Classic Riddler

25 June 2021

Riddle:

Riddler solitaire is played with 11 cards: an ace, a two, a three, a four, a five, a six, a seven, an eight, a nine, a 10 and a joker. Each card is worth its face value in points, while the ace counts for 1 point. To play a game, you shuffle the cards so they are randomly ordered, and then turn them over one by one. You start with 0 points, and as you flip over each card your score increases by that card's points—as long as the joker hasn't shown up. The moment the joker appears, the game is over and your score is 0. The key is that you can stop any moment and walk away with a nonzero score.

What strategy maximizes your expected number of points?

Extra credit: With an optimal strategy, how many points would you earn on average in a game of Riddler solitaire?

Solution:

The way to solve this problem is by calculating the expected change in score by drawing a single card. For a given current score N and number of cards C still left in the deck, the expected change in score can be relatively easily calculated. Because N and C are the only known quantities during the game, the answer must only depend on these two values. If the expected change in score $E(N, C)$ is positive for a given N and C, then the best strategy is to draw another card. Otherwise, if the expected change in score is negative, the best strategy is to stop.

I calculated $E(N, C)$ for all possible pairs of N and C in the game. The value of N ranges from 0 (start of game, no cards drawn yet) to 55 (all ten numbered cards drawn, with only the joker left). Several combinations of N and C are not possible; for example, if only one card has been drawn, any score above 10 is not possible. To calculate $E(N, C)$, first the difference of the current score (N) from 55 is calculated and divided by $C - 1$, which gives the average score added by drawing one of the remaining numbered cards. This is then weighted by a factor of $^{(C-1)}/_{C}$, which is the probability of drawing a numbered card. Then the current score is divided by C and subtracted, giving the weighted average chance of drawing the joker and dropping the current to 0. The overal result for E is

$$E(N, C) = \frac{55 - 2N}{C}$$

The results are located in `Poker.xlsx`. All possible values are checked if they are positive or negative. (Luckily, there was no result with $E = 0$, which would be ambiguous strategy-wise.) It turns out, the best strategy is to continue taking cards as long as the score is below 28. Interestingly, this strategy turns out not to depend on the number of cards left. So the solution is to $\boxed{\textbf{draw cards as long as your score is below 28, and stop at 28 or above}}$.