PHYSICS 341, Assignment # 3
Due: Friday, October 6, 2017

(1) A Cartesian coordinate system is rotated in the counter-clockwise direction by an angle θ about the z-axis.
(a) Write down the matrix $\boldsymbol{\lambda}$ that describes this rotation.
(b) Write down the transposed matrix $\boldsymbol{\lambda}^T$. What transformation does this matrix describe?
(c) Show that the product of the matrix with its transpose gives the unit matrix.
(d) Suppose a vector $\mathbf{A}$ has components A_x, A_y and A_z in the Cartesian system before the rotation is performed. The rotation changes the components to A'_x, A'_y and A'_z. Show that the magnitude of $\mathbf{A}$ is unchanged by the rotation.

(2) Two independent coordinate transformations are performed that rotate a set of Cartesian coordinates. The first, $\mathcal{R}_1$ rotates the y-axis toward the z-axis by 90° around the x-axis. The second, $\mathcal{R}_2$, rotates the x-axis toward the y-axis by 90° around the z-axis. Compute the transformation matrices that describe:
(a) A rotation $\mathcal{R}_1$ followed by a rotation $\mathcal{R}_2$.
(b) A rotation $\mathcal{R}_2$ followed by a rotation $\mathcal{R}_1$.
(c) Two successive rotations $\mathcal{R}_2$.
(d) If the original orientation of the Cartesian coordinates is given as in Figure 1, sketch the final orientation of the results of (a), (b), and (c).

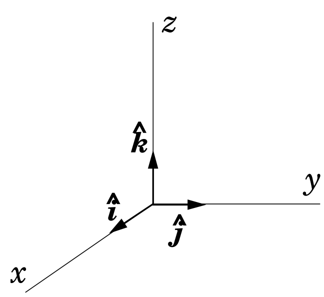

Figure 1: A right-handed 3D Cartesian coordinate system.

(3) A disk of radius R_0 rolls *without slipping* on a horizontal surface where its centre has a *constant* acceleration of $\mathbf{a}_1 = a_1\hat{\boldsymbol{\imath}}$ in the positive x-direction.
(a) Show that with respect to the centre of the disk, a point P on the rim has an instantaneous acceleration with magnitude $a_0 = (a_1^2 + v_1^4/R_0^2)^{1/2}$, where v_1 is the instantaneous forward speed of the disk's centre.
(b) If the angular position of the point P is θ measured from a vertical axis that passes through the centre C, show that the magnitude of the acceleration of the point P on the rim as measured from coordinates fixed on the origin of a Cartesian coordinate system fixed to the ground is:

$$a = a_1 \left[2 + 2\cos\theta + \frac{v_1^4}{a_1^2 R_0^2} - \frac{2v_1^2}{a_1 R_0}\sin\theta \right]^{1/2}$$

where the angular position θ time $t = 0$ is $\theta(0) = 0$. (Hint: For the geometry of the disk refer to the lecture notes `Rolling_Disk.pdf`.)

(over)

(4) A small object moves along an elliptical path in the x-y plane and its time dependent position vector is given by:

$$\mathbf{r}(t) = A\cos(\omega t)\hat{\mathbf{i}} + 2A\sin(\omega t)\hat{\mathbf{j}}$$

where both A and ω are constants. Find the *speed* v of the object as a function of time. Also determine the values of v when $t = 0$ and $t = \pi/(2\omega)$. Show that these two times are when the object is at a minimum and a maximum distance from the origin.

(5) The time derivative of the acceleration is a vector that is called the "jerk". (In Britain it is often referred to as the "jolt".) Compute $\mathbf{J} = d\mathbf{a}/dt$ in 3-D cylindrical coordinates.