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MASENO UNIVERSITY
UNIVERSITY EXAMINATIONS 2024/2025

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 225: DISCRETE MATHEMATICS II

Date: 27th January, 2025 Time: 3.30 - 6.30pm

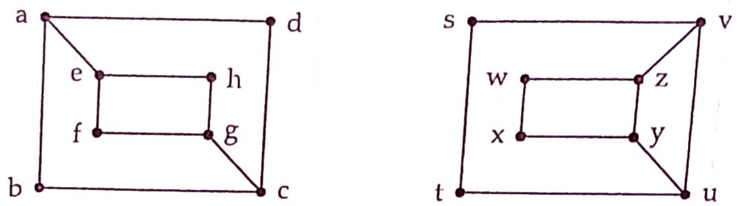
INSTRUCTIONS:

- DO NOT write anywhere on this question paper
- Answer Question ONE and any other TWO.

MMA 225: DISCRETE MATHEMATICS II

QUESTION ONE (Compulsory) [30 Marks]

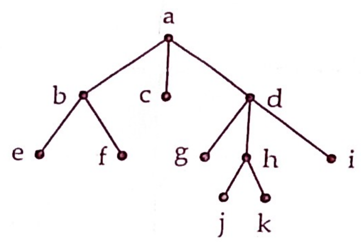
(a) Determine whether the following graphs are isomorphic or not. [3 Marks]



(b) Find the connectivity of the following graphs: K_n and C_n. [2 Marks]

(c) State and prove the handshaking lemma. Hence show that every graph has an even number of odd vertices. [6 Marks]

(d) In which order does an inorder traversal visit the vertices of the ordered rooted tree T below? [3 Marks]

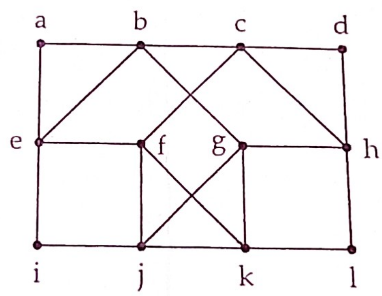


(e) Using techniques from graph theory, show that $1 + 2 + \cdots + n = n(n+1)/2$. [6 Marks]

(f) (i) State Kuratowski's theorem. [1 Mark]

(ii) Show that the complete graph K_5 is non-planar. [4 Marks]

(g) Use Depth-First search (DFS) to obtain a spanning tree of the simple graph shown below. [5 Marks]



QUESTION TWO [20 Marks]

(a) State the following: [4 Marks]

(i) A complete graph which is also a cycle graph.

(ii) A complete graph which is also a path graph.

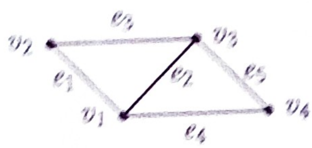
(iii) A complete graph which is also a bipartite graph.

(iv) A wheel graph which is also a regular graph.

(b) Prove that a connected graph G is Eulerian if and only if the degree of each vertex of G is even. [7 Marks]

(c) (i) State matrix-tree theorem. [1 Mark]

(ii) Find the adjacency matrix, incidence matrix and Laplacian matrix of the following graph.



Hence determine the number of spanning trees of the graph. [8 Marks]

QUESTION THREE [20 Marks]

(a) Write down the girths of the following graphs: [2 Marks]

(i) K_9

(ii) $K_{5,7}$

(b) Let G be a simple graph on n vertices. Prove that if G has k components, then the number m of edges of G satisfies $n-k \leq m \leq (n-k)(n-k+1)/2$. [6 Marks]

(c) Show that there is a gathering of five people in which there are no three people who all know each other and no three people none of whom knows either of the other two. [7 Marks]

(d) Describe the Chinese postman problem and explain how to solve this problem. [5 Marks]

QUESTION FOUR [20 Marks]

(a) State Dirac's theorem for the existence of a Hamilton circuit in a simple graph and hence use it to show that a complete graph K_n has a Hamilton circuit whenever $n \geq 3$. [3 Marks]

(b) State any two applications of graph theory in computer science. [2 Marks]

(c) Show that a planar graph is bipartite if and only if its dual is Eulerian. [7 Marks]

