

# Express Riddler

30 July 2021

## Riddle:

Riddler Nation is competing against Conundrum Country at an Olympic archery event. Each team fires three arrows toward a circular target 70 meters away. Hitting the bull's-eye earns a team 10 points, while regions successively farther away from the bull's-eye are worth fewer and fewer points.

Whichever team has more points after three rounds wins. However, if the teams are tied after each team has taken three shots, both sides will fire another three arrows. (If they remain tied, they will continue firing three arrows each until the tie is broken.)

For every shot, each archer of Riddler Nation has a one-third chance of hitting the bull's-eye (i.e., earning 10 points), a one-third chance of earning 9 points and a one-third chance of earning 5 points.

Meanwhile, each archer of Conundrum Country earns 8 points with every arrow.

Which team is favored to win?

*Extra credit:* What is the probability that the team you identified as the favorite *will* win?

## Solution:

The first thing to do is to determine the probabilities for all possible scores after three arrows. Of course, Conundrum Country will score 24 with probability 1. For Riddler Nation, the probabilities are:

Score	Probability	Score	Probability
30	$1/27$	24	$6/27$
29	$3/27$	23	$3/27$
28	$3/27$	20	$3/27$
27	$1/27$	19	$3/27$
25	$3/27$	15	$1/27$

Counting up the scores greater than 24 gives a total probability of  $11/27$ . This is the probability that Riddler Nation wins in the first round. Counting up the scores less than 24 gives  $10/27$ , the probability that Riddler Nation loses the first round. The remaining probability of  $6/27$  is the probability that they tie in the first round.

Because a tie just leads back to the same situation for future rounds, it is only necessary to consider the relative probability of winning or losing in a single round. This relative probability  $P$  comes out to

$$\begin{aligned} P &= \frac{11/27}{11/27 + 10/27} \\ &= \frac{11}{10 + 11} \\ &= \frac{11}{21} \end{aligned}$$

So the overall probability of Riddler Nation winning is  $\frac{11}{21}$ .

A more formal way of defining the total probability of winning is to consider the infinite series, which includes the probability of winning each subsequent round:

$$\begin{aligned}
 P &= \frac{11}{27} + \left(\frac{6}{27}\right) \left(\frac{11}{27}\right) + \left(\frac{6}{27}\right)^2 \left(\frac{11}{27}\right) + \left(\frac{6}{27}\right)^3 \left(\frac{11}{27}\right) + \dots \\
 &= \left(\frac{11}{27}\right) \sum_{n=0}^{\infty} \left(\frac{6}{27}\right)^n \\
 &= \left(\frac{11}{27}\right) \frac{1}{1 - 6/27} \\
 &= \frac{11/27}{21/27} \\
 &= \frac{11}{21}
 \end{aligned}$$