

Collateralized Mortgage Obligations

Capital can be understood only as motion, not as a thing at rest.

Karl Marx (1818–1883), *Das Kapital*

Mutual funds combine diverse financial assets into a portfolio and issue a single class of securities against it. CMOs reverse that process by issuing a diverse set of securities against a relatively homogeneous portfolio of assets [660]. This chapter surveys CMOs. The tax treatment of CMOs is generally covered under the provisions of the Real Estate Mortgage Investment Conduit (REMIC) rules of 1986. As a result, CMOs are often referred to as **REMICs** [162, 469].

30.1 Introduction

The complexity of a CMO arises from layering different types of payment rules on a prioritized basis. In the first-generation CMOs, the sequential-pay CMOs, each class of bond would be retired sequentially. A sequential-pay CMO with a large number of tranches will have very narrow cash flow windows for the tranches. To further reduce prepayment risk, tranches with a principal repayment schedule were introduced. They are called **scheduled bonds**. For example, bonds that guarantee the repayment schedule when the actual prepayment speed lies within a specified range are known as **planned amortization class** bonds (**PACs**). PACs were introduced in August 1986 [141]. Whereas PACs offer protection against both contraction and extension risks, some investors may desire protection from only one of these risks. For them, a bond class known as the **targeted amortization class** (**TAC**) was created.

Scheduled bonds expose certain CMO classes to less prepayment risk. However, this can occur only if the redirection in the prepayment risk is absorbed as much as possible by other classes referred to as the **support bonds** or **companion bonds**. Support bonds are a necessary by-product of the creation of scheduled tranches.

Pro rata bonds provide another means of layering. Principal cash flows to these bonds are divided proportionally, but the bonds can have different interest payment rules. Suppose the WAC of the collateral is 10%, tranche B1 receives 40% of the principal, and tranche B2 receives 60% of the principal. Given this pro rata structure, many choices of interest payment rules are possible for B1 and B2 as long as the interest payments are nonnegative and the WAC does not exceed 10%. The coupon rates can even be floating. One possibility is for B1 to have a coupon of 5% and B2 to have a coupon of 13.33%. Bonds with pass-through coupons that are higher

and lower than the collateral coupon have thus been created. Bonds like B1 are called **synthetic discount securities** and bonds like B2 are called **synthetic premium securities**. An extreme case is for B1 to receive 99% of the principal and have a 5% coupon and B2 to receive only 1% of the principal and have a 505% coupon. In fact, first-generation IOs took the form of B2 in July 1986 [55].

IOs have either a **nominal principal** or a notional principal. A nominal principal represents actual principal that will be paid. It is called “nominal” because it is extremely small, resulting in an extremely high coupon rate. A case in point is the B2 class with a 505% coupon above. A notional principal, in contrast, is the amount on which interest is calculated. An IO holder owns none of the notional principal. Once the notional principal amount declines to zero, no further payments are made on the IO.

30.2 Floating-Rate Tranches

A form of pro rata bonds are floaters and inverse floaters whose combined coupon does not exceed the collateral coupon. A floater is a class whose coupon rate varies directly with the change in the reference rate, and an inverse floater is a class whose coupon rate changes in the direction opposite to the change in the reference rate. When the coupon on the inverse floater changes by x times the amount of the change in the reference rate, this multiple x is called its **slope**. Because the interest comes from fixed-rate mortgages, floaters must have a coupon cap. Similarly, inverse floaters must have a coupon floor. Floating-rate classes were created in September 1986.

