Express Riddler

13 August 2021

Riddle:

You are very clever when it comes to solving Riddler Express puzzles. You are so clever, in fact, that you are in the top 10 percent of solvers in Riddler Nation (which, as you know, has a very large population). You don't know *where* in the top 10 percent you are—in fact, you realize that you are equally likely to be anywhere in the topmost decile. Also, no two people in Riddler Nation are equally clever.

One Friday morning, you walk into a room with nine members randomly selected from Riddler Nation. What is the probability that you are the cleverest solver in the room?

Solution:

If your cleverness is x, then the probability that you are more clever than nine other uniformly-distributed people is x^9. The probability distribution $P(x)$ of your cleverness is

$$P(x) = \begin{cases} 10 & 0.9 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

This ensures that x is evenly distributed between 0.9 and 1, and that the total probability is normalized:

$$\int_{-\infty}^{\infty} P(x)\,dx = 1$$

The probability of being the most clever among nine other people is given by the integral

$$\begin{aligned} \int_{-\infty}^{\infty} P(x)x^9\,dx &= 10\int_{0.9}^{1} x^9\,dx \\ &= x^{10}\Big|_{0.9}^{1} \\ &= 1 - \left(\frac{9}{10}\right)^{10} \\ &= \frac{10{,}000{,}000{,}000 - 3{,}486{,}784{,}401}{10{,}000{,}000{,}000} \\ &= \frac{6{,}513{,}215{,}599}{10{,}000{,}000{,}000} \end{aligned}$$

So the solution is $\boxed{\mathbf{6{,}513{,}215{,}599/10{,}000{,}000{,}000 = 0.6513215599}}$.