Lecture #22 Summary

- root-locus: a plot of how roots of a poly $\pi(s) = D(s) + KN(s)$ move around $\mathbb{C}$ as K change
- good for analysis, and to some extent, design
- simple example
- actual rules

$$\pi(s) = (s-p_1)\cdots(s-p_n) + K(s-z_1)\cdots(s-z_m)$$

$\cdots$

On the root-locus: $\pi(s) = 0$ for some value of K. $\Rightarrow \angle \frac{N(s)}{D(s)} = \angle -\frac{1}{K}$

$\Leftrightarrow \angle N(s) - \angle D(s) = \pi$

To compute the departure angle from p_i, θ_{p_i}, plug $s = p_i$ into the above angle condition.

$\angle(p_i - z_1) + \cdots + \angle(p_i - z_m) - \big(\angle(p_i - p_1) + \cdots$

$$+ (\theta_{p_i}) + \cdots + \angle(p_i - p_n)) = \pi$$

Rearrange and solve for θ_{p_i}.

To compute arrival angle at z_i, denoted θ_{z_i}, do the same.

$$\angle(z_i - z_1) + \cdots + \theta_{z_i} + \cdots + \angle(z_i - z_m) - \sum_{j=1}^{m} (z_i - p_j) = \pi$$

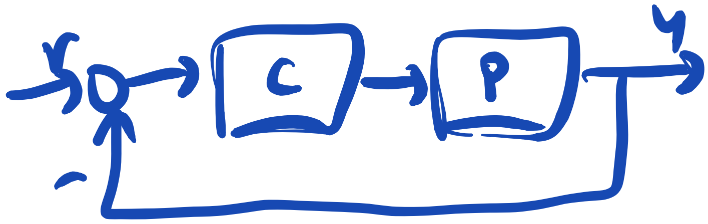
<u>Procedure for plotting root-locus:</u>

Given $\pi(s) = D(s) + KN(s)$

1. Compute the roots $\{z_1, \ldots, z_m\}$ of $N(s)$ and place o (circle) at each location.
2. Compute the roots of $\{p_1, \ldots, p_n\}$ of $D(s)$ and place x at each location.
3. Use "no-yes-no" rule (#7) to fill in the real axis.
4. Compute the centroid σ, label the point $s = \sigma + j0$ on the plot
5. Compute and draw asymptotes ($n-m$ of them)
6. Compute angles using rule 8. Usually only needed for conjugate and repeated real roots.
7. Give a reasonable guess of how it looks and draw it!

6.2. Examples

<u>e.g. 6.2.1</u> $P(s) = \dfrac{1}{s^2+2s+5}$, $C(s) = K\left(1 + \dfrac{1}{0.25s}\right)$



ch.p.

$\pi(s) = (s^2+2s+5)s + K(s+4)$

Step 1: $z_1 = -4$ $m=1$

Step 2: $D(s) = s(s^2+2s+5)$, $n=3$

$\{p_1, p_2, p_3\} = \{0, -1+j2, -1-j2\}$

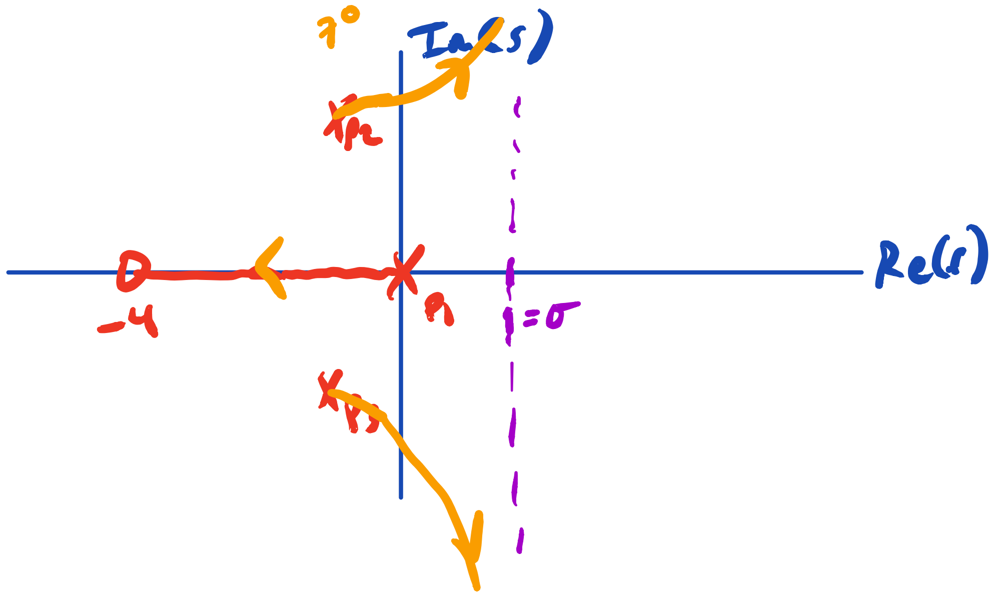
Step 3: Fill in $\mathbb{R}$ line (no-yes-no)

