

# Hedging

Does an *instantaneous* cube exist?  
H.G. Wells, *The Time Machine*

Hedging strategies appear throughout this book. This is to be expected because one of the principal uses of derivatives is in the management of risks. In this chapter, we focus on the use of non-interest-rate derivatives in hedging. Interest rate derivatives will be picked up in Chap. 21.

## 16.1 Introduction

One common thread throughout this book has been the management of risks. **Risk management** means selecting and maintaining portfolios with defined exposure to risks. Deciding which risks one is to be exposed to and which risks one is to be protected against is also an integral part of risk management. Evidence suggests that firms engaged in risk management not only are less risky but also perform better [813].

A **hedge** is a position that offsets the price risk of another position. A hedge reduces risk exposures or even eliminates them if it provides cash flows equal in magnitude but opposite in directions to those of the existing exposure. For hedging to be possible, the return of the derivative should be correlated with that of the hedged position. In fact, the more correlated their returns are, the more effective the hedge will be.

Three types of traders play in the markets. **Hedgers** set up positions to offset risky positions in the spot market. **Speculators** bet on price movements and hope to make a profit. **Arbitragers** lock in riskless profits by simultaneously entering into transactions in two or more markets, which is called arbitrage.

## 16.2 Hedging and Futures

The most straightforward way of hedging involves forward contracts. Because of daily settlements, futures contracts are harder to analyze than forward contracts. Luckily, the forward price and the futures price are generally close to each other; therefore results obtained for forwards will be assumed to be true of futures here.

### 16.2.1 Futures and Spot Prices

Two forces prevent the prevailing prices in the spot market and the futures market from diverging too much at any given time. One is the delivery mechanism, and the other is hedging. Hedging relates the futures price and the spot price through arbitrage. In fact, the futures price should equal the spot price by the carrying charges. Carrying charges, we recall, are the costs of holding physical inventories between now and the maturity of the futures contract. In practice, the futures price does not necessarily exceed the spot price by exactly the carrying charges for inventories that have what Kaldor (1908–1986) termed the convenience yield derived from their availability when buyers need them [468].

### 16.2.2 Hedgers, Speculators, and Arbitragers

A company that is due to sell an asset in the future can hedge by taking a short futures position. This is known as a **selling** or **short hedge**. The purpose is to lock in a selling price or, with fixed-income securities, a yield. If the price of the asset goes down, the company does not fare well on the sale of the asset, but it makes a gain on the short futures position. If the price of the asset goes up, the reverse is true. Clearly, a selling hedge is a substitute for a later cash market sale of the asset. A company that is due to buy an asset in the future can hedge by taking a long futures position. This is known as a **buying** or **long hedge**. Clearly, a buying hedge is used when one plans to buy the cash asset at a later date. The purpose is to establish a fixed purchase price. These strategies work because spot and futures prices are correlated.

A person who gains or loses from the difference between the spot and the futures prices is said to **speculate on the basis**. Simultaneous purchase and sale of futures contracts on two different yet related assets is referred to as a **spread**. A person who speculates by using spreads is called a **spreader** [95, 799]. A hedger is someone whose net position in the spot market is offset by positions in the futures market. A **short hedger** is long in the spot market and short in the futures market. A **long hedger** does the opposite. Those who are net long or net short are speculators. Speculators will buy (sell) futures contracts only if they expect prices to increase (decrease, respectively). Hedgers, in comparison, are willing to pay a premium to unload unwanted risk onto speculators. Speculators provide the market with liquidity, enabling hedgers to trade large numbers of contracts without adversely disrupting prices.

If hedgers in aggregate are short, speculators are net long and the futures price is set below the expected future spot price. On the other hand, if hedgers are net long, speculators are net short and the futures price is set above the expected future spot price. There seems to be evidence that short hedging exceeds long hedging in most of the markets most of the time. If hedgers are net short in futures, speculators must be net long. It has been theorized that speculators will be net long only if the futures price is expected to rise until it equals the spot price at maturity; speculators therefore extract a risk premium from hedgers. This is Keynes's theory of **normal backwardation**, which implies that the futures price underestimates the future spot price [295, 468, 470].

► **Exercise 16.2.1** If the futures price equals the expected future spot price, then hedging may in some sense be considered a free lunch. Give your reasons.

