Lecture 19 Summary

- unstable pole-zero cancellation <u>BAD</u>
- internal stability $\Rightarrow$ I.O. stability
- If $s^n + a_{n-1}s^{n-1} + \dots + a_1 s + s_0$ is Hurwitz, then $a_i > 0$ for all i

Routh-Hurwitz

(i) $\pi(s)$ is Hurwitz $\Leftrightarrow$ all elements in 1st column of Routh table have the same sign

(ii) If there are no zeros in 1st column, then

- $\#$ of sign changes $= \#$ of bad roots
- π has no roots on Im axis

e.g. 5.3.2 $\pi(s) = a_2 s^2 + a_1 s + a_0$

s^2	a_2	a_0
s^1	a_1	0
s^0	$\frac{a_1 a_0 - a_2 0}{a_1} = a_0$	

π is Hurwitz if and only if a_2, a_1, a_0 have the same sign.

$\triangle$

e.g. 5.3.3

$\pi(s) = 2s^4 + s^3 + 3s^2 + 5s + 10$

s^4	2	3	10
s^3	1	5	0
s^2	-7	10	
s^1	$\frac{45}{7}$	0	
s^0	10		

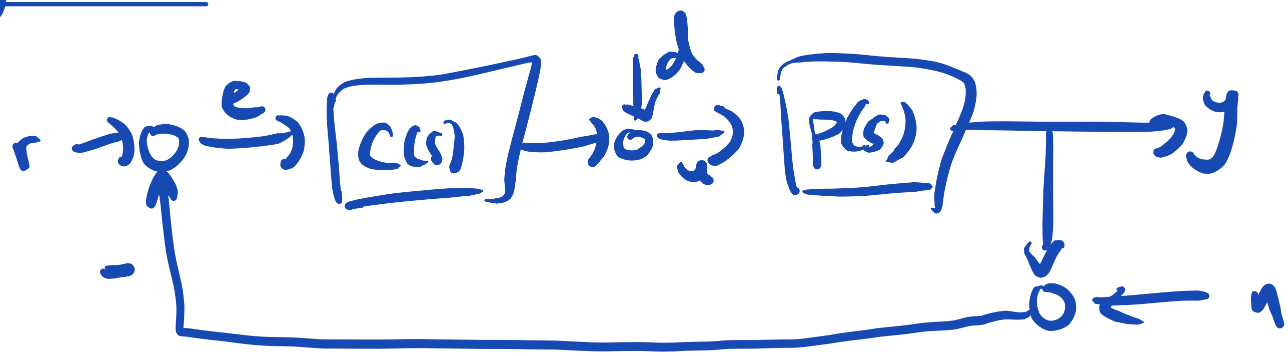
(i) π is <u>not</u> Hurwitz

(ii) The list of signs in 1st column: $\{+, +, -, +, +\}$

$\rightarrow$ 2 sign changes

$\Rightarrow$ 2 bad roots

e.g. 5.3.4



$$P(s) = \frac{1}{s^4 + 6s^3 + 11s^2 + 6s}$$

Let's see if proportional control will stabilize the loop.

$C(s) = K_p$

$\pi(s) = K_p + s^4 + 6s^3 + 11s^2 + 6s$ (Necessary: $K_p > 0$)

s^4	1	11	K_p
s^3	6	6	0
s^2	10	K_p	
s^1	$6-\frac{3}{5}K_p$	0	
s^0	K_p		

$$6-\frac{3}{5}K_p > 0$$

$$6 > \frac{3}{5}K_p$$

$$10 > K_p \iff K_p < 10$$

Conclusion:
$0 < K_p < 10$ for a stable loop with prop. controller Δ

5.4 Steady state performance

Typical design specs:
- internal stability (mandatory)
- good transient behaviour (depends, in a complicated way, on pole and zero locations)
- this section: Steady state behaviour (tracking and disturbance rejection)

e.g. 4.5.1

plant: motor speed control $\quad \dot{w} = -w + u$

w – angular velocity of shaft $\quad u$ – applied voltage

$$P(s) = \frac{1}{s+1} \qquad r(t) = r_0 \, \mathbb{1}(t)$$

Tracking error: $e(t) = r(t) - y(t)$

Try an integral controller: $C(s) = \dfrac{k_i}{s}$

Take $k_i = 1$ (valid by Routh-Hurwitz)