Suppose the floater has a principal of P_f and the inverse floater has a principal of P_i . Define $\omega_f \equiv P_f/(P_f + P_i)$ and $\omega_i \equiv P_i/(P_f + P_i)$. To make the structure self-supporting, the coupon rates of the floater, c_f , and the inverse floater, c_i , must satisfy $\omega_f \times c_f + \omega_i \times c_i = \text{WAC}$, or

$$c_i = \frac{\text{WAC} - \omega_f \times c_f}{\omega_i}.$$

The slope is clearly ω_f/ω_i . To make sure that the inverse floater will not encounter a negative coupon, the cap on the floater must be less than WAC/ω_f . In fact, caps and floors are related by

$$\text{floor} = \frac{\text{WAC} - \omega_f \times \text{cap}}{\omega_i}.$$

EXAMPLE 30.2.1 Consider a CMO deal that includes a floater with a principal of \$64 million and an inverse floater with a principal of \$16 million. The coupon rate for the floating-rate class is $\text{LIBOR} + 0.65$ and that for the inverse floater is $42.4 - 4 \times \text{LIBOR}$. The slope is thus four. The WAC of the two classes is

$$\frac{64}{80} \times \text{floater coupon rate} + \frac{16}{80} \times \text{inverse floater coupon rate} = 9\%,$$

regardless of the level of LIBOR. Consequently the coupon rate on the underlying collateral, 9%, can support the aggregate interest payments that must be made to these two classes. If we set a floor of 0% for the inverse floater, the cap on the floater is 11.25%.

A variant of the floating-rate CMO is the **superfloater** introduced in 1987. In a conventional floating-rate class, the coupon rate moves up or down on a one-to-one basis with the reference rate subject to caps and floors. A superfloater's coupon rate, in comparison, changes by some multiple of the change in the reference rate, thus magnifying any changes in the value of the reference rate. Superfloater tranches are bearish because their value generally appreciates with rising interest rates.

Suppose that the initial LIBOR is 7% and the coupon rate for a superfloater is based on this formula:

$$(\text{initial LIBOR} - 40 \text{ basis points}) + 2 \times (\text{change in LIBOR}).$$

The following table shows how the superfloater changes its coupon rate as LIBOR changes. The coupon rates for a conventional floater of LIBOR plus 50 basis points are also listed for comparison.

<i>LIBOR Change (Basis Points)</i>	<i>−300</i>	<i>−200</i>	<i>−100</i>	<i>0</i>	<i>+100</i>	<i>+200</i>	<i>+300</i>
Superfloater	0.6	2.6	4.6	6.6	8.6	10.6	12.6
Conventional floater	4.5	5.5	6.5	7.5	8.5	9.5	10.5

A superfloater provides a much higher yield than a conventional floater when interest rates rise and a much lower yield when interest rates fall or remain stable. We verify this by looking at the above table by means of spreads in basis points to LIBOR in the next table:

<i>LIBOR Change (Basis Points)</i>	<i>−300</i>	<i>−200</i>	<i>−100</i>	<i>0</i>	<i>+100</i>	<i>+200</i>	<i>+300</i>
Superfloater	−340	−240	−140	−40	60	160	260
Conventional floater	50	50	50	50	50	50	50

► **Exercise 30.2.1** Repeat the calculations in the text by using the following formula:

$$(\text{initial LIBOR} - 50 \text{ basis points}) + 1.5 \times (\text{change in LIBOR}).$$

► **Exercise 30.2.2** Argue that the maximum coupon rate that could be paid to a floater is higher than would be possible without the inclusion of an inverse floater.

30.3 PAC Bonds

PAC bonds may be the most important innovation in the CMO market [141]. They are created by calculation of the cash flows from the collateral by use of two prepayment speeds, a fast one and a slow one. Consider a **PAC band** of 100 PSA (the **lower collar**) to 300 PSA (the **upper collar**). Figure 30.1 shows the principal payments at the two collars. Note that the principal payments under the higher-speed scenario are higher in the earlier years but lower in later years. The shaded area represents the principal payment schedule that is guaranteed for every possible prepayment speed between 100% and 300% PSAs. It is calculated by taking the minimum of the principal paydowns at the lower collar and those at the upper collar. This schedule is called the **PAC schedule**. See Fig. 30.2 for a linear-time cash flow generator for a simple CMO containing a PAC bond and a support bond.

