Express Riddler

16 July 2021

Riddle:

I have three dogs: Fatch, Fetch and Fitch. Yesterday, I found a brown 12-inch stick for them to play with. I marked the top and bottom of the stick and then threw it for Fatch. Fatch, a Dalmatian, bit it in a random spot—leaving a mark—and returned it to me. In her honor, I painted the stick black from the top to the bite and white from the bottom to the bite.

I subsequently threw the stick for Fetch and then for Fitch, each of whom retrieved the stick by biting a random spot. What is the probability that Fetch and Fitch both bit the same color (i.e., both black or both white)?

Solution:

I will label the spot that Fatch marks as x. If x is expressed as a fraction of the whole stick, then $0 < x < 1$. The probability that Fetch bites below x (in the white region) is x, and the same is true for Fitch. So the total probability that both Fetch and Fitch bite below x is x^2. Similarly, the probability that both Fetch and Fitch bite above x is $(1-x)^2$. To get the total probability across all possible values of x requires integration:

$$\int_0^1 x^2 + (1-x)^2 \, dx$$

which gives the solution $\boxed{\mathbf{2/3}}$.