PHYSICS 341, Assignment # 8
Due: Monday, November 27, 2017

(1) Consider the linear ordinary second order differential equation of motion:

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega_0^2 x = 0 \tag{1}$$

(a) Find the general solutions for $x(t)$ and $v(t)$ when $\omega_0 = 0$.
(b) What is the differential equation for the phase space trajectories: $v = v(x)$ and what is the solution?
(c) Sketch the phase space portrait for equation (1) when $\gamma > 0$ and $\omega_0 = 0$. Use the solutions you found in parts (a and b).

(2) Show that when $\omega_0^2 < 0$, the phase portrait will always be a saddle about the equilibrium point, regardless of the sign of γ. That is show for both $\gamma > 0$ and for $\gamma < 0$ that the general solution for the differential equation (1) will always consist of an increasing and a decreasing exponential function of time when $\omega_0^2 < 0$.

(3) A position dependent force $F(x) = x - x^3$ acts on a particle of mass m.
(a) Write Newton's second law as a system of two first order differential equations for the position and velocity of the particle.
(b) Identify the equilibrium points in the phase space (x, v).
(c) What are the equations that describe the perturbations about the equilibrium points?
(d) Sketch a phase portrait of the dynamics determined by the force given above.

(4) A force $F(t) = F_0 e^{-\alpha t}$ acts on an undamped oscillator of mass, m, spring constant k. Using the method of undetermined coefficients compute a particular solution to the equations of motion and then determine the general solution. If the initial conditions are $x(t = 0) = x_0$ and $v(t = 0) = v_0$, compute the constants of integration for the motion.

(5) For what value of the damping factor γ will there be no longer any resonant enhancement of the steady state oscillator amplitude? That is find the critical value of γ for which the amplitude $A(\omega)$ is a monotonically decreasing function of the driving frequency ω. Give your answer in terms of the natural frequency ω_0 and the driving frequency ω. What is the phase of the steady-state solution when $\omega = \omega_0/2, \omega_0, 2\omega_0$?

NOTE: MidTerm Exam #2 will take place on Friday, Nov 24 in class