Recommender Systems: Content-based Systems & Collaborative Filtering

Mining of Massive Datasets
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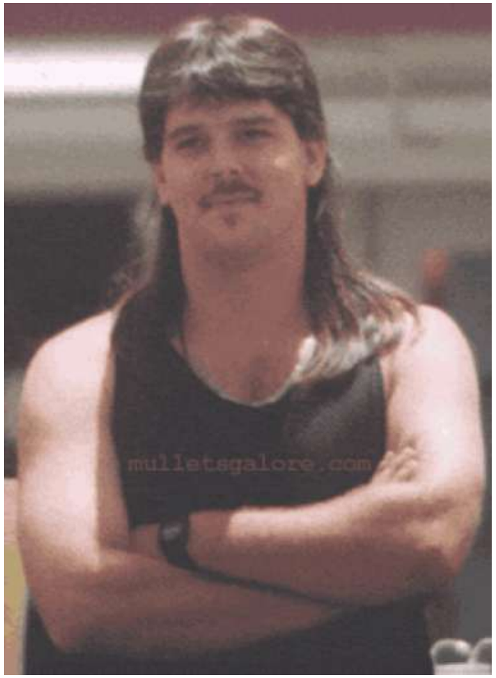
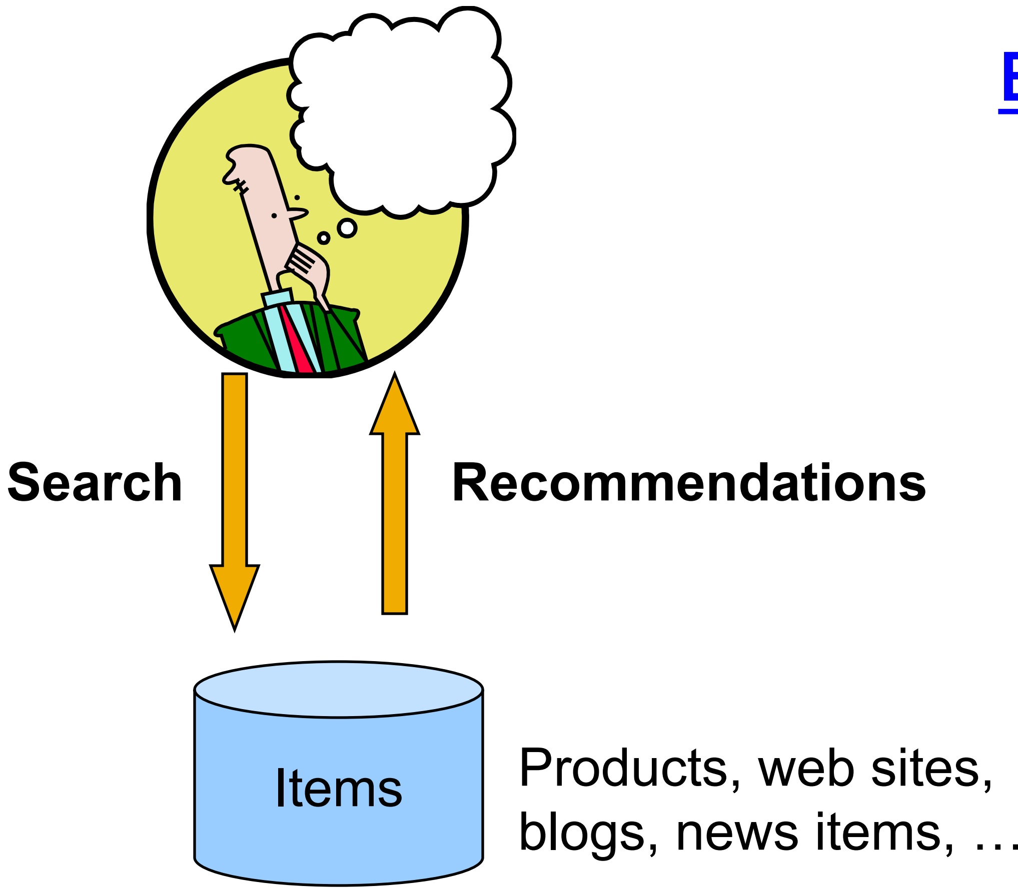
http://www.mmds.org

Example: Recommender Systems



- **Customer X**
  - Buys Metallica CD
  - Buys Megadeth CD
- **Customer Y**
  - Does search on Metallica
  - Recommender system suggests Megadeth from data collected about customer **X**

Recommendations



Examples:

From Scarcity to Abundance

- **Shelf space is a scarce commodity for traditional retailers**
  - Also: TV networks, movie theaters,…
- **Web enables near-zero-cost dissemination of information about products**
  - From scarcity to abundance
- **More choice necessitates better filters**
  - Recommendation engines
  - How **Into Thin Air** made **Touching the Void** a bestseller: http://www.wired.com/wired/archive/12.10/tail.html

Types of Recommendations

- **Editorial and hand curated**
  - List of favorites
  - Lists of “essential” items
- **Simple aggregates**
  - Top 10, Most Popular, Recent Uploads
- **Tailored to individual users**
  - Amazon, Netflix, …

Formal Model

- X = set of **Customers**
- S = set of **Items**

- **Utility function** u: $X \times S \rightarrow R$
  - R = set of ratings
  - R is a totally ordered set
  - e.g., **0-5** stars, real number in **[0,1]**

Utility Matrix

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

Key Problems

- **(1) Gathering "known" ratings for matrix**
  - How to collect the data in the utility matrix
- **(2) Extrapolate unknown ratings from the known ones**
  - Mainly interested in high unknown ratings
    - We are not interested in knowing what you don't like but what you like
- **(3) Evaluating extrapolation methods**
  - How to measure success/performance of recommendation methods

(1) Gathering Ratings

- **Explicit**
  - Ask people to rate items
  - Doesn't work well in practice – people can't be bothered
- **Implicit**
  - Learn ratings from user actions
    - E.g., purchase implies high rating
  - What about low ratings?

