Potential Outcomes Simulation Homework

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Objective

In this exercise, you will be tasked with simulating an intervention study with a pre-determined average treatment effect. The goal is for you to understand the **potential outcome framework**, and the properties of **completely randomized experiments** through simulation.

Problem Statement

The goal is to simulate a data set with a treatment effect of $\tau = 5$.

The setting for our hypothetical study is Professor Hill's Causal Inference class. After the first attempt at Quiz I, Professor Hill decides to give students an opportunity to take the quiz again. Before the second attempt of the quiz, Professor Hill randomly assigns half the class to attend an extra tutoring session to half of the class. The other half of the class does not receive any additional help. Consider the half of the class that receives tutors as the treated group. The goal is to estimate the effect of the extra tutoring session on average test scores for the retake of Quiz 1.

We are assuming that SUTVA is satisfied.

Question 1: Generating potential outcomes; Calculating ATE (all seeing/omniscient) For this section, you are a god of Statistics. That is, assume you are omniscient and know the potential outcome of $Y(0)$ and $Y(1)$ for everyone.

(a) Please simulate a dataset consistent with the assumptions below while demonstrating an average treatment effect (ATE) of approximately **5**.

Simulation assumptions The Data Generating Process (DGP) has the following features:
* Population size N is 1000.
* The pretest (Causal Quiz I score) is independent and identically distributed with a Normal distribution with mean of 65 and standard deviation of 3.
* The potential outcomes for the corresponding to Causal Quiz II score should be linearly related to the pretest quiz score. In particular they should take the form:

$$Y(0) = \beta_0 + \beta_1 X + 0 + \epsilon$$

$$Y(1) = \beta_0 + \beta_1 X + \tau + \epsilon$$

where β_0 is the intercept taking the value of **10**. β_1 is set to **1.1**. τ is 5. ϵ should be drawn from a N(0,1) distribution. Please also set the seed at 1234 before generating these draws.

```
# Population size
N = 1000
# Intercept
beta0 = 10
# Slope
```

```
beta1 = 1.1
# Population average treatment effect
tau = 5

# Setting seed for consistent draws
set.seed(1234)
# Drawing epsilon from N(0,1) distribution
epsilon = rnorm(1,0,1)
# Drawing for pretest scores
X = rnorm(N,65,3)

# Initializing all outcomes for no intervention
Y0 = rep(NA,N)
# Simulating no intervention posttest scores from pretest scores
Y0 = beta0 + beta1*X + 0 + epsilon

# Initializing all outcomes for intervention for all
Y1 = rep(NA,N)
# Simulating all intervention posttest scores from pretest scores
Y1 = beta0 + beta1*X + 5 + epsilon

# This results in three vectors of simulated data, X, Y0, Y1
```

