Express Riddler

29 October 2021

Riddle:

I have a spherical pumpkin. I carefully calculate its volume in cubic inches, as well as its surface area in square inches. Next, I got up to have a piece of Halloween candy (which, naturally, was a Reese's Peanut Butter Cup).

But when I came back to my calculations, I saw that my units—the square inches and the cubic inches—had mysteriously disappeared from my calculations. But it didn't matter, because both numerical values were the same!

What is the radius of my spherical pumpkin?

Extra credit: Let's dispense with 3D thinking. Instead, suppose I have an n-hyperspherical pumpkin. Once again, I calculate its volume (with units in^n) and surface area (with units in^{n-1}). Miraculously, the numerical values are once again the same! What is the radius of my n-hyperspherical pumpkin?

Solution:

Solving this riddle simply requires setting the volume and surface area of a sphere with r equal to each other:

$$\frac{4}{3}\pi r^3 = 4\pi r^2$$

This has a solution of $r = 3$ in whatever units are used. In the current case, the solution is $\boxed{\textbf{3 in}}$.

For the case of an n-hypersphere, the strategy is the same: set the n-dimensional "volume" and $n-1$-dimensional "surface area" equal to each other. I don't know the general formula for the volume of an n-dimensional sphere, but I do know that the surface area is the derivative of the volume. The volume is some constant term k multiplied by the r^n term. Setting this equal to its derivative yields:

$$kr^n = \frac{d}{dr} kr^n$$
$$= nkr^{n-1}$$

This has the relatively simple solution of $\boxed{r = n}$.