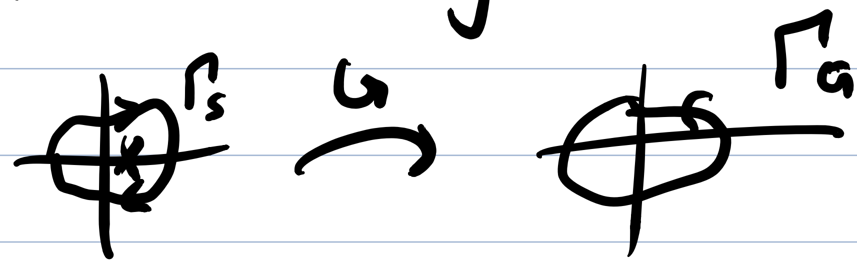
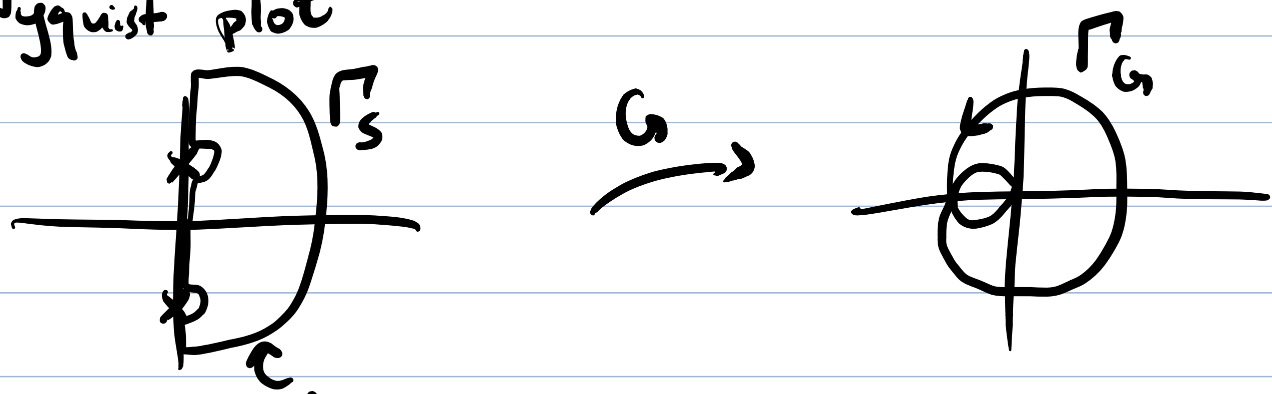
Lecture 33 Summary

- principle of the argument



- Nyquist plot



Theorem 8.2.3

- n = # of poles of CP with $Re(s) > 0$
- I.O. stability $\Leftrightarrow$ Nyquist plot of CP encircles $-\frac{1}{K}$ n times CCW

Proof

$$G(s) := \frac{KN_pN_c + D_pD_c}{D_pD_c} = 1 + KC(s)P(s)$$

Logic so far:

I.O. stability $\Leftrightarrow$ G has no zeros w/ $Re(s) \geq 0$

if $KN_pN_c + D_pD_c$ has no roots w/ $Re(s) \geq 0$

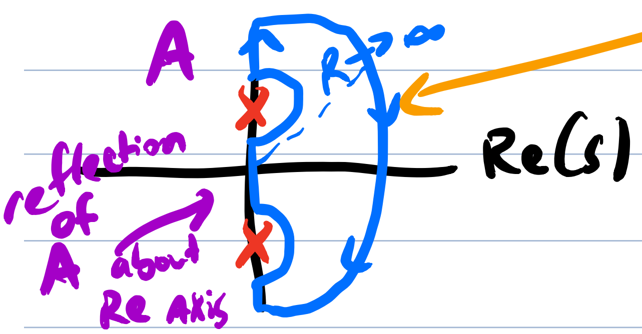
- Draw Nyquist plot of G. By principle of the argument, Γ_G will encircle the origin $n-m$ times CCW.
- Since we want no zeros of G with $\text{Re}(s) \geq 0$, we want $m = 0$.
- So, I.O. stability $\Leftrightarrow$ Γ_G encircles the origin n times CCW.

$$CP = \frac{1}{K} G - \frac{1}{K}$$

$\Rightarrow$ the Nyquist plot of CP is just a scaled translation of the Nyquist plot of G by $-\frac{1}{K}$.

<u>Conclusion:</u> I.O. stability $\Leftrightarrow$ Nyquist plot of CP encircles $-\frac{1}{K}$ n times CCW. $\square$

<u>Remark 8.2.5</u>



maps to zero ($\Rightarrow$ irrelevant)

Since $C(s)P(s)$ is strictly proper, the infinite part of the Nyquist contour is mapped to zero (the origin).

