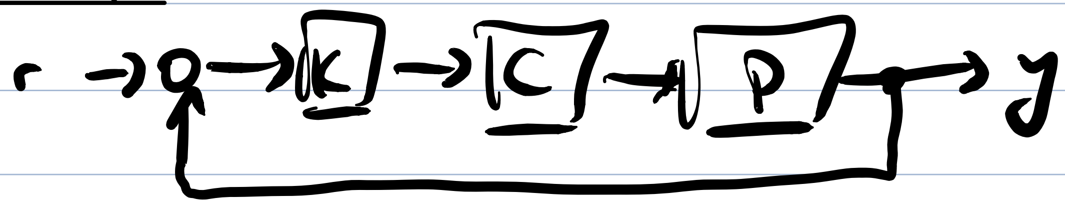
## Lecture 24 Summary

- sequence of root locus examples
  - pendulum on a cart w/
    1) proportional control
    2) P. D.
    3) "practical" P.D. (lead)
    4) P.I.
    5) P.I.D. over to you!

**MATLAB**

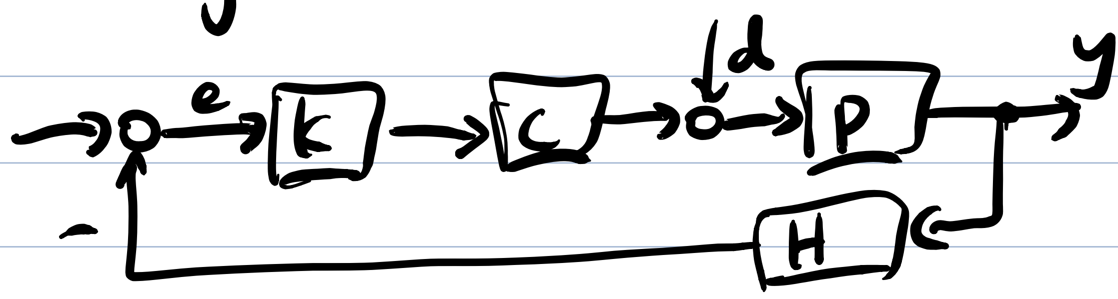



$$CP = \frac{8(s+2)}{(s+1)(s+5)(s+10)}$$

```
PC = zpk( [-2], [-1 -5 -10], 8) //zero-pole-gain
rlocus(PC)
sgrid; //put damping ratio lines (overshoot)
```

## 6.3. Non-standard problems

**Non-unity feedback**



$\pi(s) = \underbrace{D_p D_c D_h}_{D(s)} + K \underbrace{N_p N_c N_h}_{N(s)}$ → proceed as before

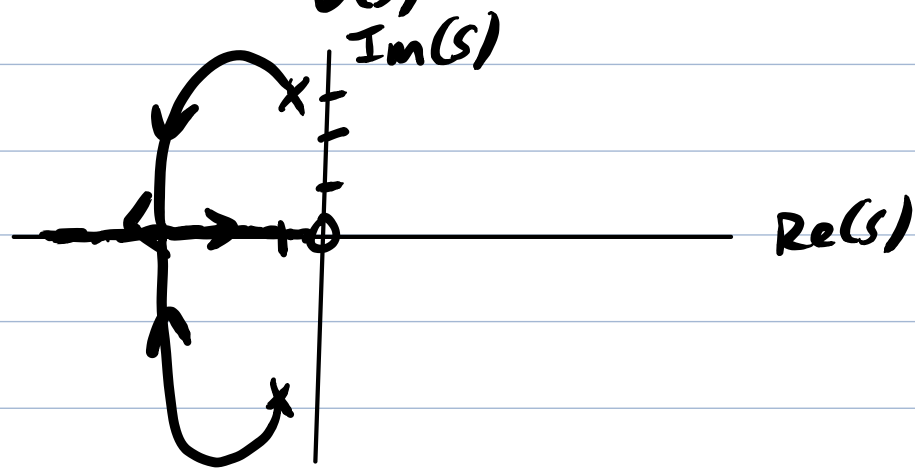
## Controller isn't linear in the gain

- even if we can't factor a $K$ from the controller, the ch.p will still have the form $\pi(s) = D(s) + KN(s)$ except that in this case, $D(s) \neq D_p D_c$ and $N(s) \neq N_p N_c$

