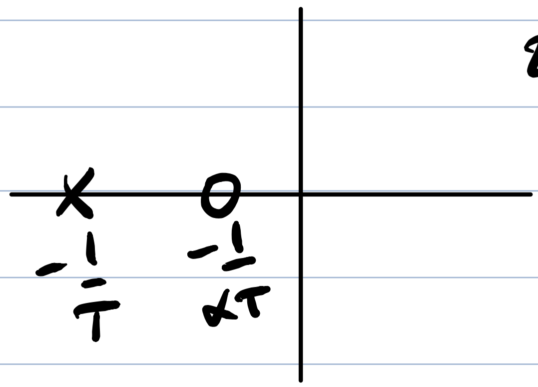
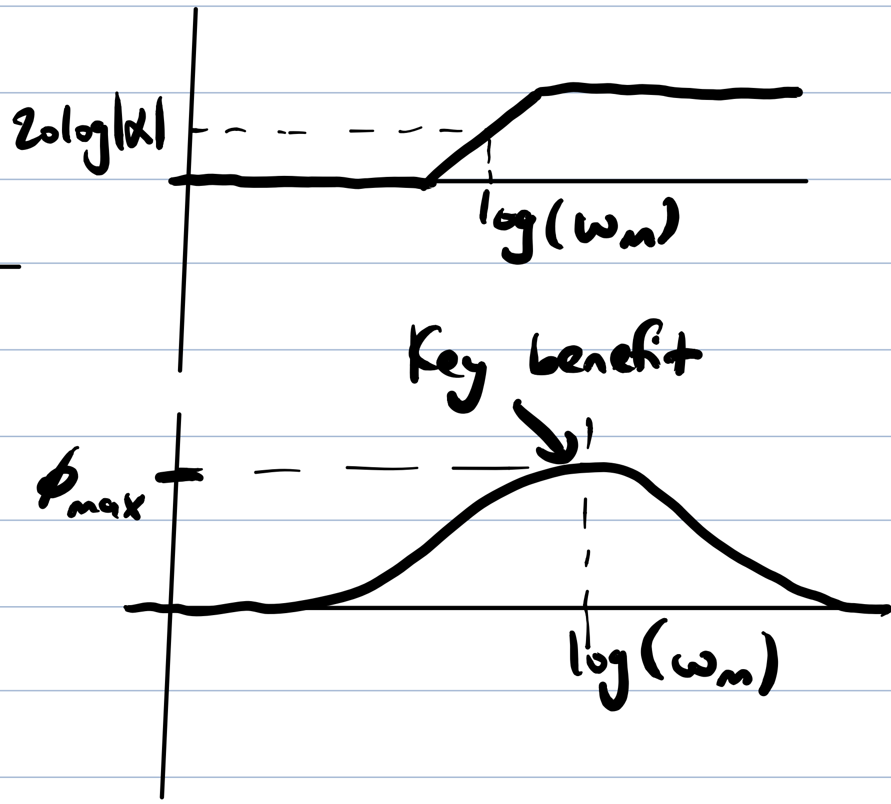
Lecture 30 Summary

- lag design procedure
- lead controllers





e.g. 4.4.1.

$$P(s) = \frac{1}{s(s+2)}$$ DC motor

Specs:
$|e_{ss}| \leq 0.05$, $r(t) = t$
$\Phi_{pm} = 45°$

Trick: Write $C(s) = K\frac{\alpha T s + 1}{Ts+1} = \frac{\hat{K}}{\sqrt{\alpha}}\frac{\alpha T s+1}{Ts+1}$

Step 1: Pick $\hat{K}$ using ss spec + FVT

$\hat{K} \geq 40$

Boost $\hat{K}$ by 10dB (a guess) to account for magnitude distortion. $\hat{K} = 40 \cdot 10^{1/2}$

Step 2: Draw Bode plot of $\hat{K}P(j\omega)$. Observe the phase margin and gain crossover frequency.

$$\Phi_{pm} = 10.2^\circ \quad \text{at} \quad \omega_{gc} = 11.2 \text{ rad/s}$$

Set $\omega_m = \omega_{gc}$

We need to add $\Phi_{pm}^{des} - \Phi_{pm} = 45^\circ - 10.2^\circ$
$= 34.8^\circ$

$\Rightarrow \phi_{max} = 34.8^\circ$

Step 3: Design equations

$$\alpha = \frac{1 + \sin(\phi_{max})}{1 - \sin(\phi_{max})} = 3.66$$

$$\omega_m = \frac{1}{T\sqrt{\alpha}} \iff T = \frac{1}{\omega_m\sqrt{\alpha}} = 0.0467$$

$$K = \frac{\hat{K}}{\sqrt{\alpha}} = 66.13$$

Step 4: Simulate the design

$$C(s) = 241.9\,\frac{s + 5.85}{s + 21.43}$$

zero closer to Im axis than pole (lead controller)

I get $\Phi_{pm} = 45^\circ$ at $\omega_{gc} = 11.1$ rad/s.

