Lecture 26 Summary

- PID control
  - discussion of each of the terms P, I, D
- Pole placement

$$C(s) = \frac{g_2 s^2 + g_1 s + g_0}{s(s+f_1)}$$ (PID controller in standard form)

$$P(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\pi^{des}(s) = s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0$$

$$\underbrace{\begin{bmatrix} 1 & b_1 & 0 & 0 \\ a_1 & b_0 & b_1 & 0 \\ a_0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & b_0 \end{bmatrix}}_{\text{depends only on plant model}} \underbrace{\begin{bmatrix} f_1 \\ g_2 \\ g_1 \\ g_0 \end{bmatrix}}_{\text{Controller gains}} = \underbrace{\begin{bmatrix} \alpha_3 - a_1 \\ \alpha_2 - a_0 \\ \alpha_1 \\ \alpha_0 \end{bmatrix}}_{\text{depends on plant and desired ch. p.}}$$

Remarks

- if $b_1 s + b_0$ and $s^2 + a_1 s + a_0$ are coprime, equation has a unique sol^n (proof in the notes, this is actually an iff!)
- can't $b_0 = 0$ in plant because it will lead to an unstable pole-zero cancellation.

- T_d is treated as a design parameter, in contrast to classical approaches
- This shows that if the plant can be adequately modelled as a 2^{nd}-order system, then PID can achieve (almost) any control specification

<u>e.g. 7.3.1.</u> $P(s) = \dfrac{2}{s^2+3s+2}$

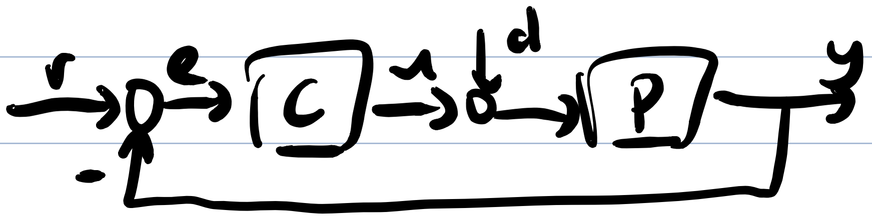
<u>Specs:</u>

$e_{ss} = 0$ for $r(t) = r_0 \mathbb{1}(t)$

$y_{ss} = 0$ for $d(t) = d_0 \mathbb{1}(t)$

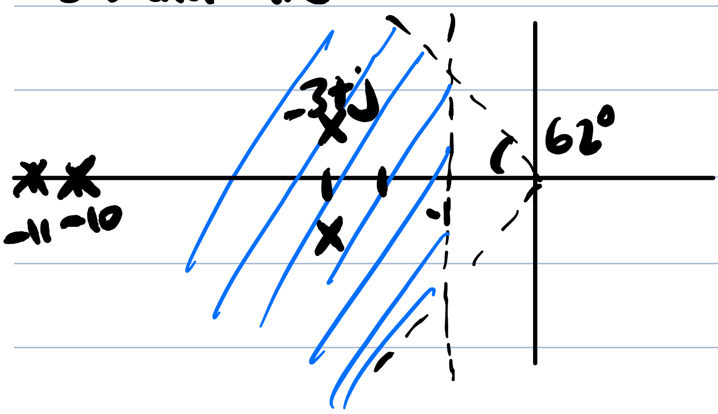
$T_s \leq 4$ sec

$\%OS \leq 0.2$



$$C(s) = K_p\left(1 + \frac{1}{T_i s} + \frac{T_d s}{T_d s + 1}\right) \quad \text{(PID)}$$

