RGB-to-Spectra Using PCA

Objectives

1. Compute the eigenspectra of paint pigments from the Munsell Color Chips database.
2. Convert a color camera into a spectral imager using principal components analysis.

Estimating Color Signal from Digital Color

Recall that the color signal $C(\lambda)$ bouncing off a surface is given by

$$C(\lambda) = \sum P(\lambda)R(\lambda) \tag{1}$$

where $P(\lambda)$ is the light source spectral power distribution and $R(\lambda)$ is the reflectance of that surface. This is the signal that reaches our eyes or camera sensor.

When the surface is imaged by a color camera, the camera outputs n digital numbers where n is the number of channels or filters the color camera has. Let q_n be the digital number of the nth camera channel. Then

$$q_n = \sum C(\lambda)S_n(\lambda) \tag{2}$$

where $S_n(\lambda)$ is the spectral sensitivity of the nth color channel.

Let's assume the color signal can be approximated as a linear superposition of m eigenspectra

$$C(\lambda) \approx \sum_{i=1}^{m} a_i e_i(\lambda) \tag{3}$$

where $e_i(\lambda)$ is the ith eigenspectra.

Now suppose we are using a typical color camera with 3 channels, R,G,B. Combining Equation (2) and (3) into a system of equations we have

$$\begin{aligned} q_R &= a_1 \sum e_1(\lambda)S_R(\lambda) + a_2 \sum e_2(\lambda)S_R(\lambda) + \ldots + a_m \sum e_m(\lambda)S_R(\lambda) \\ q_G &= a_1 \sum e_1(\lambda)S_G(\lambda) + a_2 \sum e_2(\lambda)S_G(\lambda) + \ldots + a_m \sum e_m(\lambda)S_G(\lambda) \\ q_B &= a_1 \sum e_1(\lambda)S_B(\lambda) + a_2 \sum e_2(\lambda)S_B(\lambda) + \ldots + a_m \sum e_m(\lambda)S_B(\lambda) \end{aligned} \tag{4}$$

or in matrix form ,

$$\begin{bmatrix} q_R \\ q_G \\ q_B \end{bmatrix} = \begin{bmatrix} e_1 \cdot S_R & e_2 \cdot S_R & \dots & e_m \cdot S_R \\ e_1 \cdot S_G & e_2 \cdot S_G & \dots & e_m \cdot S_G \\ e_1 \cdot S_B & e_2 \cdot S_B & \dots & e_m \cdot S_B \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}. \tag{5}$$

Note that we expressed the summation in Equation (4) as dot products. We can express Equation (5) in vector-matrix notation as

$$\mathbf{q} = \mathbf{Ta} \tag{6}$$

where $\mathbf{T}$ is the $3 \times m$ matrix in Equation 5.

Note that $\mathbf{q}$ is just the pixel color. The elements of $\mathbf{T}$ are just $\mathbf{e_i}$ which can be computed from an ensemble of spectra, and $\mathbf{S_n}$ which can be measured or obtained from the camera specs. With $\mathbf{a}$ being the only unknown, we can solve for it using Wiener estimation given by

$$\mathbf{a} = \left(\mathbf{T}^T\mathbf{T}\right)^{-1}\mathbf{T}^T\mathbf{q} \tag{7}$$

and then we can use Equation (3) to reconstruct the color signal spectrum.

We have shown that if a few eigenspectra can describe the color signal of a surface we can transform its digital color (just 3 numbers (RGB)) to a full spectrum. For this reason the technique we described is also known as ***spectral superresolution.***

Activity Procedure

In this activity you will test the spectral imaging concept using your results for the Macbeth Color Checker chart in the Spectral Imaging activity (Activity 2).

1. Download the Munsell color chips spectra from UVLE. Multiply each spectra using the same light source you used in rendering the Macbeth chart in Spectral Imaging. This will be the ensemble of color signals.
2. Compute the eigenspectra of the color signals in Step 1. Show the percentage explanation of the eigenspectra. Choose the number of eigenspectra m to be up to where you get at least 95% of the data explained.
3. Using the spectral senstivity of the camera you used in Activity 2, compute the transformation matrix $\mathbf{T}$.
4. Now pick any color patch from the Macbeth chart, get its rendered digital color from Activity 2 (its $\mathbf{q}$) and use Equation (7) to compute the eigenspectra coefficients $\mathbf{a}$.

5. Reconstruct the color signal of the patch using Equation (3) and remark how similar it is to its expected color signal. Do the same for other color patch samples.

References

1. Imai, F. H., & Berns, R. S. (1999, October). Spectral estimation using trichromatic digital cameras. In Proceedings of the International Symposium on Multispectral Imaging and Color Reproduction for Digital Archives (Vol. 42, pp. 1-8). Chiba University Chiba, Japan.

2. Soriano, M., Oblefias, W., & Saloma, C. (2002). Fluorescence spectrum estimation using multiple color images and minimum negativity constraint. *Optics Express*, *10*(25), 1458-1464.

Due in 2 weeks (Mar 30, 2022)