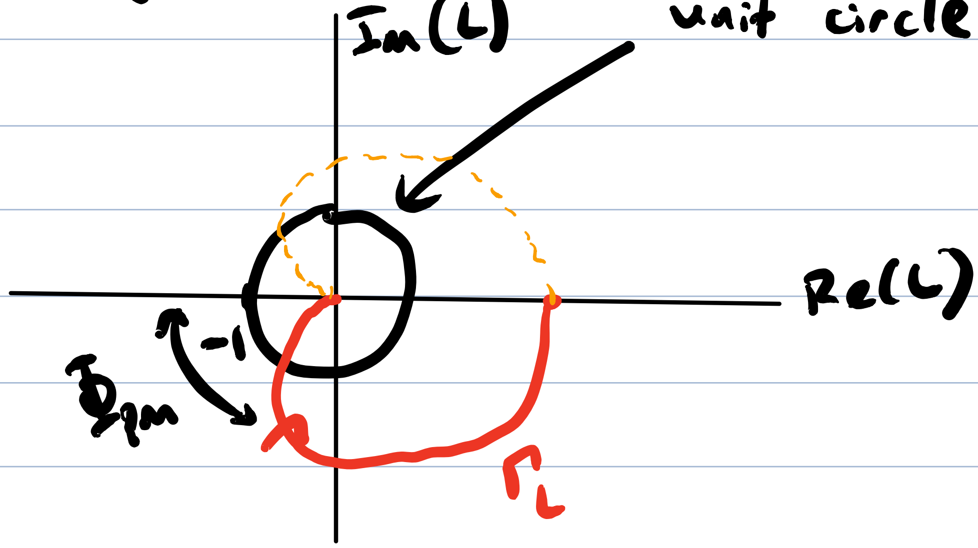
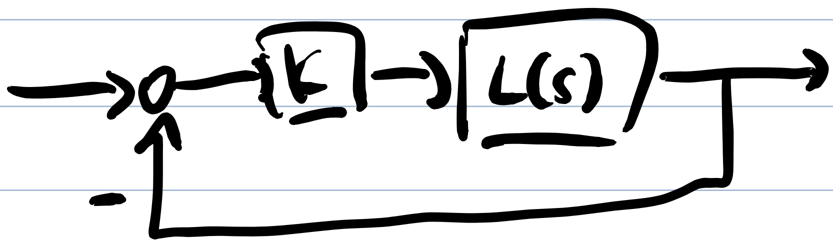
Summary Lecture 35

- Nyquist example
- Stability margin



8.4.2. Gain Margin

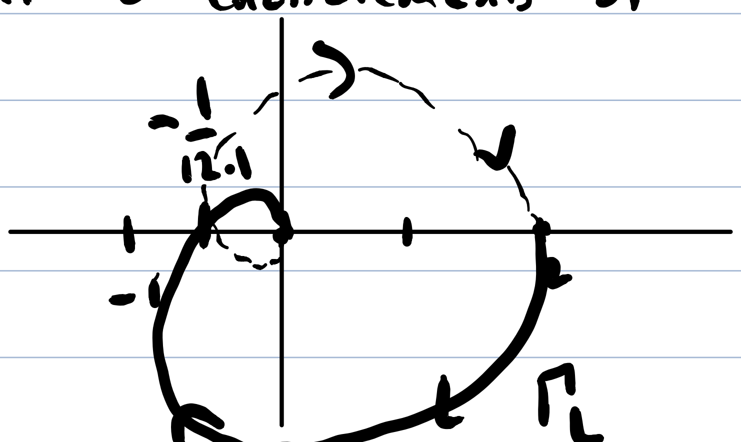


$$L = CP$$

$K=1$ Nominal model, I.O. stable

e.g. 8.4.3. $L(s) = \dfrac{2}{(s+1)^2(0.1s+1)}$

want 0 encirclements of -1.



- Since $n=0$, nominal system is I.O. stable.
- the perturbed system will remain stable so long as $-\frac{1}{K} < -\frac{1}{12.1}$.