Remark: We've designed a lag and a lead controller to meet the same specs.

Lag: $\Phi_{pm} \approx 45°$, $\omega_{gc} = 1.7$ rad/s
Lead: $\Phi_{pm} = 45°$, $\omega_{gc} = 11.1$ rad/s

- the lead controller gives a higher ω_{gc} ($\approx$ closed-loop ω_{bw}) so closed-loop system will be faster
- the lead controller will use more control effort. ▲

<u>Procedure for Lead Design</u>

$$C(s) = K\frac{\alpha T s + 1}{Ts+1}$$

Specs: (a) Φ^{pm}_{des} and (b) <u>one</u> of:

(i) steady-state tracking
(ii) desired ω_{gc}

1. Let $\hat{K} = K\sqrt{\alpha}$
   (i) Use FVT to pick $\hat{K}$ s.t. $\hat{K}P(s)$ meets spec (i) in s.s. Boost the gain by 10 dB to account for magnitude distortion.
   (ii) Pick $\hat{K}$ s.t. $\hat{K}P(s)$ has desired ω_{gc}
2. Draw Bode plot of $\hat{K}P(j\omega)$
3. Find ω_{gc} and Φ_{pm}. Set $\omega_m = \omega_{gc}$.

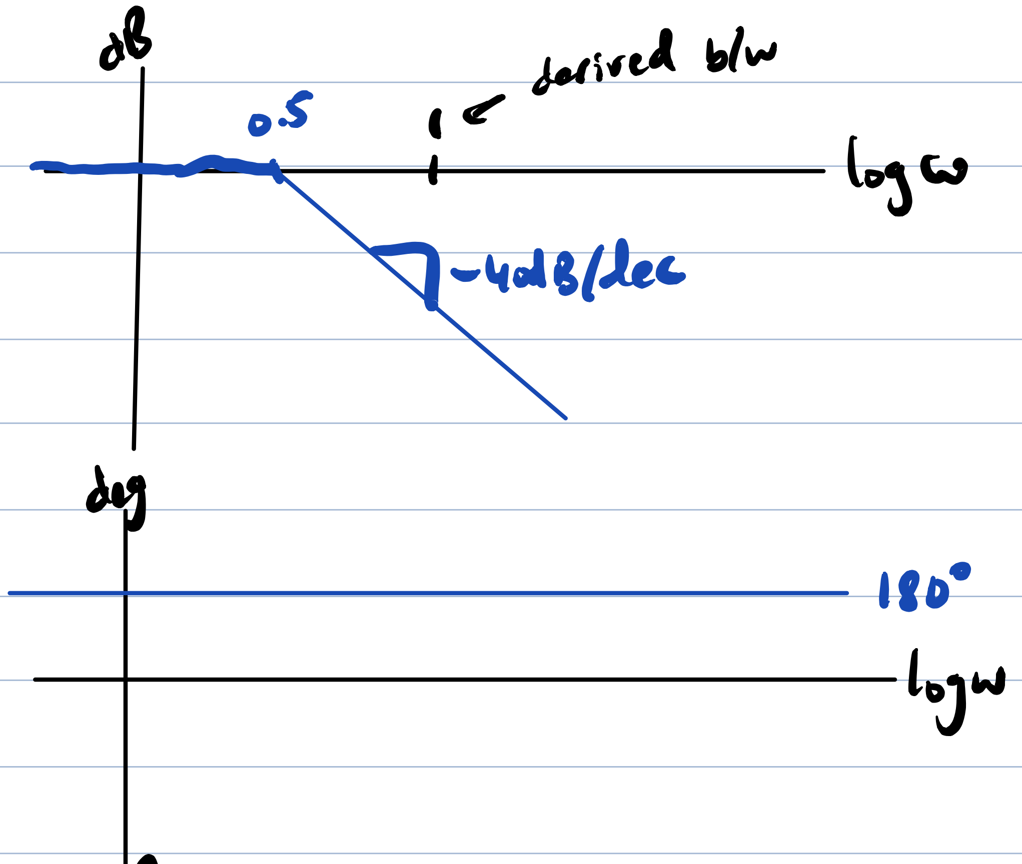
4. Determine amount of phase to add, ϕ_{max}
$= \Phi_{pm}^{des} - \Phi_{pm}$

