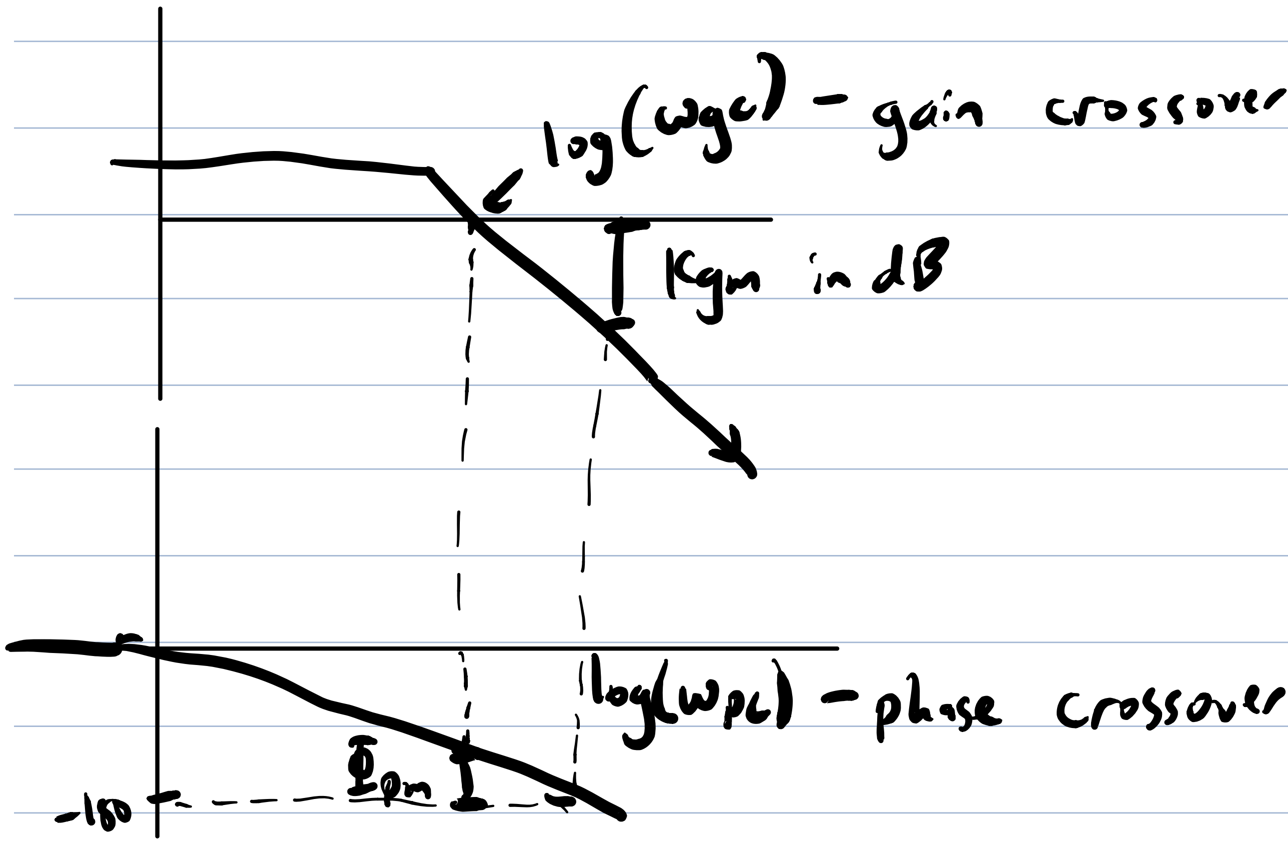
Summary Lecture #28

- gain and phase margin - a margin of robustness (tolerance in design to plant uncertainty)
- how to read from Bode plots

