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DEA as a tool for bankruptcy assessment: A comparative study with logistic regression technique

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Abstract

This paper proposes data envelopment analysis (DEA) as a quick-and-easy tool for assessing corporate bankruptcy. DEA is a non-parametric method that measures weight estimates (not parameter estimates) of a classification function for separating default and non-default firms. Using a recent sample of large corporate failures in the United States, we examine the capability of DEA in assessing corporate bankruptcy by comparing it with logistic regression (LR). We find that DEA outperforms LR in evaluating bankruptcy out-of-sample. This feature of DEA is appealing and has practical relevance for investors. Another advantage of DEA over LR is that it does not have assumptions associated with statistical and econometric methods. Furthermore, DEA does not need a large sample size for bankruptcy evaluation, usually required by such statistical and econometric approaches. The need for such a large sample size is a significant disadvantage to practitioners when investment decisions are made using small samples. DEA can bypass such a difficulty related to a sample size. Thus, DEA is a practically appealing method for bankruptcy assessment.

Keywords: Bankruptcy; Data envelopment analysis; Logit regression

1. Introduction

This study proposes use of data envelopment analysis (DEA) as a quick-and-easy tool for assessing corporate bankruptcy. In the past, portfolio decision makers have frequently used logistic regression (LR) (Ohlson, 1980; Keasey and Watson, 1991) for bankruptcy evaluation. Hence, we compare DEA with LR to examine the performance of the two models in terms of bankruptcy assessment. The DEA is a non-parametric approach for production-based performance analysis. In this research, we change the research direction from production-based performance analysis to bankruptcy assessment. Thus, this study documents a new type of DEA application for bankruptcy evaluation.[1]

Bankruptcy assessment is important because firm failure imposes significant direct and indirect costs on a firm's stakeholders. Extant evidence suggests that direct costs (such as court costs, lawyers and accountants fees) may be as low as 5% (Warner, 1977) or as high as 28% when both direct and indirect costs (such as lost sales, lost profits, higher cost of credit, inability to issue new securities and lost investment opportunities) are considered (Altman, 1984). Thus, the need for early detection of potential insolvency is very important because corporate

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[1] DEA was first proposed by Charnes et al. (1978). Cooper et al. (2006) provided complete knowledge on recent DEA developments. Gattoufi et al. (2004) listed more than 3000 previous DEA contributions.

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leaders and investors make their decisions in a world of dynamic technology development, imperfect knowledge and uncertainty.

A comprehensive survey of the literature on corporate bankruptcy assessment by Aziz and Dar (2006) rightfully points out that the literature is dominated by statistical discriminant analysis (DA) and LR (as an econometric approach). Statistical DA was first applied in bankruptcy assessment by Altman (1968). According to Collins and Green (1982), LR is as good as DA. Subsequent re-tests of the performance of Altman's model in forecasting bankruptcy on recent data reveal inadequacies in the DA model (Eisenbeis, 1977; Grice and Ingram, 2001).

The purpose of this study is to restructure the analytical feature of DEA so that it can fit within the framework of bankruptcy assessment. In this sense, the use of DEA discussed in this study is different from the conventional uses of production-based performance analysis of DEA. After discussing how the use of DEA for bankruptcy assessment is different from the previous uses of DEA, this study compares its bankruptcy evaluation capability with that of LR, which is a widely used econometric approach for corporate bankruptcy. This study also discusses the use of DEA as a quick-and-easy tool for bankruptcy assessment. Hence, it does not explore the theoretical (mathematical) aspect of DEA for bankruptcy assessment. Such a quick-and-easy use is of practical importance to investors and many individuals who are interested in corporate bankruptcy, because they often have to make quick decisions about their investments and may not have such analytical understanding to investigate corporate performance.

The remainder of this paper is organized as follows: Section 2 presents a detailed literature survey that discusses bankruptcy evaluation models. The position of this study is also discussed in this section. Section 3 describes a data set obtained from the well-known research prepared by Altman who is a pioneer of bankruptcy evaluation. This section also prepares a description of both DEA and LR. The results from the comparative analysis of the two approaches are summarized in Section 4. Section 5 concludes this study and discusses future research extensions.

2. Literature review

The extensive research on bankruptcy evaluation can be broadly grouped into three categories. First, there is research which centers on a particular model. The studies in this group test how a particular model performs in assessing corporate failure, vis-à-vis another model (Lo, 1986). Second, there is research that focuses on the selection of an appropriate set of variables on which we implement a particular model. While this may seem trivial, sample and variable selection is an important part of the bankruptcy prediction process (Keasey and Watson, 1991). Finally, there is a large body of research that deals with the bankruptcy evaluation process. The focus of this research is on the first of these three groups, i.e. testing the efficacy of alternative assessment models. Consequently, much of the literature reviewed in this paper concentrates on the history and development of bankruptcy assessment models.

Altman's (1968) seminal work introduced the first bankruptcy evaluation model. Altman used statistical DA (i.e., Fisher and Smith: See Fisher, 1936; Smith and Winakor, 1935) to discriminate between bankrupt and non-bankrupt firms. The statistical DA uses a linear combination of independent variables to assign a score (often referred to as "Z-score") to a particular firm. This score is then used to discriminate between bankrupt and non-bankrupt firms through the use of a cut-off point. While this model produced adequate results within sample, its ability to forecast out-of-sample was poor. Grice and Ingram (2001) re-tested Altman's (1968) model on a more recent sample and found that its ability to correctly classify bankrupt companies fell from 83.5% to 57.8%.

Eisenbeis (1977) outlined many statistical problems of statistical DA in financial or econometric applications. Indeed, the incorporation of *a priori* probabilities for the respective groups (bankrupt and non-bankrupt) is another important factor. One possibility, according to Eisenbeis (1977), is to assume that there will be an equally likely occurrence of both groups in a particular sample. For matched pair sampling, statistical DA may be adequate, but practitioners must be wary when implementing statistical DA on a large random sample of firms since it is unlikely that the population of potential failed firms will be 50% of a given random sample (this problem also exists with LR). It is thus difficult to satisfy all the assumptions of statistical DA. Since then several authors have made significant contributions to the literature by attempting to use other types of DA or similar techniques in classifying firms into different groups. For example, Freed and Glover's (1981, 1982, 1986) pioneering work showed how a DA problem could be formulated by a mathematical programming model and could be solved by a LP algorithm. In other later work, Retzlaff-Roberts and Puelz (1996) established the relationship between DA and DEA.

Ohlson (1980), Mensah (1983), Zavgren (1985) and Keasey and Watson (1987) used logistic or probit regression, which gave a conditional probability of an observation belonging to a particular category. Both of these regression techniques (unlike DA) do not require the independent variables to be jointly multivariate normal and do not require prior probabilities. Press and Wilson (1978) obtained superior predictive ability with LR on a holdout sample as opposed to DA in a study on breast cancer patients. In addition, Collins and Green (1982) determined that LR, while only being superior to DA at overall prediction, was far superior in determining potential failed firms in a bankruptcy assessment context.

