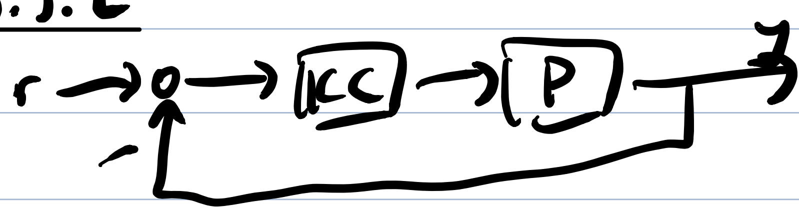
Summary Lec 34

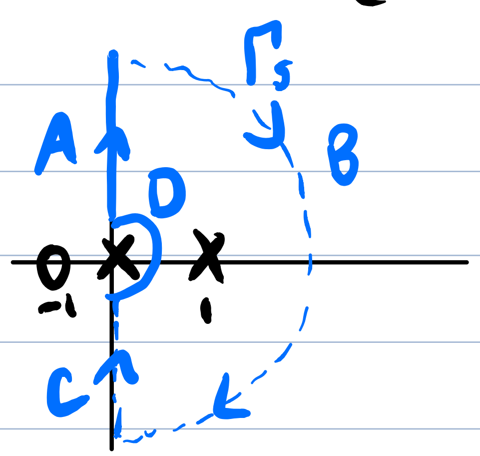
- Nyquist: - $C(s)P(s)$ has n poles with $Re(s)>0$
  - closed-loop system is I.O. stable iff Nyquist plot encircles $-\frac{1}{K}+0j$ n times in CCW direction
- example
- important parts of Nyquist contour, relation to Bode

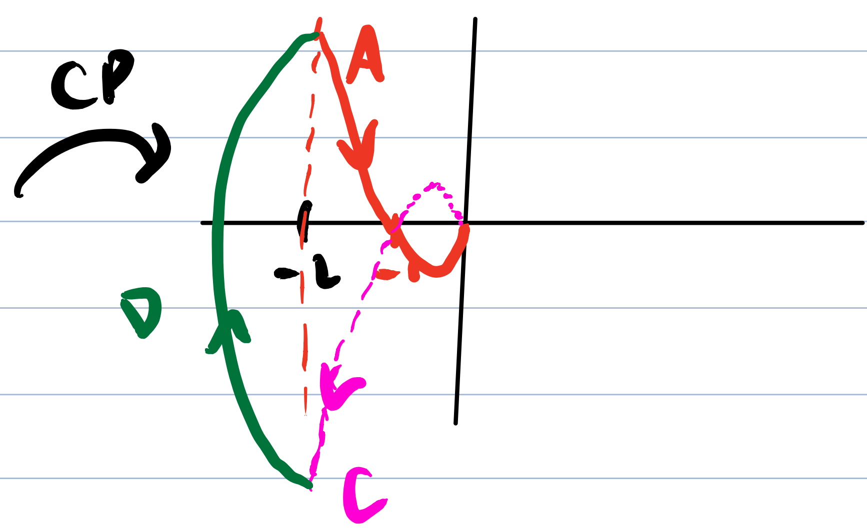
<u>note:</u> 8 questions on final, 1 T/F.
Chris will host review session the day before (to answer Qs)

<u>e.g. 8.3.2</u>



$$C(s)P(s) = \frac{s+1}{s(s-1)}$$ (like dc motor with extra zero)





$n=1$

Segment A: $s = j\omega \quad \omega \in [\varepsilon, \infty), \; 0 < \varepsilon \ll 1$

$$C(j\omega)P(j\omega) = \frac{j\omega + 1}{j\omega(j\omega - 1)} \cdot \frac{(-j)}{(-j)}$$

$$= \frac{-j + \omega}{\omega(j\omega - 1)}$$

$$= \frac{j - \omega}{\omega(1 - j\omega)} \cdot \frac{1 + j\omega}{1 + j\omega}$$

$$= \frac{j - \omega - \omega - j\omega^2}{\omega(1 + \omega^2)}$$

$$= \frac{-2}{1 + \omega^2} + j \, \frac{1 - \omega^2}{\omega(1 + \omega^2)}$$

- real part always < 0 for $\omega \in [\varepsilon, \infty)$
- when $\omega = 1$, plot crosses Re axis at $s = -1$
- when $\omega \in [\varepsilon, 1)$, Im part is positive
- when $\omega \in (1, \infty)$, Im part < 0
- when $\omega = \varepsilon$, $C(j\omega)P(j\omega) \approx -2 + j\infty$

