# MASENO UNIVERSITY
# UNIVERSITY EXAMINATIONS 2024/2025

## SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, APPLIED STATISTICS, MATHEMATICAL SCIENCES, MATHEMATICS & ECONOMICS, BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

## MAIN CAMPUS

## MMA 227: CALCULUS II

Date: 23rd January, 2025 Time: 8.30 - 11.30am

**INSTRUCTIONS:**

- Do not write anywhere on this Question paper
- Answer Question ONE and any other TWO.
- More instructions on the answer booklet.

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**Question 1 (30 points)** .

a) State the fundamental theorem of calculus, and hence evaluate the definite integral $\int_{-1}^{1}(3t^2+6)dt$. (5 pts)

c b) Evaluate the average value of the function $f(x) = x/2$ over the interval $[1, 6]$ and find $c$ such that $f(c)$ equals the average value of the function over $[1, 6]$. (5 pts)

c) Evaluate the following integrals using appropriate techniques:

i) $\int \frac{3x^2}{1+x^3}dx$, (3 pts)

ii) $\int \cos^2(x)dx$ (4 pts)

iii) $\int 3t \cdot e^{2t}dt$. (4 pts)

iv) $\int \frac{x^2+3x+3}{x+1}dx$ (4 pts)

d) Evaluate the improper integral $\int_0^2 \frac{1}{\sqrt{4-2x}}dx$ and hence state whether it converges or diverges. (5 pts)

**Question 2 (20 points)** .

a) Evaluate the integrals

(i) $\int \cos(15t)\cos(11t)dt$ (4 pts)

(ii) $\int (2t^2+\sqrt{t})dt$ (4 pts)

(iii) $\int_0^{\pi} \frac{\sin^3(x)}{\cos^2(x)}dx$ (6 pts)

b) Calculate the area bounded by the curve $y = e^x$, the $y$-axis and the line $x = 2$. (6 pts)

**Question 3 (20 points)** .

a) Express $f(x) = \frac{12x-2}{6x^2-x-2}$ as a sum of partial fractions and hence evaluate $\int_1^5 \frac{12x-2}{6x^2-x-2}dx$ (6 pts)

b) By making an appropriate substitution, evaluate $\int_0^1 3x^2 \cdot e^{x^3}dx$ (4 pts)

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b) By making an appropriate substitution, evaluate $\int_0^1 3x^2 \cdot e^{x^3}\, dx$ (4 pts)

c) Evaluate the integral $\int \cos(x) \cdot e^x \, dx$ (5 pts)

d) Calculate the length of the curve $y = 5x - 2$ between $x = 1$ and $x = 5$. Give your answer in exact form. (5 pts)

**Question 4 (20 points)** .

a) Evaluate the improper integral $\int_0^2 x \ln x \, dx$. State whether the integral converges or diverges. (7 pts)

b) By applying the method of integration by parts twice, evaluate the integral $\int 2x^2 \cdot e^x dx$. (7 pts)

b) Calculate the volume of the solid formed when the part of the curve $y = x^2$ between $x = 2$ and $x = 4$ is rotated about the $x-$axis. (6 pts)

**Question 5 (20 points)** .

a) Evaluate the integral $\int \frac{7x}{\sqrt{5 - 4x - x^2}} dx$ by completing the square and making the appropriate trigonometric substitution (7 pts)

b) Define $\mathbf{R}$ as the region bounded above by the graph of $f(x) = 2x - x^2$ and below by the $x-$axis over the interval $[0, 3]$. Calculate (by the method of cylindrical shells) the volume of the solid of revolution formed by revolving $\mathbf{R}$ around the $y-$axis. (6 pts)

c) Use Simpsons rule $S_6$ to estimate the integral $\int_1^2 x^2 \, dx$, and hence calculate the absolute error and the relative error. (7 pts)