

Express Riddler

9 October 2020

Riddle:

Every weekend, I drive into town for contactless curbside pickup at a local restaurant. Across the street from the restaurant are six parking spots, lined up in a row.

While I *can* parallel park, it's definitely not my preference. No parallel parking is required when the rearmost of the six spots is available or when there are two consecutive open spots. If there is a random arrangement of cars currently occupying four of the six spots, what's the probability that I will have to parallel park?

Solution:

This problem is small enough that it is possible to enumerate all arrangements of cars. There are only $\binom{6}{4} = 15$ ways to arrange four cars in six spots, which are listed below:

1-2-3-4	1-2-3-5	1-2-3-6
1-2-4-5	1-2-4-6	1-2-5-6
1-3-4-5	1-3-4-6	1-3-5-6
1-4-5-6	2-3-4-5	2-3-4-6
2-3-5-6	2-4-5-6	3-4-5-6

Equivalently, there are 15 ways to have two open spots among six:

1-2	1-3	1-4
1-5	1-6	2-3
2-4	2-5	2-6
3-4	3-5	3-6
4-5	4-6	5-6

Looking at these, there are six arrangements (1-3, 1-4, 1-5, 2-4, 2-5, 3-5) in which both the sixth spot is filled and there are no consecutive spots open. Thus the solution is $\frac{6}{15} = 0.4$.