PHYSICS 341, Assignment # 6
Due: Friday, November 3, 2017

(1) A simple harmonic oscillator consists of a mass $m = 0.1$ kg attached to a spring with force constant $k = 10^{-3}$ N/m. The mass is displaced 3.0 cm and released from rest. Calculate the following quantities:
(a) the natural linear frequency f_0 and period T_0
(b) the total energy, E
(c) the maximum velocity
If the same oscillator is set in motion with $v_0 = 0.1$ m/s when it is at its equilibrium position calculate:
(d) the maximum displacement
(e) the maximum value of the potential energy.

(2) Assuming that the Earth is a solid sphere of constant density, show that if a straight hole were drilled through the Earth from pole to pole an object dropped into the hole from one end would undergo simple harmonic motion. Compute the period of oscillation and show that that the period of oscillation depends only on the density of the Earth and is independent of its size. [Hint: Use Newton's law of gravity where the force on the particle is due only to the total mass located inside a radius r when the particle is located at r.]

What is the period of oscillation for the Earth given the following values for the Earth's radius and mass?
$R_\oplus = 6.38 \times 10^6$ m
$M_\oplus = 5.98 \times 10^{24}$ kg

(3) Using the properties of complex exponential functions prove the following trigonometric identities:
(a) $\cos^2\theta + \sin^2\theta = 1$
(b) $\cos^3\theta = \frac{1}{4}(3\cos\theta + \cos 3\theta)$
(c) $\sin^3\theta = \frac{1}{4}(3\sin\theta - \sin 3\theta)$

(4) The general solution for the undamped harmonic oscillator equation in terms of complex exponential functions is:

$$x(t) = Ce^{i\omega_0 t} + C^* e^{-i\omega_0 t}$$

where ω_0 is the natural frequency of the oscillator, C is a complex constant and C^* is the complex conjugate of C. Using the properties of complex exponential functions,
(a) show that the acceleration is related to the position by:

$$a(t) = -\omega_0^2 x(t)$$

(b) show that the sum of the instantaneous kinetic and potential energies is both time independent and real.

(5) A particle of mass m undergoes simple harmonic motion and has a velocity v_1 when its position (measured from the equilibrium position) is x_1. It also has a velocity v_2 when its position is x_2. Find both the amplitude and (angular) frequency of the motion.