### 16.2.3 Perfect and Imperfect Hedging

Consider an investor who plans on selling an asset  $t$  years from now. To eliminate some of the price uncertainties, the investor sells futures contracts on the same asset with a delivery date in  $T$  years. After  $t$  years, the investor liquidates the futures position and sells the asset as planned. The cash flow at that time is

$$S_t - (F_t - F) = F + (S_t - F_t) = F + \text{basis},$$

where  $S_t$  is the spot price at time  $t$ ,  $F_t$  is the futures price at time  $t$ , and  $F$  is the original futures price. The investor has replaced the price uncertainty with the smaller basis uncertainty; as a consequence, risk has been reduced. The hedge is **perfect** if  $t = T$ , that is, when there is a futures contract with a matching delivery date.

When the cost of carry and the convenience yield are known, the cash flow can be anticipated with complete confidence according to Eq. (12.13). This holds even when there is a maturity mismatch  $t \neq T$  as long as (1) the interest rate  $r$  is known and (2) the cost of carry  $c$  and the convenience yield  $y$  are constants. In this case,  $F_t = S_t e^{(r+c-y)(T-t)}$  by Eqs. (12.11). Let  $h$  be the number of futures contracts sold initially. The cash flow at time  $t$ , after the futures position is liquidated and the asset is sold, is

$$S_t - h(F_t - F) = S_t - h[S_t e^{(r+c-y)(T-t)} - F].$$

Pick  $h = e^{-(r+c-y)(T-t)}$  to make the cash flow a constant  $hF$  and eliminate any uncertainty. The number  $h$  is the hedge ratio. Note that  $h = 1$  may not be the best choice when  $t \neq T$ .

A number of factors make hedging with futures contracts less than perfect. The asset whose price is to be hedged may not be identical to the underlying asset of the futures contract; the date when the asset is to be transacted may be uncertain; the hedge may require that the futures contract be closed out before its expiration date. These problems give rise to basis risk. As shown above, basis risk does not exist in situations in which the spot price relative to the futures price moves in predictable manners.

#### Cross Hedge

A hedge that is established with a maturity mismatch, an asset mismatch, or both is referred to as a **cross hedge** [746]. Cross hedges are common practices. When firms want to hedge against price movements in a commodity for which there are no futures contracts, they can turn to futures contracts on related commodities whose price movements closely correlate with the price to be hedged.

**EXAMPLE 16.2.1** We can hedge a future purchase price of 10,000,000 Dutch guilders as follows. Suppose the current exchange rate is U.S.\$0.48 per guilder. At this rate, the dollar cost is U.S.\$4,800,000. A regression analysis of the daily changes in the guilder rate and the nearby German mark futures reveals that the estimated slope is 0.95 with an  $R^2$  of 0.92. Now that the guilder is highly correlated with the German mark, German mark futures are picked. The current exchange rate is U.S.\$0.55/DEM1; hence the commitment of 10,000,000 guilders translates to  $4,800,000/0.55 = 8,727,273$  German marks. Because each futures contract controls 125,000 marks, we trade  $0.95 \times (8,727,273/125,000) \approx 66$  contracts.

**EXAMPLE 16.2.2** A British firm expecting to pay DEM2,000,000 for purchases in 3 months would like to lock in the price in pounds. Besides the standard way of using mark futures contracts that trade in pounds, the firm can trade mark and pound futures contracts that trade in U.S. dollars as follows. Each mark futures contract controls 125,000 marks, and each pound futures contract controls 62,500 pounds. Suppose the payment date coincides with the last trading date of the currency futures contract at the CME. Currently the 3-month mark futures price is \$0.7147 and the pound futures price is \$1.5734. The firm buys  $2,000,000/125,000 = 16$  mark futures contracts, locking in a purchase price of \$1,429,400. To further lock in the price in pounds at the exchange rate of \$1.5734/£1, the firm shorts  $1,429,400/(1.5734 \times 62,500) \approx 15$  pound futures contracts. The end result is a purchase price of  $2,000,000 \times (0.7147/1.5734) = 908,478$  pounds with a cross rate of £0.45424/DEM1.

### Hedge Ratio (Delta)

In general, the futures contract may not track the cash asset perfectly. Let  $\rho$  denote the correlation between  $S_t$  and  $F_t$ ,  $\delta_S$  the standard deviation of  $S_t$ , and  $\delta_F$  the standard deviation of  $F_t$ . For a short hedge, the cash flow at time  $t$  is  $S_t - h(F_t - F)$ , whereas for a long hedge it is  $-S_t + h(F_t - F)$ . The variance is  $V \equiv \delta_S^2 + h^2\delta_F^2 - 2h\rho\delta_S\delta_F$  in both cases. To minimize risk, the hedger seeks the hedge ratio  $h$  that minimizes the variance of the cash flow of the hedged position,  $V$ . Because  $\partial V/\partial h = 2h\delta_F^2 - 2\rho\delta_S\delta_F$ ,

$$h = \rho \frac{\delta_S}{\delta_F} = \frac{\text{Cov}[S_t, F_t]}{\text{Var}[F_t]}, \quad (16.1)$$

which was called beta in Exercise 6.4.1.

