QUESTION ONE [Compulsory] [30 Marks]

a) A car dealer offers purchasers a three-year warranty on a new car. He sells two models, the Zippy and the Nifty. For the first 50 cars sold of each model the number of claims under the warranty is shown in the table below.

A A

	Claim	No Claim
Zippy	35	15
B Nifty	40	10

One of the purchasers is chosen at random. Let A be the event that no claim is made by the purchaser under the warranty and B the event that the car purchased is a Nifty.

i) Find $p(A \cap B)$. (2 Marks)

ii) Find $p(A')$. (2 Marks)

iii) Given that the purchaser chosen does not make a claim under the warranty, find the probability that the car purchased is a Zippy. (3 Marks)

iv) Show that making a claim is not independent of the make of the car purchased. Comment on this result. (3 Marks)

b) A random variable X has probability density function

$$p(x) = p(X = x) = \begin{cases} \frac{x}{c}, x = 1, 2, 3, 4 \\ 0, elsewhere \end{cases}$$

where c is a constant. Determine:

i) the value of the constant c. (3 Marks)

ii) $E[X]$. (2 Marks)

iii) $Var[X]$. (2 Marks)

iv) $F(x)$, the cdf of X. Plot $F(x)$. (3 Marks)

c) The continuous random variable X has probability density function

$$f(x) = \begin{cases} 3 - 48x^2, -0.25 \leqslant x \leqslant 0.25 \\ 0, elsewhere \end{cases}$$

Find $p(\frac{1}{8} \leqslant x \leqslant \frac{1}{4})$. (5 Marks)

d) If for a certain random variable X, $p(X < 500) = 0.5$ and $p(X > 650) = 0.0228$, find the standard deviation of X. (5 Marks)

QUESTION TWO [20 Marks]

a) A fair die has six faces numbered 1, 2, 2, 3, 3 and 3. The die is rolled twice and the number showing on the uppermost face is recorded each time. Find the probability that the sum of the two numbers recorded is at least 5. (3 Marks)

b) A discrete random variable X has a probability function as shown in the table below, where a and b are constants.

x	0	1	2	3
$p(X = x)$	0.2	0.3	b	a

Given that $E[X] = 1.7$,

i) Find the value of a and b. (5 Marks)

ii) Find $E[2X - 3]$. (2 Marks)

iii) Find $Var[X]$. (3 Marks)

iv) Evaluate $Var[2X - 3]$. (2 Marks)

c) The continuous random variable Y is normally distributed with mean 100 and variance 256, $(Y \sim N(100, 16^2))$. Find k such that $p(100 - k \leqslant Y \leqslant 100 + k) = 0.516$. (5 Marks)

QUESTION THREE [20 Marks]

a) A market researcher asked 100 adults which of the three newspapers A, B, C they read. The results showed that 30 read A, 26 read B, 21 read C, 5 read both A and B, 7 read both B and C, 6 read both C and A and 2 read all three.

i) Draw a Venn diagram to represent these data. (3 Marks)

One of the adults is then selected at random. Find the probability that she reads:

ii) at least one of the newspapers, (2 Marks)

iii) only A, (1 Mark)

iv) only one of the newspapers, (2 Marks)

v) A given that she reads only one newspaper. (2 Marks)

b) A continuous random variable X has probability density function

$$f(x) = \begin{cases} kx(x-2), 2 \leqslant x \leqslant 3 \\ 0, elsewhere \end{cases}$$

where k is a positive constant. Find

i) k. (2 Marks)

ii) $E[X]$. (2 Marks)

iii) the cumulative density function $F(x)$. (3 Marks)

c) The random variable $X \sim Bin(150, 0.02)$.

Use a suitable approximation to estimate $p(X > 7)$. (3 Marks)

QUESTION FOUR [20 Marks]

a) Consider the random variable X uniformly distributed on the interval [-2, 4]. Find $F(x)$ and compute $p(X > 2)$. (7 Marks)

Hint: For $X \sim U[a, b]$,

$$f(x) = \begin{cases} \frac{1}{b-a}, a \leqslant x \leqslant b \\ 0, elsewhere \end{cases}$$

b) Paul writes an examination in statistics. He can pass the examination or not. The random variable of interest is X, which describes whether he passes the examination $(X = 1)$ or not $(X = 0)$. The probability that Paul passes the examination is p. Write down the probability function of X, and find $E[X]$ and $Var[X]$. (6 Marks)

c) A health club lets members use, on each visit, its facilities for as long as they wish. The club's records suggest that the length of a visit can be modelled by a normal distribution with mean 90 minutes. Only 20% of members stay for more than 125 minutes. Find the:

i) standard deviation of the normal distribution. (4 Marks)

ii) probability that a visit lasts less than 25 minutes. (3 Marks)

QUESTION FIVE [20 Marks]

a) A random variable X has moment generating function

$$M_X(t) = \left(\frac{1}{3}e^t + \frac{2}{3}\right)^6, t \in \mathbb{R}$$

Find $p(2 < X \leqslant 4)$. (5 Marks)

b) The number of times a bus comes to a bus station in an hour has a Poisson distribution. If the probability of no bus showing up in an hour is $\frac{1}{500}$, find the probability of having 2 buses or more showing up in an hour. (5 Marks)

Hint: For $X \sim P_o(\lambda)$,

$$p(x) = p(X = x) = \begin{cases} \frac{\lambda^x e^{-\lambda}}{x!}, x = 0, 1, 2, \ldots \\ 0, \text{elsewhere} \end{cases}$$

c) A random variable T has an exponential distribution such that $p(T \leqslant 2) = 2p(T > 4)$. Find $Var[T]$. (10 Marks)

Hint: For $X \sim Exp(\lambda)$,

$$f(x) = \begin{cases} \lambda e^{-\lambda x}, x > 0 \\ 0, \text{elsewhere} \end{cases}$$

and $Var[X] = \frac{1}{\lambda^2}$

Standard Normal Tables

z	Distribution function									
	0	1	2	3	4	5	6	7	8	9
0.0	0.5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
0.1	0.5398	5438	5478	5517	5557	5596	5636	5675	5714	5763
0.2	0.5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
0.3	0.6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
0.4	0.6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
0.5	0.6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
0.6	0.7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
0.7	0.7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
0.8	0.7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
0.9	0.8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1.0	0.8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1.1	0.8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1.2	0.8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1.3	0.9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1.4	0.9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1.5	0.9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1.6	0.9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1.7	0.9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1.8	0.9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1.9	0.9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2.0	0.9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2.1	0.9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2.2	0.9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2.3	0.9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2.4	0.9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2.5	0.9938	9940	9941	9943	9945	9946	9948	9949	9951	9952
2.6	0.9953	9955	9956	9957	9959	9960	9961	9962	9963	9964
2.7	0.9965	9966	9967	9968	9969	9970	9971	9972	9973	9974
2.8	0.9974	9975	9976	9977	9977	9978	9979	9980	9980	9981
2.9	0.9981	9982	9983	9983	9984	9984	9985	9985	9986	9986
3.0	0.9987	9987	9987	9988	9988	9989	9989	9989	9990	9990
3.1	0.9990	9991	9991	9991	9992	9992	9992	9992	9993	9993
3.2	0.9993	9993	9994	9994	9994	9994	9994	9995	9995	9995
3.3	0.9995	9995	9995	9996	9996	9996	9996	9996	9996	9997
3.4	0.9997	9997	9997	9997	9997	9997	9997	9997	9997	9998
3.5	0.9998	9998	9998	9998	9998	9998	9998	9998	9998	9998

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