PHYSICS 341, Assignment # 2
Due: Friday, September 29, 2017

(1) Suppose the force acting on a particle is given as $\mathbf{F} = f_0(y\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}})$. Two particles begin at the origin (0,0) and follow two different paths to the same point in the x-y plane. The first particle travels along a trajectory $y = \sqrt{ax}$ and the second particle travels along a trajectory $y = bx$.
(a) At what point $(x_m,\, y_m)$ do the two particles meet?
(b) Compute the work done in moving the the first particle from the origin to $(x_m,\, y_m)$.
(b) Compute the work done in moving the the second particle from the origin to $(x_m,\, y_m)$.
(d) Is the work performed on both particles the same or different? Explain why.

(2) Two three-dimensional vectors are given as:

$$\mathbf{A} = 3\hat{\mathbf{e}}_1 - 2\hat{\mathbf{e}}_2 + \hat{\mathbf{e}}_3, \qquad \mathbf{B} = -2\hat{\mathbf{e}}_1 + \hat{\mathbf{e}}_2 + 3\hat{\mathbf{e}}_3$$

find the following:
(a) $\mathbf{A} - \mathbf{B}$ and $|\mathbf{A} - \mathbf{B}|$
(b) the component of $\mathbf{B}$ along $\mathbf{A}$
(c) the angle between $\mathbf{A}$ and $\mathbf{B}$
(d) $\mathbf{A} \times \mathbf{B}$
(e) $(\mathbf{A} - \mathbf{B}) \times (\mathbf{A} + \mathbf{B})$

(3) Using the cross-product relationships among the unit vectors $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$ and $\hat{\boldsymbol{k}}$ prove that the following triple vector product relation holds:

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}).$$

This is kbown as the *bac minus cab* rule and can be used to show that the cross product is not associative. (See Example 1.7.3 in the text.)

(4) (a) Find the transformation matrix that rotates the y-axis of a cartesian coordinate system 30° toward z around the x-axis.
(b) If the same rotation is performed twice what are the new coordinate directions ($\bar{x}$,$\bar{y}$,$\bar{z}$) in terms of the original directions (x,y,z)?
(c) Is the rotation positive or negative and why?

(5) Find the values of σ needed to make the following transformation matrix describe an orthogonal transformation.

$$\mathbf{T} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sigma & -\sigma \\ 0 & \sigma & \sigma \end{pmatrix}$$