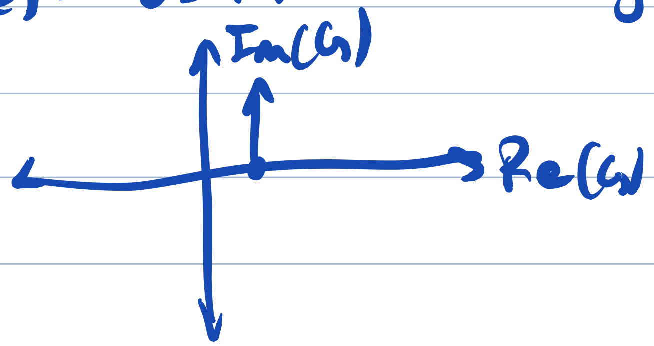
Lecture 14 Summary

- graphical representations of freq. response
- for rational TFs, we only need to know how to sketch 4 basic terms
- (i) $G(s) = K$ (ii) $G(s) = s^n$
  (iii) $G(s) = \tau s \pm 1$

(a) $G(s) = \tau s + 1$ $G(j\omega) = j\omega\tau + 1$

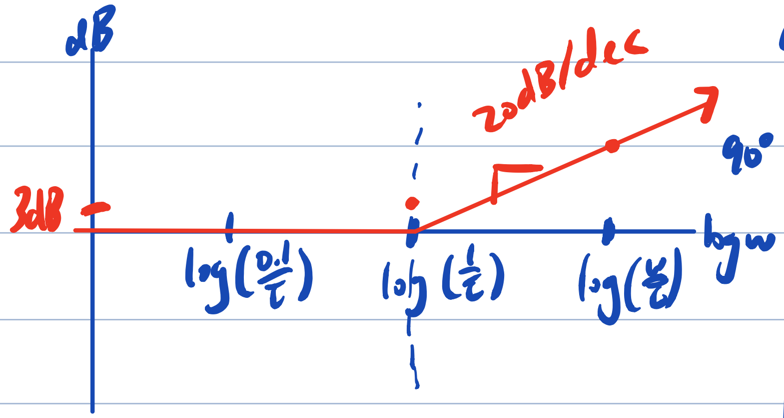


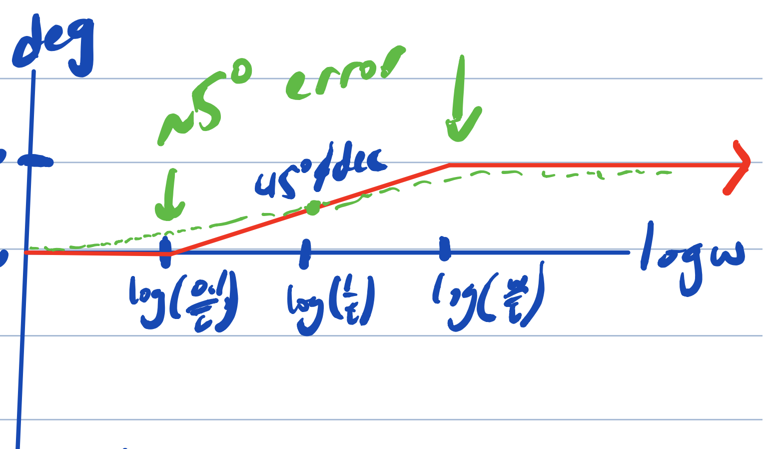
<u>Polar Plot</u>

$\omega \mapsto \begin{bmatrix} 1 \\ \omega\tau \end{bmatrix}$

$20\log|G(j\omega)|$
$= 20\log|j\omega\tau + 1|$

<u>Bode plot</u>





Approximations to sketch Bode plot:

(i) for $\omega \leq \frac{1}{\tau}$, $\mathrm{Im}(G(j\omega)) \approx 0$

$\Rightarrow (\forall \omega \leq \frac{1}{\tau})\ 20\log|G(j\omega)| \approx 20\log 1 = 0\ \mathrm{dB}$

(ii) for $\omega > 1/\tau$, $\mathrm{Re}(G(j\omega)) \approx 0$

$\Rightarrow$ (for $\omega \geq \frac{1}{\tau}$) $20\log|G(j\omega)| \approx 20\log|\tau\omega|$

$\frac{1}{\tau}$ is "break frequency"

$20\log\left|\tau \cdot \frac{10}{\tau}\right| = 20$

Error introduced by approximation is largest at the break frequency $\omega = 1/\tau$

Actual magnitude is $20\log|G(j\,1/\tau)| = 20\log|j+1| = 20\log(\sqrt{2})$ ≈ 3 dB

Approximations to <u>sketch</u> phase Bode plot:

(i) for $\omega \ll 1/\tau$, $\angle G(j\omega) \approx \angle(j0+1) = 0°$

(our convention: $\omega \ll 1/\tau$ means $\omega \leq 0.1/\tau$)

(ii) for $\omega \gg 1/\tau$ $\angle G(j\omega) \approx \angle(j\tau\omega) = 90°$

(our convention: $\omega \gg 1/\tau \Rightarrow \omega \geq 10/\tau$)