LR requires the estimation of model parameters using an *ex-post* sample (the estimation sample) of failed and non-failed firms. By extrapolating the estimated model, one can then attempt to classify failed and non-failed firms from an *ex-ante* standpoint. In a number of previous studies on bankruptcy evaluation, the estimated logistic models provide accurate classification within-sample (Platt and Platt, 1990). However, the evaluation ability of many of these models is significantly poor when implemented on a holdout sample. Given that the purpose of any evaluation model is to classify *ex-ante* firms that will fail or not fail, clearly the real test of any model is its evaluation ability out-of-sample (Johnsen and Melicher, 1994).

While statistical DA and probit/logit dominate the literature (Aziz and Dar, 2006), these are not the only methods used for bankruptcy assessment. More recent methods include neural networks (Platt et al., 1999), CUSUM methodology (Kahya and Theodossiou, 1999), options-based approaches (Charitou and Trigeorgis, 2000) and the chaos approach (Lindsay and Campbell, 1996).

2.1. Position of this research

The proposed use of DEA has methodological strengths and drawbacks. The comparison between DEA and statistical/econometric approaches characterises unique features of DEA as follows:

Non-parametric: First, DEA measures the weight estimates of a functional form for classifying observations into two groups. Each weight estimate is measured between zero and one. Thus, the weight estimate can be expressed as a percentile and it determines the relative importance of individual variables unlike the classical linear regression models. Hence, DEA is a non-parametric approach. In contrast, statistical DA and econometric methods belong to a parametric approach because they measure the parameter estimates of a functional form. A parameter estimate can take any real number in a positive or negative sign.

Distribution-free: Second, the statistical DA assumes that variables used for group classification are multivariate normally distributed. In finance, most of the variables used for statistical DA, such as firm size, loan size, and return on equity (ROE), tend to be highly skewed with an occurrence of a few extreme values. Such an occurrence makes the variables to be non-normally distributed. Violations of the assumption may bias the tests of significance and estimated error rates. In contrast, DEA is distribution-free because it does not need to specify the distribution of variables. Moreover, the computational process of DEA classifies all observations as efficient or inefficient. All efficient observations consist of an efficiency frontier. All the inefficient observations exist below the efficiency frontier. The DEA-based efficiency evaluation does not assume any distribution in relation to the degree to which each observation locates from the efficiency frontier. In this sense, this study considers that DEA is also distribution-free. Thus, the concept of "distribution-free" is different from the concept of "non-parametric" in DEA.

Third, statistical DA assumes that the group dispersion (variance–covariance) matrices are equal across all groups being investigated. Violation of this assumption affects the significance tests for the differences in group means and related significance tests. DEA does not need such a specification. However, the strength of DEA implies a drawback, as well. DEA does not have statistical tests used in statistical DA and econometric approaches. DEA researchers need to investigate an asymptotic property of the group dispersion matrix. The popularity of the statistical DA and econometric approaches is because they allow us to easily access various statistical tests. In this sense, the conventional approaches are better than DEA.

Fourth, the econometric approach requires the estimation of model parameters using an ex post sample consisting of failed and non-failed firms and then extrapolating the estimated model for predicting bankruptcies in a holdout sample. If the estimation sample is deficient or biased (unrepresentative of the true population of failed and non-failed firms), then any model estimated on this sample would be deficient or biased (Keasey and Watson, 1991). Hence, the prediction would be biased. Deficiencies such as these in the econometric model could be avoided by using DEA which circumvents the need for an estimation sample.

Finally, statistical DA-based classification rules incorporate *a priori* probabilities to account for the relative occurrence of observations in different populations and the costs of misclassification errors. There are practical difficulties in incorporating them into the rules. Therefore, we assume that the group membership is equally likely. DEA does not need such a specification on prior decision rules.

Time horizon: A major drawback of DEA proposed in this study is that it does not have a bankruptcy prediction capability, as found in mathematical models such as a rating-migration model (Altman and Rijken, 2004), a bankruptcy hazard rate model (Chava and Jarrow, 2004) and a Taylor's expansion model (Laitinen and Laitinen, 2000). Those models can estimate the bankruptcy prediction on long-term and short-term time horizons. The proposed bankruptcy assessment by DEA does not have such a capability because no DEA model can handle simultaneously both a negative value in a data set and a frontier shift over a time horizon. As an important future research extension, we need to explore the dynamic change of bankruptcy on a time horizon.[2]

[2] The CCR ratio form ("CCR" was named after Charnes et al., 1978) can measure a frontier shift among multiple periods under the assumption of constant Returns to Scale (RTS), but the model cannot deal with a negative value in a data set. Meanwhile, Range-Adjusted Measure (RAM: Cooper et al., 1999), which is an extended model of the additive model, has the property of translation invariance to a data shift under the assumption of variable RTS. Hence, it can handle the negative value in a data set. However, the DEA model cannot measure a dynamic shift of an efficiency frontier among multiple periods. Thus, no DEA model, currently available, can simultaneously handle both a negative value in a data set and a frontier shift over a time horizon. In addition, we know some previous studies that have explored a dynamic change of DEA performance in a time horizon. For instance, Sueyoshi and Sekitani (2005) explored a dynamic change of DEA in a time horizon. All the previous studies on dynamic DEA assumed that all inputs and outputs were positive. Hence, the studies on dynamic DEA do not fit within the nature of bankruptcy assessment because we must deal with a negative value.

Hereafter, this study focuses upon LR as a methodological alternative to DEA. As mentioned previously, DEA is a non-parametric approach and therefore the statistical requirements imposed on the distributional properties of the variables for group classification do not need to be satisfied. On the other hand, LR is user friendly and does not need to satisfy rigid statistical conditions (Ohlson, 1980) as found in statistical DA. Furthermore, LR performs at least as well as DA (Collins and Green, 1982). Unlike DA, LR does not require independent variables to be jointly multivariate normal and does not require prior probabilities either (Ohlson, 1980). As a result, LR (not statistical DA) has recently become the most popular tool for bankruptcy prediction among researchers and practitioners.

3. Methodology (DEA: Additive model)

This section contains a detailed description of DEA, LR and a data set used in this study together with a description of variable selection. The two models are then examined for their abilities to assess bankruptcy in Section 4.

3.1. DEA model used for bankruptcy assessment

DEA is a methodology based on linear programming, which evaluates the relative efficiencies of Decision Making Units (DMUs) with weights for inputs and outputs. Various radial and non-radial DEA models (Cooper et al., 2006) have been proposed in past literature.

