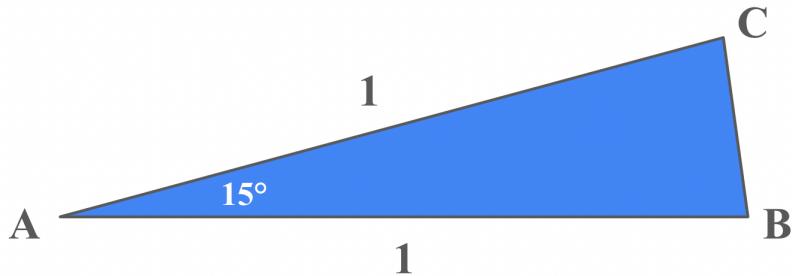


# Classic Riddler

7 January 2022

## Riddle:

Amare the ant is traveling within Triangle ABC, as shown below. Angle A measures 15 degrees, and sides AB and AC both have length 1.



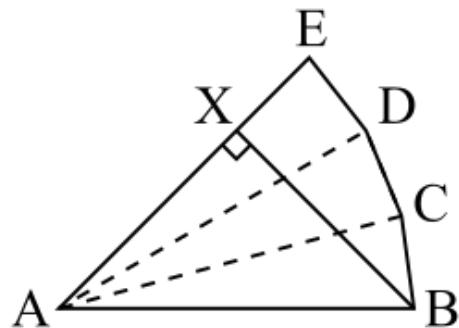
Amare starts at point B and wants to ultimately arrive on side AC. However, the queen of his colony has asked him to make several stops along the way. Specifically, his path must:

- Start at point B.
- Second, touch a point—any point—on side AC.
- Third, touch a point—any point—back on side AB.
- Finally, proceed to a point—any point—on side AC (not necessarily the same point he touched earlier).

What is the shortest distance Amare can travel to complete the queen's desired path?

## Solution:

Traveling across the triangle three times can be reimagined as traveling across three stacked triangles. I show this in the diagram below, in which  $\triangle ABC$ ,  $\triangle ACD$ , and  $\triangle ADE$  are all similar triangles.



In this case, the shortest total path length is simply a straight line that intersects  $\overline{AE}$  at a right angle. If the intersection point is X, then  $\angle AXB$  is a right angle. Because each of  $\angle BAC$ ,  $\angle CAD$ , and  $\angle DAE$  are 15°,  $\angle BAE$  and  $\angle BAX$  are 45°. Thus  $\angle ABX$  is also 45°, and  $\triangle ABX$  is an isosceles right triangle with a hypotenuse of 1. So the two legs each have length  $\sqrt{2}/2$ . So the solution is  $\boxed{\sqrt{2}/2 \approx 0.707}$ .