ELSEVIER

Journal of Economic Behavior and Organization
Vol. 28 (1995) 257–274

JOURNAL OF
Economic Behavior
& Organization

Bayesian interactions and collective dynamics of opinion: Herd behavior and mimetic contagion

André Orléan

CREA, Ecole Polytechnique, 1 rue Descartes, 75005, Paris, France

Received 20 April 1993; revised 21 November 1994

Abstract

Much recent work has been devoted to the analysis of herd behavior within sequential decision models. The present article generalizes their results to non-sequential contexts. We will show that, as soon as the hypothesis of sequentiality is dropped, a large variety of situations can be observed. Our model has been designed to study the collective learning process through which a group of interacting agents deals with environmental uncertainty. The crucial question revolves around the relative weight given by each individual to the different sources of information: his private information and his observation of the group opinion.

JEL classification: D83

Keywords: Collective learning; Mimetism; Herd behavior; Social psychology; Convention

1. Introduction

Much of the recent work devoted to the emergence and stability of institutions and conventions has put a great emphasis on the notions of collective learning and collective knowledge: in order to achieve coordination between agents, conventional rules have to rely on a certain commonality of expectations and beliefs. My object in this paper is to understand how this commonality of expectations and beliefs is produced and under which conditions this consensus will remain stable. My central concern will be to show that opinions are modified endogenously as a

SSDI 0167-2681(95)00035-6

result of interaction between individual agents. In this perspective, the role played by interpersonal influences will be pointed out. Before the eighties, very few publications were devoted to this issue. Mass psychology, social pressures and contagion were seriously taken into account only by a small group of academic researchers. The main reason for this neglect is the fact that in the eyes of the majority of economists these phenomena were seen as basically grounded on irrational motivations and thus discredited as unscientific.

This situation has changed considerably, and when one looks at the recent literature, it is easy to notice the presence of quite a few new terms, almost absent from economic theory fifteen years ago: rumors [Banerjee (1993)], herd behavior [Banerjee (1992), Froot et al. (1992)], contagion and epidemics [Arthur and Lane (1989), Kirman (1993)], mimetism and mimetic bubbles [Orléan (1989), Topol (1991)], fashions and fads [Shiller (1989)], informational cascades [Bikhchandani et al. (1992)]. In most of these papers, interpersonal influences are analyzed as the result of a rational behavior [Banerjee (1992), Bikhchandani et al. (1992), Froot et al. (1992), Harris and Raviv (1993)]. Herd behavior and mimetism are no more seen as some deviant dynamics grounded on fads, but as the direct consequence of Bayesian calculus. To obtain this quite surprising result, these models assume a sequential process: individuals enter the market one by one, individual decisions are irreversible, and each agent only observes his predecessors. Most of the time, this assumption is quite unrealistic, especially if we want to formalize financial markets or the emergence of social conventions: at each date t, every agent is present and decisions are reversible. It is not clear whether the results obtained in a sequential framework will still hold under these more realistic assumptions. The present article is devoted to this question. To handle it we will propose a new framework that does not require any more the assumption of sequentiality. The mathematical framework is very close to the ones proposed by Föllmer (1974) and Kirman (1993). The difference with respect to Kirman's model is the way changes of individual opinions are formalized. Kirman assumes a stochastic process of random meetings. In this paper we try to provide a more rigorous microeconomic foundation for the transition probabilities, based on Bayesian hypotheses.

Section 2 summarizes the ideas contained in the Bikhchandani, Hirshleifer and Welch's article [Bikhchandani et al. (1992)] that shows how mimetic influences can be understood as the result of Bayesian behavior. Section 3 integrates these ideas within a model which deals with collective dynamics. Section 4 analyzes some specific situations and shows that if the collective process is no longer assumed to be sequential, one cannot be sure that an informational cascade will appear. Nevertheless such a phenomenon will continue to be observed frequently.

2. Bayesian behavior and rational mimetism

Let us consider a group of individuals, each deciding whether to adopt or to reject a certain behavior. Adoption will be designated by (B) and rejection by (V).

All individuals have the same cost of adopting: r. R, the gain of adopting is either high (H) or low (L), with $H = r + d$, $L = r - d$ and $d > 0$. The state of the world is assumed to be either {H} or {L} with equal prior probability. At date T, the true value of R is publicly revealed, the payments are made and a new round begins.

Agent i's information set is noted I. Before time T, in order to choose between (B) and (V), the agent i will calculate γ, the posterior probability $p(H|I)$ of the event {H}, and $E[R|I]$, the conditional expected value of R. $E[R|I]$ is equal to $\gamma \cdot H + (1-\gamma) \cdot L$. If γ is greater (resp. smaller) than 1/2, $E[R|I]$ is greater (resp. smaller) than r and the agent will choose (B) (resp. (V)). If γ is equal to 1/2, $E[R|I]$ is equal to r and we will suppose that the agent chooses (B) with probability 1/2.

We assume the existence of a signal X defined as follows: X is either $\{+\}$ or $\{-\}$; and its value is linked to the true value of R through the following conditional probabilities:

$$\mathrm{Prob}(X = +|R = H) = p(+|H) = p(-|L) = p$$

and

$$p(+|L) = p(-|H) = 1 - p$$

The probability p is assumed to be greater than 1/2. Each agent i can independently observe the signal X_i before deciding between (B) or (V). Let us consider an agent who is only observing his private signal: $I = \{X_i\}$. After observing his signal, either $\{+\}$ or $\{-\}$, this agent will revise his expectations regarding the final payoff R using Bayes' rule:

$$p(H|+) = \frac{p(+|H) \cdot P(H)}{P(+)} = p(L|-) = p$$

$$p(L|+) = p(H|-) = 1 - p$$

It follows that if his signal is $\{+\}$, $\gamma = p\,(H|+) = p$ is greater than 1/2, and this agent will choose (B). If his signal is $\{-\}$, $\gamma = p(H|-) = 1 - p$ is smaller than 1/2, and this agent will choose (V).

