1. Proof of Theorem 2.2

There are 11 diagrams in Shulman's definition of symmetric monoidal double category, which we check now for decorated cospan double categories.

Diagram 1 of 11. Set up but not solved.
M_1 is given by $a \rightarrow m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \rightarrow m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \rightarrow m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
N_1 is given by $a' \rightarrow n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \rightarrow n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
N_3 is given by $c' \rightarrow n_3 \leftarrow d'$ with $y_3 \in F(n_3)$.

$$\begin{tikzcd}
((M_1 \otimes N_1)\odot(M_2 \otimes N_2))\odot(M_3 \otimes N_3) \arrow[r, "\chi \odot 1"] \arrow[d, "\alpha"'] & ((M_1 \odot M_2)\otimes(N_1 \odot N_2))\odot(M_3 \otimes N_3) \arrow[d, "\chi"] \\
(M_1 \otimes N_1)\odot((M_2 \otimes N_2)\odot(M_3 \otimes N_3)) \arrow[d, "1 \odot \chi"'] & ((M_1 \odot M_2)\odot M_3)\otimes((N_1 \odot N_2)\odot N_3) \arrow[d, "\alpha \otimes \alpha"] \\
(M_1 \otimes N_1)\odot((M_2 \odot M_3)\otimes(N_2 \odot N_3)) \arrow[r, "\chi"] & (M_1 \odot (M_2 \odot M_3))\otimes(N_1 \odot (N_2 \odot N_3))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{(m_1+_b m_2)+(n_1+_{b'}n_2),m_3+n_3}(\phi_{m_1+_b m_2,n_1+_{b'}n_2}((F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))),\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+_b m_2)+(n_1+_{b'}n_2))+_{c+c'}(m_3+n_3))$$

$$\phi_{(m_1+_b m_2)+c m_3,(n_1+_{b'}n_2+_{c'}n_3}((F(\psi\phi_{m_1+_b m_2,m_3}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),x_3),(F(\psi)\phi_{n_1+_{b'}n_2,n_3}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),y_3))) \in F(((m_1+_b m_2)+_c m_3)+((n_1+_{b'}n_2)+_{c'}n_3))$$

$$\phi_{m_1+_b(m_2+_c m_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+_c m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+_b(m_2+_c m_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

(2) Down and then right:

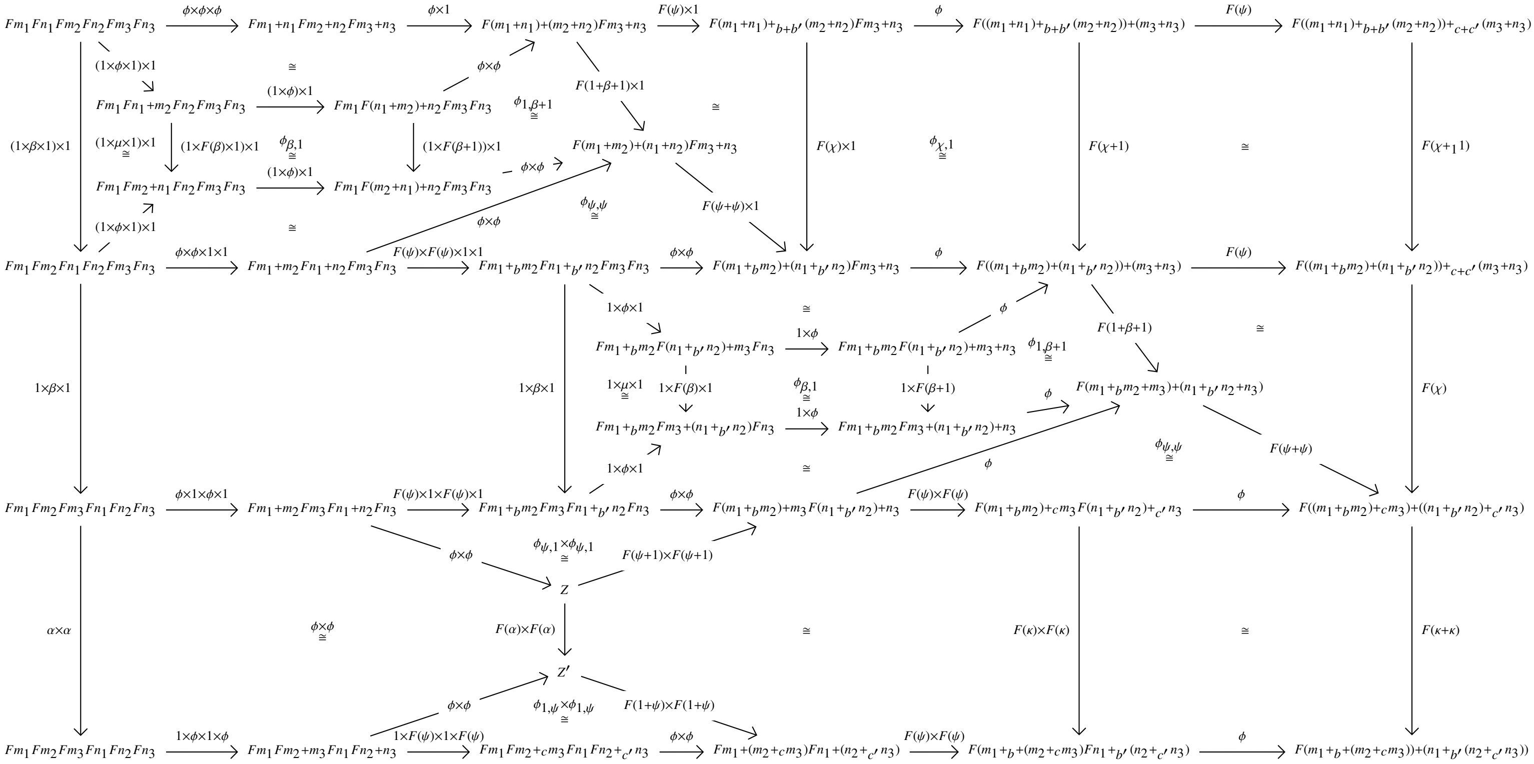
$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{m_1+n_1,(m_2+n_2)+_{c+c'}(m_3+n_3)}((\phi_{m_1,n_1}(x_1,y_1),F(\psi)\phi_{m_2+n_2,m_3+n_3}(\phi_{m_2,n_2}(x_2,y_2),\phi_{m_3,n_3}(x_3,y_3)))) \in F((m_1+n_1)+_{b+b'}((m_2+n_2)+_{c+c'}(m_3+n_3)))$$

