins Methodology

- Transform data so that a form of stationarity is attained.
 - E.g taking logarithm of the data, or other forms of scaling.
- Guess values p, d, and g respectively for ARIMA (p, d, g).
- Estimate the parameters needed for future prediction.
- Perform diagnostics using AIC, BIC, or REC (Regression Error Characteristic Curves) [Jinbo & Kristin, 2003].



Stationarity

- Constant statistical properties over time
 - Joint probability distribution is shift-invariant
 - Can occur in different moments (mean, variance, skewness, and kurtosis)
- Why is it useful?
- Mean reverting.
- Markovian assumption
 - Future is independent of the past given the present



With states (A, B, C, D), then a sequence of states occurring in a sequence $P(A, B, C, D) = P(C \mid A, B) * P(B \mid A) * P(A)$

$$= P(C | B) * P(B | A) * P(A)$$

- n: number of states, m: number of observations
- S: set of states, Y: set of measurements

$$S = \{s_1, s_2, ..., s_n\}, Y = \{y_1, y_2, ..., y_m\}$$

- $P_{S(1)} = [p_1, p_2, ..., p_n] -- prior probability$
- T = P(s(t+1) | s(t)) --- transition probability
- $P_{S(t+1)} = P_{S(t)}^* T --- stationarity$
- Mapping states to measurement, P (y|s)
- Makes it easy to compose more relationship using bayes theorem in Kalman filtering formulation (continuous states and measurements).

 There are methods for converting data from non-stationary to stationary

(implicit in algorithm formulation or explicit in data preprocessing)

- Detrending
- Differencing
- Avoid using algorithm based on stationarity assumption with non-stationary data.
- Testing for stationarity
 - Formally, use unit root test [https://faculty.washington.edu/ezivot/econ584/notes/unitroot.pdf]
 - Informally, compare means or other statistic measures between different disjoint batches.

For a fuller treatment on stationarity, see [http://www.phdeconomics.sssup.it/documents/Lesson4.pdf]

Why stocks are hard to predict

• If **efficient market hypothesis** holds, then the stock times series, X_t is i.i.d. This means that the past **cannot** be used to predict the future.

$$P(X_{t} | X_{t-1}) = P(X_{t})$$

- Efficient market hypothesis
 - Stocks are always at the right price or value
 - Cannot profit by purchasing undervalued or overvalued asset.
 - Stock selection is not guaranteed to increase returns.
 - Profit can only be achieved by seeking out riskier investments.

In summary, it is "impossible to beat the market". Very Controversial

- "Essentially, all models are wrong, but some are useful." --- George Box
 - As such, there is still **hope** with prediction, albeit a faint one.
- o Avoid transient spurious correlation. (Domain knowledge, causal inference, & statistical models). Kenneth Emeka Odoh



API

- Time difference model
- Online ARIMA
- Discrete kalman filter
- Extended kalman filter
- Unscented kalman filter

API: Online ARIMA

```
import numpy as np
from RecursiveARIMA import RecursiveARIMA

X = np.random.rand(10,5)

recArimaObj = RecursiveARIMA(p=6, d=0, q=6)

recArimaObj.init(X)

x = np.random.rand(1,5)

recArimaObj.update(x)

print "Next state"
print recArimaObj.predict()
```

API: Discrete Kalman Filter

```
from DiscreteKalmanFilter import DiscreteKalmanFilter

X = np.random.rand(2,2)

Z = np.random.rand(2,2)

dkf = DiscreteKalmanFilter()
dkf.init(X,Z) #training phase

dkf.update()

x = np.random.rand(1,2)

z = np.random.rand(1,2)

print "Next state"

print dkf.predict(x,z)
```

import numpy as np

API: Extended Kalman Filter

```
from ExtendedKalmanFilter import ExtendedKalmanFilter
X = np.random.rand(2,2)
Z = np.random.rand(2,15)

dkf = ExtendedKalmanFilter()
dkf.init(X, Z) #training phase

dkf.update()
x = np.random.rand(1,2)
z = np.random.rand(1,15)

print "Next state"
print dkf.predict(x, z)
```

import numpy as np

API: Unscented Kalman filter

```
import numpy as np
from UnscentedKalmanFilter import UnscentedKalmanFilter

X = np.random.rand(2,2)
Z = np.random.rand(2,15)

dkf = UnscentedKalmanFilter()

dkf.init(X, Z) #training phase
 dkf.update()

x = np.random.rand(1,2)
z = np.random.rand(1,15)

print "Next state"
 print dkf.predict(x, z)
```

Other Approaches

- LSTM/RNN
- HMM
- Traditional ML methods

Application: Anomaly Detection

Anomaly Detection

It is ideal for unbalanced data set

• small number of negative samples and large number of positive samples or vice versa.

What is a normal data?

• This is the holy grail of a number of anomaly detection methods.