(2) Extrapolating Utilities

- **Key problem:** Utility matrix ***U*** is **sparse**
  - Most people have not rated most items
  - **Cold start:**
    - New items have no ratings
    - New users have no history
- **Three approaches to recommender systems:**
  - **1)** Content-based
  - **2)** Collaborative
  - **3)** Latent factor based

Content-based Recommender Systems

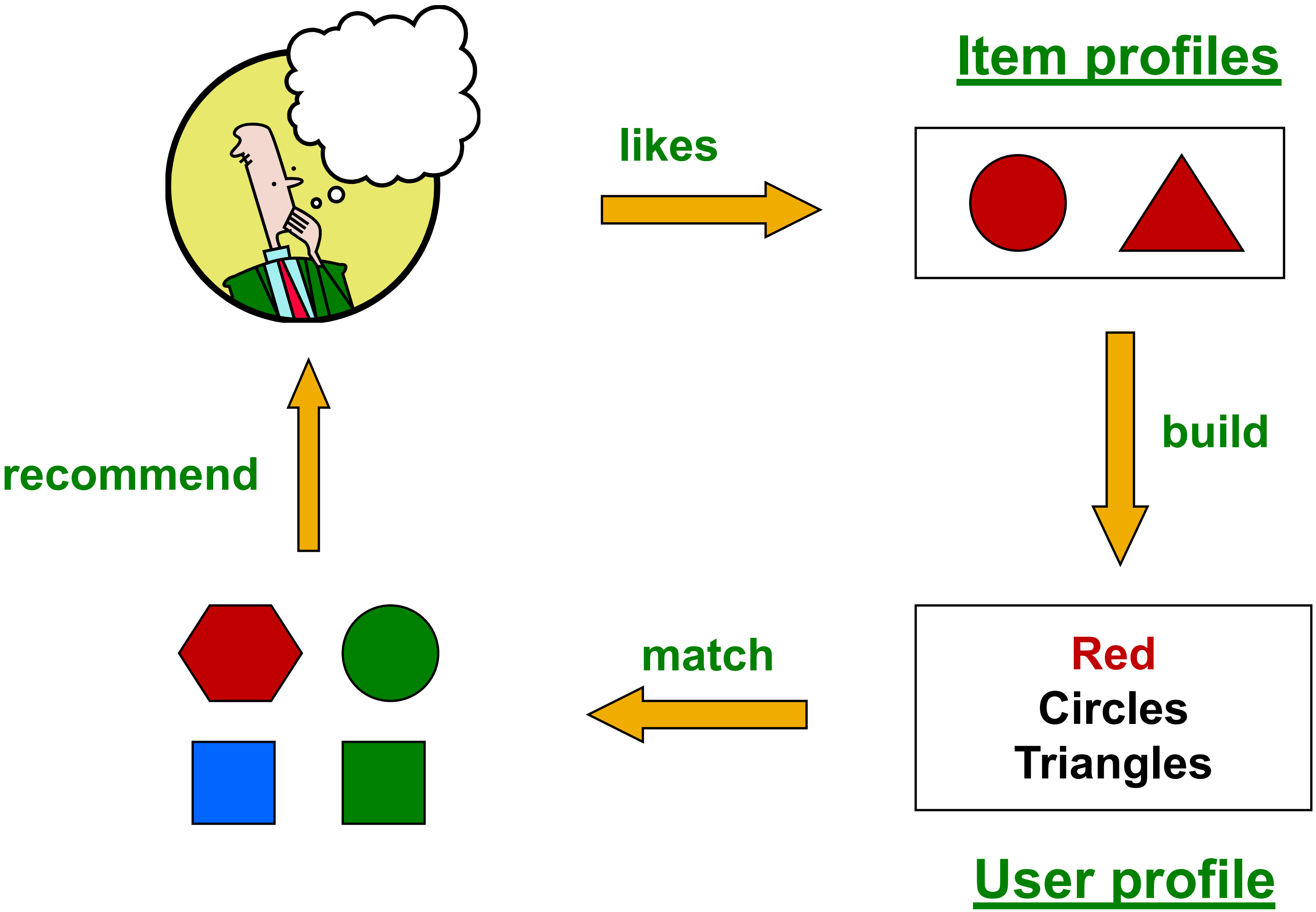
Content-based Recommendations

- **Main idea:** Recommend items to customer ***x*** similar to previous items rated highly by ***x***

Example:

- **Movie recommendations**
  - Recommend movies with same actor(s), director, genre, …
- **Websites, blogs, news**
  - Recommend other sites with "similar" content

Plan of Action



Item Profiles

- For each item, create an **item profile**
- **Profile is a set (vector) of features**
  - **Movies:** author, title, actor, director,…
  - **Text:** Set of "important" words in document
- **How to pick important features?**
  - Usual heuristic from text mining is **TF-IDF** (Term frequency * Inverse Doc Frequency)
    - **Term … Feature**
    - **Document … Item**

Sidenote: TF-IDF

f_{ij} = frequency of term (feature) i in doc (item) j

$$TF_{ij} = \frac{f_{ij}}{\max_k f_{kj}}$$

Note: we normalize TF to discount for "longer" documents

n_i = number of docs that mention term i

N = total number of docs

$$IDF_i = \log \frac{N}{n_i}$$

TF-IDF score: $w_{ij} = TF_{ij} \times IDF_i$

Doc profile = set of words with highest **TF-IDF** scores, together with their scores

