QUESTION ONE (30 MARKS)

(a) Given that I is an identity matrix find A if $(2I - A^T)^{-1} = \begin{bmatrix} 3 & 10 \\ -3 & 5 \end{bmatrix}$ (4 mks)

(b) If $A = \begin{bmatrix} 3 & 4 \\ -6 & 1 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 14 & 9 \\ -5 & 3 & -1 \end{bmatrix}$ show that $AB \neq BA$ (5 mks)

(c) Determine the values of x, y and z by reducing the given system of linear equations to echelon form. (6 mks)

$$3x - z + y = -5$$
$$2x - 2y + z = 6$$

(d) Given $\boldsymbol{p} = 2i - 7j - 3k$ and $\boldsymbol{q} = 2i + 5k$ find the projection of $\boldsymbol{p}$ on $\boldsymbol{q}$ (5 mks)

(e) Use Cramer's rule to find x_1, x_2, x_3 and x_4. (10 mks)

$$x_1 + x_2 - x_3 + x_4 = 3$$
$$2x_1 - 3x_2 - 4x_3 + x_4 = 7$$
$$3x_1 + 2x_2 + 6x_3 - 2x_4 = -1$$
$$5x_1 - x_3 + x_2 - 2x_4 = 0$$

QUESTION TWO (20 MARKS)

(a) If $detA = -22$ and $\det B = 16.2$ calculate $det(A^3B^{-1}B^2AB^T)$, given that matrices A and B are square matrices (3 mks)

(b) Given two vectors $\boldsymbol{a} = 3i - j + 2k$ and $\boldsymbol{b} = 4i - 4j + 2k$ calculate $\|2\boldsymbol{a} - 3\boldsymbol{b}\|$ (4 mks)

(c) Find the inverse of the matrix using matrix inversion algorithm

$$\begin{pmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ 1 & -3 & -2 \end{pmatrix}$$ (8 mks)

(d) Compute the determinant of $\begin{bmatrix} 2 & 4 & 1 & 5 \\ 3 & 2 & 5 & 1 \\ 1 & 2 & 1 & 4 \\ 3 & 4 & 3 & 2 \end{bmatrix}$ (5 mks)

QUESTION THREE (20 MARKS)

(a) Find the angle between the vectors $-5i + 2j - 3k$ and $4i + 3j - k$ (4 mks)

(b) Show that $\boldsymbol{a}.\boldsymbol{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\|\cos\theta$ (5 mks)

(c) Solve the system by Gauss-Jordan elimination (6 mks)

$$x + 2y + z = 4$$
$$3x - 2z + y = 3$$
$$5x + 3z + 5y = -8$$

(d) Given that $A = \begin{bmatrix} -3 & 6 & 2 \\ 1 & 5 & 4 \\ 4 & -8 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 9 & 3 \\ 2 & 10 & -4 \\ -3 & -18 & 6 \end{bmatrix}$

Prove that $\det(AB) = detAdetB$ (5 mks)

QUESTION FOUR (20 MARKS)

(a) Let $A = \begin{bmatrix} a+2x & -x & p \\ b+2y & -y & q \\ c+2z & -z & r \end{bmatrix}$ Evaluate $detA$ given that

$$det\begin{bmatrix} a & -p & x \\ b & -q & y \\ c & -r & z \end{bmatrix} = 32$$ (6 mks)

(b) Compute the rank of

$$\begin{bmatrix} 1 & -1 & 2 & -1 & 6 & 1 \\ 1 & 0 & -1 & 1 & 4 & 1 \\ 2 & 1 & 3 & -4 & -2 & 2 \\ 0 & -1 & 1 & -1 & 5 & -1 \end{bmatrix}$$ (6 mks)

(c) Find the solution of the following system of linear equations using Gaussian elimination with backward substitution (8mks)

$$x_1 + x_2 + 3x_4 = 4$$
$$2x_1 + x_2 - x_3 + x_4 = 1$$
$$3x_1 - x_2 - x_3 + 2x_4 = -3$$
$$-x_1 + 2x_2 + 3x_3 - x_4 = 4$$

QUESTION FIVE (20 MARKS)

(a) Determine if the following vectors are parallel, orthogonal or neither (4 mks)
$\langle 1,3,-1\rangle$ and $\langle 2,0,1\rangle$

(b) (i) Find β so that $6\beta i - \beta j + 8k$ and $\beta i + 2j - k$ are perpendicular. (3 mks)
(ii) A plane is defined by 3 points P(1,-1,4), Q(2,0,1) and R(0,2,3), find a vector perpendicular to the plane. (5 mks)

(c) Given the matrix

$$M = \begin{bmatrix} 1 & 2 & -1 \\ 3 & -4 & -3 \\ 6 & 5 & 0 \end{bmatrix}, \text{ Compute } (adjM)M$$ (8 mks)

QUESTION THREE (20 MARKS)

(a) Find the angle between the vectors $-5i + 2j - 3k$ and $4i + 3j - k$ (4 mks)

(b) Show that $\boldsymbol{a}.\boldsymbol{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\|cos\theta$ (5 mks)

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QUESTION FOUR (20 MARKS)

(a) Let $A = \begin{bmatrix} a + 2x & -x & p \\ b + 2y & -y & q \\ c + 2z & -z & r \end{bmatrix}$ Evaluate $detA$ given that

$$det \begin{bmatrix} a & -p & x \\ b & -q & y \\ c & -r & z \end{bmatrix} = 32$$ (6 mks)

(b) Compute the rank of

$$\begin{bmatrix} 1 & -1 & 2 & -1 & 6 & 1 \\ 1 & 0 & -1 & 1 & 4 & 1 \\ 2 & 1 & 3 & -4 & -2 & 2 \\ 0 & -1 & 1 & -1 & 5 & -1 \end{bmatrix}$$ (6 mks)

(c) Find the solution of the following system of linear equations using Gaussian elimination with backward substitution (8mks)

$$x_1 + x_2 + 3x_4 = 4$$
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