Valuation of Shout Option

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ABSTRACT

We price a Shout Option on the S&P500 using parameterized market data of the index level and traded options. We suggest possible ways to hedge a short position in this option, as well as estimating profitability and risk with a 3 point volatility spread. We highlight the implications of our methods and our model.

Keywords: Exotic Options, Valuation

INTRODUCTION

We begin with an exotic shout option. The payoff of a shout option is similar to a vanilla European option (in the case of a call, $(S-K)^+$), except that at a specified time (a 'shout date'), the holder of the option may choose to exchange their claim on this risky floating payment for a guaranteed fixed payoff F.

We chose to use a call for our simulated option. We chose to set the shout date to T = 0.5 (half a year), and the fixed payment $F = 100$ to ensure that the Shout has some appeal beyond that of the vanilla European call.

We also suggest ways to hedge the shout option by using traded options to hedge the potential losses from both upside and downside risk.

1 METHODS

1.1 Data Sources

We used Python to download market data on the S&P500 from 1/1/2001 to 11/24/2019. From this data, we calculated a historical volatility σ from the past 252 data points (approximately one year of market data), and used the last downloaded data point for the most recent index level S_0. We found the market risk free rate r and estimated S&P500 equivalent dividend yield from online sources d. We got market data on S&P500 call options from the CME on 11/24/2019.

1.2 Parametrizing to S&P real-world data

Assuming the risk-neutral measure, with distinct random numbers z_1 and z_2, we can simulate stock prices for the two periods $T = \frac{1}{2}$ and $T = 1$ as:

$$\begin{aligned} S_{\frac{1}{2}} &= S_0 e^{(r-d-\frac{\sigma^2}{2})\frac{1}{2}+\sigma z_1\sqrt{\frac{1}{2}}} \\ S_1 &= S_{\frac{1}{2}} e^{(r-d-\frac{\sigma^2}{2})\frac{1}{2}+\sigma z_2\sqrt{\frac{1}{2}}} \end{aligned} \tag{1}$$

z_1 and z_2 are sets of independent random normal numbers that incorporate the antithetic variate technique to de-mean and the control variate technique to ensure standard deviation is 1. In the current iteration of the project, we generate 25000 random numbers per set, take their negatives so we have sets of 50000 numbers with a mean of 0, and divide by the standard deviation of those so the standard deviation is 1. This is one of many attempts at variance reduction.

1.3 Shout Option Payoff Simulation

There are three possible payouts for a Shout Option: F, $S_T - K$, or 0. We postulate the existence of an optimal trigger index level Q, below which 'exercise' (to receive the fixed F at maturity) will be an

optimal choice compared to choosing to choosing to receive $S_T - K$. We determine the value of Q via our simulated values of $S_{\frac{1}{2}}$ and S_1 . For a given path and Q, if $S_{\frac{1}{2}} < Q$, we set the payoff of that path to be F; else, the payoff is $(S_1 - K)^+$. For each Q, we average the payoff over all paths. We then find the level of Q that provides the highest average payoff.

We also experimented with a secondary control variate. We can simulate a vanilla call price using the same simulated stock prices, which often will not agree with the Black-Scholes price even though it theoretically should. We subtract the path simulated call and add the analytical call to the payoff. However, the need for this is removed by later benchmarking.

1.4 Calibrating Black-Scholes Formula to Market Prices

The famous Black-Scholes model produces theoretical option prices for known parameters. Unfortunately, these prices are not observed what is observed in the market. We resolved this by regression.

CME S&P Call Options are provided for a variety of strike prices and maturities. We took call prices for December 2020 and June 2020 from 'today' from the CME. We also calculated the analytical Black-Scholes prices for these calls, for each available strike. We then regressed each set of analytical prices to the respective set of market-observed calls to obtain regression constants that can scale the analytical Black-Scholes price to a realistic price (subject to several assumptions, not accounting for discretization error).

1.5 Benchmarking the Shout Option

We perform a completely separate benchmarking to relate our simulated shout values to the values of calls with maturity $T = \frac{1}{2}$ and $T = 1$. As mentioned earlier, we simulate the value of the underlying stocks. The simulated payout of the $T = \frac{1}{2}$ call is $(S_{\frac{1}{2}} - K)^+$, and the simulated payout of the $T = 1$ call is $(S_1 - K)^+$. We calculate payouts for all of our simulated stock prices at both maturities, as well as the calculation of the shout option. The formula is shown below.

$$\hat{V_0} = \alpha + \beta_1 \hat{C_{\frac{1}{2}}} + \beta_2 \hat{C_1}$$

We do not benchmark off of the simulated value of the stock because it becomes a highly dominant term that produces the counter-intuitive result that a shout option priced at a vol spread of 3 points is actually less valuable than an option without the spread.

