Bridge of Doom

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Summary: This computational essay presents Project 1 of the QEA2 Spring 2024 course, titled "Bridge of Doom." It delves into the mathematical modeling and practical application of steering a modified Neato robot along a predetermined parametric curve. The notebook is organized into several sections for ease of understanding: Section 1 covers the plotting of the path; Section 2 details the plotting of wheel velocities; Section 3 outlines the code implementation on the Neato robot; and Section 4 focuses on graphing sensor data to compare the robot's actual path against its planned trajectory.

Section 1: Plotting the Path

```
% Define a scale factor for parameter t
beta = 0.2;

% Generate 1000 linearly spaced points between 0 and 3.2
points = linspace(0, 3.2, 1000);

% Calculate x and y coordinates for the parametric curve
x = 0.3960 * cos(2.65*(1.4 + points)); % x-coordinates based on a cosine
function
y = -0.99 * sin(1.4 + points); % y-coordinates based on a sine function

% Declare symbolic variables for further calculations
syms u r t

% Define u as a scaled version of t
u = beta * t;

% Parametric equations for x, y, and z components
r_i = 0.3960 * cos(2.65*(1.4 + u)); % x-component of the curve
r_j = -0.99 * sin(1.4 + u); % y-component of the curve
r_k = 0 * u; % z-component of the curve (zero for 2D path)

% Assume t is a real number for symbolic computation
assume(t,'real');

% Combine the components to form the parametric curve vector
r = [r_i, r_j, r_k];

% Calculate the derivative of r with respect to t
drdu = diff(r,t);

% Calculate the unit tangent vector T_hat
T_hat = simplify(drdu./norm(drdu));
```

```
% Calculate the derivative of T_hat with respect to t
dT_hatdu = diff(T_hat,t);

% Calculate the unit normal vector N_hat
N_hat = dT_hatdu/norm(dT_hatdu);
N_hat = simplify(N_hat);

% Specific points to plot tangent and normal vectors
vecs = transpose(linspace(0.5, 3, 3));

% Calculate x and y coordinates at specific points
vecs_x = 0.3960 * cos(2.65*(1.4 + vecs));
vecs_y = -0.99 * sin(1.4 + vecs);

% Evaluate N_hat (normal vector) and T_hat (tangent vector) at specific
points
N = eval(subs(N_hat, vecs/beta));
T = eval(subs(T_hat, vecs/beta));
```

Plot figure

```
% Initialize figure for plotting
f = figure();
hold on;

% Plot the parametric curve
plot(x, y);
title('Parametric Curve Visualization of a Neato Path Including Unit
Tangent and Normal Vectors');
xlabel('X Position (meters)');
ylabel('Y Position (meters)');

% Add quiver plots for tangent and normal vectors at specified points
quiver(vecs_x, vecs_y, N(:,1), N(:, 2)); % Normal vectors
quiver(vecs_x, vecs_y, T(:,1), T(:, 2)); % Tangent vectors

% Add legend to distinguish between path, tangent, and normal vectors
legend("Path", "Tangent", "Normal");
```

```
beta = 0.2000
u =
```

$$\frac{t}{5}$$

```
r_i =
```

$$\frac{99\cos\left(\frac{53\,t}{100}+\frac{371}{100}\right)}{250}$$

```
r_j =
```

$$-\frac{99 \sin\left(\frac{t}{5}+\frac{7}{5}\right)}{100}$$

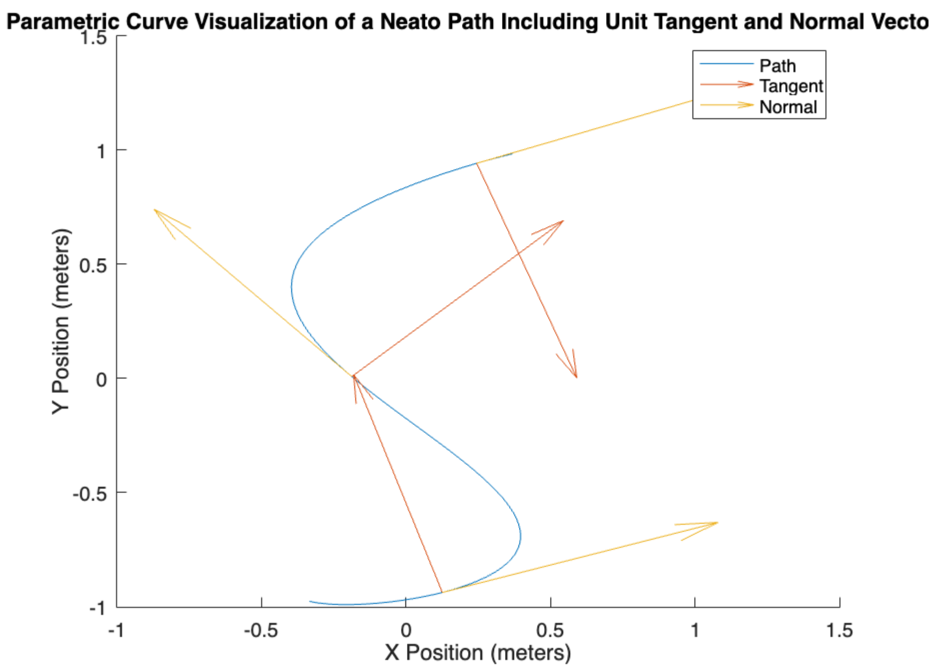
```
r_k = 0
T_hat =
```

$$\left(-\frac{53 \sin\left(\frac{53\,t}{100}+\frac{371}{100}\right)}{\sigma_1} \quad -\frac{50 \cos\left(\frac{t}{5}+\frac{7}{5}\right)}{\sigma_1} \quad 0\right)$$

where

$$\sigma_1 = \sqrt{2500 \cos\left(\frac{t}{5}+\frac{7}{5}\right)^2 + 2809 \sin\left(\frac{53\,t}{100}+\frac{371}{100}\right)^2}$$

```
N = 3×3
   -0.3061    0.9520         0
    0.7310    0.6824         0
    0.3456   -0.9384         0
T = 3×3
    0.9520    0.3061         0
   -0.6824    0.7310         0
    0.9384    0.3456         0
```



