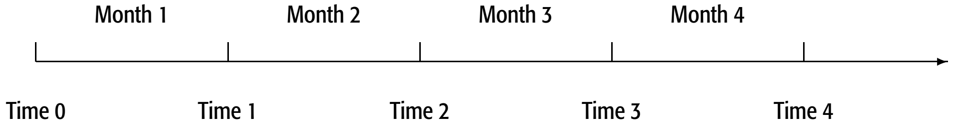
CHAPTER
TWENTY-NINE

Analysis of Mortgage-Backed Securities

Oh, well, if you cannot measure, measure anyhow.
Frank H. Knight (1885–1972)

Compared with other fixed-income securities, the MBS is unique in two respects. First, its cash flow consists of PRINCIPAL AND INTEREST (P&I). Second, the cash flow may vary because of prepayments in the underlying mortgages. This chapter covers the MBS's cash flow and valuation. We adopt the following time line when discussing cash flows:



Because mortgage payments are paid in arrears, a payment for month i occurs at time i, that is, end of month i. The end of a month is identified with the beginning of the coming month.

29.1 Cash Flow Analysis

A traditional mortgage has a fixed term, a fixed interest rate, and a fixed monthly payment. Figure 29.1 illustrates the scheduled P&I for a 30-year, 6% mortgage with an initial balance of $100,000. Figure 29.2 shows how the remaining principal balance decreases over time. In the early years, the P&I consists mostly of interest. Then it gradually shifts toward principal payment with the passage of time. However, the total P&I payment remains the same each month, hence the term *level* pay. Identical characteristics hold for the pool's P&I payments in the absence of prepayments and servicing fees.

From the discussions in Section 3.3, we know that the remaining principal balance after the kth payment is

$$C\,\frac{1-(1+r/m)^{-n+k}}{r/m}, \tag{29.1}$$

where C is the scheduled P&I payment of an n-month mortgage making m payments per year and r is the annual mortgage rate. For mortgages, $m=12$. The remaining

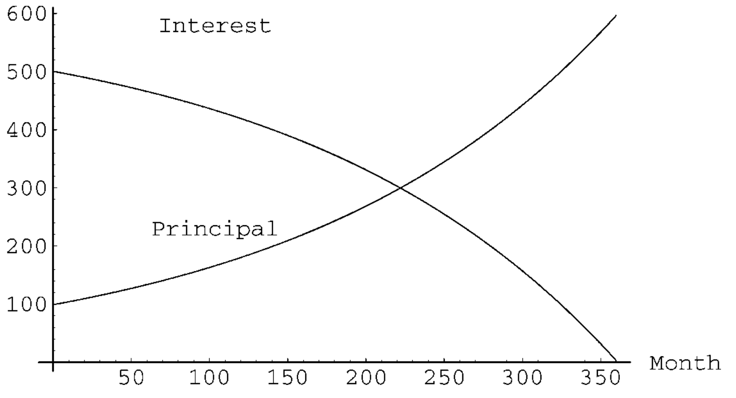


Figure 29.1: Scheduled P&I payments. The schedule is for a 30-year 6% mortgage with an original loan amount of $100,000.

principal balance after k payments can be expressed as a portion of the original principal balance; thus

$$\mathrm{Bal}_k \equiv 1 - \frac{(1+r/m)^k - 1}{(1+r/m)^n - 1} = \frac{(1+r/m)^n - (1+r/m)^k}{(1+r/m)^n - 1}. \tag{29.2}$$

We can verify this equation by dividing balance (29.1) by Bal_0. The remaining principal balance after k payments is simply

$$\mathrm{RB}_k \equiv \mathcal{O} \times \mathrm{Bal}_k,$$

where $\mathcal{O}$ is the original principal balance.

The term **factor** denotes the portion of the remaining principal balance to its original principal balance expressed as a decimal [729]. So Bal_k is the monthly factor when there are no prepayments. It is also known as the **amortization factor**. When the idea of factor is applied to a mortgage pool, it is called the **paydown factor on the pool** or simply the **pool factor** [298].

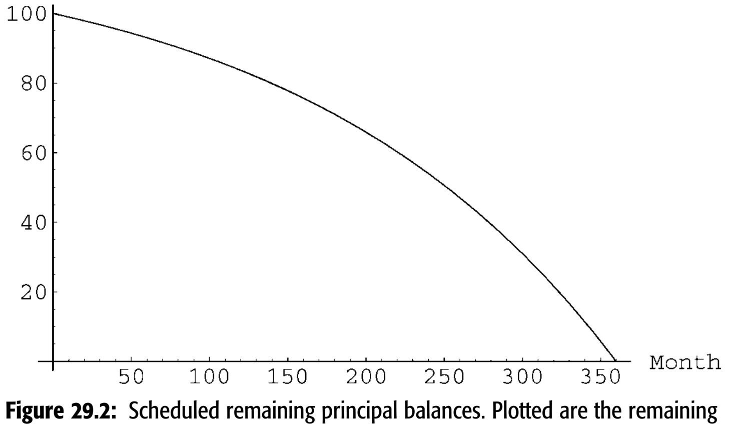


Figure 29.2: Scheduled remaining principal balances. Plotted are the remaining principal balances as percentages of par after each scheduled payment is made.

EXAMPLE 29.1.1 The remaining balance of a 15-year mortgage with a 9% mortgage rate after 54 months is

$$\mathcal{O} \times \frac{[\,1 + (0.09/12)\,]^{180} - [\,1 + (0.09/12)\,]^{54}}{[\,1 + (0.09/12)\,]^{180} - 1} = \mathcal{O} \times 0.824866.$$

In other words, roughly 82.49% of the original loan amount remains after 54 months.

By the amortization principle, the tth interest payment is

$$I_t \equiv \mathrm{RB}_{t-1} \times \frac{r}{m} = \mathcal{O} \times \frac{r}{m} \times \frac{(1 + r/m)^n - (1 + r/m)^{t-1}}{(1 + r/m)^n - 1}.$$

The principal part of the tth monthly payment is

$$P_t \equiv \mathrm{RB}_{t-1} - \mathrm{RB}_t = \mathcal{O} \times \frac{(r/m)(1 + r/m)^{t-1}}{(1 + r/m)^n - 1}. \tag{29.3}$$

The scheduled P&I payment at month t, or $P_t + I_t$, is therefore

$$(\mathrm{RB}_{t-1} - \mathrm{RB}_t) + \mathrm{RB}_{t-1} \times \frac{r}{m} = \mathcal{O} \times \left[\frac{(r/m)(1 + r/m)^n}{(1 + r/m)^n - 1} \right], \tag{29.4}$$

indeed a level pay independent of t. The term within the brackets, called the **payment factor** or **annuity factor**, represents the monthly payment for each dollar of mortgage.

EXAMPLE 29.1.2 The mortgage in Example 3.3.1 has a monthly payment of

$$250{,}000 \times \frac{(0.08/12) \times [\,1 + (0.08/12)\,]^{180}}{[\,1 + (0.08/12)\,]^{180} - 1} = 2{,}389.13$$

by Eq. (29.4), in total agreement with the number derived there.

➤ **Exercise 29.1.1** Derive Eq. (29.4) from Eq. (3.6).

➤ **Exercise 29.1.2** Consider two mortgages with identical remaining principals but different mortgage rates. Show that their remaining principal balances after the next monthly payment will be different; in fact, the mortgage with a lower mortgage rate amortizes faster.

29.1.1 Pricing Adjustable-Rate Mortgages

We turn to ARM pricing as an interesting application of derivatives pricing and the analysis above. Consider a 3-year ARM with an interest rate that is 1% above the 1-year T-bill rate at the beginning of the year. This 1% is called the **margin**. For simplicity, assume that this ARM carries annual, not monthly, payments. The T-bill rates follow the binomial process, in boldface, in Fig. 29.3, and the risk-neutral probability is 0.5. How much is the ARM worth to the issuer?