Bikhchandani et al. (1992) analyze a sequential process: each agent i observes X_i and chooses between (B) or (V) one after the other. The order of entry is exogenous. An agent i, who makes his decision at date t, also observes the actions of previous agents. The agent i will use this new source of information in order to revise his posterior probability γ. These authors show that this way of choosing between (B) and (V) leads to an "informational cascade" with a probability equal to 1: "An informational cascade occurs when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information" (p. 994). A cascade means the emergence of a general conformity whether on (B) or (V). When a cascade appears, the behavior of individuals becomes strictly imitative. They do not pay attention to their private signal any more; they just choose what the preceding agent has chosen. In this kind of situation, their action conveys no information about their private signal. Once the assumption of sequentiality is dropped, the question is: Will the process of collective decision still converge on an informational cascade? Before answering this question, let us emphasize how important the notion of informational cascade is.

Let us consider a group of 100 agents who have chosen (B). It is possible that the cascade started when the fifth agent entered the market. In this case only the actions of the first four agents give information about their private signals. Following individuals just imitated (B) without looking at their signal: the information conveyed by their action is null. "Cascades aggregate the information of only a few early individuals' actions" [Bikhchandani et al. (1992), p. 1006]. An external observer could be easily misled by such a situation: he will considerably overestimate the probability of the event $\{R = H\}$ as well as the accuracy of his estimation if he does not take into account the fact that individuals interact.

3. Individual representations and collective dynamics of opinion

Let us consider a population of N agents. At each date $t = 1, 2, \ldots, (T - 1)$, each individual's decision is either (B) or (V) depending on the value of γ. At date T, the value of R is publicly revealed and the payments are made. At $T + 1$, a new state of the world is drawn randomly and a new round begins. In our model, the process is no longer sequential: the N agents are simultaneously present. Each agent can make a decision and, some time later, can change it. The configuration of opinion at a given date t is defined by the number $n(t)$ of people having opinion (B). This variable will be called "the group opinion". It is a macroscopic datum which aggregates all the private choices. We will assume (i) that each agent makes one and only one observation of his private signal X at date 0 and (ii) that he is able to evaluate more or less accurately $n(t)$ at each date t. Why these assumptions? Why only one observation? Because what we want to understand is the role played by the collective interactions in the formation of the group opinion. It is the reason why the information set has been supposed to be given at the beginning of the period, at date 0. From $t = 1$ to $t = T - 1$, only interactions took place. What we try to understand is how these interactions will affect the group opinion: Will the ability of the group to make the right choice be improved or not? The elementary unit of time has been defined in line with this perspective: it corresponds to one interaction. It follows from these definitions that the whole period from date 0 to date T expressed in the standard units of time can be small but the number T can be very large. Let us underline that in our framework, individuals interact only indirectly through the variable $n(t)$.

To analyze the dynamics that emerges when agents make their decisions on the basis of their private signal as well as on the group opinion, we need a framework that formalizes endogenous reversible changes in opinions. The relevant concepts governing the dynamics of opinion formation will be the individual transition probabilities which are functions of the group opinion n or $f = n/N$: P_{VB}, the transition probability from opinion (V) to opinion (B) and P_{BV}, the transition probability from opinion (B) to opinion (V).

The stochastic dynamics is then defined as follows. At date t inferior to T, an agent is drawn randomly from the population. This agent does not know whether the state of the world is {H} or {L}. His information set is composed of Z, his last opinion, either (B) or (V); X, his private signal, either {+} or {−}; and $n = n(t-1)$, the group opinion that he has observed at date $t-1$: $I = \{Z, X, n\}$.

Let us consider an agent j such that his last choice was (B): $Z = (B)$. Let us consider $w = p(H|\ Z)$. What can be said about w? The theory of cognitive dissonance proposed by psychologists is a way to analyze the formation of w: because his last choice was (B), the agent i increases his prior probability of the event {H}. w is no more equal to 1/2; it is now greater than 1/2. The psychological interpretation of this mechanism says that people are able to manipulate their own beliefs in order to confirm "desired" beliefs. People tend to discard information suggesting that their last decision might be an error. Because this behavior will increase the prior probability of the event {H} if $Z = (B)$ as well as the prior probability of {L} if $Z = (V)$, the role of w can more easily be understood as a factor of inertia within our model: when w is increasing, the probability that an agent changes his opinion is decreasing. w will be assumed to be the same for all the agents. When w is assumed to be equal to 1/2, the prior probabilities are equal to 1/2. Let us now suppose that agent j has observed $\{X = +\}$. He will reestimate his prior probability of {H} according to Bayes' rule:

$$p(H|B,+) = p(H|Z = B, X = +) = \frac{p(+|H) \cdot p(H|B)}{p(+)}$$

According to the preceding definitions, we obtain:

$$p(H|B,+) = \frac{p \cdot w}{p \cdot w + (1-p) \cdot (1-w)} \tag{1}$$

Similarly, it follows that:

$$p(H|B,-) = \frac{(1-p) \cdot w}{(1-p) \cdot w + p \cdot (1-w)} \tag{2}$$

$$p(L|V,+) = \frac{(1-p) \cdot w}{p \cdot (1-w) + (1-p) \cdot w} \tag{3}$$

$$p(L|V,-) = \frac{p \cdot w}{p \cdot w + (1-p) \cdot (1-w)} \tag{4}$$

It is easy to verify that these four functions are increasing in w. Now, we can calculate $\gamma = p(H|Z, X, n)$. We can write:

$$\gamma = p(H|Z,X,n) = \frac{p(n|H) \cdot p(H|Z,X)}{p(n|H) \cdot p(H|Z,X) + p(n|L) \cdot p(L|Z,X)}$$