Section 2: Plotting the Wheel Velocities

```
% Calculate velocity vector as derivative of position vector r w.r.t t.
vel = diff(r, t);

% Calculate angular velocity vector (omega) as cross product of T_hat and
its derivative.
omega = cross(T_hat, diff(T_hat));
```

```
% Evaluate linear velocity (V_n) for points, normalizing to get speed at
each point.
V_n = sqrt(sum((eval(subs(vel, transpose(points)/beta))).^2,2));

% Evaluate angular velocity at points, focusing on z-axis component for 2D
motion.
ang_vel = eval(subs(omega, transpose(points)/beta));

% Plot linear and angular velocities.
figure; hold on;
plot(points, V_n); % Linear velocity
plot(points, ang_vel(:, 3)); % Angular velocity, z-axis component
legend("Linear Velocity", "Angular Velocity");
```

Calculating Left and Right Wheel Velocities

```
% Calculate velocities for left and right wheels based on linear and
angular velocities.
V_L = V_n - ang_vel(:, 3) * 0.245/2; % Left wheel velocity
V_R = V_n + ang_vel(:, 3) * 0.245/2; % Right wheel velocity
```

vel =

$$\left(-\frac{5247\sin\left(\frac{53\,t}{100}+\frac{371}{100}\right)}{25000}\quad -\frac{99\cos\left(\frac{t}{5}+\frac{7}{5}\right)}{500}\quad 0\right)$$

ans =

$$\left(-\frac{2809\cos\left(\frac{53\,t}{100}+\frac{371}{100}\right)}{100\,\sqrt{\sigma_2}}-\frac{53\,\sigma_3\,\sigma_1}{2\,{\sigma_2}^{3/2}}\quad \frac{10\sin\left(\frac{t}{5}+\frac{7}{5}\right)}{\sqrt{\sigma_2}}-\frac{25\cos\left(\frac{t}{5}+\frac{7}{5}\right)\sigma_1}{{\sigma_2}^{3/2}}\quad 0\right)$$

where

$$\sigma_1 = 1000\cos\left(\frac{t}{5}+\frac{7}{5}\right)\sin\left(\frac{t}{5}+\frac{7}{5}\right)-\frac{148877\cos\left(\frac{53\,t}{100}+\frac{371}{100}\right)\sigma_3}{50}$$

$$\sigma_2 = 2500\cos\left(\frac{t}{5}+\frac{7}{5}\right)^2+2809\,{\sigma_3}^2$$

$$\sigma_3 = \sin\left(\frac{53\,t}{100}+\frac{371}{100}\right)$$

omega =

$$\left(0 \quad 0 \quad -\frac{50\,\sigma_4\left(\frac{2809\cos\left(\frac{53\,t}{100}+\frac{371}{100}\right)}{100\,\sqrt{\sigma_2}}+\frac{53\,\sigma_3\,\sigma_1}{2\,{\sigma_2}^{3/2}}\right)}{\sqrt{\sigma_2}}-\frac{53\,\sigma_3\left(\frac{10\sin\left(\frac{t}{5}+\frac{7}{5}\right)}{\sqrt{\sigma_2}}-\frac{25\,\sigma_4\,\sigma_1}{{\sigma_2}^{3/2}}\right)}{\sqrt{\sigma_2}}\right)$$

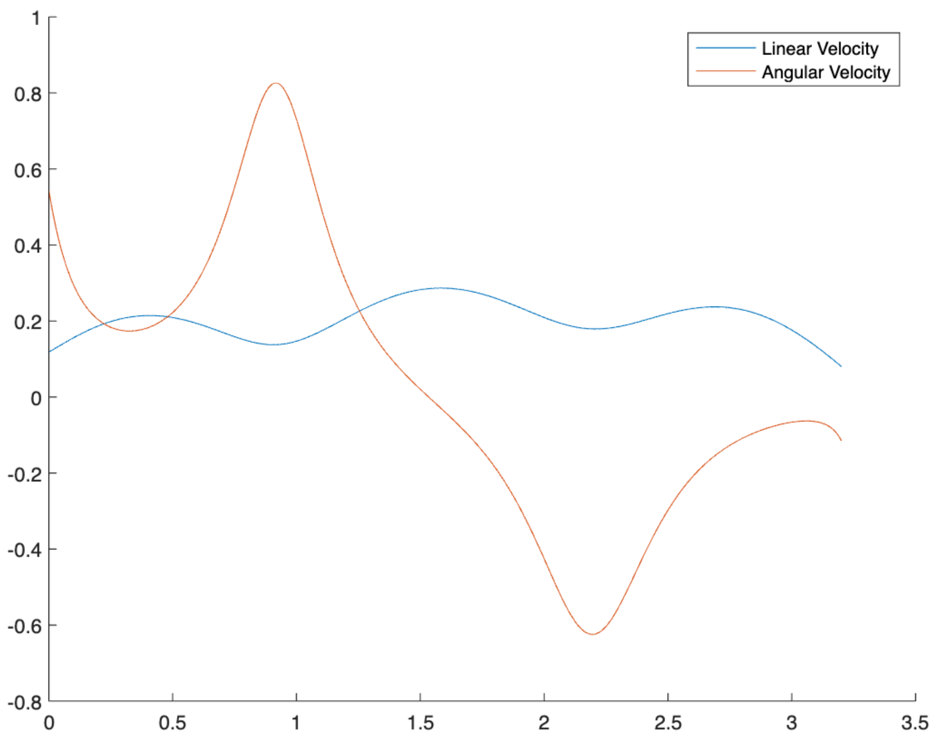
where

$$\sigma_1 = 1000\,\sigma_4\sin\left(\frac{t}{5}+\frac{7}{5}\right)-\frac{148877\cos\left(\frac{53\,t}{100}+\frac{371}{100}\right)\sigma_3}{50}$$

$$\sigma_2 = 2500\,{\sigma_4}^2+2809\,{\sigma_3}^2$$

$$\sigma_3 = \sin\left(\frac{53\,t}{100}+\frac{371}{100}\right)$$

$$\sigma_4 = \cos\left(\frac{t}{5}+\frac{7}{5}\right)$$




```
V_L = 1000×1
    0.0512
    0.0540
    0.0567
    0.0594
    0.0621
    0.0647
    0.0672
```

