Question 1: Compulsory (30 marks)

a) Find the area of a parallelogram whose adjacent sides are $\mathrm{i} - 2\mathrm{j} + 3\mathrm{k}$ and $2\mathrm{i} + \mathrm{j} - 4\mathrm{k}$. (4mks)

b) A constant force of $\vec{F} = 10\mathrm{i} + 2\mathrm{j} - \mathrm{k}$ newtons displaces an object from $\vec{OA} = \mathrm{i} + \mathrm{j} + \mathrm{k}$ to $\vec{OB} = 2\mathrm{i} - \mathrm{j} + 3\mathrm{k}$ (in metres). Find the work done in newton metres. (4mks) 16

c) The equation $\frac{2x-1}{3} = \frac{y+4}{3} = \frac{-z+5}{2}$ represents a straight line. Express this in vector form. (4mks) 24

d) If $\phi = 3x^2y - y^3z^2$, find grad Φ at the point $(1, -2, -1)$. (4mks)

e) Find $\frac{d^2\vec{r}}{dt^2}$ given that $\vec{r} = t^3\mathrm{i} + (t^2 + \frac{1}{t})\mathrm{j} + (3t^2 + 1)\mathrm{k}$. (2mks)

f) If $\vec{A} = (3x^2 + 6y)\mathrm{i} - 14yz\mathrm{j} + 20xz^2\mathrm{k}$, evaluate the line integral $\oint \vec{A}.d\vec{r}$ from $(0, 0, 0)$ to $(1, 1, 1)$ along the curve C with $x = t$; $y = t^2$; $z = t^3$. (7mks)

g) Determine the angle between $\vec{a}$ and $\vec{b}$ when $\vec{a} = \mathrm{i} + 2\mathrm{j} - 3\mathrm{k}$ and $\vec{b} = 2\mathrm{i} - \mathrm{j} + 4\mathrm{k}$. (5mks) 5

h If $\vec{a} = 2\mathrm{i} - 7\mathrm{j} + \mathrm{k}$, find the unit vector in the direction of $\vec{a}$. 2mks

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Question 2 (20 marks)

a) Find the moment and the magnitude of the moment of a force of $(\mathrm{i}+2\mathrm{j}-3\mathrm{k})$ newtons about point B having coordinates $(0, 1, 1)$ when the force acts on a line through A whose coordinates are $(1, 3, 4)$. (7mks)

b) A particle moves along the curve $\vec{r} = (t^3 - 4t)\mathrm{i} + (t^2 + 4t)\mathrm{j} + (8t^2 - 3t^3)\mathrm{k}$, where t is the time. Find the magnitude of the tangential components of its acceleration at $t = 2$. (9mks) 9

c) Find the unit normal to the surface $xy^3z^2 = 4$ at $(-1,-1,2)$. (4mks)

Question 3 (20 marks)

a) Given that $\vec{a} = \mathrm{i} + 4\mathrm{j} - \mathrm{k}$ and $\vec{b} = 2\mathrm{i} - 3\mathrm{j} + \mathrm{k}$, find:

$P \; \frac{a \cdot b}{b \cdot b} \, b$

i) the projection of $\vec{a}$ in the direction of $\vec{b}$

ii) the projection of $\vec{b}$ in the direction of $\vec{a}$

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(4mks)

b) If a vector field is given by $\vec{F} = (x^2 - y^2 + x)\mathrm{i} - (2xy + y)\mathrm{j}$, is this field irrotational? If so, find its scalar potential. (8mks)

c) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (8mks)

Question 4 (20 marks)

a) A vector field is given by $\vec{F} = (2y + 3)\mathrm{i} + xz\mathrm{j} + (yz - x)\mathrm{k}$. Evaluate $\oint_c \vec{F}.d\vec{r}$ along the path C with $x = 2t; y = t; z = t^3$ from $t = 0$ to $t = 1$. (8mks)

b) Evaluate $\int\int_S \vec{A}.\hat{n}dS$ where $\vec{A} = 18z\mathrm{i} - 12\mathrm{j} + 3y\mathrm{k}$ and S is the part of the plane $2x + 3y + 6z = 12$ included in the first octant. (12mks)

Question 5 (20 marks)

a) If U_1, U_2, U_3 ar curvilinear coordinates such that $x = \frac{1}{2}(U_1 - 3), y = U_2 + 4$ and $z = U_3 - 2$, prove that the system is orthogonal. Find the scale factor and the expression for $(ds)^2$. (12mks)

b) Evaluate $\oint_c \vec{F}.d\vec{r}$ by Stoke's theorem, where $\vec{F} = y^2\mathrm{i} + x^2\mathrm{j} - (x + z)\mathrm{k}$ and c is the boundary of the triangle with vertices at $(0,0,0)$, $(1,0,0)$ and $(1,1,0)$. (8mks)