Classic Riddler

20 November 2020

Riddle:

To celebrate Thanksgiving, you and 19 of your family members are seated at a circular table (socially distanced, of course). Everyone at the table would like a helping of cranberry sauce, which happens to be in front of you at the moment.

Instead of passing the sauce around in a circle, you pass it randomly to the person seated directly to your left or to your right. They then do the same, passing it randomly either to the person to *their* left or right. This continues until everyone has, at some point, received the cranberry sauce.

Of the 20 people in the circle, who has the greatest chance of being the *last* to receive the cranberry sauce?

Solution:

I suppose there's a way to solve this analytically, but I don't want to bother with that. I wrote a script to simulate this process many times; the code can be found in `cranberry_pass.C`. It basically counts the number of times each person gets the sauce last. I numbered the people 2–20 (1 is the first person, and doesn't really count). Some sample results from the 100,000,000 simulations are:

Position	**Count**
2	5266781
3	5258326
4	5262347
5	5260950
6	5264241
7	5269333
8	5266353
9	5262939
10	5257483
11	5261289
12	5258953
13	5265369
14	5264188
15	5270231
16	5264248
17	5262048
18	5262265
19	5256353
20	5266303

Clearly, each person got the sauce last with very nearly the same frequency, which is about 5,260,000/100,000,000=0.0526. Since there is no obvious coding error (the numbers are different, and in any case sum to 10,000,000), I'm going to claim this is proof that the frequencies are in fact equal. Therefore the solution is that each other person (besides the first person) has an equal probability of being served last, with probability $\boxed{\mathbf{1/19 = 0.5263\ldots}}$.