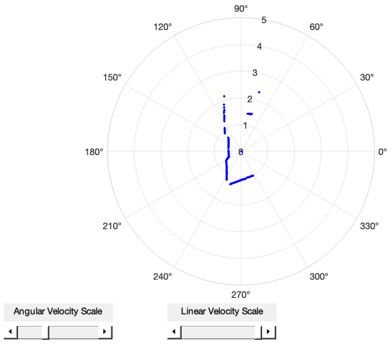
```
    0.0698
    0.0722
    0.0747
      ⋮
V_R = 1000×1
    0.1846
    0.1843
    0.1841
    0.1839
    0.1838
    0.1837
    0.1837
    0.1837
    0.1837
    0.1838
      ⋮
```

Section 3: Code Implementation on the Neato Robot

```
% Initial setup

% Assuming V_L and V_R are your vectors of length 1000
new_length = 1000 / 10; % Target length after averaging

% Data collection storage space initialization
max_points = new_length;

fig = gcf;
```

```
Setup complete. Press enter to start.
```

```
drivetime = 0.2; % s for each velocity pair
encoder_data = zeros(max_points,3);
iter = 0;

% Shorten V_L and V_R by averaging every 10 elements
V_L_short = mean(reshape(V_L, [10, length(V_L)/10]));
V_R_short = mean(reshape(V_R, [10, length(V_R)/10]));

% Now, incorporate V_L_short and V_R_short into the modified robot code
% [sensors,vels] = neatoSim(); % Uncomment for simulator
[sensors,vels] = neato('192.168.16.67'); % Uncomment for physical neato
```




```
fig = gcf;

for idx = 1:length(V_L_short) % Using shortened vectors
    vl = V_L_short(idx);
    vr = V_R_short(idx);
    disp(['Running with vl = ', num2str(vl), ' and vr = ', num2str(vr)]);
    tic;
    t = toc;
    while t < drivetime
        t = toc; % Update t
        vels.lrWheelVelocitiesInMetersPerSecond = [vl, vr];
    end
    sensors.encoders
    if isempty(sensors.encoders) == 0
        iter=iter+1;
        if iter<=max_points

encoder_data(iter,:)=[t,sensors.encoders(1),sensors.encoders(2)];
        end
    end




    % Stop the robot after the drive time for the current velocity pair
    vels.lrWheelVelocitiesInMetersPerSecond = [0, 0];
    %pause(0.1); % Pause to stabilize before the next iteration
end
```

```
Running with vl = 0.063206 and vr = 0.18391
ans =

     []
Running with vl = 0.08721 and vr = 0.1849
ans =

     []
Running with vl = 0.10764 and vr = 0.18868
ans =

     []
Running with vl = 0.12518 and vr = 0.1941
```

```
ans =

     []
Running with vl = 0.14026 and vr = 0.20034
ans = 1×2
    0.0580    0.0820
Running with vl = 0.15318 and vr = 0.20682
ans = 1×2
    0.1630    0.2290
Running with vl = 0.16411 and vr = 0.21314
ans = 1×2
    0.2230    0.3110
Running with vl = 0.17316 and vr = 0.21901
ans = 1×2
    0.2550    0.3520
Running with vl = 0.18041 and vr = 0.22423
ans = 1×2
    0.2880    0.3960
Running with vl = 0.18589 and vr = 0.22866
ans = 1×2
    0.3220    0.4380
Running with vl = 0.18963 and vr = 0.23222
ans = 1×2
    0.3590    0.4850
Running with vl = 0.19164 and vr = 0.23485
ans = 1×2
    0.3980    0.5330
Running with vl = 0.19194 and vr = 0.23654
ans = 1×2
    0.4340    0.5760
Running with vl = 0.19054 and vr = 0.23734
ans = 1×2
    0.4710    0.6220
Running with vl = 0.18746 and vr = 0.23728
ans = 1×2
    0.5100    0.6690
Running with vl = 0.18271 and vr = 0.23648
ans = 1×2
    0.5460    0.7140
Running with vl = 0.17631 and vr = 0.23506
ans = 1×2
    0.5850    0.7630
Running with vl = 0.1683 and vr = 0.23319
ans = 1×2
    0.6210    0.8100
Running with vl = 0.15871 and vr = 0.23108
ans = 1×2
    0.6540    0.8560
Running with vl = 0.14759 and vr = 0.22899
ans = 1×2
    0.6870    0.9040
Running with vl = 0.13502 and vr = 0.22718
ans = 1×2
    0.7510    0.9970
Running with vl = 0.12116 and vr = 0.22597
ans = 1×2
    0.7770    1.0390
Running with vl = 0.10625 and vr = 0.22563
ans = 1×2
    0.8040    1.0850
Running with vl = 0.090682 and vr = 0.22637
ans = 1×2
    0.8280    1.1330
```

```
Running with vl = 0.075123 and vr = 0.22824
ans = 1×2
    0.8490    1.1750
Running with vl = 0.060558 and vr = 0.23103
ans = 1×2
    0.8670    1.2210
Running with vl = 0.048314 and vr = 0.23422
ans = 1×2
    0.8810    1.2630
Running with vl = 0.03991 and vr = 0.23703
ans = 1×2
    0.8930    1.3090
Running with vl = 0.036696 and vr = 0.23872
ans = 1×2
    0.9030    1.3560
Running with vl = 0.039392 and vr = 0.23889
ans = 1×2
    0.9100    1.4020
Running with vl = 0.047806 and vr = 0.23774
ans = 1×2
    0.9180    1.4480
Running with vl = 0.060933 and vr = 0.23596
ans = 1×2
    0.9270    1.4960
Running with vl = 0.077356 and vr = 0.23442
ans = 1×2
    0.9350    1.5410
Running with vl = 0.095678 and vr = 0.23383
ans = 1×2
    0.9480    1.5910
Running with vl = 0.11478 and vr = 0.23459
ans = 1×2
    0.9650    1.6380
Running with vl = 0.13388 and vr = 0.23677
ans = 1×2
    0.9820    1.6810
Running with vl = 0.15248 and vr = 0.24023
ans = 1×2
    1.0080    1.7310
Running with vl = 0.17026 and vr = 0.24471
ans = 1×2
    1.0340    1.7750
Running with vl = 0.18705 and vr = 0.2499
