Lec 13 Summary

- steady state gain $G(0)$ (G BIBO stable)
- freq response $u \to \boxed{G(s)} \to y$
  BIBO stable

$u(t) = \cos(\omega t) \Rightarrow y(t) = |G(j\omega)| \cos(\omega t + \angle G(j\omega))$

- frequency response $\mathbb{R} \to \mathbb{C}$
  $\omega \mapsto G(j\omega)$ $\omega \geq 0$ without loss of info

Bode plot

(1) Magnitude

$20\log|G(j\omega)|$ vs. $\log \omega$

(2) Phase

$\angle G(j\omega)$ vs. $\log \omega$

Polar plot

$$\omega \mapsto \begin{bmatrix} \mathrm{Re}(G(j\omega)) \\ \mathrm{Im}(G(j\omega)) \end{bmatrix}$$

- to sketch Bode plot of a rational TF we only have to know how to sketch 4 basic terms

(i) Pure gain term $G(s) = K$

(ii) roots at the origin $G(s) = s^n$

(iii) 1st order polynomials
$G(s) = \tau s + 1$, $\tau \geq 0$

(iv) complex conjugate roots
$G(s) = s^2 + 2\zeta\omega_n s + \omega_n^2$

$$= \omega_n^2\left(\frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n}s + 1\right)$$

<u>e.g. 3.8.5</u>

$$G(s) = \frac{40s^2(s-2)}{(s+5)(s^2+4s+100)}$$

$$= \frac{40(2)}{5(100)} \frac{s^2\left(\frac{s}{2}-1\right)}{\left(\frac{s}{5}+1\right)\left(\frac{s^2}{10^2}+\frac{4s}{10^2}+1\right)}$$

- once factored, we obtain the frequency response by setting $s=j\omega$

<u>e.g. 3.8.6</u> $G(s)$ from last example

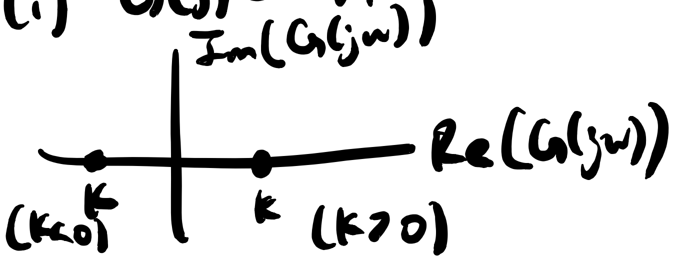
$$20\log|G(j\omega)| = 20\log\left|\frac{80}{500}\right| + 20\log|j\omega^2|$$

$$+ 20\log\left|\frac{j\omega}{2}-1\right| - 20\log\left|\frac{j\omega}{5}+1\right| - 20\log\left|\frac{(j\omega)^2}{100}+\frac{4j\omega}{100}+1\right|$$

$$\angle G(j\omega) = \angle\frac{80}{500} + \angle(j\omega)^2 + \angle\left(\frac{j\omega}{2}-1\right)$$

$$- \angle\left(\frac{j\omega}{5}+1\right) - \angle\left(\frac{(j\omega)^2}{100}+\frac{4j\omega}{100}+1\right)$$