- so we can increase K from 1 up to 12.1 before losing stability.
- 12.1 is K_{gm}, the gain margin.

$$-\frac{1}{12.1} = L(j\omega_{pc})$$

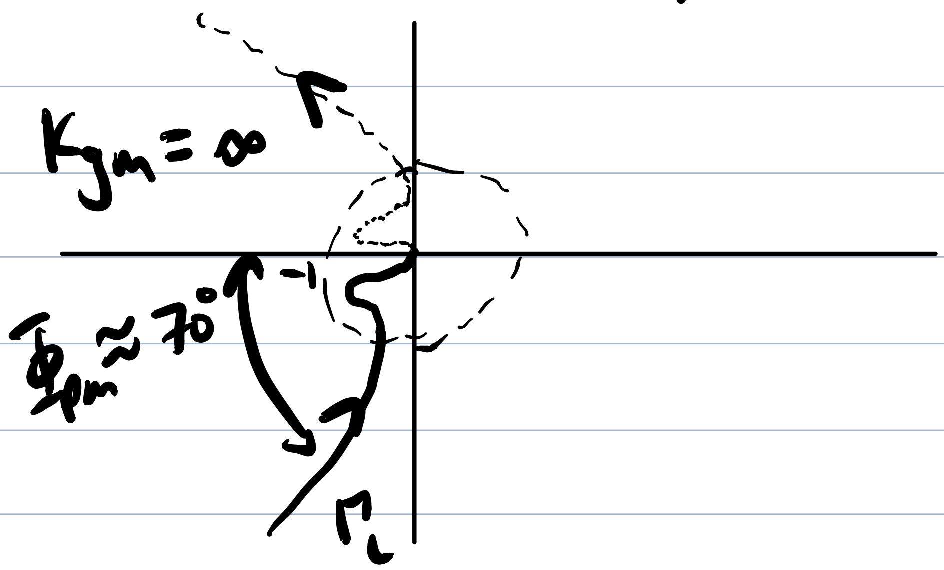
8.4.3. Stability Margin

- phase margin depends on the distance from the Nyquist plot to -1 along the unit circle.
- gain margin depends on the distance along the negative real line/axis.
- more generally, a good measure of robustness is the minimum distance from -1 to Nyquist plot

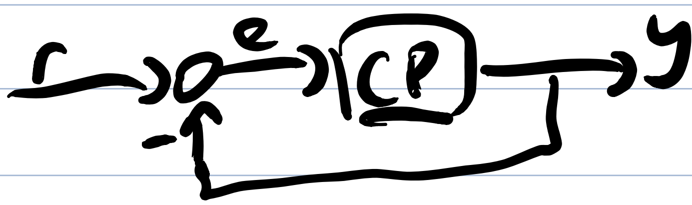
eg. 8.4.5

$$L(s) = \frac{0.38(s^2+0.1s+0.55)}{s(s+1)(s^2+0.06s+0.5)}$$

$N=0 \Rightarrow$ I.O. stability



Very high Φ_{pm} and K_{gm}! But the system is not really robust.
Based on Φ_{pm}, K_{gm}, we'd say this is a robust design. However, the Nyquist plot is dangerously close to -1. ▲



, assume I.O. stable.

$$\frac{E(s)}{R(s)} = \frac{1}{1+C(s)P(s)} =: S(s)$$

↳ sensitivity function

- the distance from -1 to the Nyquist plot of $L(s) = C(s)\,P(s)$

$$\min_{\omega \in \mathbb{R}} |-1 - L(j\omega)|$$

$$= \min_{\omega \in \mathbb{R}} |1 + C(j\omega)\,P(j\omega)|$$

$$= \max_{\omega \in \mathbb{R}} \left| \frac{1}{1 + C(j\omega)P(j\omega)} \right|$$

$$= \max_{\omega \in \mathbb{R}} |S(j\omega)|$$

Conclusion: Stability margin = peak of Bode plot of $S(s)$ (in dB).

Reasonable number: 0.5 to 0.8 (not in dB)

helicopter demo (slides on LEARN)

END SE 380