Express Riddler

11 October 2019

Riddle:

An auditorium with 200 seats, numbered from 1 to 200, is filled to capacity. A speaker, who happens to be a mathematician, steps up to the podium overlooking the audience and pauses for a moment. "You know," she says, "I'm thinking of a rather large whole number. Every seat number in this auditorium evenly divides my number, except for two of them—and those two seats happen to be next to each other."

As you'd expect, adjacent seats in the auditorium have consecutive numbers. Which two numbers was the speaker referring to?

Solution:

The two adjacent numbers which are excluded must be powers of prime numbers. Otherwise, their factorization will still show up in the final product. For example, if $35 \ (= 5 \times 7)$ is excluded, the factors of 5 and 7 would still show up because 5 and 7 themselves are not excluded. Further, the excluded numbers must be the highest possible powers that are less than 200. For example, if the number $27 \ (= 3^3)$ is excluded, all the 3s would still show up because $81 \ (= 3^4)$ is not excluded.

All of the highest prime powers below 200 are listed below.

128 (2^7)	169 (13^2)	31	53	73	101	127	151	179	199
81 (3^4)	17	37	59	79	103	131	157	181	
125 (5^3)	19	41	61	83	107	137	163	191	
49 (7^2)	23	43	67	89	109	139	167	193	
121 (11^2)	29	47	71	97	113	149	173	197	

The only two adjacent numbers on this list, and the solution, are $\boxed{\textbf{127 and 128}}$.

There isn't a particular answer to the "rather large" whole number about which the speaker is thinking. However, it must have all of the other factors above at least, in addition to the remaining factors of 2 that shows up in 64. Plugging those into Wolfram Alpha gives me 1,327,927,515,090,260,884,407,345,538,562,367,745,796,828,278,681,721, 394,601,759,928,808,007,945,120,777,126,248,000, a number with 88 digits. Any multiple of this number could be the speaker's number, as long as that multiple itself isn't divisible by 2 or 127.