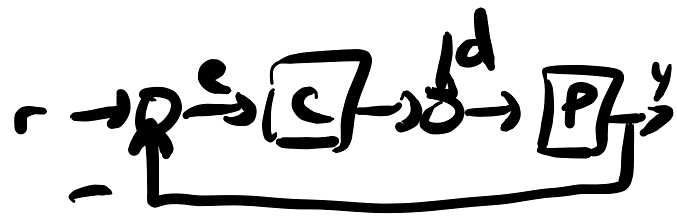
Lecture 20 Summary

- Routh-Hurwitz examples
- steady-state specs



$$Y(s) = \frac{PC}{1+PC} R(s) + \frac{P}{1+PC} D(s)$$

We want, as $t \to \infty$, $y(t) \to r(t)$ despite $d(t)$

More generally, if $C(s)$ provides I.O. stability, and $R(s)$ is a step, then the FVT applies.

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$

$$= \lim_{s \to 0} \frac{s}{1+CP} \frac{r_0}{s}$$

$$= \lim_{s\to 0} \frac{r_0}{1+C(s)P(s)}$$

So $e_{ss}=0 \iff \lim_{s\to 0} C(s)P(s) = \infty$ (CP has pole at origin)

$\boxed{\text{Conclusion: Integral control is fundamental to perfect step tracking.}}$

If $P(s)$ already has a pole at $s=0$, then

$C(s)$ just has to stabilize the loop to get perfect step tracking.

Otherwise, it is common to pick $C(s) = \frac{1}{s} C_1(s)$ and design $C_1(s)$ to get I.O. stability.

Internal model principle

Theorem 5.4.3

Assume P is strictly proper, $C(s)$ proper, and the feedback loop is I.O. stable. If $C(s)P(s)$ contains an internal model of the unstable part of $R(s)$, then perfect asymptotic tracking occurs.

Say $\mathcal{L}\{r(t)\} = R(s) = \dfrac{N_r(s)}{D_r(s)} = \dfrac{N_r(s)}{D_r^-(s) D_r^+(s)}$

roots of D_r^- have $Re(s) < 0$. Roots of $D_r^+(s)$ have $Re(s) \geq 0$.

The IMP says that if

$$C(s)P(s) = \frac{N(s)}{D(s) D_r^+(s)}$$

and we have I.O. stability, then $e_{ss} = 0$.

To apply FVT, $sE(s)$ can't have poles in $\overline{\mathbb{C}^+}$.

$$sE(s) = s\frac{1}{1+PC}R(s) = \frac{s D(s) \cancel{D_r^+(s)} N_r(s)}{\underbrace{\pi(s) D_r^-(s)}_{\text{poles in } \mathbb{C}^-} \cancel{D_r^+(s)}}$$

So FVT applies and $e_{ss} = 0$, as expected.

<u>e.g. 5.4.3.</u> $P(s) = \frac{1}{s+1}$ $\quad r(t) = r_0 \sin(t)$, $\underset{\text{constant}}{r_0 \in \mathbb{R}}$

$$\mathcal{L}\{r(t)\} = \frac{r_0}{s^2+1} \Rightarrow D_r^+(s) = s^2+1 \quad D_r^-(s) = 1$$

This suggests that the controller $C(s) = \frac{1}{D_r^+(s)} C_1(s)$

$$= \frac{1}{s^2+1} C_1(s).$$

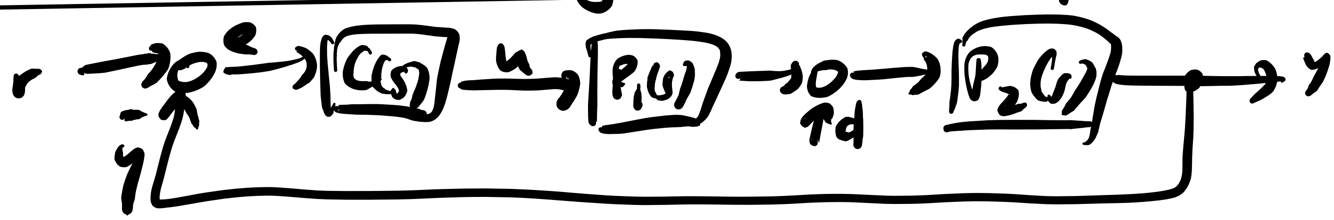
C_1 must be chosen to ensure I.O. stability.

Let's keep it simple, try $C_1(s) = a_2 s^2 + a_1 s + a_0$.

$$\pi(s) = (s+1)(s^2+1) + a_2 s^2 + a_1 s + a_0$$
$$= s^3 + (a_2+1)s^2 + (a_1+1)s + (a_0+1)$$

You can check that $a_2 = a_0 = 0$, $a_1 = 1$ works. $\triangle$

<u>5.4.2. Disturbance Rejection</u> (Steady State)



Generalized Disturbance model

$P_1(s) = 1 \Rightarrow$ input disturbance

$P_2(s) = 1 \Rightarrow$ output disturbance

$$Y(s) = P_2(s)\left(D(s) + P_1(s)U(s)\right)$$ and

$$\frac{Y(s)}{D(s)} = \frac{P_2(s)}{1 + C(s)P_1(s)P_2(s)}$$ (verify!)

Best case scenario: $\lim_{t\to\infty} y(t) = 0$

(disturbance has no lasting effects on the output)

Write $\mathcal{L}\{d(t)\} = D(s) = \frac{N_d(s)}{D_d^-(s)\,D_d^+(s)}$

Assume $P_1 P_2$ strictly proper, C proper, and I.O. stability.

Let's see when FVT will apply:

$$y_{ss} = \lim_{t\to\infty} y(t) \overset{?}{=} \lim_{s\to 0} sY(s)$$

$$= \lim_{s\to 0} s\frac{P_2}{1+CP_1P_2}\,\frac{N_D}{D_d^- D_d^+}$$

$$= \lim_{s\to 0} \frac{s D_c D_{p_1} N_{p_2}}{\underbrace{D_c D_{p_1} D_{p_2} + N_c N_{p_1} N_{p_2}}_{\text{Hurwitz by assumption}}} \cdot \frac{N_d}{D_d^- D_d^+}$$

This suggests, to reject disturbances reliably, the controller should contain an internal model $D_d^+(s)$.

$$C(s) = \underbrace{\frac{1}{D_d^+(s)}}_{\text{internal model}} \underbrace{C_1(s)}_{\text{chosen for feedback stability}}$$