**EXAMPLE 16.2.3** Suppose that the standard deviation of the change in the price per bushel of corn over a 3-month period is 0.4 and that the standard deviation of the change in the soybeans futures price over a 3-month period is 0.3. Assume further that the correlation between the 3-month change in the corn price and the 3-month change in the soybeans futures price is 0.9. The optimal hedge ratio is  $0.9 \times (0.4/0.3) = 1.2$ . Because the size of one soybeans futures contract is 5,000 bushels, a company expecting to buy 1,000,000 bushels of corn in 3 months can hedge by buying  $1.2 \times (1,000,000/5,000) = 240$  futures contracts on soybeans.

The hedge ratio can be estimated as follows. Suppose that  $S_1, S_2, \dots, S_t$  and  $F_1, F_2, \dots, F_t$  are the daily closing spot and futures prices, respectively. Define  $\Delta S_i \equiv S_{i+1} - S_i$  and  $\Delta F_i \equiv F_{i+1} - F_i$ . Now estimate  $\rho, \delta_S$ , and  $\delta_F$  with Eqs. (6.2) and (6.18).

► **Exercise 16.2.2** Show that if the linear regression of  $s$  on  $f$  based on the data

$$(\Delta S_1, \Delta F_1), (\Delta S_2, \Delta F_1), \dots, (\Delta S_{t-1}, \Delta F_{t-1})$$

is  $s = \beta_0 + \beta_1 f$ , then  $\beta_1$  is an estimator of the hedge ratio in Eq. (16.1).

### 16.2.4 Hedging with Stock Index Futures

Stock index futures can be used to hedge a well-diversified portfolio of stocks. According to the Capital Asset Pricing Model (CAPM), the relation between the return

on a portfolio of stocks and the return on the market can be described by a parameter  $\beta$ , called beta. Approximately,

$$\Delta_1 = \alpha + \beta \times \Delta_2,$$

where  $\Delta_1$  ( $\Delta_2$ ) is the change in the value of \$1 during the holding period if it is invested in the portfolio (the market index, respectively) and  $\alpha$  is some constant. The change in the portfolio value during the period is therefore  $S \times \alpha + S \times \beta \times \Delta_2$ , where  $S$  denotes the current value of the portfolio. The change in the value of one futures contract that expires at the end of the holding period is approximately  $F \times \Delta_2$ , where  $F$  is the current value of one futures contract. Recall that the value of one futures contract is equal to the futures price multiplied by the contract size. For example, if the futures price of the S&P 500 is 1,000, the value of one futures contract is  $1,000 \times 500 = 500,000$ .

The uncertain component of the change in the portfolio value,  $S \times \beta \times \Delta_2$ , is approximately  $\beta S/F$  times the change in the value of one futures contract,  $F \times \Delta_2$ . The number of futures contracts to short in hedging the portfolio is thus  $\beta S/F$ . This strategy is called **portfolio immunization**. The same idea can be applied to change the beta of a portfolio. To change the beta from  $\beta_1$  to  $\beta_2$ , we short

$$(\beta_1 - \beta_2) \frac{S}{F} \quad (16.2)$$

contracts. A perfectly hedged portfolio has zero beta and corresponds to choosing  $\beta_2 = 0$ .

**EXAMPLE 16.2.4** Hedging a well-diversified stock portfolio with the S&P 500 Index futures works as follows. Suppose the portfolio in question is worth \$2,400,000 with a beta of 1.25 against the returns on the S&P 500 Index. So, for every 1% advance in the index, the expected advance in the portfolio is 1.25%. With a current futures price of 1200,  $1.25 \times [2,400,000/(1,200 \times 500)] = 5$  futures contracts are sold short.

➤ **Exercise 16.2.3** Redo Example 16.2.4 if the goal is to change the beta to 2.0.

