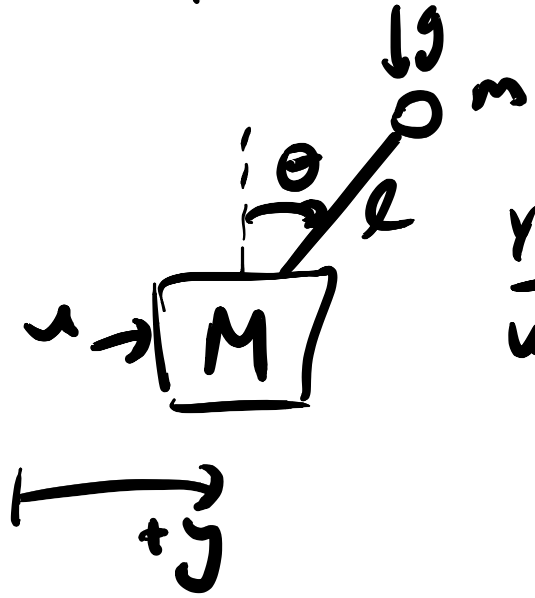
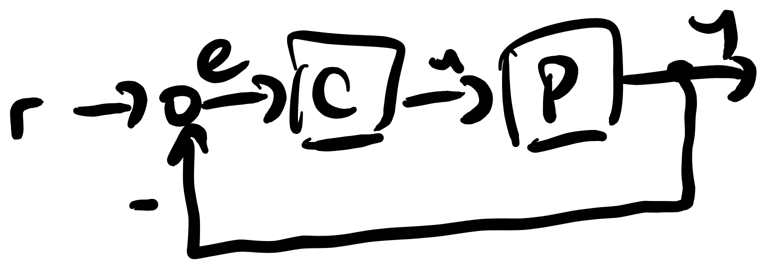
Summary Lecture #23

- root-locus examples

e.g. 6.2.3



$$\frac{Y(s)}{U(s)} = P(s) = \frac{\frac{1}{M\ell}}{s^2 - \frac{g}{M\ell}(M+m)}$$



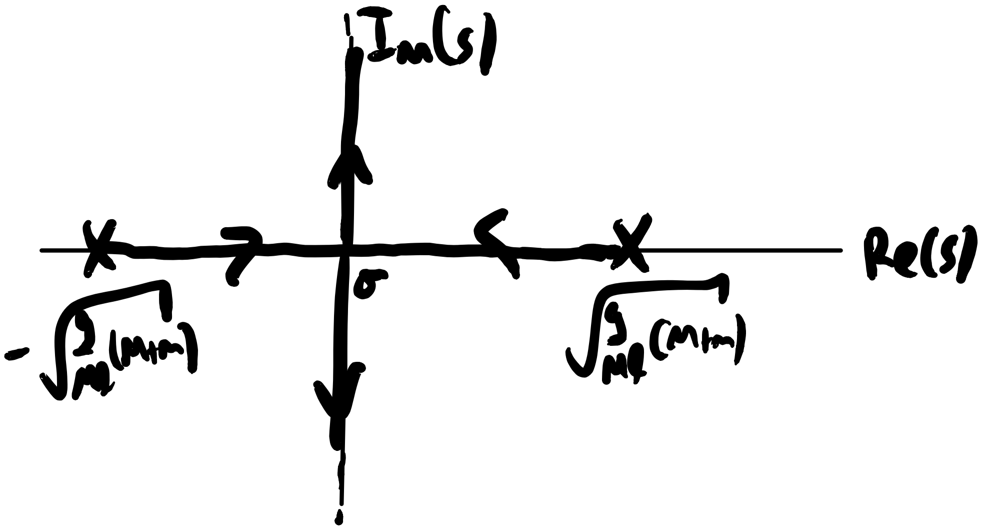
- let's see if a proportional controller can stabilize the loop.

