Express Riddler

5 March 2021

Riddle:

You have three coins in your pocket, each of which can be a penny, nickel, dime or quarter with equal probability. You might have three different coins, three of the same coin or two coins that are the same and one that is different.

Each of these coins can buy you a string whose length in centimeters equals the value of the coin in cents, i.e., the penny buys 1 cm of string, the nickel buys 5 cm of string, etc. After purchasing your three lengths of string, what is the probability that they can be the side lengths of a triangle?

Solution:

If the three lengths of string are labeled a, b, and c, then the requirements to form a triangle are:

$$a + b > c \qquad a + c > b \qquad b + c > a$$

The only available lengths are 1, 5, 10, and 25. It is immediately clear that the only time these requirements can be filled is if $a = b = c$, which means that all the coins must be the same denomination. The probability of this occuring is $1/4 \cdot 1/4$, or $\boxed{\mathbf{1/16}}$.