(b) Write a function to generate the data generating process (DGP) for pretest, Y0, and Y1 with arguments for sample size, the coefficient on the pretest, and the random seed. Then use this function to simulate a data set with sample size equal to *100*, seed equal to 1234, and the coefficient on the covariate set to 1.1. The probability of being assigned to treatment should be equal to .5.

```
# Start function, with inputs N, beta1, and seed
DGPGod = function(n,beta1,seed){
  # Setting seed for consistent draws
  set.seed(seed)
  # Drawing epsilon from N(0,1) distribution
  epsilon = rnorm(n,0,1)
  # Drawing for pretest scores, reset the seed
  set.seed(seed)
  X = rnorm(n,65,3)
  # Drawing for treatment assignment, reset the seed
  set.seed(seed)
  z = rbinom(n,1,0.5)

  # Initializing all outcomes for no intervention
  Y0 = rep(NA,n)
  # Simulating no intervention posttest scores from pretest scores
  Y0 = beta0 + beta1*X + 0 + epsilon

  # Initializing all outcomes for intervention for all
  Y1 = rep(NA,N)
  # Simulating all intervention posttest scores from pretest scores
  Y1 = beta0 + beta1*X + 5 + epsilon

  df1 = data.frame(X, z, Y0, Y1)

  return(df1)
```

```r
  # Return data frame
}

data1 = DGPGod(100,1.1,1234)
data1
```

```
##             X z       Y0       Y1
## 1   61.37880 0 76.30962 81.30962
## 2   65.83229 1 82.69295 87.69295
## 3   68.25332 1 86.16310 91.16310
## 4   57.96291 1 71.41350 76.41350
## 5   66.28737 1 83.34524 88.34524
## 6   66.51817 1 83.67604 88.67604
## 7   63.27578 0 79.02862 84.02862
## 8   63.36010 0 79.14948 84.14948
## 9   63.30664 1 79.07286 84.07286
## 10  62.32989 1 77.67284 82.67284
## 11  63.56842 1 79.44807 84.44807
## 12  62.00484 1 77.20694 82.20694
## 13  62.67124 0 78.16211 83.16211
## 14  65.19338 1 81.77717 86.77717
## 15  67.87848 0 85.62582 90.62582
## 16  64.66914 1 81.02577 86.02577
## 17  63.46697 0 79.30266 84.30266
## 18  62.26641 0 77.58186 82.58186
## 19  62.48848 0 77.90016 82.90016
## 20  72.24751 0 91.88809 96.88809
## 21  65.40226 0 82.07658 87.07658
## 22  63.52794 0 79.39005 84.39005
## 23  63.67836 0 79.60564 84.60564
## 24  66.37877 0 83.47623 88.47623
## 25  62.91884 0 78.51700 83.51700
## 26  60.65539 1 75.27272 80.27272
## 27  66.72427 1 83.97145 88.97145
## 28  61.92903 1 77.09828 82.09828
## 29  64.95459 1 81.43491 86.43491
## 30  62.19215 0 77.47542 82.47542
## 31  68.30689 0 86.23988 91.23988
## 32  63.57322 0 79.45495 84.45495
## 33  62.87168 0 78.44941 83.44941
## 34  63.49623 1 79.34459 84.34459
## 35  60.11272 0 74.49490 79.49490
## 36  61.49714 1 76.47924 81.47924
## 37  58.45988 0 72.12583 77.12583
## 38  60.97702 0 75.73373 80.73373
## 39  64.11712 1 80.23454 85.23454
## 40  63.60231 1 79.49664 84.49664
## 41  69.34849 1 87.73283 92.73283
## 42  61.79407 1 76.90484 81.90484
## 43  62.43391 0 77.82193 82.82193
## 44  64.15813 1 80.29332 85.29332
## 45  62.01698 0 77.22434 82.22434
## 46  62.09446 1 77.33539 82.33539
## 47  61.67805 1 76.73853 81.73853
```

```
## 48   61.24404 0 76.11646 81.11646
## 49   63.42852 0 79.24754 84.24754
## 50   63.50945 1 79.36355 84.36355
## 51   59.58191 0 73.73407 78.73407
## 52   63.25377 0 78.99707 83.99707
## 53   61.67333 1 76.73177 81.73177
## 54   61.95511 1 77.13566 82.13566
## 55   64.51307 0 80.80207 85.80207
## 56   66.68917 1 83.92114 88.92114
## 57   69.94345 0 88.58562 93.58562
## 58   62.67994 1 78.17458 83.17458
## 59   69.81773 0 88.40541 93.40541
## 60   61.52657 1 76.52142 81.52142
## 61   66.96977 1 84.32333 89.32333
## 62   72.64697 0 92.46066 97.46066
## 63   64.89572 0 81.35053 86.35053
## 64   62.99110 0 78.62058 83.62058
## 65   64.97719 0 81.46730 86.46730
## 66   70.33125 1 89.14146 94.14146
## 67   61.58418 0 76.60399 81.60399
## 68   69.10348 1 87.38166 92.38166
## 69   68.98869 0 87.21713 92.21713
## 70   66.00942 1 82.94683 87.94683
## 71   65.02068 0 81.52964 86.52964
## 72   63.63359 1 79.54148 84.54148
## 73   63.90043 0 79.92395 84.92395
## 74   66.94486 1 84.28763 89.28763
## 75   71.21081 0 90.40216 95.40216
## 76   64.53980 1 80.84039 85.84039
## 77   60.82790 0 75.51999 80.51999
## 78   62.82925 0 78.38860 83.38860
## 79   65.77479 0 82.61053 87.61053
## 80   64.04882 1 80.13665 85.13665
## 81   64.46663 1 80.73550 85.73550
## 82   64.49002 0 80.76903 85.76903
## 83   60.88309 0 75.59910 80.59910
## 84   64.47864 1 80.75272 85.75272
## 85   67.55070 0 85.15600 90.15600
## 86   67.09283 1 84.49972 89.49972
## 87   66.64999 0 83.86499 88.86499
## 88   63.79180 0 79.76825 84.76825
## 89   64.42522 0 80.67615 85.67615
## 90   61.41642 1 76.36353 81.36353
## 91   64.84052 0 81.27142 86.27142
## 92   65.76559 1 82.59734 87.59734
## 93   70.11789 0 88.83565 93.83565
## 94   68.00454 0 85.80651 90.80651
## 95   63.51325 0 79.36899 84.36899
## 96   66.06665 1 83.02887 88.02887
## 97   61.59618 0 76.62119 81.62119
## 98   67.63461 0 85.27628 90.27628
## 99   67.91875 0 85.68354 90.68354
## 100  71.36335 1 90.62080 95.62080
```

```r
# Use the function
```

