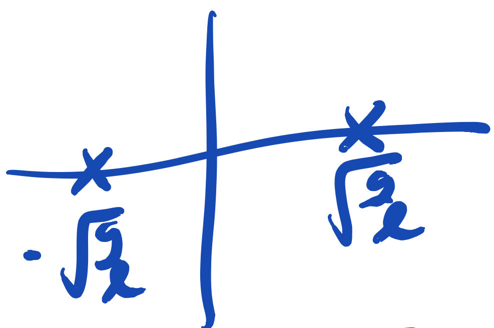
Summary Lecture 16

- Bode plots for complex conjugate roots
- for asymptotic plots (sketches by hand), assume $\zeta = 1$
- be aware that error between sketch & exact is unbounded as $\zeta \to 0$

Ch. 5

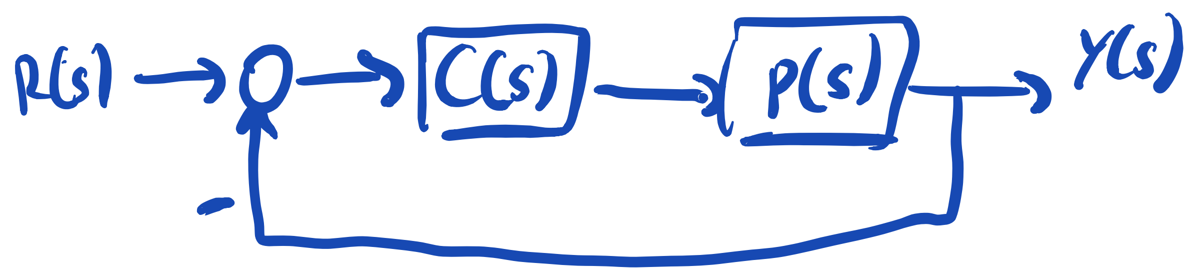
Transfer fcn from $U(s) = \mathcal{L}\{\delta u\}$ to $Y(s) = \mathcal{L}\{\delta y\}$

$$\frac{Y(s)}{U(s)} = \frac{1}{M\ell^2} \frac{1}{s^2 - g/\ell} =: P(s) \text{ (plant)}$$



Poles of P $\quad s = \pm\sqrt{\frac{g}{\ell}}$

Assume $\frac{g}{\ell} = 1$, $M\ell^2 = 1$



- One controller that stabilizes the upright position is:

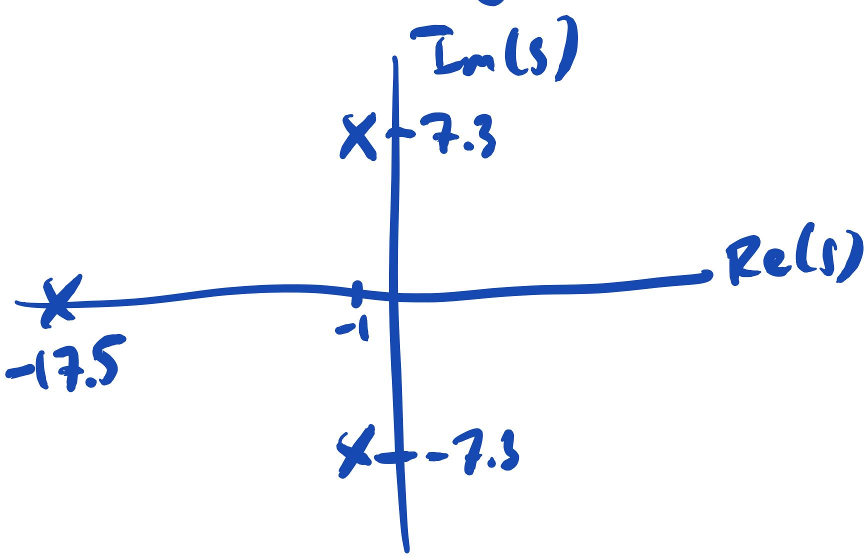
$$C(s) = 100\,\frac{s+10}{s+20} \qquad (\text{"lead controller"})$$