Each new coupon rate at the reset date determines the level mortgage payment for the months until the next reset date as if the ARM were a fixed-rate loan with the new coupon rate and a maturity equal to that of the ARM. This implies, for example, that in the interest rate tree of Fig. 29.3 the scenario $\mathrm{A} \to \mathrm{B} \to \mathrm{E}$ will leave our 3-year ARM with a remaining principal at the end of the second year different from

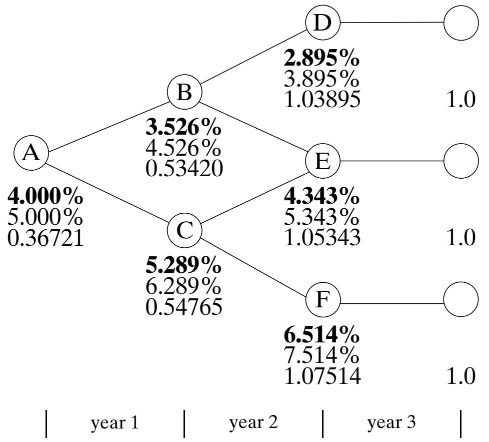


Figure 29.3: ARM's payment factors under stochastic interest rates. Stacked at each node are the T-bill rate, the mortgage rate (which is 1% above the T-bill rate), and the payment factor for a mortgage initiated at that node and ending at the end of year three (based on the mortgage rate at the same node, of course). The short rates are from Fig. 23.8.

that under the scenario $A \to C \to E$ (see Exercise 29.1.2). This path dependency calls for care in algorithmic design to avoid exponential complexity.

The idea is to attach to each node on the binomial tree the annual payment per \$1 of principal for a mortgage initiated at that node and ending at the end of year three – in other words, the payment factor [546]. At node B, for example, the annual payment factor can be calculated by Eq. (29.4) with $r = 0.04526, m = 1$, and $n = 2$ as

$$\frac{0.04526 \times (1.04526)^2}{(1.04526)^2 - 1} = 0.53420.$$

The payment factors for other nodes in Fig. 29.3 are calculated in the same manner.

We now apply backward induction to price the ARM (see Fig. 29.4). At each node on the tree, the net value of an ARM of value \$1 initiated at that node and ending at the end of the third year is calculated. For example, the value is zero at terminal nodes because the ARM is immediately repaid. At node D, the value is

$$\frac{1.03895}{1.02895} - 1 = 0.0097186,$$

which is simply the NPV of the payment 1.03895 next year (note that the issuer makes a loan of \$1 at D). The values at nodes E and F can be computed similarly. At

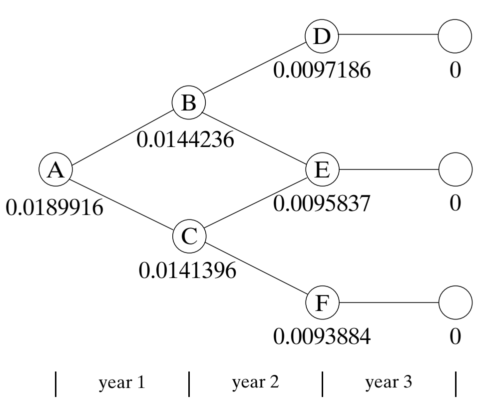


Figure 29.4: Backward induction for ARMs.

node B, we first figure out the remaining principal balance after the payment 1 year hence as

$$1-(0.53420-0.04526)=0.51106,$$

because \$0.04526 of the payment of \$0.53426 constitutes interest. The issuer will receive \$0.01 above the T-bill rate next year, and the value of the ARM is either \$0.0097186 or \$0.0095837 per \$1, each with probability 0.5. The ARM's value at node B thus is

$$\frac{0.51106\times(0.0097186+0.0095837)/2+0.01}{1.03526}=0.0144236.$$

The values at nodes C and A can be calculated similarly as

$$\frac{[\,1-(0.54765-0.06289)\,]\times(0.0095837+0.0093884)/2+0.01}{1.05289}=0.0141396,$$
$$\frac{[\,1-(0.36721-0.05)\,]\times(0.0144236+0.0141396)/2+0.01}{1.04}=0.0189916,$$

respectively. The value of the ARM to the issuer is hence \$0.0189916 per \$1 of loan amount. The complete algorithm appears in Fig. 29.5. The above idea of **scaling** has wide applicability for pricing certain classes of path-dependent securities [449, 546].

ARMs are indexed to publicly available indices such as LIBOR, the constant-maturity Treasury (CMT) rate, and the Cost of Funds Index (COFI). The CMT rates are based on the daily CMT yield curve constructed by the Federal Reserve Bank

Algorithm for pricing ARMs:

```
input:   n, r[n][n], s;
real     P[n], f, p;
integer  i, j;
for (j = 0 to n − 1) { // Nodes at time n − 1.
        f := 1 + r[n − 1][j] + s;   //(29.4) with n = 1.
        P[j] := f/(1 + r[n − 1][j]) − 1;
}
for (i = n − 2 down to 0) // Nodes at time i.
        for (j = 0 to i)  {
            f := (r[i][j] + s)(1 + r[i][j] + s)^(n−i) ×
                ((1 + r[i][j] + s)^(n−i) − 1)^(−1);  //See (29.4).
            p := 1 − (f − r[i][j] − s);
            P[j] := (p × (P[j] + P[j + 1]) × 0.5 + s) ×
                (1 + r[i][j])^(−1);
        }
return P[0];
```

Figure 29.5: Algorithm for pricing ARMs. $r[\,i\,][\,j\,]$ is the $(j+1)$th T-bill rate for period $i+1$, the ARM has n periods to maturity, s is the margin, f stores the payment factors, and p stores the remaining principal amounts. All rates are measured by the period. In general, the floating rate may be based on the k-period Treasury spot rate plus a spread. Then Programming Assignment 29.1.3 can be used to generate the k-period spot rate at each node.

of New York and published weekly in the Federal Reserve's *Statistical Release* H.15 [525]. Cost of funds for thrifts indices are calculated based on the monthly weighted average interest cost for thrifts. The most popular cost of funds index is the 11th Federal Home Loan Bank Board District COFI [325, 330, 820].

If the ARM coupon reflects fully and instantaneously current market rates, then the ARM security will be priced close to par and refinancings rarely occur. In reality, adjustments are imperfect in many ways. At the reset date, a margin is added to the benchmark index to determine the new coupon. ARMs also often have **periodic rate caps** that limit the amount by which the coupon rate may increase or decrease at the reset date. They also have **lifetime caps** and **floors**. To attract borrowers, mortgage lenders usually offer a below-market initial rate (the "teaser" rate). The **reset interval**, the time period between adjustments in the ARM coupon rate, is often annual, which is not frequent enough. Note that these terms are easy to incorporate into the pricing algorithm in Fig. 29.5.

➢ **Programming Assignment 29.1.3** Given an n-period binomial short rate tree, design an $O(kn^2)$-time algorithm for generating k-period spot rates on the nodes of the tree. This tree documents the dynamics of the k-period spot rate.

➢ **Programming Assignment 29.1.4** Implement the algorithm in Fig. 29.5. The binomial T-bill rate tree and the mortgage rate as a spread over the T-bill rate are parts of the input.

➢ **Programming Assignment 29.1.5** Consider an IAS with an amortizing schedule that depends solely on the prevailing k-period spot interest rate. This swap's cash flow depends on only the prevailing principal amount and the prevailing k-period spot interest rate. Design an efficient algorithm to price this swap on a binomial short rate tree.

29.1.2 Expressing Prepayment Speeds

The cash flow of a mortgage derivative is determined from that of the mortgage pool. The single most important factor complicating this endeavor is the unpredictability of prepayments. Recall that prepayment represents the principal payment made in excess of the scheduled principal amortization. We need only compare the amortization factor Bal_t of the pool with the reported factor to determine if prepayments have occurred. The amount by which the reported factor exceeds the amortization factor is the prepayment amount.

Single Monthly Mortality

An SMM of ω means that $\omega\%$ of the scheduled remaining balance at the end of the month will prepay. In other words, the SMM is the percentage of the remaining balance that prepays for the month. Suppose the remaining principal balance of an MBS at the beginning of a month is \$50,000, the SMM is 0.5%, and the scheduled principal payment is \$70. Then the prepayment for the month is $0.005 \times (50{,}000 - 70) \approx 250$ dollars. If the same monthly prepayment speed s is maintained since the issuance of the pool, the remaining principal balance at month i will be $\text{RB}_i \times (1 - s/100)^i$. It goes without saying that prepayment speeds must lie between 0% and 100%.

EXAMPLE 29.1.3 Take the mortgage in Example 29.1.1. Its amortization factor at the 54th month is 0.824866. If the actual factor is 0.8, then the SMM for the initial period of 54 months is

$$100 \times \left[1 - \left(\frac{0.8}{0.824866} \right)^{1/54} \right] = 0.0566677.$$

In other words, roughly 0.057% of the remaining principal is prepaid per month.