Segment B: to the origin

Segment C: just a reflection

Segment D: $s = \varepsilon e^{j\theta}, \quad \theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$$C(\varepsilon e^{j\theta})P(\varepsilon e^{j\theta}) = \frac{\varepsilon e^{j\theta} + 1}{\varepsilon e^{j\theta}(\varepsilon e^{j\theta} - 1)}$$

$$\approx \frac{1}{\varepsilon e^{j\theta}(-1)}$$

$$= \frac{1}{\varepsilon} e^{-j\theta} \cdot (-1)$$

$$= \frac{1}{\varepsilon} e^{j(\pi - \theta)}$$

Count encirclements:

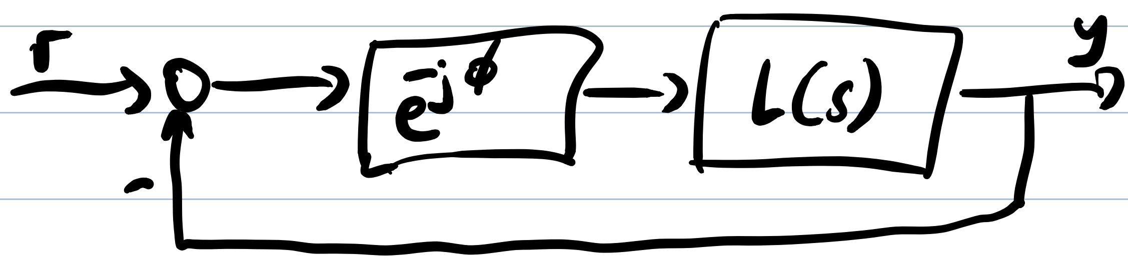
	$-\infty < -\frac{1}{k} < -1$	$-1 < -\frac{1}{k} < 0$	$0 < -\frac{1}{k} < \infty$
N	-1	1	0