Answer the following questions based on the DGP or using your simulated data set. Remember that you are still all-seeing.

(a) What is your interpretation of tau?

In a nonsensical world, given omnipotence, if a student receives tutoring, their post test score can be expected to be 5 points higher than if they were not to receive tutoring. Given the assumption of omnipotence, this tau is the population average treatment effect.

(b) How would you interpret the intercept in the DGP for $Y(0)$ and $Y(1)$?

In a nonsensical world, given omnipotence, an intercept in the DGP of 10 means we can expect a student with pretest score of 0 to score Y(0) = 10 and Y(1) = 15.

(c) Consider: How would you interpret the β_1 coefficient?

In a nonsensical world, given omnipotence, for any change of 1 in student pretest score, we can expect the post test score to increase by beta1 times the pretest score, holding all other variables constant

Question 2: Calculating ATE (all seeing/omniscient) Answer this question using the simulated dataset from above.

(a) The Sample Average Treatment Effect (SATE) is the average of individual treatment effects in the sample. Calculate it for your sample.

```r
# Calculating SATE with difference of means
SATEGod = (mean(subset(data1,data1[,2]==1)[,4])-mean(subset(data1,data1[,2]==0)[,3]))
SATEGod
```

```
## [1] 4.907522
```

Question 3: Estimating SATE (not all seeing/researchers'view) For Questions 3 and 4, you are a **mere** researcher! Return your god-vision goggles and use only the data available to the researcher (that is, you will not have access to the counterfactual outcomes for each student).

(a) Using the same simulated dataset used in the previous case where $\tau = \mathbf{5}$, please randomly assign students to treatment and control groups (remember, this is something a research would do in practice). The probability of being assigned to treatment should be equal to .5. One way to do this is by using the following command to generate treatment assignment:

Note that an alternative method is the following. . . . think about what difference this might make in practice. . .

