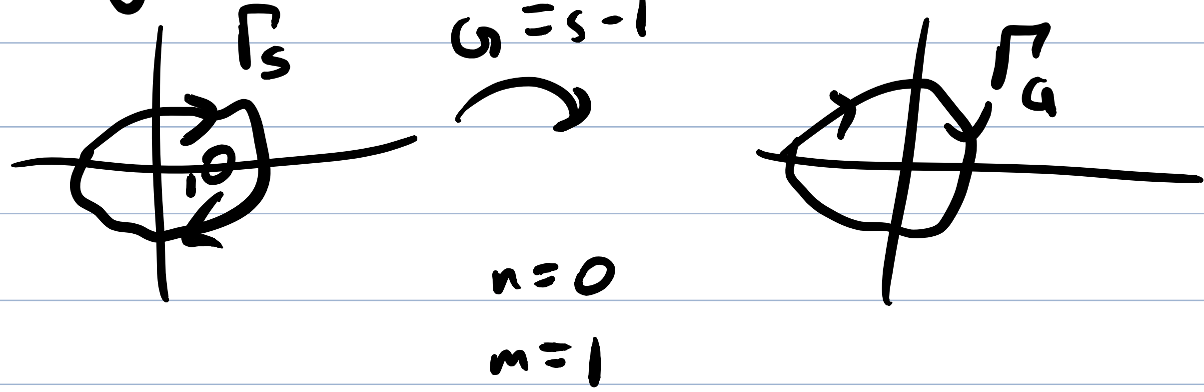
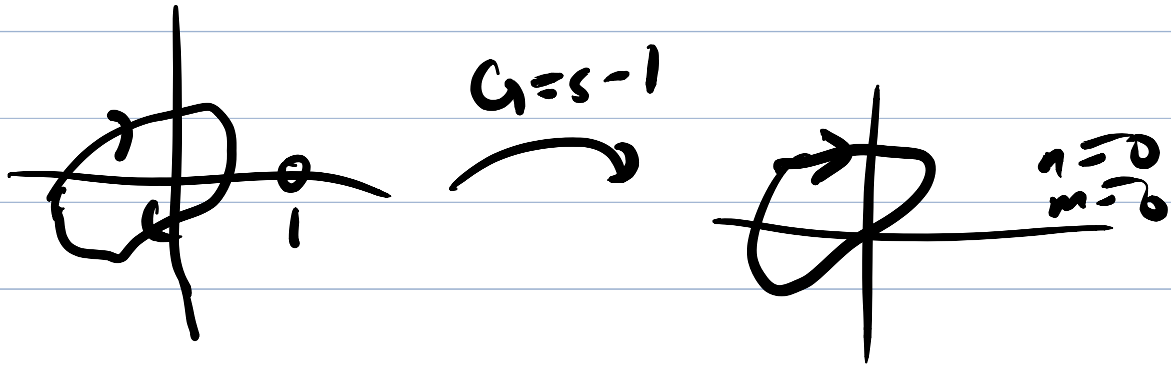
Summary Lecture 32

1.



2.



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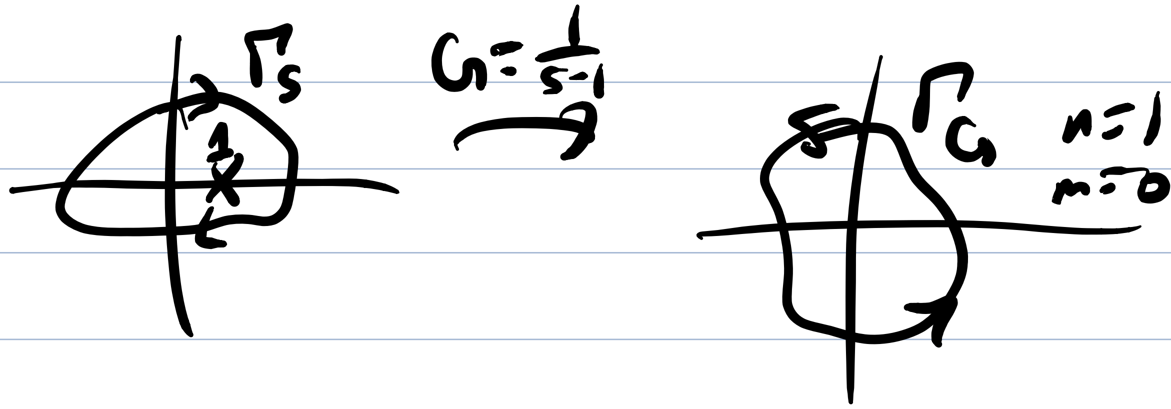


Theorem 8.1.1. (Principle of the Argument)
Suppose $G(s)$ has no poles on Γ_s and Γ_s encloses n poles and m zeros of G. Then Γ_G has $n-m$ CCW encirclements of the origin.

