

Express Riddler

17 July 2020

Riddle:

Riddler Township is having its quadrennial presidential election. Each of the town's 10 "shires" is allotted a certain number of electoral votes: two, plus one additional vote for every 10 citizens (rounded to the nearest 10).

The names and populations of the 10 shires are summarized in the table below.

Shire	Population	Electoral Votes
Oneshire	11	3
Twoshire	21	4
Threeshire	31	5
Fourshire	41	6
Fiveshire	51	7
Sixshire	61	8
Sevenshire	71	9
Eightshire	81	10
Nineshire	91	11
Tenshire	101	12

As you may know, under this sort of electoral system, it is quite possible for a presidential candidate to lose the popular vote and still win the election.

If there are two candidates running for president of Riddler Township, and every single citizen votes for one or the other, then what is the *lowest* percentage of the popular vote that a candidate can get while still winning the election?

Solution:

There are 75 total electoral votes in this scenario. The best way to get the minimum number of actual votes is to just barely get a majority of electoral votes, which is 38. Whatever combination of shires gets to 38 electoral votes, those shires should also just barely get a majority of votes to swing their shires. The remaining shires should have 100% of their actual votes go the other way. The majority combination of shires should also be preferentially lower-population shires. For example, although Eightshire has as many electoral votes as Twoshire and Fourshire, it represents a larger actual voter population, so in this case, Twoshire and Fourshire would be preferred to Eightshire in forming an electoral majority.

By my non-exhaustive calculations, the best way to get to 38 electoral votes is with Oneshire (3), Threeshire (5), Fourshire (6), Fiveshire (7), Sixshire (8), and Sevenshire (9). With each shire winning only a majority by one actual vote, that totals $6 + 16 + 21 + 26 + 31 + 36 = 136$ electoral votes. The total population is 560, so my solution is $136/560 \approx \boxed{24.29\%}$.