Next, create the observed data set which must include pretest scores, treatment assignment and observed Y.

```r
# Start function, with inputs N, beta1, and seed
DGP = function(n,beta1,seed){
  # Setting seed for consistent draws
  set.seed(seed)
  # Drawing epsilon from N(0,1) distribution
  epsilon = rnorm(n,0,1)
  # Drawing for pretest scores, reset the seed
  set.seed(seed)
  X = rnorm(n,65,3)
  # Drawing for treatment assignment, reset the seed
  set.seed(seed)
  z = rbinom(n,1,0.5)
  # Make intermediate data frame to subset in the next step
```

```r
  Y = beta0 + beta1 * X + 5*z + epsilon


  df2 = data.frame(X, z, Y)

  return(df2)
  # Return data frame
}

data2 = DGP(1000,1.1,1234)
data2
```

```
##             X z        Y
## 1    61.37880 0 76.30962
## 2    65.83229 1 87.69295
## 3    68.25332 1 91.16310
## 4    57.96291 1 76.41350
## 5    66.28737 1 88.34524
## 6    66.51817 1 88.67604
## 7    63.27578 0 79.02862
## 8    63.36010 0 79.14948
## 9    63.30664 1 84.07286
## 10   62.32989 1 82.67284
## 11   63.56842 1 84.44807
## 12   62.00484 1 82.20694
## 13   62.67124 0 78.16211
## 14   65.19338 1 86.77717
## 15   67.87848 0 85.62582
## 16   64.66914 1 86.02577
## 17   63.46697 0 79.30266
## 18   62.26641 0 77.58186
## 19   62.48848 0 77.90016
## 20   72.24751 0 91.88809
## 21   65.40226 0 82.07658
## 22   63.52794 0 79.39005
## 23   63.67836 0 79.60564
## 24   66.37877 0 83.47623
## 25   62.91884 0 78.51700
## 26   60.65539 1 80.27272
## 27   66.72427 1 88.97145
## 28   61.92903 1 82.09828
## 29   64.95459 1 86.43491
## 30   62.19215 0 77.47542
## 31   68.30689 0 86.23988
## 32   63.57322 0 79.45495
## 33   62.87168 0 78.44941
## 34   63.49623 1 84.34459
## 35   60.11272 0 74.49490
## 36   61.49714 1 81.47924
## 37   58.45988 0 72.12583
## 38   60.97702 0 75.73373
## 39   64.11712 1 85.23454
## 40   63.60231 1 84.49664
## 41   69.34849 1 92.73283
```

```
## 42   61.79407 1 81.90484
## 43   62.43391 0 77.82193
## 44   64.15813 1 85.29332
## 45   62.01698 0 77.22434
## 46   62.09446 1 82.33539
## 47   61.67805 1 81.73853
## 48   61.24404 0 76.11646
## 49   63.42852 0 79.24754
## 50   63.50945 1 84.36355
## 51   59.58191 0 73.73407
## 52   63.25377 0 78.99707
## 53   61.67333 1 81.73177
## 54   61.95511 1 82.13566
## 55   64.51307 0 80.80207
## 56   66.68917 1 88.92114
## 57   69.94345 0 88.58562
## 58   62.67994 1 83.17458
## 59   69.81773 0 88.40541
## 60   61.52657 1 81.52142
## 61   66.96977 1 89.32333
## 62   72.64697 0 92.46066
## 63   64.89572 0 81.35053
## 64   62.99110 0 78.62058
## 65   64.97719 0 81.46730
## 66   70.33125 1 94.14146
## 67   61.58418 0 76.60399
## 68   69.10348 1 92.38166
## 69   68.98869 0 87.21713
## 70   66.00942 1 87.94683
## 71   65.02068 0 81.52964
## 72   63.63359 1 84.54148
## 73   63.90043 0 79.92395
## 74   66.94486 1 89.28763
## 75   71.21081 0 90.40216
## 76   64.53980 1 85.84039
## 77   60.82790 0 75.51999
## 78   62.82925 0 78.38860
## 79   65.77479 0 82.61053
## 80   64.04882 1 85.13665
## 81   64.46663 1 85.73550
## 82   64.49002 0 80.76903
## 83   60.88309 0 75.59910
## 84   64.47864 1 85.75272
## 85   67.55070 0 85.15600
## 86   67.09283 1 89.49972
## 87   66.64999 0 83.86499
## 88   63.79180 0 79.76825
## 89   64.42522 0 80.67615
## 90   61.41642 1 81.36353
## 91   64.84052 0 81.27142
## 92   65.76559 1 87.59734
## 93   70.11789 0 88.83565
## 94   68.00454 0 85.80651
## 95   63.51325 0 79.36899
```