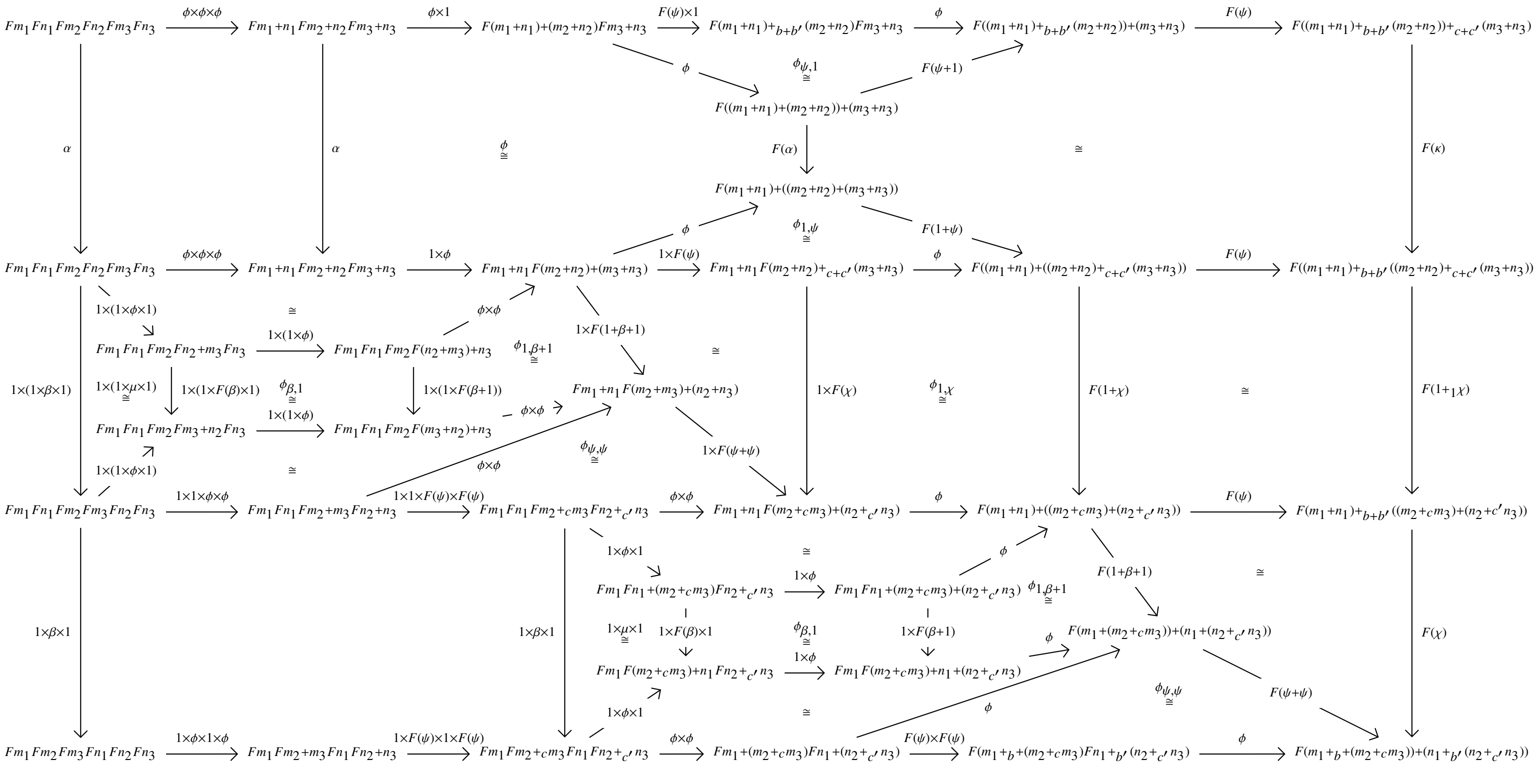
$$F(\psi)\phi_{m_1+n_1,(m_2+c m_3)+(n_2+_{c'}n_3)}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2+c m_3,n_2+_{c'}n_3}((F\psi)\phi_{m_2,m_3}(x_2,x_3),F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+n_1)+_{b+b'}((m_2+_c m_3)+(n_2+_{c'}n_3)))$$

$$\phi_{m_1+_b(m_2+_c m_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+_c m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+_b(m_2+_c m_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

