Express Riddler

19 November 2021

Riddle:

After realizing that your friends on μετα (The Riddler Social Network) are more popular than you, you decide to make some new friends at your local gym, which is open daily from 5 p.m. to 8 p.m.

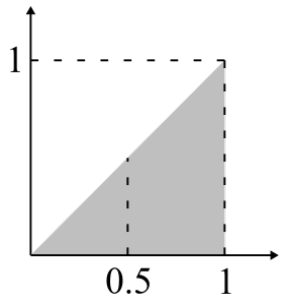
Some gym members attend very often. Others barely show up at all. As a matter of fact, there's a uniform distribution for how often the different members are in the gym—from 0 percent of the time that the gym is open to 100 percent of the time.

As a new member, you plan to be in the gym 50 percent of the time that it's open. While working out, you decide to make friends with the first person you see. (Aw!)

What is the probability that this person visits the gym more often than you?

Solution:

If the percent of time a member spends at the gym is x, then the probability that that person is at the gym at any given time is of course x. Thus, the probability distribution is simply a triangle bounded by a positively-sloped line (with a particular normalization factor). Randomly meeting a person at the gym is equivalent to randomly selecting a point from inside the triangle. To determine if that person visits the gym more often than 50% of the time, it is necessary to find the proportion of the triangle to the right of the 50% mark. This is represented by the graph below:



This can be solved geometrically or with calculus. Using calculus, the probability is simply the ratio:

$$\frac{\int_{0.5}^{1} x\,dx}{\int_{0}^{1} x\,dx} = 0.75$$

So the solution is **0.75**.