$$L(s) = C(s)P(s)H(s)$$



$[K_{gm}, \Phi_{pm}, w_{pc}, w_{gc}] = \text{margin}(SYS)$

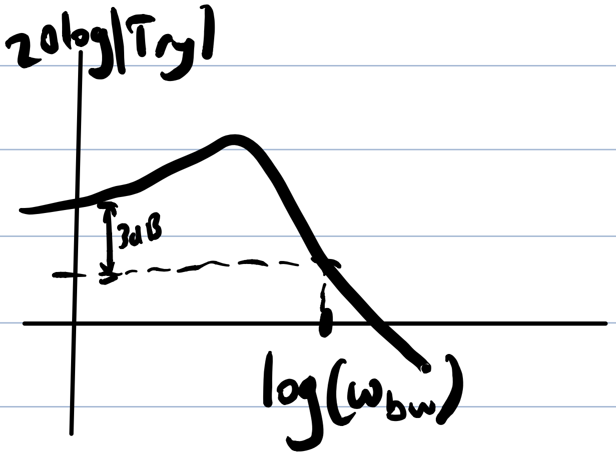
- why do we care about the freq. at which $|L(j\omega)| = 1$ (ω_{gc}) and at which $\angle L(j\omega) = \pi$ (ω_{pc})?
- consider $\dfrac{Y(s)}{R(s)} = \dfrac{CP}{1+CPH} = \dfrac{CP}{1+L(s)}$
- K_{gm} and Φ_{gm} provide a "distance" from $L(j\omega) = -1$

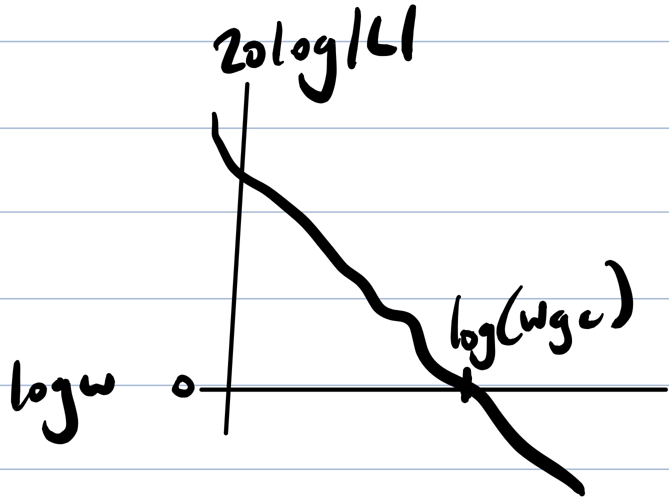
9.2. Performance Specs

this chapter: 1) stability
2) adequate step/ramp tracking
3) bandwidth
4) phase margin

Relationship between ω_{gc} and closed-loop bandwidth

$$\frac{Y(s)}{R(s)} = \frac{C(s)P(s)}{1+L(s)} =: T_{ry}(s)$$





Rule of thumb: $\omega_{gc} < \omega_{bw} < 2\omega_{gc}$

For design, $\boxed{\omega_{bw} = \omega_{gc}}$

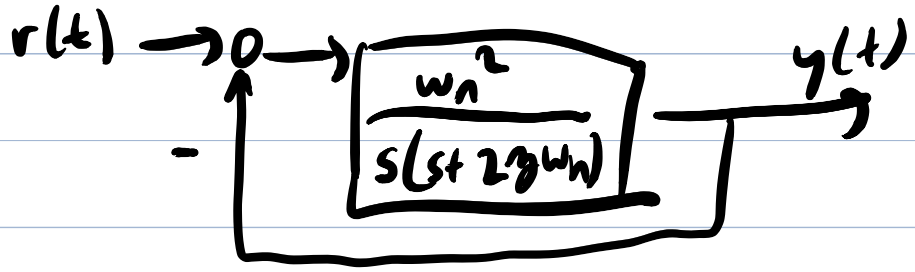
- $T_s^{max} \rightarrow$ all poles with $\mathrm{Re}(s) \leq -\dfrac{4}{T_s^{max}}$

$\rightarrow \omega_{bw} \geq \dfrac{4}{T_s^{max}} \rightarrow \omega_{gc} \geq \dfrac{4}{T_s^{max}}$

