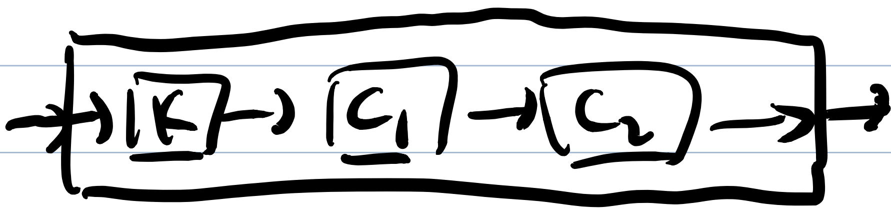
Lecture 31 Summary

- lead controller design, procedure/examples
- lead-lag

$$C(s) \triangleq K \underbrace{C_1(s)}_{\text{lead}} \underbrace{C_2(s)}_{\text{lag}} = K \frac{\alpha_1 T_1 s + 1}{T_1 s + 1} \frac{\alpha_2 T_2 s + 1}{T_2 s + 1}$$

$\alpha_1 > 1$ $\qquad$ $0 < \alpha_2 < 1$



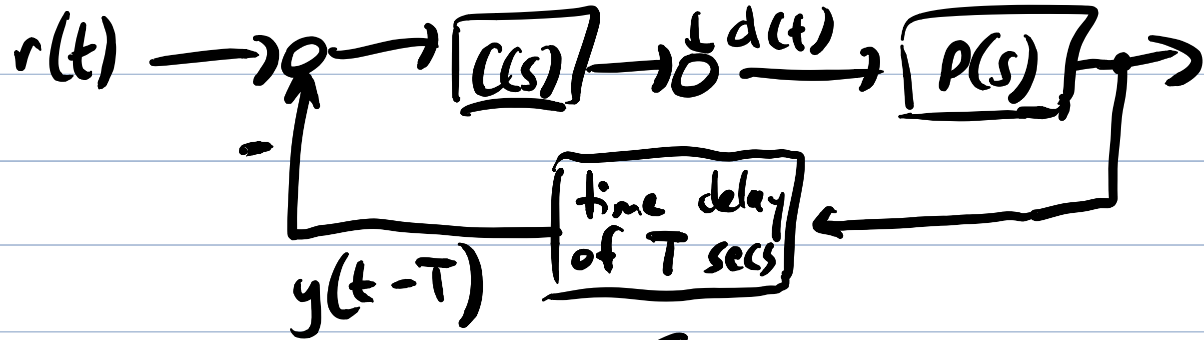
Procedure for lead-lag design

Specs: (a) steady-state tracking or disturbance rejection

(b) Φ_{pm}^{des}

1. Use FVT to pick K and meet spec (a).
2. Draw Bode of $KP(j\omega)$. Get Φ_{pm}, ω_{gc}.
3. Split $\Phi_{pm}^{des} = \Phi_{pm,1}^{des} + \Phi_{pm,2}^{des}$ into two roughly equal parts.
4. Design lag controller C_2 to achieve $\Phi_{pm} = \Phi_{pm,2}^{des}$
5. Draw Bode plot of $KC_2(j\omega)P(j\omega)$.
6. Design Lead controller C_1 to get $\Phi_{pm} = \Phi_{pm,1}^{des}$.

Phase-margin and time delay



- TF of delay is e^{-sT}
- Bode plot of delay

$$|e^{-j\omega T}| = 1$$
$$\angle e^{-j\omega T} = -\omega T$$

$\Rightarrow$ time delay only affects phase plot, not gain plot.