$\pi(s) = s^2 + s + 1 \Rightarrow$ I.O. stable

- let's compute the s.s. tracking error

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

$$= \lim_{s \to 0} s \left(\frac{1}{1+PC} \right) R(s)$$

$$= \lim_{s \to 0} \cancel{s} \frac{1}{1+PC} \cdot \frac{r_0}{\cancel{s}} = \lim_{s \to 0} \frac{r_0}{1+PC} = \lim_{s \to 0} \frac{s(s+1)}{s^2+s+1} r_0$$

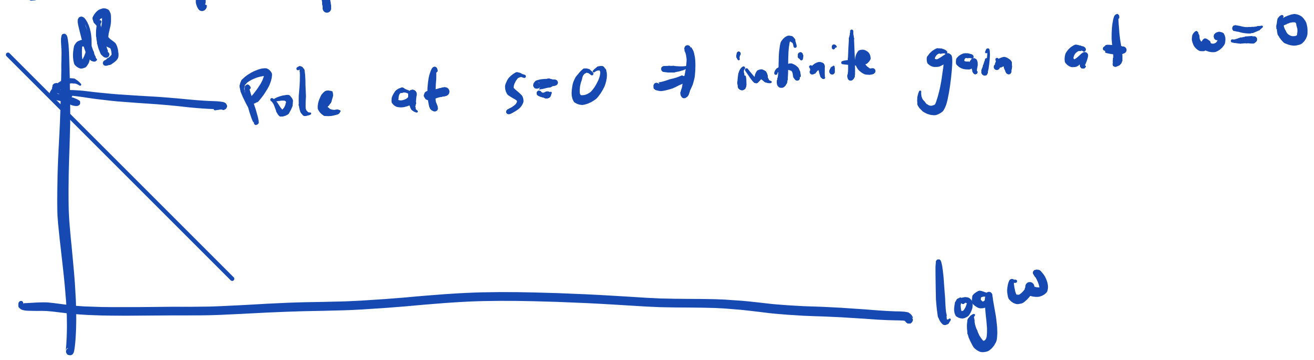
$$= 0$$

We get perfect asymptotic step tracking.

<u>Why it works?</u> $C(s)$ has an <u>"internal model"</u> of $R(s)$ (an integrator). This places a zero at $s = 0$ in the TF $\dfrac{E(s)}{R(s)}$

Other interpretations on why this works:

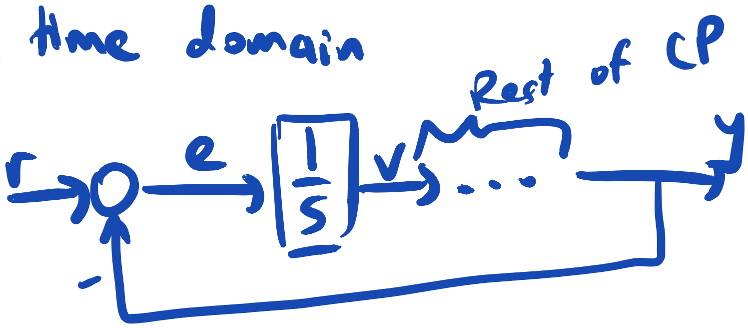
(i) Frequency domain



Bode of $P(j\omega)\,C(j\omega) = \dfrac{1}{j\omega(j\omega+1)}$

→ As $\omega \to 0$, $\dfrac{E(j\omega)}{R(j\omega)} = \dfrac{1}{1+P(j\omega)\,C(j\omega)} \to 0$

(ii) time domain



If system is I.O. stable, constant r produces, in steady state, constant y, e, v.

⇒ e must go to 0 to make v be constant

⇒ **Need** integral control to track constant signals.