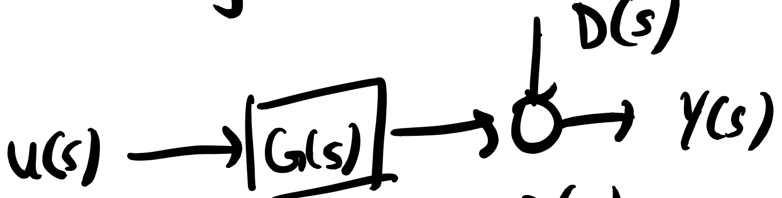
Summary Lec 6

- transfer functions
- get TF from ODE model, e.g. $\dot{y} + y = u$

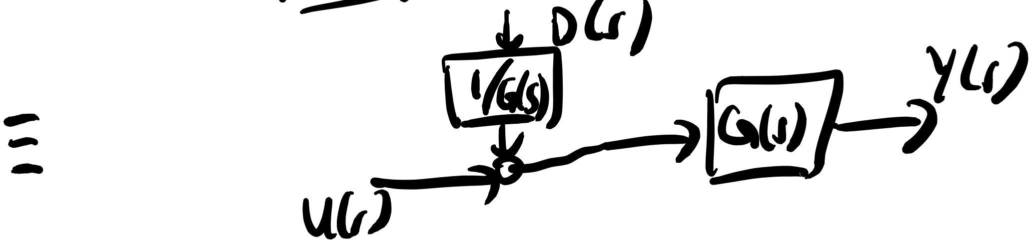
$$\Rightarrow \frac{Y(s)}{U(s)} = \frac{1}{s+1}$$

- common examples (integrators, time delays, etc.)
- terminology (poles, proper, etc.)
- ss2tf: $(A, B, C, D) \mapsto C(s-IA)^{-1}B + D$
- block diagrams

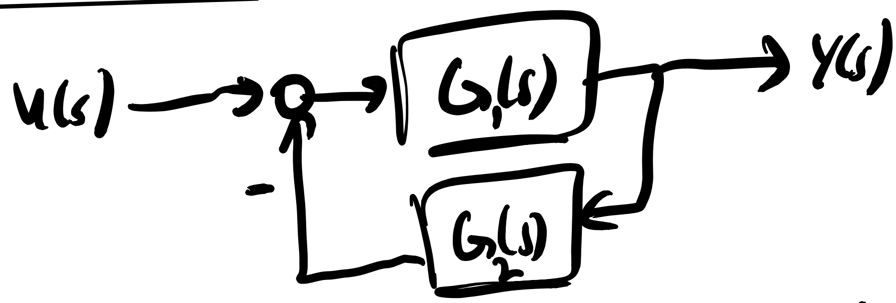
(iii) Moving blocks



$Y(s) = D(s) + G(s)U(s)$



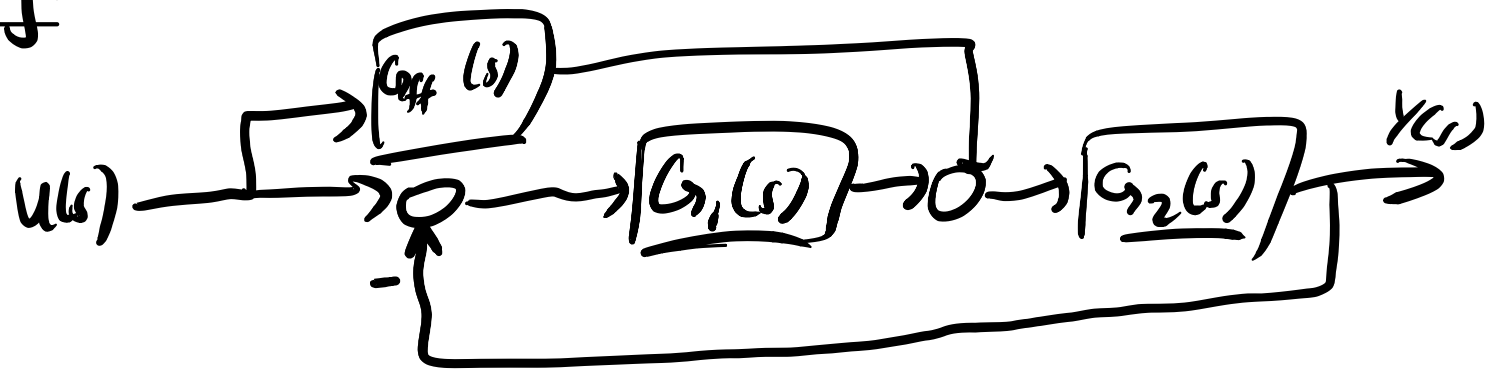
(iv) Feedback



$Y(s) = G_1(s)\left(U(s) - G_2(s)Y(s)\right)$

$Y(s)\left(1 + G_2(s)G_1(s)\right) = G_1(s)U(s)$

e.g.



