Classic Riddler

23 October 2020

Riddle:

This season, on the way to winning her fourth WNBA championship in her 17-year career, Sue Bird made approximately 50 percent of her field goal attempts. Suppose she and Seattle Storm teammate Breanna Stewart are interested in testing whether Bird has a "hot hand"—that is, if her chances of making a basket depend on whether or not her previous shot went in. Bird happens to know that her chances of making any given shot is always 50 percent, independent of her shooting history, but she agrees to perform an experiment.

In each trial of the experiment, Bird will take three shots, while Stewart will record which shots Bird made or missed. Stewart will then look at all the trials that had at least one shot that was immediately preceded by a made shot. She will randomly pick one of these trials and then randomly pick a shot that was preceded by a made shot. (If there was only one such shot to pick from, she will choose that shot.)

What is the probability that Bird made the shot that Stewart picked?

Solution:

For three shots in each trial, there are only 8 ($= 2^3$) possible outcomes for each trial. I list them below, using 1s for scores and 0s for misses:

$$
\begin{array}{ll}
0\text{-}0\text{-}0 & 0\text{-}0\text{-}1 \\
0\text{-}1\text{-}0 & 0\text{-}1\text{-}1 \\
1\text{-}0\text{-}0 & 1\text{-}0\text{-}1 \\
1\text{-}1\text{-}0 & 1\text{-}1\text{-}1
\end{array}
$$

Each of these possible outcomes is equally likely for any trial. Six of these have a made shot preceding some other shot. Of these six, three (0-1-0, 1-0-0, and 1-0-1) have a made shot followed only by a missed shot, so by choosing one of these trials, Stewart will pick a missed shot. Two (0-1-1 and 1-1-1) have made shots followed only by made shots, and by choosing either of these trials, Stewart will pick a made shot. The fifth (1-1-0) has both a missed and a made shot after a made shot. Since the choice of shot within a trial is evenly distributed, Stewart has a 50% chance of picking either a missed or made shot. So in a sense, there is a $3\,1/2$ out of 6 chance, or $7/12$, of picking a missed shot, and a $2\,1/2$ out of 6 chance, or $5/12$, of picking a made shot. So the solution is $\boxed{5/12}$.