Classic Riddler

9 October 2020

Riddle:

Parking cars is one thing—parking trucks is another thing entirely. Suppose I'm driving a *very long* truck (with length L) with two front wheels and two rear wheels. (The truck is so long compared to its width that I can consider the two front wheels as being a single wheel, and the two rear wheels as being a single wheel.)

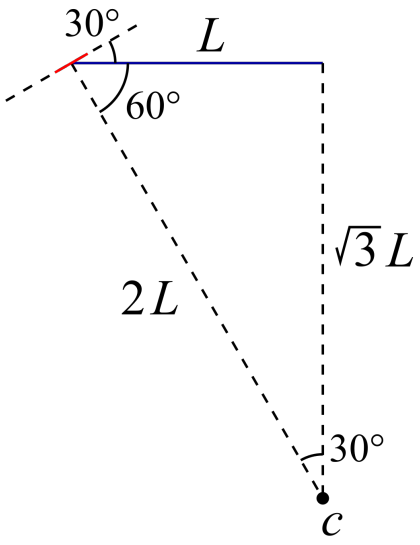
Question 1: Suppose I can rotate the front wheels up to 30 degrees in either direction (right or left), but the rear wheels do not turn. What is the truck's turning radius?

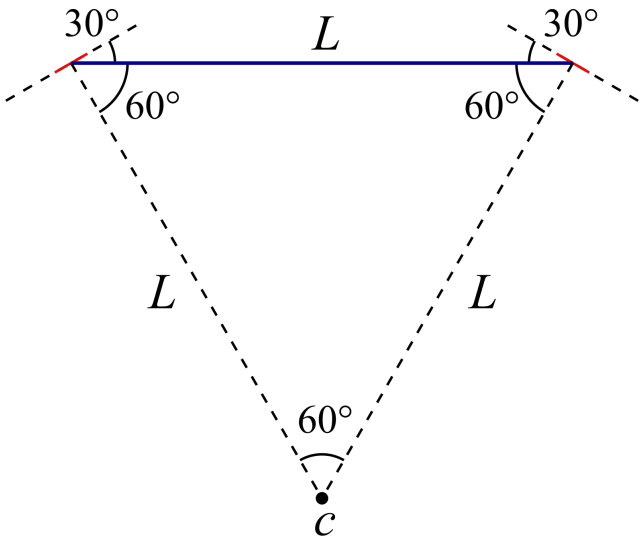
Question 2: Suppose I can also rotate the rear wheels—independently from the front wheels—30 degrees in either direction. *Now* what is the truck's turning radius?

Solution:

This is a basic geometry problem. In principle, for an infinitely thin vehicle with just two infinitely thin wheels (one each at the front and back), the wheels move exactly along their orientation. With this motion, the centers of the wheels (i.e., where the wheels touch the ground) trace out concentric circles. The center of the circles is where lines normal to the wheels meet, and these lines are the radii of the circles. For a real car, this is only approximate; while each wheel moves in a circle, its orientation is not exactly normal to the radius (and thus the motion of the each wheel is not strictly along its own orientation).

For the specific cases in this riddle, I have created the two diagrams below. The truck (of length L is in blue, and the turned wheels are in red. The two normal lines originating from the wheels come to a point at c, the mutual center of the circles the wheels trace out, and these lines along with the truck form a triangle. Using basic geometry, it is possible to identify the lengths of all the sides of the triangle.





According to the link from the riddle, the turning radius is defined as the outermost radius that a vehicle uses to make a full turn. Thus, the solution to the riddle is the length of the longest triangle side. So the solutions are $\boxed{\mathbf{2}\boldsymbol{L}}$ for the first question and $\boxed{\boldsymbol{L}}$ for the second question.