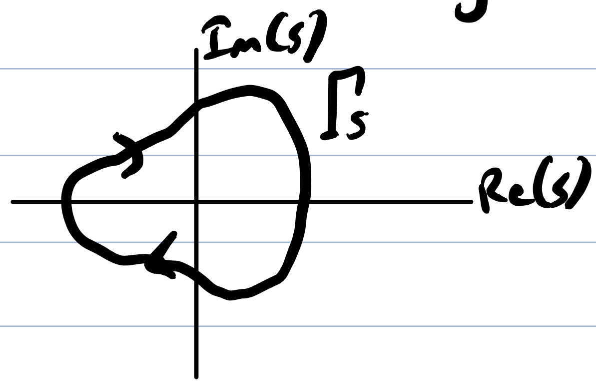
- suppose that when $T=0$, system has a phase margin of Φ_{pm} (radians) at ω_{gc}.
- so the system will go unstable if the phase is reduced by Φ_{pm} radians at freq ω_{gc}.
- therefore, the maximum delay we can tolerate before losing stability satisfies

$$\omega_{gc} T_{max} = \Phi_{pm}$$
$$\Leftrightarrow \boxed{T_{max} = \frac{\Phi_{pm}}{\omega_{gc}}}$$

Ch.8. Freq. Domain Stability Analysis

8.1. Cauchy's Principle of the Argument

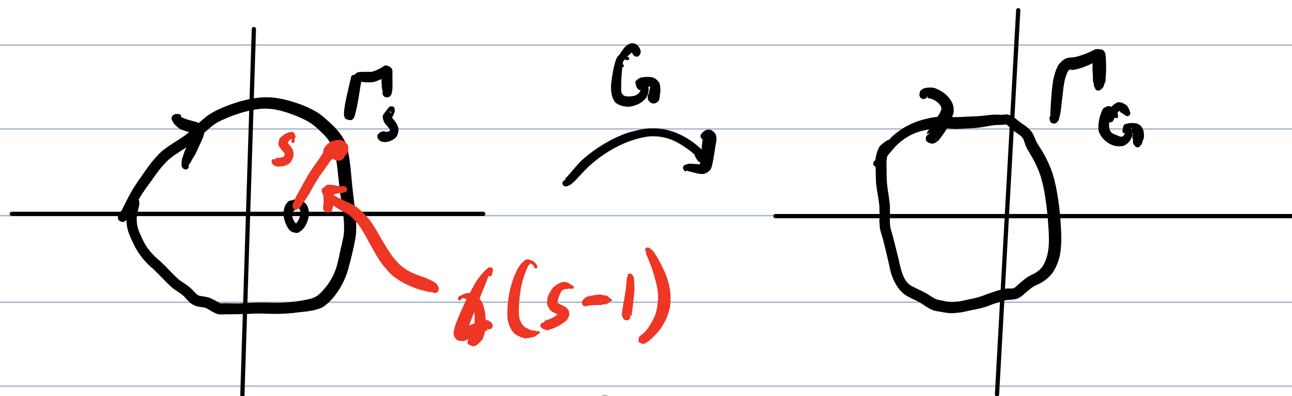
Consider a closed curve in $\mathbb{C}$ with no self-intersections with negative (cw) orientation.



Let $G(s)$ be rational. For each $s \in \mathbb{C}$, $G(s) \in \mathbb{C}$.
$G: \mathbb{C} \to \mathbb{C}$

If Γ_s doesn't pass through any poles or zeros of G, then as s makes a circuit around Γ_s, $G(s)$ will trace out a different closed curve. Call it Γ_G.