ans = 1×2
    1.0640    1.8200
Running with vl = 0.20274 and vr = 0.25547
ans = 1×2
    1.0980    1.8670
Running with vl = 0.21725 and vr = 0.26114
ans = 1×2
    1.1330    1.9120
Running with vl = 0.23054 and vr = 0.26663
ans = 1×2
    1.1740    1.9610
Running with vl = 0.24256 and vr = 0.27172
ans = 1×2
    1.2170    2.0110
Running with vl = 0.2533 and vr = 0.2762
ans = 1×2
    1.2640    2.0610
Running with vl = 0.26273 and vr = 0.27992
ans = 1×2
    1.3110    2.1100
```

```
Running with vl = 0.27084 and vr = 0.28272
ans = 1×2
    1.3560    2.1610
Running with vl = 0.27763 and vr = 0.2845
ans = 1×2
    1.4130    2.2230
Running with vl = 0.28312 and vr = 0.28517
ans = 1×2
    1.4610    2.2710
Running with vl = 0.28732 and vr = 0.28466
ans = 1×2
    1.5130    2.3210
Running with vl = 0.29027 and vr = 0.28291
ans = 1×2
    1.6590    2.4630
Running with vl = 0.29202 and vr = 0.27989
ans = 1×2
    1.7630    2.5620
Running with vl = 0.29264 and vr = 0.2756
ans = 1×2
    1.8710    2.6660
Running with vl = 0.29221 and vr = 0.27002
ans = 1×2
    1.9710    2.7650
Running with vl = 0.29083 and vr = 0.26318
ans = 1×2
    2.0240    2.8180
Running with vl = 0.28863 and vr = 0.2551
ans = 1×2
    2.1350    2.9290
Running with vl = 0.28575 and vr = 0.24584
ans = 1×2
    2.1910    2.9850
Running with vl = 0.28233 and vr = 0.23546
ans = 1×2
    2.2480    3.0410
Running with vl = 0.27857 and vr = 0.22404
ans = 1×2
    2.3600    3.1420
Running with vl = 0.27466 and vr = 0.21172
ans = 1×2
    2.4120    3.1850
Running with vl = 0.2708 and vr = 0.19862
ans = 1×2
    2.4660    3.2310
Running with vl = 0.26719 and vr = 0.18495
ans = 1×2
    2.5200    3.2730
Running with vl = 0.264 and vr = 0.17097
ans = 1×2
    2.6220    3.3500
Running with vl = 0.26139 and vr = 0.15701
ans = 1×2
    2.6710    3.3840
Running with vl = 0.25942 and vr = 0.1435
ans = 1×2
    2.7200    3.4160
Running with vl = 0.25809 and vr = 0.13098
ans = 1×2
    2.7700    3.4470
Running with vl = 0.25727 and vr = 0.12006
ans = 1×2
    2.8190    3.4760
Running with vl = 0.25676 and vr = 0.1114
```

```
ans = 1×2
    2.8690    3.5020
Running with vl = 0.25634 and vr = 0.10557
ans = 1×2
    2.9180    3.5260
Running with vl = 0.2558 and vr = 0.10298
ans = 1×2
    2.9680    3.5480
Running with vl = 0.25506 and vr = 0.10377
ans = 1×2
    3.0170    3.5700
Running with vl = 0.25416 and vr = 0.10774
ans = 1×2
    3.0660    3.5900
Running with vl = 0.25325 and vr = 0.11443
ans = 1×2
    3.1130    3.6110
Running with vl = 0.25255 and vr = 0.12323
ans = 1×2
    3.1630    3.6320
Running with vl = 0.25221 and vr = 0.13343
ans = 1×2
    3.2150    3.6560
Running with vl = 0.25234 and vr = 0.14441
ans = 1×2
    3.3150    3.7070
Running with vl = 0.25292 and vr = 0.15562
ans = 1×2
    3.3640    3.7350
Running with vl = 0.25388 and vr = 0.1666
ans = 1×2
    3.4150    3.7640
Running with vl = 0.25508 and vr = 0.17703
ans = 1×2
    3.4640    3.7950
Running with vl = 0.25633 and vr = 0.18664
ans = 1×2
    3.5150    3.8280
Running with vl = 0.25746 and vr = 0.19524
ans = 1×2
    3.5630    3.8630
Running with vl = 0.25826 and vr = 0.20268
ans = 1×2
    3.6080    3.8990
Running with vl = 0.25856 and vr = 0.20886
ans = 1×2
    3.6430    3.9380
Running with vl = 0.25822 and vr = 0.21368
ans = 1×2
    3.6820    3.9790
Running with vl = 0.25707 and vr = 0.2171
ans = 1×2
    3.7180    4.0180
Running with vl = 0.25502 and vr = 0.21907
ans = 1×2
    3.7580    4.0590
Running with vl = 0.25197 and vr = 0.21955
ans = 1×2
    3.7990    4.1020
Running with vl = 0.24784 and vr = 0.21853
ans = 1×2
    3.8800    4.1850
Running with vl = 0.24259 and vr = 0.21601
ans = 1×2
```

```
    3.9210    4.2280
Running with vl = 0.23617 and vr = 0.212
ans = 1×2
    3.9620    4.2710
Running with vl = 0.22857 and vr = 0.20651
ans = 1×2
    4.0000    4.3120
Running with vl = 0.2198 and vr = 0.19956
ans = 1×2
    4.0420    4.3560
Running with vl = 0.20988 and vr = 0.19119
ans = 1×2
    4.0840    4.3980
Running with vl = 0.19885 and vr = 0.18145
ans = 1×2
    4.1250    4.4410
Running with vl = 0.18677 and vr = 0.17037
ans = 1×2
    4.1670    4.4800
Running with vl = 0.17372 and vr = 0.15799
ans = 1×2
    4.2070    4.5160
Running with vl = 0.15981 and vr = 0.14435
ans = 1×2
    4.2450    4.5480
Running with vl = 0.1452 and vr = 0.12946
ans = 1×2
    4.2860    4.5800
Running with vl = 0.13012 and vr = 0.11326
ans = 1×2
    4.3250    4.6090
Running with vl = 0.11495 and vr = 0.095567
ans = 1×2
    4.3560    4.6340
Running with vl = 0.10041 and vr = 0.075882
ans = 1×2
    4.3820    4.6560
```