$$C(s) = K_p$$

$$\pi(s) = \frac{K_p}{M\ell} + s^2 - \frac{g}{M\ell}(M+m)$$

$$D(s) := s^2 - \frac{g}{M\ell}(M+m) \qquad N(s) := 1$$

$$K := \frac{K_p}{M\ell}$$



Check that $\sigma = 0$

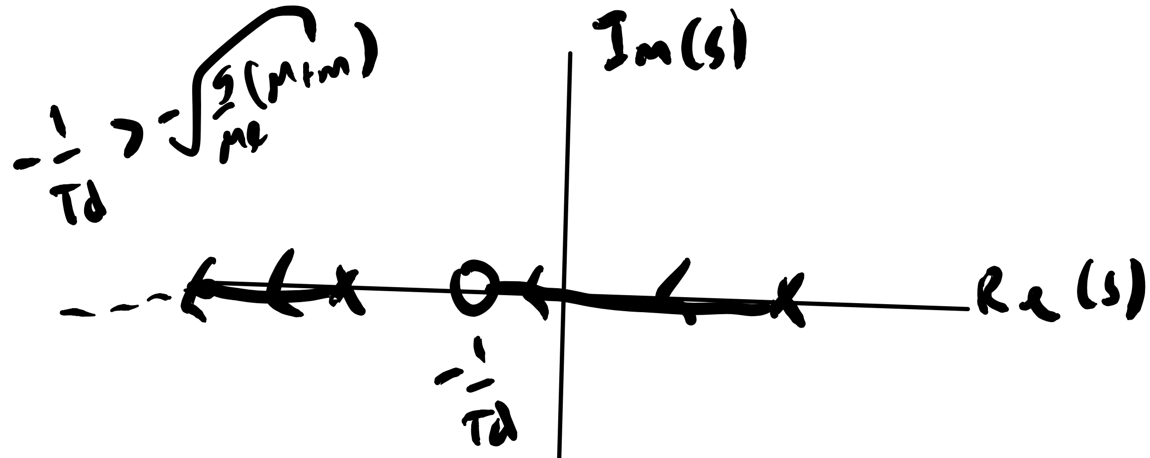
$n - m = 2$

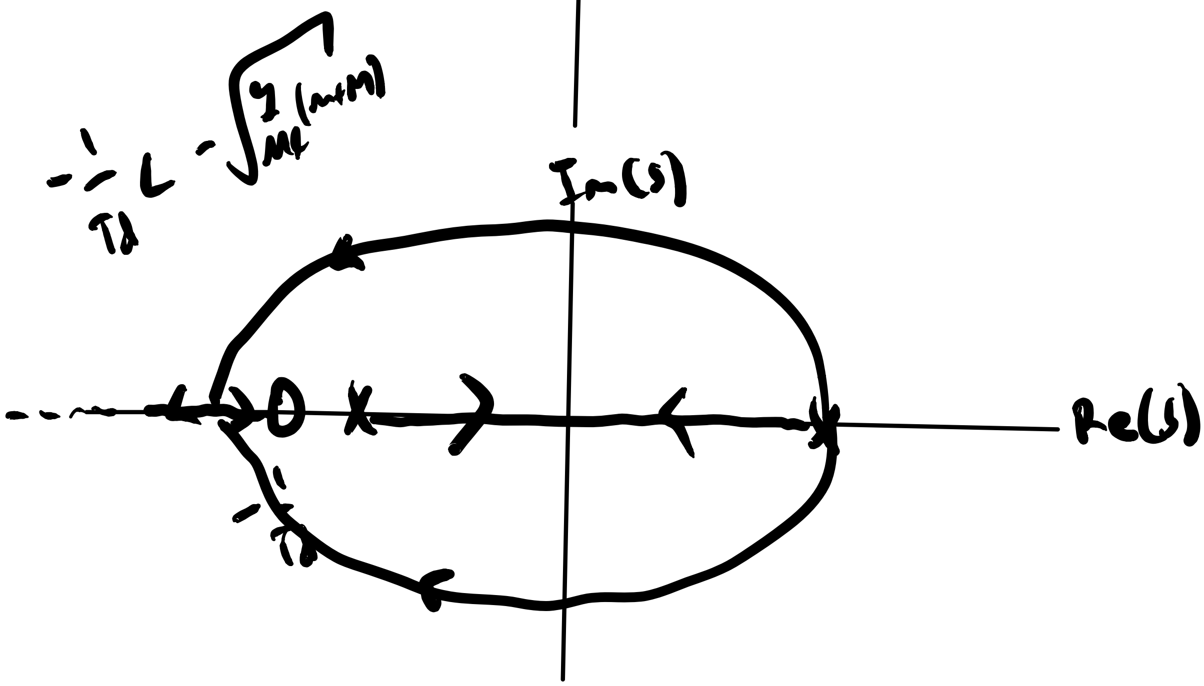
$\Rightarrow$ Proportional won't work.