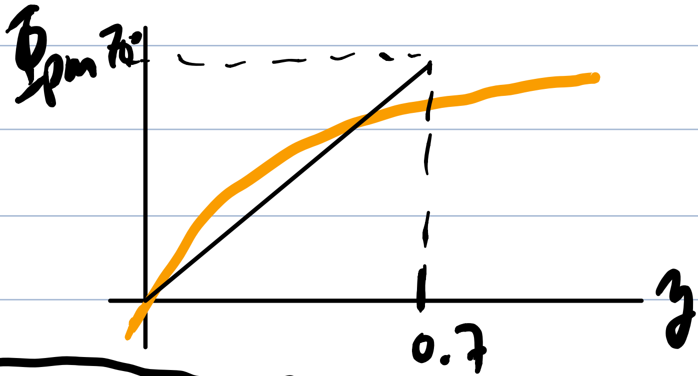
Relationship between damping ratio and Φ_{pm}

- we claimed that good gain and phase margins give good transient performance.

- for the following system we can get an exact relationship between ζ and Φ_{pm}.



- after some algebra, $\Phi_{pm} = \operatorname{atan}\left[2\zeta\left[(1+4\zeta^4)^{1/2} - 2\zeta^2\right]^{1/2}\right]$



$\boxed{\Phi_{pm} \approx 100\zeta}$, $0 < \zeta < 0.7$

- $\%OS^{max} \rightarrow \zeta^{min} \rightarrow \Phi_{pm}^{min}$

we present 3 simple control designs:

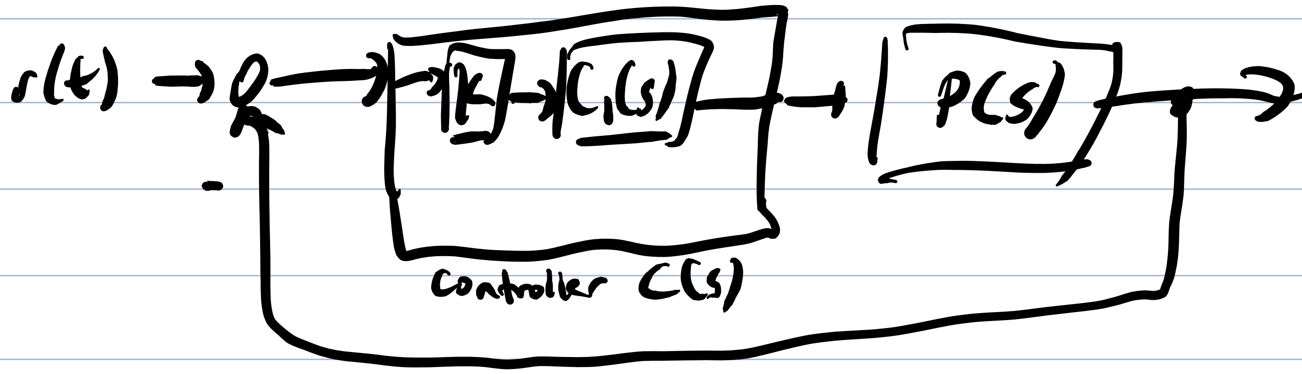
1) lag 2) lead 3) lead-lag

The design methods we present work well with "nice plants"

(1) open-loop stable, or, at worst, one pole at $s=0$

(2) only one set of crossover frequencies $(\omega_{gc}, \omega_{pc})$

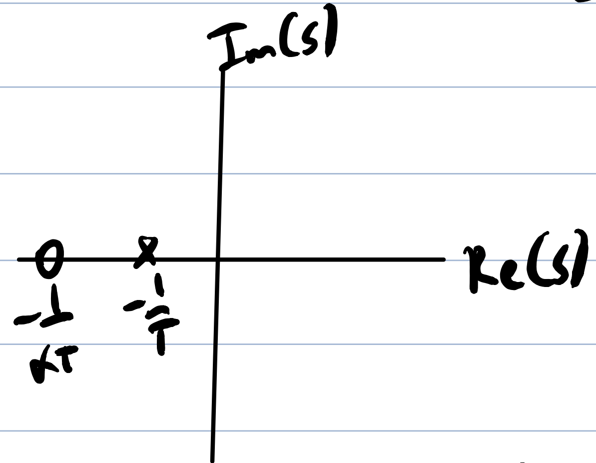
9.3. Lag Controllers



Definition (Lag Controller)

