Express Riddler

10 December 2021

Riddle:

My condo complex has a single elevator that serves four stories: the garage (G), the first floor (1), the second floor (2) and the third floor (3). Unfortunately, the elevator is malfunctioning and stopping at every single floor, no matter what. The elevator always goes G, 1, 2, 3, 2, 1, G, 1, 2, etc.

I want to board the elevator on a random floor (with all four floors being equally likely). As I round the corner to approach the elevator, I hear that its doors have closed, but I have no further information about which floor it's on or whether the elevator is going up or down. The doors might have just closed on my floor, for all I know.

On average, how many stops will the elevator make until it opens on my floor (including the stop on your floor)? For example, if I am waiting on the second floor, and I heard the doors closing on the garage level, then the elevator would open on my floor in two stops.

Extra credit: Instead of four floors, suppose my condo had N floors. On average, how many stops will the elevator make until it opens on my floor?

Solution:

For the four-floor path (G-1-2-3-2-1-G...), there are six possible elevator starting points. If you are on the garage floor, the starting points are 6, 5, 4, 3, 2, and 1 stop away. Those have an average of $3/2$ stops. Similary, if you are on the third floor, the average number of stops is also $3/2$. If you are on the first (or second) floor, the starting points are 1, 4, 3, 2, 1, and 2 stops away. Those have an average of $13/6$ stops. Averaging these numbers of stops over all four of your possible locations, the solution is $\boxed{\mathbf{17/6}}$.