We thus obtain α, β_1, and β_2. We can then benchmark off of market traded options $C_{\frac{1}{2}}$ and C_1:

$V_0 = \alpha + \beta_1 C_{\frac{1}{2}} + \beta_2 C_1$

One run of the program calculates the value of the option to be $\$189.99 \pm 0.27$ with an optimal shout level of \$3107. (Uncertainties are given by maximum likely error: $MLE = 2\frac{\sigma}{\sqrt{N}}$. For benchmarked values, this is given by the propagation of error from the estimators in accordance with $\sigma^2 = s_e^2 \beta^T (X^T X)^{-1} \beta$.)

Note that the simulation does not appear to be numerically stable, so subsequent runs of the code may produce differing values.

1.6 Pricing with a 3 point volatility spread

In order to calculate over a 3 point volatility spread, we simulate the stock with $\sigma + 3\%$ instead of σ.

We have two options in assessing risk. We can either price the option at the average value of our simulations or the maximum value output by our simulation. Strictly speaking, if we attempt to sell for the maximum simulated value, the counterparty may not wish to buy at that price. However, this maximum simulated price is perhaps a built-in worse-case scenario. It would be less risky to use the maximum price.

In one run of our code, we calculated the average value of our option at the spread to be $\$190.45 \pm 0.24$. The profitability of selling the option at the spread at the average price ends up being $\approx \$0.46 \pm 0.52$ per contract. It is usually only optimal to exercise this option at $Q \approx \$3050$, which is $\approx \$50$ less than the option without the spread.

1.6.1 Risk Discussion

While the calculation of value is not numerically stable, for a given set of common random numbers value of the Shout Option with the spread is always greater than without. The profit between the average shout prices is usually between \$0.20 and \$0.70. In one run of the code, the profitability was $\approx \$0.46 \pm 0.52$ per contract. In this case, it is entirely possible to realize a loss.

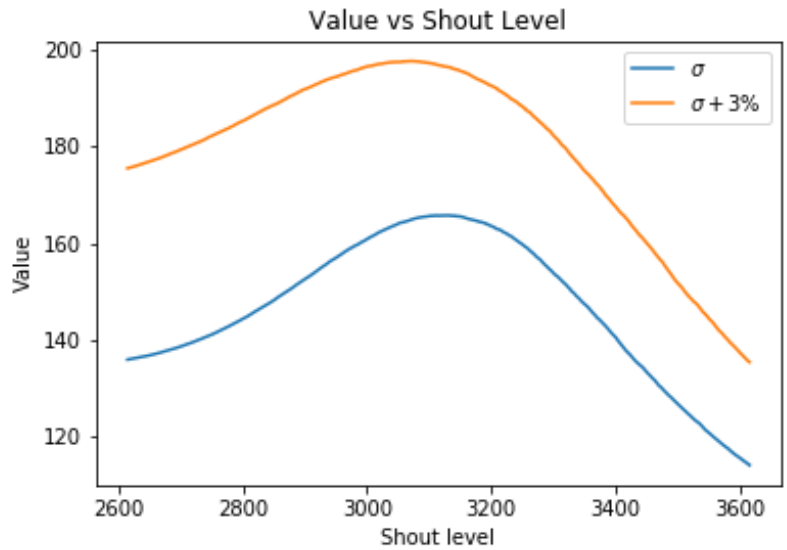

Figure 1. Unbenchmarked shout payoffs at varying shout levels

Practically speaking, we wish to sell the option for as high as you can get away with. That means selling at the highest price that will fall within the buying counterparty's simulated maximum error.

If volatility is significantly higher than the calculated historical volatility (even with the spread), then our pricing of the option is below fair value.

1.6.2 Benchmarking of the shout at spread

We benchmark the shout with a spread in much the same way that we benchmark the shout without a spread, which is to say we use market data. However, this causes the two options to become much closer in value.

Without benchmarking, the shout at historical vol is worth $\approx$ \$170. With benchmarking, as discussed before, it is worth \$ $\approx$ \$190

Without benchmarking, the shout at spread is worth $\approx$ \$180. With benchmarking, it is worth \$ $\approx$ \$191.

Attempts to correct this would likely rely on dubious assumptions. For example, instead of benchmarking on the actual market option prices, we could calculate what the market prices *should be* if volatility was 3 points higher, using the Black-Scholes formula and our earlier benchmarking of market prices to the Black-Scholes formula. However, this would assume that changes in volatility would have a linear response in price, which is probably not true.

