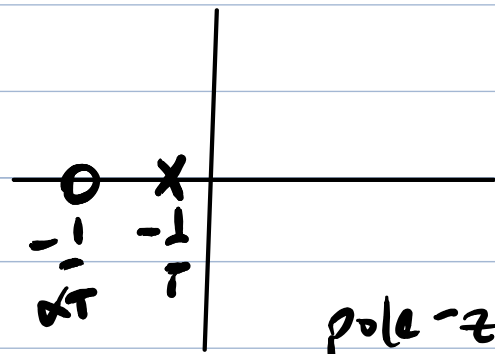
Lecture 29 Summary

- Converting specs $T_s \to \omega_{gc}$ $\quad \%OS \to \Phi_{pm}$
- design for 1) lag (~PI) 2) lead (~PD) 3) lead-lag (~PID)
  - works well for "nice" plants
- lag $C(s) = K\dfrac{\alpha T s + 1}{T s + 1}$ $\quad 0 < \alpha < 1$



pole-zero

e.g. 9.3.1. $P(s) = \dfrac{1}{s(s+2)}$

Specs: 1) $|e_{ss}| \leq 0.05$ for $r(t) = t\mathbb{1}(t)$
2) $\Phi_{pm}^{des} = 45°$

Step 1: $K \geq 40$. I pick $K = 40$.

Step 2: Draw Bode plot of partially compensated plant $KP(s) = \dfrac{40}{s(s+2)}$ (on exam: given Bode plot of P)

- we observe from Bode $\omega_{gc} = 6.17$ rad/s, $\Phi_{pm} = 18°$

Step 3: Design $C_1(s) = \dfrac{\alpha T s + 1}{T s + 1}$

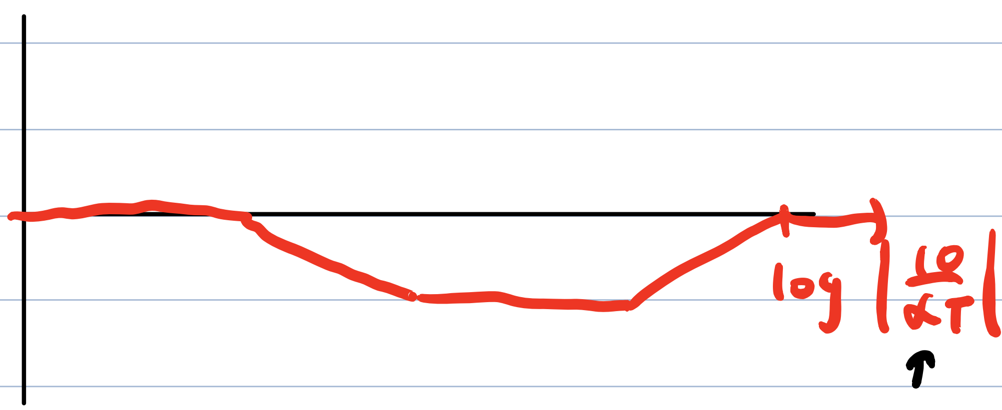
- We want $\Phi_{pm} = 45°$. We'll aim for $50°$ since $\measuredangle C_1(jw)$ only approaches 0 asymptotically
- From the Bode plot, we observe that

  $\Phi_{pm}^{des} = 50° = 180° - \measuredangle KP(jw)$ when $w = 1.7$ rad/s
- So let's reduce the gain at $w = 1.7$ rad/s so that 1.7 becomes the gain crossover frequency, while not changing the phase at $w = 1.7$ rad/s
- the gain of $KP(jw)$ at $w = 1.7$ is 19dB (from Bode)

$$20\log|\alpha| = -19\text{dB}$$

$$\iff \boxed{\alpha = \frac{1}{9}}$$

Bode of lag:



- pick T so that $\measuredangle C(j\omega) \approx 0$ at $\omega = 1.7$ rad/s

$\frac{10}{\alpha T} \leq 1.7$ I'll pick $T = 52.7$ (equality)

$$C(s) = 40 \frac{\left(\frac{1}{9}\right)(52.7)s + 1}{52.7 s + 1} = 4.45 \frac{s + 0.17}{s + 0.019}$$