Apart from the base models, DEA has some significant new developments which are worthwhile mentioning here. For example, Lozano and Villa (2006) introduced a new DEA model which could explicitly take into account integer inputs and outputs instead of the alternative approach where the real-valued results from the DEA model are rounded off. Appa and Williams (2006) proposed a new formula to obtain the efficiency values of primal and dual optimal solutions by substituting inputs and outputs of each DMU into the formula. This approach eliminates the hassle of running the DEA model for each and every DMU in the sample. Asmild et al. (2006) proposed methods for evaluating non-marginal tradeoffs between variables used in a DEA model. These methods are capable of handling scalar as well as additive changes. In addition to these contributions, Cooper et al. (2007) explored the property of input and output aggregations because all DEA models suffer from the difficulty in the input and output aggregations. See Sueyoshi and Sekitani (2007a,b).

In this paper, we do not discuss such production-based performance analyses found in recent DEA contributions. Rather, this study documents how to use the additive model to discriminate healthy firms from those that are more liable for bankruptcy. Thus, the proposed use of the DEA model is very different from the previous DEA studies directed toward production analysis.

The additive model (Charnes et al., 1985) evaluates the relative efficiency of the specific oth firm as follows:

$$
\begin{aligned}
\text{Max} \quad & es^- + es^+ \\
\text{subject to:} \quad & X\lambda + s^- = x_o \\
& Y\lambda - s^+ = y_o \\
& e\lambda = 1 \\
& \lambda \geqslant 0, \quad s^- \geqslant 0, \quad s^+ \geqslant 0.
\end{aligned}
\tag{1}
$$

Here, n is the number of DMUs, k is the number of inputs, m is the number of outputs, $X = (x_j) \in R^{k\times n}$ is a $k \times n$ matrix of inputs, $Y = (y_j) \in R^{m\times n}$ is a $m \times n$ matrix of outputs, e is a row vector with all elements equal to 1, s^- is a vector of input slacks, s^+ is a vector of output slacks, x_o is the column vector of inputs of the oth DMU, y_o is column vector of outputs of the oth DMU, and $\lambda \in R^n$ is an intensity variable vector connecting inputs and outputs.

By using the input and output variables (see Section 3 for details) related to each firm (DMU), we solve the additive model (1) for each firm by changing x_o and y_o into the corresponding input and output vectors of the firm whose efficiency value is calculated. The optimal value of the objective function corresponding to each firm identifies the healthy and financially distressed firm by examining whether there is at least positive slack on optimality of (1).

The following four comments are useful for understanding the use of (1) in terms of bankruptcy assessment: First, in the past, researchers have used DEA for bankruptcy evaluation. For example, Cielen et al. (2004) applied DEA for bankruptcy assessment and compared DEA with mathematical programming-based DA methods (Freed and Glover, 1981, 1982, 1986). In bankruptcy assessment, their use of DEA has a major problem. They used the CCR ratio form that cannot deal with negative values usually found in financial factors. Their use of DEA is very limited in bankruptcy assessment, because some of the financial ratios that we deal with in bankruptcy assessment models have negative values. In contrast, this research selects the additive model from many radial and non-radial DEA models because the additive model has the property of translation invariance.[3] This property allows negative values in inputs and outputs. This is a major extension of this study over Cielen et al. (2004).

Second, in addition to Cielen et al. (2004), we acknowledge the existence of other important research efforts related to this study. For example, Kao and Liu (2004) documented an application of DEA for bankruptcy

[3] Ali and Seiford (1990, p.404) have proved that the additive model is translation invariant to a data shift so that "it does not alter the efficient frontier, and the classification of firms as efficient or inefficient (the objective function value) is invariant to translations of data." However, they did not discuss whether the additive model is translation invariant in terms of technical efficiency. The additive model is translation-variant in the measurement of technical efficiency. We are not interested in the measurement of technical efficiency in this study, but whether a firm exists on a frontier for bankruptcy assessment. Hence, the property of translation invariance is effective in this study.

evaluation. Cheng et al. (2006) and Ravi et al. (2008) documented recent developments on bankruptcy assessment. Ravikumar and Ravi (2007) have provided the most recent review on bankruptcy evaluation.

Third, the additive model used in this paper focuses on the estimation of Pareto–Koopman's efficient empirical production functions. The selection of input–output variables in the DEA model is guided by expert opinion, past experience and economic theory. The selection is identical with the variable selection in a regression model. Moreover, the efficiency of a specific DMU is determined by examining slacks only. This unique feature of the additive model is different from the radial models, such as the ratio form (used in Cielen et al., 2004), because the radial models (CCR and BCC: Banker et al., 1984) need to examine both a DEA efficiency score and slacks. Moreover, an efficiency score measured by the ratio form used in Cielen et al. (2004) depends upon input-based or output-based measurement. The input-based efficiency score is different from that of the output-based one. In contrast, the additive model incorporates both input and output slacks in the efficiency measurement so that it can avoid the problem associated with the ratio form. Consequently, the determination of efficiency status is more easily determined by the additive model than the radial model.

Finally, it is important to mention that Sueyoshi (2004, 2006) has incorporated the non-parametric feature of DEA into DA. This combined approach is referred to as "DEA-DA (DEA-discriminant analysis)" and is a non-parametric and distribution-free approach. The DEA-DA can solve various DA problems using a computer intensive algorithm on a modern personal computer. It is necessary to compare the DEA-DA with the proposed additive model in terms of bankruptcy assessment. Such a research task is an important future extension of this study.

3.2. Input and output variable selection

In the context of bankruptcy assessment, the smaller (inferior) values in the financial ratios, which could possibly cause financial distress, are considered to be input variables. In contrast, the larger (superior) values in those ratios, which could cause financial distress, are classified as output variables. The classification indicates that default firms tend to be a value equal to zero or close to zero for the objective function of the additive model (1) and non-default firms tend to be values greater than zero. Consequently, this study expects that default firms consist of a frontier and non-default firms are inside the frontier.

A difficulty associated with the proposed selection of inputs and outputs is that we do not have a clear threshold to determine inputs and outputs. The selection is usually determined by a DEA user. Consequently, different DEA users may select different combinations of inputs and outputs. That is a shortcoming of DEA. Hence, this study provides a guideline regarding the selection of inputs and outputs, as mentioned above.

3.3. Bankruptcy frontier and computation of probabilities

3.3.1. Bankruptcy frontier

This analytical feature due to the selection of inputs and outputs is different from the conventional use of DEA. In the conventional DEA-based production analysis, productive performers consist of an efficiency frontier and insufficient performers exist within a production possibility set shaped by the efficiency frontier. In contrast, this study takes an approach that is opposite to the conventional production analysis because we are interested in bankruptcy assessment. The frontier used in this study is a "bankruptcy frontier" (not an efficiency frontier found in the conventional use of DEA-based production analysis) which contains many default firms (poor performers). Non-default (healthy) firms are expected to exist inside a "bankruptcy possibility set", which is shaped by the bankruptcy frontier. The mathematical definition of the bankruptcy possibility set is the same as the production possibility set in production economics. However, the nature of outputs in bankruptcy evaluation is opposite to that of production analysis. That is, larger is better in production analysis. In contrast, smaller is better in the bankruptcy evaluation. In addition, an opposite description can be applied to the use of inputs.