Proof
Write $G(s) = K \prod_i (s-z_i) \Big/ \prod_i (s-p_i)$.

K is a real gain, $\{z_i\}$ are zeros, and $\{p_i\}$ are poles.

For each s on Γ_s,

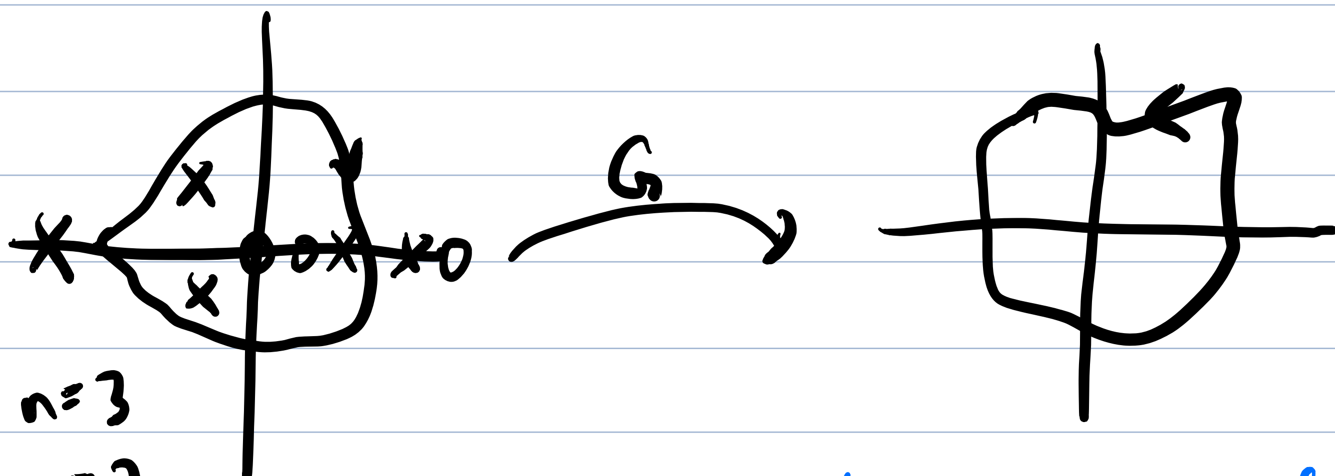
$$\measuredangle G(s) = \measuredangle K + \sum_i \measuredangle(s - z_i) - \sum_i \measuredangle(s - p_i)$$

- if a zero z_i is enclosed by Γ_s, the net change after one circuit around Γ_s in the term corresponding to that zero, is -2π.
  - if z_i isn't enclosed by Γ_s, the net change of $\measuredangle(s - z_i)$ is zero.
- so the net change in $\measuredangle G$ is

$$m(-2\pi) - n(-2\pi) = (n-m)2\pi$$

- So Γ_G encircles the origin $n-m$ times in the C.C.W. direction. ∎

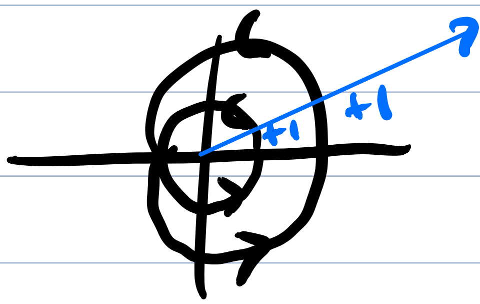
e.g.



$n=3$

$m=2$

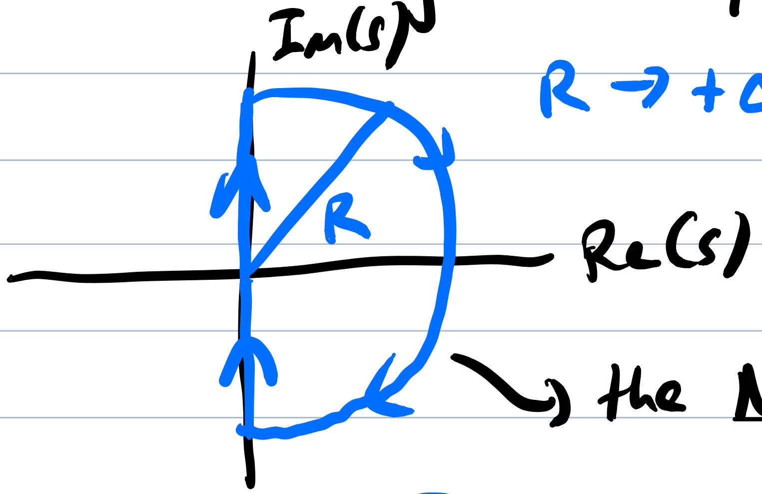
Aside:
(Trick)