User Profiles and Prediction

- **User profile possibilities:**
  - Weighted average of rated item profiles
  - **Variation:** weight by difference from average rating for item
  - …
- **Prediction heuristic:**
  - Given user profile $\boldsymbol{x}$ and item profile $\boldsymbol{i}$, estimate
    $$u(\boldsymbol{x}, \boldsymbol{i}) = \cos(\boldsymbol{x}, \boldsymbol{i}) = \frac{\boldsymbol{x} \cdot \boldsymbol{i}}{||\boldsymbol{x}|| \cdot ||\boldsymbol{i}||}$$

Pros: Content-based Approach

- **+: No need for data on other users**
  - No cold-start or sparsity problems
- **+: Able to recommend to users with unique tastes**
- **+: Able to recommend new & unpopular items**
  - No first-rater problem
- **+: Able to provide explanations**
  - Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

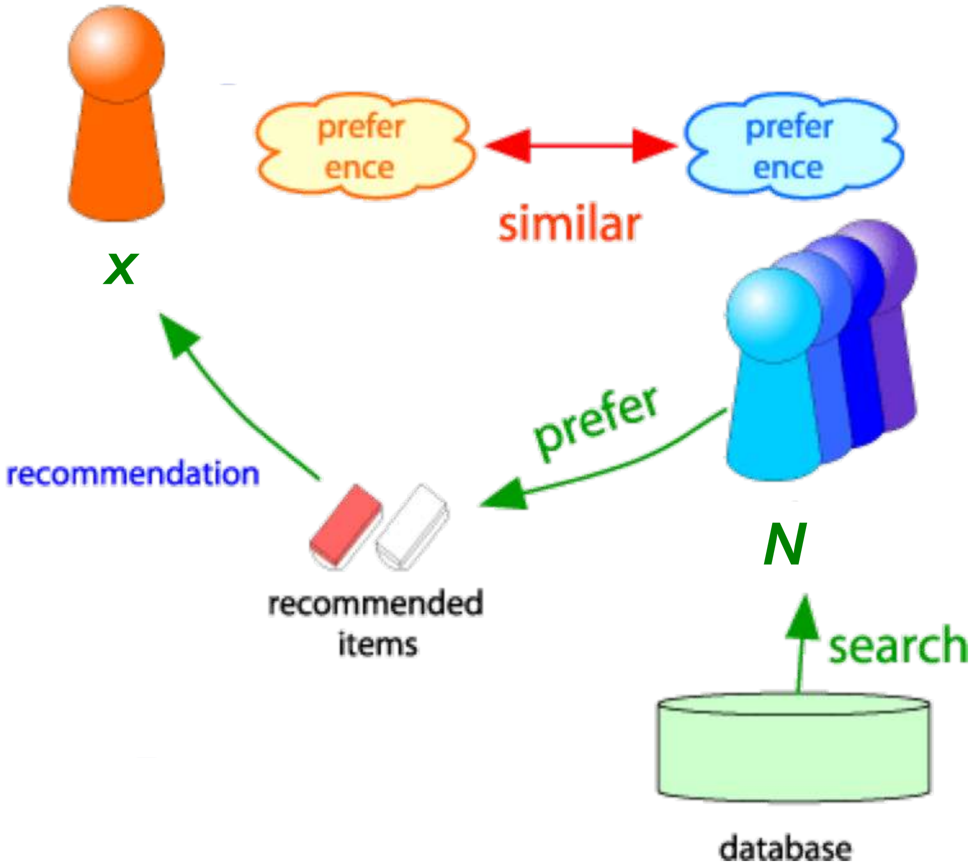
- **–: Finding the appropriate features is hard**
  - E.g., images, movies, music
- **–: Recommendations for new users**
  - **How to build a user profile?**
- **–: Overspecialization**
  - Never recommends items outside user's content profile
  - People might have multiple interests
  - **Unable to exploit quality judgments of other users**

Collaborative Filtering

Harnessing quality judgments of other users

Collaborative Filtering

- Consider user ***x***
- Find set ***N*** of other users whose ratings are "**similar**" to ***x***'s ratings
- Estimate ***x***'s ratings based on ratings of users in ***N***



Finding "Similar" Users

$r_x = [*, \_, \_, *, ***]$
$r_y = [*, \_, **, **, \_]$

- Let r_x be the vector of user x's ratings
- **Jaccard similarity measure**
  - **Problem:** Ignores the value of the rating
- **Cosine similarity measure**
  - sim(x, y) = cos(r_x, r_y) = $\frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$
  - **Problem:** Treats missing ratings as "negative"
- **Pearson correlation coefficient**
  - S_{xy} = items rated by both users x and y

$$sim(x,y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})(r_{ys} - \overline{r_y})}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \overline{r_x})^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \overline{r_y})^2}}$$

r_x, r_y as sets:
r_x = {1, 4, 5}
r_y = {1, 3, 4}

r_x, r_y as points:
r_x = {1, 0, 0, 1, 3}
r_y = {1, 0, 2, 2, 0}

$\overline{r_x}$, $\overline{r_y}$ … avg. rating of **x**, **y**

Similarity Metric

Cosine sim:
$$sim(x,y) = \frac{\sum_i r_{xi} \cdot r_{yi}}{\sqrt{\sum_i r_{xi}^2} \cdot \sqrt{\sum_i r_{yi}^2}}$$

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

- **Intuitively we want:** **sim(*A*, *B*) > sim(*A*, *C*)**
- **Jaccard similarity:** 1/5 < 2/4
- **Cosine similarity:** 0.386 > 0.322
  - Considers missing ratings as "negative"
  - **Solution: subtract the (row) mean**

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	$2/3$			$5/3$	$-7/3$		
B	$1/3$	$1/3$	$-2/3$				
C				$-5/3$	$1/3$	$4/3$	
D		0					0

sim A,B vs. A,C:
0.092 **>** -0.559

Notice cosine sim. is correlation when data is centered at 0

Rating Predictions

From similarity metric to recommendations:

