



# MASENO UNIVERSITY UNIVERSITY EXAMINATIONS 2024/2025

**SECOND YEAR FIRST SEMESTER EXAMINATION FOR  
THE DEGREE OF BACHELOR OF SCIENCE IN  
MATHEMATICAL SCIENCES AND BACHELOR OF SCIENCE  
IN MATHEMATICS AND COMPUTER SCIENCE**

## MAIN CAMPUS

### MMA 223: NUMERICAL ANALYSIS I

Date: 21<sup>st</sup> January, 2025

Time: 3.30 - 6.30pm

#### INSTRUCTIONS:

- Do not write anywhere on this Question paper
- Answer Question ONE and any other TWO
- Start each question on a fresh page
- Indicate question numbers clearly at the top of each page.

## SECTION ONE (COMPULSORY) [30 Marks]

- a) Define the following terms
- Absolute error [2mks]
  - Relative error [2mks]
  - Round-off error [2mks]
- b) What is the order of convergence of the Newton-Raphson method? Explain briefly how it is derived. [5mks]
- c) Explain the Bisection method. Why is the method guaranteed to converge and how can you estimate the number of iterations required to reach a desired accuracy? [5mks]
- d) Construct a forward difference table for the following data [6mks]
- | x    | 1 | 2 | 3  | 4  | 5   | 6   | 7   | 8   |
|------|---|---|----|----|-----|-----|-----|-----|
| f(x) | 1 | 8 | 27 | 64 | 125 | 216 | 343 | 512 |
- e) Given the function  $f(x) = \sin x$ , approximate  $f'(1)$  using the forward difference formula with step size  $h = 0.1$ . Compare the result with the exact derivative  $f'(1) = \cos(1)$  [8mks]
- ### QUESTION TWO [20 Marks]
- a) Use the Newton-Raphson method to approximate the root of the equation  $f(x) = x^2 - 2 = 0$ , starting from  $x_0 = 1$ . Perform six iterations, correct to 6 d.p. [10mks]
- b) Use the Bisection method to find an approximation to the root of the equation  $f(x) = x^3 - 4x + 1 = 0$  in the interval  $[1, 2]$ . Perform four iterations. [10mks]
- ### QUESTION THREE [20 Marks]
- a) Use the Trapezoidal Rule to approximate the integral  $\int_0^2 (x^2 + 1)dx$ . [10mks]
- Divide the interval into 4 equal subintervals and compute the numerical solution.

- b) Use Central Difference Formula to approximate the derivative of the function  $f(x) = e^x$  at  $x = 1$  with a step size  $h = 0.01$ . Estimate the error. [10mks]

### QUESTION FOUR [20 Marks]

- a) The following data gives the melting point of an alloy of lead and zinc, where  $t$  is the temperature in degrees Celsius and  $P$  is the percentage of lead in the alloy.

P	40	50	60	70	80	90
t	180	204	226	250	276	304

Find the melting point of the alloy containing 84% lead.

[10mks]

- b) The following are the population of a district

Year(x)	1981	1991	2001	2011	2021	2031
Population(y)	363	391	421	?	467	501

Find the population of the year 2011.

[10mks]

### QUESTION FIVE [20 Marks]

- a) Find the root of the equation  $x^2 - 2x - 3 = 0$  using the fixed point iteration method.

Use the initial guess  $x_0 = 4$  and iterate until  $|x_{n+1} - x_n| < 0.001$  [10mks]

- b) Estimate the integral  $I = \int_0^1 (x^2 + 1)dx$  using the Romberg Integration Method.

[10mks]

END

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