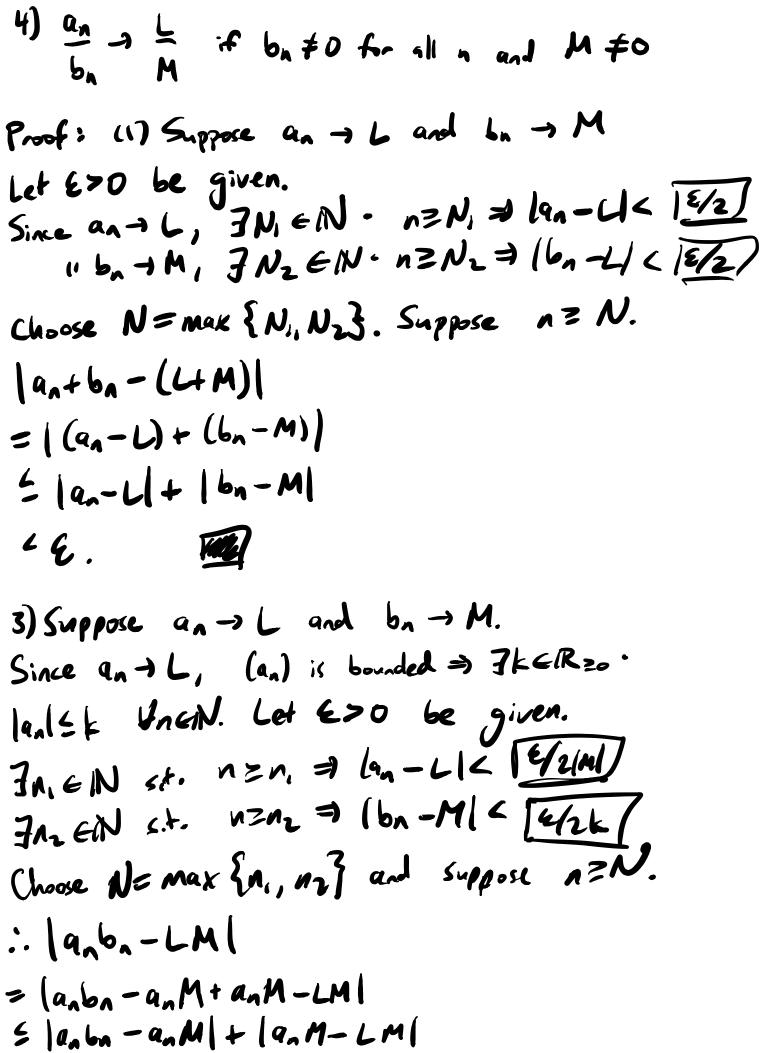
Week 2 A1 due, AZ posted tomorrow 2.5. Properties of Limits Definition We say (an) is bounded if {an: nEN} is bounded. i.e. lan I M the N Proposition: If an -> L then (an) is bounded. Proof: Suppose an - L. For E=1, there exists NEN s.t. 19n-L/21 when AZN :. -1 < an-L < 1 : -1+L < an < 1+L Take m = min { a, a, ..., a, ..., L-15, M = max {a, az, ..., an -1, L+1} : m = an = M Une N :. (an) is bounded. eg. $(a_n) = (\sin(n))$ $(a_n) = (-1)^n$ bounded but not conveyent (converse not the) Proposition and L, bn + M 1) ant bn -> L+M

2) xa, > xL, xER 3) a, b, > LM



= | an(bn-M) | + | M(Gn-L)| = | aal | bn - M + | M | an - L/ = K | 6n - M | + | M | 19n - L | 16. Note: If IMI = 0, take I an - LI 61 instead of 6 2/M1. Hist to prove (4) - first prove on limit law then use product rule 2.6. Monotone Convergence Theorem <u>Definition</u> We say (an) is: Ostrictly monotone increasing 1) Monotone increasing if ansanti Unes if and anti trew 4) strictly m. dec. 3) mondone decreasing if an > anti Vacill if an Zani theN Theorem (Monotone Convergence Theorem): Every monotone increasing sequence that is

bounded above converges. Same fir mon.

dec. 5e7. bounded below.

4. Sup)

4. Lift

Definition

We say (an) is monotone if it is monotone increasing or monotone decreasing.

Proof next time -