Fig. 1 depicts the bankruptcy frontier and the bankruptcy possibility set located below the bankruptcy frontier. The figure has a single input (x) and a single output (y) for descriptive convenience. The symbol (○) indicates the performance of a default firm and the symbol ($\times$) indicates the performance of a non-default firm in the $x-y$ coordinates.

3.3.2. Computation for default and non-default probabilities

A benefit of the selection of inputs and outputs proposed in this study is that we have two groups (default and non-default) of firms and another two groups (firms on the bankruptcy frontier and firms not on the frontier).

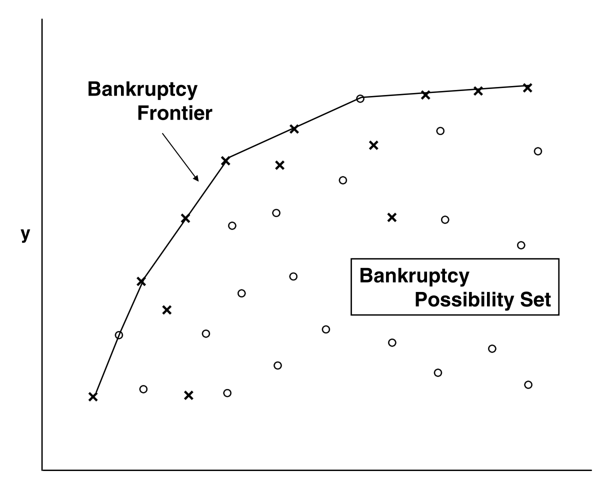

Fig. 1. Bankruptcy frontier and bankruptcy possibility set. The symbol (○) indicates a non-default firm and the symbol ($\times$) indicates a default firm.

Consequently, after applying the additive model (1) to an observed data set, we can have four (2×2) groups of firms. Based upon these four groups, we can evaluate the default/non-default status. This type of bankruptcy assessment capability cannot be found in both the previous DEA works and the previous business and economic research on bankruptcy assessment. Moreover, the probability computation is quick and easy. Hence, the proposed use of DEA is methodologically more practical in bankruptcy assessment.

To explain the bankruptcy assessment capability, this study returns to Fig. 1 and discusses the capability in the following manner:

Step 1: Apply the additive model (1) to a firm that is sampled from an observed data set (in relation to default and non-default firms).

Step 2: Classify the firm based upon whether it has zero in all slacks (S^+ and S^-) on optimality of (1). If all slacks are zero, then the firm is on the bankruptcy frontier. Otherwise (at lease one slack is positive), the firm is not on the bankruptcy frontier.

Step 3: Confirm whether all the firms are examined by (1). If yes, go to Step 4. Otherwise, go to Step 1.

Step 4: Classify all of the firms into the following four groups: (a) default firms on the bankruptcy frontier, (b) default firms not on the bankruptcy frontier, (c) non-default firms not on the bankruptcy frontier and (d) non-default firms on the bankruptcy frontier.

Step 5: Compute the numbers of firms belonging to the four groups. Then, compute the following probabilities:
(a) $P(\mathrm{BR}/\mathrm{BR})$ = the number of default firms on the bankruptcy frontier divided by the total number of default firms. (b) $P(\mathrm{NBR}/\mathrm{BR})$ = the number of default firms not on the bankruptcy frontier divided by the total number of default firms. (c) $P(\mathrm{NBR}/\mathrm{NBR})$ = the number of non-default firms not on the bankruptcy frontier divided by the total number of non-default firms. (d) $P(\mathrm{BR}/\mathrm{NBR})$ = the number of non-default firms on the bankruptcy frontier divided by the total number of non-default firms.

Step 6: Stop.

The misclassification rate is determined by $P(\mathrm{NBR}/\mathrm{BR})$ plus $P(\mathrm{BR}/\mathrm{NBR})$, while the correct classification rate is determined by P (BR/BR) plus $P(\mathrm{NBR}/\mathrm{NBR})$. This study considers that if a firm is on the bankruptcy frontier, the firm belongs to the status of bankruptcy.[4]

[4] According to Altman (1968, p.599), $P(\mathrm{NBR}/\mathrm{BR})$ is Type I error and $P(\mathrm{BR}/\mathrm{NBR})$ is Type II error.

3.3.3. Training and validation samples along with tests

Fig. 2 visually describes the conventional rationale relating to how to examine the methodological validity of a classification method. As depicted in Fig. 2, an observed sample set is usually classified into a training sample and a validation sample. The validation sample implies a group of hold-out observations, while the training sample is the remaining observations. The training sample is used to obtain parameter (weight) estimates and compute the probabilities related to bankruptcy. The hit rate (correct classification) is used as a criterion in this stage. The DEA bankruptcy assessment is within the scope of a training sample only. The proposed DEA approach cannot be extended to the examination of a validation sample. Moreover, if we include an observation (belonging to the validation sample) in the training sample, then the observation may change the location of the bankruptcy frontier in the input–output data space. Consequently, the proposed DEA approach needs to repeat the whole computation process to obtain the four probabilities relating to bankruptcy. Hence, the proposed DEA approach does not have a learning process that constantly updates a database for the training sample. In contrast, LR can identify the group membership of an observation in the validation sample. In a similar manner, the LR can predict the bankruptcy probability of a newly sampled observation. In this sense, the proposed use of DEA is for "bankruptcy assessment," not for "bankruptcy prediction."

Admitting that such a drawback is related to DEA, we return to the purpose of this study. As mentioned at the beginning of the introduction, the purpose of this study is to develop a quick-and-easy tool for predicting corporate bankruptcy for corporate leaders and managers. They usually do not have the time and analytical skill to examine the bankruptcy assessment of a target company. In this case, they may use the four bankruptcy probabilities measured by DEA as an initial assessment for bankruptcy because the proposed approach has computational tractability and efficiency. It is managerially (not mathematically) acceptable for them to predict the bankruptcy of a target company based upon the four DEA-based bankruptcy probabilities after considering the business similarities between the target firm and other firms used in the training sample for DEA computation. Thus, the bankruptcy assessment may fit for the purpose of this article.