As has been shown before, the decision of the agent (Z, X) depends only on the value of γ. He will choose (B) if γ is greater than $1/2$. This condition is equivalent to:

$$\frac{p(n|H)}{p(n|L)} > \frac{p(L|Z,X)}{p(H|Z,X)} = h(Z,X,w) \tag{5}$$

From equations (1), (2), (3) and (4), we can easily derive h(Z, X, w).

To calculate p(n| H) would be easy if the individuals were taking their decisions independently, on the sole basis of their own private signal. This is the situation described at the beginning of section 2. We have shown that this kind of agent chooses {B} when he observes {+} and chooses {V} when he observes {−}: $\tilde{n}$, the random variable equal to the number of agents having chosen {B}, is following a binomial law. Its expected value is equal either to pN when the state of nature is {H} or to $(1-p)N$ when the state of nature is {L}. Then p(n| H) can be easily calculated. When the assumption of independence is no more valid, the estimation of p(n| H) becomes very difficult. For instance, a large n can be associated with two very different situations. n can be large because a lot of individuals have observed {X = +}, or because most of the agents were imitators. Whether you believe in the first explanation or in the second, your decision will be quite different. The analysis presented by Bikhchandani et al. has shown that, within a sequential context, the second explanation is not incompatible with the assumption of individual rationality.

Within the present framework we are building, the situation is even more complex because p(n| H), i.e. the probability law followed by $\tilde{n}$ when the state of nature is {H}, is precisely what the model is trying to calculate! A first way to handle this problem would be to look at the set of probability laws that are "fixed points" of our model: p^* is called a fixed point if, when p^* is used to calculate p(n| H) and p(n| L), the resulting probability according to our model is indeed p^*. This approach could be seen as a generalization of the rational expectations hypothesis. Our feeling is that this kind of approach does not constitute the right answer to our question, i.e. to study the collective learning process through which a group of interacting agents deals with environmental uncertainty. It seems to us more fruitful to lessen the constraint of perfect rationality and to put the emphasis on the representations the agents have constructed to analyze the collective behavior. We will introduce these representations in our model and we will study the collective dynamics that is generated. The fact that the ex post probability law will differ from the ex ante assumptions is not crucial here. What is more important is to show which parameters are central in the determination of the dynamics.

It will be assumed that individuals agents agree with the following representation about the way the group opinion is formed. They believe that the group is split between two types of agents: the independent ones and the imitators. Independent agents make their decision on the sole basis of their private signal. Imitators have no private information; they just copy the choice of the independent agents.

According to this representation, the greater the number of independent agents, the more precise the information conveyed by the group opinion n. We will assume that every agent makes the same subjective estimation of the total number of independent agents. We will note N_i this unanimous evaluation. N_i will then be considered as a parameter. This parameter is central because it measures the weight the individuals will put on n in forming their opinion. It measures the state of confidence of the population in the value of n. If N_i is small, this means that agents believe that the group opinion is of little value in estimating the probabilities of the states of nature {H} and {L}.

Agents who believe in such a representation are able to calculate $p(n_i| H)$ and $p(n_i| L)$ where n_i is the number of independent agents having chosen (B). For an independent agent, the probability of choosing (B) when the state of nature is {H} (respectively {L}) is equal to p (respectively $1 - p$). It follows that:

$$p(n_i|H) = \binom{N_i}{n_i} \cdot p^{n_i} \cdot (1-p)^{N-n_i} \tag{6}$$

$$p(n_i|L) = \binom{N_i}{n_i} \cdot (1-p)^{n_i} \cdot p^{N-n_i} \tag{7}$$

We can write:

$$\frac{p(n_i|H)}{p(n_i|L)} = \theta^{2n_i - N_i} \tag{8}$$

with:

$$\theta = \frac{p}{1-p} > 1 \quad \text{and} \quad \log\theta > 0$$

Within this framework, agents can calculate $p(H| Z, X, n_i)$. Replacing n by n_i in the inequality (5), it follows that the individual (Z,X) whose last choice is Z and private information is X, will choose (B) if:

$$f_i = \frac{n_i}{N_i} > \frac{1}{2} + \frac{1}{2N_i} \cdot \frac{\log h(Z,X,w)}{\log\theta} \tag{9}$$

We will note:

$$f^*(Z,X,w) = \frac{1}{2} + \frac{1}{2N_i} \cdot \frac{\log h(Z,X,w)}{\log \theta} \tag{10}$$

It appears that the condition (9) is interesting because it integrates through the variable $\log(h)/N_i$, the effect of the relative magnitude of the accuracy of (Z, X), the prior information, compared with N_i, the accuracy of the group opinion (n).