Right and then down (omitting morphisms emanating out of 1 on the left due to space restrictions):



Down and then right (omitting morphisms emanating out of 1 on the left due to space restrictions):



Diagrams 2 and 3 of 11. Set up but not solved.
$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} a'$ with $x \in F(m)$.
$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} b'$ with $y \in F(n)$.
$U_a \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{\mathrm{id}} a \xleftarrow{\mathrm{id}} a$ with $\perp_a \in F(a)$.

$$
\begin{tikzcd}
\begin{array}{c} a{+}b{\to}(a{+}b){+}_{a+b}(m{+}n){\leftarrow}a'{+}b' \\ F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\perp_{a+b}){\in}F((a{+}b){+}_{a+b}(m{+}n)) \end{array} & (M{\otimes}N){\odot}(U_{a+b}) \arrow[r, "\mathrm{id}\odot\mu"] \arrow[d, "r"'] & (M{\otimes}N){\odot}(U_a{\otimes}U_b) \arrow[d, "\chi"] & \begin{array}{c} a{+}b{\to}(a{+}b){+}_{a+b}(m{+}n){\leftarrow}a'{+}b' \\ F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\phi_{a,b}(\perp_a,\perp_b)){\in}F((a{+}b){+}_{a+b}(m{+}n)) \end{array} \\
\begin{array}{c} a{+}b{\to}m{+}n{\leftarrow}a'{+}b' \\ \phi_{m,n}(x,y){\in}F(m{+}n) \end{array} & M{\otimes}N & (M{\odot}U_a){\otimes}(N{\odot}U_b) \arrow[l, "r\otimes r"'] & \begin{array}{c} a{+}b{\to}(a{+}_am){+}(b{+}_bn){\leftarrow}a'{+}b' \\ \phi_{a+_am,b+_bn}(F(\psi)\phi_{a,m}(\perp_a,x),F(\psi)\phi_{b,n}(\perp_b,y)){\in}F((a{+}_am){+}(b{+}_bn)) \end{array}
\end{tikzcd}
$$

Down:

$$
\begin{tikzcd}[column sep=small]
& F(0){\times}(F(m){\times}F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0){\times}(F(m{+}n) \arrow[r, "F(!_{a+b})\times 1"] \arrow[dr, "\phi_{0,m+n}"'] & F(a{+}b){\times}F(m{+}n) \arrow[r, "\phi_{a+b,m+n}"] \arrow[d, phantom, "\overset{\phi_{!_{a+b},1}}{\cong}"] & F((a{+}b){+}(m{+}n)) \arrow[r, "F(\psi)"] & F((a{+}b){+}_{a+b}(m{+}n)) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0\times(x\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\overset{\lambda_{M\otimes N}}{\cong}"] & & & F(0{+}(m{+}n)) \arrow[ur, "F(!_{a+b}+1)"] \arrow[drr, "F(\ell)"'] \arrow[rr, phantom, "\cong"] & & \\
& F(m){\times}F(n) \arrow[rrrr, "\phi_{m,n}"] & & & & F(m{+}n)
\end{tikzcd}
$$

Right, down and then left:

$$
\begin{tikzcd}[column sep=small]
& F(0){\times}(F(m){\times}F(n)) \arrow[r, "1\times\phi_{m,n}"] \arrow[d, phantom, "\overset{\phi_{!_a,!_b}}{\cong}", shift left=8ex] & F(0){\times}F(m{+}n) \arrow[rr, "F(!_{a+b})\times 1"] & & F(a{+}b){\times}F(m{+}n) \arrow[r, "\phi_{a+b,m+n}"] \arrow[d, phantom, "\cong", shift left=10ex] & F((a{+}b){+}(m{+}n)) \arrow[r, "F(\psi)"] & F((a{+}b){+}_{a+b}(m{+}n)) \arrow[d, "F(\mathrm{id})"] \\
& (F(0){\times}F(0)){\times}(F(m){\times}F(n)) \arrow[u, "\phi_{0,0}\times 1"'] \arrow[r, "(F(!_a)\times F(!_b))\times 1"] & (F(a){\times}F(b)){\times}(F(m){\times}F(n)) \arrow[urr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[rr, "\phi_{a,b}{\times}\phi_{m,n}"] \arrow[d, "1\times\phi\times 1"'] \arrow[dddd, bend right=60, "1\times\beta\times 1"'] \arrow[dr, phantom, "\cong"] & & F(a{+}b){\times}F(m{+}n) \arrow[r, "\phi_{a+b,m+n}"] & F((a{+}b){+}(m{+}n)) \arrow[r, "F(\psi)"] \arrow[dl, "F(1+\beta+1)"'] \arrow[dr, phantom, "\cong"] & F((a{+}b){+}_{a+b}(m{+}n)) \arrow[dddd, "F(\kappa')"] \\
& & F(a){\times}F(b{+}m){\times}F(n) \arrow[r, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times 1"'] \arrow[ddr, phantom, "\overset{\phi_{\beta,1}}{\cong}"] \arrow[d, phantom, "\overset{1\times\mu\times 1}{\cong}", shift right=12ex] & F(a){\times}F((b{+}m){+}n) \arrow[dd, "1\times F(\beta+1)"] \arrow[urr, "\phi"] \arrow[dr, phantom, "\overset{\phi_{1,\beta+1}}{\cong}"] & & & \\
1 \arrow[uuur, bend left, "\phi_0\times(x\times y)"] \arrow[uur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[ddr, "(\phi_0\times x)\times(\phi_0\times y)"'] \arrow[ddddr, bend right, "x\times y"'] \arrow[uuur, phantom, "\overset{\phi}{\cong}"] & & & & F((a{+}m){+}(b{+}n)) \arrow[ddrr, bend left, "F(\psi+\psi)"] \arrow[ddr, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"] & & \\
& & F(a){\times}F(m{+}b){\times}F(n) \arrow[r, "1\times\phi"] & F(a){\times}F((m{+}b){+}n) \arrow[ur, "\phi"] & & & \\
& (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "(F(!_a)\times 1)\times(F(!_b)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] \arrow[dd, phantom, "\overset{\lambda_M\times\lambda_N}{\cong}"] & (F(a){\times}F(m)){\times}(F(b){\times}F(n)) \arrow[u, "1\times\phi\times 1"] \arrow[rr, "\phi_{a,m}\times\phi_{b,n}"] \arrow[ur, phantom, "\cong"] \arrow[d, phantom, "\overset{\phi_{!_a,1}\times\phi_{!_b,1}}{\cong}"] & & F(a{+}m){\times}F(b{+}n) \arrow[uu, "\phi\times\phi"] \arrow[r, "F(\psi)\times F(\psi)"] & F(a{+}_am){\times}F(b{+}_bn) \arrow[r, "\phi_{a+_am,b+_bn}"] & F((a{+}_am){+}(b{+}_bn)) \arrow[dd, "F(\kappa'')"] \\
& & F(0{+}m){\times}F(0{+}n) \arrow[urr, "F(!_a+1)\times F(!_b+1)"] \arrow[rr, "F(\ell)\times F(\ell)"'] & & F(m){\times}(n) \arrow[drr, "\phi_{m,n}"] \arrow[rr, phantom, "\cong"] & & \\
& F(m){\times}F(n) \arrow[rrrrr, "\phi_{m,n}"] & & & & & F(m{+}n)
\end{tikzcd}
$$