3.4. Output variables

We have used nine financial variables (two outputs and seven inputs) that proxy for the financial strength/weakness and potential insolvency of firms. These financial ratios have commonly been used in past bankruptcy literature and are considered to be the most efficient ones (Altman, 1968) and some of the variables are common with those used by Cielen et al. (2004). This sub-section and

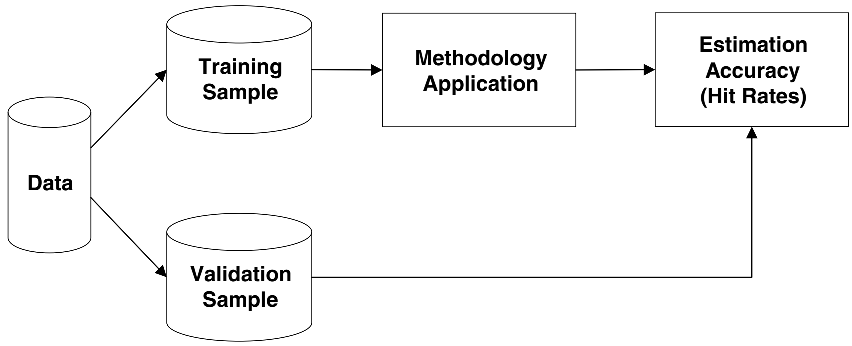

Fig. 2. Tests for methodological validation: (a) An observed sample set is classified into a training sample and a validation sample. The validation sample implies a group of hold-out observations, while the training sample is the remaining observations and (b) The hit rate is measured by the number of correct classified observations divided by the total number of observations.

Section 3.5 give a detailed description of these variables and explain how they contribute to the assessment of financial distress or bankruptcy in the worst case.

3.4.1. Total debt/total assets (TDTA)

This variable is used as a leverage measure that indicates a long-term financial obligation. An increase in leverage will increase the probability of financial distress. With high leverage there might be some future states of the world where the cash flow is insufficient to service debt, resulting in bankruptcy. Larger values for TDTA would result in a value of zero for the objective function of the additive model for firms which are likely to go bankrupt and hence appear in the bankruptcy frontier. On the other hand, firms with small TDTA are less likely to go bankrupt and would result in a positive value for the objective function of the additive model.

3.4.2. Current liabilities/total assets (CLTA)

A firm with a high CLTA ratio would have difficulty in meeting short-term debt obligations. This indicates a lack of cash flow to fund firm operations. A firm in this situation would have difficulty in running its day-to-day operations because of a reduced flexibility in obtaining working capital. Larger values for CLTA would result in a value of zero for the objective function of the additive model. Hence, firms with high CLTA may appear on the bankruptcy frontier. On the other hand, firms with small CLTA are less likely to go bankrupt and would result in a positive value for the objective function.

3.5. Input variables

We define the following input variables in such a way that smaller values for these variables (i.e. firms more likely to go bankrupt) would result in a value of zero for the objective function of the additive model and hence appear on the bankruptcy frontier. Consequently, we use financial ratios that proxy for the financial strength and hence solvency of firms. Firms with poor liquidity measures such as cash flows, net income, working capital, operating earnings and interest coverage are expected to be financially more vulnerable to insolvency and hence more likely to go bankrupt. The following measures of liquidity are used in the analysis.

- CFTA = cash flow/total assets.
- NITA = net income/total assets.
- WCTA = working capital/total assets.
- CATA = current assets/total assets.
- EBTA = earnings before interest and taxes/total assets.
- EBIE = earnings before interest and taxes/interest expense.

We also use a market-based growth measure as an input variable to capture market participants' collective assessment of a firm's financial strength. The interpretation of the impact of this variable on the DEA model's predictions follows the description for other input variables. The following growth measure is used:

- MVCE = market value of equity/book value of common equity.

Summary statistics of all the input and output variables, computed separately for bankrupt and non-bankrupt match firms, are presented in Table 1.

Since most variables seem skewed, we use the median for the purpose of comparison. Results from a two-way Wilcoxon rank-sum test indicate that the median values for the bankrupt and non-bankrupt firms are significantly different for all variables except CATA. These results suggest that the variables used in the analysis are appropriate in classifying the firms as bankrupt or non-bankrupt.

3.5.1. Comment on the selection of CATA

We need to comment on the selection of variables in Table 1. For example, the medians of bankrupt and non-bankrupt groups are not significantly different in the CATA, as shown in Table 1. Regardless of whether a particular variable is significant or insignificant in the univari-

Table 1
Summary financial statistics (1991–2004)

Sample selection	Statistic	Input variables							Output variables	
		CFTA	NITA	WCTA	CATA	EBTA	EBIE	MVCE	TDTA	CLTA
Non-bankrupt firms	Mean	−0.040	−0.120	0.170	0.410	−0.040	−24.060	1.900	0.320	0.250
	Median	0.056	0.003	0.153	0.391	0.370	1.188	1.497	0.262	0.202
	Standard deviation	0.370	0.440	0.260	0.240	0.260	396.040	16.630	0.280	0.200
	Skewness	−4.790	−4.810	−1.060	0.270	−3.910	−15.100	−12.060	1.390	3.310
Bankrupt firms	Mean	−0.270	−0.410	−0.230	0.400	−0.140	−4.490	0.450	0.710	0.630
	Median	−0.106	−0.180	−0.008	0.379	−0.052	−0.726	0.281	0.586	0.392
	Standard deviation	0.530	0.760	0.640	0.200	0.370	27.480	5.540	0.520	0.630
	Skewness	−3.210	−4.900	−2.300	0.250	−5.420	−8.750	−6.110	1.930	2.330
Two sample Wilcoxon rank – sum test between bankrupt and non-bankrupt medians (Z)		8.298	8.129	7.563	0.101	5.828	4.629	8.212	−9.157	−8.855
p-value > (Z)		0.000	0.000	0.000	0.920	0.000	0.000	0.000	0.000	0.000

ate analysis of Table 1, it may still appear significant or insignificant in the logit model. On the other hand, we do not know the exact contribution of a particular variable in DEA. Therefore, we retain CATA, since current assets constitute that a portion of the total assets can readily be used to measure a company's ability to meet its short-term debt obligations. Hence, CATA is a measure of a firm's liquidity and an important determinant of a firm's solvency. The purpose of this analysis is to investigate whether, overall, the selected variables are capable of distinguishing between the two groups. The individual difference between the two groups is not important in this type of study.

3.6. Logistic regression model

The logistic regression model is the most widely-used technique among practitioners to predict bankruptcy. As demonstrated in the study of Collins and Green (1982) who studied default and non-default credit unions, LR performs at least as well as DA while using fewer statistical assumptions. To ensure a proper comparison, both DEA and LR (see Eq. (2)) are formulated using the same set of variables as defined above. The bankruptcy probability is computed by the following equation:

$$P_j = \frac{1}{1+e^{-Z_i}} = E\left(Y_i \middle| \begin{array}{l} \mathrm{TDTA}_i, \mathrm{CLTA}_i, \mathrm{CFTA}_i, \mathrm{NITA}_i, \mathrm{WCTA}_i, \\ \mathrm{CATA}_i, \mathrm{EBTA}_i, \mathrm{EBIE}_i \ \&\ \mathrm{MVCE}_i \end{array} \right). \tag{2}$$

Here, Y_i is 1 if the firm is bankrupt and 0 otherwise,

$$\begin{aligned} Z_i = \beta_1 + \beta_2\mathrm{TDTA} + \beta_3\mathrm{CLTA} + \beta_4\mathrm{CFTA} + \beta_5\mathrm{NITA} \\ + \beta_6\mathrm{WCTA} + \beta_7\mathrm{CATA} + \beta_8\mathrm{EBTA} + \beta_9\mathrm{EBIE} \\ + \beta_{10}\mathrm{MVCE} \end{aligned}$$

and P_j is the probability that a firm will go bankrupt.