To completely determine agents' choices, we have to understand how they form their subjective evaluation of f_i. This depends on the hypotheses they made concerning the imitators. Several hypotheses are plausible. If agents believe that the probability for an imitator to choose (B) is equal to f_i, their resulting estimation of f_i will follow the law $\tilde{f}_i$ given by the Eq. (11):

$$\tilde{f}_i = f + \tilde{\varepsilon} \quad \text{with} \quad E(\tilde{\varepsilon}) = 0 \tag{11}$$

where $\tilde{\varepsilon}$ can be understood as a pure noise or as the consequence of the fact that the determination of f_i, on the basis of the observation of f, introduces an element of interpretation. $\tilde{\varepsilon}$ will then be analyzed as describing the existence of a variability in the way people understand the world. To choose a random agent j at date t means that a random drawing ε_j has been made which specifies agent j's subjective interpretation of f. The condition of transition from (Z, X, w) to (B) [Eq. (9)] can be rewritten as follows:

$$\tilde{f}_i = f + \tilde{\varepsilon} > f^*$$

For the sake of simplicity, we will assume that $\tilde{f}_i$ follows the distribution represented in Fig. 1, where $\tilde{f}_i \in [f-\alpha, f+\alpha]$. α is a measure of the heterogeneity of individual beliefs about $\tilde{f}_i$, f being given.

It is then possible to calculate the transition probability from choice (V) to (B). From now on, we will assume that the state of the world is {H}. An agent such that

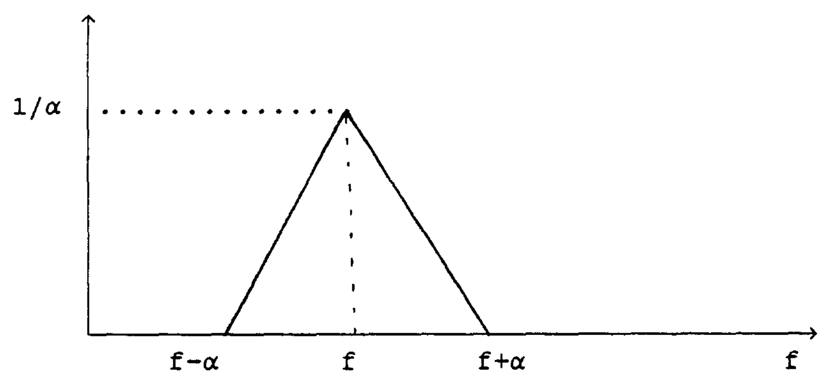

Fig. 1.

Z = (V) will receive the signal {+} with the probability $p(+| H) = p$. He will decide to take the choice (B) if and only if: $\tilde{f}_i > f^*(V,+)$. This will happen with the probability: $\text{prob}(\tilde{f}_i > f^*(V,+))$. With probability $(1 - p)$, the agent (V) will receive the signal {−} and will take the action (B) with the probability: $\text{prob}(\tilde{f}_i > f^*(V,-))$. So, if the state of nature is {H}, we have:

$$\begin{aligned} P_{VB} &= p \cdot \text{prob}\left(\tilde{f}_i > f^*(V,+)\right) + (1-p) \cdot \text{prob}\left(\tilde{f}_i > f^*(V,-)\right) \\ &= P_{VB}(f). \end{aligned} \tag{12}$$

Similarly, for an agent (B), we find:

$$\begin{aligned} P_{BV} &= p \cdot \text{prob}\left(\tilde{f}_i < f^*(B,+)\right) + (1-p) \cdot \text{prob}\left(\tilde{f}_i < f^*(B,-)\right) \\ &= P_{BV}(f). \end{aligned} \tag{13}$$

Equations (12) and (13) provide a complete description of the collective dynamics of the group opinion n(t). The Appendix contains the exact determination of these transition probabilities. In accordance with intuition, P_{VB} will be an increasing function of f and P_{BV} a decreasing function of f. Nevertheless, if α is large, these relationships will be weakened because individual estimations of f_i will be widely dispersed.

4. The dynamics of opinion

The collective configuration of opinions will be defined either by the unidimensional state variable n, the number of agents having chosen (B,) or by the state variable $f = n/N$. Assuming that the population is composed of N agents, there are $(N + 1)$ possible states of collective opinion. At each date t the collective configuration of the group is defined by p(n; t), a probability law defined on S, the set of the $(N + 1)$ configurations of opinion. At date t, one agent is drawn randomly from the overall population and has to choose a new opinion. (This assumption is made only for the sake of simplicity; we could have assumed that L agents are drawn randomly with $L << N$. The results would not have been qualitatively modified). If the actual state of opinion is $f = f(t)$, we can determine the probability W_+ that during the considered unit of time, n turns to $n + 1$: it is equal to the probability of drawing an agent having opinion (V) multiplied by the probability that this agent changes his opinion to opinion (B):

$$\text{prob}(n \to n+1) = W_+(f) = (1-f) \cdot P_{VB}(f)$$

and, in the same manner, $W_-(f)$:

$$\text{prob}(n \to n-1) = W_-(f) = f \cdot P_{BV}(f)$$

These probabilities determine a stochastic process. As Kirman (1993) emphasized, the equilibrium concept within such a framework is not a particular state of opinion f^*. The equilibrium is the limit distribution of the stochastic process under consideration: "this is not a situation with multiple equilibria, in the normal sense, since *every* state is always revisited, and there is no particular convergence to any particular state" (p. 147). The stochastic process is said to be "ergodic" when the probabilistic dynamics converges to a unique stationary distribution, P_{st}. We now have to compute the stationary distribution (when the property of ergodicity holds) and study its relationship to herd behavior. To do that, we will analyze two polar cases: (i) N_i large: in taking their decision, the agents mostly rely on the group opinion; (ii) N_i small: the agents do not believe in the informational relevance of the group opinion; they mostly rely on their private information. (All the corresponding proofs can be found in the Appendix).