<u>e.g. 8.1.1.</u> $G(s) = s - 1$

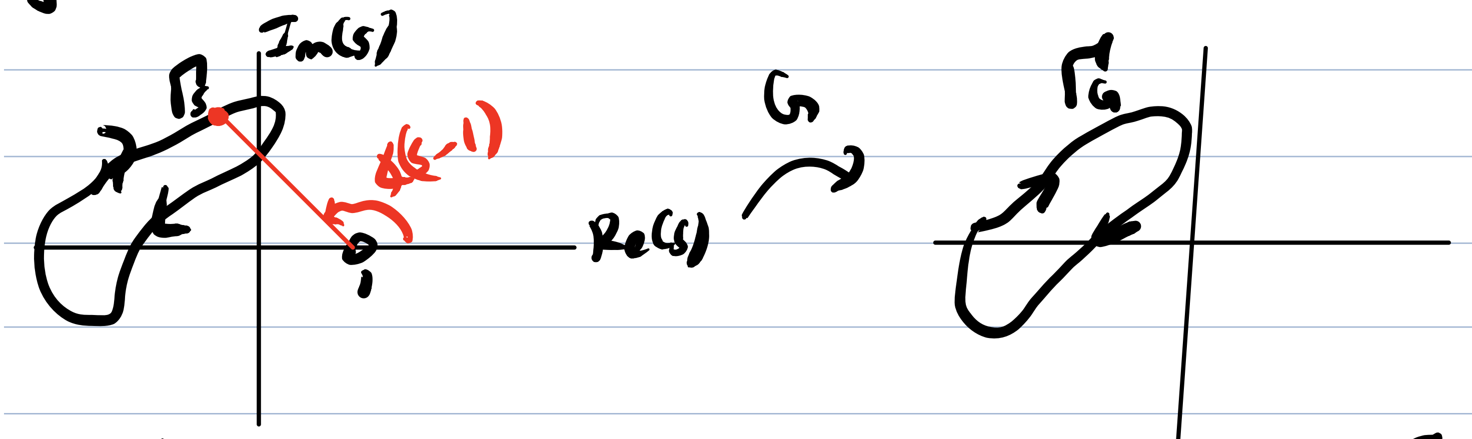


- Γ_G is a shift of Γ_s to left by 1.
- note $\measuredangle(s-1) = \measuredangle G(s)$.
- the net change in $\measuredangle(s-1)$ is -2π as s travels around Γ_s.

$\Rightarrow$ net change of $\measuredangle G(s) = -2\pi$ as s travels around Γ_s.

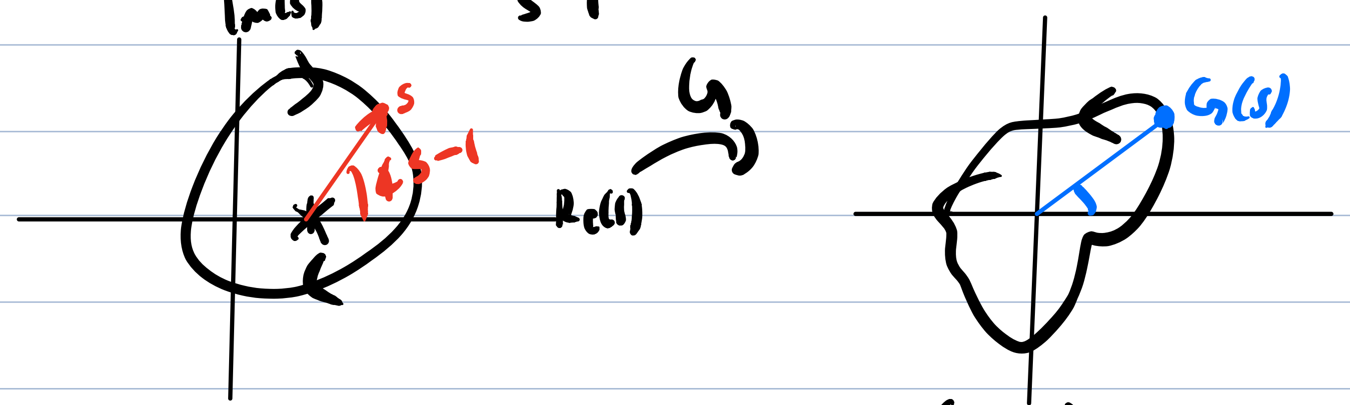
- So Γ_G encircles the origin once, in the CW direction. Δ

e.g. 8.1.2. $G(s) = s-1$



Net change is zero as s travels around Γ_s.
thus Γ_G doesn't encircle the origin. Δ

e.g. 8.1.3. $G(s) = \dfrac{1}{s-1}$



In this example, $\measuredangle G(s) = -\measuredangle(s-1)$
$\Rightarrow$ the net change as s travels around Γ_s is $+2\pi$ in $\measuredangle G$

Theorem (Cauchy's Principle of the Argument)

Suppose Γ_s encircles n poles of G and m zeros of G. Then Γ_G encircles the origin $n-m$ times in the counterclockwise direction.