- Let $\boldsymbol{r_x}$ be the vector of user $\boldsymbol{x}$'s ratings
- Let $\boldsymbol{N}$ be the set of $\boldsymbol{k}$ users most similar to $\boldsymbol{x}$ who have rated item $\boldsymbol{i}$
- **Prediction for item *s* of *user x*:**
  - $r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$
  - $r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$
  - Other options?
- **Many other tricks possible…**

Shorthand:
$\boldsymbol{s_{xy} = sim(x, y)}$

Item-Item Collaborative Filtering

- **So far: User-user collaborative filtering**
- **Another view: Item-item**
  - For item ***i***, find other similar items
  - Estimate rating for item ***i*** based on ratings for similar items
  - Can use same similarity metrics and prediction functions as in user-user model

$$\boxed{r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}}$$

s_{ij}… similarity of items ***i*** and ***j***
r_{xj}…rating of user ***u*** on item ***j***
N(i;x)… set items rated by ***x*** similar to ***i***

Item-Item CF (|N|=2)

users

movies	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3			5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

- unknown rating

- rating between 1 to 5

Item-Item CF (|N|=2)

movies \ users	1	2	3	4	**5**	6	7	8	9	10	11	12
1	1		3		?	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

- estimate rating of movie **1** by user **5**

Item-Item CF (|N|=2)

users

movies	1	2	3	4	**5**	6	7	8	9	10	11	12	sim(1,m)
1	1		3		?	5			5		4		1.00
2			5	4			4			2	1	3	-0.18
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		-0.10
5			4	3	4	2					2	5	-0.31
6	1		3		3			2			4		**0.59**

Neighbor selection:
Identify movies similar to movie **1**, **rated by user 5**

Here we use Pearson correlation as similarity:

1) Subtract mean rating m_i from each movie i

$m_1 = (1+3+5+5+4)/5 = 3.6$

row 1: *[-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]*

2) Compute cosine similarities between rows

Item-Item CF (|N|=2)

users; movies (rows)

movies	1	2	3	4	5	6	7	8	9	10	11	12	sim(1,m)
1	1		3		?	5			5		4		**1.00**
2			5	4			4			2	1	3	**-0.18**
3	2	4		1	2		3		4	3	5		**0.41**
4		2	4		5			4			2		**-0.10**
5			4	3	4	2					2	5	**-0.31**
6	1		3		3			2			4		**0.59**

Compute similarity weights:

$s_{1,3}=0.41$, $s_{1,6}=0.59$

Item-Item CF (|N|=2)

users

movies	1	2	3	4	5	6	7	8	9	10	11	12
1	1		3		**2.6**	5			5		4	
2			5	4			4			2	1	3
3	2	4		1	2		3		4	3	5	
4		2	4		5			4			2	
5			4	3	4	2					2	5
6	1		3		3			2			4	

Predict by taking weighted average:

$r_{1.5}$ = (0.41*2 + 0.59*3) / (0.41+0.59) = 2.6

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

CF: Common Practice

Before:

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

- Define **similarity** s_{ij} of items i and j
- Select k nearest neighbors $N(i; x)$
  - Items most similar to i, that were rated by x
- Estimate rating r_{xi} as the weighted average:

$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

- μ = overall mean movie rating
- b_x = rating deviation of user x
  = *(avg. rating of user x)* $- \mu$
- b_i = rating deviation of movie i

Item-Item vs. User-User

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

- **In practice, it has been observed that <u>item-item</u> often works better than user-user**
- **Why?** Items are simpler, users have multiple tastes

Pros/Cons of Collaborative Filtering

- **+ Works for any kind of item**
  - No feature selection needed
- **- Cold Start:**
  - Need enough users in the system to find a match
- **- Sparsity:**
  - The user/ratings matrix is sparse
  - Hard to find users that have rated the same items
- **- First rater:**
  - Cannot recommend an item that has not been previously rated
  - New items, Esoteric items
- **- Popularity bias:**
  - Cannot recommend items to someone with unique taste
  - Tends to recommend popular items

Hybrid Methods

- **Implement two or more different recommenders and combine predictions**
  - Perhaps using a linear model
- **Add content-based methods to collaborative filtering**
  - Item profiles for new item problem
  - Demographics to deal with new user problem

Evaluation & Practical Tips

- Evaluation
- Error metrics

Evaluation

movies

users

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Evaluation

movies

users

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

Test Data Set

Evaluating Predictions

- **Compare predictions with known ratings**
  - **Root-mean-square error** (RMSE)
    - $\sqrt{\sum_{xi}\left(r_{xi} - r_{xi}^{*}\right)^{2}}$ where $\boldsymbol{r_{xi}}$ is predicted, $\boldsymbol{r^{*}_{xi}}$ is the true rating of $\boldsymbol{x}$ on $\boldsymbol{i}$
  - **Precision at top 10**:
    - % of those in top 10
  - **Rank Correlation**:
    - Spearman's *correlation* between system's and user's complete rankings
- **Another approach: 0/1 model**
  - **Coverage:**
    - Number of items/users for which system can make predictions
  - **Precision:**
    - Accuracy of predictions
  - **Receiver operating characteristic** (ROC)
    - Tradeoff curve between false positives and false negatives

Problems with Error Measures

- **Narrow focus on accuracy sometimes misses the point**
  - Prediction Diversity
  - Prediction Context
  - Order of predictions
- **In practice, we care only to predict high ratings:**
  - RMSE might penalize a method that does well for high ratings and badly for others