Diagram 4 of 11. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
P_1 is given by $a'' \to p_1 \leftarrow b''$ with $z_1 \in F(p_1)$.
P_2 is given by $b'' \to p_2 \leftarrow c''$ with $z_2 \in F(p_2)$.

$$\begin{tikzcd}
((M_1 \otimes N_1) \otimes P_1) \odot ((M_2 \otimes N_2) \otimes P_2) \arrow[r, "\alpha \odot \alpha"] \arrow[d, "\chi"'] & (M_1 \otimes (N_1 \otimes P_1)) \odot (M_2 \otimes (N_2 \otimes P_2)) \arrow[d, "\chi"] \\
((M_1 \otimes N_1) \odot (M_2 \otimes N_2)) \otimes (P_1 \odot P_2) \arrow[d, "\chi \otimes 1"'] & (M_1 \odot M_2) \otimes ((N_1 \otimes P_1) \odot (N_2 \otimes P_2)) \arrow[d, "1 \otimes \chi"] \\
((M_1 \odot M_2) \otimes (N_1 \odot N_2)) \otimes (P_1 \odot P_2) \arrow[r, "\alpha"] & (M_1 \odot M_2) \otimes ((N_1 \odot N_2) \otimes (P_1 \odot P_2))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2)) \in F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$F(\psi)\phi_{m_1+(n_1+p_1),m_2+(n_2+p_2)}(\phi_{m_1,n_1+p_1}(x_1,\phi_{n_1,p_1}(y_1,z_1)),\phi_{m_2,n_2+p_2}(x_2,\phi_{n_2,p_2}(y_2,z_2))) \in F((m_1+(n_1+p_1))+_{b+(b'+b'')}(m_2+(n_2+p_2)))$$

$$\phi_{m_1+_b m_2,(n_1+p_1)+_{b'+b''}(n_2+p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)(\phi_{n_1+p_1,n_2+p_2}(\phi_{n_1,p_1}(y_1,z_1),\phi_{n_2,p_2}(y_2,z_2))) \in F((m_1+_b m_2)+((n_1+p_1)+_{b'+b''}(n_2+p_2)))$$

$$\phi_{m_1+_b m_2,(n_1+_{b'}n_2)+(p_1+_{b''}p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'}n_2,p_1+_{b''}p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2))) \in F((m_1+_b m_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2)))$$

(2) Down and then right:

