A4 – Measuring Area from Images

Introduction

Area measurement from images has application in cancer research (cancerous cells are generally bigger than healthy cells), remote sensing (estimation of land area) and automated product inspection (solder leads in circuit boards, grains) to name a few. Equations of area are known for regular, geometric shapes. However, in most real samples, shapes are arbitrary.

There are two techniques to obtain the area of images. The first is using Green's Theorem, and the other is through morphological operations. Morphological operations consider connected pixels as blobs, area estimation is then done by counting pixels in the blob. Technically, the method described in this activity is most appropriate for 2D objects that are scanned using a flat-bed scanner or captured using a camera aligned to the object's normal axis. Examples of 2D objects are photographs, paper documents, artworks on canvas and other flat media, textiles, maps, microscope slides, x-rays, etc.

If the object of interest can be segmented from the background, the number of pixels can be counted using `find()`.

Area from edge: Green's Theorem

Green's Theorem relates a double integral to a line integral. Let R be a region whose area we are to estimate and let F_1 and F_2 be functions that are continuous and have continuous partial derivatives $\partial F_1/\partial y$ and $\partial F_2/\partial x$ everywhere in some region or domain containing R.

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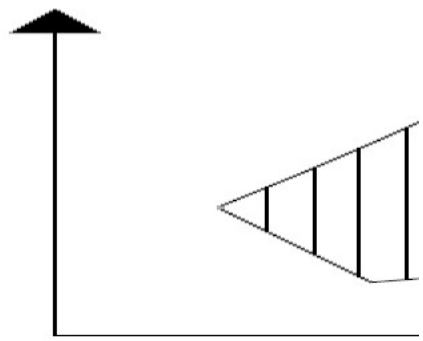

Figure 1: Region R with countour taken in the counterclockwise direction.

Then

$$\iint \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y} \right) dx\, dy = \oint (F_1\, dx + F_2\, dy) \tag{1}$$

R is to the left of the boundary if it the boundary is traversed in a counterclockwise direction. Let C be the collection of points (x,y) that form the contour or boundary of the region R.

If we choose F_1=0, F_2=x and F_1=−y, F_2=0, then Eq. 1 gives

$$\iint_R dx\, dy = \oint_C x dy \tag{2}$$

and

$$\iint_R dx\, dy = -\oint_C y\, dx \tag{3}$$

The right side of each equation above is the area A of the region R. By addition and averaging we have

$$A = \frac{1}{2} \oint (x dy - y dx). \tag{4}$$

In summation (discrete) form, if there are N_b pixels in the boundary or contour of R then the area of R is

$$A = \frac{1}{2} \sum_{i=1}^{N_b} [x_i\, y_{i+1} - y_i\, x_{i+1}] \tag{5}$$

Another Proof

So long as a closed curve is convex (no concavities) its area may be computed by summing the area of its “pie slices” as shown in Figure 2 below.

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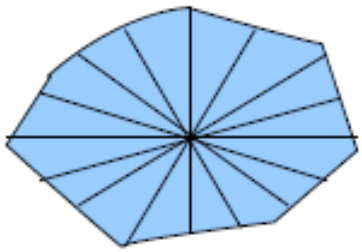

Figure 2: Convex region divided into "pie slices".

If we zoom in on one slice, we can compute its area by subtracting triangles from the largest rectangle that encloses the pie slice. From figure 3

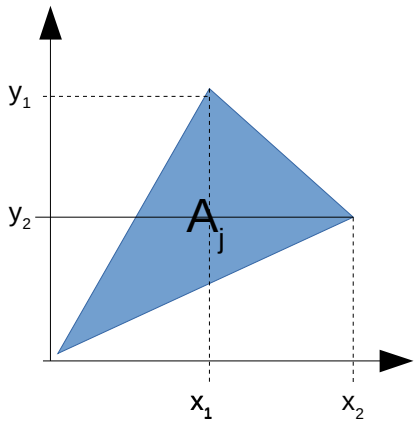

Figure 3: Area of one pie slice.

$$A_j = x_1 y_2 - \frac{1}{2} x_1 y_1 - \frac{1}{2} x_2 y_2 - \left[\frac{1}{2} (y_2 - y_1)(x_1 - x_2) \right] \tag{6}$$

$$A_j = \frac{1}{2} \left[x_1 y_2 - x_2 y_1 \right] \tag{7}$$

Procedure

1. Create black and white synthetic images of regular geometric shape such as rectangles, triangles or circles in Paint (or in Scilab or Matlab). Background should be black, shape should be white. The area of the image must be analytically known. Save the image as BMP or PNG. DO NOT SAVE IT AS JPEG. (Remember to use **`imwrite`** in Scilab or Matlab.)
2. Load the image in Scilab,Matlab or Python and obtain the edge image using **`edge`**.
3. Extract the edge pixel coordinates of the shape using **`find`** and then sort the list in order of increasing angle. To do so, find the pixel position of the centroid of the shape and subtract this from the x,y coordinates of the edge. Next convert the pixel position into r and θ

using $r=\sqrt{x^2+y^2}$, $\theta=atan(y,x)$. Sort the [x,y] pairs according to increasing θ . The resulting list of x,y coordinates now define the contour of the edge in order

4. Use the x,y values of the contour implement Green's theorem to get the image area (in number of pixels). Compare with the analytic value of area and comment on the accuracy of the technique.
5. Next, go to Google Earth or Google Maps and seek out the location of a place of interest, e.g. CS Atrium, your house. Using Green's Theorem, Paint (optional) and Scilab, delineate the edge and find its lot area in square meters.Note: Google map has an absolute scale which you can use for conversion. Explain how you can check the accuracy of your estimation.

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