Latent Factor Models

The Netflix Prize

- **Training data**
  - 100 million ratings, 480,000 users, 17,770 movies
  - 6 years of data: 2000-2005
- **Test data**
  - Last few ratings of each user (2.8 million)
  - **Evaluation criterion:** Root Mean Square Error (RMSE) $= \frac{1}{|R|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$
  - **Netflix's system RMSE: 0.9514**
- **Competition**
  - 2,700+ teams
  - **$1 million** prize for 10% improvement on Netflix

The Netflix Utility Matrix *R*

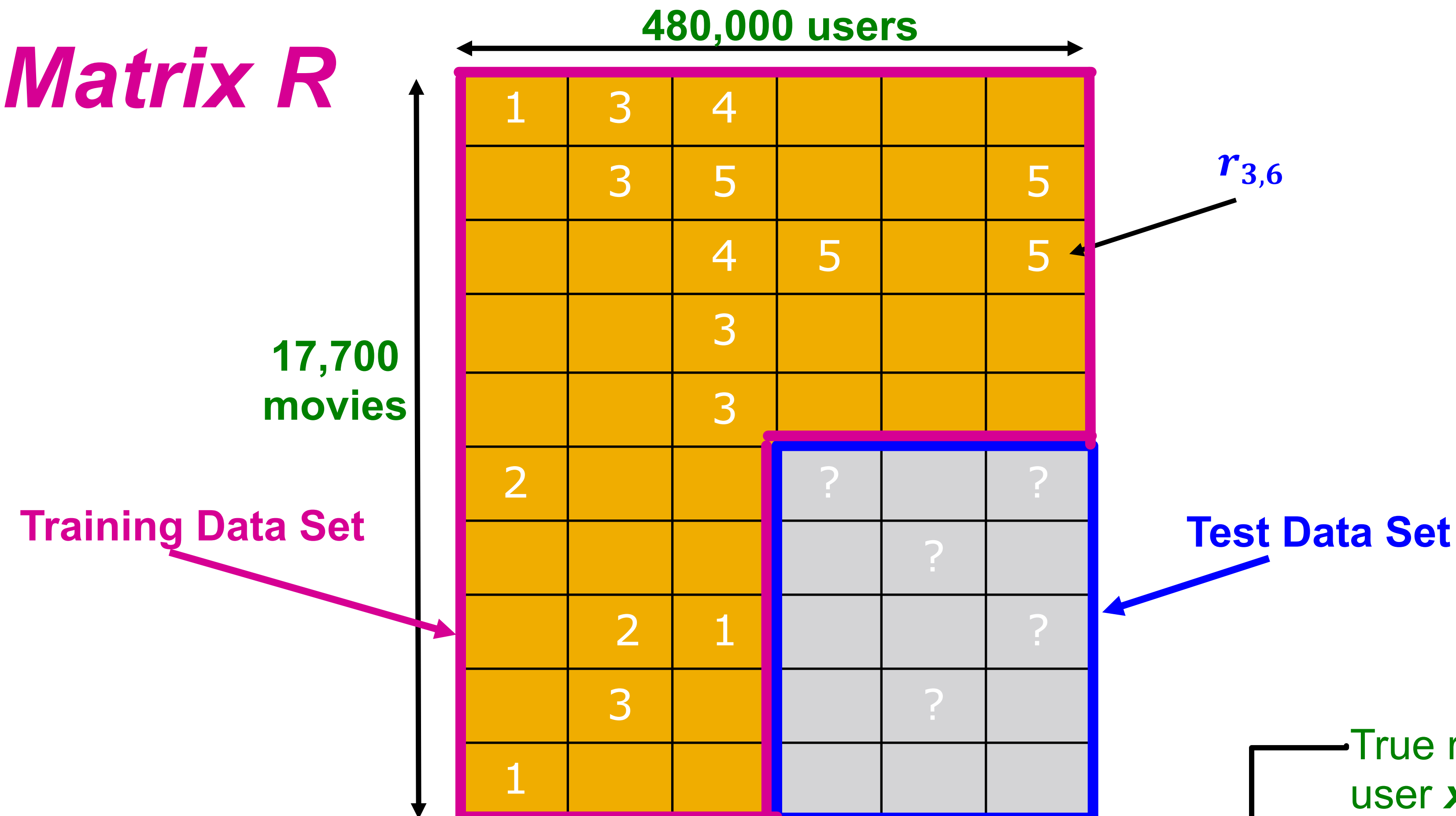
Matrix R

480,000 users

17,700 movies

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Utility Matrix *R*: Evaluation



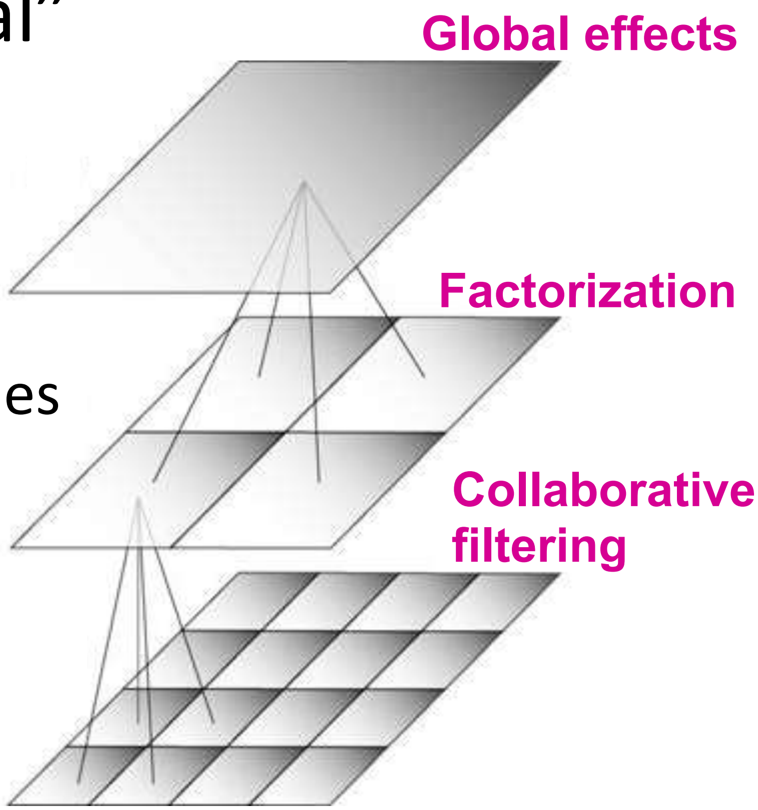
True rating of user ***x*** on item ***i***

