Express Riddler

30 October 2020

Riddle:

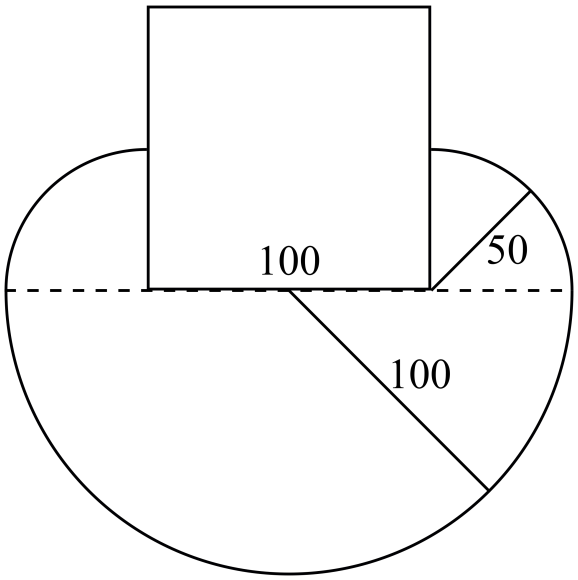
While waiting in line to vote early last week, I overheard a discussion between election officials. Apparently, there may have been a political sign that was within 100 feet of the polling place, against New York State law.

This got me thinking. Suppose a polling site is a square building whose sides are 100 feet in length. An election official takes a string that is also 100 feet long and ties one end to a door located in the middle of one of the building's sides. She then holds the other end of the string in her hand.

What's the area of the region outside of the building she can reach while holding the string?

Solution:

The path traced out by the end of the string is a circular arc. When the string hits the edge of the building, it continues in a smaller arc along the sides of the building. The shape is shown below:



The region traced out by the string forms a semicircle and two quarter circles. The radius of the large semicircle is of course 100 ft, and the radius of the quarter circles is 50 ft. So the area A is

$$A = \frac{1}{2}\pi 100^2 + 2\frac{1}{4}\pi 50^2$$

The solution is therefore (approximately) **19,635 sq ft**.