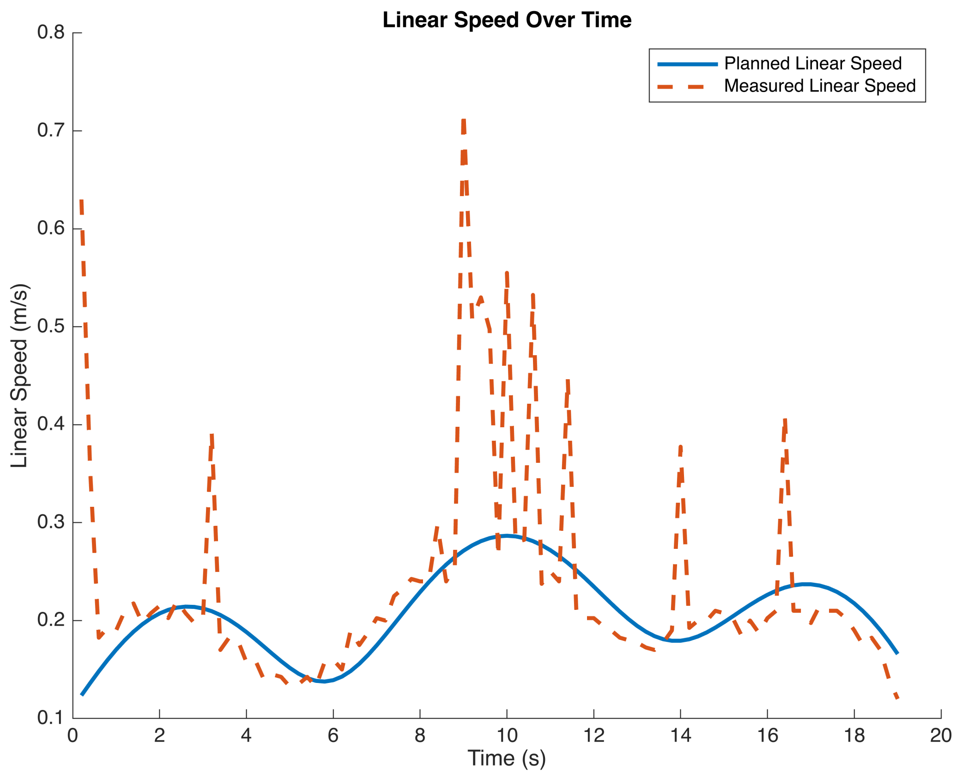
```
% Clean up
close(fig); % Close the figure at the end of all iterations
disp('Data collection complete.');
```

```
Data collection complete.
```