4.1. N_i large

A large N_i corresponds to a situation where the agents believe that the accuracy of the collective configuration of opinion is infinitely greater than the accuracy of their private signal. In such a situation a rational agent will not take into account the information he gets from observing his private signal X, either $\{+\}$ or $\{-\}$. He will make his choice on the sole basis of the observation of f. If we assume $1/N_i \cong 0$, we find that $f^*(Z, X, W)$ is equal to $1/2$ whatever the values of Z, X and w are: agents just follow the choice of the majority. Inserting this value of f^* in the definition of the transition probabilities [Equations (12) and (13)], we find that $f = 1/2$ is always an extremum of the stationary distribution. But it is either a maximum or a minimum, according to the value of α. α will be considered as a parameter of control that measures the diversity in the way agents interpret f. We will examine the modifications of the stationary distribution of opinions for different values of α, N_i being held constant. Fig. 2 (2.1–2.4) shows the stationary distributions $P_{st}(f)$ respectively for α equal to 10.0, 1.2, 0.8 and 0.6. The probability p remains equal to 0.6. N has been assumed to be equal to 100.

A large α means that the values of $\tilde{f}_i$ that each agent infers from the observation of f are widely dispersed [Equation (11)]. This effect counteracts the natural reinforcement effect which is directly integrated in the dynamics through the positive relation that links P_{VB} to f and P_{BV} to $(1 - f)$. A large α means that a non-negligible part of the agents interpret a high f in terms of a low f_i. As α increases, the probability of deviant interpretations of the data increases too. When agents make a mistake in interpreting the group opinion, the link between P_{VB} and P_{BV} on one hand, and f on the other hand, is reduced; thus the reinforcement effect loses some of its power. At the limit, when $\alpha \to \infty$, the transition probabilities do not depend any more on f. The interpretation of the agents is becoming completely independent of f: $P_{VB} = P_{BV} = 1/2$. Fig. 2 (2.1) corresponding to α equal to 10 gives us a good approximation of such a situation. The stationary distribution is unimodal with an important peak at $\{f = 0.5\}$. Here everyone is following the

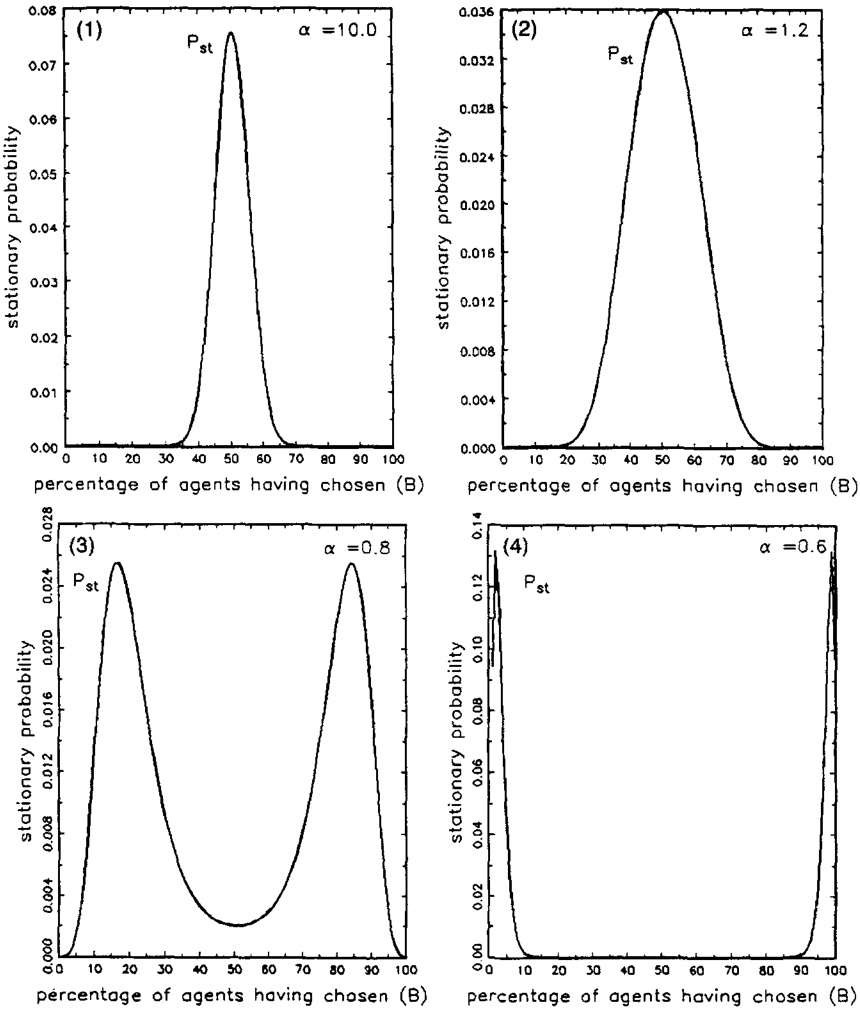

Fig. 2.

choice made by the majority of the independent agents, but half of the individuals believe it is (B) and the other half it is (V).