Step 4: Verify design in simulation
(Lsim command for ramp, don't need step() in MATLAB)

- from Bode plot of $L(s) = C(s)P(s)$ we get $\Phi_{pm} = 44.6°$ at $\omega_{gc} = 1.7$ rad/s. Close enough. ▲

<u>Procedure for Lag design</u>

$$C(s) = K \frac{\alpha T s + 1}{T s + 1} \qquad K, T > 0$$
$$0 < \alpha < 1$$

<u>Specs:</u>

(a) steady-state tracking / dist. rejection
(b) Φ_{pm}^{des}

1. Use FVT to fix K and meet spec (a)
2. Draw Bode plot of $KP(j\omega)$
3. If spec (b) is met, STOP. Else, find ω s.t.

$$180° + \measuredangle P(j\omega) = \underbrace{\Phi_{pm}^{des}}_{\text{given}} + \underbrace{\delta°}_{5°}$$

4. Shift the gain of $KP(s)$ down at ω from Step 3 s.t. ω becomes ω_{gc}.

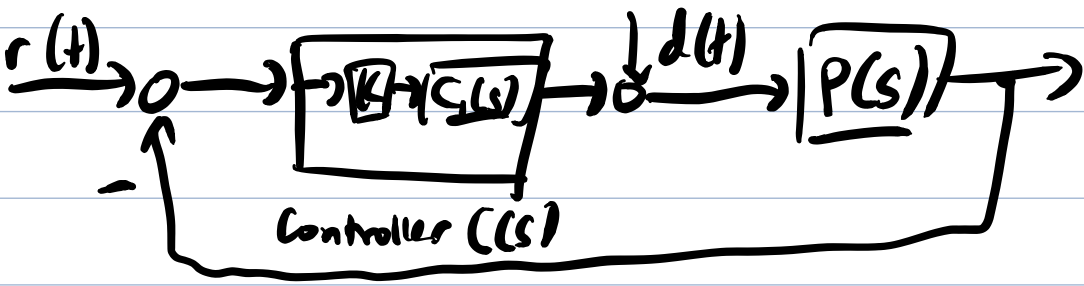
$$\alpha = \frac{1}{KP(j\omega)}$$

5. Put the zero far away from ω s.t. the phase isn't affected very much

$$\frac{10}{\alpha T} \leq \omega$$

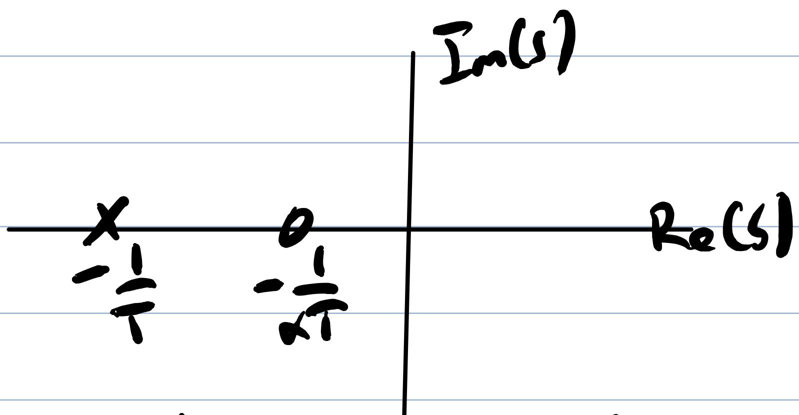
6. Simulate design.