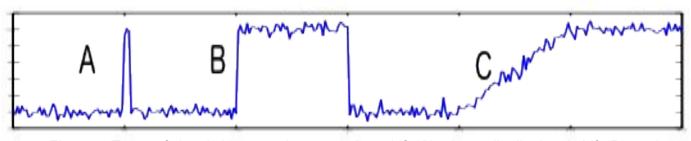


Figure 7: Types of signal changes: abrupt transient shift (**A**), abrupt distributional shift (**B**), and gradual distributional shift (**C**) [kelvin & william, 2012].

Data Stream

- Time and space constraints.
- Online algorithms
 - Detecting concept drift.
 - Forgetting unnecessary history.
 - Revision of model after significant change in distribution.
- Time delay to prediction.

Handling Data Stream

There are a number of window techniques:

- Fixed window
- Adaptive window (ADWIN)
- Landmark window
- Damped window

[Zhu & Shasha, 2002]

$$\begin{array}{ll} \textbf{Require:} & X_t, \hat{X}_t, \hat{\sigma}_t, T, t \\ \textbf{Ensure:} & P_t, \hat{X}_{t+1}, \hat{\sigma}_{t+1} \\ & Z_t \leftarrow \frac{X_t - \hat{X}_t}{\hat{\sigma}_t} & \text{\#Z-score} \\ & P_t \leftarrow \frac{1}{\sqrt{2\pi}} \exp(-\frac{Z_t^2}{2}) & \text{\#normal distribution pdf} \\ & \textbf{if } t \leq T \textbf{ then} & \text{\#define factor} \\ & \alpha_t \leftarrow 1 - 1/t & \text{\#weighing factor} \\ & \textbf{else} \end{array}$$

weighing factor

Weighted Moving

average

Weighted Moving

 $E(X^2)$

 $\alpha_t \leftarrow (1 - \beta P_t) \alpha$

 $s_1 \leftarrow \alpha_t s_1 + (1 - \alpha_t) X_t$

 $s_2 \leftarrow \alpha_t s_2 + (1 - \alpha_t) X_t^2$

 $\hat{\sigma}_{t+1} \leftarrow \sqrt{s_2 - s_1^2}$ # Moving standard deviation Algorithm 1: Probabilistic Exponential Moving Average [kelvin & william, 2012].

Exponentially Weighted Moving Average (EWMA)

- Add a forgetting parameter to weight recent items more or less.
- Volatile to abrupt transient change and bad with distributional shifts.

$$\mu_t = \alpha \mu_{t-1} + (1 - \alpha) X_t$$

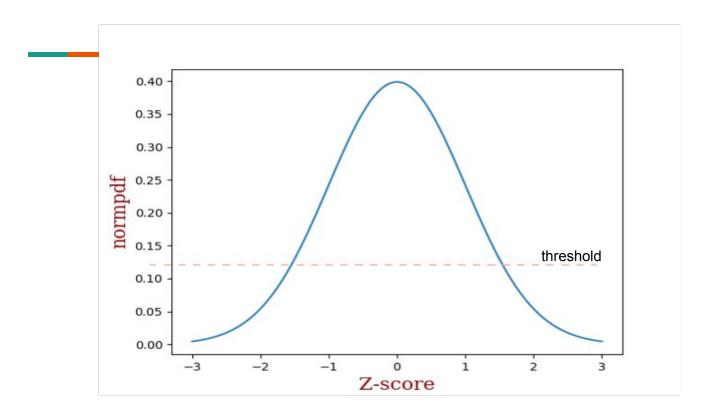
Probabilistic Exponentially Weighted Moving Average (PEWMA)

- Retains the forgetting parameter of EWMA with an added extra parameter to include probability of data.
- Works with every kind of shift.

$$\mu_t = \alpha (1 - \beta P_t) \mu_{t-1} + (1 - \alpha (1 - \beta P_t)) X_t$$

 $X_{t+1} \leftarrow s_1$

end if



Kenneth Emeka Odoh

Figure 8: Normal Distribution Curve.

Exercise [Group Activity]

 Modify Algorithm 1 to work for the multivariate case?

Deployment

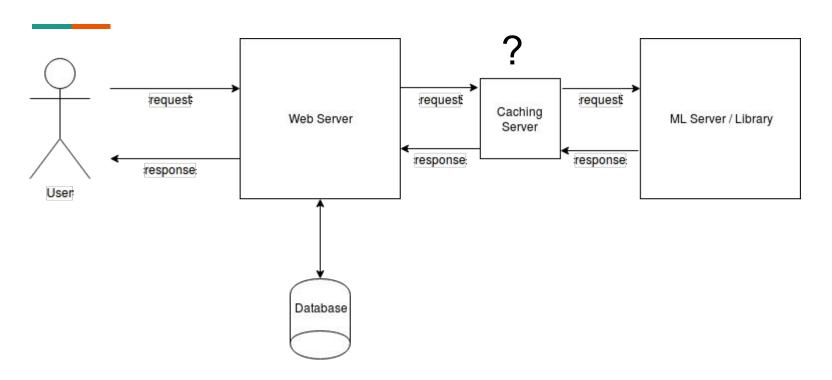
This aims to show a streaming architecture for serving content.