This result is surprising. It shows that even in a situation where agents essentially rely on the group opinion, neither informational cascade, nor herd behavior, nor unanimity emerge! The existence of an important diversity in the way people interpret f lessens the ex post relation linking their choice to f_i.

When α is decreasing ($\alpha = 1.2$), the shape of the stationary distribution remains unimodal, but its variance becomes greater (Fig. 2 (2.2)). In other words, although the most probable state remains $\{f=0.5\}$, large deviations from the balanced situation can be expected. It should be stressed that these fluctuations do not result from any randomness of fundamental informations (X_i) as measured by its variance: they are produced by the Bayesian interactions among the members of the group. Thus we are confronted with an *endogenous* variability produced by the interactions between agents. It seems to us that this effect may be at the basis of the excess volatility that has often been observed on financial markets.

When α is becoming smaller than 1 ($\alpha=0.8$), the stationary distribution $P_{st}(f)$ is qualitatively altered. The opinion configuration determined by $\{f=0.5\}$ falls to a minimum and becomes highly improbable (Fig. 2 (2.3)). Meanwhile, two maxima, f_+ and f_-, appear. What produces these configurations, f_+ and f_-, is a process in which collective beliefs become self-validating. Having no confidence in their private signal, individuals look for supplementary information, and find it by observing the group opinion itself. When the diversity of individual interpretations becomes low, i.e., α becomes smaller, the role played by average opinion in the formation of individual beliefs leads to a self reinforcing dynamics. Beliefs are polarized on the same evaluation of the frequency f_i, and such a dynamics gives rise to two symmetrical peaks.

When α is still decreasing ($\alpha=0.6$), the form of the distribution does not change, but the large probability peaks shift to $\{f=0\}$ and $\{f=1\}$, which are the situations of unanimity (Fig. 2 (2.4)). When α is smaller than 0.5, the process is no longer ergodic. We have two stationary distributions: δ_0 and δ_1, the Dirac distributions in $\{f=0\}$ and $\{f=1\}$. These two situations correspond to pure situations of herd behavior and informational cascades: the positive feedback effect is strong enough to give rise to unanimity. But this unanimity can either be on (B) or on (V).

4.2. N_i small

We will now consider the opposite situation where N_i, the agents' subjective evaluation of the accuracy of the group opinion, is very small. We will assume that N_i is so small that no transition can occur if X, the private information, is in accordance with Z, the last choice, whatever the value of f is:

$$P_{B+V}=0 \text{ and } P_{V-B}=0 \quad \forall f$$

with P_{ZXV}, the probability that an agent having observed X switches from Z to V. It follows that an agent can change his opinion only if he receives the good signal. We will assume that a V-agent (resp. a B-agent) will switch to opinion (B) (resp. (V)) if and only if he receives the signal $\{+\}$ (resp. $\{-\}$), whatever the value of f is.

In this situation, the value taken by f plays no role in the individual choices. This is completely consistent with our assumption that N_i is small; i.e., the degree

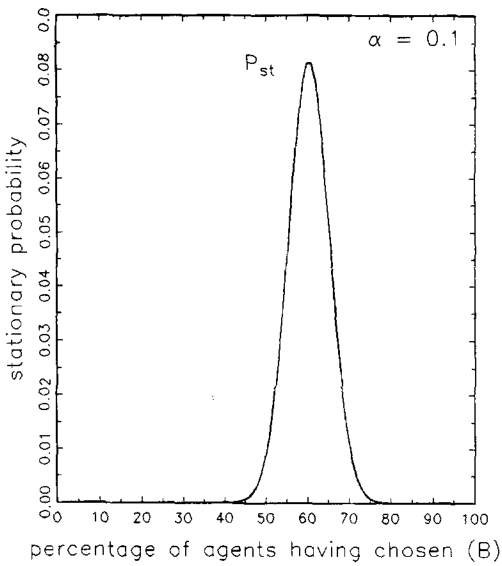

Fig. 3.

of accuracy of the group opinion is so low that all the agents ignore it. The transition probabilities can be written as follows:

$$P_{VB}(f) = p \cdot P_{V+B}(f) = p \quad \forall f$$

$$P_{BV}(f) = (1-p) \cdot P_{B-V}(f) = (1-p) \quad \forall f$$

Fig. 3 shows the stationary distribution we obtain for N_i small and $\alpha = 0.1$. This distribution is unimodal and $\{f = p\}$ is its mode. The distribution of the agents' choices is then identical to the binomial law of parameter p for N agents. As individuals do not use the group opinion, they behave exactly as if they were isolated and independent. What happens if N_i is assumed to be larger, holding constant the preceding hypotheses on the transition probabilities?

For a N_i larger, the problem is getting more complex because the process is no longer ergodic. Strictly speaking, we get two stationary distributions: δ_0 and δ_1, the Dirac distributions in $\{f = 0\}$ and $\{f = 1\}$. Here we find again our informational cascades. The process can converge either on the right value or on the wrong one. But we have to interpret this result with care because the process is no longer ergodic. One can show that there is a competition between two dynamics (see the Appendix).