**e.g. 6.3.1** $P(s) = \frac{1}{s(s+2)}$ $\quad C(s) = 10(1 + T_d s)$

ch.p. $\pi(s) = s(s+2) + 10(1 + T_d s)$

$$= \underbrace{s^2 + 2s + 10}_{D(s)} + \underbrace{10T_d}_{K} \underbrace{s}_{N(s)}$$




## Improper loop gain

$\pi(s) = D(s) + KN(s) \quad \deg(N) > \deg(D)$

Observe $\pi(s) = 0 \iff N(s) + \frac{1}{K} D(s) = 0 \quad (K \neq 0)$

Define $\hat{D}(s) := N(s)$, $\hat{N}(s) = D(s)$, $\hat{K} := \frac{1}{K}$

Now do the usual root-locus for $\hat{\pi}(s) = \hat{D}(s) + \hat{k} N(s)$

- after drawing the plot for $\hat{\pi}$:
  1. switch o to x
  2. Switch x to o
  3. Reverse arrows

<u>e.g. 6.3.2</u> $P(s) = \dfrac{1}{s(s+1)}$ $C(s) = \dfrac{s+3}{\tau s+1}$

Compute $\pi(s) = s(s+1)(\tau s+1) + s + 3$

$$= \tau s^3 + s^2 + \tau s^2 + 2s + 3$$

$$= s^2 + 2s + 3 + \tau s^2(s+1)$$

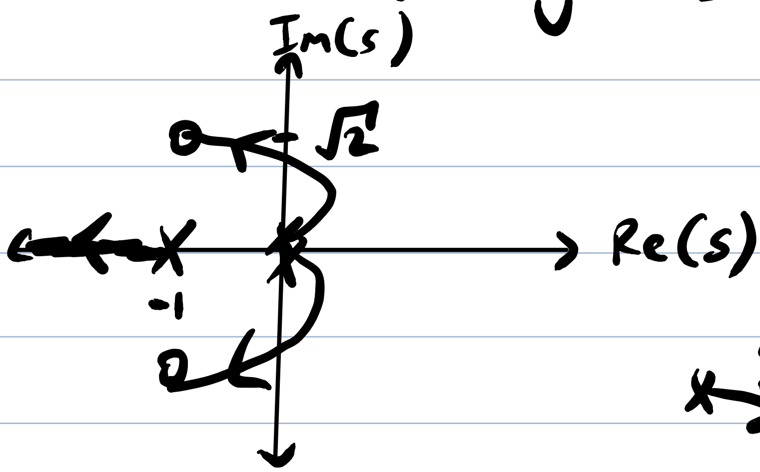
$n - m = -1 \Rightarrow$ Use trick above