Conditional Prepayment Rate

The **conditional prepayment rate** (**CPR**) is the annualized equivalent of an SMM:

$$\text{CPR} = 100 \times \left[1 - \left(1 - \frac{\text{SMM}}{100} \right)^{12} \right].$$

Conversely,

$$\text{SMM} = 100 \times \left[1 - \left(1 - \frac{\text{CPR}}{100} \right)^{1/12} \right].$$

For example, the SMM of 0.0566677 in Example 29.1.3 is equivalent to a CPR of

$$100 \times \left\{ 1 - \left[1 - \left(\frac{0.0566677}{100} \right)^{12} \right] \right\} = 0.677897.$$

Roughly 0.68% of the remaining principal is prepaid annually. Figure 29.6 plots the P&I cash flows under various prepayment speeds. Observe that with accelerated prepayments, the principal cash flow is shifted forward in time.

PSA

In 1985 the Public Securities Association (PSA) standardized a prepayment model. The PSA standard is expressed as a monthly series of CPRs and reflects the increase in CPR that occurs as the pool seasons [619]. The PSA standard postulates the following prepayment speeds: The CPR is 0.2% for the first month, increases thereafter by 0.2% per month until it reaches 6% per year for the 30th month, and then stays at 6% for the remaining years. (At the time the PSA proposed its standard, a seasoned 30-year GNMA's typical prepayment speed was ~6% CPR [260].) The PSA benchmark is also referred to as **100 PSA**. Other speeds are expressed as some percentage of PSA. For example, 50 PSA means one-half the PSA CPRs, 150 PSA means one-and-a-half

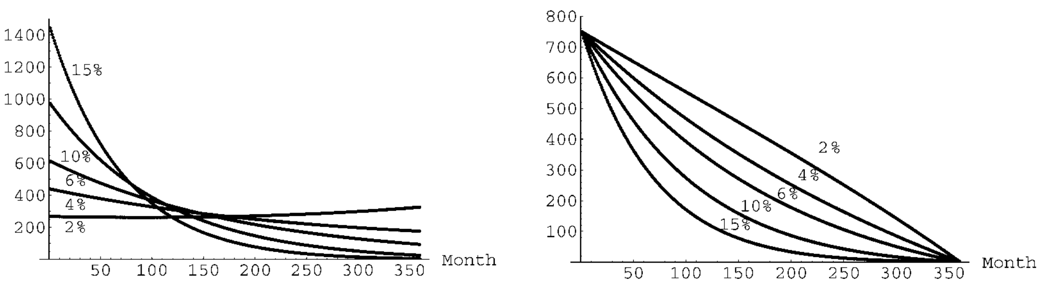


Figure 29.6: Principal (left) and interest (right) cash flows at various CPRs. The 6% mortgage has 30 years to maturity and an original loan amount of $100,000.

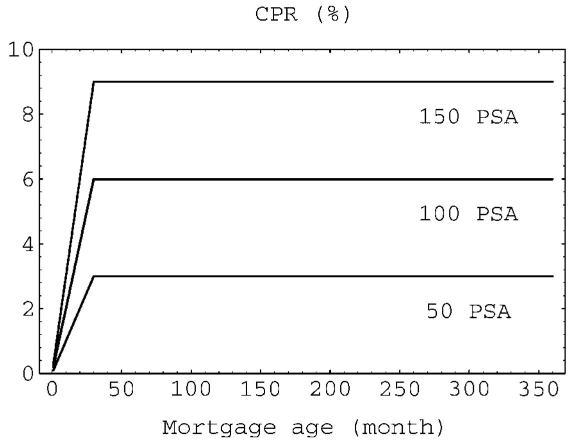


Figure 29.7: The PSA prepayment assumption.

the PSA CPRs, and so on. Mathematically,

$$\text{CPR} = \begin{cases} 6\% \times \frac{\text{PSA}}{100}, & \text{if the pool age exceeds 30 months} \\ 0.2\% \times m \times \frac{\text{PSA}}{100}, & \text{if the pool age } m \leq 30 \text{ months} \end{cases}. \tag{29.5}$$

See Fig. 29.7 for an illustration and Fig. 29.8 for the cash flows at 50 and 100 PSAs. Conversely,

$$\text{PSA} = \begin{cases} 100 \times \frac{\text{CPR}}{6}, & \text{if the pool age exceeds 30 months} \\ 100 \times \frac{\text{CPR}}{0.2 \times m}, & \text{if the pool age } m \leq 30 \text{ months} \end{cases}.$$

See Fig. 29.9 for the conversion algorithm.

Conversion between PSA and CPR/SMM requires knowing the age of the pool. A prepayment speed of 150 PSA implies a CPR of $0.2\% \times 2 \times (150/100) = 0.6\%$ if the pool is 2 months old, but a CPR of $6\% \times 1.5 = 9\%$ if the pool age exceeds 30 months.

➤ **Exercise 29.1.6** Consider the following PSA numbers:

Month	6	12	18	24	30	36
PSA	100	130	154	230	135	125

Compute their equivalent CPRs.

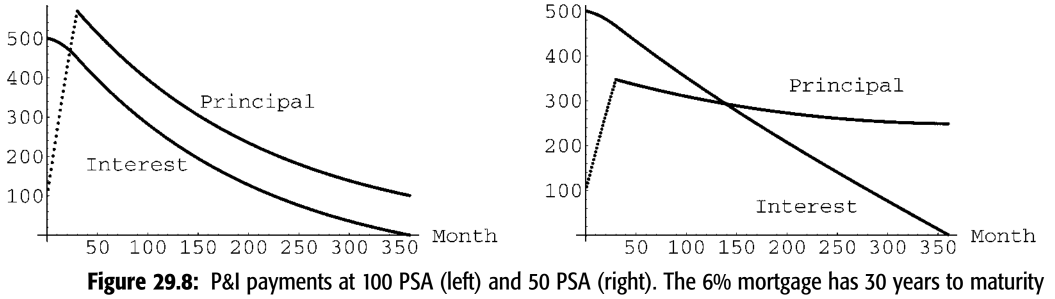


Figure 29.8: P&I payments at 100 PSA (left) and 50 PSA (right). The 6% mortgage has 30 years to maturity and an original loan amount of $100,000.

PSA-to-SMM algorithm:

```
input:    n, PSA, age;
real      SMM[1..n], cpr;
integer   i;
PSA := PSA/100;
for (i = 1 to n) {
        if [i + age ≤ 30]
              cpr := 0.2 × (i + age) × PSA/100;
         else cpr := 6.0 × PSA/100;
         SMM[i] := 1 − (1 − PSA × cpr)^(1/12);
}
return SMM[ ];
```

Figure 29.9: PSA-to-SMM conversion. The pool has n more monthly cash flows, PSA is the prepayment speed, and age is the number of months since the pool's inception. SMM[ ] stores the prepayment vector in decimal, the ith of which denotes the SMM during month i as seen from now.

➤ **Exercise 29.1.7** Is the SMM assuming 200 PSA twice the SMM assuming 100 PSA?

29.1.3 Prepayment Vector and Cash Flow Analysis

Although it tries to capture, if crudely, how prepayments vary with age, the PSA should be viewed as a market convention rather than as a model. Instead of a single PSA number, a vector of PSAs generated by a prepayment model should be used to describe the monthly prepayment speed through time. The monthly cash flows can be derived thereof.

Similarly, the CPR should be seen purely as a measure of speed rather than a model. When we treat a single CPR number as the true prepayment speed, that number will be called the **constant prepayment rate** for obvious reasons. This simple model fails to address the empirical fact that pools with new production loans typically prepay at a slower rate than seasoned pools. As in the PSA case, a vector of CPRs should be preferred. In practice, a vector of CPRs or SMMs is easier to work with than a vector of PSAs because of the lack of dependence on the pool age. In any case, a CPR vector can always be converted into an equivalent PSA vector and vice versa.

To price an MBS, we start with its cash flow, that is, the periodic P&I under a static prepayment assumption as given by a prepayment vector. The invoice price is now $\sum_{i=1}^{n} C_i/(1+r)^{\omega-1+i}$, where C_i is the cash flow at time i, n is the **weighted average maturity (WAM)**,[1] r is the discount rate, and ω is the fraction of period from settlement until the first P&I payment date. The WAM is the weighted average remaining term of the mortgages in the pool, where the weight for each mortgage is the remaining balance. The r that equates the above with the market price is called the (**static**) **cash flow yield**. The **implied PSA** is the single PSA speed producing the same cash flow yield.