- use transient specs to define a <u>good region</u> in the complex plane where the roots of $\pi(s)$ should lie



$$\pi^{des}(s) = (s+3-j)(s+3+j)\underbrace{(s+10)(s+11)}_{\text{non-dominant}}$$

$$= s^4 + \underset{\alpha_3}{27}s^3 + \underset{\alpha_2}{246}s^2 + \underset{\alpha_1}{870}s + \underset{\alpha_0}{1100}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} f_1 \\ g_2 \\ g_1 \\ g_0 \end{bmatrix} = \begin{bmatrix} \alpha_3 - a_1 \\ \alpha_2 - a_0 \\ \alpha_1 \\ \alpha_0 \end{bmatrix} = \begin{bmatrix} 27-3 \\ 246-2 \\ 870 \\ 1100 \end{bmatrix}$$

$\Rightarrow f_1 = 24$

$3 \cdot 24 + 2 \cdot g_2 = 246 - 2$

$\ldots$

$$\Rightarrow \begin{bmatrix} f_1 \\ g_2 \\ g_1 \\ g_0 \end{bmatrix} = \begin{bmatrix} 24 \\ 86 \\ 411 \\ 550 \end{bmatrix}$$

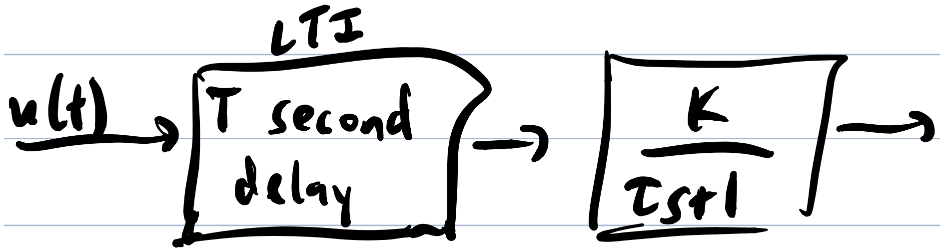
$$C(s) = \frac{86s^2 + 411s + 550}{s(s+24)}$$

$\Rightarrow K_p = 16.17$; $T_i = 0.7056$; $T_d = 0.1799$; $\tau_d = 0.0417$

You should check that the actual $\pi(s) = \pi^{des}(s)$.

(Will want to simulate this and draw Bode + Nyquist plots). $\triangle$

- another common model, for which PID is effective, is first-order + time delay.



$$P(s) = \frac{K}{\tau s+1} e^{-sT}$$

Padé approximation of irrational term:

$$e^{-sT} \approx \frac{-s\frac{T}{2}+1}{s\frac{T}{2}+1} \qquad \text{(MATLAB pade)}$$

Now $P(s) \approx \frac{K}{\tau s+1} \frac{-sT+2}{sT+2}$ (2^{nd} order)

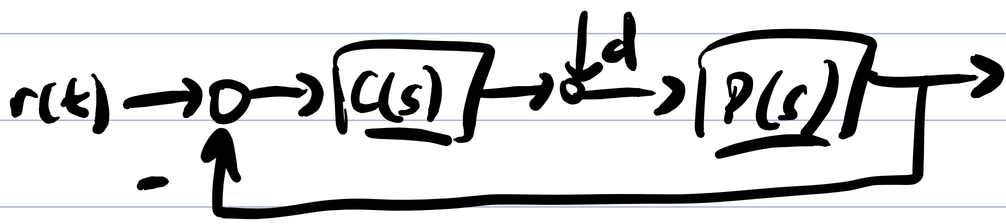
Remark:

- time-delay of T seconds acts like a <u>nonminimum phase zero</u> at $s = \frac{2}{T}$.
  - → bigger delay ⇒ "worse" zeros (bigger <u>undershoot</u>)

Takeaway:

- PID is nothing new (just another TF)
- but it's popular, so we studied them separately

Ch. 9. Intro to control design in frequency domain



- design specs will be in the frequency domain
  e.g. bandwidth, gain margin, phase margin

- since our real interest is how the system behaves in physical time, in this chapter, we will use the relationship between time and frequency domains
- we'll take time domain specs like T_s, %OS, and convert them to freq. domain specs.

Intro to gain and phase margin

- describe degree of stability (no longer a binary question)