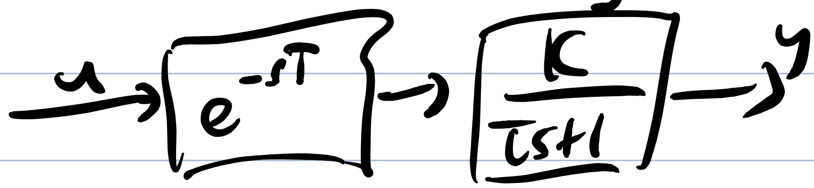
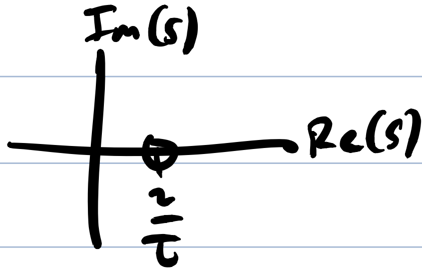
Lecture 27 Summary

- Pole placement for PID design
- plants with time delay



  - approx. e^{-sT} using rational TF (Padé)
  - delay puts a <u>zero</u> at $s = \frac{2}{T}$

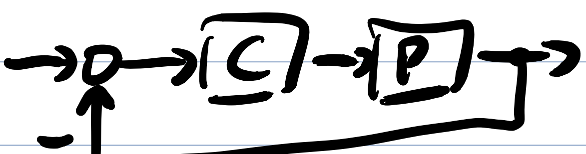


- intro to freq. domain design

<u>Intro to gain and phase margins</u>

If a system is stable, how stable is it?
This depends on our plant model and how much certainty there is in the model.

- gain and phase margins are a way to quantify how much uncertainty our design can tolerate.
  → best understood using Nyquist plots (Ch. 8)
- in this intro, we'll use Bode plots to describe them.



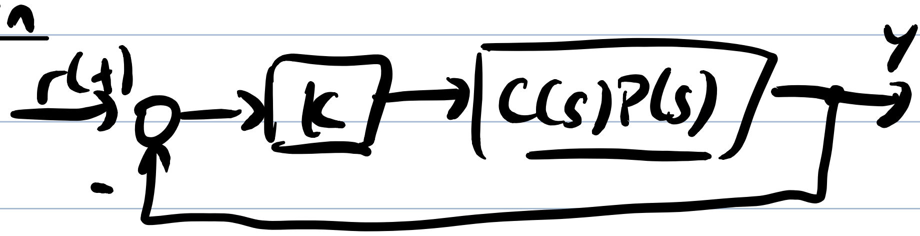
P(s) – plant model used for design

$$P(s)\cdot\underbrace{\Delta P(s)} \xrightarrow{s=j\omega} |P(j\omega)\,\Delta P(j\omega)| = |P(j\omega)|\,|\Delta P(j\omega)|$$

uncertainty in plant model

$$\measuredangle P(j\omega)\,\Delta P(j\omega) = \measuredangle P(j\omega) + \measuredangle \Delta P(j\omega)$$

<u>Gain margin</u>

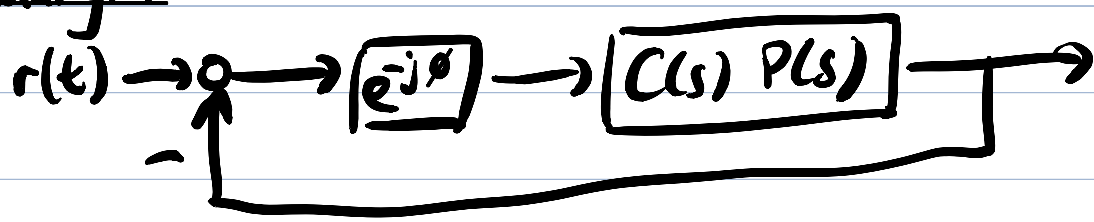


- think of $K=1$ as your nominal design

<u>Definition</u>

$$K_{gm} := \sup\{\bar{K} > 1 : \text{closed-loop stability for } K \in [1, \bar{K})\}$$

<u>Phase margin</u>



- think of $\phi = 0$ as the nominal design.

