Express Riddler

1 October 2021

Riddle:

I recently purchased a new Velo-ton stationary bike and took it for a spin. The bike records three key metrics throughout the ride: cadence (how fast I'm riding), resistance (how hard I have to push the pedals to keep moving) and output (the power I produce).

With a little experimentation, I determine that the power (in watts) is equal to the product of the cadence and resistance values divided by 20. For example, if my cadence is 64 and my resistance is 25, then my power output is $(64 \cdot 25)/20$, or 80 watts.

Whenever I ride, I always make sure that my resistance is between 20 and 60, while my cadence is between 60 and 100. After a particularly grueling 30-minute workout, I notice that my average resistance was 40, while my average cadence was 80. (Note that these averages are computed per unit of time, rather than per unit of distance traveled.)

At first, I figure my average power was $(40 \cdot 80)/20$, or 160 watts. But I soon realize other values are also possible. What is the maximum average power that I could have produced? What is the minimum?

Solution:

In order to produce maximum and minimum output product, the individual cadence and resistance must be maximized and minimized, which occur at the individual upper and lower limits. For the resistance to average out to 40, it would be at 20 for half the time, and 60 for the other half, i.e.,

$$\frac{1}{2}(20) + \frac{1}{2}(60) = 40$$

Similarly the cadence would be at 60 and 100 each for half of the time:

$$\frac{1}{2}(60) + \frac{1}{2}(100) = 80$$

To maximize the power, the maximum cadence and resistance would occur together for the same half of the workout, and the minimum cadence and resistance would occur for the other half:

$$\frac{\frac{1}{2}(20)(60) + \frac{1}{2}(60)(100)}{20} = 180$$

On the other hand, to minimize the power, the maximum cadence would occur at the same half as the minimum resistance, and the minumum cadence and maximum resistance would occur in the other half:

$$\frac{\frac{1}{2}(20)(100) + \frac{1}{2}(60)(60)}{20} = 140$$

So the maximum and minimum average power rates are **180 watts** and **140 watts**, respectively.