

Express Riddler

26 November 2021

Riddle:

On the Food Network's latest game show, Cranberries or Bust, you have a choice between two doors: A and B. One door has a lifetime supply of cranberry sauce behind it, while the other door has absolutely nothing behind it. And boy, do you love cranberry sauce.

Of course, there's a twist. The host presents you with a coin with two sides, marked A and B, which correspond to each door. The host tells you that the coin is weighted in favor of the cranberry door—without telling you which door that is—and that door's letter will turn up 60 percent of the time. For example, if the sauce is behind door A, then the coin will turn up A 60 percent of the time and B the remaining 40 percent of the time.

You can flip the coin twice, after which you must make your selection. Assuming you optimize your strategy, what are your chances of choosing the door with the cranberry sauce?

Extra credit: Instead of two flips, what if you are allowed three or four flips? Now what are your chances of choosing the door with the cranberry sauce?

Solution:

With two flips, there are four possible outcomes: AA, AB, BA, and BB. The optimal strategy is simple: if you get the same side both flips, choose that door; if you get each side once, choose a door randomly. This strategy works for either door being correct. Without loss of generality, assume that the correct door is A. Flipping AA occurs with probability $0.6^2 = 0.36$; if you flip this, you would be correct also with probability 0.36. Flipping BB occurs with probability $0.4^2 = 0.16$; if you flip this, you would lose with probability 0.16. There is a combined probability of $2 \cdot 0.6 \cdot 0.4 = 0.48$ of flipping AB or BA; if you flip this, you would win probability $0.48/2 = 0.24$. Thus your total probability of winning is $0.36 + 0.24 = \boxed{0.6}$. Interestingly, this is the same probability as flipping the coin a single time.

Similarly, the probability of winning with three or four flips is the same. For the three-flip case, you should pick whichever side flips over at least twice. For the four-flip case, you should pick whichever side flips over at least twice out of the first three. The probability of flipping AAA is $0.6^3 = 0.216$. The probability of flipping any of AAB, ABA, or BAA is $3 \cdot 0.6^2 \cdot 0.4 = 0.432$. Thus the total probability of winning in this case is $\boxed{0.648}$.