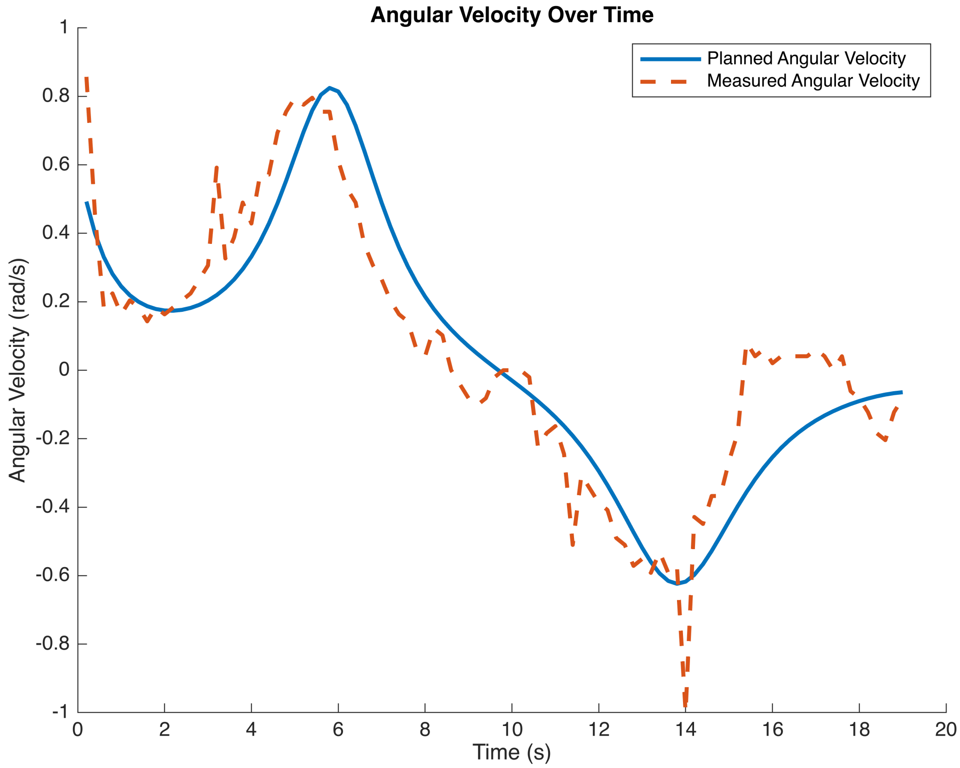
Section 4: Graphing Sensor Data to Compare the Robot's Actual Path against Its Planned Trajectory.

```
% Parameters
wheel_base = 0.245; % Distance between the wheels in meters

% Initialization
x_pos = 0; % Initial x position
y_pos = 0; % Initial y position
theta = 0; % Initial orientation angle in radians

% Calculate measured linear speed and angular velocity
measured_linear_speeds = zeros(max_points-1, 1);
```

```
measured_angular_velocities = zeros(max_points-1, 1);

for i = 2:max_points
    delta_left = (encoder_data(i,2) - encoder_data(i-1,2));
    delta_right = (encoder_data(i,3) - encoder_data(i-1,3));
    delta_time = drivetime; % Assuming constant drive time between
measurements

    % Linear speed is the average of the left and right wheel speeds
    measured_linear_speeds(i-1) = (delta_left + delta_right) / (2 *
delta_time);

    % Angular velocity is the difference in wheel speeds divided by the
wheelbase, considering the direction
    measured_angular_velocities(i-1) = (delta_right - delta_left) /
(wheel_base * delta_time);
end

% Time array for plotting
time_array = (1:max_points-1) * drivetime;

% Plot Linear Speed
% subplot(2, 1, 1); % Two rows, one column, first subplot
figure;
hold on;
V_n_short = mean(reshape(V_n, [10, length(V_n)/10]));
plot(time_array(1:95), V_n_short(1:95), 'LineWidth', 2); % Planned linear
speed in solid line
plot(time_array(1:95), measured_linear_speeds(1:95), '--', 'LineWidth', 2);
% Measured linear speed in dashed line
title('Linear Speed Over Time');
xlabel('Time (s)');
ylabel('Linear Speed (m/s)');
legend('Planned Linear Speed', 'Measured Linear Speed');
hold off;
```

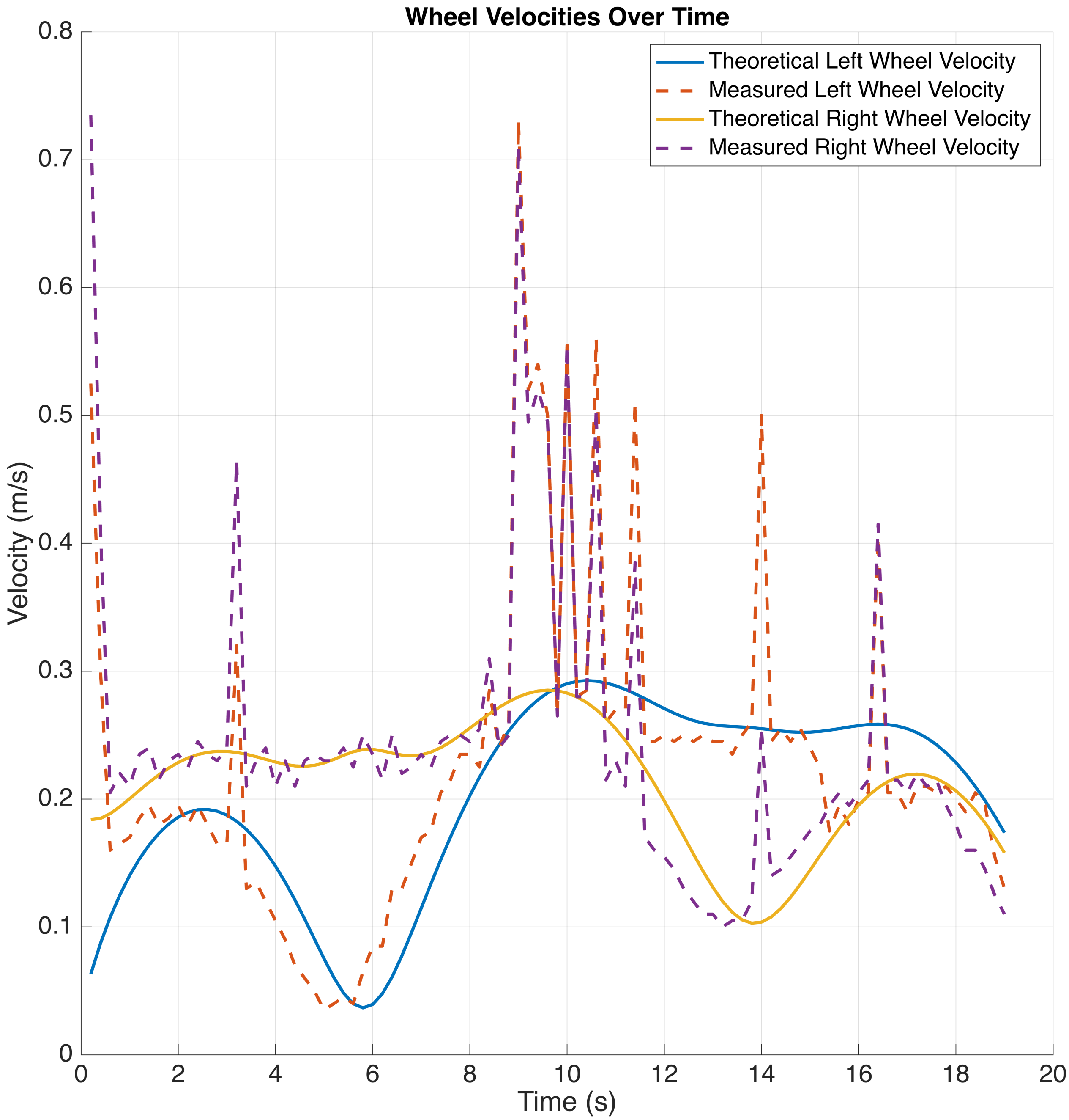



```
% Plot Angular Velocity
% subplot(2, 1, 2); % Two rows, one column, second subplot
figure;
hold on;
planned_angular_velocities = mean(reshape(ang_vel(:, 3), [10,
