Master of Technology

U2/6: Computational Intelligence I

To clarify slide 43 and 44

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An easy optimization case

Consider the following constraint optimization problem to minimize

$$\min_{w} \left(\frac{1}{2} \|\mathbf{w}\|^2 \right), \quad \text{subject to} \quad g_i(w) = 0, i = 1, 2, \dots, l.$$
 (1)

We can apply the method of Lagrange multipliers to solve it. In this method, we define the Lagrangian to be

$$L(w,\alpha) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^{l} \alpha_i g_i(w).$$
 (2)

Here, the α_i are called the *Lagrange multipliers*. We would then find and set partial derivatives of L to be zero as

$$\frac{\partial L}{\partial w} = 0, \frac{\partial L}{\partial \alpha_i} = 0. \tag{3}$$

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A difficult optimization case

Consider the following constraint optimization problem to minimize

$$\min_{w} \left(\frac{1}{2} \|\mathbf{w}\|^2 \right), \quad \text{subject to} \quad (g_i(w) \ge 0, i = 1, 2, \dots, l.$$
 (4)

Again, we define the Lagrangian to be below with all $\alpha_i \geq 0$

$$L(w,\alpha) = \frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^{l} \alpha_i g_i(w).$$
 (5)

We further define a function θ_p (p stands for primal)

$$\theta_p(w) = \max_{\alpha_i \ge 0} L(w, \alpha) = \max_{\alpha_i \ge 0} \left(\frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^l \alpha_i g_i(w) \right). \tag{6}$$

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Detailed study of (6)

If the constraints are satisfied for a particular value of w, that means,
g_i(w) ≥ 0, then

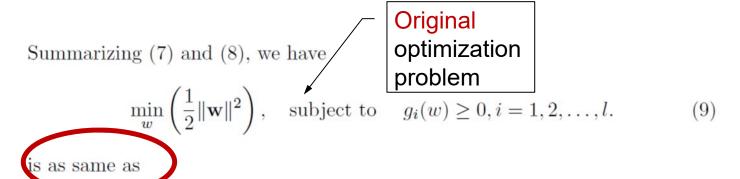
$$\theta_p(w) = \max_{\alpha_i \ge 0} L(w, \alpha) = \max_{\alpha_i \ge 0} \left(\frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^l \alpha_i \overbrace{g_i(w)}^{\text{positive}} \right) = \frac{1}{2} \|\mathbf{w}\|^2. \quad (7)$$

If the constraints are NOT satisfied for a particular value of w, that means,
g_i(w) < 0, then

$$\theta_p(w) = \max_{\alpha_i \ge 0} L(w, \alpha) = \max_{\alpha_i \ge 0} \left(\frac{1}{2} \|\mathbf{w}\|^2 - \sum_{i=1}^l \alpha_i \underbrace{g_i(w)}^{\text{negative}} \right) = \infty.$$
 (8)

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Trick 1: Lagrangian function



$$\min_{w} \theta_{p}(w) = \min_{w} \max_{\alpha_{i} \ge 0} L(w, \alpha) = \min_{w} \max_{\alpha_{i} \ge 0} \left(\frac{1}{2} \|\mathbf{w}\|^{2} - \sum_{i=1}^{l} \alpha_{i} g_{i}(w) \right).$$
(10)

Revised optimization problem

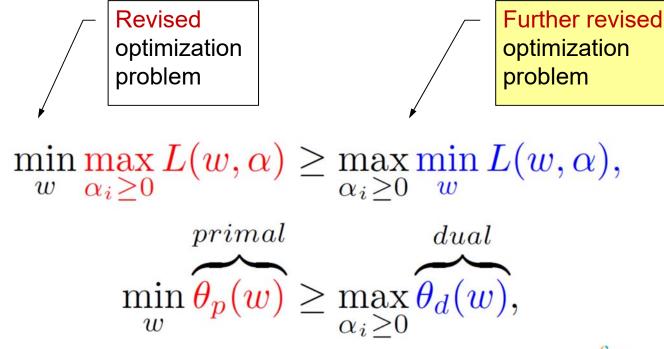
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Trick 2: Dual representation



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