

①

P: is it fake  
Q: is it liked  
R: is it new

p: probability a story is fake  
q: probability a story is liked  
r: probability a story is new

	P	Q	R	probability	Payoff
1	T	T	T	$pqr$	a
2	T	T	F	$pq(1-r)$	b
3	T	F	T	$p(1-q)r$	c
4	T	F	F	$p(1-q)(1-r)$	d
5	F	T	T	$(1-p)qr$	e
6	F	T	F	$(1-p)q(1-r)$	f
7	F	F	T	$(1-p)(1-q)r$	g
8	F	F	F	$(1-p)(1-q)(1-r)$	h

ignore this case as it will not be propagated in social network

let us assume that 1/6 of all news items are fake news.

$$P(k \text{ events in interval}) = \frac{\lambda^k e^{-\lambda}}{k!}$$



~~$$P(k \text{ in interval } t) = \lambda^k e^{-\lambda t}$$~~

$$\frac{(rt)^k e^{-rt}}{k!}$$

$$\lambda = rt$$

$\lambda$ : average number of events

Expected utility,  $u(p, q, r)$

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$$u(p, q, r) = pqr * a + pq(1-r) * b + p(1-q)r * c + p(1-q)(1-r) * d \\ + (1-p)qr * e + (1-p)q(1-r) * f + (1-p)(1-q)r * g + (1-p)(1-q)(1-r) * h$$

$$E[q] = \frac{U+1}{U+D+2}$$

$U$ : number of apples in a news  
 $D$ : number of downvotes of a news

$$E(1-q) = \frac{D+1}{U+D+2}$$

$r = e^{-\lambda s}$  (probability that it is at least  $s$  seconds since last reload)

$P = \frac{\lambda^k e^{-\lambda}}{k!}$  (probability that fake news is at 1% of news item in an interval  
 $k = 1\%$  of element in interval)

set the utility

$$h=0$$

$$g=0$$

otherwise set  $a, b, c, d, e, f = 1$

$$u(p, q, r) = pqr + pq(1-r) + p(1-q)r + p(1-q)(1-r) + (1-p)qr + (1-p)q(1-r) \\ = p(qr + q(1-r) + (1-q)r + (1-q)(1-r)) + (1-p)(qr + q(1-r)) \\ = p(qr + q(1-r) + (1-q)r + (1-q)(1-r)) + (1-p)q(x + 1-x) \\ \frac{\lambda^k e^{-\lambda}}{k!} \left( \frac{U+1}{U+D+2} e^{-\lambda s} + \frac{U+1}{U+D+2} (1-e^{-\lambda s}) + \frac{D+1}{U+D+2} e^{-\lambda s} + \frac{(D+1)(1-e^{-\lambda s})}{U+D+2} \right) + \left( 1 - \frac{\lambda^k e^{-\lambda}}{k!} \right) \frac{U+1}{U+D+2}$$

$$\left( \frac{\cancel{Ue} + \cancel{e} + \cancel{U} - \cancel{Ue} + \cancel{1} - \cancel{e} + \cancel{De} + \cancel{e} + \cancel{D} - \cancel{De} + \cancel{1} - \cancel{e}}{U+D+2} \right)$$

$$\left( \frac{\cancel{U+D+1+1}}{\cancel{U+D+2}} \right) \Rightarrow \frac{\lambda^k e^{-\lambda}}{k!} + \left( 1 - \frac{\lambda^k e^{-\lambda}}{k!} \right) \frac{U+1}{U+D+2}$$

(3)

$$\frac{\lambda^k e^{-\lambda}}{k!} + \frac{u+1}{u+D+2} - \frac{\lambda^k e^{-\lambda}}{k!} \left( \frac{u+1}{u+D+2} \right)$$

$$\frac{\lambda^k e^{-\lambda} (u+D+2) + k! (u+1) - \lambda^k e^{-\lambda} (u+1)}{k! (u+D+2)}$$

$$\cancel{\lambda^k e^{-\lambda} (u+1)}$$

$$\frac{\cancel{\lambda^k e^{-\lambda} u} + \lambda^k e^{-\lambda} D + 2\lambda^k e^{-\lambda} - \cancel{\lambda^k e^{-\lambda} u} - \cancel{\lambda^k e^{-\lambda}}}{k! (u+D+2)}$$

$$= \frac{\lambda^k e^{-\lambda} D + 2\lambda^k e^{-\lambda} - \lambda^k e^{-\lambda}}{k! (u+D+2)}$$

$$= \frac{\lambda^k e^{-\lambda} D + \lambda^k e^{-\lambda}}{k! (u+D+2)}$$

$$= \frac{\lambda^k e^{-\lambda} (D+1)}{k! (u+D+2)}$$

(expected utility of a news)  
Considering the presence of fake news