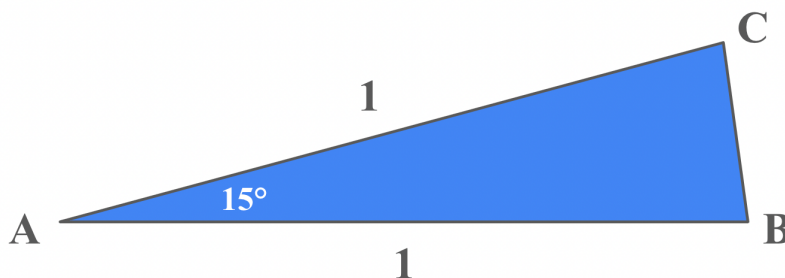


Classic Riddler

7 January 2022

Riddle:

Amare the ant is traveling within Triangle ABC, as shown below. Angle A measures 15 degrees, and sides AB and AC both have length 1.



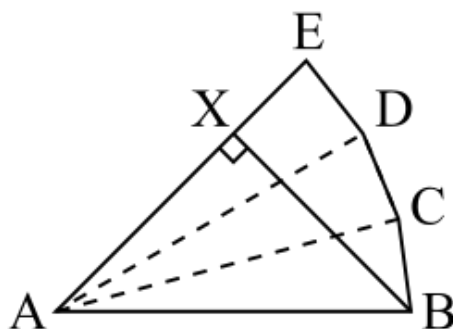
Amare starts at point B and wants to ultimately arrive on side AC. However, the queen of his colony has asked him to make several stops along the way. Specifically, his path must:

- Start at point B.
- Second, touch a point—any point—on side AC.
- Third, touch a point—any point—back on side AB.
- Finally, proceed to a point—any point—on side AC (not necessarily the same point he touched earlier).

What is the shortest distance Amare can travel to complete the queen's desired path?

Solution:

Traveling across the triangle three times can be reimagined as traveling across three stacked triangles. I show this in the diagram below, in which $\triangle ABC$, $\triangle ACD$, and $\triangle ADE$ are all similar triangles.



In this case, the shortest total path length is simply a straight line that intersects \overline{AE} at a right angle. If the intersection point is X, then $\angle AXB$ is a right angle. Because each of $\angle BAC$, $\angle CAD$, and $\angle DAE$ are 15° , $\angle BAE$ and $\angle BAX$ are 45° . Thus $\angle ABX$ is also 45° , and $\triangle ABX$ is an isosceles right triangle with a hypotenuse of 1. So the two legs each have length $\sqrt{2}/2$. So the solution is $\sqrt{2}/2 \approx 0.707$.