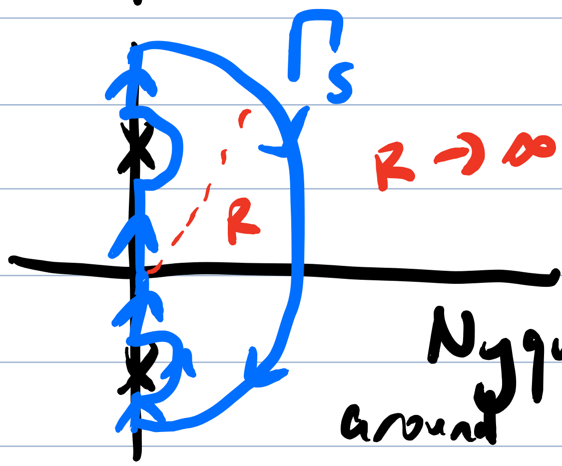


arbitrary ray to infinity
2 CCW encirclements

8.1.1 Nyquist Contour

- Nyquist's bright idea: Take Γ_s to encircle the entire right-half plane.





Definition (Nyquist Plot)

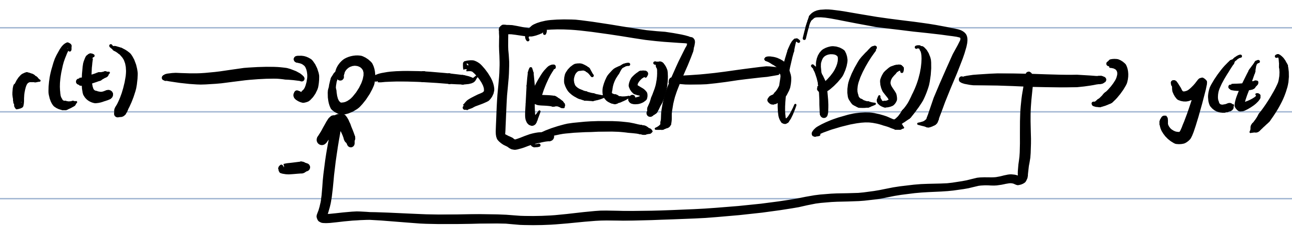
For this choice of Γ_s, the corresponding Γ_G curve is called the Nyquist plot of G.

If $G(s)$ has no poles/zeros on the Nyquist contour, then the Nyquist plot of G will encircle the origin $n-m$ times in the ccw direction.

n = # of poles of G with $Re(s) > 0$
m = " zeros " "

- G will have no poles on Γ_s iff G has no poles on the Im axis and G is proper.
- if G has poles on $j\mathbb{R}$, we'll indent to the right.
- zeros don't matter because we won't be counting encirclements of the origin.

8.2 Nyquist stability criterion



Assumptions:

1. P and C proper, PC strictly proper.
2. No unstable pole-zero cancellations
3. $K \neq 0$

Key idea: If the feedback system is input/output stable, then

$$\frac{Y(s)}{R(s)} = \frac{KP(s)C(s)}{1 + KP(s)C(s)} \quad \text{is BIBO stable.}$$

We will work with $G(s) := 1 + KC(s)P(s)$.

$= 1 + K \frac{N_c N_p}{D_c D_p}$

$= \frac{D_c D_p + K N_c N_p}{D_c D_p}$

Theorem 8.2.3 (The Nyquist criterion)
Let n = # of poles of $C(s)P(s)$ with $Re(s) > 0$. Construct the Nyquist plot of $C(s)P(s)$ indenting to the right around poles on $j\mathbb{R}$.
The feedback system is I.O. stable if and only if the Nyquist plot doesn't pass through $-\frac{1}{K}$ and encircles it exactly n times C.C.W.

Proof

$$\frac{Y}{R} = \frac{KCP}{G}$$

Since we assumed no unstable cancellations, IO stability is equivalent to G having no zeros with $Re(s) \geq 0$. The poles of G = the poles of CP. So G has n poles with $Re(s) > 0$.
Since Γ_s indents around poles on $j\mathbb{R}$ and since G is proper,

Γ_s doesn't pass through poles of G.
- By principle of the argument, Γ_G will encircle the origin $n-m$ times CCW.