MBSs are quoted in the same manner as U.S. Treasury notes and bonds. For example, a price of 94-05 means $94\frac{5}{32}\%$ of par value. Sixty-fourth of a percent is expressed by appending "+" to the price. Hence, the price 94-05+ represents $94\frac{11}{64}\%$ of par value.

Cash Flow

Each cash flow is composed of the principal payment, the interest payment, and the principal prepayment. Let B_k denote the actual remaining principal balance at month k. Given the pool's actual remaining principal balance at time $i-1$ (i.e., B_{i-1}), the P&I payments at time i are

$$\overline{P_i} \equiv B_{i-1}\left(\frac{\mathrm{Bal}_{i-1}-\mathrm{Bal}_i}{\mathrm{Bal}_{i-1}}\right) = B_{i-1}\,\frac{r/m}{(1+r/m)^{n-i+1}-1}, \tag{29.6}$$

$$\overline{I_i} \equiv B_{i-1}\,\frac{r-\alpha}{m}, \tag{29.7}$$

where α is the **servicing spread** (or servicing fee rate), which consists of the servicing fee for the servicer as well as the guarantee fee. The prepayment at time i is

$$\mathrm{PP}_i = B_{i-1}\,\frac{\mathrm{Bal}_i}{\mathrm{Bal}_{i-1}} \times \mathrm{SMM}_i,$$

where SMM_i is the prepayment speed for month i. If the total principal payment from the pool is $\overline{P_i} + \mathrm{PP}_i$, the remaining principal balance is

$$\begin{aligned} B_i &= B_{i-1} - \overline{P_i} - \mathrm{PP}_i \\ &= B_{i-1}\left[1 - \left(\frac{\mathrm{Bal}_{i-1}-\mathrm{Bal}_i}{\mathrm{Bal}_{i-1}}\right) - \frac{\mathrm{Bal}_i}{\mathrm{Bal}_{i-1}} \times \mathrm{SMM}_i\right] \\ &= \frac{B_{i-1} \times \mathrm{Bal}_i \times (1-\mathrm{SMM}_i)}{\mathrm{Bal}_{i-1}}. \end{aligned} \tag{29.8}$$

Equation (29.8) can be applied iteratively to obtain

$$B_i = \mathrm{RB}_i \times \prod_{j=1}^{i}(1-\mathrm{SMM}_j). \tag{29.9}$$

Define $b_i \equiv \prod_{j=1}^{i}(1-\mathrm{SMM}_j)$. Then the scheduled P&I is

$$\overline{P_i} = b_{i-1} P_i, \quad \overline{I_i} = b_{i-1} I_i' \tag{29.10}$$

where $I_i' \equiv \mathrm{RB}_{i-1} \times (r-\alpha)/m$ is the scheduled interest payment. The scheduled cash flow and the b_is determined from the prepayment vector are therefore all that are needed to calculate the projected actual cash flows. Note that if the servicing fees do not exist (that is, $\alpha = 0$), the projected monthly payment *before* prepayment at month i becomes

$$\overline{P_i} + \overline{I_i} = b_{i-1}(P_i + I_i) = b_{i-1} C, \tag{29.11}$$

where C is the scheduled monthly payment on the original principal. See Fig. 29.10 for a linear-time algorithm for generating the mortgage pool's cash flow.

Servicing and guarantee fees are deducted from the gross **weighted average coupon** (**WAC**) of the aggregate mortgage P&I to obtain the **pass-through rate**. The WAC is the weighted average of all the mortgage rates in the pool, in which the weight used for each mortgage is the remaining balance. The servicing spread

Mortgage pool cash flow under prepayments:

input: $n, r\ (r > 0)$, SMM[1..n];
real $B[n+1]$, $P[1..n]$, $\overline{I}[1..n]$, PP[1..n], b;
integer i;
$b := 1$;
$B[0] := 1$;
for ($i = 1$ to n) {
  $b := b \times (1 - \text{SMM}[i])$; //See (29.9).
  $B[i] := b \times \frac{(1+r)^n - (1+r)^i}{(1+r)^n - 1}$; // See (29.2).
  $P[i] := B[i-1] - B[i]$;
  $\overline{I}[i] := B[i-1] \times r$; //See (29.7).
  PP $[i] := B[i] \times \text{SMM}[i]/(1 - \text{SMM}[i])$;
}
return $B[\ ]$, $P[\ ]$, $\overline{I}[\ ]$, PP[ ];

Figure 29.10: Mortgage pool cash flow under prepayments. SMM is the prepayment vector, and the mortgage rate r is a monthly rate. The pool has n monthly cash flows, and its principal balance is \$1. B stores the remaining principals, P are the principal payments (prepayments included), $\overline{I}$ are the interest payments, and PP are the prepayments. The prepayments are calculated based on Exercise 29.1.9, part (1).

for an MBS represents both the guarantee fee and the actual servicing fee itself. For example, a Ginnie Mae MBS with a 10.5% pass-through rate has a total servicing of 0.50%, of which 0.44% is retained by the servicer and 0.06% is remitted to Ginnie Mae. The figure most visible to the investor is the pass-through rate, but the amortization of P&I is a function of the gross mortgage rate of the individual loans making up the pool.

➤ **Exercise 29.1.8** Show that the scheduled monthly mortgage payment at month i is

$$B_{i-1} \frac{(r/m)(1+r/m)^{n-i+1}}{(1+r/m)^{n-i+1} - 1}.$$

➤ **Exercise 29.1.9** Verify that (1) $\text{PP}_i = B_i [\,\text{SMM}_i / (1 - \text{SMM}_i)\,]$ and (2) the actual principal payment $\overline{P_i} + \text{PP}_i$ is $b_{i-1}(P_i + \text{RB}_i \times \text{SMM}_i)$ (not $b_i P_i$).

➤ **Exercise 29.1.10** Verify Eqs. (29.9) and (29.10).

➤ **Exercise 29.1.11** Derive Eq. (29.11) by using Eqs. (29.2) and (29.4).

➤ **Exercise 29.1.12** Derive the PVs of the PO and IO strips based on current-coupon mortgages under constant SMM and zero servicing spread.

➤ **Exercise 29.1.13** Show that a pass-through backed by traditional mortgages with a mortgage rate equal to the market yield is priced at par regardless of prepayments. Assume either zero servicing spread or a pass-through rate equal to the market yield. (Prices of par-priced pass-throughs are hence little affected by variations in the prepayment speed.)

➢ **Programming Assignment 29.1.14** Implement the algorithm in Fig. 29.10.

29.1.4 Pricing Sequential-Pay CMOs

Consider a three-tranche sequential-pay CMO backed by \$3,000,000 of mortgages with a 12% coupon and 6 months to maturity. The three tranches are called A, B, and Z. All three tranches carry the same coupon rate of 12%. The Z tranche consists of **Z bonds**. A Z bond receives no payments until all previous tranches are retired. Although a Z bond carries an explicit coupon rate, the owed interest is accrued and added to the principal balance of that tranche. For that reason, Z bonds are also called **accrual bonds** or **accretion bonds**. When a Z bond starts receiving cash payments, it becomes a pass-through instrument.

Assume that the ensuing monthly interest rates are 1%, 0.9%, 1.1%, 1.2%, 1.1%, and 1.0%. Assume further that the SMMs are 5%, 6%, 5%, 4%, 5%, and 6%. We want to calculate the cash flow and the fair price of each tranche.