(i) $G(s) = K$

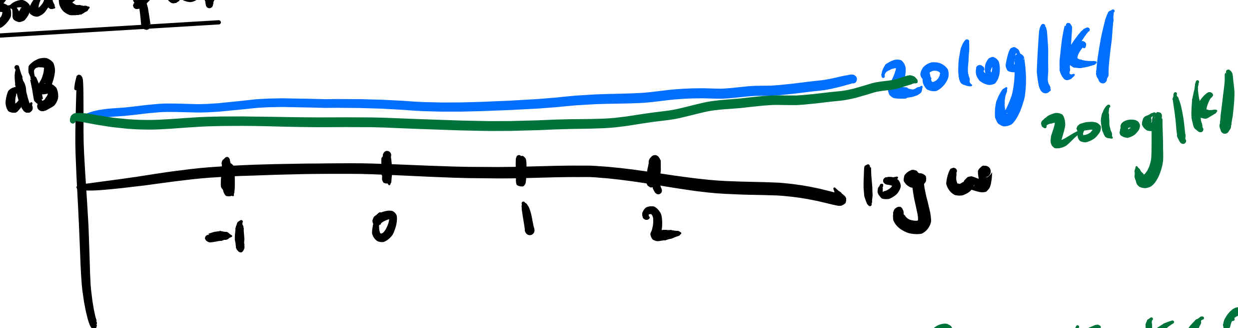


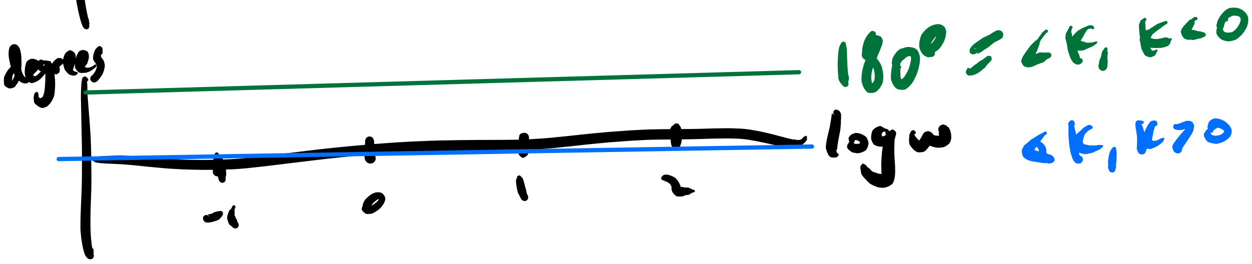
Polar Plot

$$\omega \mapsto \begin{bmatrix} Re(G(j\omega)) \\ Im(G(j\omega)) \end{bmatrix}$$

— $K > 0$

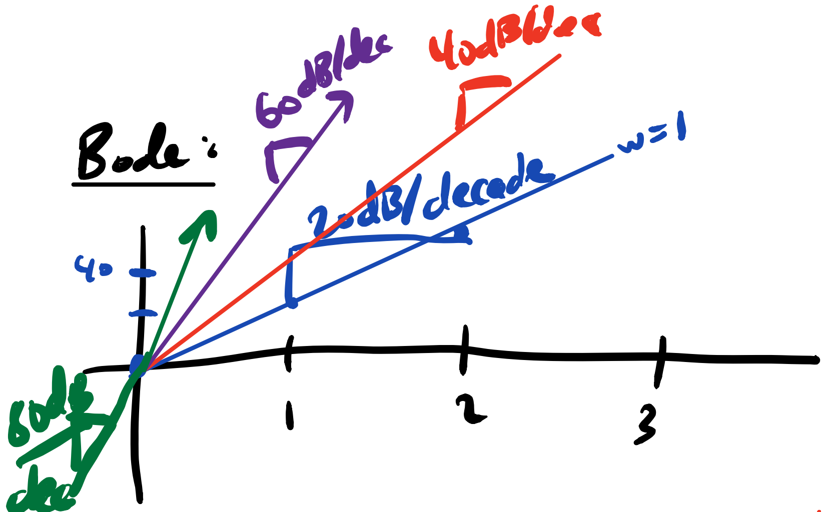
Bode plot

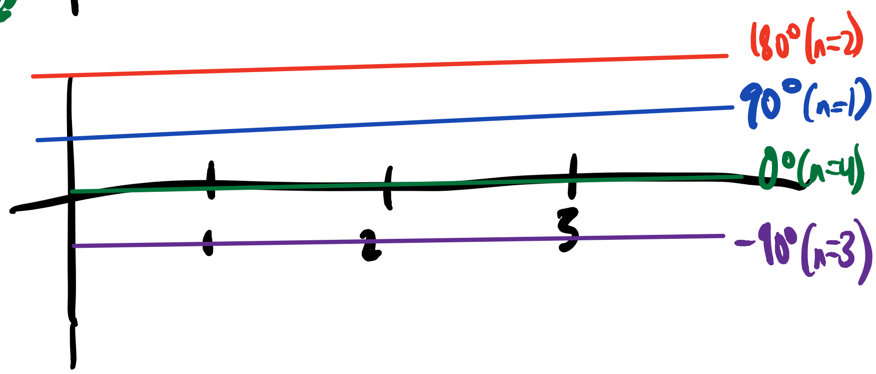




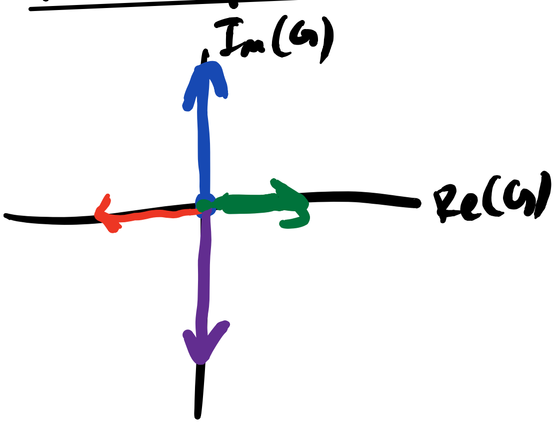
(ii) $G(s) = s^n$

Bode:





Polar plot:



$n=1$: $G(j\omega) = j\omega$ —

$n=2$: $G(j\omega) = -\omega^2$ —

$n=3$: $G(j\omega) = -j\omega^3$ —

$n=4$: $G(j\omega) = \omega^4$ —

(iii) First order $G(s) = \tau s \pm 1, \quad \tau > 0$

(a) $\tau s + 1$

Polar plot

$G(j\omega) = \tau j\omega + 1$