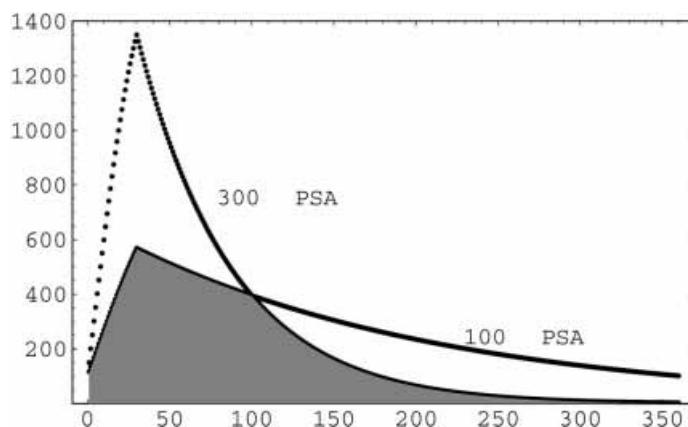


Figure 30.1: PAC schedule. The underlying mortgages are 30-year ones with a total original loan amount of \$100,000,000 (the numbers on the y axis are in thousands) and a coupon rate of 6%. The PAC schedule is determined by the principal payments at 100 PSA and 300 PSA.

Adherence to the amortization schedule of the PAC takes priority over those of all other bonds. The cash flow of a PAC bond is therefore known as long as its support bonds are not fully paid off. Whether this happens depends to a large extent on the CMO structure, such as priority and the relative sizes of PAC and non-PAC classes. For example, a relatively small PAC is harder to break than a larger PAC, other things being equal.

If the actual prepayment speed is 150 PSA, the principal payment pattern of the PAC bond adheres to the PAC schedule. The cash flows of the support bond “flow around” the PAC bond (see Fig. 30.3). The cash flows are neither sequential nor pro rata; in fact, the support bond pays down *simultaneously* with the PAC bond. Because more than one class of bonds may be receiving principal payments at the same time, structures with PAC bonds are called **simultaneous-pay CMOs**. At the lower prepayment speed of 100 PSA, far less principal cash flow is available in the early years of the CMO. As all the principal cash flows go to the PAC bond in the early years, the principal payments on the support bond are deferred and the support bond extends. The support bond does, however, receive more interest payments.

If prepayments move outside the PAC band, the PAC schedule may not be met. At 400 PSA, for example, the cash flows to the support bond are accelerated. After the support bond is fully paid off, all remaining principal payments go to the PAC bond, shortening its life. See Fig. 30.4 for an illustration. The support bond thus absorbs part of the contraction risk. Similarly, should the actual prepayment speed fall below the lower collar, then in subsequent periods the PAC bond has priority on the principal payments. This reduces the extension risk, which is again absorbed by the support bond.

The PAC band guarantees that if prepayments occur at any single constant speed within the band *and* stay there, the PAC schedule will be met. However, the PAC schedule may not be met even if prepayments on the collateral always vary within the band over time. This is because the band that guarantees the original PAC schedule can expand and contract, depending on actual prepayments. This phenomenon is known as **PAC drift**.

PAC cash flow generator:

