Express Riddler

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Riddle:

The American League Championship Series of Riddler League Baseball determines one of the teams that will compete in the Riddler World Series. This year's teams—the Tampa Bay Lines and the Minnesota Twin Primes—are evenly matched. In other words, both teams are equally likely to win each game of the best-of-seven series.

On average, how many games will the series last? (Note that the series ends as soon as one team has won four games.)

Solution:

I designate a completed Championship Series as a list of Ts and Ms corresponding to the winner of each game in order. For example, if the Tampa Bay Lines win the first four games, this would be TTTT; similarly, if the Minnesota Twin Primes win the first four games, this would be MMMM. Because the probability of either team winning a single games is fixed at $1/2$, the probability of winning four games is $(1/2)^4 = 1/16$. Because there are two ways for the series to end in four games, the total probability to end in four games is $2(1/16) = 1/8$.

There are four ways for either team to win in the fifth game. For Tampa Bay to win, these are:

TTTMT TTMTT TMTTT MTTTT

Similarly, there are four ways for Minnesota to win in the fifth game. So there are eight ways for the series to end in five games, and the total probability is $8(1/2)^5 = 1/4$.

There are ten ways for Tampa Bay to win in the sixth game:

TTTMMT	TTMTMT	TMTTMT	MTTTMT	TTMMTT
TMTMTT	MTTMTT	TMMTTT	MTMTTT	MMTTTT

So the total probability to end in six games is $20(1/2)^6 = 5/16$.

Finally, there are 20 ways for Tampa Bay to win in the seventh game:

TTTMMMT	TTMTMMT	TMTTMMT	MTTTMMT	TTMMTMT
TMTMTMT	MTTMTMT	TMMTTMT	MTMTTMT	MMTTTMT
TTMMMTT	TMTMMTT	MTTMMTT	TMMTMTT	MTMTMTT
MMTTMTT	TMMMTTT	MTMMTTT	MMTMTTT	MMMTTTT

So the total probability to end in six games is $40(1/2)^7 = 5/16$.

The average number of games N in the series is the weighted sum:

$$N = 4\left(\frac{1}{8}\right) + 5\left(\frac{1}{4}\right) + 6\left(\frac{5}{16}\right) + 7\left(\frac{5}{16}\right)$$

which gives a sum of **$93/16 = 5.8125$**.