```
## 96   66.06665 1 88.02887
## 97   61.59618 0 76.62119
## 98   67.63461 0 85.27628
## 99   67.91875 0 85.68354
## 100  71.36335 1 95.62080
## 101  66.24357 0 83.28245
## 102  63.57584 1 84.45871
## 103  65.19798 0 81.78377
## 104  63.49257 0 79.33935
## 105  62.52200 0 77.94821
## 106  65.50097 0 82.21805
## 107  62.31121 0 77.64606
## 108  65.50456 0 82.22320
## 109  66.06490 0 83.02636
## 110  64.84368 0 81.27595
## 111  64.41220 1 85.65748
## 112  63.05279 0 78.70900
## 113  61.67070 1 81.72800
## 114  67.54782 0 85.15188
## 115  65.06709 0 81.59616
## 116  67.49342 1 90.07390
## 117  61.26714 1 81.14956
## 118  65.50708 0 82.22681
## 119  67.01950 0 84.39462
## 120  64.92117 1 86.38701
## 121  64.42582 1 85.67701
## 122  62.65428 1 83.13780
## 123  71.17449 1 95.35010
## 124  67.25150 1 89.72716
## 125  70.47262 0 89.34410
## 126  65.24018 0 81.84426
## 127  63.10577 0 78.78494
## 128  60.46014 1 79.99286
## 129  63.09170 0 78.76477
## 130  65.67890 0 82.47310
## 131  68.04107 1 90.85887
## 132  65.75825 1 87.58683
## 133  61.48416 0 76.46062
## 134  67.00614 0 84.37547
## 135  60.04970 1 79.40457
## 136  63.90244 0 79.92684
## 137  64.05165 1 85.14069
## 138  59.15526 1 78.12254
## 139  67.76017 1 90.45625
## 140  63.13139 1 83.82165
## 141  63.99789 1 85.06364
## 142  69.18544 1 92.49914
## 143  66.91002 0 84.23770
## 144  64.67470 0 81.03374
## 145  66.54129 0 83.70918
## 146  66.19782 1 88.21687
## 147  69.98857 1 93.65028
## 148  65.82768 1 87.68634
## 149  66.51882 1 88.67697
```

```
## 150  66.04266 1 87.99447
## 151  63.86829 0 79.87788
## 152  65.29286 0 81.91976
## 153  69.91623 0 88.54660
## 154  62.37322 1 82.73495
## 155  65.36528 0 82.02357
## 156  69.08639 1 92.35716
## 157  64.29614 1 85.49113
## 158  61.83985 1 81.97045
## 159  62.39065 0 77.75993
## 160  63.82962 1 84.82245
## 161  62.45795 0 77.85639
## 162  64.21808 1 85.37925
## 163  63.75674 0 79.71800
## 164  64.45085 0 80.71288
## 165  66.22117 0 83.25034
## 166  66.87390 1 89.18592
## 167  70.03462 0 88.71628
## 168  64.79392 0 81.20462
## 169  64.03748 1 85.12039
## 170  69.41302 0 87.82532
## 171  70.11299 1 93.82862
## 172  65.12973 1 86.68595
## 173  64.00203 1 85.06957
## 174  59.53329 0 73.66439
## 175  69.23379 0 87.56843
## 176  62.48725 1 82.89840
## 177  61.62871 0 76.66782
## 178  74.13130 1 99.58819
## 179  65.70506 0 82.51059
## 180  64.90022 0 81.35699
## 181  56.80334 0 69.75146
## 182  64.70063 0 81.07090
## 183  67.92810 1 90.69694
## 184  66.24161 0 83.27964
## 185  67.73697 1 90.42299
## 186  70.95120 0 90.03005
## 187  68.50733 1 91.52717
## 188  63.47379 1 84.31243
## 189  67.11254 0 84.52797
## 190  64.40475 1 85.64681
## 191  63.38579 1 84.18630
## 192  56.43272 1 74.22024
## 193  62.63106 0 78.10452
## 194  66.46344 1 88.59760
## 195  71.50410 1 95.82254
## 196  66.50208 1 88.65299
## 197  66.86063 1 89.16690
## 198  62.10229 1 82.34662
## 199  65.48796 1 87.19942
## 200  58.76529 1 77.56358
## 201  66.45568 1 88.58648
## 202  67.09031 1 89.49611
## 203  65.55654 0 82.29771
```

```