length(ang_vel(:, 3))/10]));
plot(time_array(1:95), planned_angular_velocities(1:95), 'LineWidth', 2); %
Planned angular velocity in solid line
plot(time_array(1:95), measured_angular_velocities(1:95), '--',
'LineWidth', 2); % Measured angular velocity in dashed line
title('Angular Velocity Over Time');
xlabel('Time (s)');
ylabel('Angular Velocity (rad/s)');
legend('Planned Angular Velocity', 'Measured Angular Velocity');
hold off;
```




```
% Assuming wheel_radius and encoder_resolution are defined as before
% Calculate measured left and right wheel velocities from encoder data
measured_V_L = zeros(max_points-1, 1);
measured_V_R = zeros(max_points-1, 1);

for i = 2:max_points
    delta_left = (encoder_data(i,2) - encoder_data(i-1,2));
    delta_right = (encoder_data(i,3) - encoder_data(i-1,3));
    delta_time = drivetime; % Time interval between measurements

    % Wheel velocities in meters per second
    measured_V_L(i-1) = delta_left / delta_time;
    measured_V_R(i-1) = delta_right / delta_time;
end


% Plotting
figure; % Start a new figure
figure('Position',[300 50 1200 1200]);

hold on; % Hold on for multiple plots
% Theoretical Left Wheel Velocity
plot(time_array(1:95), V_L_short(1:95), 'LineWidth', 2, 'DisplayName',
'Theoretical Left Wheel Velocity');
```

```
% Measured Left Wheel Velocity
plot(time_array(1:95), measured_V_L(1:95), '--', 'LineWidth', 2,
'DisplayName', 'Measured Left Wheel Velocity');

% Theoretical Right Wheel Velocity
plot(time_array(1:95), V_R_short(1:95), 'LineWidth', 2, 'DisplayName',
'Theoretical Right Wheel Velocity');

% Measured Right Wheel Velocity
plot(time_array(1:95), measured_V_R(1:95), '--', 'LineWidth', 2,
'DisplayName', 'Measured Right Wheel Velocity');

% Configure the plot
title('Wheel Velocities Over Time');
fontsize(16,"points");
xlabel('Time (s)');
ylabel('Velocity (m/s)');
legend('show');
grid on;
hold off;
```