$$\textbf{RMSE} = \frac{1}{|\mathrm{R}|}\sqrt{\sum_{(i,x)\in R}(\hat{r}_{xi} - r_{xi})^2}$$

Predicted rating

BellKor Recommender System

- The winner of the Netflix Challenge!
- Multi-scale modeling of the data: Combine top level, “regional” modeling of the data, with a refined, local view:
  - Global:
    - Overall deviations of users/movies
  - Factorization:
    - Addressing “regional” effects
  - Collaborative filtering:
    - Extract local patterns



Modeling Local & Global Effects

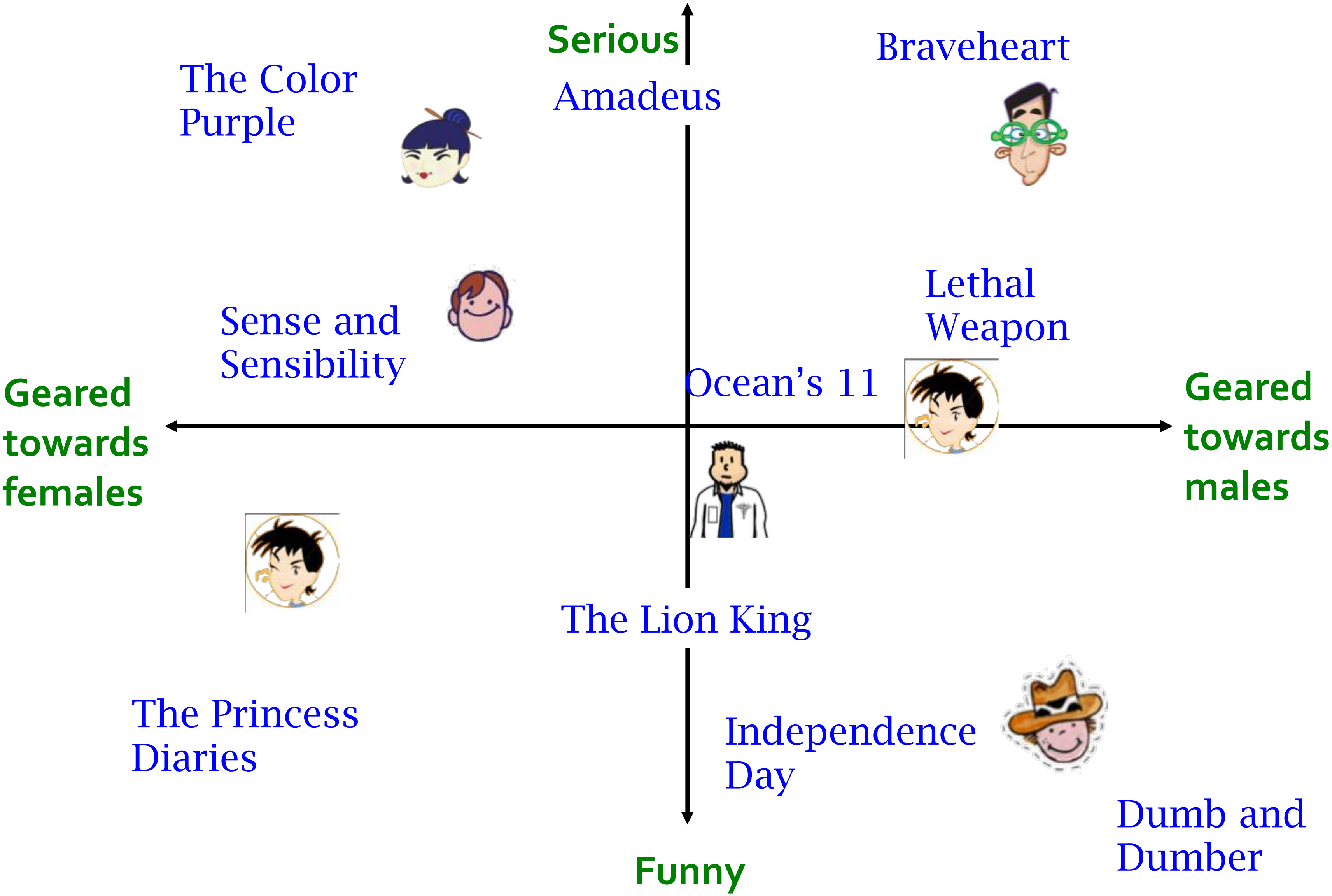
- **Global:**
  - Mean movie rating: **3.7 stars**
  - *The Sixth Sense* is **0.5** stars above avg.
  - Joe rates **0.2** stars below avg.

