Classic Riddler

2 October 2020

Riddle:

I have 10 chocolates in a bag: Two are milk chocolate, while the other eight are dark chocolate. One at a time, I randomly pull chocolates from the bag and eat them—that is, until I pick a chocolate of the other kind. When I get to the other type of chocolate, I put it back in the bag and start drawing again with the remaining chocolates. I keep going until I have eaten all 10 chocolates.

For example, if I first pull out a dark chocolate, I will eat it. (I'll always eat the first chocolate I pull out.) If I pull out a second dark chocolate, I will eat that as well. If the third one is milk chocolate, I will not eat it (yet), and instead place it back in the bag. Then I will start again, eating the first chocolate I pull out.

What are the chances that the *last* chocolate I eat is milk chocolate?

Solution:

To solve this, I will define the probability $P(n_d, n_m, last)$ of eating milk chocolate last given that there are n_d remaining dark pieces, n_m remaining milk pieces, and that I most recently ate type $last$. For my notation, I let $last$ be m (milk), d (dark), or x (none/replacement step).

I can define P for the possible final states, and build up the probabilities up to the case for $P(8, 2, x)$. I can start with the end cases:

$$P(0, 0, d) = 0$$
$$P(0, 0, m) = 1$$

Even more generally, I can write

$$P(n_d > 0, 0, x) = P(n_d > 0, 0, d) = P(n_d > 0, 0, m) = 0$$
$$P(0, n_m > 0, s) = P(0, n_m > 0, d) = P(0, n_m > 0, m) = 1$$

From here, the rules for the probabilities are as follows (with $n_d, n_m > 0$):

$$P(n_d, n_m, x) = \left(\frac{n_d}{n_d + n_m}\right) P(n_d - 1, n_m, d) + \left(\frac{n_m}{n_d + n_m}\right) P(n_d, n_m - 1, m)$$
$$P(n_d, n_m, d) = \left(\frac{n_d}{n_d + n_m}\right) P(n_d - 1, n_m, d) + \left(\frac{n_m}{n_d + n_m}\right) P(n_d, n_m, x)$$
$$P(n_d, n_m, m) = \left(\frac{n_d}{n_d + n_m}\right) P(n_d, n_m, x) + \left(\frac{n_m}{n_d + n_m}\right) P(n_d, n_m - 1, m)$$

Carrying these calculations out gives a rather surprising result. For any combinations of dark and milk chocolates (with both being more than zero), the probability of finishing with either chocolate is always $1/2$. So the answer to the specific riddle with $n_d = 8$ and $n_m = 2$ (i.e., $P(8, 2, x)$) is $\boxed{\mathbf{1/2}}$.