The goal is that the deployment should not be **AWS specific** to prevent getting locked down in a proprietary vendor.

- Flask
- Redis
- Gevent
- Scikit-learn / keras / numpy
- PostgreSQL

Machine learning as a service

- Web Server
 - Nginx (web server), Flask (lightweight web framework), and Gunicon (load balancer), Gevent (non-blocking I/O)
- Caching Server
 - Redis (as a transportation medium and caching)
- ML Server
 - Custom algorithm and third party libraries
- Database
 - PostgreSQL



Kenneth Emeka Odoh

Figure 9: An example of a deployment architecture.

Database

- Authentication
- metadata
- User management
 - Subscription
 - Managing limiting rates

Other Considerations

- Rather than serve json in your rest API, think of using Google protobuf.
- Use the database as a service, rather than tight coupling in your application.
- Flask is an easy to use web framework, think about using in MVP.
- Load balancer and replicated database.
- Make a Version control of your model to enhance reproducibility.
 - If necessary, map each prediction interval to a model.

Lessons

- "Prediction is very difficult, especially if it's about the future." -- Niels Bohr.
- "If you make a great number of predictions, the ones that were wrong will soon be forgotten, and the ones that turn out to be true will make you famous." -- Malcolm Gladwell.
- "I have seen the future and it is very much like the present, only longer." -- Kehlog Albran.



- Recursive ARIMA
 - Not ideal for chaotic or irregularly spaced time series.
- Higher orders and moment are necessary in the model, making unscented Kalman filter ideal for chaotic time series.
- Time difference model tends to overfit.
- MLE based method tend to under-estimates the variance.
- ARIMA requires domain knowledge to choose ideal parameters.
- Regularizers can be used in the RLS to reduce overfitting.
- Kalman gain ⇔ damped window

Kenneth Emeka Odoh

Thanks for listening



https://github.com/kenluck2001?tab=repositories



@kenluck2001



https://www.linkedin.com/in/kenluck2001/



kenneth.odoh@gmail.com



- 1. Castanon, D., & Karl, C. W. SC505: Stochastic processes. Retrieved 06/15, 2017, from http://www.mit.edu/people/hmsallum/GradSchool/sc505notes.pdf
- 2. Wellstead, P. E. & Karl, C. W. (1991). Self-tuning Systems: Control and Signal Processing. Chichester, United Kingdom: John Wiley & Sons Ltd.
- 3. Hamilton, J. D. (1994). Time series analysis. Chichester, United Kingdom: Princeton University Press.
- 4. Julier, S. J. The scaled unscented transformation. Retrieved 06/15, 2017, from https://www.cs.unc.edu/~welch/kalman/media/pdf/ACC02-IEEE1357.PDF
- 5. Terejanu, G. A. Extended kalman filter tutorial. Retrieved 06/15, 2017, from https://www.cse.sc.edu/~terejanu/files/tutorialEKF.pdf
- 6. Wan, E. A., & Merwe, R. (2000). The unscented kalman filters for nonlinear estimation. IEEE Adaptive Systems for Signal Processing, Communications, and Control Symposium, pp. 153-158.
- 7. Kevin M. Carter & William W. Streilein (2012). Probabilistic reasoning for streaming anomaly detection. In Proceedings of the Statistical Signal Processing Workshop, pp. 377–380.
- 8. Y.Zhu & D. Shasha (2002). Statstream: Statistical monitoring of thousands of data streams in real time. In Proceedings of the 28th international conference on Very Large Data Bases, 358–369. VLDB Endowment.
- 9. Welch, G., & Bishop, G. An introduction to the kalman filter. Retrieved 06/15, 2017, from https://www.cs.unc.edu/~welch/media/pdf/kalman_intro.pdf
- 10. Andrew Fraser (2008). Hidden markov models and Dynamical System. Retrieved 06/15, 2017, from http://www.siam.org/books/ot107/
- 11. Jinbo Bi & Kristin P. Bennett (2003). Regression Error Characteristic Curves. Retrieved 06/15, 2017, from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.566.4289&rep=rep1&type=pdf



Saturday, March 24, 2018

Study Group: Reliable and Distributed Algorithms



Hosted by Drew and Kenneth Emeka Odoh From Vancouver Tech Meetup

Upcoming Meetups

March 24th, 2018 | Sharing Memory

April 7th, 2018 | Consensus (paxo)

Venue: Vancouver public library, Downtown

Time: 2pm - 4pm