Express Riddler

21 May 2021

Riddle:

The Riddler Cheese Company is producing what are called "craft triples"—triangular slices of cheese whose side lengths are Pythagorean triples, when measured in inches.

However, the company's slicing machine recently malfunctioned and produced a stock of square slices with side lengths of 5 inches. To salvage this situation, what is the greatest number of whole Pythagorean slices that can be made from each 5-inch square? (Note: You can only cut pieces out of the square. No melting or gluing pieces together!)

Extra credit: What is the smallest square of cheese such that 100 percent of the square can be partitioned into craft triples?

Solution:

My hand-drawn, not-to-scale solution is shown on the following page. The 5×5 triangle can be divided into four 3-4-5 triangles. Each triangle has a hypotenuse on an edge of the square, and there is a 1×1 square leftover in the center. The triangles fit because the 3-5 and 4-5 angles of the triangle are complimentary. Further, the area of each triangle is 1/2 * 3 * 4 = 6, so together they take up 24 of the 25 square inches of cheese. The remaining 1 square inch is formed by the leftover lengths of sides: with each 3-inch side lined up against a 4-inch side leaves the 1 inch each time. So the solution to the riddle is 4 triangles.

For the extra credit, I believe that the answer is a 12-inch square. This would be formed by doubling each 3-4-5 triangle, matching the hypotenuses to form 3×4 rectangles. Arranging these 3×4 rectangles 4 across and 3 high would achieve a total length of 12 on each side. I can't prove this is the answer, though.

