PHYSICS 341, Assignment # 5
Due: Friday, October 27, 2017

(1) A particle is released from rest at a position x_0 on the x-axis (measured from the origin) at $t = 0$. The particle is attracted to the origin by a force

$$F(x) = -kx^{-2}.$$

Show that the time required to reach the origin is

$$t_c = \pi \left(\frac{mx_0^3}{8k} \right)^{1/2}.$$

(2) A particle moving along the x-axis is subject to a force

$$F(x, v) = cvx$$

where c is a positive copnstant. If the particle passes through the origin with speed v_0 at time $t = 0$ find x as a function of t.

(3) A particle of mass m is constrained to to move along a horizontal frictionless plane. The motion is governed by a force $F(x) = -kx + kx^3/A^2$ where A and k are positive constants. The initial conditions at $t = 0$ are such that the particle is moving in the positive x direction with a kinetic energy $T_0 = \frac{1}{2}kA^2$ when it is located at $x = 0$.
(a) Find the potential energy function for the force, F, and sketch it as a function of x (In the sketch you can set $k = A = 1$).
(b) Compute the kinetic energy of the particle as a function of position.
(c) Compute the total energy of the particle as a function of position.
(d) Compute the location of the turning points of the particle.

(4) A particle of mass m moving is one-dimension has a potential energy function

$$V(x) = V_0 \left[\left(\frac{a}{x} \right)^4 - 2 \left(\frac{a}{x} \right)^2 \right].$$

where V_0 and a are constants.
(a) Sketch the potential $V(x)$ for $x \geq 0$.
(b) Determine the equilibrium position of the particle.
(b) What is the value of the potential at the equilibrium position?
(c) At what frequency will the particle oscillate between turning points if it has a kinetic energy just large enough to perturb it away from its equilibrium state?

(5) A particle of mass $m = .01$ kg moves under the influence of a potential energy function:

$$V(x) = k \left[x^2 + 2\alpha \ln \left(1 - \frac{x}{a} \right) \right]$$

where $k = 1.0$ Joules/m^2, $a = 10$ cm, $\alpha = 16$ cm^2.
(a) Find the force exerted on the particle.
(b) Compute the location of the equilibrium points and determine their stability type.
(c) What is the maximum kinetic energy of the particle such that the motion remains bounded?

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