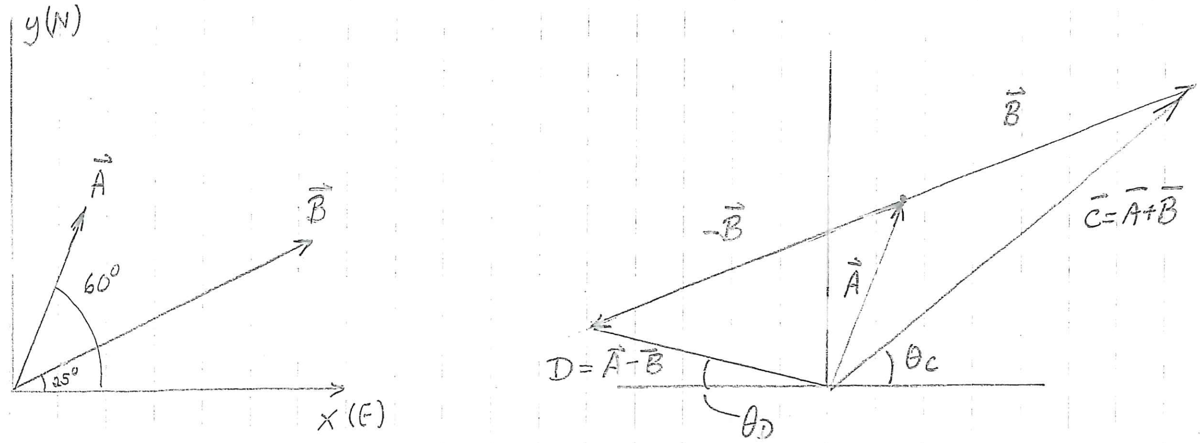
(1)

(a)



$\vec{A} = 3cm(\cos 60°)\hat{\imath} + 3cm \sin(60°)\hat{\jmath} = [1.50\hat{\imath} + 2.60\hat{\jmath}]\ cm$

$\vec{B} = 5cm(\cos 25°)\hat{\imath} + 5cm \sin(25°)\hat{\jmath} = [4.53\hat{\imath} + 2.11\hat{\jmath}]\ cm$

$\vec{C} = \vec{A}+\vec{B} = (1.50+4.53)cm\,\hat{\imath} + (2.60+2.11)cm\,\hat{\jmath} = [6.03\hat{\imath} + 4.71\hat{\jmath}]\ cm$

(b)

$|C| = \sqrt{(6.03)^2 + (4.71)^2}\ cm = \underline{7.65\ cm}$

(c) $\vec{D} = \vec{A}-\vec{B} = (1.50-4.53)cm\,\hat{\imath} + (2.60-2.11)\,cm\,\hat{\jmath} = [-3.03\hat{\imath} + 0.49\hat{\jmath}]\ cm$

$|D| = \sqrt{(3.03)^2 + (0.49)^2}\ cm = 3.07\ cm$

(e) $\theta_c = \tan^{-1}\left(\frac{C_y}{C_x}\right) = \tan^{-1}\left(\frac{4.71}{6.03}\right) = 37.99°$ (.6631) rad

North of East

$\theta_c = \cos^{-1}\left(\frac{\vec{C}\cdot\hat{\imath}}{|C|}\right) = \cos^{-1}\left(\frac{6.03}{7.65}\right) = 37.98° = (.6629)$ rad

(f) $\theta_D = \tan^{-1}\left(\frac{D_y}{D_x}\right) = \tan^{-1}\left(\frac{.49}{3.03}\right) = 9.19° = (0.1603)$ rad N of West

$\theta_c = \cos^{-1}\left(\frac{\vec{D}\cdot\hat{\imath}}{|D|}\right) = \cos^{-1}\left(\frac{-3.03}{3.07}\right) = \underline{170.74°} = (2.980)$ rad N of East

$= \underline{9.25°}$ N of West

(2) $\vec{O} = 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$ $\quad \vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ $\quad \vec{B} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$

(a) $\vec{O}+\vec{A} = (0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}) + (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k})$ definition

$= (0+A_x)\hat{\imath} + (0+A_y)\hat{\jmath} + (0+A_z)\hat{k}$ concatination

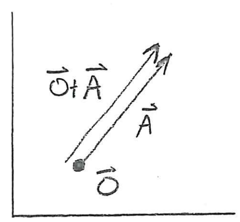
$= (A_x+0)\hat{\imath} + (A_y+0)\hat{\jmath} + (A_z+0)\hat{k}$ commutation

$= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) + (0\hat{\imath} + 0\hat{\jmath} + 0\hat{k})$ seperation

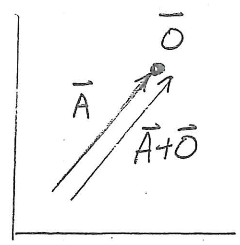
$= \vec{A} + \vec{O}$ definition

$= A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$ additive identity

$= \vec{A}$ definition



tail of $\vec{A}$ coincides with head & tail of $\vec{O}$



head of $\vec{A}$ coincides with head and tail of $\vec{O}$

(b) $\vec{A}+(-\vec{A}) = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) + [(-A_x\hat{\imath}) + (-A_y\hat{\jmath}) + (-A_z\hat{k})]$ definition

$= [A_x + (-A_x)]\hat{\imath} + [A_y + (-A_y)]\hat{\jmath} + [A_z + (-A_z)]\hat{k}$ concatination

$= [A_x - A_x]\hat{\imath} + [A_y - A_y]\hat{\jmath} + [A_z - A_z]\hat{k}$ subtraction definition

$= 0\hat{\imath} + 0\hat{\jmath} + 0\hat{k}$ additive inverse

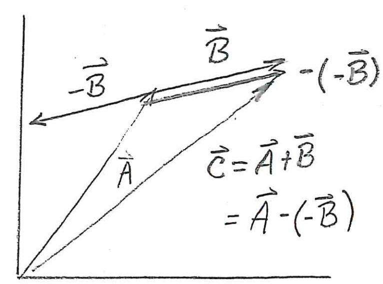
(c) $\vec{A} - (-\vec{B}) = (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) - (-B_x\hat{\imath} - B_y\hat{\jmath} - B_z\hat{k})$ def.

$= [A_x - (-B_x)]\hat{\imath} + [A_y - (-B_y)]\hat{\jmath} + [A_z - (-B_z)]\hat{k}$ concat.

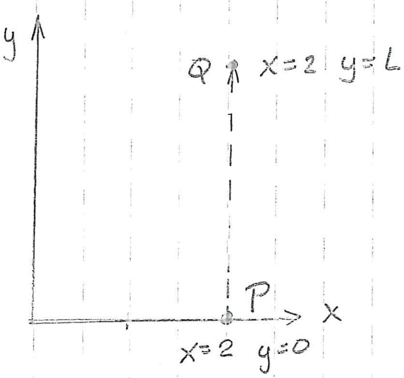
$= [A_x + B_x]\hat{\imath} + [A_y + B_y]\hat{\jmath} + (A_z + B_z)\hat{k}$ subtraction

$= (A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}) + (B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k})$ separation

$= \vec{A} + \vec{B}$ def.



(3) $F = F_0 \dfrac{x\hat{\imath} + y\hat{\jmath}}{\sqrt{x^2+y^2}}$



$x = 2 \quad dx = 0$ therefore

$d\vec{s} = dx\hat{\imath} + dy\hat{\jmath} = dy\,\hat{\jmath}$

$$W = \int_P^Q \vec{F} \cdot d\vec{s} = F_0 \int_P^Q \frac{(2\hat{\imath} + y\hat{\jmath})}{\sqrt{2^2+y^2}} \cdot dy\,\hat{\jmath}$$

$$= F_0 \int_{P_y}^{Q_y} \frac{y\,dy}{\sqrt{y^2+4}} \qquad Q_y = L \quad P_y = 0$$

$$= F_0 \sqrt{y^2+4}\,\Big|_0^L$$

$$= F_0 \left(\sqrt{L^2+4} - \sqrt{4} \right) = F_0 \left[(L^2+4)^{1/2} - 2 \right]$$