$\hat{D}(s) := s^2(s+1)$ $\hat{N}(s) = s^2 + 2s + 3$

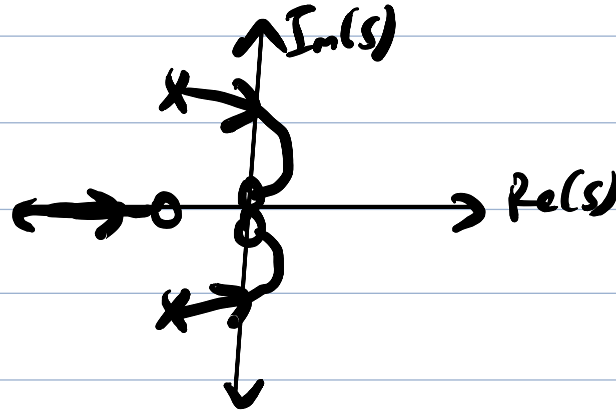
$\hat{n} = 3$ $\hat{m} = 2$

$\{0, 0, -1\}$ $\{-1 \pm j\sqrt{2}\}$

$\hat{\pi}$



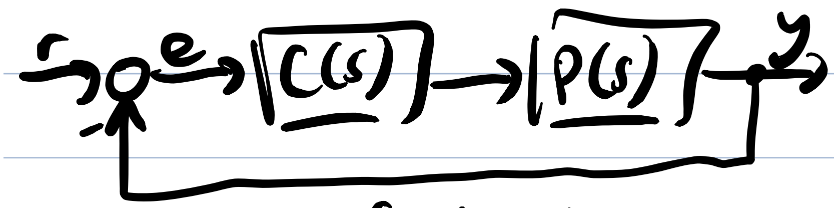

$\pi$



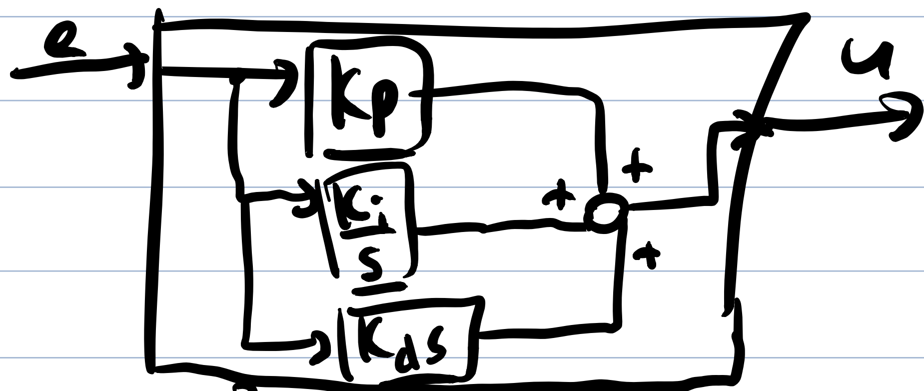
See notes for $K < 0$.

## Ch. 7 PID Controllers

### 7.1. Classical PID




error feedback




P.I.D. controller

$$\frac{U(s)}{E(s)} = K_p + \frac{K_i}{s} + K_d s = \frac{K_d s^2 + K_p s + K_i}{s}$$

time domain: $u(t) = K_p e(t) + K_i \int_0^t e(\tau)\,d\tau$

$$+ K_d \frac{de(t)}{dt}$$

Sometimes expressed as:

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right)$$

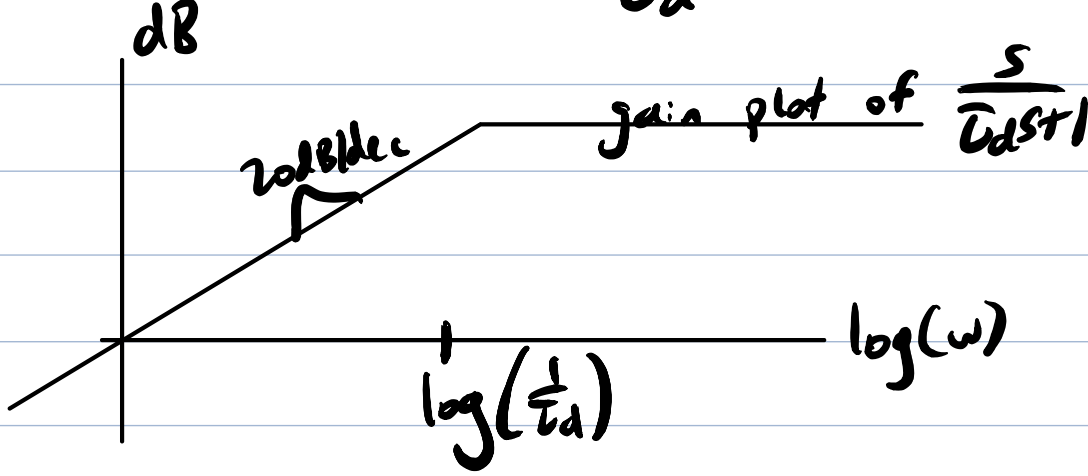
$T_i$ - integral time constant

$T_d$ - derivative time constant

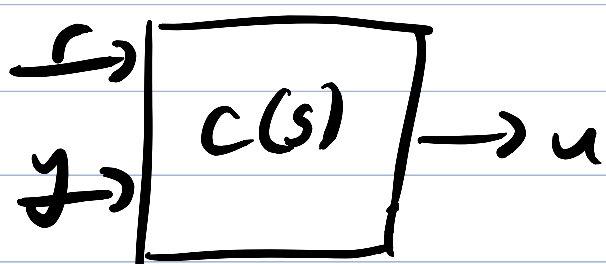
## Refinements to basic PID

1) usually, the derivative term is rolled off.

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{\tau_d s + 1}\right)$$




2) since r is often a sequence of steps, we avoid differentiating and applying $K_p$ to it




two-degrees-of-freedom