Ken Sible — August 6, 2018

$$\Psi_{uv}(u_i, v_j) = \frac{\Psi(u_{i+1}, v_{j+1}) - \Psi(u_{i+1}, v_j) - \Psi(u_i, v_{j+1}) + \Psi(u_i, v_j)}{\Delta u \Delta v} + O(\Delta u, \Delta v)$$

$$\frac{\partial^n}{\partial u^n}\frac{\partial^m}{\partial v^m}\Psi(u_i, v_j) \to \Psi_{i,j}^{(n,m)} \Rightarrow \Psi_{i,j}^{(1,1)} = \frac{\Psi_{i+1,j+1} - \Psi_{i+1,j} - \Psi_{i,j+1} + \Psi_{i,j}}{\Delta u \Delta v} + O(\Delta u, \Delta v)$$

$$\frac{\Psi_{i+1,j+1} - \Psi_{i+1,j} - \Psi_{i,j+1} + \Psi_{i,j}}{\Delta u \Delta v} = \Psi_{i,j}^{(1,1)} + \frac{1}{2}\left[\Psi_{i,j}^{(2,1)}\Delta u + \Psi_{i,j}^{(1,2)}\Delta v\right] + \frac{1}{6}\left[\Psi_{i,j}^{(3,1)}\Delta u^2 + \Psi_{i,j}^{(2,2)}\Delta u \Delta v + \Psi_{i,j}^{(1,3)}\Delta v^2\right] + \dots$$

$$\frac{\Psi_{i+2,j+2} - \Psi_{i+2,j} - \Psi_{i,j+2} + \Psi_{i,j}}{(2\Delta u)(2\Delta v)} = \Psi_{i,j}^{(1,1)} + \frac{1}{2}\left[\Psi_{i,j}^{(2,1)}(2\Delta u) + \Psi_{i,j}^{(1,2)}(2\Delta v)\right] + \frac{1}{6}\left[\Psi_{i,j}^{(3,1)}(2\Delta u)^2 + \Psi_{i,j}^{(2,2)}(2\Delta u)(2\Delta v) + \Psi_{i,j}^{(1,3)}(2\Delta v)^2\right] + \dots$$

$$\Psi_{i,j}^{(1,1)} = \frac{8(\Psi_{i+1,j+1} - \Psi_{i+1,j} - \Psi_{i,j+1} + \Psi_{i,j}) - (\Psi_{i+2,j+2} - \Psi_{i+2,j} - \Psi_{i,j+2} + \Psi_{i,j})}{4\Delta u \Delta v} + O(\Delta u^2, \Delta v^2, \Delta u \Delta v)$$

$$-4\Psi_{uv} = V_l(r)\Psi$$

$$8(\Psi_{i+1,j+1} - \Psi_{i+1,j} - \Psi_{i,j+1} + \Psi_{i,j}) - (\Psi_{i+2,j+2} - \Psi_{i+2,j} - \Psi_{i,j+2} + \Psi_{i,j}) = -\Delta u \Delta v V_l(r_c)\Psi_{i,j}$$

$$\Psi_{i+2,j+2} = 8(\Psi_{i+1,j+1} - \Psi_{i+1,j} - \Psi_{i,j+1} + \Psi_{i,j}) + (\Psi_{i+2,j} + \Psi_{i,j+2} - \Psi_{i,j}) + \Delta u \Delta v V_l(r_c)\Psi_{i,j}$$

$$\Psi_{ij} \to \Psi'_{ij} = \Psi_{i+1,j} + \Psi_{i,j+1} - \frac{1}{2}\left[\Psi_{i+2,j} + \Psi_{i,j+2}\right] + O(\Delta u^2, \Delta v^2)$$

$$\boxed{\Psi_{i+2,j+2} = 8(\Psi_{i+1,j+1} - \Psi_{i+1,j} - \Psi_{i,j+1} + \Psi_{i,j}) + (\Psi_{i+2,j} + \Psi_{i,j+2} - \Psi_{i,j}) + \Delta u \Delta v V_l(r_c)\Psi'_{i,j}}$$