PHYSICS 341, Assignment # 9
Due: Thursday, December 7, 2017

(1) A damped harmonic oscillator is driven by a periodic force $F_{\text{drive}} = F_0 \sin \omega t$. Show that the amplitude and the phase of the steady-state motion is exactly the same as that for a driving force $F_{\text{drive}} = F_0 \cos \omega t$.

(2) The resonance frequency for the velocity of a driven damped oscillator does not coincide with the resonant frequency for the position.
(a) Show that the resonance frequency for the position of a damped, driven oscillator (i.e. the frequency where the position is a maximum) is given by:

$$\omega_r = \sqrt{\omega_0^2 - 2\gamma^2}.$$

(b) Show that resonance (i.e. maximum amplitude) for the particle's velocity occurs exactly at the natural frequency of the undamped oscillator. This means that the velocity is maximized when the frequency of the driving force matches the natural frequency of the oscillator.

(3) A particle of mass m moves under an attractive central force given by:

$$\mathbf{F} = -\frac{k}{r^5}\hat{\mathbf{e}}_r, \qquad \text{where } \; k > 0$$

(a) Show that a trajectory $r(\theta) = 2r_0 \cos\theta$ is due to such a force.
(b) Show that the particle's orbit is circular with diameter $2r_0$.
(c) Show that the particle orbit passes through the force centre.
(d) Sketch the particle's orbit.

(4) The orbit of Halley's comet highly elliptic ($e = 0.967$) and has a period of 76 years. Compute the furthest distance from the Sun reached by the comet. Compute the closest distance the comet comes to the Sun. In addition, compute the orbital angular momentum of the comet.

(5) A particle moves in an elliptic orbit under the influence of a central force $F(r) = -k/r^2$. Prove that the product of the maximum speed with the minimum speed is equal to $(2\pi a/\tau)^2$, where a is the semi-major axis of the ellipse, and τ is the period of the particle's motion.

The masses of Halley's comet and the Sun are:
$M_{\text{Halley}} = 3.0 \times 10^{15}$ kg
$M_{\odot} = 2.0 \times 10^{30}$ kg