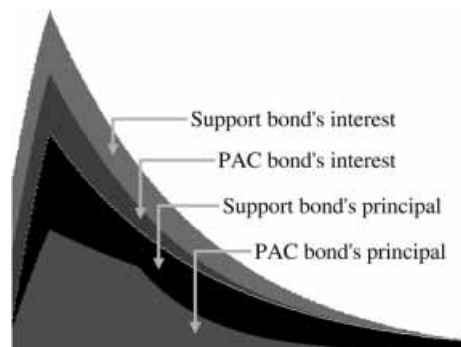
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input:   $n, r$  ( $r > 0$ ),  $SMM[1..n]$ ,  $PSA_u$ ,  $PSA_\ell$ ,  $\mathcal{O}[2]$ ;
real    $P[1..n]$ ,  $\bar{T}[1..n]$ ; // Pool cash flows.
real    $smm[1..n]$ ,  $B[2][n+1]$ ,  $P[2][1..n]$ ,  $\bar{T}[2][1..n]$ ,  $P, I$ ;
integer  $i$ ;
Call the algorithm in Fig. 29.10 with  $SMM[1..n]$  for
     $P[1..n]$  and  $\bar{T}[1..n]$ ; // Pool cash flows.
Call the algorithm in Fig. 29.9 for  $smm[1..n]$  based on  $PSA_u$ ;
Call the algorithm in Fig. 29.10 with  $smm[1..n]$  and
    store the principal cash flow in  $P[0][1..n]$ ;
Call the algorithm in Fig. 29.9 for  $smm[1..n]$  based on  $PSA_\ell$ ;
Call the algorithm in Fig. 29.10 with  $smm[1..n]$  and
    store the principal cash flow in  $P[1][1..n]$ ;
for ( $i = 1$  to  $n$ ) {  $P[0][i] := \min(P[0][i], P[1][i]);$  }
// PAC schedule per one dollar of original principal:
Normalize  $P[0][1..n]$  so that the  $n$  elements sum to one;
 $B[0][0] := \mathcal{O}[0]$ ;  $B[1][0] := \mathcal{O}[1]$ ; // Original balances.
for ( $i = 1$  to  $n$ ) { // Month  $i$ .
     $P := P[i]$ ;  $I := \bar{T}[i]$ ; // Pool P&I for month  $i$ .
     $P[1][i] := \min(0, P - \mathcal{O}[0] \times P[0][i], B[1][i-1])$ ;
     $B[1][i] := B[1][i-1] - P[1][i]$ ;
     $\bar{T}[1][i] := B[1][i-1] \times r$ ; // Support bond done.
     $P := P - P[1][i]$ ;
     $P[0][i] := P$ ;
     $B[0][i] := B[0][i-1] - P$ ;
     $\bar{T}[0][i] := I - \bar{T}[1][i]$ ; // PAC bond done.
}
return  $B[ ][ ]$ ,  $P[ ][ ]$ ,  $\bar{T}[ ][ ]$ ;

```

Figure 30.2: PAC cash flow generator. $SMM[]$ stores the actual prepayment speeds, PSA_u and PSA_ℓ form the PAC band, and the mortgage rate r is a monthly rate. The pool has n monthly cash flows, and its principal balance is assumed to be \$1. \mathcal{O} stores the original balances of individual bonds as fractions of \$1; in particular, bond 0 is the PAC bond, bond 1 is the support bond, and $\mathcal{O}[0] + \mathcal{O}[1] = 1$. B stores the remaining principals, P are the principal payments (prepayments included), and \bar{T} are the interest payments. The CMO deal contains one PAC tranche and one support tranche.

Figure 30.3: Cash flows of a PAC bond at 150 PSA. The mortgage rate is 6%, the PAC band is 100 PSA to 300 PSA, and the actual prepayment speed is 150 PSA.



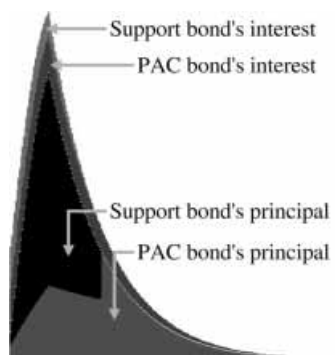


Figure 30.4: Cash flows of a PAC bond at 400 PSA. The mortgage rate is 6%, the PAC band is 100 PSA to 300 PSA, and the actual prepayment speed is 400 PSA.

PACs can be divided sequentially to provide narrower paydown structures. These **sequential PACs** narrow the range of years over which principal payments occur. See Fig. 30.5 for an illustration. Although these bonds are all structured with the same band, the actual range of speeds over which their schedules will be met may differ. We can take a CMO bond and further structure it. For example, the sequential PACs could be split by use of a pro rata structure to create high and low coupon PACs. We can also replace the second tranche in a four-tranche ABCZ sequential CMO with a PAC class that amortizes starting in year four, say. But note that tranche C may start to receive prepayments that are in excess of the schedule of the PAC bond. It may even be retired earlier than tranche B.