We can compute the pool's cash flow by invoking the algorithm in Fig. 29.10 with $n = 6$, $r = 0.01$, and $\text{SMM} = [\,0.05, 0.06, 0.05, 0.04, 0.05, 0.06\,]$. We can derive individual tranches' cash flows and remaining principals thereof by allocating the pool's P&I cash flows based on the CMO structure. See Fig. 29.11 for the breakdown. Note that the Z tranche's principal is growing at 1% per month until all previous tranches are retired. Before that time, the interest due the Z tranche is used to retire A's and B's principals. For example, the \$10,000 interest due tranche Z at month one is directed to tranche A instead, reducing A's remaining principal from \$386,737 to \$376,737 while increasing Z's from \$1,000,000 to \$1,010,000. At month four, the interest amount that goes into tranche Z, \$10,303, is exactly what is required of Z's remaining principal of \$1,030,301. The tranches can be priced

Month		*1*	*2*	*3*	*4*	*5*	*6*
Interest rate		1.0%	0.9%	1.1%	1.2%	1.1%	1.0%
SMM		5.0%	6.0%	5.0%	4.0%	5.0%	6.0%
Remaining principal (B_i)							
	3,000,000	2,386,737	1,803,711	1,291,516	830,675	396,533	0
A	1,000,000	376,737	0	0	0	0	0
B	1,000,000	1,000,000	783,611	261,215	0	0	0
Z	1,000,000	1,010,000	1,020,100	1,030,301	830,675	396,533	0
Interest ($\overline{I_i}$)		30,000	23,867	18,037	12,915	8,307	3,965
A		20,000	3,767	0	0	0	0
B		10,000	20,100	18,037	2,612	0	0
Z		0	0	0	10,303	8,307	3,965
Principal		613,263	583,026	512,195	460,841	434,142	396,534
A		613,263	376,737	0	0	0	0
B		0	206,289	512,195	261,215	0	0
Z		0	0	0	199,626	434,142	396,534

Figure 29.11: CMO cash flows. Month-i numbers reflect the ith monthly payment. "Interest" and "Principal" denote the pool's P&I and distributions to individual tranches. Interest payments may be used to make principal payments to tranches A, B, and C. The Z bond thus protects earlier tranches from extension risk.

as follows:

$$\text{tranche A} = \frac{20000 + 613263}{1.01} + \frac{3767 + 376737}{1.01 \times 1.009} = 1000369,$$

$$\text{tranche B} = \frac{10000 + 0}{1.01} + \frac{20100 + 206289}{1.01 \times 1.009} + \frac{18037 + 512195}{1.01 \times 1.009 \times 1.011} + \frac{2612 + 261215}{1.01 \times 1.009 \times 1.011 \times 1.012} = 999719,$$

$$\text{tranche Z} = \frac{10303 + 199626}{1.01 \times 1.009 \times 1.011 \times 1.012} + \frac{8307 + 434142}{1.01 \times 1.009 \times 1.011 \times 1.012 \times 1.011} + \frac{3965 + 396534}{1.01 \times 1.009 \times 1.011 \times 1.012 \times 1.011 \times 1.01} = 997238.$$

This CMO has a total theoretical value of $2,997,326, slightly less than its par value of $3,000,000. See the algorithm in Fig. 29.12.

We have seen that once the interest rate path and the prepayment vector for that interest rate path are available, a CMO's cash flow can be calculated and the CMO priced. Unfortunately, the remaining principal of a CMO under prepayments is, like an ARM, path dependent. For example, a period of high rates before dropping to the current level is not likely to result in the same remaining principal as a period of low rates before rising to the current level. This means that if we try to price a 30-year CMO on a binomial interest rate model, there will be $2^{360} \approx 2.35 \times 10^{108}$ paths to consider! As a result, Monte Carlo simulation is the computational method of choice. It works as follows. First, one interest rate path is generated. Based on that path, the prepayment model is applied to generate the pool's principal, prepayment, and interest cash flows. Now the cash flows of individual tranches can be generated and their PVs derived. The above procedure is repeated over many interest rate scenarios. Finally, the average of the PVs is taken.

➤ **Exercise 29.1.15** Calculate the monthly prepayment amounts for Fig. 29.11.

➢ **Programming Assignment 29.1.16** Implement the algorithm in Fig. 29.12 for the cash flows of a four-tranche sequential CMO with a Z tranche. Assume that each tranche carries the same coupon rate as the underlying pool's mortgage rate. Figures 29.13 and 29.14 plot the cash flows and remaining principal balances of one such CMO.

29.1.5 Weighted Average Life

The **weighted average life** (**WAL**) of an MBS is the average number of years that each dollar of unpaid *principal* due on the mortgages remains outstanding. It is computed by

$$\text{WAL} \equiv \frac{\sum_{i=1}^{m} i\, P_i}{12 \times P},$$

where m is the remaining term to maturity in months, P_i is the principal repayment i months from now, and P is the current remaining principal balance.[2] See Fig. 29.15 for an illustration. Usually, the greater the anticipated prepayment rate, the shorter

Sequential CMO cash flow generator:

```
input:   n, r (r > 0), SMM[1..n], O[1..4];
real     B[n+1], P[1..n], Ī[1..n]; // Pool cash flows.
real     B[1..4][n+1], P[1..4][1..n], Ī[1..4][1..n];
real     P, I;
integer  i, j;
Call the algorithm in Fig. 29.10 for B[n+1], P[1..n], Ī[1..n];
for (j = 1 to 4) { B[j][0] := O[j]; } // Original balances.
for (i = 1 to n) { // Month i.
        P := P[i]; I := Ī[i]; // Pool P&I for month i.
        for (j = 1 to 3)  { // Tranches A, B, C.
            Ī[j][i] := B[j][i-1] × r;  // Interest due tranche j.
            I := I - Ī[j][i];
            if [B[j][i-1] ≤ P]  { // Retire it.
                P := P - B[j][i-1]; P[j][i] := B[j][i-1];
                B[j][i] := 0;
            } else {
                B[j][i] := B[j][i-1] - P; P[j][i] := P; P := 0;
            }
        }
        for (j = 1 to 3)  { // Interest as prepayment for A, B, C.
            if [B[j][i] ≤ I]  { // Retire it.
                Ī[j][i] := Ī[j][i] + B[j][i]; I := I - B[j][i];
                B[j][i] := 0;
            } else {
                B[j][i] := B[j][i] - I; Ī[j][i] := Ī[j][i] + I;
                I := 0;
            }
        }
        // Tranche Z.
        Ī[4][i] := I; P[4][i] := P;
        B[4][i] := B[4][i-1] × (1 + r) - P - I;
}
return B[][], P[][] Ī[][];
```

Figure 29.12: Sequential CMO cash flow generator. SMM is the prepayment vector, and the mortgage rate r is a monthly rate. The pool has n monthly cash flows, and its principal balance is assumed to be \$1. B stores the remaining principals, P are the principal payments (prepayments included), and $\overline{I}$ are the interest payments. Tranche 1 is the A tranche, tranche 2 is the B tranche, and so on. $\mathcal{O}$ stores the original balances of individual tranches as fractions of \$1.

the average life. Given a static prepayment vector, the WAL increases with coupon rates because a larger proportion of the payment in early years is then interest, delaying the repayment of principal. The implied PSA is sometimes defined as the single PSA speed that gives the same WAL as the static prepayment vector.

29.2 Collateral Prepayment Modeling

The interest rate level is the most important factor in influencing prepayment speeds. The MBS typically experiences accelerating prepayments after a lag when the prevailing mortgage rate becomes 200 basis points below the WAC. This event is known

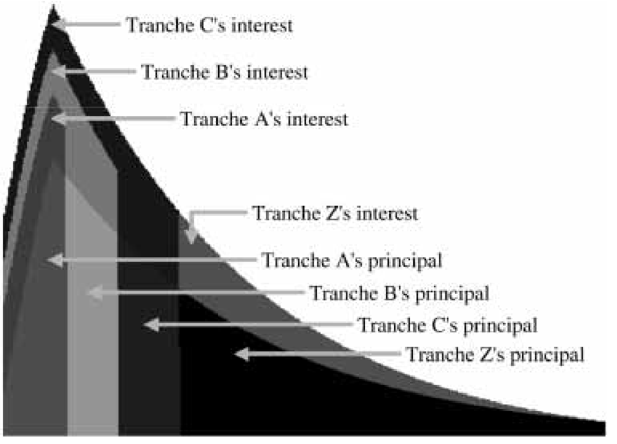


Figure 29.13: Cash flows of a four-tranche sequential CMO. The mortgage rate is 6%, the actual prepayment speed is 150 PSA, and each tranche has an identical original principal amount.

as the **threshold for refinancing**. The prepayment speed accelerates rapidly and then tends to "**burn out**" and settle at a lower speed. The subsequent times when rates fall through the refinancing threshold will not produce the same response. Over time, the pool is left mostly with mortgagors who do not refinance under any circumstances, and the pool's interest rate sensitivity falls. Next to refinancing incentive, loan size is also critical as the monetary savings are proportional to it [4].

The age of a pool has a general impact on prepayments. Refinancing rates are generally lower for new loans than seasoned ones. Interest rate changes and other human factors have little impact on prepayment speeds for the early years of the pool's life. Afterwards, the pool begins to experience such factors that can lead to higher prepayment speeds, such as the sale of the house. This increase in prepayment speeds will stabilize to a steady state with age. We must add that given sufficient refinancing incentives, prepayment speeds can rise sharply even for new loans.

