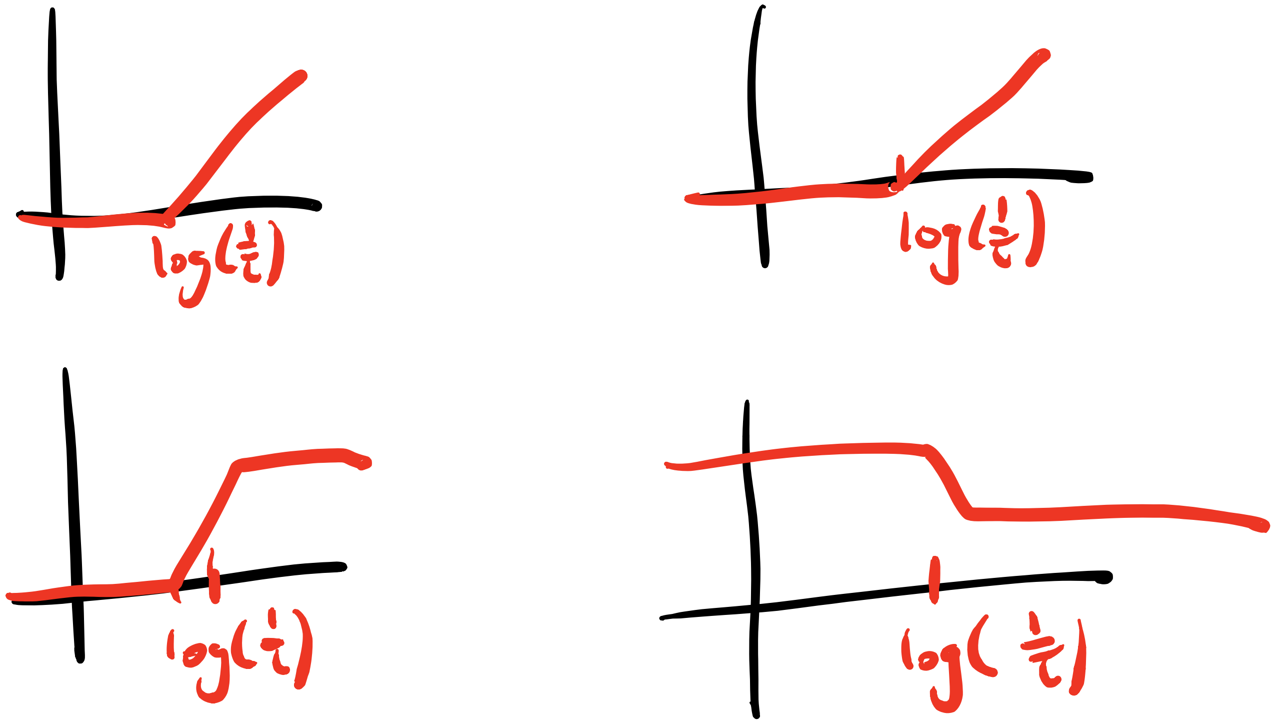
Lecture 15 Summary

- asymptotic Bode plot of $G(s) = \tau s \pm 1,\ \tau > 0$



$\tau s + 1$ $\qquad\qquad$ $\tau s - 1$

(iv) Complex conjugate roots

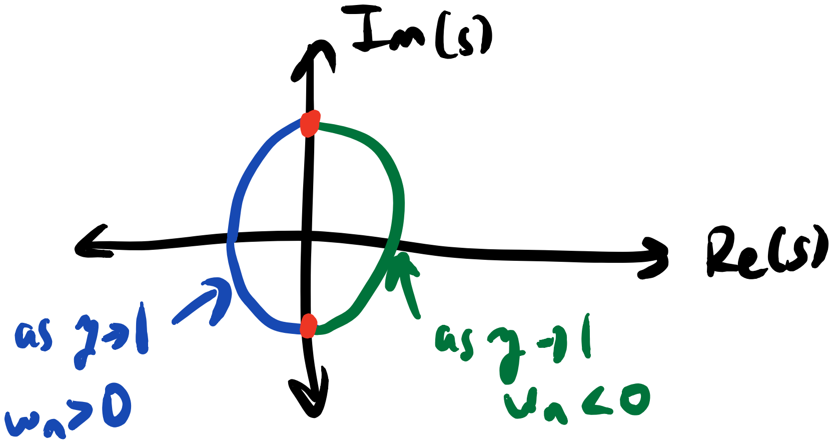
$$G(s) = \frac{s^2}{\omega_n^2} + \frac{2\zeta}{\omega_n} s + 1, \qquad \zeta \in [0, 1),\ \omega_n \neq 0$$

