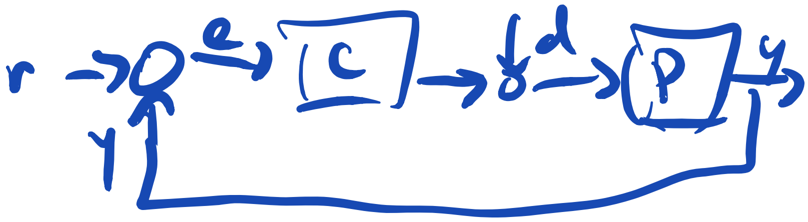
Summary Lecture 21

- internal model principle



- tracking $CP = \dfrac{N(s)}{D(s)\underbrace{D_r^+(s)}_{\text{unstable poles of } R(s)}}$

- disturbance rejection $C(s) = \dfrac{N(s)}{D_c(s)\underbrace{D_d^+(s)}_{\text{unstable poles of } D(s)}}$

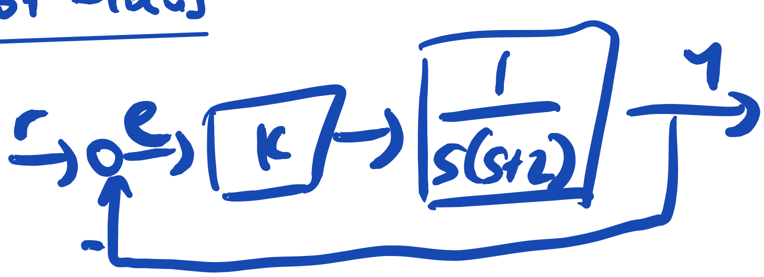
Q: What if I want to track steps and reject disturbances of 1 rad/s? What should C look like?

A: $C(s) = \dfrac{N_c(s)}{D_c(s)\cdot s\cdot(s^2+1)}$

Ch.6 Root-Locus method

6.1. Basic root-locus

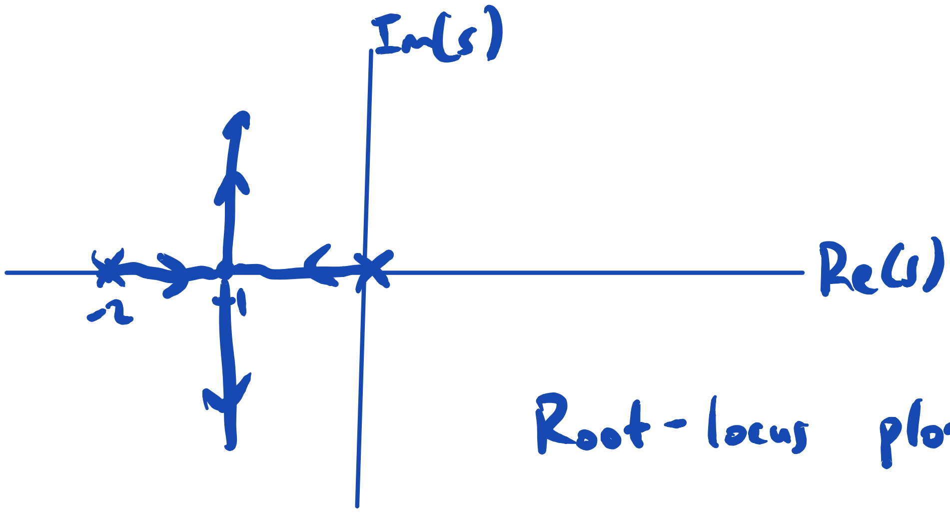
e.g. 6.1:



$\pi(s) = K + s^2 + 2s$

roots are $s = -1 \pm \sqrt{1-K}$

- observe:
  - if $K \leq 0$, we lose I.O. stability
  - if $0 < K \leq 1$ we have 2 real roots
  - if $K > 1$ then there are 2 complex conjugate roots with real part equal to -1



Root-locus plot

- picture tells us if system goes unstable as we vary K
- for a good step response, we should pick K so that $\mathrm{Im}(s)$ of poles $\neq 0$ but is not too big. $\triangle$

MATLAB rltool, sisotool, rlocus, rlocusplot

$$\pi(s) = \underbrace{D_p D_c}_{D(s)} + \underbrace{K N_p N_c}_{N(s)}$$

root-locus: drawing of how the roots of π vary as we vary K.

$$\pi(s) = (s-p_1)\cdots(s-p_n) + K(s-z_1)\cdots(s-z_m)$$

Assumptions: $n := \deg(D)$ $m := \deg(N)$

1. $m \leq n$ (CP is proper)
2. $K \geq 0$
3. D and N is monic

Construction Rules

1. Roots of π are symmetric about the real axis.
2. There are n "branches" (paths) of the root locus (since $\deg(D) = n$)
3. The roots of $\pi(s) = D(s) + KN(s)$ are a continuous function of K.
4. When $K=0$, the roots of π equal the roots of D.
5. As $K \to +\infty$, m branches approach the roots of $N(s)$ ($\pi(s)=0 \leftrightarrow N(s)/D(s) = -1/K \to 0$ as $K \to +\infty$, so they approach roots of N).
6. The remaining $n-m$ branches tend to ∞.

