Classic Riddler

30 October 2020

Riddle:

Instead of playing hot potato, you and 60 of your closest friends decide to play a socially distanced game of hot pumpkin.

Before the game starts, you all sit in a circle and agree on a positive integer N. Once the number has been chosen, you (the leader of the group) start the game by counting "1" and passing the pumpkin to the person sitting directly to your left. She then declares "2" and passes the pumpkin one space to *her* left. This continues with each player saying the next number in the sequence, wrapping around the circle as many times as necessary, until the group has collectively counted up to N. At that point, the player who counted "N" is eliminated, and the player directly to his or her left starts the next round, again proceeding to the same value of N. The game continues until just one player remains, who is declared the victor.

In the game's first round, the player 18 spaces to your left is the first to be eliminated. Ricky, the next player in the sequence, begins the next round. The second round sees the elimination of the player 31 spaces to Ricky's left. Zach begins the third round, only to find himself eliminated in a cruel twist of fate. (Woe is Zach.)

What was the smallest value of N the group could have used for this game?

Extra credit: Suppose the players were numbered from 1 to 61, with you as Player No. 1, the player to your left as Player No. 2 and so on. Which player won the game?

Extra extra credit: What's the smallest N that would have made you the winner?

Solution:

I went about this problem in a few different ways. To answer the first question, I set up a script that searches for and checks possible solutions. Based on the description, the number N must satisfy the following equations:

$$\begin{aligned} N &= 19 \mod 61 \\ N &= 32 \mod 60 \\ N &= 1 \mod 59 \end{aligned}$$

The script I set up can be found at the beginning of `hot_pumpkin.C`. It starts counting integers i and calculates $N = 61i + 19$. Then it checks the modulus for both 60 and 59. If it's a match, then it outputs the results. The first few results are 136,232; 352,172; 568,112; and 999,992. The solutions are all of the form $N = 136,232 + 215,940n$ for any non-negative integer n. Notably, $215,940 = 61 * 60 * 59$. So the solution to the first part is **136,232**.

For the second part, I decided to play out the game by hand. My work can be seen in `Hot_pumpkin.xlsx`. The calculation was made much easier, because it's only necessary to pass the pumpkin based on the modulus for each round. The modulus ($N \mod$ round#) is calculated in the file. I also set up the entire game in my C++ script. In both cases, the player that won (and the solution) is **player 58**.

For the third part, I extended my work from part two in the C++ script. It starts counting N from 1, and calculates the winner for a game with 61 players. For fun, I outputted all the winners up through a win for player 1. The solution is that player 1 wins for $\boldsymbol{N = 120}$. All the other winners for a 61-player game are shown in the table below. Interestingly, the original game could have been shortened by a whole lot by choosing $N = 84$, since player 58 would have won anyway.

N	Winner	N	Winner	N	Winner	N	Winner
1	61	31	12	61	33	91	60
2	59	32	59	62	19	92	22
3	10	33	4	63	14	93	25
4	27	34	24	64	6	94	19
5	49	35	28	65	13	95	23
6	7	36	22	66	7	96	33
7	35	37	57	67	39	97	18
8	46	38	20	68	39	98	23
9	48	39	35	69	30	99	50
10	22	40	56	70	44	100	50
11	14	41	30	71	30	101	7
12	6	42	31	72	5	102	12
13	61	43	12	73	31	103	33
14	10	44	30	74	28	104	40
15	15	45	41	75	20	105	41
16	53	46	59	76	26	106	48
17	19	47	4	77	48	107	18
18	25	48	42	78	20	108	24
19	18	49	12	79	19	109	12
20	20	50	51	80	7	110	31
21	13	51	2	81	40	111	51
22	53	52	41	82	15	112	20
23	56	53	24	83	59	113	42
24	16	54	34	84	58	114	25
25	35	55	44	85	47	115	27
26	16	56	46	86	13	116	37
27	47	57	51	87	11	117	17
28	49	58	49	88	21	118	53
29	49	59	2	89	20	119	3
30	3	60	37	90	5	120	1