(iii) linear interpolation between $\frac{0.1}{\tau}$ and $\frac{10}{\tau}$

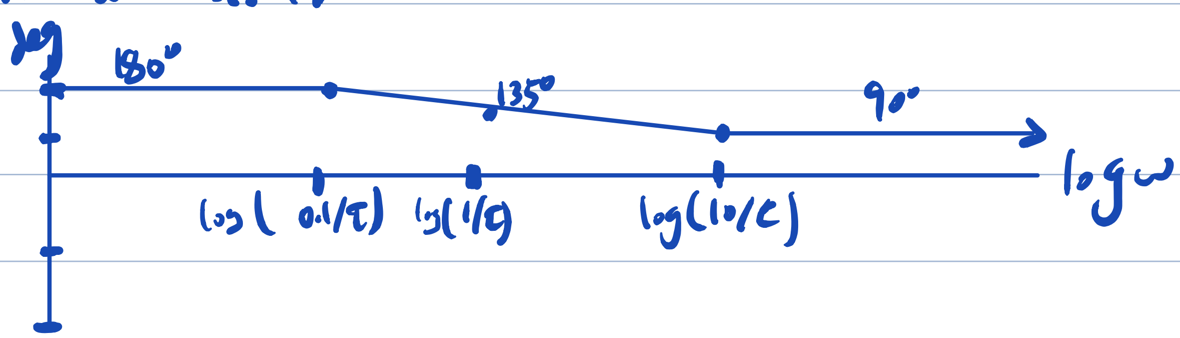
(b) $G(s) = \tau s - 1$

Polar plot:

Bode plot:

Same magnitude pot as (a) $\tau s + 1$

phase plot changes - goes from 180° at l.f. to 90° at h.f.



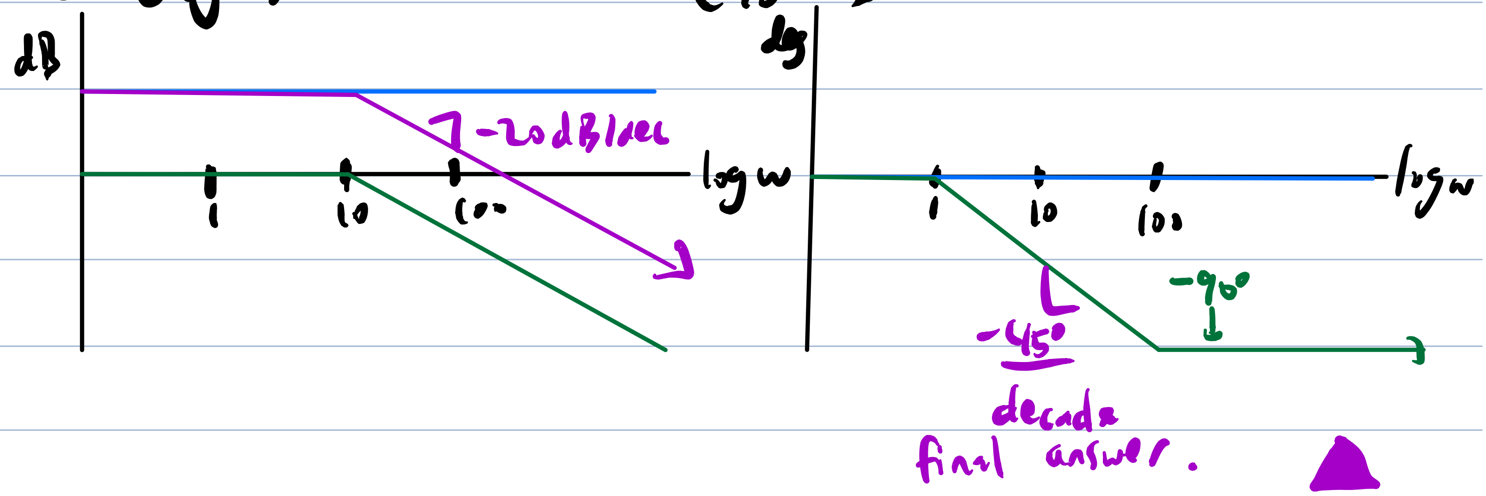
e.g. $G(s) = \dfrac{100}{s+10} = \dfrac{10}{\frac{s}{10}+1}$

Frequency response $G(j\omega) = \dfrac{10}{\left(\frac{j\omega}{10}+1\right)}$

(a) (b)

$$20\log|G(j\omega)| = 20 - \tfrac{1}{2}\log\left(\frac{\omega^2}{100}+1\right)$$

$$\angle G(j\omega) = \angle 10 - \angle\left(\frac{j\omega}{10}+1\right)$$



The <u>bandwidth</u> of this system is the smallest frequency at which $|G(j\omega)_{BW}| = \frac{1}{\sqrt{2}}|G(0)|$

<u>In dB:</u> $20\log|G(j\omega)_{BW}| = 20\log\left(\frac{1}{\sqrt{2}}\right) - 20\log|G(0)|$

$= 20\log|G(0)| - 3$

Easy to read from Bode plot: in this case

$\omega_{BW} = \omega_0 = 10$ rad/s