Remark 8.2.6

Since $C(s)P(s)$ is rational we get:

$$|C(j\omega)P(j\omega)| = |C(-j\omega)P(-j\omega)|$$
$$\measuredangle C(j\omega)P(j\omega) = -\measuredangle C(-j\omega)P(-j\omega)$$

So the image of Γ_s corresponding to the negative imaginary axis is a reflection of the positive Im axis about the Re axis.

Remark 8.2.7

So these remarks show that the Nyquist plot of CP is, aside from indentations, just a polar plot of $C(j\omega)P(j\omega)$ as $\omega: 0 \to \infty$.

- We already know that the Bode plot of CP is just $20\log|C(j\omega)P(j\omega)|$ and $\measuredangle C(j\omega)P(j\omega)$ vs. ω. So Nyquist plots and Bode plots are closely related.

Procedure

1. Pick Γ_s as the Nyquist contour. Indent to the right of poles on $j\mathbb{R}$
2. Draw image of Γ_s under $C(s)P(s)$. Based on our Remarks, we only need to worry about Γ_s along the non-negative Im axis.

3. Observe N, the # of CCW encirclements of $-1/K$ made by the plot from step 2.

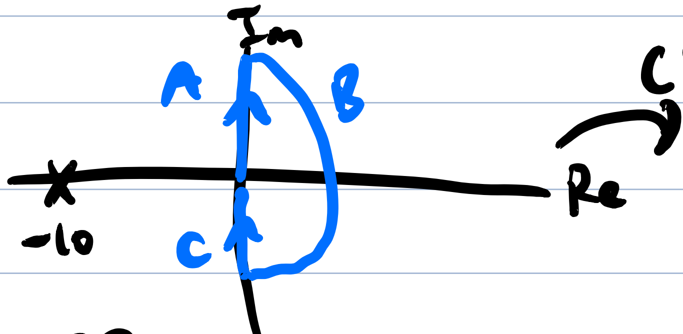
4. Principle of the Argument $N = n - m$.
where n = # of poles of CP with $Re(s) > 0$ (known). m = # of zeros of CP with $Re(s) \geq 0$. (unknown).

5. Stability $\Leftrightarrow$ $N = n$.

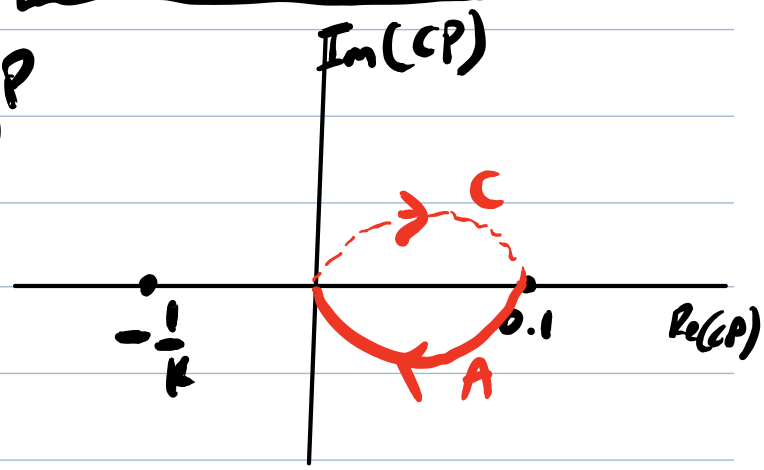
8.3. Examples

e.g. $C(s)P(s) = \dfrac{1}{s+10}$

1.



$n = 0$



2. Segment A: $s = j\omega$, ω from $0 \to +\infty$

$$C(j\omega)P(j\omega) = \frac{1}{j\omega + 10} = \frac{10 - j\omega}{\omega^2 + 10^2}$$

$$= \frac{10}{\omega^2 + 10^2} - j\frac{\omega}{\omega^2 + 10}$$

- real part > 0 so no Im axis crossings
- im part ≤ 0 so no Re axis crossings
- so image of segment A stays in 4th quadrant

Segment B: mapped to zero

Segment C: Reflection of segment A

3,4,5. Want $N = n = 0$ encirclements.

	$-\frac{1}{K} \in (-\infty, 0)$	$-\frac{1}{K} \in (0, 0.1)$	$-\frac{1}{K} \in (0.1, \infty)$
N	0	-1	0

I.O. stability $\iff K > -10$. ▲

e.g. Compare Nyquist and Bode