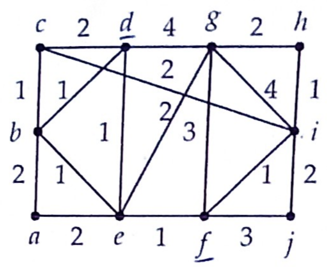
(d) Let G be a plane drawing of a connected planar graph, and let v, e and f denote respectively number of vertices, edges and faces of G. Prove that $v - e + f = 2$. [8 Marks]

QUESTION FIVE [20 Marks]

(a) Obtain the chromatic numbers of the following graphs:

(i) complete graph, K_n. (ii) wheel, W_n. [3 Marks]

(b) Consider the weighted graph below with vertices, $a, b, c, d, e, f, g, h, i$, and j:



(i) Use Kruskal's algorithm to find a minimum spanning tree of the weighted graph. What is the total weight of the spanning tree? [5 Marks]

(ii) Use Dijkstra's algorithm to find the shortest path between d and f. What is the length of the shortest path? [4 Marks]

(c) Describe the use of graph theory in solving the Königsberg bridge problem. [3 Marks]

(d) Using alphabetical order, construct a binary search tree for the words in the sentence "The quick brown fox jumps over the lazy dog." [5 Marks]

MASENO UNIVERSITY
UNIVERSITY EXAMINATIONS 2024/2025

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICAL SCIENCES AND BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE

MAIN CAMPUS

MMA 223: NUMERICAL ANALYSIS I

Date: 21st January, 2025 Time: 3.30 - 6.30pm

INSTRUCTIONS:

- Do not write anywhere on this Question paper
- Answer Question ONE and any other TWO
- Start each question on a fresh page
- Indicate question numbers clearly at the top of each page.

QUESTION ONE (COMPULSORY) [30 Marks]

a) Define the following terms

i. Absolute error [2mks

ii. Relative error [2mks

iii. Round-off error [2mks

b) What is the order of convergence of the Newton-Raphson method? Explain briefly how it is derived. [5mks

c) Explain the Bisection method. Why is the method guaranteed to converge and how can you estimate the number of iterations required to reach a desired accuracy?[5mks]

d) Construct a forward difference table for the following data [6mks]

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

e) Given the function $f(x) = \sin x$, approximate $f'(1)$ using the forward difference formula with step size $h = 0.1$. Compare the result with the exact derivative $f'(1) = \cos(1)$ [8mks]

QUESTION TWO [20 Marks]

a) Use the Newton-Raphson method to approximate the root of the equation $f(x) = x^2 - 2 = 0$, starting from $x_0 = 1$. Perform six iterations, correct to 6 d.p. [10mks]

b) Use the Bisection method to find an approximation to the root of the equation $f(x) = x^3 - 4x + 1 = 0$ in the interval $[1, 2]$. Perform four iterations. [10mks]

QUESTION THREE [20 Marks]

a) Use the Trapezoidal Rule to approximate the integral

$$\int_0^2 (x^2 + 1)dx.$$

Divide the interval into 4 equal subintervals and compute the numerical solution. [10mks]

b) Use Central Difference Formula to approximate the derivative of the function $f(x) = e^x$ at $x = 1$ with a step size $h = 0.01$. Estimate the error. [10mks]

QUESTION FOUR [20 Marks]

a) The following data gives the melting point of an alloy of lead and zinc, where t is the temperature in degrees Celsius and P is the percentage of lead in the alloy.

P	40	50	60	70	80	90
t	180	204	226	250	276	304

Find the melting point of the alloy containing 84% lead. [10mks]

b) The following are the population of a district

Year(x)	1981	1991	2001	2011	2021	2031
Population(y)	363	391	421	?	467	501

Find the population of the year 2011. [10mks]

QUESTION FIVE [20 Marks]

a) Find the root of the equation $x^2 - 2x - 3 = 0$ using the fixed point iteration method. Use the initial guess $x_0 = 4$ and iterate until $|x_{n+1} - x_n| < 0.001$ [10mks]

b) Estimate the integral $I = \int_0^1 (x^2 + 1)dx$ using the Romberg Integration Method. [10mks]

END

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SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE, APPLIED STATISTICS, MATHEMATICAL SCIENCES, MATHEMATICS & ECONOMICS, BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 227: CALCULUS II

Date: 23rd January, 2025 Time: 8.30 - 11.30am

INSTRUCTIONS:

- Do not write anywhere on this Question paper
- Answer Question ONE and any other TWO.
- More instructions on the answer booklet.

