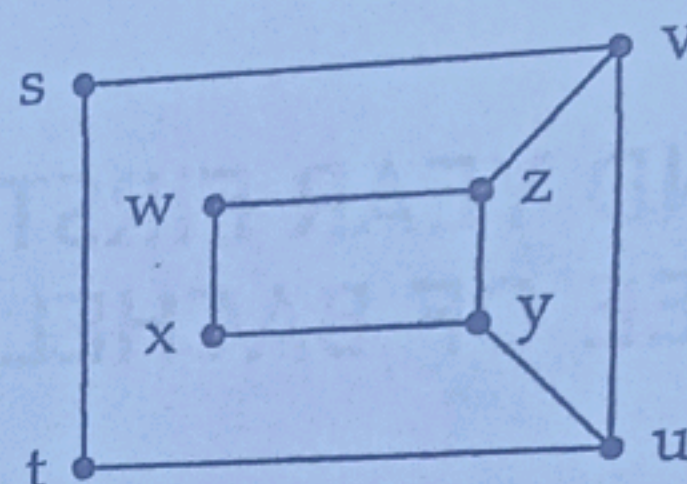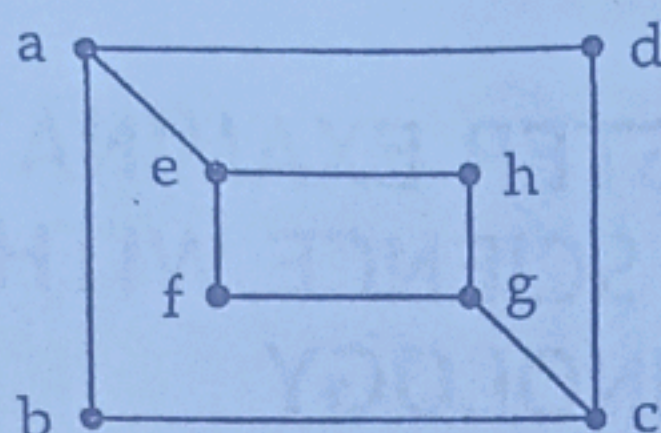# MMA 225: DISCRETE MATHEMATICS II

QUESTION ONE (Compulsory)   [30 Marks]

(a) Determine whether the following graphs are isomorphic or not.   [3 Marks]

(b) Find the connectivity of the following graphs: K_n and C_n.   [2 Marks]

(c) State and prove the handshaking lemma. Hence show that every graph has an even number of odd vertices.   [6 Marks]

(d) In which order does an inorder traversal visit the vertices of the ordered rooted tree T below?   [3 Marks]
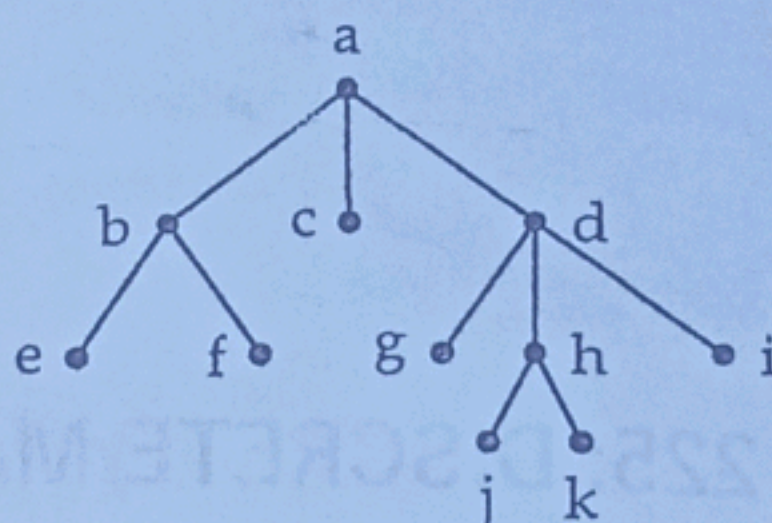
(e) Using techniques from graph theory, show that $1+2+\cdots+n = n(n+1)/2$.   [6 Marks]

(f) (i) State Kuratowski's theorem.   [1 Mark]

(ii) Show that the complete graph K_5 is non-planar.   [4 Marks]

(g) Use Depth-First search (DFS) to obtain a spanning tree of the simple graph shown below.   [5 Marks]
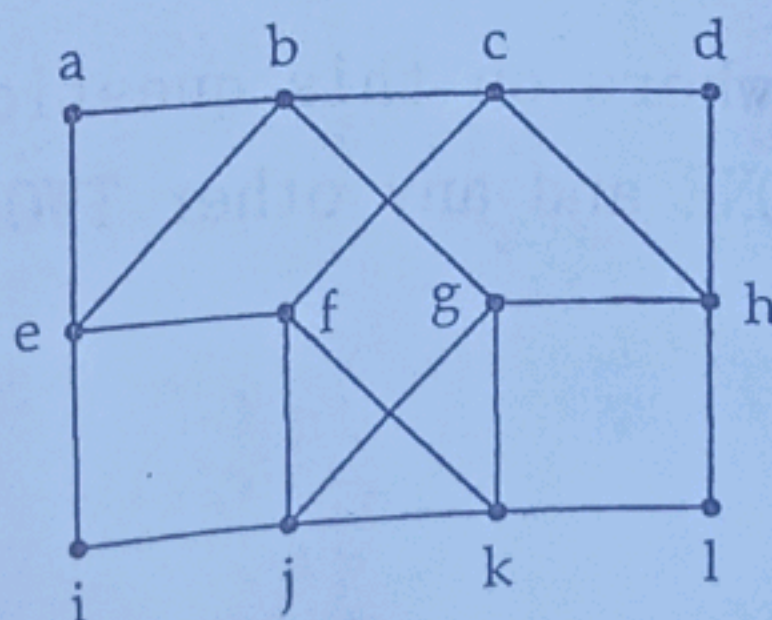
QUESTION TWO
[20 Marks]