Refinancing is not the only reason prepayments accelerate when interest rates decline. Lower interest rates make housing more affordable and may trigger the trade-up to a bigger house. However, by and large, very high prepayment speeds are primarily due to refinancings, not housing turnover.

In prepayment modeling, the WAC instead of the pass-through rate is the governing factor. To start with, MBSs with identical pass-through rate may have different WACs, which almost surely result in different prepayment characteristics. The

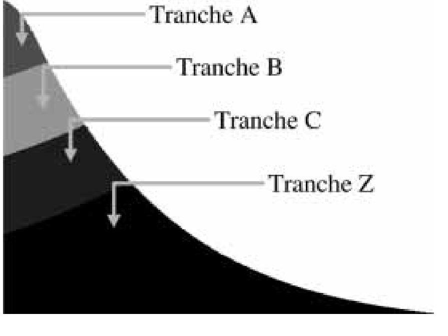


Figure 29.14: Remaining principal balances of a four-tranche sequential CMO. The CMO structure is identical to the one in Fig. 29.13. Tranche Zs principal balance grows until it becomes the current-pay tranche.

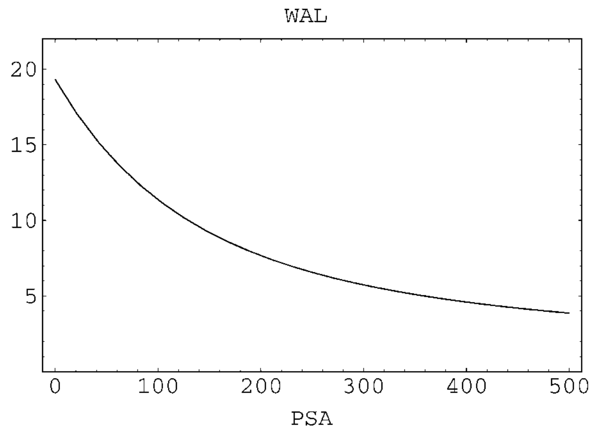


Figure 29.15: WAL under various PSAs. The underlying mortgages have 30 years to maturity and a 6% coupon rate.

WAC may also change over time because, absent prepayments, mortgages with lower coupons amortize faster than those with higher coupons (see Exercise 29.1.2). This makes the WAC increase over time. With prepayments, however, mortgages with higher coupons prepay faster, making the pool's WAC decline over time.

Each mortgage type (government-insured, conventional, and so forth) has a different prepayment behavior. For example, Freddie Mac and Fannie Mae pass-throughs seem to take longer to season than Ginnie Maes, and prepayment rates for 15-year mortgage pass-throughs usually exceed those of comparable-coupon 30-year pass-throughs [54, 325].

From the analysis above, a prepayment model needs at least the following factors: current and past interest rates, state of the economy (especially the housing market), WAC, current coupon rate, loan age, loan size, agency and pool type, month of the year, and burnout. Although we have discussed prepayment speeds at the pool level, a model may go into individual loans to generate the pool's cash flow if such information is available and the benefits outweigh the costs [4, 259]. A long-term average of the projected speeds is typically reported as the model's projected prepayment vector. This projection can be a weighted average of the projected speeds, the single speed that gives the same weighted average life as the vector, or the single speed that gives the same yield as the vector [433].

A PO is purchased at a discount. Because its cash flow is returned at par, a PO's dollar return is simply the difference between the par value and the purchase price. The faster that dollar return is realized, the higher the yield. Prepayments are therefore beneficial to POs. In declining mortgage rates, not only do prepayments accelerate, the cash flow is also discounted at a lower rate; consequently, POs appreciate in value. The opposite happens when mortgage rates rise (see Fig. 29.16). In summary, POs have positive duration and do well in bull markets.

An IO, in contrast, has no par value. Any prepayments reduce the pool principal and thus the interest as well. When mortgage rates decline and prepayments accelerate, an IO's price usually declines even though the cash flow will be discounted at a lower rate. If mortgage rates rise, the cash flow improves. However, beyond a certain point, the price of an IO will decline because of higher discount rates (see Fig. 29.17). An IO's price therefore moves in the same direction as the change in mortgage rates

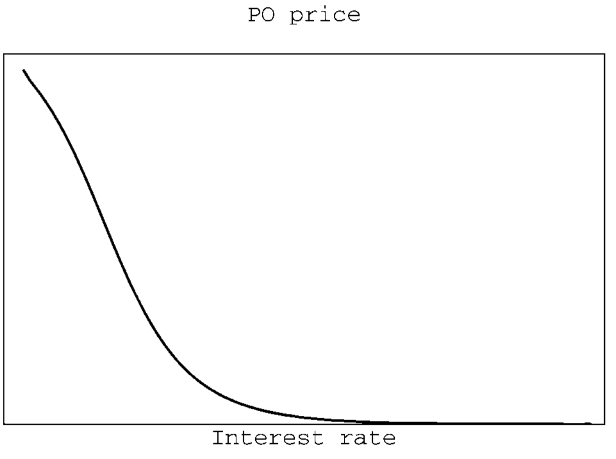


Figure 29.16: Price of PO.

over certain ranges (negative duration, in other words). Unlike most fixed-income securities, IOs do best in bear markets.

SMBSs are extremely sensitive to changes in prepayment speeds (see Exercise 29.2.2). These securities are often combined with other types of securities to alter the return characteristics. For example, because the PO thrives on the acceleration of prepayment speeds, it serves as an excellent hedge against MBSs whose price flattens or declines if prepayments accelerate, whereas IOs can hedge the interest rate risk of securities with positive duration.

➤ **Exercise 29.2.1** Divide the borrowers into slow and fast refinancers. (More refined classification is possible.) The slow refinancers are assumed to respond to refinancing incentive at a higher rate than fast refinancers. Describe how this setup models burnout.

➤ **Exercise 29.2.2** From Exercise 29.1.12, show that the prices of PO and IO strips are extremely sensitive to prepayment speeds.

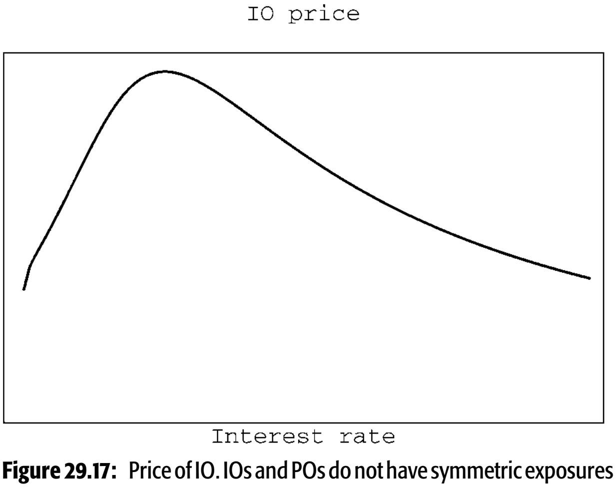


Figure 29.17: Price of IO. IOs and POs do not have symmetric exposures to rate changes.

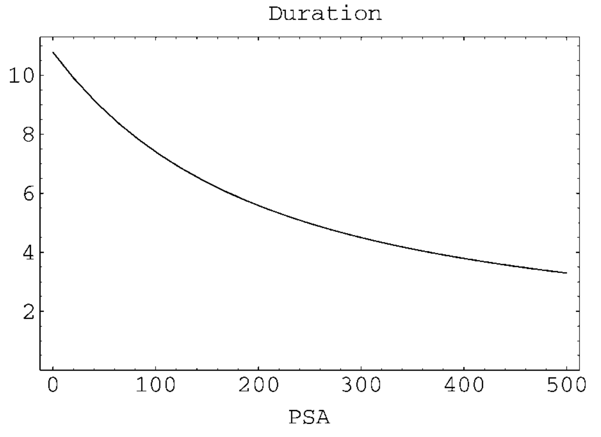


Figure 29.18: MD under various PSAs. The coupon rate and the market yield are assumed to be 6%. The underlying mortgages have 30 years to maturity.

➤ **Exercise 29.2.3** Firms that derive income from servicing mortgages can be viewed as taking a long position in IOs. Why?

29.3 Duration and Convexity

Duration is more important for the evaluation of pass-throughs than the WAL, which measures the time to the receipt of the principal cash flows [247, 619]. Figure 29.18 illustrates the Macaulay duration (MD) of a pass-through under various prepayment assumptions. The MD derived under a static prepayment vector, which does not change as yields change, is also called **static duration** or **cash flow duration**.