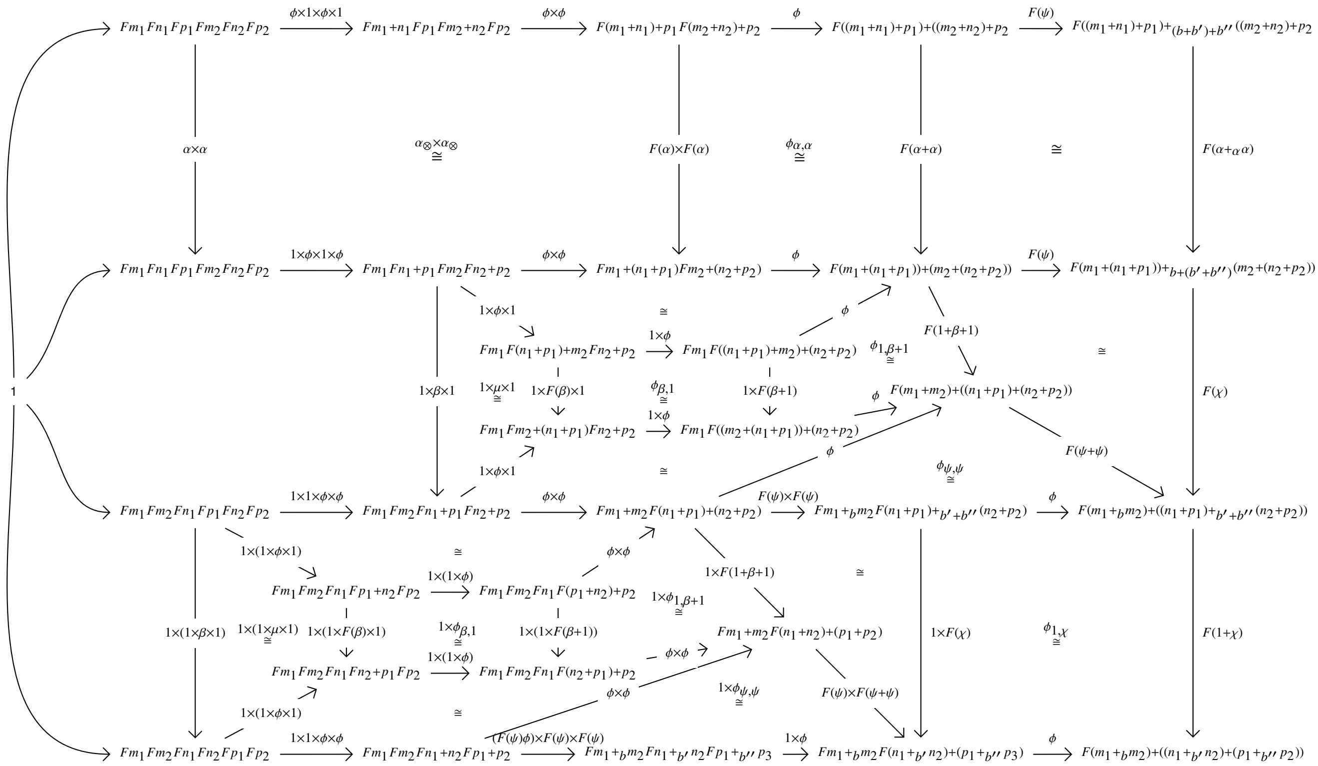
$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2)) \in F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),p_1+_{b''}p_2}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+(p_1+_{b''}p_2))$$

$$\phi_{(m_1+_b m_2)+(n_1+_{b'}n_2),p_1+_{b''}p_2}(\phi_{m_1+_b m_2,n_1+_{b'}n_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2)) \in F(((m_1+_b m_2)+(n_1+_{b'}n_2))+(p_1+_{b''}p_2))$$

$$\phi_{m_1+_b m_2,(n_1+_{b'}n_2)+(p_1+_{b''}p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'}n_2,p_1+_{b''}p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2))) \in F((m_1+_b m_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2)))$$

Right and then down:



Down and then right:

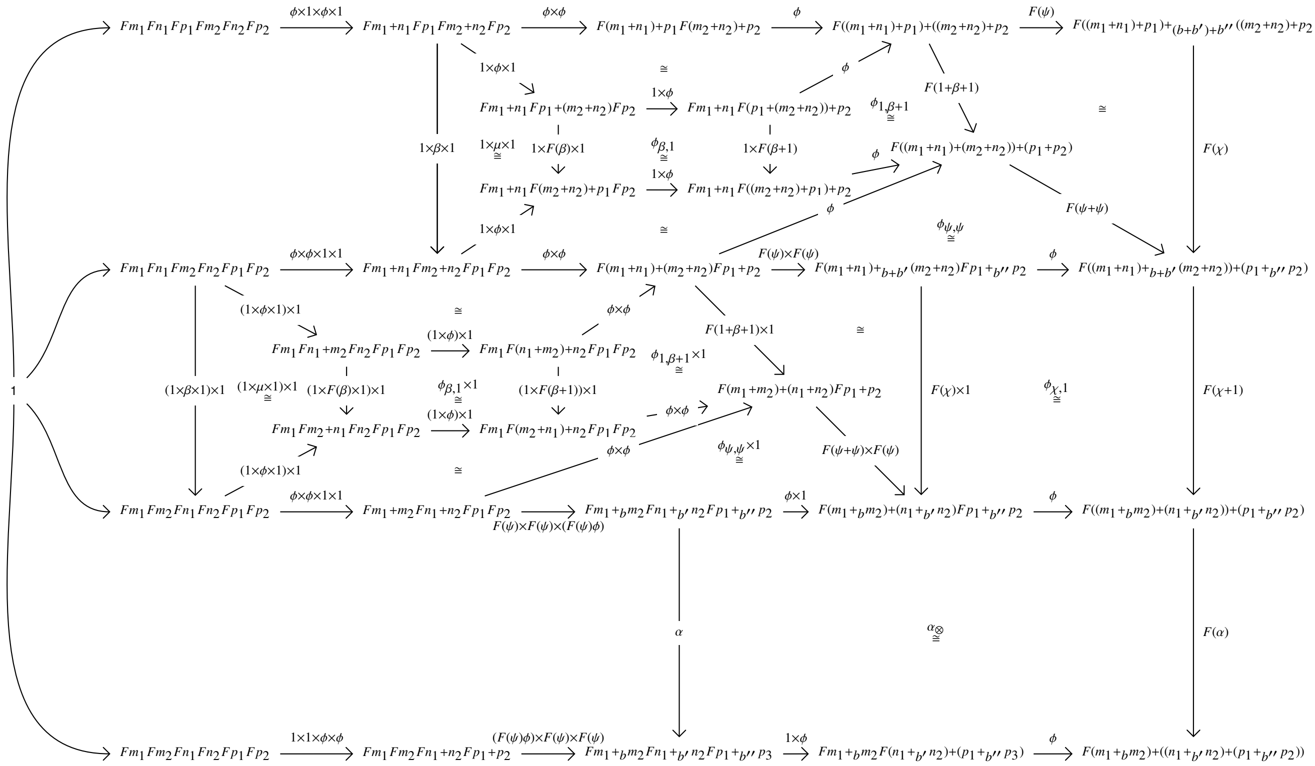


Diagram 5 of 11. Set up but not solved.