Support bonds themselves can have cash flows prioritized so as to reduce prepayment risk. Support bonds with schedules, also referred to as **PAC II bonds**, are supported by other support bonds without schedules. PACs in a structure in which there are PAC II level bonds are called **PAC I bonds**.

- **Programming Assignment 30.3.1** Implement the cash flow generator in Fig. 30.2.
- **Programming Assignment 30.3.2** Implement the cash flow generator for sequential PAC bonds.

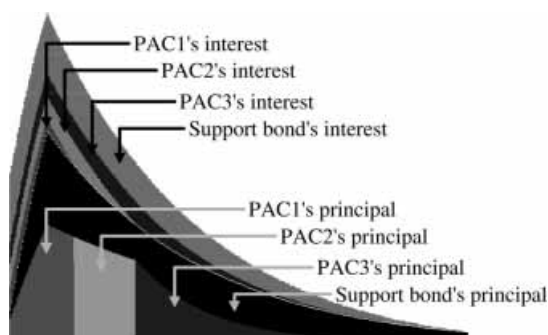


Figure 30.5: Cash flows of sequential PAC bonds. The mortgage rate is 6%, the PAC band is 100 PSA to 300 PSA, and the actual prepayment speed is 150 PSA. The three PAC bonds have identical original principal amounts.

30.4 TAC Bonds

Created in 1986, TAC bonds, just as PAC bonds, have priority over other bond classes that do not have a schedule for principal repayment. PACs have a higher priority over TACs, however. TAC bonds have a single PSA prepayment speed over which the principal repayment schedule is guaranteed. When prepayments exceed the speed, the excess principal is paid to the support bonds first. However, when prepayments fall short of the speed, TAC bonds will extend. TACs are therefore designed to provide protection against contraction risk but not extension risk.

30.5 CMO Strips

A class in a CMO structure can be a **CMO strip**. A CMO strip that is created when an IO is stripped from a CMO bond is called a **bond IO**. For example, this stripping mechanism creates an **inverse IO** from an inverse floater and a **PAC IO** from a PAC bond. Bond IOs lower the coupon of the CMO tranche. Some people call bond IOs **IOettes** to distinguish them from IO strips created off the entire collateral. A PO class that is neither a PAC nor a TAC is called a **super PO**. Like a PO strip, such bonds are purchased at a substantial discount from par and are returned at par. When prepayments accelerate as interest rates decline, “super” performance follows, hence the name.

30.6 Residuals

All CMOs contain a residual interest composed of the excess of collateral cash flows plus any reinvestment income over the payments for principal, interest, and expenses. This excess cash flow is called the **CMO residual**. The residual arises in part because credit rating agencies require CMOs to be overcollateralized in order to receive AAA credit rating: The cash flows must be sufficient to meet all the obligations under any prepayment scenario.

Another source of residual cash flow is reinvestment income. There is usually a delay between the time the payments from the collateral are received and the time they are remitted to the CMO bondholders. For example, whereas the mortgages in the collateral pay monthly, most CMOs pay quarterly or semiannually. The CMO trustee is therefore able to reinvest the pool cash flows before distribution dates. To be conservative in calculating the funds needed to meet future obligations, the rating agencies require that the trustee assume a relatively low reinvestment rate. CMO trustees have been able to reinvest at higher rates, and the excess is retained as a residual.

Additional Reading

See [14, 54, 161, 260, 325, 439, 758] for in-depth analyses of CMOs.