Hedging Strategies

1.6.3 Why sell this option?

We would sell this option if it is our belief that $S_{\frac{1}{2}} > Q$ and $S_1 < K$. This implies that the drift in the market (if it exists) is not large enough to cross these boundaries, and that volatility is small enough to make jumps across these boundaries unlikely. Our two concerns are that either the stock drops beneath Q at T = 0.5, or that the stock rises past K at T = 1.

We're also going to assume that transaction costs won't completely wipe out the profitability of this hedge.

1.6.4 Hedging

From a theoretical standpoint, being short a Shout Option is similar to being short a binary call with strike Q and a vanilla European call. A naive hedge is to therefore to enter in opposite trades with smaller cost than the option is sold for. In this case, that would be to go long some numbers of European calls on the index with strike K and long some number of binary calls with strike Q.

Without hedging, we are facing unlimited upside loss when the stock prices increase if the trigger price Q is exceeded at $t = 0.5$. We definitely loss $F - V_0$ (V_0 is the our portfolio's premium) if it does

not price at $t = 0.5 < Q$. In the best case, we can only earn our shout option's premium which is v_0. The losses without hedging is shown on the table below, which accounts for our sale of the option as a negative loss.

Table 1. Potential Losses without Hedging

	$S_{0.5} < Q$	$S_{0.5} > Q$
$S_1 > K$	$F - V_0$	$S_1 - K - V_0$
$S_1 < K$	$F - V_0$	$-V_0$

The largest risk is in the case where $S_{0.5} > Q$ and $S_1 > K$, as in this case there is a potentially infinite loss from positive market movements.

Another naive approach to hedge this risk is to create a covered call position by purchasing the underlying shares of stocks (represented by S_0). At the end of the period, our wealth will have changed by $S_1 - S_0$. This will put a cap on possible loss. However, this is a costly hedge because buying the entire index is expensive and buying other peoples' ETFs of the index introduces basis risk.

Table 2. Potential Loss, Hedging with Stock

	$S_{0.5} < Q$	$S_{0.5} > Q$
$S_1 > K$	$F - V_0 + (S_0 - S_1)$	$S_0 - K - V_0$
$S_1 < K$	$F - V_0 + (S_0 - S_1)$	$(S_0 - S_1) - V_0$

The above strategy is costly, and in every case where $S_1 < S_0$ we adopt an additional risk; we have traded a potentially infinite risk for a cost of S_0. This is still pretty bad.

Instead, we can buy a call option at value C with the same strike price and maturity date at time 0. This will restrict our downside loss to C but negate our potential adverse losses. In this case, we have a fixed profit $V_0 - C$ in the situations where the shout is not triggered, as we price our shout to be worth more than the vanilla call with the same parameters. Of the cases where the shout is triggered, in one we lose $F + C - V_0$. Given the data we parameterized our model on, this will be a positive loss.

Table 3. Potential Loss, Hedging with Calls

	$S_{0.5} < Q$	$S_{0.5} > Q$
$S_1 > K$	$F + (K - S_1) + (C - V_0)$	$C - V_0$
$S_1 < K$	$F + C - V_0$	$C - V_0$

Finally, to perfectly hedge, we recommend entering a position in binary options with a total future value of F at $T = 0.5$, with a strike at Q. This will nullify the potential loss of F entirely at the cost of however much the OTC binary options cost. As long as the cost B of these options is such that $V_0 - C - B > 0$, the trade will still be worth it. The CBOE offers such a binary option with ticker BSZ.

1.7 Everything Dubious: A Disclosure

Here is a discussion of everything wrong with our implementation:

- Almost every number in this summary is merely one instance of many simulations, because there is a high lack of numerical stability in our implementation.
- Including the S&P500 as a benchmark asset causes the benchmarked value of the shout at plus 3 points to become less than the shout at historical volatilities.
- Benchmarking with market data causes the shout at plus 3 points and the shout to become much closer in value. The benchmarked shout at spread will be worth \$190-191 and the benchmarked shout without spread will be worth \$189-190, but the respective unbenchmarked options will be worth $\approx$ \$180 and $\approx$ \$160.
- Benchmarking causes the MLE of the option with volatility to increase relative to the unbenchmarked value. This would suggest that benchmarking does not help in pricing the spread. However, not benchmarking would undervalue the spread.

In conclusion, this model would make a validator cry. We hope that by pointing out these obvious errors it becomes clear that we're not ignorant of the implications of our assumptions but merely budgeting our time.