When specifying variables for (2), we need to pay attention to the fact that DEA classifies all variables into either outputs or inputs. For example, TDTA and CLTA are treated as outputs in DEA. Meanwhile, the two variables are used as independent variables (not outputs) in (2) because the logistic regression does not need such a separation between inputs and outputs. The independent variable in the logistic regression does not imply that the variable is an input or an output in DEA.

3.7. Data

The primary data set on large US bankruptcies was obtained from Altman's bankruptcy database maintained at New York University. The original file contained 609 firms of which 130 had complete financial and stock returns data for the nine variables used in this study. This sample contained bankruptcies over the period 1991–2004 and represented the full spectrum of industries. A random sample of 100 firms was drawn from these 130 for analysis. We obtained input and output variables used in the two models from data retrieved from Standard and Poor's Compustat and the Centre for Research in Security Prices (CRSP) databases. Since the models are used to predict bankruptcy, all variables are computed at the end of the fiscal year immediately preceding the year of bankruptcy.

Samples used for analysis contained bankrupt and matching non-default firms. Every default firm was matched with a healthy non-default firm belonging to the same 3-digit Standard Industrial Classification (SIC) code and with a market value within 20% of the market value of the bankrupt firm. Inclusion of matched pairs reduces the possibility of a model discriminating against firms with a different size or industry code as opposed to discriminating against bankrupt and non-default firms. This could occur since smaller firms are more risky and thus more likely to experience financial distress. Both DEA and LR were tested on several samples by varying the ratio of bankrupt to non-default match firms in the range from 0.055 to 1. However, in the case of LR, predictions were made both within- sample as well as out-of-sample.

4. Results and discussion

In this section, we compare the two approaches (DEA and LR) numerically in order to assess the prediction capability of the two models in relation to bankruptcy. Table 2 contains results of our first experiment where the capability of DEA in evaluating bankruptcy is tested by using 50 different samples, each containing one bankrupt and several matching non-default firms.

A sample of 910 non-default matching firms, in the range of ±20% of the market value of the bankrupt firm, was used to construct the samples. The ratio of default to non-default firms in each sample ranged from 0.025 to 0.1. Sample sizes ranged from 11 to 41. This experiment enabled us to assess the strength of DEA in identifying a single bankrupt firm from among several firms in a sample.

As Table 2 shows, 43 of 50 bankrupt firms appeared in the bankruptcy frontier, whereas 294 of 910 non-bankrupt firms appeared in the bankruptcy frontier. Based upon the DEA result, we estimate the conditional probabilities to assess an occurrence of bankruptcy as follows: (a) $P(\mathrm{BR}/\mathrm{BR}) = 86\%$ (=43/50), (b) $P(\mathrm{NBR}/\mathrm{BR}) = 14\%$ (=7/50), (c) $P(\mathrm{BR}/\mathrm{NBR}) = 32.31\%$ (=294/910) and (d) $P(\mathrm{NBR}/\mathrm{NBR}) = 67.69\%$ (=616/910). Here, BR and NBR indicate bankruptcy and non-bankruptcy, respectively.

Results of this experiment are of immense importance to portfolio decision makers who are considering the purchase/sale of financial assets. For example, the recent bankruptcy of Delta Airlines in the face of record fuel costs was preceded by speculation in the financial media about the ability of airlines to withstand the high operating costs in a highly competitive market place with no flexibility to pass additional costs onto the consumer. Application of the proposed DEA model to all major airlines in the United States would have shed light on which firms were more vulnerable to bankruptcy and hence aided in portfolio decision making. LR, on the other hand, would need an estimation sample over a prior period to estimate the model parameters. This clearly indicates that LR would not helpful in the case.

We next extended the previous experiment by including more than one bankrupt firm in each sample to analyse the robustness of the additive model. In this robustness check, four samples, each containing nine bankrupt firms but a different (decreasing) number of non-bankrupt matching firms, were drawn from the pool of matching firms so that the BR/NBR ratio varied from 0.25 to 1.00. Sample sizes ranged from 18 to 48. Furthermore, to test the impact of the size constraint, a fifth sample containing 50 bankrupt and 1533 non-bankrupt firms (BR/NBR ratio = 0.033) was drawn where the non-bankrupt firms were randomly drawn from the same industry without imposing the ±20% size constraint. Results using the additive model on these five samples are presented in Table 3.

Table 3 reveals that the probability of correctly identifying default firms remains constant (88.89%) for the first four samples. This shows that the default firms continue to remain in the bankruptcy frontier even when the number of non-bankrupt firms was gradually increased in the sample. In other words, the DEA model performs remarkably well in identifying bankrupt firms. The probability, or $P(\mathrm{NBR}/\mathrm{NBR})$, gradually decreases (except in the fourth sample) while $P(\mathrm{BR}/\mathrm{NBR})$ gradually decreases, implying that some of the newly added non-default firms are wrongly classified by the DEA model as bankrupt firms. This result shows that DEA is much more powerful in correctly evaluating default firms than non-default firms. Table 3 also reveals that the probability of overall correct predictions (i.e. default firms as bankrupt and non-default firms as non-bankrupt) of the model is 74– 86%. The DEA model's percentage of overall correct evaluations rises to its maximum (86%) when the BR/NBR ratio in the sample is 1. When the BR/NBR ratio decreases, the percentage of overall correct evaluations also gradually decreases. Even when the BR/NBR in the sample is extremely low (0.033) and the sample size is extremely large, DEA's overall correct evaluations remain at 77%.

We next turn our attention to comparing the results obtained using the DEA model with those obtained using the LR technique. For the purpose of comparison, samples are drawn so that the ratio of BR/NBR varies from 0.053 to 1 across the samples. These samples (columns 2–6) were used for predictions within-sample. In addition, a sixth sample (last column in Table 4) is drawn to test the accuracy of the LR model in evaluating out-of-sample. A cut-off point of 0.5 is used to classify default and non-default firms.