$$a, b, c \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
\bot_{(a+b)+c} \in F((a+b)+c) & U_{(a+b)+c} \arrow[rr, "\mu_\alpha"] \arrow[d, "\mu"'] & & U_{a+(b+c)} \arrow[d, "\mu"] & \bot_{a+(b+c)} \in F(a+(b+c)) \\
\phi_{a+b,c}(\bot_{a+b},\bot_c) \in F((a+b)+c) & U_{a+b} \otimes U_c \arrow[d, "\mu"'] & & U_a \otimes U_{b+c} \arrow[d, "\mu"] & \phi_{a,b+c}(\bot_a,\bot_{b+c}) \in F(a+(b+c)) \\
\phi_{a+b,c}(\phi_{a,b}(\bot_a,\bot_b),\bot_c) \in F((a+b)+c) & (U_a \otimes U_b) \otimes U_c \arrow[rr, "\alpha'"] & & U_a \otimes (U_b \otimes U_c) & \phi_{a,b+c}(\bot_a,\phi_{b,c}(\bot_b,\bot_c)) \in F(a+(b+c))
\end{tikzcd}$$

All the left cospans are $(a + b) + c \to (a + b) + c \leftarrow (a + b) + c$ and all the right cospans are $a + (b + c) \to a + (b + c) \leftarrow a + (b + c)$.

Right and then down:

$$\begin{tikzcd}
& F(0) \arrow[rrr, "F(!_{(a+b)+c})"] \arrow[rrrd, phantom, "\cong"] & & & F((a+b)+c) \arrow[d, "F(\alpha)"] \\
& F(0) \arrow[rrr, "F(!_{a+(b+c)})"] \arrow[rrrdd, phantom, "\overset{\phi_{!_a,!_{b+c}}}{\cong}"] & & & F(a+(b+c)) \arrow[dd, "F(\mathrm{id})"] \\
1 \arrow[ur, "\phi_0"] \arrow[dr, "\phi_0 \times \phi_0"'] \arrow[uur, bend left, "\phi_0"] \arrow[ddr, bend right, "\phi_0\times(\phi_0\times\phi_0)"'] \arrow[r, phantom, "\overset{\phi}{\cong}"] & {} & & & \\
& F(0)\times F(0) \arrow[uu, "\phi_{0,0}"] \arrow[r, "F(!_a) \times F(!_{b+c})"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_c}}{\cong}"] & F(a)\times F(b+c) \arrow[rr, "\phi_{a,b+c}"] & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times(F(0)\times F(0)) \arrow[u, "1 \times \phi_{0,0}"] \arrow[r, "F(!_a) \times (F(!_b) \times F(!_c))"] \arrow[ul, phantom, "\overset{\phi}{\cong}"] & F(a)\times(F(b)\times F(c)) \arrow[u, "1 \times \phi_{b+c}"] \arrow[r, "1 \times \phi_{b+c}"] & F(a)\times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
& F(0) \arrow[rrr, "F(!_{(a+b)+c})"] \arrow[rrrd, phantom, "\overset{\phi_{!_{a+b},!_c}}{\cong}"] & & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_{a+b}) \times F(!_c)"] \arrow[ul, phantom, "\overset{\phi}{\cong}"] & F(a+b)\times F(c) \arrow[rr, "\phi_{a+b,c}"] & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, "\phi_0"] \arrow[dr, "\phi_0^2 \times \phi_0"'] \arrow[uur, bend left, "\phi_0"] \arrow[ddr, bend right, "\phi_0\times(\phi_0\times\phi_0)"'] \arrow[r, phantom, "\overset{\phi}{\cong}"] & {} \arrow[r, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & {} & & \\
& (F(0)\times F(0))\times F(0) \arrow[uu, "\phi_{0,0}"] \arrow[r, "(F(!_a) \times F(!_b)) \times F(!_c)"] \arrow[d, "\alpha'"] & (F(a)\times F(b))\times F(c) \arrow[uu, "\phi_{a,b} \times 1"] \arrow[r, "\phi_{a,b} \times 1"] \arrow[d, "\alpha'"] \arrow[dr, phantom, "\cong"] & F(a+b)\times F(c) \arrow[r, "\phi_{a+b,c}"] & F(a+(b+c)) \arrow[d, "F(\alpha)"] \\
& F(0)\times(F(0)\times F(0)) \arrow[r, "F(!_a) \times (F(!_b) \times F(!_c))"] & F(a)\times(F(b)\times F(c)) \arrow[r, "1 \times \phi_{b+c}"] & F(a)\times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Diagrams 6 and 8 of 11. Set up but not solved.
$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} c$ with $y \in F(n)$.
$U_0 \in F\mathbb{C}\mathbf{sp}$ is given by $0 \xrightarrow{!} 0 \xleftarrow{!} 0$ with $\bot_0 \in F(0)$.