## 16.3 Hedging and Options

### 16.3.1 Delta Hedge

The delta (hedge ratio) of a derivative is defined as  $\Delta \equiv \partial f / \partial S$ . Thus  $\Delta f \approx \Delta \times \Delta S$  for relatively small changes in the stock price,  $\Delta S$ . A delta-neutral portfolio is hedged in the sense that it is immunized against small changes in the stock price. A trading strategy that dynamically maintains a delta-neutral portfolio is called **delta hedge**.

Because delta changes with the stock price, a delta hedge needs to be rebalanced periodically in order to maintain delta neutrality. In the limit in which the portfolio is adjusted continuously, perfect hedge is achieved and the strategy becomes self-financing [294]. This was the gist of the Black–Scholes–Merton argument in Subsection 15.2.1.

For a non-dividend-paying stock, the delta-neutral portfolio hedges  $N$  short derivatives with  $N \times \Delta$  shares of the underlying stock plus  $B$  borrowed dollars such that

$$-N \times f + N \times \Delta \times S - B = 0.$$

This is called the **self-financing condition** because the combined value of derivatives, stock, and bonds is zero. At each rebalancing point when the delta is  $\Delta'$ , buy  $N \times (\Delta' - \Delta)$  shares to maintain  $N \times \Delta'$  shares with a total borrowing of  $B' = N \times \Delta' \times S' - N \times f'$ , where  $f'$  is the derivative's prevailing price. A delta hedge is a discrete-time analog of the continuous-time limit and will rarely be self-financing, if ever.

► **Exercise 16.3.1** (1) A delta hedge under the BOPM results in perfect replication (see Chap. 9). However, this is impossible in the current context. Why? (2) How should the value of the derivative behave with respect to that of the underlying asset for perfect replication to be possible?

### A Numerical Example

Let us illustrate the procedure with a hedger who is short European calls. Because the delta is positive and increases as the stock price rises, the hedger keeps a long position in stock and buys (sells) stock if the stock price rises (falls, respectively) in order to maintain delta neutrality. The calls are replicated well if the cumulative cost of trading stock is close to the call premium's FV at expiration.

Consider a trader who is short 10,000 calls. This call's expiration is 4 weeks away, its strike price is \$50, and each call has a current value of  $f = 1.76791$ . Because an option covers 100 shares of stock,  $N = 1,000,000$ . The underlying stock has 30% annual volatility, and the annual riskless rate is 6%. The trader adjusts the portfolio weekly. As  $\Delta = 0.538560$ ,  $N \times \Delta = 538,560$  shares are purchased for a total cost of  $538,560 \times 50 = 26,928,000$  dollars to make the portfolio delta-neutral. The trader finances the purchase by borrowing

$$B = N \times \Delta \times S - N \times f = 26,928,000 - 1,767,910 = 25,160,090$$

dollars net. The portfolio has zero net value now.

At 3 weeks to expiration, the stock price rises to \$51. Because the new call value is  $f' = 2.10580$ , the portfolio is worth

$$-N \times f' + 538,560 \times 51 - Be^{0.06/52} = 171,622 \quad (16.3)$$

before rebalancing. That this number is not zero confirms that a delta hedge does not replicate the calls perfectly; it is not self-financing as \$171,622 can be withdrawn. The magnitude of the **tracking error** – the variation in the net portfolio value – can be mitigated if adjustments are made more frequently, say daily instead of weekly. In fact, the tracking error is positive ~68% of the time even though its expected value is essentially zero. It is furthermore proportional to vega [109, 537]. In practice tracking errors will cease to decrease to beyond a certain rebalancing frequency [45].

With a higher delta  $\Delta' = 0.640355$ , the trader buys  $N \times (\Delta' - \Delta) = 101,795$  shares for \$5,191,545, increasing the number of shares to  $N \times \Delta' = 640,355$ . The cumulative cost is

$$26,928,000 \times e^{0.06/52} + 5,191,545 = 32,150,634,$$

and the net borrowed amount is<sup>1</sup>

$$B' = 640,355 \times 51 - N \times f' = 30,552,305.$$

The portfolio is again delta-neutral with zero value. Figure 16.1 tabulates the numbers.