Parameters of the LR model are estimated using the first five samples and the estimated model are used to assess bankruptcies within the same sample (i.e. within-sample prediction). Table 4 shows that the probabilities of overall correct evaluations are very high (ranging from 84% to 95%), but such an outcome may not be very appealing to the practitioner. This result, however, compares very favourably with the DEA model. The sixth column in Table 4 contains results for out-of-sample evaluation. In this case, the parameters of the LR model are estimated using a different sample of 135 firms (50 defaults and 85 non-defaults) and then the estimated model is used to assess bankruptcies in the sixth sample i.e. to make out-of-sample evaluations. In sharp contrast to the in-sample

Table 2
Summary of the DEA results for fifty large bankruptcies

	Appeared in the frontier (F)	Not appeared in the frontier (NF)	Total
No. of bankrupt firms (BR)	43	7	50
No. of non-bankrupt firms (NBR)	294	616	910
Total	337	623	960

a. This table contains results obtained with the DEA model over samples containing one bankrupt firm and several non-bankrupt match firms belonging to the same 3-digit SIC industry classification and with a market value within ±20% of the bankrupt firm.

Table 3
Bankruptcy assessments from the DEA model

	Samples with matching non-bankrupt firms				Sample with non matching non-bankrupt firms
Sample size	48	38	28	18	1583
No. of bankruptcy firms (BR)	9	9	9	9	50
Ratio: no. of bankrupt firms (BR)/ no. of non bankrupt firms (NBR)	0.25	0.33	0.50	1.00	0.033
P(BR/BR)	88.89%	88.89%	88.89%	88.89%	84%
P (NBR/BR)	11.11%	11.11%	11.11%	11.11%	16%
P(NBR/NBR)	71.79%	68.97%	68.42%	83.33%	76.58%
P(BR/NBR)	28.21%	31.03%	31.58%	16.67%	23.42%
P(overall correct predictions)	75%	74%	75%	86%	77%

a. This table contains results obtained with the DEA model over samples containing both bankrupt firms and several non-bankrupt match firms belonging to the same 3-digit SIC industry classification and with a market value with +/−20% of the bankrupt firm. The last column depicts results using a matching technique that relaxes the size constraint. The composition of the samples was altered by varying the number of bankrupt firms while keeping the number of non-bankrupt firms fixed. *P*(BR/BR) is the probability of correctly assessing a bankrupt firm as bankrupt and *P*(NBR/BR) is the complement i.e. incorrectly picking a non-bankrupt firm as bankrupt. Other probabilities have similar interpretation.

evaluations, the overall predictive ability of the LR model drops to 67% which makes it far less efficient than the DEA model.

In all the LR models used in Table 4, the global null hypothesis: H_0:BETA = 0 is rejected at the 5% level of significance using the Wald's test. This implies that the variables selected in the model do contribute to classifying the firms as bankrupt and non-bankrupt. The evaluation power of the LR models is assessed using the normal approximation (large sample case) of Kendall's Tau test and the null hypothesis $H_0{:}\gamma = 0$ is rejected at the 5% level of significance in every case.

4.1. Comparison between the two models in these performances

In the case of the LR model, the overall correct evaluations within-sample range from 84% to 95% compared to 74–86% for the DEA model. The LR model performs extremely well in the within-sample case, but these predictions are of little use to the practitioner. A practitioner may have a sample of firms that he wishes to invest in and wishes to ascertain which firms in the sample are more likely to go bankrupt in the near future. In this case, prior information on default and non-default firms is not available in the sample. Therefore, another sample (called "the estimation sample") of default and non-default firms has to be used to estimate the model. This drawback of the LR is evident from the recent string of bankruptcies in the airline industry in the face of an adverse cost structure resulting from record oil prices. In such cases, the practitioner faces several problems. First, it would be very difficult to obtain a large enough sample of past airline bankruptcies to estimate the model. Second, more importantly, even if one could collect such an estimation sample, the economic conditions are not likely to match current conditions. This renders calibration of the model of little use for the purposes of out-of-sample predictions. In such

Table 4
Bankruptcy predictions using the logistic regression

	Within sample predictions					Out of sample prediction
Size of the estimation sample	1001	48	38	75	100	135
No. of bankrupt firms (BR)	50	9	9	25	50	50
Ratio: no. of bankrupt firms (BR)/no. of non bankrupt firms (NBR)	0.053	0.25	0.33	0.5	1	0.588
P(BR/BR)	16%	44.44%	66.67%	68%	78%	64%
P(NBR/BR)	84%	55.55%	33.33%	32%	22%	36%
P(NBR/NBR)	99.47%	94.87%	89.66%	94%	92%	69.30%
P(BR/NBR)	0.53%	5.13%	10.34%	6%	8%	30.70%
P(overall correct predictions)	95%	85%	84%	85%	85%	67%
Kendall's Tau Test (Z=)	<.0001	<.0001	<.0001	<.0001	<.0001	<.0001
Global null hypothesis test BETA = 0						
Wald (p-value)	<.0001	0.0039	0.0045	<.0001	0.0098	0.0026

a. This table contains results obtained with the Logistic Regression model over samples containing both bankrupt firms and several non-bankrupt match firms belonging to the same 3-digit SIC industry classification and with a market value with +/−20% of the bankrupt firm. The composition of the samples was altered by varying the number of bankrupt firms while keeping the number of non-bankrupt firms fixed. *P*(BR/BR) is the probability of correctly predicting a bankrupt firm as bankrupt and *P*(NBR/BR) is the complement i.e. incorrectly picking a non-bankrupt firm as bankrupt. Other probabilities have similar interpretation. Columns under "within sample prediction" contain results for cases where estimation of the logit model and predictions are made using the same sample. The last column contains results for cases where predictions are made using a model calibrated using a separate mode.

situations, the DEA model outperforms and is more practically useful than the LR model, producing 74–86% overall correct predictions compared to the LR model, which has a corresponding figure of 67%.

Another interesting result that emerges from Tables 3 and 4 is that in 84–89% of cases, the DEA model performs extremely well in correctly identifying bankrupt firms compared to the LR model, with corresponding values of 16–64%. Practitioners are more interested in knowing possible bankruptcies before their investment decisions are made and therefore this result would be very appealing to them. On the other hand, the LR model performs extremely well in identifying non-bankrupt firms correctly (within-sample case) in the range of 69.3–99.47% compared to the DEA model with corresponding values ranging from 68.42% to 83.33%.

Overall, if we compare the LR model and the DEA model in out-of-sample evaluations, which is more appealing and useful to the practitioner, the DEA model outperforms with overall correct evaluations in 74–86% cases compared to 67% of cases for the LR model. Furthermore, the LR model uses choice-based sampling instead of random sampling. This means that we need to observe the dependent variable (i.e. the firm selected is bankrupt or not) and then a sample could be drawn based on this knowledge. This sampling approach violates the random sampling design assumption causing the parameter and probability estimates of the logit model to be asymptotically biased (Manski and Lerman, 1977). On the other hand, DEA is a non-parametric and distribution-free approach. The choice-based sampling does not have any effect on the results in this case, as this approach circumvents the need for an estimation sample. This is another advantage of the DEA model.

At the end of this section, the following five comments may be useful to understand our comparison between DEA and LR.

