PHYSICS 341, Assignment # 7
Due: Wednesday, November 15, 2017

(1) An underdamped harmonic oscillator has an amplitude that drops by a factor of $1/e$ of its initial value after n complete cycles. Show that the ratio of the period of the damped oscillator, T_1 to the period of the undamped oscillator, T_0 is given by:

$$\frac{T_1}{T_0} = \left(1 + \frac{1}{4\pi^2 n^2}\right)^{\frac{1}{2}} \approx 1 + \frac{1}{8\pi^2 n^2}$$

where the approximate value given in the last expression is valid for large n. (Hint: consider a binomial expansion for the function $f(x) = (1+x)^n$ where $x \ll 1$.)

(2) Express both the displacement $x(t)$ and the velocity $v(t)$ for the overdamped harmonic oscillator in terms of hyperbolic functions.

(3) Determine at what time the displacement maxima of the underdamped harmonic oscillator occur. Then show that the ratio of successive maxima is a constant. (Note: The maxima do not occur at the points where the displacement curve intersects the exponential envelope $x = Ae^{-\gamma t}$.)

(4) Consider the energy associated with an underdamped harmonic oscillator.
(a) Derive an expression for the total energy E as a function of time.
(b) Derive an expression for the time derivative of the total energy dE/dt as a function of time.
(c) For a *lightly* damped oscillator where the exponential decay is very small what is the *average rate* at which the damped oscillator loses energy? (i.e. compute the time average over one cycle.)

(5) A simple plane pendulum consists of a mass m suspended from a fixed point by means of a string of length ℓ and is immersed in a viscous medium. The medium provides a retarding force proportional to the velocity $F_{\text{ret}} = -2mv\sqrt{g/\ell}$ where g is the acceleration due to gravity. At $t = 0$ the pendulum mass is released from rest with an angular displacement of θ_0 from the vertical. Find the angular displacement, θ and angular velocity, $\omega = \dot{\theta}$ as a function of time.