Question 1 (30 points)

a) State the fundamental theorem of calculus, and hence evaluate the definite integral $\int_{-1}^{1} (3t^2 + 6)dt$ (5 pts)

b) Evaluate the average value of the function $f(x) = x/2$ over the interval $[1, 6]$ and find c such that $f(c)$ equals the average value of the function over $[1, 6]$. (5 pts)

c) Evaluate the following integrals using appropriate techniques:

i) $\int \frac{3x^2}{1+x^3}dx$, (3 pts)

ii) $\int \cos^2(x)dx$ (4 pts)

iii) $\int 3t \cdot e^{2t}dt$. (4 pts)

iv) $\int \frac{x^2+3x+3}{x+1}\,dx$ (4 pts)

d) Evaluate the improper integral $\int_0^2 \frac{1}{\sqrt{4-2x}}dx$ and hence state whether it converges or diverges. (5 pts)

Question 2 (20 points) .

a) Evaluate the integrals

(i) $\int \cos(15t)\cos(11t)dt$ (4 pts)

(ii) $\int (2t^2 + \sqrt{t})dt$ (4 pts)

(iii) $\int_0^{\pi} \frac{\sin^3(x)}{\cos^2(x)}dx$ (6 pts)

b) Calculate the area bounded by the curve $y = e^x$, the y-axis and the line $x = 2$. (6 pts)

Question 3 (20 points) .

a) Express $f(x) = \frac{12x-2}{6x^2-x-2}$ as a sum of partial fractions and hence evaluate $\int_1^5 \frac{12x-2}{6x^2-x-2}\,dx$ (6 pts)

b) By making an appropriate substitution, evaluate $\int_0^1 3x^2 \cdot e^{x^3}\,dx$ (4 pts)

c) Evaluate the integral $\int \cos(x) \cdot e^x \, dx$ (5 pts)

d) Calculate the length of the curve $y = 5x - 2$ between $x = 1$ and $x = 5$. Give your answer in exact form. (5 pts)

Question 4 (20 points) .

a) Evaluate the improper integral $\int_0^2 x \ln x \, dx$. State whether the integral converges or diverges. (7 pts)

b) By applying the method of integration by parts twice, evaluate the integral $\int 2x^2 \cdot e^x dx$. (7 pts)

b) Calculate the volume of the solid formed when the part of the curve $y = x^2$ between $x = 2$ and $x = 4$ is rotated about the $x-$axis. (6 pts)

Question 5 (20 points) .

a) Evaluate the integral $\int \frac{7x}{\sqrt{5 - 4x - x^2}} dx$ by completing the square and making the appropriate trigonometric substitution (7 pts)

b) Define R as the region bounded above by the graph of $f(x) = 2x - x^2$ and below by the $x-$axis over the interval $[0, 3]$. Calculate (by the method of cylindrical shells) the volume of the solid of revolution formed by revolving R around the $y-$axis. (6 pts)

c) Use Simpsons rule S_6 to estimate the integral $\int_1^2 x^2 \, dx$, and hence calculate the absolute error and the relative error. (7 pts)

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UNIVERSITY EXAMINATIONS 2024/2025

SECOND YEAR FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

MMA 229: VECTOR ANALYSIS

Date: 22nd January, 2025 Time: 8.30 - 11.30am

INSTRUCTIONS:

- Do not write anywhere on this Question paper
- Answer Question ONE and any other TWO.