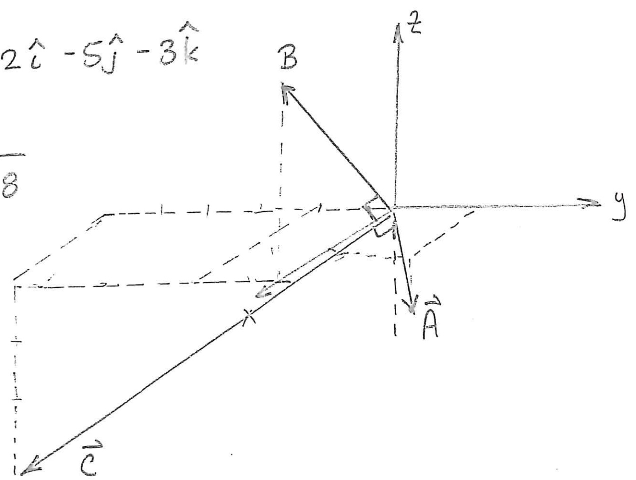
(4) $\vec{A} = \hat{\imath} + \hat{\jmath} - \hat{k}$ $\quad \vec{B} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$

compute $\vec{C} = \vec{A} \times \vec{B}$ or $\vec{B} \times \vec{A} = -\vec{C}$ since $\vec{C} \perp \vec{A}$ and $\vec{C} \perp \vec{B}$

$$\vec{C} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & -1 \\ 2 & -1 & 3 \end{vmatrix} = \hat{\imath}\,(3-1) - \hat{\jmath}\,(+3+2) + \hat{k}\,(-1-2)$$

$$= 2\hat{\imath} - 5\hat{\jmath} - 3\hat{k}$$

$$|C| = \sqrt{4+25+9} = \sqrt{38}$$



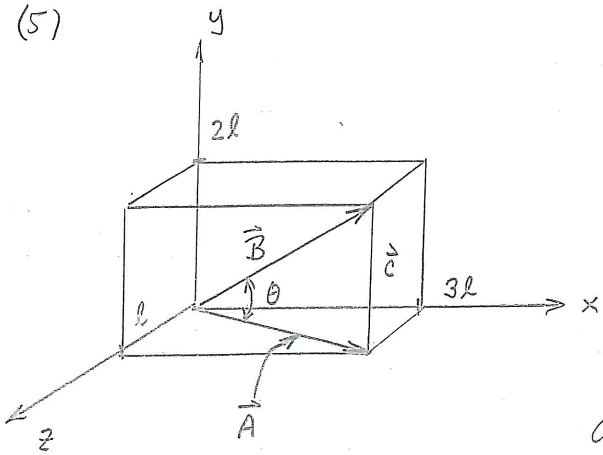
$$\hat{n}_c = \frac{\vec{C}}{|C|} = \pm \frac{[2\hat{\imath} - 5\hat{\jmath} - 3\hat{k}]}{\sqrt{38}}$$

check that $\vec{C}$ is perpendicular to $\vec{A}$ and $\vec{B}$

$$\vec{C} \cdot \vec{A} = (2)(1) + (-5)(1) + (-3)(-1) = 2 - 5 + 3 = 0$$

$$\vec{C} \cdot \vec{B} = (2)(2) + (-5)(-1) + (3)(-3) = 4 + 5 - 9 = 0$$

(5)



$\vec{A} = 3\ell\hat{i} + 0\hat{j} + \ell\hat{k}$

$\vec{B} = 3\ell\hat{i} + 2\ell\hat{j} + \ell\hat{k}$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}|\,|\vec{B}|}$$

$$\vec{A}\cdot\vec{B} = (3\ell\hat{i} + 0\hat{j} + \ell\hat{k})\cdot(3\ell\hat{i} + 2\ell\hat{j} + \ell\hat{k})$$

$$= 9\ell^2 + \ell^2 = 10\ell^2$$

$$|\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}} = \sqrt{9+1}\,\ell = \sqrt{10}\,\ell$$

$$|\vec{B}| = \sqrt{\vec{B}\cdot\vec{B}} = \sqrt{9+4+1}\,\ell = \sqrt{14}\,\ell$$

$$\cos\theta = \frac{10\ell^2}{\sqrt{10}\sqrt{14}\,\ell^2} = \sqrt{\frac{10}{14}} = \sqrt{\frac{5}{7}}$$

$$\theta = \cos^{-1}\left(\frac{5}{7}\right)^{1/2} = 0.5639 = 32.31^\circ$$

alternative $\quad \vec{C} = \vec{B} - \vec{A} = 2\ell\hat{j} \quad |\vec{C}| = 2\ell$

$$\tan\theta = \frac{|\vec{C}|}{|\vec{A}|}$$

$$\theta = \tan^{-1}\left(\frac{|C|}{|A|}\right) = \tan^{-1}\left(\frac{2\ell}{\sqrt{10}\,\ell}\right) = \tan^{-1}\sqrt{2/5} = 32.3^\circ$$