$$\begin{tikzcd}
\begin{array}{c} 0{+}a \to (0{+}m){+}_{0+b}(0{+}n) \leftarrow 0{+}c \\ (\bot_0 \oplus y)\odot(\bot_0 \oplus x) \in F((0{+}m){+}_{0+b}(0{+}n)) \end{array} & (U_0 \otimes N)\odot(U_0 \otimes M) \arrow[r, "\chi"] \arrow[d, "\lambda_N \odot \lambda_M"'] & (U_0 \odot U_0)\otimes(N \odot M) \arrow[d, "\lambda_{U_0} \otimes \mathrm{id}_{N\odot M}"] & \begin{array}{c} 0{+}a \to (0{+}_0 0){+}(m{+}_b n) \leftarrow 0{+}c \\ (\bot_0 \odot \bot_0)\oplus(y \odot x) \in F((0{+}_0 0){+}(m{+}_b n)) \end{array} \\
\begin{array}{c} a \to m{+}_b n \leftarrow c \\ y \odot x \in F(m{+}_b n) \end{array} & N \odot M & U_0 \otimes (N \odot M) \arrow[l, "\lambda_{N\odot M}"'] & \begin{array}{c} 0{+}a \to 0{+}(m{+}_b n) \leftarrow 0{+}c \\ \bot_0 \oplus (y \odot x) \in F(0{+}(m{+}_b n)) \end{array}
\end{tikzcd}$$

Down:

$$\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\phi_{!_0,1}\times\phi_{!_0,1} \cong"] & F(0{+}m)\times F(0{+}n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] & F((0{+}m){+}(0{+}n)) \arrow[r, "F(\psi)"] \arrow[dd, "F(\ell+\ell)"] & F((0{+}m){+}_{0+b}(0{+}n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] & \lambda_M\times\lambda_N \cong & F(0{+}m)\times F(0{+}n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"'] \arrow[dr, "F(\ell)\times F(\ell)"] & \cong \quad\quad \phi_{\ell,\ell} \cong & \cong & \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m{+}n) \arrow[r, "F(\psi)"] & F(m{+}_b n)
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rr, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, "1\times\phi\times 1"] \arrow[ddd, bend right=60, "1\times\beta\times 1"'] & & F(0{+}m)\times F(0{+}n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[d, phantom, "\cong"] & F((0{+}m){+}(0{+}n)) \arrow[r, "F(\psi)"] \arrow[dl, "F(1+\beta+1)"] & F((0{+}m){+}_{0+b}(0{+}n)) \arrow[ddd, "F(\chi)"] \\
& & F(0)\times F(m{+}0)\times F(n) \arrow[r, "1\times\phi"] \arrow[d, "1\times F(\beta)\times 1"'] \arrow[d, phantom, "1\times\mu\times 1 \cong", shift left=8] & F(0)\times F((m{+}0){+}n) \arrow[urr, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[d, phantom, "\phi_{\beta,1}\cong", shift right=8] & \phi_{1,\beta+1} \cong & \cong & \\
& & F(0)\times F(0{+}m)\times F(n) \arrow[r, "1\times\phi"] & F(0)\times F((0{+}m){+}n) \arrow[r, "\phi"] \arrow[d, phantom, "\cong"] & F((0{+}0){+}(m{+}n)) \arrow[rr, "F(\psi+\psi)"] \arrow[r, phantom, "\phi_{\psi,\psi}\cong", shift right=10] & & \\
& (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times 1)"'] \arrow[ddr, "\phi_{0,0}\times 1"'] & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[u, "1\times\phi\times 1"] \arrow[rr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[dd, phantom, "\phi_{!_0,!_0}\cong"] & & F(0{+}0)\times F(m{+}n) \arrow[u, "\phi"] \arrow[r, "F(\psi)\times F(\psi)"] & F(0{+}_0 0)\times F(m{+}_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[ddd, "F(\kappa)\times 1"] \arrow[ddd, phantom, "\phi_{\kappa,1}\cong", shift left=12] & F((0{+}_0 0){+}(m{+}_b n)) \arrow[ddd, "F(\kappa+1)"] \\
& & & & \cong & & \\
1 \arrow[uuuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[uur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"] \arrow[dddr, bend right, "x\times y"'] & \lambda_{U_0}\cong & F(0{+}0)\times(F(m)\times F(n)) \arrow[uurr, "F(!_0+!_0)\times\phi_{m,n}"'] \arrow[drrr, "F(\ell)\times(F(\psi)\phi_{m,n})"] & & & & \\
& F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times F(m{+}n) \arrow[rr, "1\times F(\psi)"] \arrow[d, phantom, "\lambda_{N\odot M}\cong"] & & F(0)\times F(m{+}_b n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m+_b n}"'] & F(0)\times F(m{+}_b n) \arrow[r, "\phi_{0,m+_b n}"] \arrow[d, phantom, "\phi_{!_0,1}\cong"] & F(0{+}(m{+}_b n)) \arrow[dd, "F(\ell)"] \\
& & & & & F(0{+}(m{+}_b n)) \arrow[ur, "F(!_0+1)"'] \arrow[dr, "F(\ell)"] \arrow[r, phantom, "\cong"] & \\
& F(m)\times F(n) \arrow[rrr, "1"] & & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m{+}n) \arrow[r, "F(\psi)"] & F(m{+}_b n)
\end{tikzcd}$$

(Scratch work) As a check, let us compute the decoration maps $\lambda_N\colon U_0 \otimes N \to N$ and $\lambda_M\colon U_0 \otimes M \to M$ and compose them via $\odot$. The maps λ_N and λ_M are given explicitly in the paper.