Question 1: Compulsory (30 marks)

a) Find the area of a parallelogram whose adjacent sides are i − 2j + 3k and 2i + j − 4k. (4mks)

b) A constant force of $\vec{F} = 10\mathrm{i} + 2\mathrm{j} - \mathrm{k}$ newtons displaces an object from $\vec{OA} = \mathrm{i} + \mathrm{j} + \mathrm{k}$ to $\vec{OB} = 2\mathrm{i} - \mathrm{j} + 3\mathrm{k}$ (in metres). Find the work done in newton metres. (4mks)

c) The equation $\frac{2x-1}{3} = \frac{y+4}{3} = \frac{-z+5}{2}$ represents a straight line. Express this in vector form. (4mks)

d) If $\phi = 3x^2y - y^3z^2$, find grad Φ at the point $(1, -2, -1)$. (4mks)

e) Find $\frac{d^2\vec{r}}{dt^2}$ given that $\vec{r} = t^3\mathrm{i} + (t^2 + \frac{1}{t})\mathrm{j} + (3t^2 + 1)\mathrm{k}$. (2mks)

f) If $\vec{A} = (3x^2 + 6y)\mathrm{i} - 14yz\mathrm{j} + 20xz^2\mathrm{k}$, evaluate the line integral $\oint \vec{A}.d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve C with $x = t$; $y = t^2$; $z = t^3$. (7mks)

g) Determine the angle between $\vec{a}$ and $\vec{b}$ when $\vec{a} = \mathrm{i} + 2\mathrm{j} - 3\mathrm{k}$ and $\vec{b} = 2\mathrm{i} - \mathrm{j} + 4\mathrm{k}$. (5mks)

h If $\vec{a} = 2\mathrm{i} - 7\mathrm{j} + \mathrm{k}$, find the unit vector in the direction of $\vec{a}$. 2mks

Question 2 (20 marks)

a) Find the moment and the magnitude of the moment of a force of $(\mathrm{i}+2\mathrm{j}-3\mathrm{k})$ newtons about point B having coordinates $(0, 1, 1)$ when the force acts on a line through A whose coordinates are $(1, 3, 4)$. (7mks)

b) A particle moves along the curve $\vec{r} = (t^3 - 4t)\mathrm{i} + (t^2 + 4t)\mathrm{j} + (8t^2 - 3t^3)\mathrm{k}$, where t is the time. Find the magnitude of the tangential components of its acceleration at $t = 2$. (9mks)

$\tilde{x}.\tilde{F}$
$\tilde{r}.\tilde{r}$

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c) Find the unit normal to the surface $xy^3z^2 = 4$ at $(-1, -1, 2)$. (4mks)

Question 3 (20 marks)

a) Given that $\vec{a} = \mathrm{i} + 4\mathrm{j} - \mathrm{k}$ and $\vec{b} = 2\mathrm{i} - 3\mathrm{j} + \mathrm{k}$, find:

i) the projection of $\vec{a}$ in the direction of $\vec{b}$

ii) the projection of $\vec{b}$ in the direction of $\vec{a}$

(4mks)

b) If a vector field is given by $\vec{F} = (x^2 - y^2 + x)\mathrm{i} - (2xy + y)\mathrm{j}$, is this field irrotational? If so, find its scalar potential. (8mks)

c) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (8mks)

Question 4 (20 marks)

a) A vector field is given by $\vec{F} = (2y + 3)\mathrm{i} + xz\mathrm{j} + (yz - x)\mathrm{k}$. Evaluate $\oint_c \vec{F}.d\vec{r}$ along the path C with $x = 2t; y = t; z = t^3$ from $t = 0$ to $t = 1$. (8mks)

b) Evaluate $\int\int_S \vec{A}.\hat{n}dS$ where $\vec{A} = 18z\mathrm{i} - 12\mathrm{j} + 3y\mathrm{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ included in the first octant. (12mks)

Question 5 (20 marks)

a) If U_1, U_2, U_3 ar curvilinear coordinates such that $x = \frac{1}{2}(U_1 - 3), y = U_2 + 4$ and $z = U_3 - 2$, prove that the system is orthogonal. Find the scale factor and the expression for $(ds)^2$. (12mks)

b) Evaluate $\oint_c \vec{F}.d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2\mathrm{i} + x^2\mathrm{j} - (x + z)\mathrm{k}$ and c is the boundary of the triangle with vertices at $(0, 0, 0), (1, 0, 0)$ and $(1, 1, 0)$. (8mks)