Duration is supposed to reveal how a change in yields affects the price, that is,

$$\text{percentage price change} \approx -\text{effective duration} \times \text{yield change}. \tag{29.12}$$

Relation (29.12) has obvious applications in hedging. However, static duration is inadequate for that purpose because the cash flow of an MBS depends on the prevailing yield. The most relevant measure of price volatility is the effective duration,

$$\frac{\partial P}{\partial y} \approx \frac{P_- - P_+}{P_0(y_+ - y_-)},$$

where P_0 is the current price, P_- is the price if yield is decreased by Δy, P_+ is the price if yield is increased by Δy, y is the initial yield, $y_+ \equiv y + \Delta y$, and $y_- \equiv y - \Delta y$. Figure 29.19 plots the effective duration of an MBS. For example, it says that the effective duration is approximately six at 9%; a 1% change in yields will thus move the price by roughly 6%. The prices P_+ and P_- are often themselves expected values calculated by simulation. To save computation time, either $(P_- - P_0)/(P_0 \Delta t)$ or $(P_0 - P_+)/(P_0 \Delta t)$ may be used instead, as only one of P_- and P_+ needs to be calculated then.

Similarly, convexity $\partial^2 P/\partial y^2$ can be approximated by the effective convexity:

$$\frac{P_+ + P_- - 2 \times P_0}{P_0[\,0.5 \times (y_+ - y_-)\,]^2}.$$

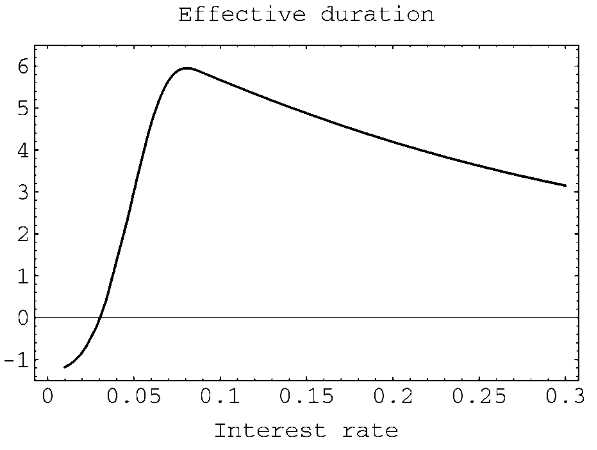


Figure 29.19: Effective duration. The MBS is from Fig. 28.8.

See Fig. 29.20 for an illustration. Convexity can improve first-order formula (29.12) by adding second-order terms,

$$\text{percentage price change} \approx -\text{effective duration} \times \text{yield change} \\ + 0.5 \times \text{convexity} \times (\text{yield change})^2.$$

We saw in Fig. 28.8 that an MBS's price increases at a decreasing rate as the yield falls below the cusp because of accelerating prepayments, at which point it starts to decrease. This negative convexity is evident in Fig. 29.20. Therefore, even if the MD, which is always positive, is acceptable for current-coupon and moderately discount MBSs, it will not work for premium-priced MBSs.

➤ **Exercise 29.3.1** Suppose that MBSs are priced based on the premise that there are no prepayments until the 12th year, at which time the pool is repaid completely. This is called the **FHA 12-year prepaid-life concept**. Argue that premium-priced MBSs are overvalued and discount MBSs are undervalued if prepayments occur before the 12th year. (Studies have shown that the average life is much shorter than 12 years [577].)

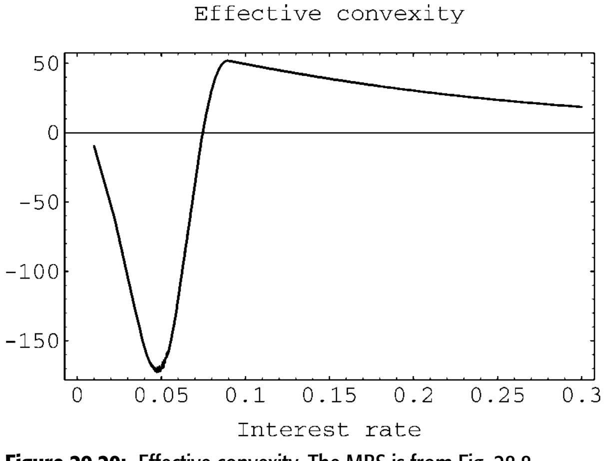


Figure 29.20: Effective convexity. The MBS is from Fig. 28.8.

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➤ **Exercise 29.3.2** Modified duration $(1/P) \sum_{i=1}^{n} i C_i (1+y)^{-(i+1)}$ cannot be negative for pass-throughs. On the other hand, effective duration, which approximates modified duration, can be negative, as shown in Fig. 29.19. Why?

➤ **Exercise 29.3.3** A hedger takes a long position in MBSs and hedges it by shorting T-bonds. Assess this strategy.

➤ **Exercise 29.3.4** Consider options on mortgage pass-through forwards. Argue that Black's model tends to overstate the call value and to underestimate the put value.

29.4 Valuation Methodologies

Mortgage valuation involves modeling the uncertain cash flow and computing its PV. As in Section 27.4, the three basic approaches to valuing MBSs are static cash flow yield, option modeling, and OAS. Because their valuation is more technical and relies more on judgment than do other fixed-income securities, not to mention such issues as prepayment risk, credit quality, and liquidity, MBSs are priced to a considerable yield spread over the Treasuries and corporate bonds.

29.4.1 The Static Cash Flow Yield Methodology

When an internal rate of return is calculated with the static prepayment assumption over the life of the security, the result is the (static) cash flow yield, we recall. The static cash flow yield methodology compares the cash flow yield on an MBS with that on "comparable" bonds. For this purpose, it is inappropriate to use the stated maturity of the MBS because of prepayments. Instead, either the MD or the WAL under the same prepayment assumption can be used.

Although simple to use, this methodology sheds little light on the relative value of an MBS. Its problems, besides being static, are that (1) the projected cash flow may not be reinvested at the cash flow yield, (2) the MBS may not be held until the final payout date, and (3) the actual prepayment behavior is likely to deviate from the assumptions.

The static spread methodology goes beyond the cash flow yield by incorporating the Treasury yield curve. The static spread to the Treasuries is the spread that makes the PV of the projected cash flow from the MBS when discounted at the spot rate plus the spread equal its market price (review Section 5.4).

29.4.2 The Option Pricing Methodology

Virtually all mortgage loans give the homeowner the right to prepay the mortgage at any time. The homeowner in effect holds an option to call the mortgage. The totality of these rights to prepay constitutes the embedded call option of the pass-through. Because the homeowner has the right to call a pro rata portion of the pool, the MBS investor is short the embedded call; therefore,

$$\text{pass-through price} = \text{noncallable pass-through price} - \text{call option price}.$$

The option pricing methodology prices the call option by an option pricing model. It then estimates the market price of the noncallable pass-through by

$$\text{noncallable pass-through price} = \text{pass-through price} + \text{call option price}.$$

The preceding price is finally used to compute the yield on this theoretical bond that does not prepay. This yield is called the option-adjusted yield.

The option pricing methodology was criticized in Subsection 27.4.2. It has additional difficulties here. Prepayment options are often "irrationally" exercised. Furthermore, a partial exercise is possible as the homeowner can prepay a portion of the loan; there is not one option but many, one per homeowner. Finally, valuation of the call option becomes very complicated for CMO bonds.

29.4.3 The Option-Adjusted-Spread Methodology

The OAS methodology has four major parts [382]. The interest rate model is the first component. Then there is the prepayment model, which is the single most important component. Although the prepayment model may be deterministic or stochastic, there is evidence showing that deterministic models that are accurate on average are good enough for pass-throughs, IOs, and POs [428, 433]. The **cash flow generator** is the third component. It calculates the current coupon rates for the interest rate paths given by the interest rate model. It then generates the P&I cash flows for the pool as well as allocating them for individual securities based on the prepayment model and security information such as CMO rules. Note that the same pool cash flow drives many securities. Finally, the equation solver calculates the OAS. Because several paths of interest rates are used, many statistics are often computed as well. See Fig. 29.21 for the overall structure.

