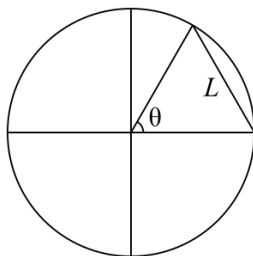# Express Riddler

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Riddle:

Help, there's a cricket on my floor! I want to trap it with a cup so that I can safely move it outside. But every time I get close, it hops exactly 1 foot in a random direction.

I take note of its starting position and come closer. Boom—it hops in a random direction. I get close again. Boom—it takes another hop in a random direction, independent of the direction of the first hop.

What is the *most probable* distance between the cricket's current position after two random jumps and its starting position? (Note: This puzzle is not asking for the *expected* distance, but rather the *most probable* distance. In other words, if you consider the probability distribution over all possible distances, where is the peak of this distribution?)

Solution:

I can assume that the first jump moves from the edge of a circle (with 1-ft radius) to the center of the circle. Then the second jump will land somewhere on the edge of the circle. I call the angle between the two jumps θ, and the distance (in ft) between the initial and final positions L. This is shown below:

The distance L can be written as

$$L(\theta) = \sqrt{(1 - \cos\theta)^2 + (\sin\theta)^2}$$
$$= \sqrt{2 - 2\cos\theta}$$

Finding the most probably distance requires maximizing this equation. Because L is positive, it is maximized at the same points as L^2, which is easier to differentiate:

$$L^2 = 2 - 2\cos\theta$$
$$\frac{d}{d\theta}L^2 = 2\sin\theta$$

Because of symmetry, this only needs to be evaluated in the range $[0, \pi]$. The zeroes are at $\theta = 0$ and π, which conveniently are also the edges of the range. Evaluating these shows that the maximum is at $\theta = \pi$, with a most-likely distance of $\boxed{\textbf{2 ft}}$.