- With this choice of C, the TF from R to Y is

$$\frac{Y(s)}{R(s)} = \frac{C(s)\,P(s)}{1 + C(s)\,P(s)}$$

$$= \frac{100s + 1000}{s^3 + 20s^2 + 99s + 980}$$

Poles: $\{-17.5,\ -1 \pm j\,7.3\}$

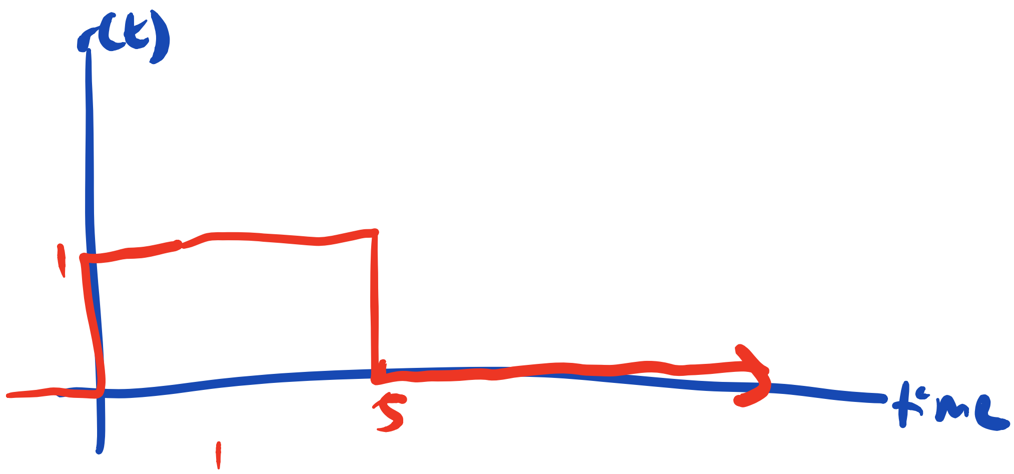


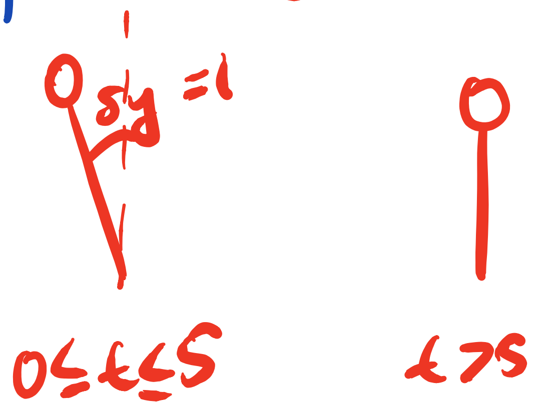
Observations:

- We have stabilized the upright position
- The dominant time-domain response comes from

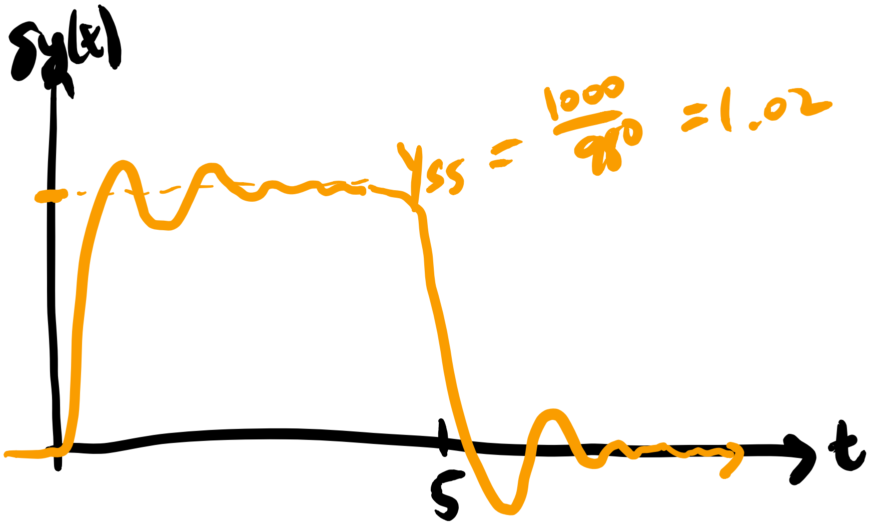
the 2 poles closest to Im(s) axis.

- Based on the 2 dominant poles, we'd expect a relatively large settling time; a lot of overshoot; oscillations at around 7.3 rad/s





- move pendulum to $\delta y = 1$ (57°) for $0 \le t \le 5$ seconds then back to upright $\delta y = 0$ for $t > 5$



