Lecture #25 Summary

- non-standard root-locus problems
  1) non-unity feedback
  2) controller can't be factored as $KC(s)$
  3) $\pi(s) = D(s) + KN(s)$ for $\deg(N) > \deg(D)$
  4) $K < 0$ (see notes)

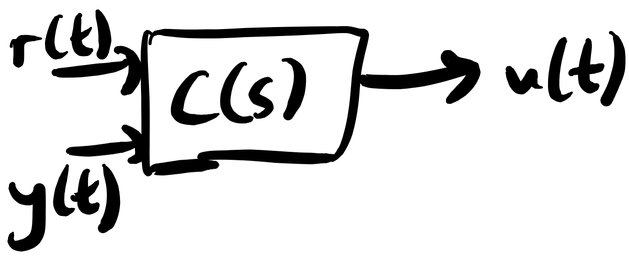
- Ch. 7 PID control $\qquad e \to \boxed{C(s)} \xrightarrow{u}$
  - Classical $C(s) = K_p + \frac{K_i}{s} + K_d s$

- refinement #1: STANDARD FORM

$$\boxed{C(s) = K_p + \frac{K_p}{T_i s} + \frac{K_p T_d s}{T_d s + 1}}$$

(P) (I) ($\approx$ D for frequencies $\leq \frac{1}{T_d}$ rad/s)

- refinement #2:



two-degree-of-freedom architecture

$$U(s) = \frac{K_i}{s}(R(s) - Y(s)) - \left(K_p + \frac{K_d s}{T_d s + 1}\right) Y(s)$$

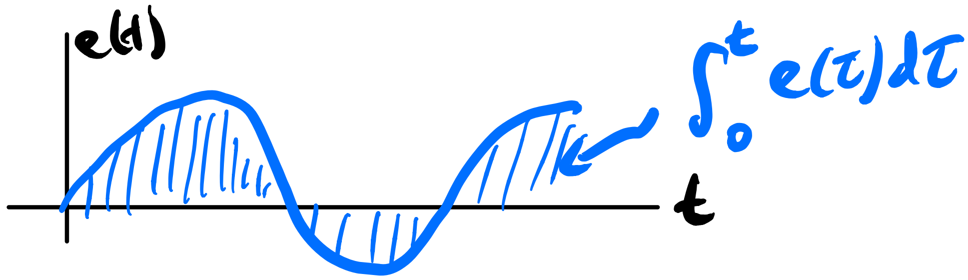
$$= C(s) \begin{bmatrix} R(s) \\ Y(s) \end{bmatrix}$$

- refinement #3 : Anti-windup
  - deals with actuator constraints
  - idea is when actuator hits its limits, stop integrating
  - see section 7.4

Consider $u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(\tau)\,d\tau + K_p T_d \frac{de}{dt}$

- proportional part:
  - can feedback stabilize open-loop stable plants
  - depends on the <u>current value</u> of $e(t)$
  - high K_p values usually improve steady-state performance
  - if K_p is too high, may lose stability

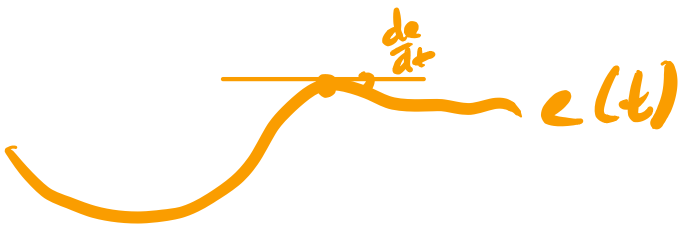
- integral part:
  - gives perfect step tracking
  - acts on the <u>accumulated error</u>



- derivative part:

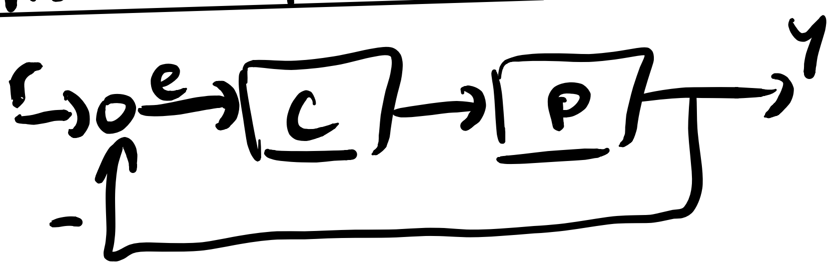
- penalizes fast changes in $e(t)$, tends to smooth out transients
- sometimes called the "predictive" part of PID
- why? Consider PD

