Express Riddler

26 February 2021

Riddle:

This is the fourth and final week of CrossProduct™ puzzles—for now. This time, there are *four* four-digit numbers—each belongs in a row of the table below, with one digit per cell. The products of the four digits of each number are shown in the rightmost column. Meanwhile, the products of the digits in the thousands, hundreds, tens and ones places, respectively, are shown in the bottom row.

				1,458
				128
				2,688
				360
				125
960	384	630	270	

Can you find all four four-digit numbers and complete the table?

Solution:

The riddle is essentially asking for 16 digits to be placed in a 4×4 table. Generally, the first step is to decompose each number into either four (not-necessarily-prime) single-digit factors. For this riddle, though, I only decomposed the rows into factors. I list all possible sets of factors for each row below:

$$\begin{aligned} 1458 &= 2 \cdot 9 \cdot 9 \cdot 9 = 3 \cdot 6 \cdot 9 \cdot 9 \\ 128 &= 1 \cdot 2 \cdot 8 \cdot 8 = 1 \cdot 4 \cdot 4 \cdot 8 = 2 \cdot 2 \cdot 4 \cdot 8 = 2 \cdot 4 \cdot 4 \cdot 4 \\ 2688 &= 6 \cdot 7 \cdot 8 \cdot 8 \\ 125 &= 1 \cdot 5 \cdot 5 \cdot 5 \end{aligned}$$

My first step is that 384 is not divisible by 5, so the bottom row must have a 1 in the second column. The rest of the row is 5s. Therefore the bottom row is 5,1,5,5.

The only products which have a factor of 7 are 2,688 and 630, so the third row, third column must be 7. Since the 630 already has factors of $5 \cdot 7 = 35$, the remaining product is 18. The only factor in common with 128 is 2, so the second row of the third column must be 2, leaving 9 for the top row.

The 270 has a factor of 5, so its remaining product is 54. The 2,688 only has remaining factors of $6 \cdot 8 \cdot 8$, and the only factor in common with 54 is 6, which goes in the last column of the third row. This leaves 8s for the first and second column. The third row is therefore 8,8,7,6.

The 960 has factors of 5 and 8, leaving it with a remaining product of 24. The 384 has factors of 1 and 8, leaving it with a remaining product of 48. The 270 has factors of 5 and 6, leaving it with a remaining product of 9. With a 9 already placed in the top row, there is only one remaining product with a factor of 9, so the factorization of $2 \cdot 9 \cdot 9 \cdot 9$ is ruled out. The 9 must go in the last column, leaving factors of 3 and 6. The 6 must go in the second column, leaving 3 in the first column. The first row is therefore 3,6,9,9.

The only remaining factors for the columns are 8, 8, and 1. Therefore the second row is 8,8,2,1.

The final solution is

3	6	9	9
8	8	2	1
8	8	7	6
5	1	5	5