<i>Weeks to expiration</i>	<i>Stock price S</i>	<i>Option value f</i>	<i>Delta Δ</i>	<i>Gamma Γ</i>	<i>Change in delta (3)–(3'')</i>	<i>No. shares bought N × (5)</i>	<i>Cost of shares (1) × (6)</i>	<i>Cumulative cost FV(8'') + (7)</i>
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	
4	50	1.76791	0.538560	0.0957074	–	538,560	26,928,000	26,928,000
3	51	2.10580	0.640355	0.1020470	0.101795	101,795	5,191,545	32,150,634
2	53	3.35087	0.855780	0.0730278	0.215425	215,425	11,417,525	43,605,277
1	52	2.24272	0.839825	0.1128601	–0.015955	–15,955	–829,660	42,825,960
0	54	4.00000	1.000000	0.0000000	0.160175	160,175	8,649,450	51,524,853

**Figure 16.1:** Delta hedge. The cumulative cost reflects the cost of trading stocks to maintain delta neutrality. The total number of shares is 1,000,000 at expiration (trading takes place at expiration, too). A doubly primed number refers to the entry from the previous row of the said column.

At expiration, the trader has 1,000,000 shares, which are exercised against by the in-the-money calls for \$50,000,000. The trader is left with an obligation of

$$51,524,853 - 50,000,000 = 1,524,853,$$

which represents the replication cost. Compared with the FV of the call premium,

$$1,767,910 \times e^{0.06 \times 4/52} = 1,776,088,$$

the net gain is  $1,776,088 - 1,524,853 = 251,235$ . The amount of money to start with for perfect replication should converge to the call premium \$1,767,910 as the position is rebalanced more frequently.

► **Exercise 16.3.2** (1) Repeat the calculations in Fig. 16.1 but this time record the weekly tracking errors instead of the cumulative costs. Verify that the following numbers result:

<i>Weeks to expiration</i>	<i>Net borrowing (B)</i>	<i>Tracking error</i>
4	25,160,090	–
3	30,552,305	171,622
2	42,005,470	367
1	41,428,180	203,874
0	50,000,000	–125,459

(This alternative view looks at how well the call is hedged.) (2) Verify that the FVs at expiration of the tracking errors sum to \$251,235.

► **Exercise 16.3.3** A broker claimed the option premium is an arbitrage profit because he could write a call, pocket the premium, then set up a replicating portfolio to hedge the short call. What did he overlook?

► **Programming Assignment 16.3.4** Implement the delta hedge for options.



<i>Weeks to expiration</i>	<i>Stock price <math>S</math></i>	<i>Option value <math>f_2</math></i>	<i>Delta <math>\Delta_2</math></i>	<i>Gamma <math>\Gamma_2</math></i>
4	50	1.99113	0.543095	0.085503
3	51	2.35342	0.631360	0.089114
2	53	3.57143	0.814526	0.070197
1	52	2.53605	0.769410	0.099665
0	54	4.08225	0.971505	0.029099

**Figure 16.2:** Hedging option used in delta–gamma hedge. This option is the same as the one in Fig. 16.1 except that the expiration date is 1 week later.

### 16.3.2 Delta–Gamma and Vega-Related Hedges

A delta hedge is based on the first-order approximation to changes in the derivative price  $\Delta f$ , which is due to small changes in the stock price  $\Delta S$ . When  $\Delta S$  is not small, the second-order term, gamma  $\Gamma \equiv \partial^2 f / \partial S^2$ , can help. A **delta–gamma hedge** is like delta hedge except that zero portfolio gamma, or **gamma neutrality**, is maintained. To meet this extra condition, in addition to self-financing and delta neutrality, one more security needs to be brought in.

The hedging procedure will be illustrated for the scenario in Fig. 16.1. A hedging call is brought in, and its properties along the same scenario are in Fig. 16.2. With the stock price at \$50, each call has a value of  $f = 1.76791$ , delta  $\Delta = 0.538560$ , and gamma  $\Gamma = 0.0957074$ , whereas each hedging call has value  $f_2 = 1.99113$ ,  $\Delta_2 = 0.543095$ , and  $\Gamma_2 = 0.085503$ . Note that the gamma of the stock is zero. To set up a delta–gamma hedge, we solve

$$\begin{aligned} -N \times f + n_1 \times 50 + n_2 \times f_2 - B &= 0 \quad (\text{self-financing}), \\ -N \times \Delta + n_1 + n_2 \times \Delta_2 - 0 &= 0 \quad (\text{delta neutrality}), \\ -N \times \Gamma + 0 + n_2 \times \Gamma_2 - 0 &= 0 \quad (\text{gamma neutrality}). \end{aligned}$$

The solutions are  $n_1 = -69,351$ ,  $n_2 = 1,119,346$ , and  $B = -3,006,695$ . We short 69,351 shares of stock, buy 1,119,346 hedging calls, and lend 3,006,695 dollars. The cost of shorting stock and buying calls is  $n_1 \times 50 + n_2 \times f_2 = -1,238,787$  dollars.