## 204  67.10220 1 89.51315
## 205  65.93504 1 87.84023
## 206  67.28139 1 89.76999
## 207  70.52739 0 89.42259
## 208  68.33709 0 86.28316
## 209  65.09799 0 81.64046
## 210  61.65665 1 81.70787
## 211  66.25417 1 88.29765
## 212  63.79929 0 79.77899
## 213  69.48048 0 87.92202
## 214  60.17876 1 79.58955
## 215  63.75274 1 84.71227
## 216  66.26603 1 88.31464
## 217  64.54479 1 85.84753
## 218  63.18155 1 83.89355
## 219  64.08584 0 80.18970
## 220  66.88861 1 89.20701
## 221  67.68552 0 85.34924
## 222  66.98064 0 84.33891
## 223  71.82045 0 91.27598
## 224  68.52049 0 86.54604
## 225  65.86313 0 82.73715
## 226  63.02069 0 78.66299
## 227  73.75742 1 99.05230
## 228  67.03225 0 84.41289
## 229  62.94704 1 83.55742
## 230  65.55948 0 82.30192
## 231  64.02682 1 85.10511
## 232  64.17589 1 85.31877
## 233  62.19949 0 77.48594
## 234  65.35054 1 87.00243
## 235  65.95748 1 87.87239
## 236  61.76737 0 76.86657
## 237  55.30054 1 72.59745
## 238  64.23538 0 80.40404
## 239  65.08855 0 81.62693
## 240  66.78282 1 89.05538
## 241  65.17741 0 81.75428
## 242  66.24020 0 83.27762
## 243  61.70668 0 76.77958
## 244  67.13353 1 89.55805
## 245  67.15667 1 89.59122
## 246  65.75495 0 82.58210
## 247  69.07182 1 92.33628
## 248  66.21341 1 88.23921
## 249  65.79309 1 87.63677
## 250  65.80413 1 87.65259
## 251  66.31079 1 88.37880
## 252  68.18037 1 91.05853
## 253  66.35657 0 83.44442
## 254  66.98960 0 84.35175
## 255  61.59088 0 76.61359
## 256  63.88851 1 84.90686
## 257  69.43091 0 87.85097
```

```
## 258  61.32829 0 76.23721
## 259  65.77421 1 87.60969
## 260  66.21501 1 88.24151
## 261  67.92741 1 90.69595
## 262  63.95337 0 79.99983
## 263  65.47588 1 87.18209
## 264  59.71023 0 73.91800
## 265  66.01579 0 82.95596
## 266  63.00030 0 78.63377
## 267  64.28406 1 85.47382
## 268  61.43670 0 76.39261
## 269  66.15481 0 83.15522
## 270  66.99974 1 89.36629
## 271  64.08616 0 80.19016
## 272  70.47503 1 94.34755
## 273  67.01168 0 84.38341
## 274  67.84590 1 90.57912
## 275  71.14821 0 90.31243
## 276  63.04666 1 83.70021
## 277  67.42586 1 89.97706
## 278  67.95974 1 90.74230
## 279  64.98149 0 81.47347
## 280  65.95716 0 82.87193
## 281  61.96453 0 77.14917
## 282  66.41050 0 83.52172
## 283  62.89709 1 83.48583
## 284  67.44105 1 89.99884
## 285  62.56571 1 83.01085
## 286  65.95819 0 82.87341
## 287  62.46043 0 77.85995
## 288  64.26271 0 80.44322
## 289  60.34142 1 79.82271
## 290  65.38530 1 87.05227
## 291  67.95633 1 90.73741
## 292  65.54974 0 82.28796
## 293  59.70131 1 78.90521
## 294  63.13840 0 78.83171
## 295  69.96813 1 93.62099
## 296  70.42942 1 94.28216
## 297  61.47489 1 81.44734
## 298  63.89989 0 79.92318
## 299  66.06088 1 88.02059
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## 854  69.19717 0 87.51594
## 855  62.34398 1 82.69304
## 856  63.48101 1 84.32278
## 857  65.49000 0 82.20233
## 858  63.98857 0 80.05028
## 859  61.89215 1 82.04541
## 860  66.22801 1 88.26015
## 861  63.42733 0 79.24584
## 862  66.51950 0 83.67795
## 863  63.48583 0 79.32969
## 864  62.12825 0 77.38382
## 865  64.71610 0 81.09307
## 866  66.94832 1 89.29259
## 867  65.72697 0 82.54198
## 868  64.82320 1 86.24658
## 869  59.15662 0 73.12449
## 870  69.35433 1 92.74121
## 871  64.88018 1 86.32826
## 872  62.40801 1 82.78481
## 873  64.39493 1 85.63273
## 874  65.78377 0 82.62340
## 875  60.44495 1 79.97109
## 876  67.15193 0 84.58443
## 877  65.74050 0 82.56138
## 878  68.60830 0 86.67190
## 879  67.95812 1 90.73997
## 880  63.38618 1 84.18686
## 881  63.52915 1 84.39178
## 882  62.01345 0 77.21928
## 883  