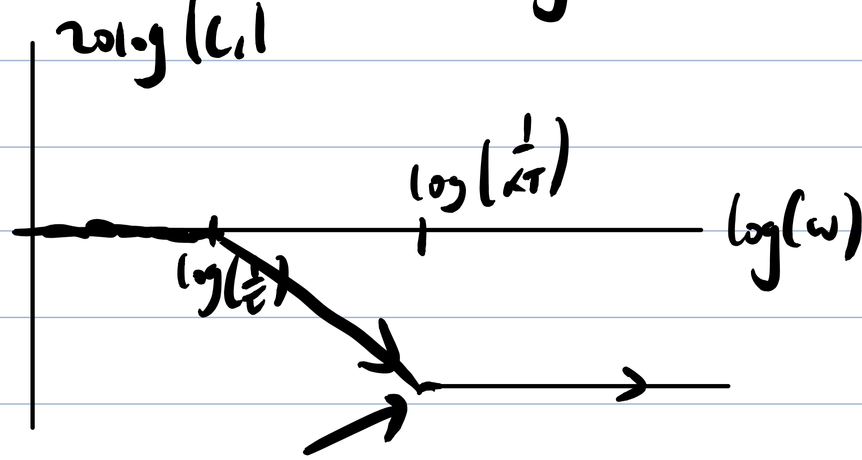
$$C(s) = K C_1(s) = K \frac{\alpha \tau s + 1}{\tau s + 1} \qquad 0 < \alpha < 1$$

$$K, T > 0$$

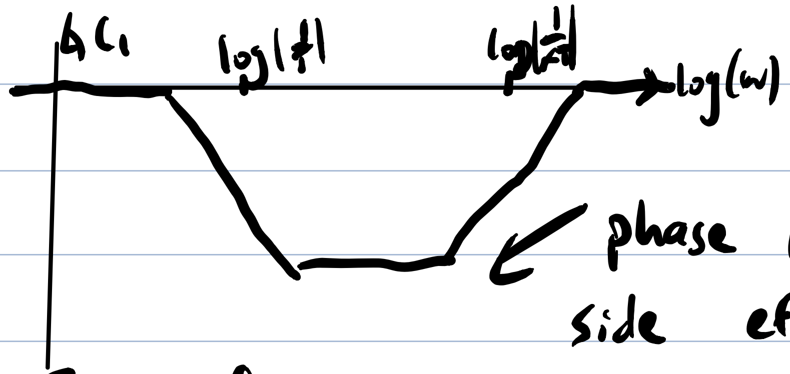


pole-zero locations of $C(s)$

$C(0) = K$ = steady-state gain



Main benefit of a lag controller: reduces high frequency gain w/o changing low-frequency gain



phase lag is an undesirable side effect

See Figure 9.7 in the notes for the "loop shaping" idea behind lag design.

<u>e.g. 9.3.1.</u> $P(s) = \dfrac{1}{s(s+2)}$

specs: 1) $r(t) = t\mathbb{1}(t) \Rightarrow |e_{ss}| \leq 0.05$
2) $\Phi_{pm}^{des} = 45°$ (for good damping)

1. Pick K to meet steady-state spec
- assuming our final design achieves stability

$$e_{ss} = \lim_{s\to 0} sE(s) = \lim_{s\to 0} s \frac{1}{1+C(s)P(s)} R(s)$$

$$\lim_{s\to 0} s \frac{1}{1+K\left(\frac{\alpha Ts+1}{Ts+1}\right)\left(\frac{1}{s(s+2)}\right)} \frac{1}{s^2}$$

$$= \frac{2}{K}$$

spec 1 is met if $K \geq 40$. Pick $\boxed{K=40}$