HW Assignment #7

Consider the heat transfer problem

with initial and Dirichlet boundary conditions

and function The analytic solution to this problem is

Write a 1D Galerkin code to solve this problem. Use N = 11 nodes, but make the number of nodes general. Use the 1D Lagrange basis functions. Keep your code general so that different f(x,t) and/or boundary conditions can be implemented. Use 2nd order Gaussian quadrature to numerically integrated the f(x,t) (right hand side) weak integral.

**\*\*Build the elemental mass and stiffness matrices by mapping from the x-space to xi-space, integrating in parent space and assemble to global as done in class \*\***

1. Solve first by using a forward Euler time derivative discretization with a time-step of . Plot the results at the final time. Increase the time-step until you find the instability. What dt does this occur at? How does the solution change as N decreases?
2. Solve the same problem with the same time-steps using an implicit backward Euler. What happens as the time-step is equal to or greater than the spatial step size? Explain why.