    ⇒ Baseline estimation:

    ***Joe* will rate *The Sixth Sense* 4 stars**
- **Local neighborhood (CF/NN):**
  - *Joe* didn't like related movie *Signs*
  - **⇒ Final estimate:**

    ***Joe* will rate *The Sixth Sense* 3.8 stars**

Latent Factor Models (e.g., SVD)



Latent Factor Models

SVD: $A = U \Sigma V^T$

- "SVD" on Netflix data: $\mathbf{R} \approx \boldsymbol{Q} \cdot \boldsymbol{P}^T$

users

items

1		3			5			5		4	
		5	4			4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

R

≈

factors

items

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

Q

users

factors

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6

$\boldsymbol{P}^T$

- For now let's assume we can approximate the rating matrix $\boldsymbol{R}$ as a product of "thin" $\boldsymbol{Q} \cdot \boldsymbol{P}^T$
  - $\boldsymbol{R}$ has missing entries but let's ignore that for now!
    - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x = \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
p_x = column x of P^T

users (columns) × items (rows):

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

Q — items (rows) × factors (columns):

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

•

P^T — factors (rows) × users (columns):

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x = \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
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users (columns) / items (rows)

1		3			5			5		4	
		5	4	?		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

Q — items (rows) / factors (columns)

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

•

P^T — factors (rows) / users (columns)

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Ratings as Products of Factors

- **How to estimate the missing rating of user *x* for item *i*?**

$$\hat{r}_{xi} = q_i \cdot p_x$$

$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q

p_x = column x of P^T

users (columns) × items (rows):

1		3			5			5		4	
		5	4	2.4		4			2	1	3
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

Q — items (rows) × f factors (columns):

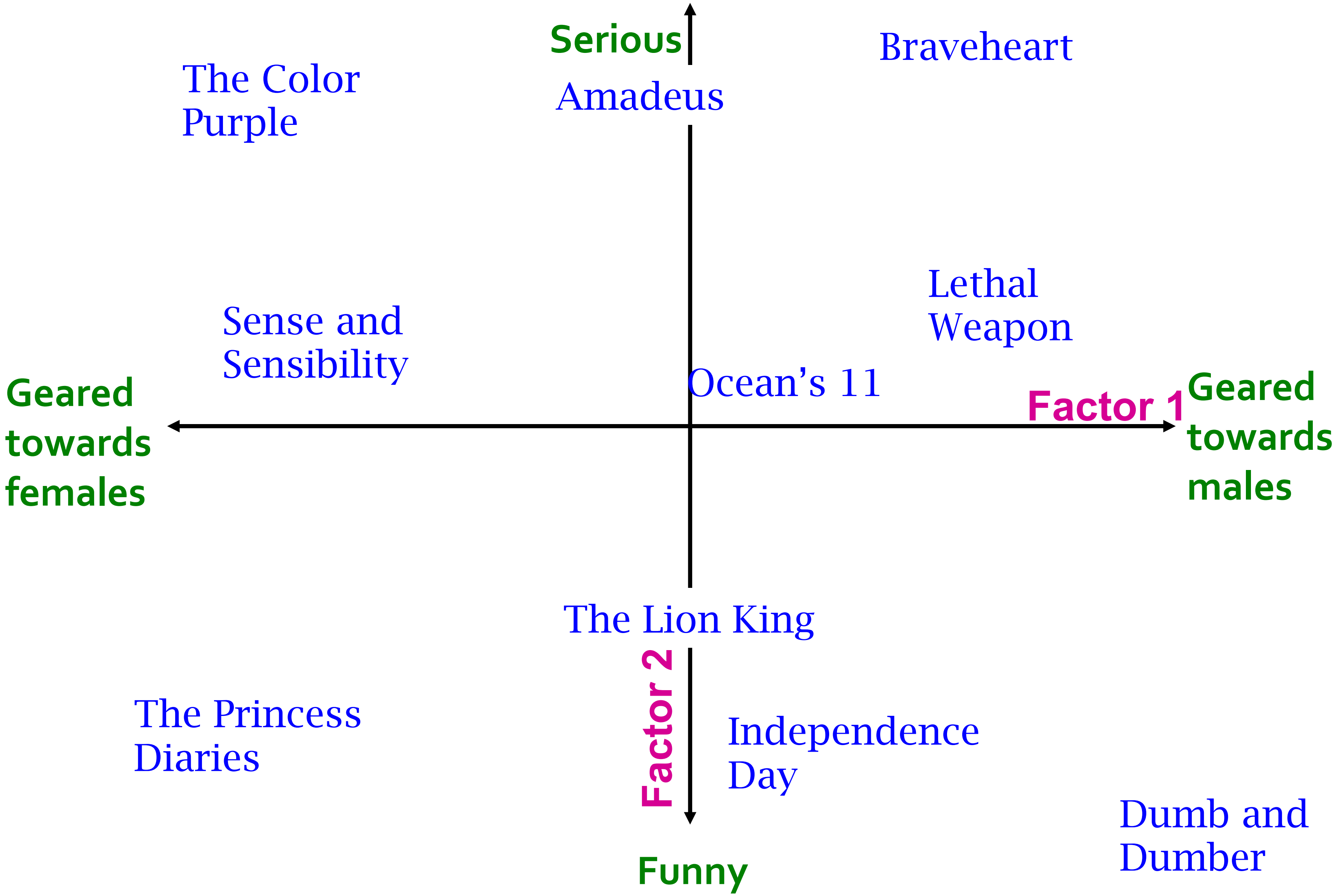
.1	-.4	.2
-.5	**.6**	**.5**
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