The first one pushes the collective opinion towards $\{f = p\}$ when f(0) is not too far from this point. The rationale of this tendency is easy to understand. It is exactly the same that the one described in Fig. 3. More precisely, there exists a certain interval $[f_s, f_T]$ (see Fig. 4 in the Appendix), such that if we consider the "associated deterministic dynamics" (see Appendix), when f(0) is within this interval, f(t) will converge towards the point {f = p}. But if f(0) is belonging to the interval $[0,f_s]$, the process is converging to {0}; and if f(0) is belonging to $[f_T,1]$, it converges to {1}. We are here confronted with a second dynamics which is based on the reinforcement effect produced by the use of the group opinion as a source of information. When f is close to {0} or to {1}, the reinforcement effect dominates the first dynamics. Why then do we not observe a stationary distribution with three modes corresponding to the three "deterministic equilibria"? Because {f = 0} and {f = 1} are absorbing states. Once the process reaches these points, the opinion configuration is locked-in: it cannot escape from these points. So, if we take into account stochastic fluctuations, it appears that n(t) can remain in the vicinity of {f = p} during very long periods. But, with probability one, we can be sure that f will be found outside the central interval $[f_s,f_T]$ and then will converge towards one of the two absorbing states, remaining there for ever.

5. Conclusion

The present article generalizes to a non-sequential context previous results concerning herd behavior and informational cascades. As soon as the hypothesis of sequentiality is dropped, that a very large variety of situations can be observed. Nevertheless, informational cascades are obtained in very different contexts. This phenomenon seems to be closely linked to non-ergodicity.

These results have been obtained within a general model where collective opinion emerges from the interactions of individual beliefs. This type of formalization should enable us to analyze the role played by interpersonal influences and mimetism in financial dynamics (e.g., Keynes (1936), Kindleberger (1978)).

Appendix A

We have assumed that $\tilde{f}$ follows the distribution shown in figure 1. If a part of the interval $[f^* - \alpha, f^* + \alpha]$ is outside [0,1], we put the weight of this part on 0 or 1. Therefore the formula given by the following equations are true for f belonging to [0,1] even if, for example, f^*, $f^* + \alpha$ or $f^* - \alpha$ are not included between 0 and 1. Let us consider the agent (Z, X, w) and the associated value f^* (Equation (10)). It is easy to see that, for $f \in [0,1]$, the probability of transition from (Z, X) to (B) is equal to:

if $f > f^* + \alpha$

$P_{(Z,X)B} = 1$

if $f^* \leq f \leq f^* + \alpha$

$$P_{(Z,X)B} = 1 - \frac{1}{2\alpha^2} \cdot [(f^* + \alpha) - f]^2$$

if $f^* - \alpha \leq f \leq f^*$

$$P_{(Z,X)B} = \frac{1}{2\alpha^2} \cdot [f - (f^* - \alpha)]^2$$

if $f \leq f^* - \alpha$

$$P_{(Z,X)B} = 0$$

It is easy to infer from these equations the values of the transition probabilities to (V), because: $P_{(Z,X)V} = 1 - P_{(Z,X)B}$. When the state of nature is {H}, the values of P_{VB} and P_{BV} are then determined on the basis of P_{V+B}, P_{V-B}, P_{B+V} and P_{B-V}:

$$P_{VB} = p \cdot P_{V+B} + (1-p) \cdot P_{V-B}$$

$$P_{BV} = p \cdot P_{B+V} + (1-p) \cdot P_{B-V}$$

The transition probabilities depend on (Z, X, w) through the value of f^* [Equation (10)]. It can be shown that:

$$f^*(B,+) = 0.5 - u < 0.5 \quad \text{with } u > 0$$

$$f^*(V,-) = 0.5 + u > 0.5$$

$$f^*(B,-) = 0.5 - v$$

$$f^*(V,+) = 0.5 + v \quad \text{with } -u < v < u$$

where u and v are functions of w.

When the state of opinion is $n = n(t)$, we know that:

$$\text{prob}\,(n \to n+1) = W_+(f) = (1-f) \cdot P_{VB}(f)$$

$$\text{prob}\,(n \to n-1) = W_-(f) = f \cdot P_{BV}(f)$$

When $W_+(f)$ is greater (smaller) than $W_-(f)$, the expected change is an increase (decrease) of f. So, the associated ''deterministic'' dynamics is determined by the following drift coefficient:

$$K(f) = W_+(f) - W_-(f)$$

Let us consider the potential V(f) defined by the following relation:

$$K(f) = -\frac{dV(f)}{df}$$

The deterministic motion generated by the potential V(f) will be called the deterministic dynamics associated to our stochastic process. When K(f) is computed, it is very easy to understand how this deterministic dynamics works. f is increasing (resp. decreasing) when K(f) is positive (resp. negative). So the fixed points of this deterministic dynamics are defined by K(f) equal to 0. But to be a stable fixed point, K′(f) must be negative. When we consider the stochastic process, these properties are transformed. The states f^*, such that $\{K(f^*) = 0\}$, are extrema of the stationary distribution of probability. They are maxima if $K'(f^*)$ is negative; they are minima if $K'(f^*)$ is positive (Weidlich and Haag (1983))