63.91489 0 79.94468
## 884  67.83306 1 90.56073
## 885  70.37662 1 94.20649
## 886  67.78523 1 90.49216
## 887  59.51019 0 73.63127
## 888  67.46563 0 85.03408
## 889  68.10881 0 85.95596
## 890  66.85226 0 84.15491
## 891  62.43521 1 82.82380
## 892  68.19353 1 91.07740
## 893  67.18844 1 89.63677
## 894  66.98797 0 84.34942
## 895  63.85578 0 79.85995
## 896  67.53768 0 85.13733
## 897  65.33478 1 86.97985
## 898  64.79669 1 86.20859
## 899  69.46129 1 92.89452
## 900  67.27167 1 89.75606
## 901  56.37787 1 74.14162
## 902  63.72037 0 79.66587
## 903  60.53778 1 80.10415
## 904  66.76637 0 84.03180
## 905  63.67220 1 84.59682
```

```
## 906  67.70391 0 85.37561
## 907  66.40349 0 83.51166
## 908  65.06612 0 81.59477
## 909  64.74468 1 86.13404
## 910  63.91620 0 79.94655
## 911  65.59852 0 82.35788
## 912  61.67524 0 76.73451
## 913  67.01838 0 84.39301
## 914  63.81766 0 79.80531
## 915  62.76810 0 78.30094
## 916  64.02166 0 80.09771
## 917  70.62401 0 89.56108
## 918  59.72600 1 78.94060
## 919  63.95342 1 84.99990
## 920  65.31977 1 86.95833
## 921  63.38702 1 84.18807
## 922  63.76619 0 79.73154
## 923  66.14705 0 83.14410
## 924  65.66754 0 82.45681
## 925  62.97874 1 83.60286
## 926  66.70230 0 83.93996
## 927  65.53845 1 87.27178
## 928  67.05903 1 89.45127
## 929  61.01821 0 75.79277
## 930  62.69625 0 78.19796
## 931  64.42189 0 80.67138
## 932  67.87715 1 90.62392
## 933  63.41291 0 79.22517
## 934  61.90134 1 82.05859
## 935  67.31524 0 84.81851
## 936  66.39607 1 88.50103
## 937  61.32299 0 76.22961
## 938  71.14003 1 95.30071
## 939  69.49602 0 87.94430
## 940  60.83315 1 80.52752
## 941  71.46013 1 95.75952
## 942  59.81265 1 79.06480
## 943  65.30613 1 86.93878
## 944  61.57616 1 81.59250
## 945  63.86897 1 84.87886
## 946  63.95365 1 85.00024
## 947  65.96866 0 82.88841
## 948  61.62698 0 76.66534
## 949  56.72885 0 69.64469
## 950  64.71257 0 81.08801
## 951  62.75990 1 83.28919
## 952  65.63234 1 87.40636
## 953  62.45447 0 77.85141
## 954  67.37679 1 89.90674
## 955  65.08756 1 86.62551
## 956  66.81016 0 84.09457
## 957  62.01518 1 82.22176
## 958  64.36600 0 80.59127
## 959  66.03206 0 82.97929
```

```
## 960  64.73186 1 86.11567
## 961  69.96211 0 88.61236
## 962  59.11127 0 73.05949
## 963  67.26838 1 89.75134
## 964  65.04326 1 86.56200
## 965  67.83714 0 85.56657
## 966  64.70357 0 81.07512
## 967  56.78074 1 74.71906
## 968  66.26092 0 83.30732
## 969  64.34486 1 85.56097
## 970  62.03193 0 77.24577
## 971  63.03059 1 83.67718
## 972  65.16699 1 86.73935
## 973  72.35619 0 92.04387
## 974  65.56452 1 87.30915
## 975  62.06091 0 77.28731
## 976  70.71083 1 94.68553
## 977  63.68444 1 84.61436
## 978  62.51082 0 77.93218
## 979  61.25831 1 81.13691
## 980  62.97857 1 83.60262
## 981  70.80550 1 94.82122
## 982  67.71968 1 90.39821
## 983  67.20958 0 84.66707
## 984  65.43988 1 87.13049
## 985  65.84936 1 87.71742
## 986  66.76783 0 84.03389
## 987  65.64755 0 82.42815
## 988  65.46516 1 87.16673
## 989  67.86148 0 85.60145
## 990  64.50244 0 80.78683
## 991  65.76501 0 82.59652
## 992  62.30758 1 82.64087
## 993  69.75191 1 93.31107
## 994  69.17828 0 87.48886
## 995  59.56323 1 78.70729
## 996  64.35104 0 80.56983
## 997  66.64957 1 88.86438
## 998  66.44821 0 83.57577
## 999  67.28255 1 89.77165
## 1000 63.66303 0 79.58367
```

```
# Use the function
```