5. $\alpha = \dfrac{1 + \sin\phi_{max}}{1 - \sin\phi_{max}}$

6. $T = \dfrac{1}{\omega_m \sqrt{\alpha}}$, $K = \dfrac{\hat{K}}{\sqrt{\alpha}}$

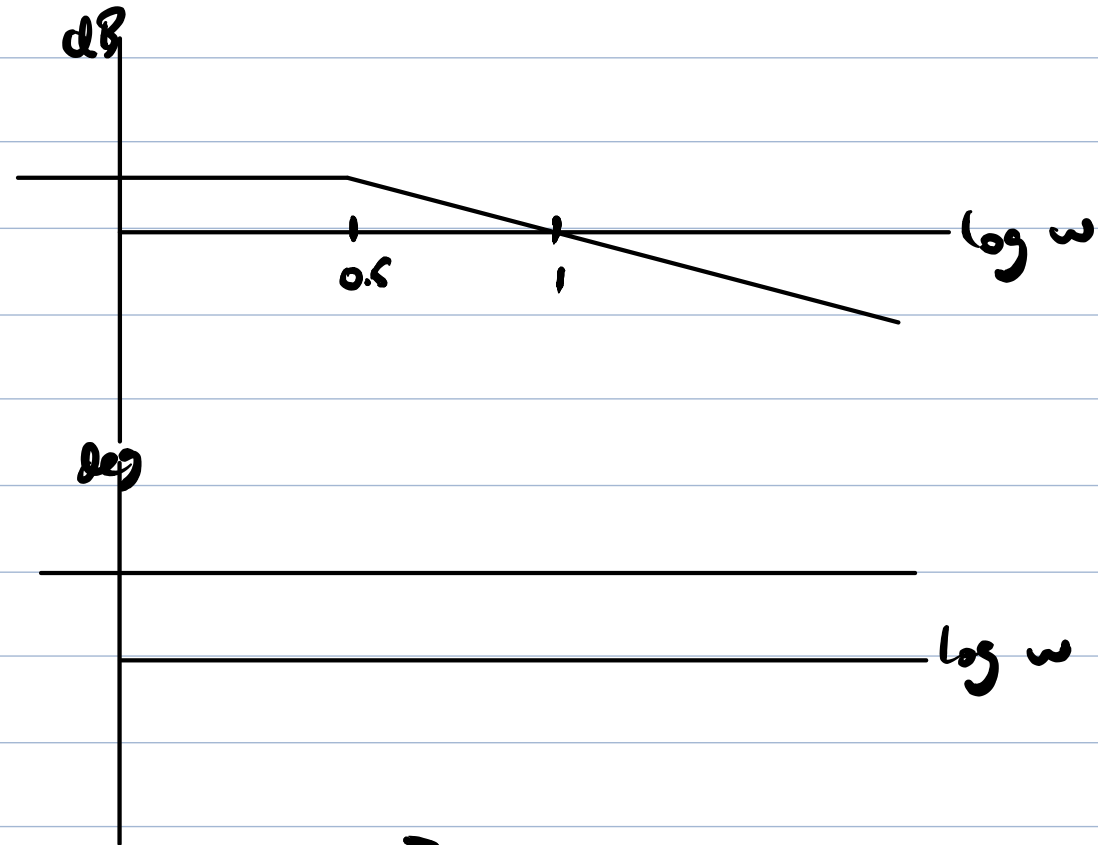
7. Simulate and iterate if needed, boost $\hat{K}$ more.

e.g. $P(s) = \dfrac{10}{s^2 - 10}$ Specs: $\Phi_{pm}^{des} = 50^\circ$
closed-loop bandwidth
$\omega_{bw} = 10$



1. Pick $\hat{K}$ s.t. $\omega_{gc} = 10$
$20\log\hat{K} = 20\text{ dB}$
$\Rightarrow \hat{K} = 10$

2. Draw Bode plot of KP.



3. $\omega_{gc} = 10$, $\Phi_{pm} = 0$, $\omega_m = 10$

4. Phase to be added: $\phi_{max} = 50° - 0° = 50°$

5. $\alpha = 7.55°$

6. $T = 0.0364$, $K = 3.64$

7. Simulate

$$C(s) = 27.4 \cdot \frac{s + 3.65}{s + 27.47}$$

In this example, we get $\omega_{bw} = 11.5$ rad/s.
so approximation $\omega_{gc} \approx \omega_{bw}$ worked well.

9.5. Lead-lag Controllers

$$C(s) = K \underbrace{C_1(s)}_{\text{lead}} \underbrace{C_2(s)}_{\text{lag}} = K \left(\frac{\alpha_1 T_1 s + 1}{T_1 s + 1}\right)\left(\frac{\alpha_2 T_2 s + 1}{T_2 s + 1}\right)$$

$\alpha_1 > 1, \quad 0 < \alpha_2 < 1$