

Classic Riddler

15 October 2021

Riddle:

Over in the National League Championship Series, the Washington Nationals and the St. Louis Cardinals (known as the “Ords” for short) are also evenly matched. Again, both teams are equally likely to win each game of the best-of-seven series.

You enter a competition in which you must predict the winner of each of the seven games before the series begins. If any or all of the fifth, sixth or seventh game are not played, you are not credited with predicting a winner.

You win the competition if you predict *at least two* games correctly. If you optimize your strategy for picking winners, what is the probability you will win the competition?

Extra credit: You enter a second competition in which you must pick the winner of the first game and then of each next game, knowing who won in all the previous games. Again, if you optimize your strategy, *now* what is the probability you will predict at least two games correctly?

Solution:

I designate a completed Championship Series as a list of Ws and Ss corresponding to the winner of each game in order. For example, if the Washington Nationals win the first four games, this would be WWWW; similarly, if the St. Louis Cardinals win the first four games, this would be SSSS.

Because there are up to seven possible games to predict, there are $2^7 = 128$ possible predictions. However, the actual strategy does not depend on the prediction of the first game, only on the pattern after the first game. Any strategy that predicts Washington as the winner of the first game has a counterpart strategy that predicts St. Louis as the winner of the first game; these counterpart strategies have the same probability of winning the competition. Additionally, the prediction for the seventh game doesn't matter; if the championship goes to seven games, each team will win with 50% probability, so both predictions are equal. Therefore, there are only $2^5 = 32$ strategies to consider.

As for the game-by-game outcomes of the series, there are only 70 possible outcomes. Two outcomes (WWWW and SSSS) last only four games, and each have a probability of $(1/2)^4 = 1/16$ of occurring. There are eight outcomes that last five games, each with probability $1/32$. For a Washington win, these are

WWWSW WWSWW WSWWW SWWWW

and similar for St. Louis. There are eight outcomes that last five games, each with probability $1/64$. For a Washington win, these are

WWWSWW WWSWSW WSWWSW SWWWSW WWSSWW
WSWSWW SWWSWW WSSWWW SWSWWW SSWWWW

and similar for St. Louis. Finally, there are forty outcomes that last five games, each with probability $1/128$. For a Washington win, these are

WWWSWW WWSWSW WSWWSW SWWWSW WWSSWW
WSWSWW SWWSWW WSSWWW SWSWWW SSWWWW
WWSSWW WWSWSW WSWWSW SWWWSW WWSSWW
WSWSWW SWWSWW WSSWWW SWSWWW SSWWWW

and similar for St. Louis.

The number of strategies and outcomes is small enough that it is possible to type them all out and make a tally of correct predictions for each strategy. I have done so in **Predictions.xlsx**. Using W as the prediction for the first and last games, this shows that the best strategy is WWWSSW, with a probability of winning the competition of **0.90625**.