Riddler Express

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Riddle:

The infamous 1984 World Chess Championship match between the reigning world champion Anatoly Karpov and 21-year-old Garry Kasparov was supposed to have been played until either player had won six games. Instead, it went on for 48 games: Karpov won five, Kasparov won 3, and the other 40 games each ended in a draw. Alas, the match was controversially terminated without a winner.

We can deduce from the games Karpov and Kasparov played that, independently of other games, Karpov's chances of winning each game were 5/48, Kasparov's chances were 3/48, and the chances of a draw were 40/48. Had the match been allowed to continue indefinitely, what would have been Kasparov's chances of eventually winning the match?

Solution:

There are two interpretations to this riddle.

First I assumed that the match was started from scratch (i.e., from a score of 0-0-0). Then I just need to calculate what are the chances to win at least 6 games outright. It is clear that the probability of drawing has no impact on the calculations; a draw is essentially a do-over. Thus, the problem is reduced to starting a match at 0-0, with probabilities of winning 5/8 and 3/8 for Karpov and Kasparaov, respectively. The match is now best of 6, out of 11 matches. The total probability is a binomial sum:

$$P(\text{Kasparov}) = \sum_{n=6}^{11} \binom{11}{n} \left(\frac{3}{8}\right)^n \left(\frac{5}{8}\right)^{11-n}$$

This gives a solution of 0.19434..., or $\boxed{\textbf{19.4\%}}$.

The second interpretation is that the match continued from 3-5-48. Again, draws have no impact, so this is essentially a match starting at 3-5, with the same probabilities for the remaining games. The math is much simpler, since Kasparov must win three games in a row, with probability $(3/8)^3$. This solution is then 0.052734375, or $\boxed{\textbf{5.3\%}}$.