→ $u(t) = K_p(t)\left(e(t) + T_d \frac{de(t)}{dt}\right)$



→ a prediction of the tracking error T_d seconds in the future

7.3. Pole placement



Proposition 7.3.1

Any controller of the form $C(s) = \frac{g_2 s^2 + g_1 s + g_0}{s^2 + f_1 s}$ is a PID controller in standard form. $K_p = \frac{g_1 f_1 - g_0}{f_1^2}$, $K_i = \frac{g_1 f_1 - g_0}{g_0 f_1}$,

$$T_d = \frac{g_0 - g_1 f_1 - g_2 f_1^2}{f_1 (g_1 f_1 - g_0)} \qquad \tau_d = \frac{1}{f_1}$$

Assumption 7.3.2: The plant $P(s)$ is

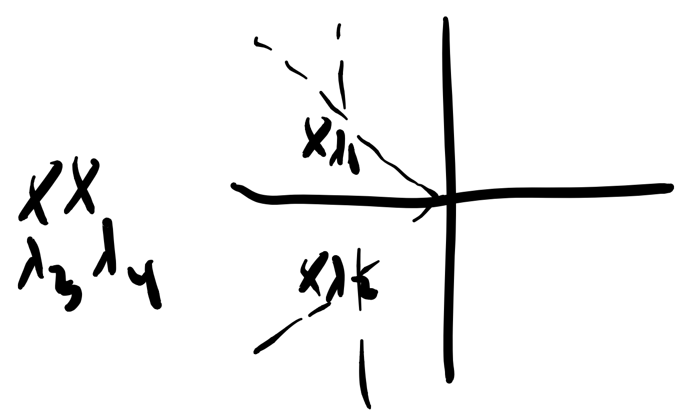
$$\frac{b_1 s + b_0}{s^2 + a_1 s + a_0}, \quad b_0 \neq 0$$

↳ needed to avoid unstable pole-zero cancellation

$$\pi(s) = D_p D_c + N_p N_c$$
$$= s^4 + (a_1 + f_1 + b_1 g_2)s^3 + (a_0 + a_1 f_1 + b_1 g_1 + b_2 g_2)s^2$$
$$+ (a_0 f_1 + b_1 g_0 + b_0 g_1)s + b_0 g_0$$

Now say we want the closed-loop poles to be located at $\{\lambda_1, \lambda_2, \lambda_3, \lambda_4\} \subseteq \mathbb{C}^-$

- the desired poles are picked based on desired settling time, % OS, T_p, etc.



- desired pole locations determine a desired ch.p.

$$\pi^{des}(s) = (s-\lambda_1)(s-\lambda_2)(s-\lambda_3)(s-\lambda_4)$$
$$=: s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0, \quad \alpha_i \in \mathbb{R}$$

MATLAB: conv.

Equate coefficients of π and π^{des}:

$$\begin{bmatrix} 1 & b_1 & 0 & 0 \\ a_1 & b_0 & b_1 & 0 \\ a_0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & b_0 \end{bmatrix} \begin{bmatrix} f_1 \\ g_2 \\ g_1 \\ g_0 \end{bmatrix} = \begin{bmatrix} \alpha_3 - a_1 \\ \alpha_2 - a_0 \\ \alpha_1 \\ \alpha_0 \end{bmatrix}$$

$$P(s) = \frac{b_1 s + b_0}{a_2 s^2 + a_1 s + a_0}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x = \begin{bmatrix} 3 \\ 0 \end{bmatrix} \qquad \begin{aligned} &Ax = b \\ &\text{rank}\,[A \;\; b] = \text{rank}\,A \end{aligned}$$

Remarks:

1) If N_p and D_p are coprime, then equation has a unique solution