They do so along asymptotes.

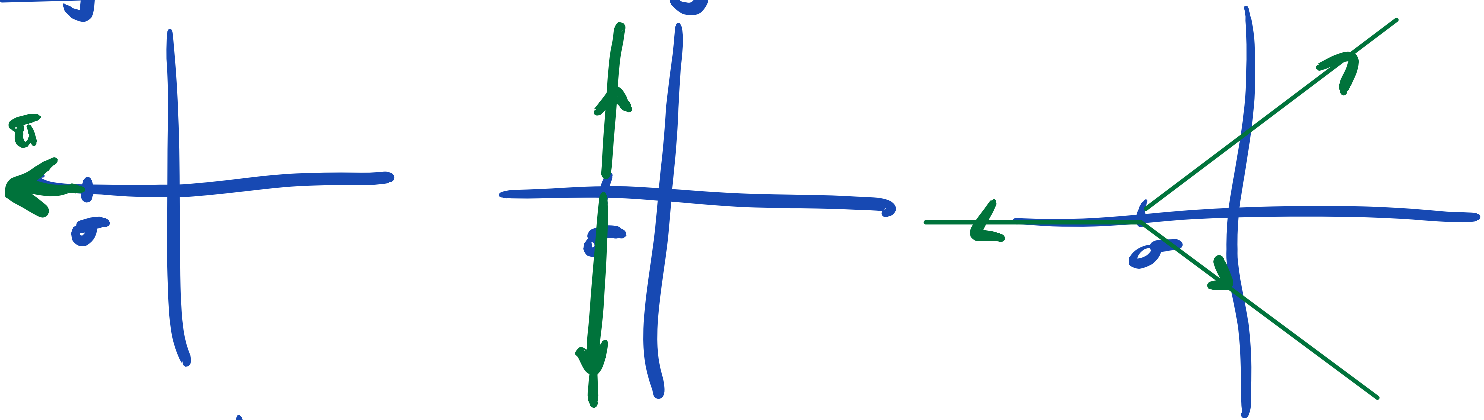
<u>Asymptotes:</u> originate at $s = \sigma + j0$

$$\sigma = \frac{\sum \text{roots of } D - \sum \text{roots of } N}{n - m}$$

and make angles $\phi_1, \ldots, \phi_{n-m}$ with the real axis given by

$$\phi_i = \frac{(2i-1)\pi}{n-m}$$

<u>Asymptote Patterns (Fig 6.4)</u>

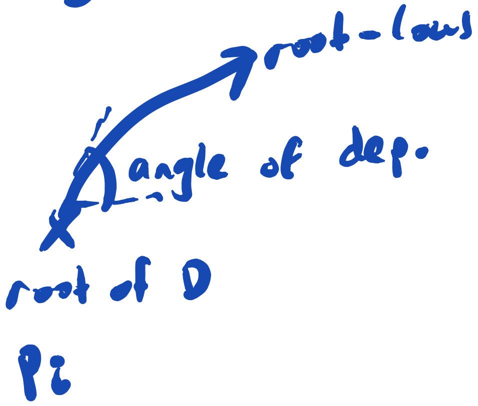


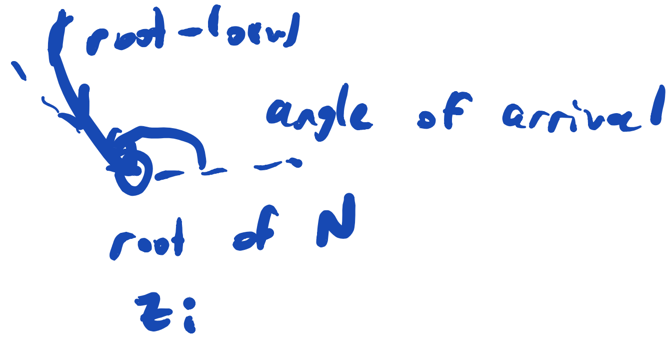
$n-m=1$ $\qquad$ $n-m=2$ $\qquad$ $n-m=3$

7. ("no-yes-no" rule): A point s_0 on the real axis is on the root locus if, and only if, s_0 is to the left of an odd number of roots of D, N

(follows from the fact that, on the Root-locus $\angle N(s_0) - \angle D(s_0) = \angle(-1/K) = \pi$)

8. (angles of arrival/departure)





- to compute, use the fact that $\angle N(s) - \angle D(s) = \pi$ on the root locus.

$$\angle(s-z_1) + \cdots + \angle(s-z_m) - \angle(s-p_1) - \cdots - \angle(s-p_n) = \pi$$