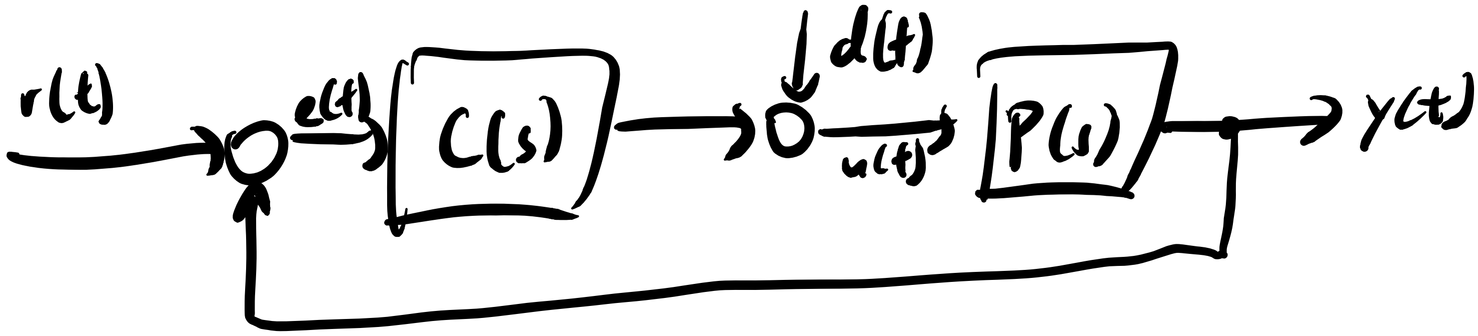
$$\frac{Y(s)}{R(s)} = \frac{100s + 1000}{s^3 + 20s^2 + 99s + 980}$$

To implement $C(s)$ on a computer, we discretize the ODE relating $e(t) = r(t) - y(t)$ to $u(t)$

$$u[k] = \frac{1}{2+20T}\Big((120T - 2)\,u[k-1] + 100\,(2+10T)\,e[k] + 100(10T-2)\,e[k-1] \Big)$$

(difference equation)

5.2 Stability of Feedback systems

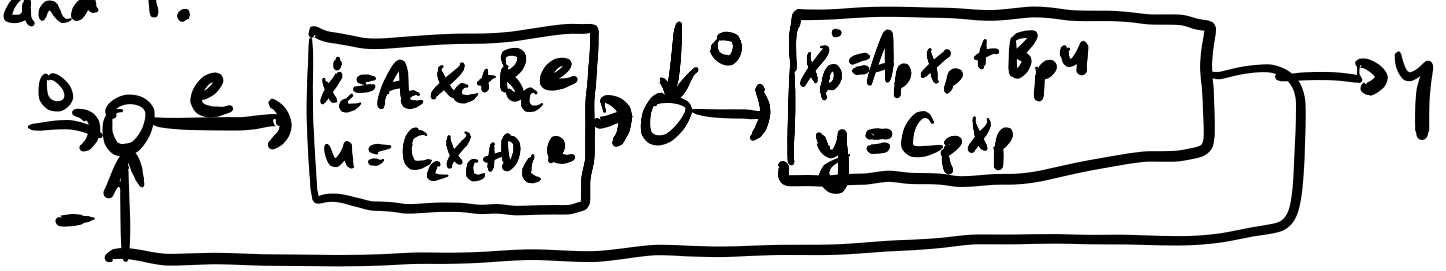


What does it mean for this system to be stable?

Assume: P and C are rational; P is strictly proper, C is proper.

5.2.1. Internal stability

Set $r(t) = d(t) = 0$. Bring in state models for C and P.



Closed loop state model

$$\dot{x}_c = A_c x_c + B_c e$$

$$\dot{x}_p = A_p x_p + B_p u$$

$$u = C_c x_c + D_c e, \quad e = -C_p x_p$$

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_c \end{bmatrix} = \left[\begin{array}{c|c} A_p - B_p D_c C_p & B_p C_c \\ \hline -B_c C_p & A_c \end{array}\right] \begin{bmatrix} x_p \\ x_c \end{bmatrix} \qquad \dot{x}_{cl} = A_{cl} x_{cl}$$

$$y_{cl} = C_{cl} x_{cl}$$

<u>Definition 5.2.2</u>

The closed-loop system is <u>internally stable</u> if

$\dot{x}_{cl} = A_{cl} x_{cl}$ is asymptotically stable.

e.g. $\dot{x}_c = -15 x_c + e$

$$u = -1000 x_c + 100 e$$

$$\dot{x}_p = \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} x_p + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} x_p$$

$$x_{cl} = \begin{bmatrix} x_p \\ x_c \end{bmatrix}, \quad A_{cl} = \left[\begin{array}{c|c} \begin{bmatrix} 0 & 1 \\ 0 & -5 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix}(100)\begin{bmatrix} 1 & 0 \end{bmatrix} & \begin{bmatrix} 0 \\ 1 \end{bmatrix}(-1000) \\ \hline -\begin{bmatrix} 1 & 0 \end{bmatrix} & -15 \end{array}\right]$$