(a) State the following: [4 Marks]

   (i) A complete graph which is also a cycle graph.

   (ii) A complete graph which is also a path graph.

   (iii) A complete graph which is also a bipartite graph.
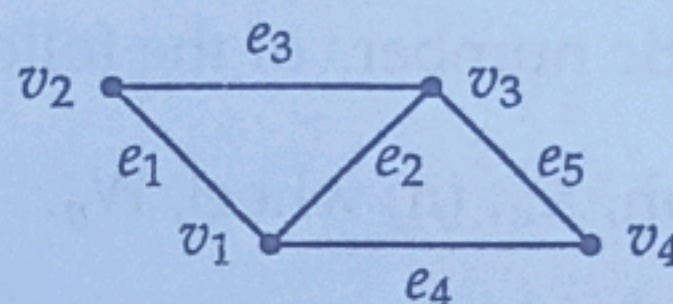
   (iv) A wheel graph which is also a regular graph.

(b) Prove that a connected graph G is Eulerian if and only if the degree of each vertex of G is even. [7 Marks]

(c) (i) State matrix-tree theorem. [1 Mark]

   (ii) Find the adjacency matrix, incidence matrix and Laplacian matrix of the following graph.

Hence determine the number of spanning trees of the graph. [8 Marks]

QUESTION THREE
[20 Marks]

(a) Write down the girths of the following graphs: [2 Marks]

   (i) K_9

   (ii) $K_{5,7}$

(b) Let G be a simple graph on n vertices. Prove that if G has k components, then the number m of edges of G satisfies $n - k \le m \le (n - k)(n - k + 1)/2$. [6 Marks]

(c) Show that there is a gathering of five people in which there are no three people who all know each other and no three people none of whom knows either of the other two. [7 Marks]

(d) Describe the Chinese postman problem and explain how to solve this problem. [5 Marks]

QUESTION FOUR [20 Marks]

(a) State Dirac's theorem for the existence of a Hamilton circuit in a simple graph and hence use it to show that a complete graph K_n has a Hamilton circuit whenever $n \geq 3$. [3 Marks]

(b) State any two applications of graph theory in computer science. [2 Marks]

(c) Show that a planar graph is bipartite if and only if its dual is Eulerian. [7 Marks]

(d) Let G be a plane drawing of a connected planar graph, and let v, e and f denote respectively number of vertices, edges and faces of G. Prove that $v - e + f = 2$.

[8 Marks]

QUESTION FIVE [20 Marks]

(a) Obtain the chromatic numbers of the following graphs:

(i) complete graph, K_n. (ii) wheel, W_n. [3 Marks]

(b) Consider the weighted graph below with vertices, $a, b, c, d, e, f, g, h, i$, and j:
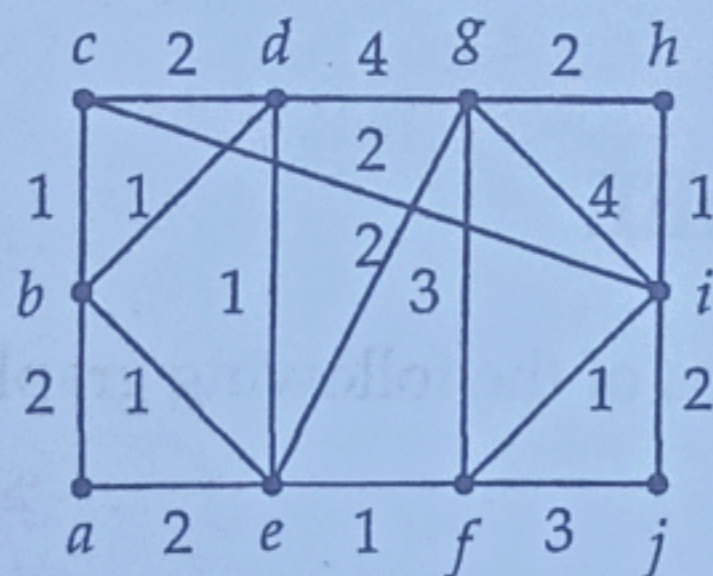
(i) Use Kruskal's algorithm to find a minimum spanning tree of the weighted graph. What is the total weight of the spanning tree? [5 Marks]

(ii) Use Dijkstra's algorithm to find the shortest path between d and f. What is the length of the shortest path? [4 Marks]

(c) Describe the use of graph theory in solving the Königsberg bridge problem. [3 Marks]

(d) Using alphabetical order, construct a binary search tree for the words in the sentence "The quick brown fox jumps over the lazy dog." [5 Marks]