Strategy 1: Write equation for $Y(s)$ and rearrange

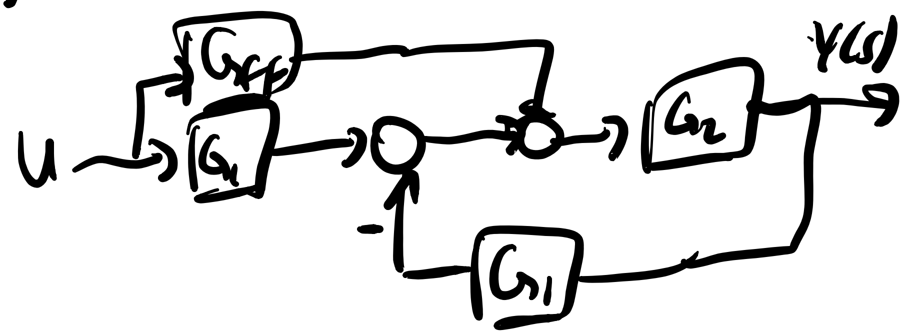
$$Y(s) = G_2(s)\left(G_{ff}(s)U(s) + G_1(s)\left[U(s) - Y(s)\right]\right)$$

$$Y(s)\left(1 + G_2(s)G_1(s)U(s)\right) = G_2(s)G_{ff}(s)U(s) + G_2(s)G_1(s)U(s)$$

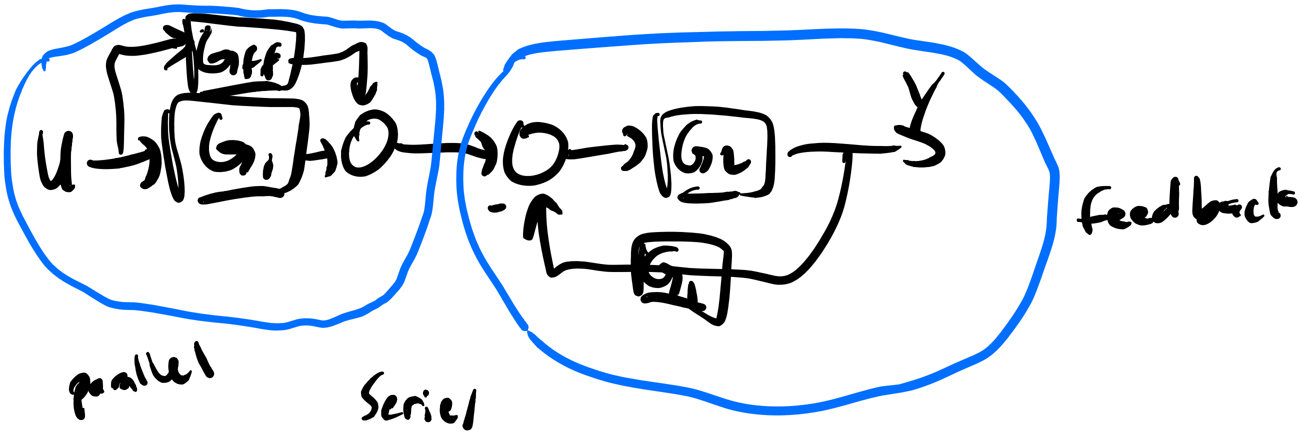
$$\frac{Y(s)}{U(s)} = \frac{G_2(s)G_{ff}(s) + G_2(s)G_1(s)}{1 + G_1(s)G_2(s)}$$

