Applied Real Anal PMATH 331

1. Assignments 15% short weekly best 10/11
2. Midterm 30% Oct 24 (wed)
3. Final 55%

Textbook: Real Analysis and Applications
Davidson and Donsig

analysis - study of continuous structure through closer approx.

real analysis - analysis in the $\mathbb{R}$ world

applied real analysis - always living in $\mathbb{R}^n$
- 2-3 weeks of applications
  (use in other areas of math)

→ series convergence using Fourier analysis
→ dynamical systems
→ polynomial approximations.

Chapter 2

2.1: Intro

$\mathbb{N} = \{1, 2, 3, \ldots\}$

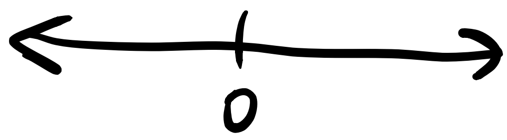
$\mathbb{Z} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$

$\mathbb{Q} = \left\{\frac{a}{b} : a, b \in \mathbb{Z}, b \neq 0\right\}$

These are all subsets of $\mathbb{R}$.

Q: How do you visualize $\mathbb{R}$?

A: The real number line.



Now, to visualize $\mathbb{R}$ in this way, we are assuming:

① **Orderability**

$\forall x, y \in \mathbb{R} \cdot x \leq y \vee y \leq x$

② **Completeness**

$\mathbb{R}$ has no gaps.

2.2: Decimal Expansions

Definition: A decimal expansion is a function $f : \mathbb{N} \cup \{0\} \to \mathbb{Z}$ such that $f(n) \in \{0, 1, \dots, 9\}$ for all $n \in \mathbb{N}$.

If f is a decimal exp. s.t. $f(i) = a_i \ \forall i \in \mathbb{N} \cup \{0\}$ we write $f = a_0 . a_1 a_2 a_3 \cdots$

Remark:

① We say f is a finite decimal exp. if $\exists N \in \mathbb{N}$ s.t. $f(n) = 0 \ \forall n \geq N$

We write $a_0 . a_1 \cdots a_{N-1}$ instead of $a_0 . a_1 \cdots a_{N-1} 000\ldots$ (omit trailing zeroes)

② We say f is <u>eventually periodic</u> if

$f = a_0 . a_1 a_2 \cdots a_n \overline{b_1 b_2 \cdots b_m}$ and say it has period m.

<u>Proposition</u>

If f is finite or eventually periodic, then f is a rational number.

<u>Proof</u>

① Let $x = a_0 . a_1 a_2 \cdots a_n$

$\therefore 10^n x = a_0 a_1 \cdots a_n \in \mathbb{Z}$

$\therefore x = \dfrac{a_0 a_1 \cdots a_n}{10^n} \in \mathbb{Q}$

② Let $y = a_0 . a_1 \cdots a_n \overline{b_1 \cdots b_m}$.

$\therefore 10^{n+m} y = a_0 a_1 \cdots a_n b_1 \cdots b_m . \overline{b_1 \cdots b_m}$

and $10^n y = a_0 a_1 \cdots a_n . \overline{b_1 \cdots b_m}$

$\therefore 10^{n+m} y - 10^n y = a_0 a_1 \cdots a_n b_1 \cdots b_m - a_0 a_1 \cdots a_n \in \mathbb{Z}$

$\therefore y = \dfrac{''}{10^{n+m} - 10^n} \in \mathbb{Q}$ $\blacksquare$

<u>Remark</u>: Let $x = 0.\overline{9}$

$10x = 9.\overline{9}$

$9x = 9$ $\nearrow$ $x = 1$

Definition

Let X be the set of decimal expansions. We say $f \sim g$, $f, g \in X$ if $f = g$ or $f = a_0 . a_1 \cdots a_k \overline{9}$ and $g = a_0 . a_1 \cdots (a_k + 1)$. This forms an equivalence relation.