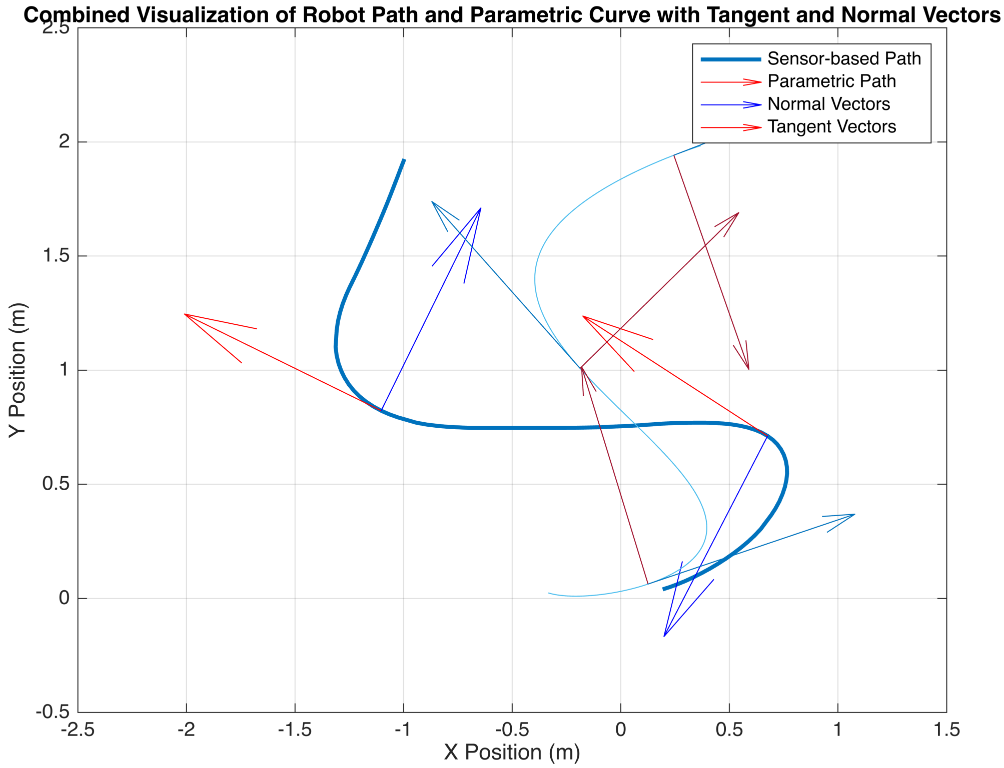

```
% Storage for position and orientation
x_trajectory = zeros(max_points, 1);
y_trajectory = zeros(max_points, 1);
theta_trajectory = zeros(max_points, 1);

% Calculate the trajectory
for i = 2:max_points
    % Calculate the distance each wheel traveled in meters
```

```
    delta_left = (encoder_data(i,2) - encoder_data(i-1,2));
    delta_right = (encoder_data(i,3) - encoder_data(i-1,3));

    % Avg distance traveled
    delta_center = (delta_left + delta_right) / 2;

    % Change in orientation
    delta_theta = (delta_right - delta_left) / wheel_base;

    % Update orientation
    theta = theta + delta_theta;

    % Update position
    x_pos = x_pos + delta_center * cos(theta);
    y_pos = y_pos + delta_center * sin(theta);

    % Store trajectory
    x_trajectory(i) = x_pos;
    y_trajectory(i) = y_pos;
    theta_trajectory(i) = theta;
end

% Number of points in the trajectory
numPointsSensorPath = length(x_trajectory);

% Preallocate arrays for derivatives and vectors for the entire sensor path
deltaXSensorPath = zeros(numPointsSensorPath, 1);
deltaYSensorPath = zeros(numPointsSensorPath, 1);
tangentVectorXAllPoints = zeros(numPointsSensorPath, 1);
tangentVectorYAllPoints = zeros(numPointsSensorPath, 1);
normalVectorXAllPoints = zeros(numPointsSensorPath, 1);
normalVectorYAllPoints = zeros(numPointsSensorPath, 1);

% Calculate discrete derivatives (differences) for x and y for the entire
sensor path
for index = 2:numPointsSensorPath-1
    deltaXSensorPath(index) = (x_trajectory(index+1) -
x_trajectory(index-1))/2;
    deltaYSensorPath(index) = (y_trajectory(index+1) -
y_trajectory(index-1))/2;
end

% Calculate unit tangent vectors for the entire sensor path
for index = 1:numPointsSensorPath
    magnitude = sqrt(deltaXSensorPath(index)^2 + deltaYSensorPath(index)^2);
    if magnitude ~= 0
        tangentVectorXAllPoints(index) = deltaXSensorPath(index) /
magnitude;
        tangentVectorYAllPoints(index) = deltaYSensorPath(index) /
magnitude;
```

```
    end
end

% Calculate derivatives of the tangent vectors to get the change in tangent
vectors across the path
deltaTangentVectorX = diff(tangentVectorXAllPoints);
deltaTangentVectorY = diff(tangentVectorYAllPoints);
deltaTangentVectorX = [deltaTangentVectorX; 0]; % Adjust size to match
original array
deltaTangentVectorY = [deltaTangentVectorY; 0]; % Adjust size to match
original array

% Calculate unit normal vectors for the entire sensor path
for index = 1:numPointsSensorPath
    magnitudeNormal = sqrt(deltaTangentVectorX(index)^2 +
deltaTangentVectorY(index)^2);
    if magnitudeNormal ~= 0
        normalVectorXAllPoints(index) = deltaTangentVectorX(index) /
magnitudeNormal;
        normalVectorYAllPoints(index) = deltaTangentVectorY(index) /
magnitudeNormal;
    end
end
```

```
% Plotting the sensor path
figure;
plot(x_trajectory(3:90), y_trajectory(3:90), 'LineWidth', 2);
hold on;

% Select a specific point to plot tangent and normal vectors
specificIndexForVectors = 30;

% Define the vectors for the chosen point
specificTangentVectorX = tangentVectorXAllPoints(specificIndexForVectors);
specificTangentVectorY = tangentVectorYAllPoints(specificIndexForVectors);
specificNormalVectorX = normalVectorXAllPoints(specificIndexForVectors);
specificNormalVectorY = normalVectorYAllPoints(specificIndexForVectors);

% Add quivers for unit tangent and normal vectors at the specified point
quiver(x_trajectory(specificIndexForVectors),
y_trajectory(specificIndexForVectors), specificTangentVectorX,
specificTangentVectorY, 'r', 'AutoScale', 'off', 'MaxHeadSize', 2,
'DisplayName', 'Tangent Vector at Specific Point');
quiver(x_trajectory(specificIndexForVectors),
y_trajectory(specificIndexForVectors), specificNormalVectorX,
specificNormalVectorY, 'b', 'AutoScale', 'off', 'MaxHeadSize', 2,
'DisplayName', 'Normal Vector at Specific Point');

% Select a specific point to plot tangent and normal vectors
```

```
specificIndexForVectors = 60;

% Define the vectors for the chosen point
specificTangentVectorX = tangentVectorXAllPoints(specificIndexForVectors);
specificTangentVectorY = tangentVectorYAllPoints(specificIndexForVectors);
specificNormalVectorX = normalVectorXAllPoints(specificIndexForVectors);
specificNormalVectorY = normalVectorYAllPoints(specificIndexForVectors);

% Add quivers for unit tangent and normal vectors at the specified point
quiver(x_trajectory(specificIndexForVectors),
y_trajectory(specificIndexForVectors), specificTangentVectorX,
specificTangentVectorY, 'r', 'AutoScale', 'off', 'MaxHeadSize', 2,
'DisplayName', 'Tangent Vector at Specific Point');
quiver(x_trajectory(specificIndexForVectors),
y_trajectory(specificIndexForVectors), specificNormalVectorX,
specificNormalVectorY, 'b', 'AutoScale', 'off', 'MaxHeadSize', 2,
'DisplayName', 'Normal Vector at Specific Point');

% Plot original parametric curve
plot(x, y+1);


% Add quivers for unit tangent and normal vectors at specified points
quiver(vecs_x, vecs_y+1, N(:,1), N(:, 2), 'AutoScale', 'off');
quiver(vecs_x, vecs_y+1, T(:,1), T(:, 2), 'AutoScale', 'off');

% Configure the plot
xlabel('X Position (m)');
ylabel('Y Position (m)');
title('Combined Visualization of Robot Path and Parametric Curve with
Tangent and Normal Vectors');
legend('Sensor-based Path', 'Parametric Path', 'Normal Vectors', 'Tangent
Vectors');
% axis equal;
grid on;

% Release the hold
hold off;
```



Combined Visualization of Robot Path and Parametric Curve with Tangent and Normal Vectors
Sensor-based Path
Parametric Path
Normal Vectors
Tangent Vectors
Y Position (m)
X Position (m)
2.5
2
1.5
1
0.5
0
-0.5
-2.5
-2
-1.5
-1
-0.5
0
0.5
1
1.5