Strategy 2: Rearrange blocks to reveal common configurations

(i) move G_1 left of junction



(ii) swap order of summing nodes

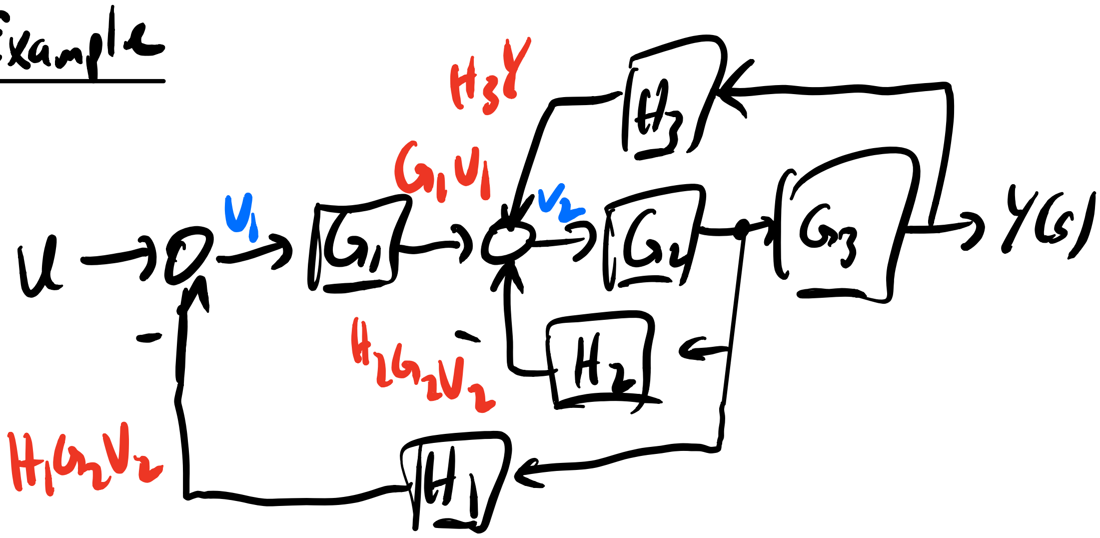


$$Y(s) = (G_1 + G_{ff})\left(\frac{G_2}{1+G_1G_2}\right)U(s)$$

Systematic method

1) Introduce new variables $\{v_1, \ldots,\}$ at the output of each summing junction
2) Write expressions for the <u>inputs</u> to the summing nodes in terms of $\{U, Y, v_1, \ldots\}$
3) Write equations for each summer and y.
4) Eliminate $\{v_1, v_2, \ldots\}$

Example



Step 1: (blue)

Step 2: (red)

Step 3: (purple)

$$Y = G_3 G_2 V_2$$

$$V_2 = G_1 V_1 + H_3 Y - H_2 G_2 V_2$$

$$V_1 = U - H_1 G_2 V_2$$

Step 4: (green)

Solve for Y: Cramer's Rule

Matrix notation

$$\begin{bmatrix} 1 & H_1 G_2 & 0 \\ G_1 & 1+H_2G_2 & -H_3 \\ 0 & -G_3G_2 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ Y \end{bmatrix} = \begin{bmatrix} U \\ 0 \\ 0 \end{bmatrix}$$

$$Y = \frac{\begin{vmatrix} 1 & H_1G_2 & U \\ -G_1 & 1+H_2G_2 & 0 \\ 0 & -G_3G_2 & 0 \end{vmatrix}}{\begin{vmatrix} 1 & H_1G_2 & 0 \\ -G_1 & 1+H_2G_2 & -H_3 \\ 0 & -G_3G_2 & 1 \end{vmatrix}} = \frac{U G_1 (G_3 G_2)}{(1+H_2G_2 - H_3G_2G_3) - ((-G_1)H_1G_2))}$$

Ch. 4: 1st and 2nd order systems

First-order:

$$\tau\dot{y} + y = Ku \quad \text{or} \quad \frac{Y(s)}{U(s)} = \frac{K}{\tau s+1} \quad \text{or}$$

$\tau, K \in \mathbb{R}$

$$\dot{x} = \frac{-1}{\tau}x + \frac{K}{\tau}u$$

$$y = x$$

Second-order:

$$\ddot{y} + 2\zeta\omega_n\dot{y} + \omega_n^2 y = K\omega_n^2 u \quad \text{or} \quad \frac{Y}{U} = \frac{K\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$\zeta, \omega_n, K \in \mathbb{R}$

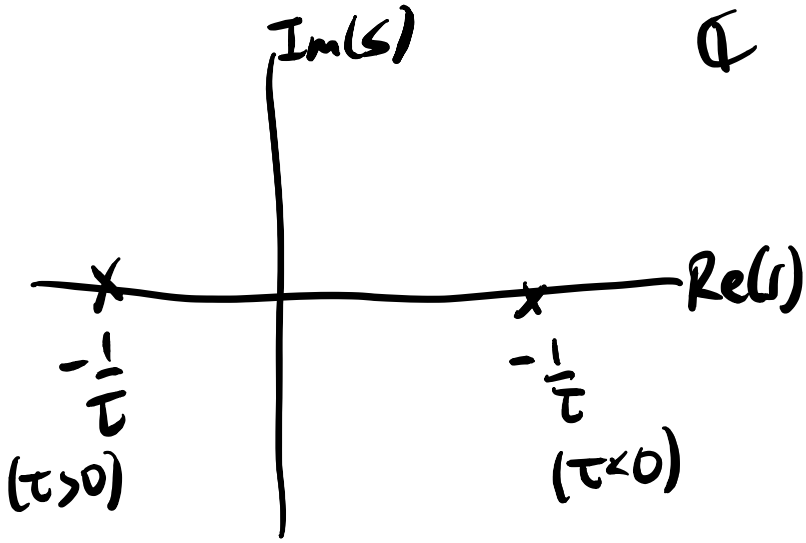
or $\quad x = \begin{bmatrix} y \\ \dot{y} \end{bmatrix} \quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} x + \begin{bmatrix} 0 \\ K\omega_n^2 \end{bmatrix} u$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x$$

Objective: Understand relationship between pole locations and time-domain behaviour

4.1 First-Order

Pole at $s = \frac{-1}{\tau}$; No zeros



Steady-state gain: K

Bandwidth: $1/\tau$

$$Y(s) = G(s)U(s)$$

$U(s) = 1$ when $u(t)$ is an impulse

So the impulse response is

$$\mathcal{L}^{-1}\{G(s)\} := g(t) = \frac{K}{\tau}e^{-t/\tau}, \quad t \geq 0$$