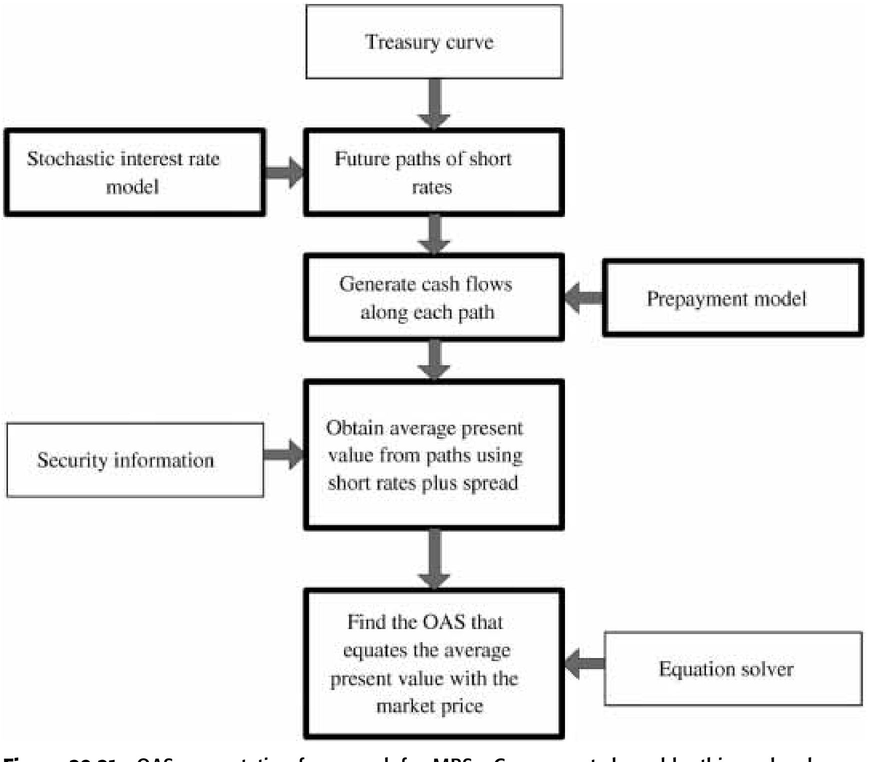

Figure 29.21: OAS computation framework for MBSs. Components boxed by thinner borders are supplied externally.

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The general valuation formula for uncertain cash flows can be written as

$$\text{PV} = \lim_{N\to\infty} \frac{1}{N} \sum_{N \text{ paths } r^*} \frac{C_n^*}{(1+r_1^*)(1+r_2^*)\cdots(1+r_n^*)}, \tag{29.13}$$

where r^* denotes a risk-neutral interest rate path for which r_i^* is the ith one-period rate and C_n^* is the cash flow at time n under this scenario. The summation averages over a large number of scenarios whose distribution matches the interest rate dynamics. The average over scenarios must also match the current spot rate curve, i.e.,

$$\frac{1}{(1+f_1)(1+f_2)\cdots(1+f_n)} = \lim_{N\to\infty} \frac{1}{N} \sum_{N \text{ paths } r^*} \frac{1}{(1+r_1^*)(1+r_2^*)\cdots(1+r_n^*)},$$

$n = 1, 2, \ldots,$ where f_i are the implied forward rates.

The Monte Carlo valuation of MBSs is closely related to Eq. (29.13). The interest rate model randomly produces a set of risk-neutral rate paths. The cash flow is then generated for each path. Finally, we solve for the spread s that makes the average discounted cash flow equal the market price:

$$P = \lim_{N\to\infty} \frac{1}{N} \sum_{N \text{ paths } r^*} \frac{C_n^*}{(1+r_1^*+s)(1+r_2^*+s)\cdots(1+r_n^*+s)}.$$

This spread s is the OAS. The implied cost of the embedded option is then calculated as

$$\text{option cost} = \text{static spread} - \text{OAS}.$$

A common alternative averages the cash flows first and then calculates the OAS as the spread that equates this average cash flow with the market price. Although this approach is more efficient, it will generally give a different spread.

OAS calculation is very time consuming. The majority of the cost lies in generating the cash flows. This is because CMOs can become arbitrarily complex in their rules for allocating the cash flows. Such complexity requires special **data structures** in software design. The computational costs are then multiplied by the many runs of the Monte Carlo simulation.

OAS can be seen to measure the risk premium for bearing systematic risks in the mortgage market. Under this interpretation, the OAS methodology identifies investments with the best potential for excess returns. Being statistically derived, the prepayment model will always be out of date and provide only a crude forecast for future conditions. Therefore an alternative interpretation is that no such risk premium exists: A nonzero OAS simply implies that the market is trading off a different set of prepayment assumptions [34]. This view suggests that one investigate the implied prepayment assumptions [188].

➤ **Exercise 29.4.1** Argue that the OAS with zero interest rate volatility, called the **zero-volatility OAS**, corresponds to the static spread.

➢ **Programming Assignment 29.4.2** Implement the OAS computation for the four-tranche sequential CMO under the BDT model. Assume a constant SMM.

Duration and Convexity

Effective duration and convexity can be computed if the OAS is held constant. The results are called the OAS duration and the **OAS convexity**, respectively [323, 325]. Key rate durations, introduced in Section 27.5 and calculated like the OAS duration, are most useful in identifying the segments of the yield curve that most affect the MBS value [260]. Note that the OAS duration is at least twice as expensive as the OAS in terms of computation time because at least one of P_+ and P_- has to be computed by simulation. The OAS convexity is three times as expensive because both P_+ and P_- have to be computed.

Prepayment risk can represent the risk that the market price reflects prepayment assumptions that are different from the model. An interesting measure of prepayment risk is the **prepayment duration**. It is the percentage change in price, with the OAS held constant, for a given percentage deviation in speeds from some base level projection (see Exercise 29.2.2) [198, 328, 433, 815].

Holding Period Returns

The HPR assesses the MBS over a holding period. The FV at the horizon consists of the projected P&I cash flows, the interest on the reinvestment thereof, and the projected horizon price. The monthly total return is

$$\left(\frac{\text{total future amount}}{\text{price of the MBS}}\right)^{1/\text{number of months}} - 1.$$

To calculate the preceding return, prepayment assumptions, reinvestment rates, and interest rate dynamics are all needed. These assumptions are not independent.

The OAS can be combined with the HPR analysis. First we create a few static interest rate and prepayment scenarios for the holding period. The prepayment assumptions are in the form of prepayment vectors. We then calculate the HPR for each scenario by assuming that the OAS remains unchanged at the horizon.

Additional Reading

See [54, 55, 260, 330, 829] for more information on MBSs, [54, 55, 124, 259, 260, 276, 297, 323, 325, 330, 595, 619, 649, 788, 789, 818, 896] for the valuation of MBSs, [134, 188, 197, 198, 438, 454] for OAS analysis, and [142, 715] for the Monte Carlo valuation of MBSs. Monte Carlo simulation typically provides an unbiased estimate [478]. Application of the variance-reduction techniques and quasi-Monte Carlo methods in Chap. 18 can result in less work [197, 354]. Parallel processing for much faster performance has been convincingly demonstrated [601, 794, 892, 893]. Additional information on duration measures can be found in [33, 258, 272, 394, 429, 504, 889]. Many yield concepts are discussed in [406]. See [118, 220, 268, 361, 411, 430, 431, 433, 540] for prepayment models, [260] for a historical account, and [296] for early models. Factors used in prepayment modeling are considered in [54, 330, 430, 433]. The FHA 12-year prepaid-life concept is discussed in [54, 363]. Valuation of MBSs may profit from two-factor models because prepayments tend to depend more on the long-term rate [456]. See [316] for the prepayments of multifamily MBSs and [203, 742, 864] for the empirical analysis of prepayments. Burnout modeling is discussed in [199]. The refinancing waves of 1991–1993 cast some doubts on the burnout concept, however

[433]. Consult [524] for hedging MBSs and [460] for options on MBSs. The 11th District COFI is analyzed in [347, 684], and the CMT rates are compared with the on-the-run yields in [525].

NOTES

1. Also known as the **weighted average remaining maturity** (**WARM**).
2. **Payment delays** should be incorporated in the WAL calculation: 14 (actual) or 45 days (stated) for GNMA Is, 19 (actual) or 50 days (stated) for GNMA IIs, 24 (actual) or 55 (stated) for Fannie Mae MBSs, 44 (actual) or 75 (stated) for Freddie Mac non-Gold PCs, and 14 (actual) or 45 (stated) for Freddie Mac Gold PCs. The **stated payment delay** denotes the number of days between the first day of the month and the date the servicer actually remits the P&I to the investor [54, 330].