Classic Riddler

30 October 2020

Riddle:

Instead of playing hot potato, you and 60 of your closest friends decide to play a socially distanced game of hot pumpkin.

Before the game starts, you all sit in a circle and agree on a positive integer N. Once the number has been chosen, you (the leader of the group) start the game by counting "1" and passing the pumpkin to the person sitting directly to your left. She then declares "2" and passes the pumpkin one space to *her* left. This continues with each player saying the next number in the sequence, wrapping around the circle as many times as necessary, until the group has collectively counted up to N. At that point, the player who counted "N" is eliminated, and the player directly to his or her left starts the next round, again proceeding to the same value of N. The game continues until just one player remains, who is declared the victor.

In the game's first round, the player 18 spaces to your left is the first to be eliminated. Ricky, the next player in the sequence, begins the next round. The second round sees the elimination of the player 31 spaces to Ricky's left. Zach begins the third round, only to find himself eliminated in a cruel twist of fate. (Woe is Zach.)

What was the smallest value of N the group could have used for this game?

Extra credit: Suppose the players were numbered from 1 to 61, with you as Player No. 1, the player to your left as Player No. 2 and so on. Which player won the game?

Extra extra credit: What's the smallest N that would have made you the winner?

Solution:

I went about this problem in a few different ways. To answer the first question, I set up a script that searches for and checks possible solutions. Based on the description, the number N must satisfy the following equations:

$$\begin{aligned} N &= 19 \mod 61 \\ N &= 32 \mod 60 \\ N &= 1 \mod 59 \end{aligned}$$

The script I set up can be found at the beginning of `hot_pumpkin.C`. It starts counting integers i and calculates $N = 61i + 19$. Then it checks the modulus for both 60 and 59. If it's a match, then it outputs the results. The first few results are 136,232; 352,172; 568,112; and 999,992. The solutions are all of the form $N = 136,232 + 215,940n$ for any non-negative integer n. Notably, $215,940 = 61 * 60 * 59$. So the solution to the first part is **136,232**.

For the second part, I decided to play out the game by hand. My work can be seen in `Hot_pumpkin.xlsx`. The calculation was made much easier, because it's only necessary to pass the pumpkin based on the modulus for each round. The modulus ($N \mod$ round#) is calculated in the file. I also set up the entire game in my C++ script. In both cases, the player that won (and the solution) is **player 58**.

For the third part, I extended my work from part two in the C++ script. It starts counting N from 1, and calculates the winner for a game with 61 players. For fun, I outputted all the winners up through a win for player 1. The solution is that player 1 wins for $\boldsymbol{N = 140}$. All the other winners for a 61-player game are shown in the table below. Interestingly, the original game could have been shortened by *a whole lot* by choosing $N = 26$, since player 58 would have won anyway.

N	Winner	N	Winner	N	Winner	N	Winner	N	Winner
1	61	31	58	61	52	91	53	121	36
2	59	32	55	62	58	92	40	122	37
3	44	33	30	63	28	93	23	123	22
4	39	34	35	64	19	94	10	124	8
5	16	35	53	65	6	95	46	125	53
6	59	36	5	66	38	96	36	126	38
7	19	37	32	67	12	97	10	127	53
8	18	38	9	68	9	98	44	128	52
9	39	39	48	69	30	99	32	129	28
10	36	40	37	70	16	100	20	130	10
11	16	41	16	71	4	101	58	131	49
12	10	42	54	72	2	102	49	132	48
13	35	43	18	73	15	103	60	133	58
14	18	44	24	74	5	104	8	134	58
15	18	45	56	75	8	105	36	135	40
16	8	46	32	76	50	106	22	136	36
17	26	47	25	77	20	107	14	137	3
18	31	48	13	78	11	108	52	138	59
19	53	49	22	79	44	109	15	139	30
20	32	50	40	80	18	110	25	140	1
21	12	51	9	81	50	111	55		
22	56	52	57	82	50	112	49		
23	36	53	34	83	23	113	18		
24	45	54	37	84	26	114	6		
25	57	55	5	85	35	115	41		
26	58	56	27	86	31	116	25		
27	16	57	14	87	23	117	2		
28	17	58	60	88	53	118	45		
29	48	59	41	89	41	119	38		
30	40	60	52	90	34	120	43		