- three cases to consider

1) roots in $\mathbb{C}^-$
   $\omega_n > 0$
   $\zeta \neq 0$

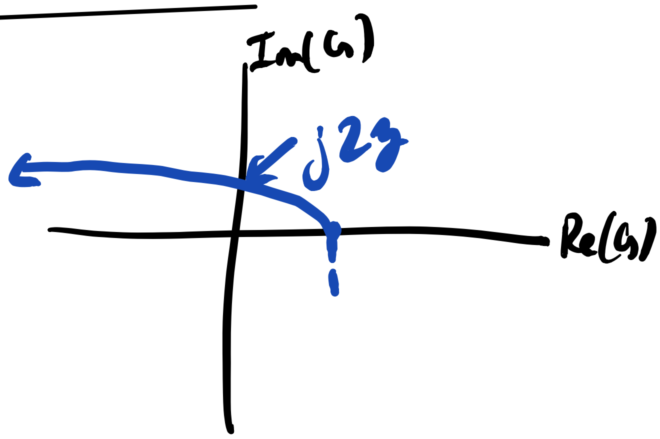
2) roots in $\mathbb{C}^+$
   $\omega_n < 0$
   $\zeta \neq 0$

3) roots on $j\mathbb{R}$
   $\zeta = 0$

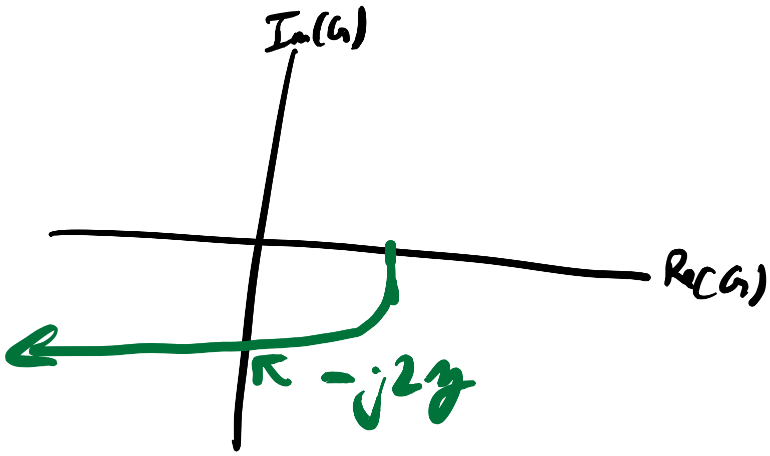


roots of G(s)

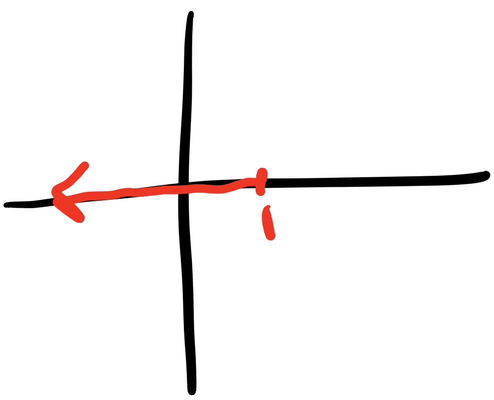
Polar Plots



$\omega_n > 0$
$\zeta \neq 0$



$\omega_n < 0$
$\zeta \neq 0$



$\zeta = 0$

$$G(j\omega) = \left(1 - \frac{\omega^2}{\omega_n^2}\right) + j\,2\zeta\frac{\omega}{\omega_n}$$

Observation: For very large $\omega \gg |\omega_n|$,

$|G(j\omega)| \approx \dfrac{\omega^2}{\omega_n^2}$ and $\angle G(j\omega) \approx 180°$

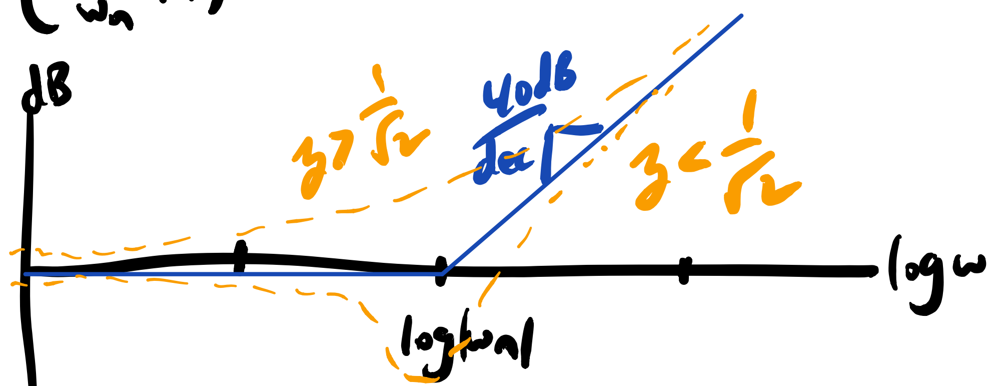
For small $\omega \ll |\omega_n|$, $|G(j\omega)| \approx 1$ and $\angle G(j\omega) \approx 0$