<u>Definition</u>

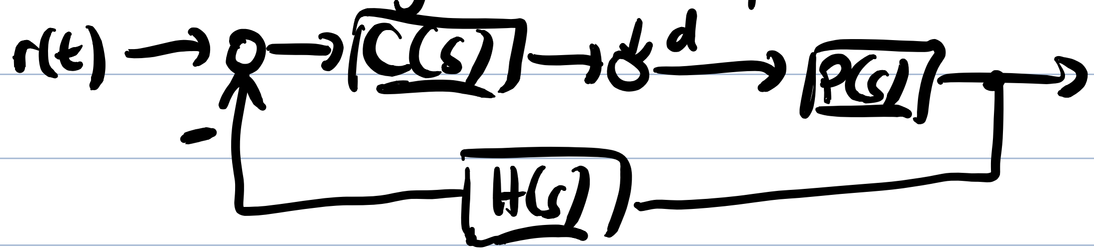
$$\Phi_{pm} := \sup\{\bar{\phi} > 0 : \text{closed-loop stability for } \phi \in [0, \bar{\phi})\}$$

- big gain and phase margins usually ensure robustness.

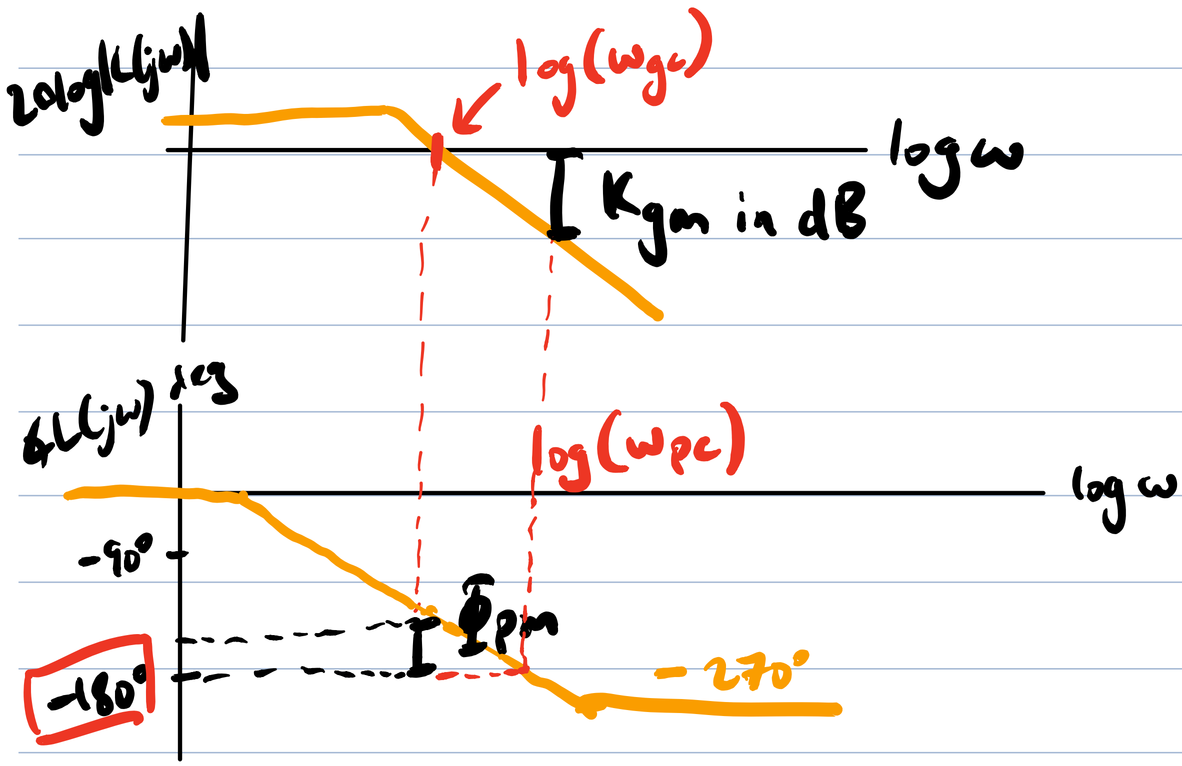
→ also gives good transient performance

Intuition: a system with small margins K_{gm} and Φ_{pm} is close to being unstable. (poles close to Im axis) which means oscillatory and slow dynamics

How to read K_{gm} and Φ_{pm} from Bode plots



$L(s) := C(s)\,P(s)\,H(s)$



w_{gc} - gain crossover frequency. Freq. at which we measure the phase margin Φ_{pm}

w_{pc} - phase crossover frequency. Freq. at which we measure the gain margin.

$$\frac{PC}{1+L}$$

$\uparrow$ -1