One week later, the stock price climbs to \$51. The new call values are  $f' = 2.10580$  and  $f'_2 = 2.35342$  for the hedged and the hedging calls, respectively, and the portfolio is worth

$$-N \times f' + n_1 \times 51 + n_2 \times f'_2 - Be^{0.06/52} = 1,757$$

before rebalancing. As this number is not zero, a delta–gamma hedge is not self-financing. Nevertheless, it is substantially smaller than delta hedge's 171,622 in Eq. (16.3). Now we solve

$$\begin{aligned} -N \times f' + n'_1 \times 51 + n'_2 \times f'_2 - B' &= 0, \\ -N \times \Delta' + n'_1 + n'_2 \times \Delta'_2 - 0 &= 0, \\ -N \times \Gamma' + 0 + n'_2 \times \Gamma'_2 - 0 &= 0. \end{aligned}$$

The solutions are  $n'_1 = -82,633$ ,  $n'_2 = 1,145,129$ , and  $B' = -3,625,138$ . The trader therefore purchases  $n'_1 - n_1 = -82,633 + 69,351 = -13,282$  shares of stock and



Weeks to expiration	Stock price $S$	No. shares bought $n'_1 - n_1$	Cost of shares $(1) \times (2)$	No. options bought $n'_2 - n_2$	Cost of options $(4) \times f_2$	Net borrowing $B$	Cumulative cost $FV(7'') + (3) + (5)$
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
4	50	−69,351	−3,467,550	1,119,346	2,228,763	−3,006,695	−1,238,787
3	51	−13,282	−677,382	25,783	60,678	−3,625,138	−1,856,921
2	53	91,040	4,825,120	−104,802	−374,293	810,155	2,591,762
1	52	−39,858	−2,072,616	92,068	233,489	−1,006,346	755,627
0	54	1,031,451	55,698,354	−1,132,395	−4,622,720	50,000,000	51,832,134

**Figure 16.3:** Delta–gamma hedge. The cumulative cost reflects the cost of trading stock and the hedging call to maintain delta–gamma neutrality. At expiration, the number of shares is 1,000,000, whereas the number of hedging calls is zero.

$n'_2 - n_2 = 1,145,129 - 1,119,346 = 25,783$  hedging calls for  $-13,282 \times 51 + 25,783 \times f'_2 = -616,704$  dollars. The cumulative cost is

$$-1,238,787 \times e^{0.06/52} - 616,704 = -1,856,921.$$

The portfolio is again delta-neutral and gamma-neutral with zero value. The remaining steps are tabulated in Fig. 16.3.

At expiration, the trader owns 1,000,000 shares, which are exercised against by the in-the-money calls for \$50,000,000. The trader is then left with an obligation of

$$51,832,134 - 50,000,000 = 1,832,134.$$

With the FV of the call premium at \$1,776,088, the net loss is \$56,046, which is smaller than the \$251,235 with the delta hedge.

If volatility changes, a delta–gamma hedge may not work well. An enhancement is the **delta–gamma–vega hedge**, which maintains also **vega neutrality**, meaning zero portfolio vega. As before, to accomplish this, one more security has to be brought into the process. Because this strategy does not involve new insights, it is left to the reader. In practice, the **delta–vega hedge**, which may not maintain gamma neutrality, performs better than the delta hedge [44].

➤ **Exercise 16.3.5** Verify that any delta-neutral gamma-neutral self-financing portfolio is automatically theta-neutral.

➤ **Programming Assignment 16.3.6** Implement the delta–gamma hedge for options.

➤ **Programming Assignment 16.3.7** Implement the delta–gamma–vega hedge for options.

### 16.3.3 Static Hedging

Dynamic strategies incur huge transactions costs. A **static strategy** that trades only when certain rare events occur addresses this problem. This goal has been realized for hedging European barrier options and look back options with standard options [157, 158, 159, 270].

➤ **Exercise 16.3.8** Explain why shorting a bull call spread can in practice hedge a binary option statically.

### Additional Reading

The literature on hedging is vast [470, 514, 569, 746]. Reference [809] adopts a broader view and considers instruments beyond derivatives. See [891] for mathematical programming techniques in risk management. They are essential in the presence of trading constraints or market imperfections. Consult [365, 369, 376, 646, 647] for more information on financial engineering and risk management.

### NOTE

1. Alternatively, the number could be arrived at by  $Be^{0.06/52} + 5,191,545 + 171,622 = 30,552,305$ .