TANH Equation

The TANH equation describes the relationship between the partial pressure of oxygen (p) and saturation of haemoglobin (s) in human blood.[1]

$$y = y_0 + x - x_0 + h \cdot \tanh(k_0 \cdot (x - x_0)) \tag{1}$$

where:

$$x = \ln \frac{p}{p_0} \tag{2}$$

$$y = \ln \frac{s}{1-s} \tag{3}$$

$$x_0 = a + b \tag{4}$$

$$h = h_0 + a \tag{5}$$

The terms y_0, p_0, k_0, and h_0 are all constants.

The terms a and b represent the chemical and thermal affinity parameters that cause the curve to shift position.

Hence, expanded:

$$y = y_0 + x - a - b + [(h_0 + a) \cdot \tanh(k_0 \cdot (x - a - b))] \tag{6}$$

$$\ln \frac{s}{1-s} = y_0 + \ln\left(\frac{p}{p_0}\right) - a - b + \left[(h_0 + a) \cdot \tanh(k_0 \cdot (\ln\left(\frac{p}{p_0}\right) - a - b))\right] \tag{7}$$

Problems

1. Find a when all other terms are known.
2. Find p when all other terms are known.

No explicit algebraic solution exist to express the equation in terms of a or p, and hence an iterative approach is required. The literature suggests using a Newton-Raphson method.

[1] It's actually a little more complicated, as p is the combined partial pressure of oxygen and carbon monoxide, and s is the combined saturation of oxygen and carbon monoxide, but as carbon monoxide tends to zero, we can assume $p \approx pO_2$ and $s \approx sO_2$.

Find a

This method is used to find a, the chemical affinity parameter that influences the position of the curve, from a single paired measurement of p and s (e.g. from an ABG sample).

1. Measure p and s.
2. Estimate a temporary value for a.
   Sensible initial guesses include $a = 0$ for a 'standard' curve ($pH = 7.40, pCO_2 = 5.33, cDPG = 5.00$, with no dyshaemoglobins), or $a = -0.00475$ for a standard curve with trace dyshaemoglobins.
3. Use this temporary a and the measured p to calculate a temporary s (Eqns (3) and (1)).
4. The difference between the temporary and the measured s allows the calculation of a new temporary parameter a, using a Newton-Raphson iteration procedure.
5. Iteration continues until the temporary and the measured s matches within a given limit ($\epsilon = 0.000001$).

Find p

This method is used to find p_{50} by defining $s = 0.5$, when a and b are known – either having been measured directly or calculated previously (e.g. in the above procedure)

1. Measure/define s, and calculate the affinity parameters a and b.
2. Guess a temporary p (e.g. $p = 7$) and calculate a temporary s (Eqns (1) and then rearrange (3)).
3. The difference between the temporary s and the measured/defined s allows the calculation of a new temporary p using a fast Newton-Raphson procedure.
4. The procedure is iterated until the difference between the temporary s and the measured/defined s is less than a given limit ($\epsilon = 0.000001$).