Express Riddler

3 December 2021

Riddle:

Tonight marks the sixth night of Hanukkah, which means it's time for some more Menorah Math!

I have a most peculiar menorah. Like most menorahs, it has nine total candles—a central candle, called the shamash, four to the left of the shamash and another four to the right. But unlike most menorahs, the eight candles on either side of the shamash are numbered. The two candles adjacent to the shamash are both "1," the next two candles out from the shamash are "2," the next pair are "3," and the outermost pair are "4."

The shamash is always lit. How many ways are there to light the remaining eight candles so that sums on either side of the menorah are "balanced"? (For example, one such way is to light candles 1 and 4 on one side and candles 2 and 3 on the other side. In this case, the sums on both sides are 5, so the menorah is balanced.)

Solution:

Using the numbers 1–4, the possible sums are 0–10. With this range, it is possible to list out all of the possible balanced sums:

0	1	2	3	4	5	6	7	8	9	10
(empty)	1-1	2-2	1,2-1,2	1,3-1,3	1,4-1,4	2,4-2,4	3,4-3,4	1,3,4-1,3,4	2,3,4-2,3,4	1,2,3,4-1,2,3,4
			1,2-3	1,3-4	1,4-2,3	2,4-1,2,3	3,4-1,2,4			
			3-1,2	4-1,3	2,3-1,4	1,2,3-2,4	1,2,4-3,4			
			3-3	4-4	2,3-2,3	1,2,3-1,2,3	1,2,4-1,2,4			

Thus, there are **26** possible balanced sums.