Step 4: $\sigma = \dfrac{\sum p_i - \sum z_i}{n-m}$

$$= \frac{(0 - 1 + j2 - 1 - j2) - (-4)}{3-1}$$

$$= 1$$

Step 5: $n-m=2 \Rightarrow$ Asymptotes are $\overset{\phi_1}{\frac{\pi}{2}}, \overset{\phi_2}{-\frac{\pi}{2}}$



Step 6: Departure angle from p_2:

$$\angle(p_2+4) - \left(\angle p_2 + \left(\theta_{p_2} \right) + \angle(p_2+1+j2)\right) = \pi$$

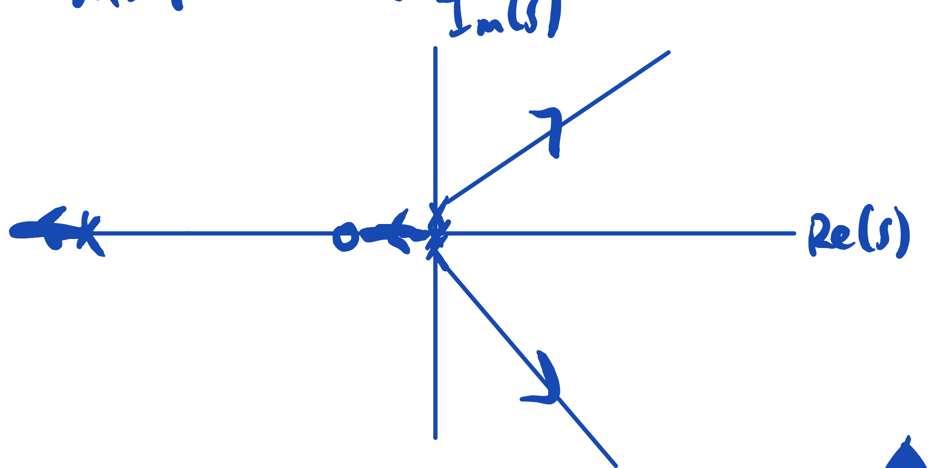
$$\theta_{p_2} = \angle(-1+j2+4) - \angle(-1+j2) - \angle(j4) - \pi$$

$$= 33.7^\circ - 116^\circ - 90^\circ - 180^\circ$$

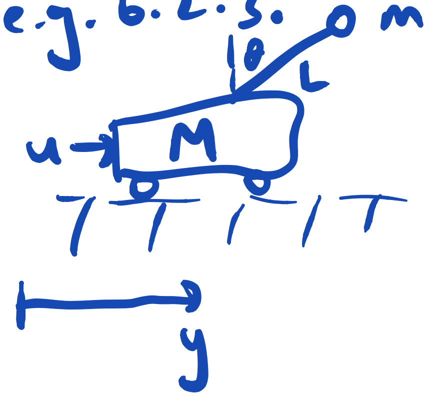
$$= -352.9^\circ \sim 7.125^\circ$$

<u>e.g. 6.2.2.</u> $T(s) = s^3(s+4) + K(s+1)$

$m = 1 \qquad n = 4$



e.g. 6.2.3.



Linearized model at $\theta = 0$ is

$$P(s) = \frac{1}{M\ell s^2 + g(M+m)}$$

$$= \frac{1/M\ell}{s^2 + \frac{g(M+m)}{M\ell}}$$