(b) Estimate SATE using a difference in mean outcomes.

```
# Calculating SATE with difference of means
SATE = (mean(subset(data2,data2[,2]==1)[,3])-mean(subset(data2,data2[,2]==0)[,3]))
SATE
```

```
## [1] 5.165079
```

(c) Is this estimate close to the true SATE? Divide the difference between SATE and estimated SATE by the standard deviation of the observed outcome, Y to express this conditional bias in standard deviation units. This helps you understand the practical significance of this difference.

```r
condbias = (SATE-SATEGod)/sd(data2[,3])
condbias
```

```
## [1] 0.05145739
```

(d) Consider: Why is $\hat{SATE}$ different from SATE and τ ?

Since we are no longer god, we cannot control the even spread of treament assignments nor see the counterfactuals to get a perfectly true SATE.

Question 4: Use Linear Regression to estimate the treatment effect

(a) Now we will use linear regression to estimate SATE for the observed data set created by Question 2. With this set up, we will begin to better understand some fundamental assumptions crucial for the later R homework assignments.

```r
reg1 = lm(Y~X+z,data2)
summary(reg1)
```

```
##
## Call:
## lm(formula = Y ~ X + z, data = data2)
##
## Residuals:
##        Min         1Q     Median         3Q        Max
## -2.662e-14 -8.230e-15 -3.270e-15  2.340e-15  2.921e-12
##
## Coefficients:
##               Estimate Std. Error    t value Pr(>|t|)
## (Intercept) -1.167e+01  6.385e-14 -1.827e+14   <2e-16 ***
## X            1.433e+00  9.822e-16  1.459e+15   <2e-16 ***
## z            5.000e+00  5.879e-15  8.505e+14   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 9.287e-14 on 997 degrees of freedom
## Multiple R-squared:      1,  Adjusted R-squared:      1
## F-statistic: 1.451e+30 on 2 and 997 DF,  p-value: < 2.2e-16
```

(b) Consider: What is gained by using linear regression to estimate ATE instead of the mean difference estimation from above?

The standard error for estimate ATE is smaller.

Challenge Question: Treatment Effect Heterogenity

(a) Based on the following function: Simulate the following "response surfaces" (relationship betwee the mean of each potential outcome and the covariate(s)"), E[$Y(0) \mid X$] and E[$Y(1) \mid X$]. Plot them on the same plot (that is make a plot with X on the x-axis and $Y(0)/Y(1)$ on the y-axis. Also simulate Y(0) and Y(1) (that is, the expected values plus"noise").

Note: X is the same pretest score used before.

$$
\begin{aligned}
\mathrm{E}[Y(0) \mid X] &= \beta_0^0 + \beta_1^0 X \\
Y(0) &= \mathrm{E}[Y(0) \mid X] + \epsilon^0 \\
Y(0) &= \beta_0^0 + \beta_1^0 X + \epsilon^0 \\
\mathrm{E}[Y(1) \mid X] &= \beta_0^1 + \beta_1^1 X \\
Y(1) &= \mathrm{E}[Y(1) \mid X] + \epsilon^1 \\
Y(1) &= \beta_0^1 + \beta_1^1 X + \epsilon^1
\end{aligned}
$$

where β_0^0 is set to **35**, β_1^0 is set to .6, β_0^1 is set to **15**, β_1^1 is set to 1. First generate a vector of predicted $Y(0)$ and $Y(1)$ (that is $\mathrm{E}[Y(1) \mid X]$. Then generate $Y(0)$ and $Y(1)$ with noise added as ϵ^0 or ϵ^1 from a distribution of N(0,1). Again, please also set seed at 1234.

(b) Comment on your findings. In particular, note that there is no longer a tau included in the DGP. Is there still a SATE? Can we calculate SATE? (Remember I have to be omniscient to do this!) What is it? Consider: How do we interpret the average treatment effect in this setting?

(c) Is the treatment effect the same for all students? If not, is there a pattern to the way it varies? Consider: Why do we care about treatment effect heterogeneity?

(d) Now generate a similar plot from the initial DGP in Question 1 to reinforce the differences between a setting with constant treatment effect and a setting with heterogeneous treatment effects.