Try a PD controller.

$$C(s) = K_p(1 + T_d s)$$
$$= K_p T_d\left(s + \frac{1}{T_d}\right)$$

$$\Pi(s) = \underbrace{s^2 - \frac{g}{Ml}(M+m)}_{\substack{D(s) \\ n=2}} + \underbrace{\frac{K_p T_d}{Ml}}_{K}\underbrace{\left(s + \frac{1}{T_d}\right)}_{\substack{N(s) \\ m=1}}$$





$n - m = 1$

- both stabilize the loop. I'd pick the right case for settling time reasons. ▲

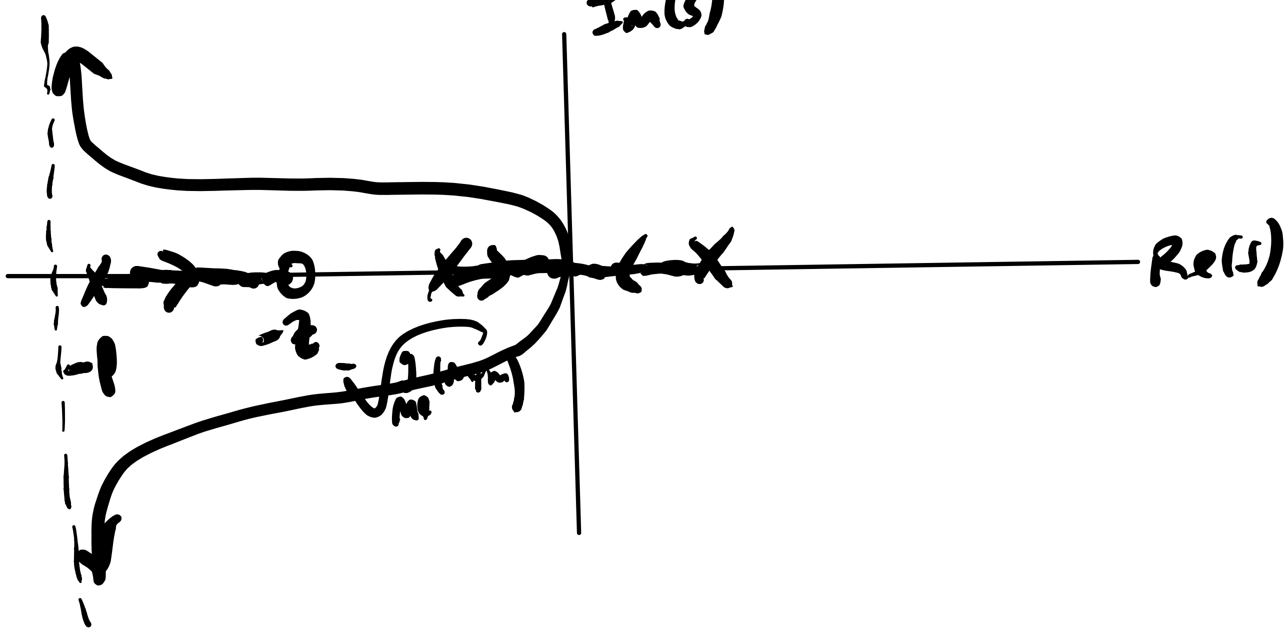
<u>e.g.</u> (Practical P.D., Lead Controller)

$$P(s) = \frac{\frac{1}{M\ell^2}}{s^2 - \frac{g}{M\ell}(m+M)} \qquad C(s) = K_p + K_d \frac{s}{\tau_d s + 1}$$

