

Express Riddler

24 January 2020

Riddle:

Derek Jeter and Larry Walker were just elected to the Baseball Hall of Fame! That got Stephanie thinking. Suppose there are 20 players on the ballot and 400 voters in a given year. Each voter can select up to 10 players for induction without voting for any given player more than once. To gain entry, a player must have been selected on at least 75 percent of the ballots.

Under these circumstances, what is the *maximum* number of players that can be inducted into the Hall of Fame?

Solution:

There are 400 total ballots, and I am assuming that each ballot must be cast, so that the 75% threshold means that a player must receive 300 votes to be inducted. The first way to determine the maximum is to divide the total votes by the 300-vote threshold. Dividing 400 votes by 300 gives $13.\overline{3}$, which means that in principle, 13 players can be inducted. But is there a way that it works out for 13 players?

The answer is yes. For this argument, I will label the 13 lucky players A–M. First, voters 1–300 each vote for player A. Then voters 301–400 and 1–200 vote for player B. Next, voters 201–400 and 1–100 vote for player C, and finally voters 101–400 vote for player D. In this scenario, A, B, C, and D have each received exactly 300 votes, and each voter has used 3 votes. This process repeats for players E–H and I–L. Now 12 players have each received the necessary votes, and each voter has voted nine times. Player M now just needs any 300 (or all 400) of the remaining votes. At most, there are only 100 votes remaining for player N, so the estimate above holds true, that the fourteenth player cannot be inducted. So the answer to the riddle is 13 players.