9.4. Lead Controllers

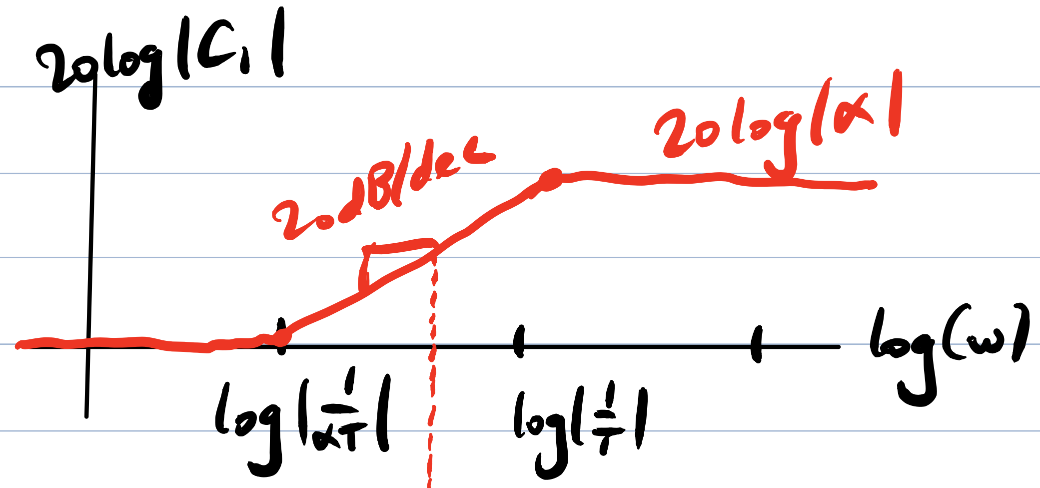


Definition (Lead Controller)

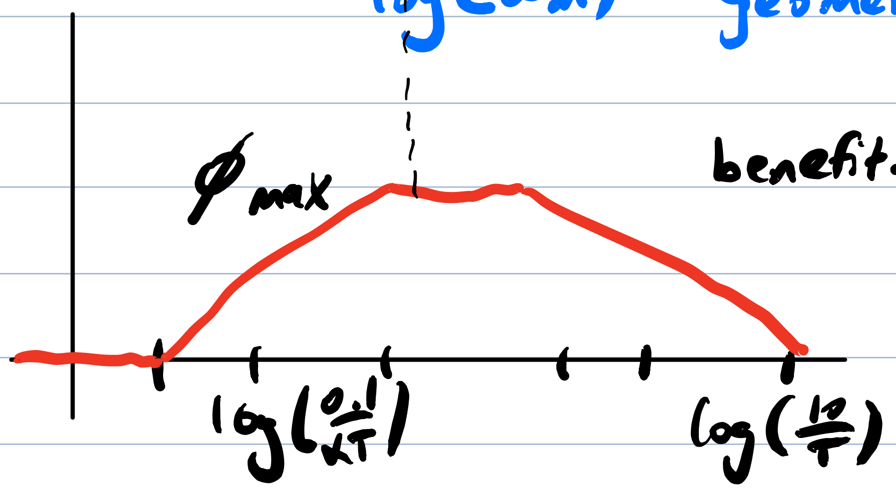
$$C(s) = KC_1(s) = K\frac{\alpha Ts+1}{Ts+1} \qquad \alpha > 1,\ K, T > 0$$



pole/zero of lead controller



$\log(\omega_m)$ - geometric mean



benefit: increase phase

Use of lead controllers

1) increase Φ_{pm} by directly adding phase at the appropriate freq.
2) increase closed-loop bandwidth by increasing ω_{gc}.

Design equations

i) $\omega_m = \dfrac{1}{T\sqrt{\alpha}}$

ii) $|C_1(j\omega_m)| = \sqrt{\alpha}$

iii) $\phi_{max} = \operatorname{asin}\left(\frac{\alpha-1}{\alpha+1}\right)$

$$\Leftrightarrow \quad \alpha = \frac{1+\sin(\phi_{max})}{1-\sin(\phi_{max})}$$

e.g. 9.4.1. (Slightly modified design procedure)

$P(s) = \frac{1}{s(s+2)}$ Specs: 1) $|e_{ss}| \leq 0.05$ for $r(t) = t$

2) $\Phi_{pm}^{des} = 45^\circ$

<u>Trick:</u> Express lead controller in the form

$$C(s) = K\,\frac{\alpha T s + 1}{T s + 1}$$

$$= \frac{\hat{K}}{\sqrt{\alpha}}\,\frac{\alpha T s + 1}{T s + 1}$$

<u>Step 1:</u> Use FVT to fix $\hat{K}$ s.t. $\hat{K}P(s)$ meets steady-state spec.

$\hat{K} \geq 40$ from before, by FVT (9.3.1 ex.)

We pick $\boxed{\hat{K} = 40.}$