

Express Riddler

22 May 2020

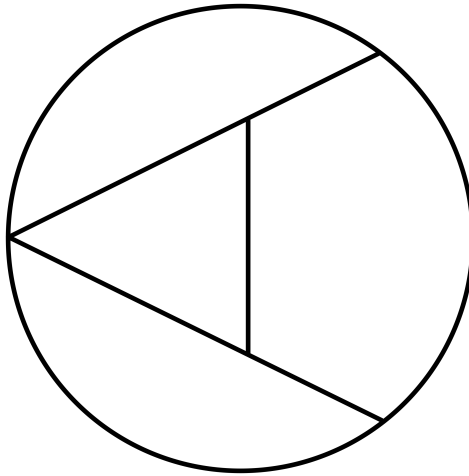
Riddle:

To share a cylindrical muffin equally with his two toddlers, Robert makes three vertical cuts in a “Y” pattern, producing three equal pieces.

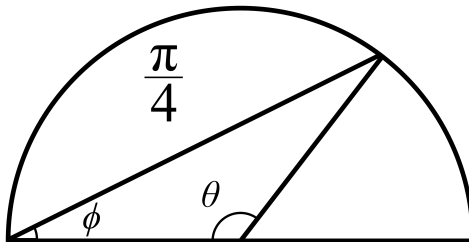
The next morning, his wife wants in on the fun. But before he can cut the muffin into quarters with an “X” pattern, one of his children suggests using an “A” pattern. If Robert were to produce equal fourths in this manner, what would be the ratio of length of the A’s middle bar to the radius of the muffin?

Solution:

This is a pretty straightforward geometry problem. The cuts basically look like this (not to scale):

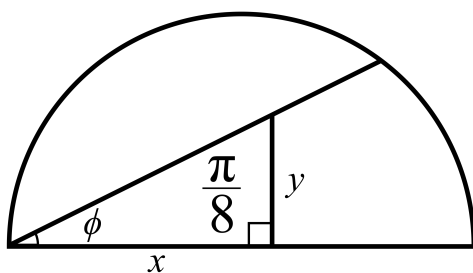


The first thing to calculate is the angle of the two secant cuts, which create two rounded cuts, each with area $\pi/4$. Although the area of these two regions is not immediately clear, it is simply the difference of two more easy-to-calculate areas. For this calculation I used the following scheme:



For an obtuse triangle sitting along the radius of a unit circle, its base is simply 1, and the height is just $\sin \theta$. Thus the area of the triangle is $(1/2) \sin \theta$. The area of the whole arc including the triangle is $(\theta/2\pi)\pi = \theta/2$. Then the difference of the two areas, which is what we need, is $\theta/2 - (1/2) \sin \theta = \pi/4$, which gives $\theta \approx 132.5^\circ$.

Now it is important to note that because the triangle is isosceles, the angle ϕ can be calculated as $\theta + 2\phi = 180^\circ$, which gives $\phi \approx 23.83^\circ$. From here, the third can be calculated, which divides the remaining center piece in half. I have used these dimensions to calculate the size of this cut:



We need to solve for y because that is the length about which the riddle is asking. It is easy to see that $x = y \cot \phi$. Then the area of the triangle is $(1/2)y^2 \cot \phi = \pi/8$, which gives $y \approx 0.5890$. The full length of the cut is twice this value, so the solution is (approximately) 1.178.