Express Riddler

16 April 2021

Riddle:

You are creating a variation of a Romulan pixmit deck. Each card is an equilateral triangle, with one of the digits 0 through 9 (written in Romulan, of course) at the base of each side of the card. No number appears more than once on each card. Furthermore, every card in the deck is unique, meaning no card can be rotated so that it matches (i.e., can be superimposed on) any other card.

What is the greatest number of cards your pixmit deck can have?

Extra credit: Suppose you allow numbers to appear two or three times on a given card. Once again, no card can be rotated so that it matches any other card. Now what is the greatest number of cards your pixmit deck can have?

Solution:

I think it is a safe assumption that the cards are one-sided, which means that if a given card is mirrored, it produces a different card. To choose three different numbers for the sides of each card, there are 10 available numbers for the first side, 9 for the second side, and 8 for the third side. This gives a total of $10 \times 9 \times 8 = 720$ possible ways to number a card. But because rotating the card doesn't change it, there are actually fewer possibilities. The card could be rotated $1/3$ or $2/3$ of the way around, so each card actually accounts for 3 possible sets of three numbers (the two rotations in addition to the original set of three numbers). So the correct solution is that there are $10 \times 9 \times 8/3 =$ $\boxed{240}$ possible cards.

When including cards with two numbers, there are now 10 options for the number which appears twice, and 9 resulting options for the number that appears once. This adds $10 \times 9 = 90$ new cards. This number does not change, because rotating one of these sets of numbers does create a new set of numbers.

When including cards with a single number, there are simply 10 additional cards: one for each digit 0–9. Considering all these together, there would be $\boxed{340}$ total cards.