4.2. Matching on industry and firm size

From the original Altman's bankruptcy database (609 firms), 130 firms had complete financial and stock returns data for the nine variables used in this study. A random sample of 100 firms was drawn from these 130 and this sample of 100 bankrupt firms was retained for analysis in the study. For each one of these 100 bankrupt firms, matching non-bankrupt firms belonging to the same 3-digit Standard Industrial Classification (SIC) code and with a market value within a range of ±20% of the market value of the bankrupt firm were initially screened. Matching on industry and firm size in this way is a standard practice in the finance literature and ensures that the bankrupt firm is evaluated against non-bankrupt firms with otherwise similar characteristics. It should be noted that factors such as the size of the industry and availability of data results in the number of matched non-bankrupt firms varying across the 100 bankrupt firms.

4.3. Construction method of various samples and sub-samples

For the first experiment, we selected 50 bankrupt firms randomly out of the 100 firms together with their corresponding matching firms. There were 50 samples, each containing one bankrupt firm and several non-bankrupt firms. Their sample sizes ranged from 11 to 41. This experiment enabled us to assess the strength of the DEA in identifying a single bankrupt firm from among several firms. The results corresponding to the DEA analysis are reported in Table 2.

For the second experiment, samples were created in such a way that the ratio of BR/NBR matching firms varied from to 0.25 to 1. In addition, a fifth sample was created by using 50 bankrupt firms and a random sample of 1533 non-matching (i.e., without regard for industry or size match) firms which had complete data for the nine variables. The results corresponding to the DEA analysis are reported in Table 3.

For the LR analysis in Table 4, five samples were created using a similar procedure so that the ratio of BR/NBR matching firms varied from 0.053 to 1. In addition, an out-of-sample of 50 bankrupt and 85 non-bankrupt firms were also created.

4.4. Overlap time periods

To examine the ability of the LR model in Table 4 for predicting an out-of-sample, a sample of 50 bankrupt and 85 matching non-bankrupt firms were drawn to estimate the parameters of the model. The estimated model was then used to predict out-of-sample using a sample of bankrupt and non-bankrupt firms (135 firms consisting of the remaining 50 bankrupt firms and the corresponding matching non-bankrupt firms). It is well known that extrapolation of regression models always leads to risky evaluations. Estimating the LR parameters and evaluating over the same time period (1991–2004) in fact gives the LR model an opportunity to perform at its best. Therefore the overlapping time periods used in this analysis is not likely to bias our conclusions.

4.5. Nature of data

The frequency of all data was annual. All the data for the nine input and output variables were obtained from Standard and Poor's Compustat annual data files immediately prior to the year of bankruptcy announcement. We did not take into account the time series variability of the financial ratios in this study because the objective was to use the values of the financial ratios immediately prior to bankruptcy for bankruptcy assessment.

4.6. Comparison using the same out-of-sample

A shortcoming of the comparison is that this study did not compare the DEA-additive model with the LR model, using the same out-of-sample. This research agenda will be an important future research task. For that purpose, we need to compare the DEA model with various other methods that are developed in other disciplines such as econometrics, statistics and computer science, as found in Sueyoshi (2006).

We need to mention that the out-of-sample in this study (last column in Table 4) has the ratio 0.588 between bankrupt and non-bankrupt firms. The ratio is close to the ratio 0.5 of the third sample in Table 3. Therefore, we should not expect the performance of DEA on the out-of-sample to be significantly lower than 75%, as observed in Table 3. A close examination of the results in Table 3 (for DEA) suggests that, regardless of the size of the sample or the ratio between the number of bankrupt and non-bankrupt firms, the performance of the DEA model never drops below 74%. In fact, this level of performance seems to persist across all samples except in the case where the ratio of bankrupt to non-bankrupt firms is 1. In this case, the performance goes up to 86%. Therefore, it is believed that running DEA on the out-of-sample will not produce inferior results to those reported in Table 3.

5. Conclusion and future extensions

This paper examines DEA as an alternative model for assessing corporate failures compared to the LR technique. The proposed use of DEA provides corporate leaders and investors with quick-and-easy information related to bankruptcy evaluation.

Under sampling conditions, where the LR is unable to obtain a ‘good’ estimated equation, the DEA is shown to be superior. These conditions occur most often when there is no estimation sample or the estimation sample does not represent the holdout sample. In within-sample evaluations, the LR appears to be superior to the DEA, but this is hardly of interest and use to the practitioner if a representative estimation sample cannot be obtained. The DEA obviates the need for an estimation sample unlike LR and this constitutes a significant advantage over the LR. Furthermore, the LR model uses choice-based sampling causing bias in the parameter and probability estimates. Since the DEA model is a non-parametric and distribution-free approach, choice-based sampling does not have any effect on the results, compared to the LR model.

The results in this paper show that in 84–89% of cases, the DEA model performs extremely well in correctly identifying the bankrupt firms compared to the LR model where the corresponding values range over 16– 64%. On the other hand, the LR model performs extremely well in correctly identifying the non-bankrupt firms (within-sample) in the range of 69.30–99.47% compared to the DEA model, which has corresponding values ranging from 68.42% to 83.33%. However, comparing the two models for out-of-sample predictions, which is more appealing and useful to the practitioner, the DEA model significantly outperforms the LR model with total correct predictions in 74–86% cases compared to 67% using the LR model.

This study has three major shortcomings, all of which need to be explored as future extensions of this research. First, this study empirically examines large bankruptcies in the United States that occurred over a wide range of industries during the most recent 15 annual periods starting from 1991. However, we need to admit that the business implication obtained in this study is limited as scientific evidence because we do not cover all the bankruptcy cases. Moreover, this study does not explore industry-specific bankruptcy characteristics because the sample data set is comprehensive in the industries examined. An investigation of this perspective is an important future research task.

Second, as mentioned previously, the proposed DEA approach does not incorporate a time horizon factor. To incorporate the time horizon, DEA needs to assume constant RTS to measure a dynamic change of bankruptcy in the time horizon. This assumption on constant RTS does not fit within the additive model (1). Hence, due to the analytical problem of DEA, we do not incorporate the time horizon factor in this study on bankruptcy assessment. The incorporation of a time horizon in DEA-based bankruptcy prediction is another important future research extension of this study.

Finally, this study provided an illustrative example regrading how corporate leaders and managers should select inputs and outputs for their bankruptcy assessments. However, the research effort did not clearly provide a benchmark regarding how to categorize financial ratios and measure into inputs and outputs. Furthermore, we did not specify how to select the best combination of inputs and outputs. Moreover, the proposed approach estimated bankruptcy probabilities by counting both the number of firms on the bankruptcy frontier and the number of firms within the frontier. We need to examine how financial analysts utilize the proposed bankruptcy assessment in their various decisional cases and ex-ante planning. These issues are important future agendas of this study. It is necessary for us to investigate the proposed approach further from the reality of financial assessment on bankruptcy in order to make suggestions on these research issues.

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