For N_i large, we know that $f^*(Z, X, w)$ is equal to 0.5, whatever the values of Z, X, and w are. Then we can calculate K(f):

$$\text{if } f < \frac{1}{2} - \alpha$$

$$P_{VB} = 0 \text{ and } P_{BV} = 1$$

$$K = (1-f)\cdot P_{VB} - f\cdot P_{BV} = -f$$

$$\text{if } f \in \left[\frac{1}{2} - \alpha, \frac{1}{2}\right]$$

$$K(f) = \frac{1}{2\alpha^2}\cdot\left[f - \left(\frac{1}{2} - \alpha\right)\right]^2 - f$$

$$\text{if } f \in \left[\frac{1}{2}, \frac{1}{2} + \alpha\right]$$

$$K(f) = -\frac{1}{2\alpha^2}\cdot\left[\left(\frac{1}{2} + \alpha\right) - f\right]^2 + (1-f)$$

$$\text{if } f > \frac{1}{2} + \alpha$$

$$P_{VB} = 1 \text{ and } P_{BV} = 0$$

$$K = (1-f)\cdot P_{VB} - f\cdot P_{BV} = (1-f)$$

It follows that $K(1/2) = 0$. Then 1/2 is always an extremum, but is it a maximum or a minimum? To answer this question, we must calculate $K'(1/2)$:

$$K'(f) = \frac{1}{\alpha^2}\cdot\left[f - \left(\frac{1}{2} - \alpha\right)\right] - 1 \quad \text{for } f \in \left[\frac{1}{2} - \alpha, \frac{1}{2}\right]$$

$$K'(1/2) = \frac{1}{\alpha} - 1$$

$K'(1/2)$ is negative for α superior to 1. The process is ergodic for α superior to 0.5. Then, for values of α inferior to 1 and superior to 0.5, the process is ergodic and 1/2 is a minimum. For α superior to 1, $K'(1/2)$ is negative and 1/2 is a maximum.

To analyze N_i small, we have to assume that: $f^*(B,+,w) + \alpha < 0$ which

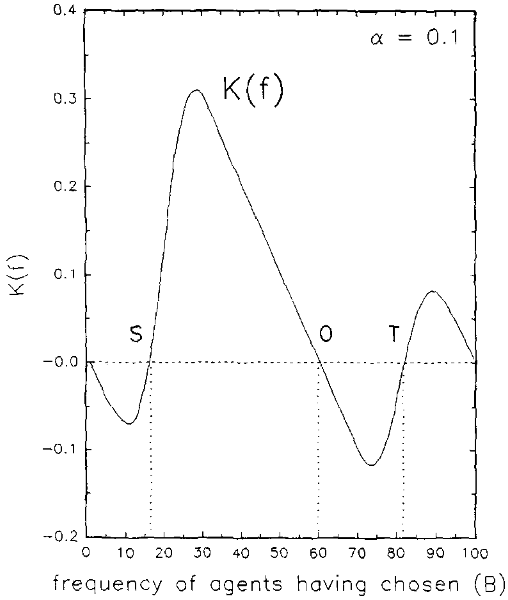

Fig. 4.

implies $f^*(V,-,w) - \alpha > 1$. This means that $P_{B+V} = 0$ and $P_{V-B} = 0$ for $f \in [0, 1]$. It follows that:

$$P_{BV} = (1 - p) \cdot P_{B-V}$$

$$P_{VB} = p \cdot P_{V+B}$$

Let us now consider $f^*(B,-,w)$ and let us assume that this variable is greater than $1 - \alpha$. If this is true, the $(V,-)$ agent will change his opinion to (B) if and only if he receives the signal $\{+\}$.

$$P_{VB} = p \cdot P_{V+B} = p$$

$$P_{BV} = (1 - p) \cdot P_{B-V} = (1 - p)$$

$$K(f) = (1 - f) \cdot P_{VB} - f \cdot P_{BV} = p(1 - f) - f(1 - p) = -f + p$$

When N_i is getting larger and when the state of nature remains {H}, K(f) is given by the Fig. 4. We have assumed that $p = 0.6$. We note that K(0.6) is still equal to 0 and K′(0.6) is still negative, but the process is no longer ergodic (on the x axis, p is written in percentage). It converges to the Dirac distributions in $\{f = 0\}$ and $\{f = 1\}$. We can note that between the points S and T, the associated deterministic process would have led the dynamics towards $\{f = p\}$.

References

Arthur, Brian W. and David A. Lane, 1989, Information contagion, Mimeo, 18 pages.
Banerjee, Abhijit V., 1992, A simple model of herd behavior, Quarterly Journal of Economics 110, 3, 797–817.
Banerjee, Abhijit V., 1993, The economics of rumours, Review of Economic Studies 60, 309–327.
Bikhchandani, Sushil, David Hirshleifer and Ivo Welch, 1992, A theory of fads, fashion, custom, and cultural change as informational cascades, Journal of Political Economy 100, 5, 992–1026.
Föllmer, Hans, 1974, Random economies with many interacting agents, Journal of Mathematical Economics 1, 51–62.
Froot, Kenneth A., David S. Scharfstein and Jeremy C. Stein, 1992, Herd on the street: Informational inefficiencies in a market with short-term speculation, Journal of Finance XLVII, 4, 1461–1484.
Harris, Milton and Artur Raviv, 1993, Differences of opinion make a horse race, The Review of Financial Studies 6, 3, 473–506.
Keynes, John M., 1936, The general theory of employment, interest and money, New York, Harcourt; 1973 reprinted as vol. 7 of The collected writings of John Maynard Keynes, London, Macmillan.
Kindleberger, Charles, 1978, Manias, panics and crashes, London, Macmillan.
Kirman, Alan, 1993, Arts, rationality, and recruitment, Quarterly Journal of Economics 111, 137–156.
Orléan, André, 1989, Mimetic contagion and speculative bubbles, Theory and Decision 27 (1–2), 63–92.
Shiller, Robert J., 1989, Market volatility, MIT Press.
Topol, Richard, 1991, Bubbles and volatility of stock prices: Effect of mimetic contagion, The Economic Journal, 101, 407, 786–800.
Weidlich, Wolfgang and Günter Haag, 1983, Concepts and models of a quantitative sociology, Springer-Verlag.