Since $n = 1$, we need $N = 1$ for I.O. stability

$\iff \boxed{K > 1}$

8.4. Stability Margins

8.4.1. Phase Margin

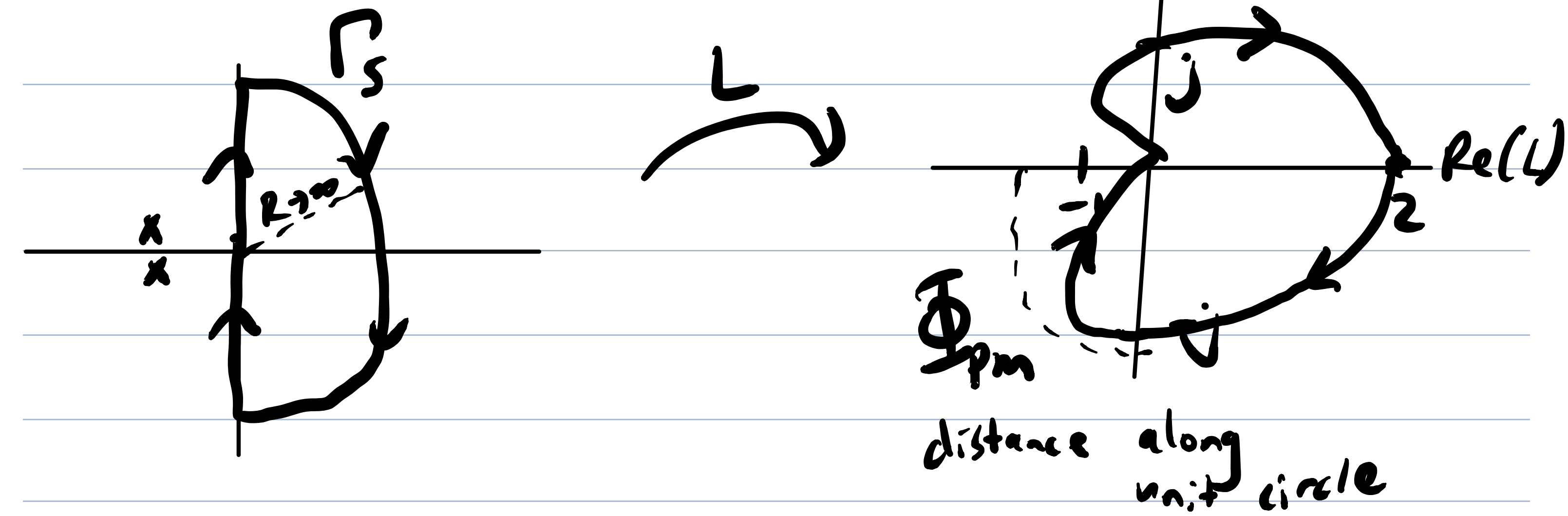


$\phi = 0$ nominal design, stable

e.g. 8.4.1. $L(s) = \frac{2}{(s+1)^2}$, $n = 0$, no bad poles

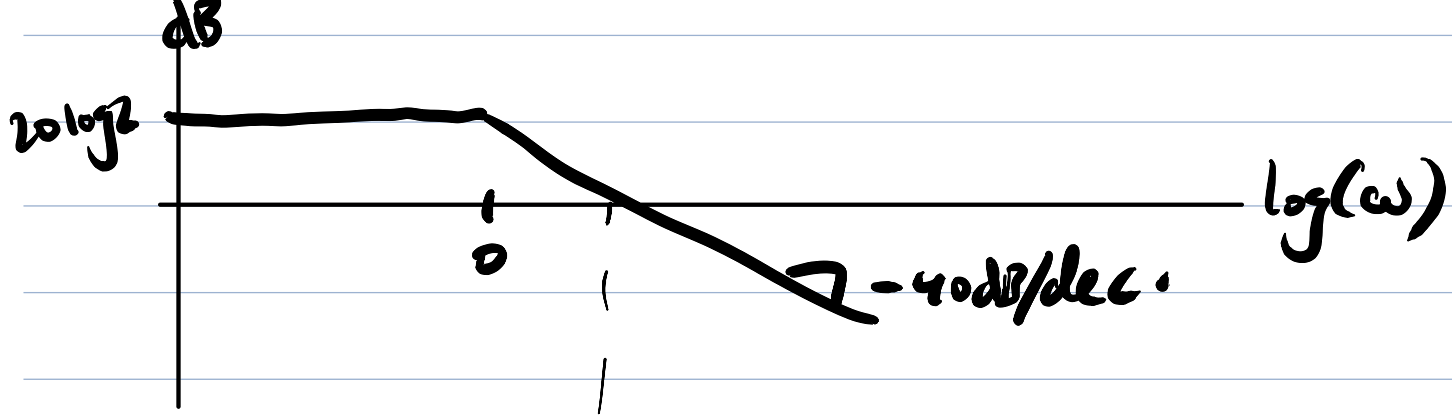
- for feedback stability, we need $N = 0$ ccw encirclements of -1

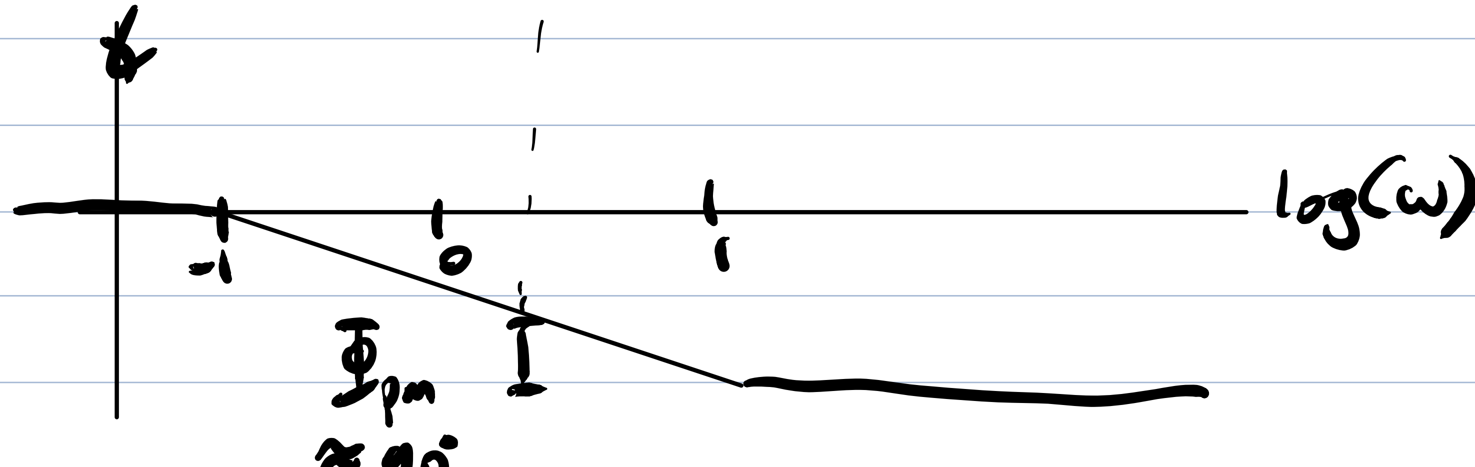
Im(L)



$$L(j\omega) = \frac{2}{(j\omega+1)^2} = \frac{2}{-\omega^2+2\omega j+1}$$
$$= \dots = \frac{2}{(\omega^2+1)^2}\left[(1-\omega^2) - j2\omega\right]$$

- if we rotate Nyquist by $-\frac{\pi}{2}$, we lose stability
- $\Phi_{pm} = \frac{\pi}{2}$





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