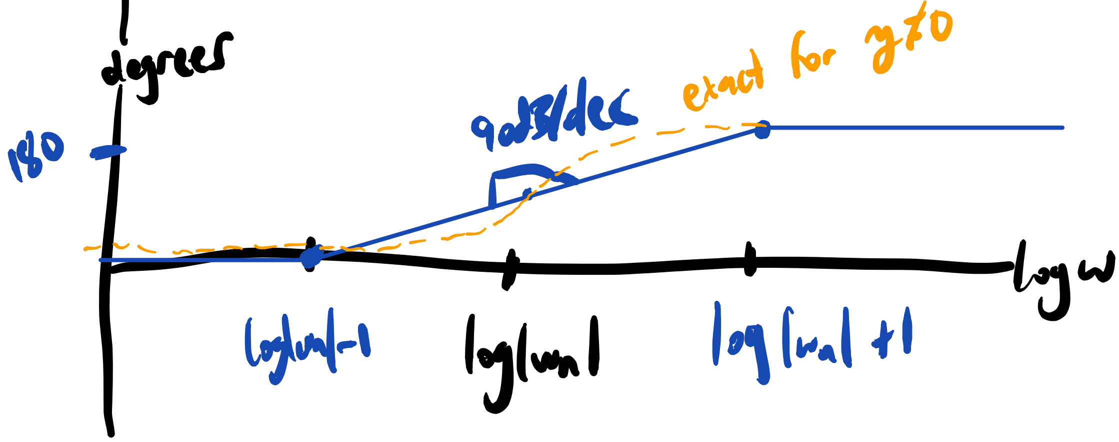
The observations we've made are also the properties of two, repeated real roots at $s = -\omega_n$

This motivates for <u>asymptotic Bode plots</u>, that we treat complex conjugate terms as two 1st-order terms with roots at $s = -\omega_n$

$$G(s) = \frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1 \approx \frac{s^2}{\omega_n^2} + \frac{2s}{\omega_n} + 1$$

$$= \left(\frac{s}{\omega_n} + 1\right)^2$$





- for $\omega_n < 0$, the magnitude plot is unaffected
- the phase plot is reflected about the horizontal axis

Ch.3. Summary

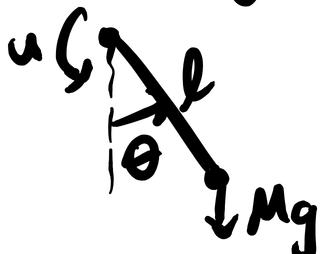
- definition of asymptotic stability for state-space models $\dot{x} = Ax$ + how to test
- definition of BIBO stability + how to test
- relationship between these concepts
- steady-state gain of a system, final-value theorem
- physical meaning of the frequency response + how to draw asymptotic Bode plots

$$\begin{tikzcd}[column sep=large, row sep=huge]
\begin{array}{c}\text{time-domain}\\ \text{(impulse response } g(t))\end{array} \arrow[rr, shift left, "\mathcal{L}"] \arrow[dr, shift left, "\mathcal{F}"] & & \begin{array}{c}G(s)\\ \text{s-domain}\end{array} \arrow[ll, shift left, "\mathcal{L}^{-1}"] \arrow[dl, shift left, "s=j\omega"] \\
& \begin{array}{c}G(j\omega)\\ \text{frequency domain}\end{array} \arrow[ul, shift left, "\mathcal{F}^{-1}"] \arrow[ur, shift left, "\text{see notes (app. 3A)}"] &
\end{tikzcd}$$

- State model: $g(t) = Ce^{At}B\mathbb{1}(t) + D\,\delta(t)$

Chapter 5: Feedback Control Theory

5.1 Closing the Loop



Design a controller to keep the pendulum upright.

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\frac{g}{\ell}\sin(x_1) + \frac{1}{M\ell^2}u$$

$$x = \begin{bmatrix}\theta \\ \dot{\theta}\end{bmatrix}$$

$$y = \theta$$

$$y = x_1$$

The equilibrium configuration corresponding to the upright position is:

$$(\bar{x}, \bar{u}) = \left(\begin{bmatrix}\pi \\ 0\end{bmatrix}, 0\right]$$

Linearization at $(\bar{x}, \bar{u})$:

$$\delta\dot{x} = \begin{bmatrix}0 & 1 \\ g/\ell & 0\end{bmatrix}\delta x + \begin{bmatrix}0 \\ 1/M\ell^2\end{bmatrix}\delta u$$

$$\delta y = \begin{bmatrix}1 & 0\end{bmatrix}\delta x$$

TF from δu to δy is:

$$\frac{\Delta Y(s)}{\Delta U(s)} = \frac{1}{M\ell^2}\,\frac{1}{s^2 - g/\ell}$$