•

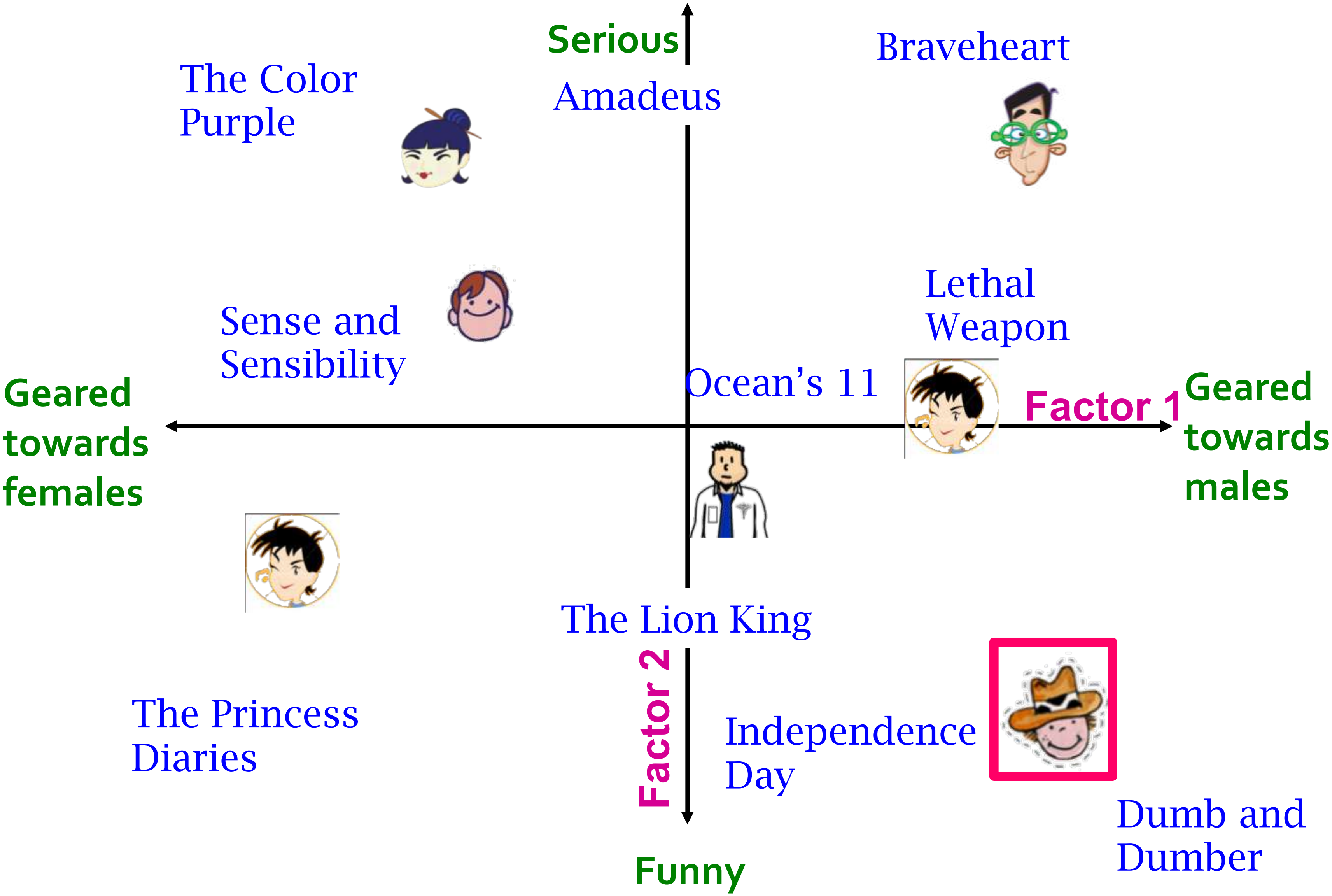
P^T — f factors (rows) × users (columns):

1.1	-.2	.3	.5	**-2**	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	**.3**	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	**2.4**	.9	-.3	.4	.8	.7	-.6	.1

Latent Factor Models

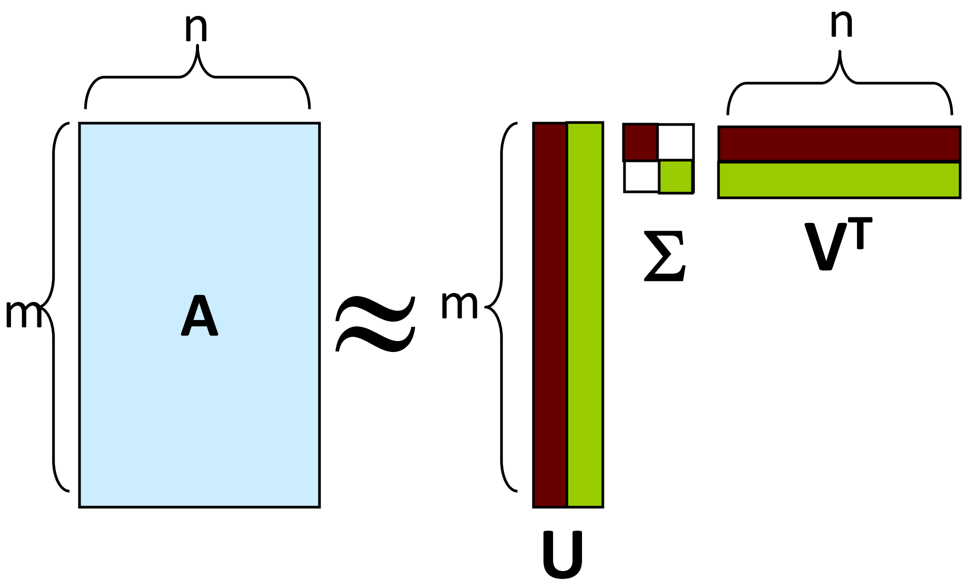


Latent Factor Models



Recap: SVD

- **Remember SVD:**
  - **A**: Input data matrix
  - **U**: Left singular vecs
  - **V**: Right singular vecs
  - Σ: Singular values



- **So in our case:**
  "SVD" on Netflix data: $R \approx Q \cdot P^T$
  $A = R,\ \ Q = U,\ \ P^T = \Sigma\, V^T$

$$\hat{r}_{xi} = q_i \cdot p_x$$