$$C(s) = \frac{(K_p + K_d/\tau_d)s + K_p/\tau_d}{s + \frac{1}{\tau_d}}$$

$$= \frac{1}{M\ell}K\frac{(s+z)}{(s+p)}$$

$$\pi(s) = (s+p)\left(s^2 - \frac{g}{M\ell}(m+M)\right) + K(s+z)$$



e.g. 6.2.5

$$P(s) = \frac{\frac{1}{M\ell}}{s^2 - \frac{g}{M\ell}(m+M)}$$

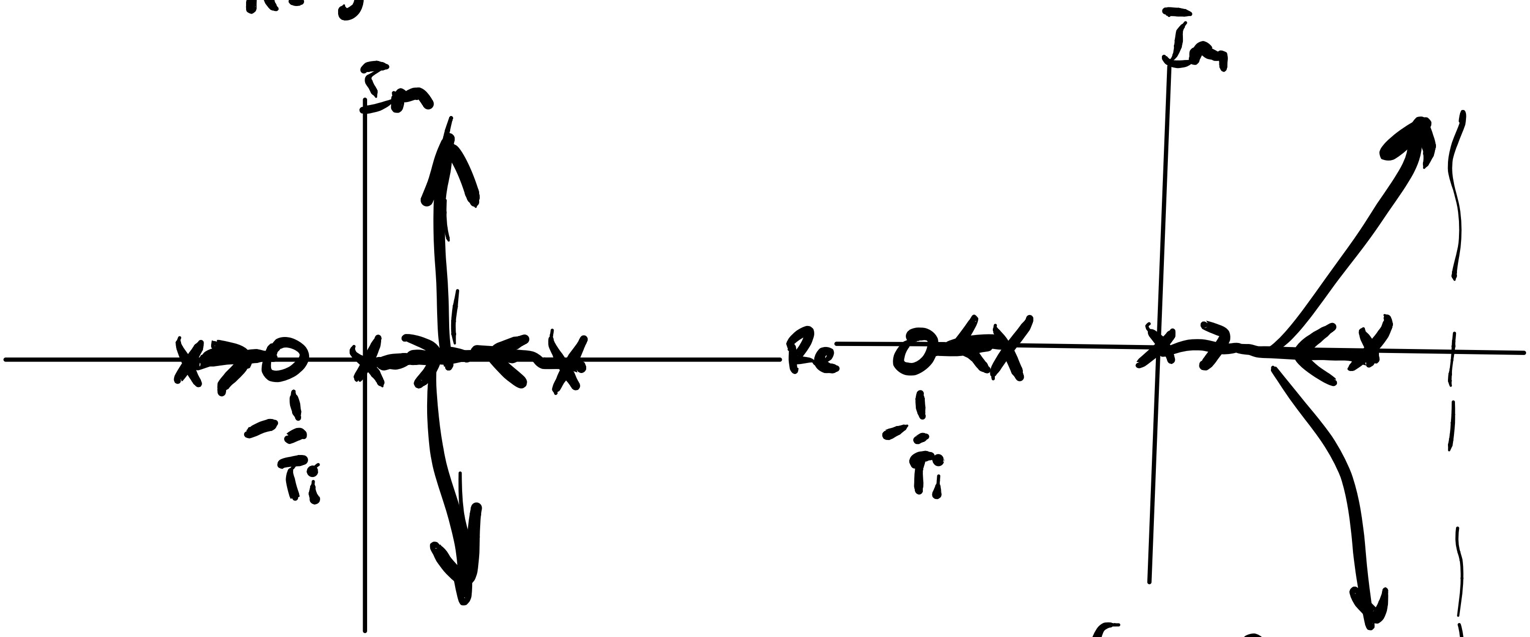
ℓ might be squared but doesn't affect root-locus shape

P.I. control

$$C(s) = K_p\left(1 + \frac{1}{T_i s}\right)$$

$$= K_p\frac{1}{s}(s + 1/T_i)$$

$$\pi(s) = \underbrace{s\left(s^2 - \frac{g}{Ml}(m+M)\right)}_{\substack{D(s) \\ n=3}} + \underbrace{\frac{K_p}{Ml}}_{K}\underbrace{\left(s + \frac{1}{T_i}\right)}_{\substack{N \\ m=1}}$$



Case 1

Case 2

need to check centroid location

$$\sigma = \frac{0 - \left(-\frac{1}{T_i}\right)}{3-1} = \frac{1}{2T_i} \rightarrow \text{Case 1}$$

$$\sigma = \frac{1}{2T_i}$$

Case 2