$\lambda_M \colon U_0 \otimes M \to M$

$$\begin{tikzcd}
& F(0)\times F(m) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m}"'] & F(0)\times F(m) \arrow[r, "\phi_{0,m}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+m) \arrow[dd, "F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)"] \arrow[dr, bend right, "x"'] \arrow[rr, phantom, "\cong"] & & F(0+m) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & \\
& F(m) \arrow[rr, "1"] & & F(m)
\end{tikzcd}$$

$\lambda_N \colon U_0 \otimes N \to N$

$$\begin{tikzcd}
& F(0)\times F(n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,n}"'] & F(0)\times F(n) \arrow[r, "\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+n) \arrow[dd, "F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times y)"] \arrow[dr, bend right, "y"'] \arrow[rr, phantom, "\cong"] & & F(0+n) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & \\
& F(n) \arrow[rr, "1"] & & F(n)
\end{tikzcd}$$

To construct $\lambda_N \odot \lambda_M \colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$, we first tensor the above two diagrams:

$$\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n)
\end{tikzcd}$$

Next, we paste with a square due to pseudonaturality of ϕ:

$$\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \arrow[r, "\phi_{0+m,0+n}"] \arrow[ddr, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0+m)+(0+n)) \arrow[dd, "F(\ell+\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & & \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n)
\end{tikzcd}$$

Finally, we paste with a square due to pseudonaturality of F to obtain the map $\lambda_N \odot \lambda_M \colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$:

$$\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \arrow[r, "\phi_{0+m,0+n}"] \arrow[ddr, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0+m)+(0+n)) \arrow[dd, "F(\ell+\ell)"] \arrow[r, "F(\psi)"] \arrow[ddr, phantom, "\cong"] & F((0+m)+_{0+b}(0+n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & & & \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}$$

Next, we compute the right, down and then left route:

$$
\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rrr, "\phi_{0,m}\times\phi_{0,n}"] \arrow[dr, "1\times\phi\times 1"] \arrow[ddd, bend right, "1\times\beta\times 1"'] & & & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"'] & F((0+m)+_{0+b}(0+n)) \arrow[ddd, "F(\chi)"] \\
& & & F(0)\times F(m+0)\times F(n) \arrow[r, "1\times\phi"] \arrow[d, "1\times F(\beta)\times 1"'] & F(0)\times F((m+0)+n) \arrow[urr, "\phi"] \arrow[d, "1\times F(\beta+1)"] & & & \\
& & & F(0)\times F(0+m)\times F(n) \arrow[r, "1\times\phi"] & F(0)\times F((0+m)+n) \arrow[r, "\phi"] & F((0+0)+(m+n)) \arrow[drr, bend left, "F(\psi+\psi)"] & & \\
& (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times 1)"'] \arrow[dr, "\phi_{0,0}\times 1"] & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[rrr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[ur, "1\times\phi\times 1"'] & & & F(0+0)\times F(m+n) \arrow[r, "F(\psi)\times F(\psi)"] \arrow[u, "\phi"] & F(0+_0 0)\times F(m+_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[dd, "F(\kappa)\times 1"'] & F((0+_0 0)+(m+_b n)) \arrow[dd, "F(\kappa+1)"] \\
1 \arrow[uuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"'] \arrow[dddr, bend right, "x\times y"'] & & F(0+0)\times(F(m)\times F(n)) \arrow[urrr, "F(!_0+!_0)\times\phi_{m,n}"] \arrow[drrrr, "F(\lambda_0)\times(F(\psi)\phi_{m,n}"] & & & & & \\
& F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m+n) \arrow[rrr, "1\times F(\psi)"] & & & F(0)\times F(m+_b n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m+_b n}"'] & F(0)\times F(m+_b n) \arrow[r, "\phi_{0,m+_b n}"] & F(0+(m+_b n)) \arrow[dd, "F(\ell)"] \\
& & & & & & F(0+(m+_b n)) \arrow[ur, "F(!_0+1)"'] \arrow[dr, "F(\ell)"] & \\
& F(m)\times F(n) \arrow[rrrr, "1"] & & & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\arrow[from=1-5, to=2-5, phantom, "\cong"]
\arrow[from=2-3, to=2-4, phantom, "\overset{1\mu\times 1}{\cong}"]
\arrow[from=2-4, to=3-5, phantom, "\overset{\phi_{\beta,1}}{\cong}"]
\arrow[from=2-5, to=3-6, phantom, "\overset{\phi_{1,\beta+1}}{\cong}"]
\arrow[from=3-5, to=4-5, phantom, "\cong"]
\arrow[from=3-6, to=4-7, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"]
\arrow[from=1-7, to=3-8, phantom, "\cong"]
\arrow[from=4-3, to=5-4, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"]
\arrow[from=5-1, to=5-3, phantom, "\overset{\lambda_{M\odot N}}{\cong}"]
\arrow[from=4-6, to=6-6, phantom, "\cong"]
\arrow[from=4-7, to=6-8, phantom, "\overset{\phi_{\kappa,1}}{\cong}"]
\arrow[from=6-3, to=8-4, phantom, "\overset{\lambda_{N\odot M}}{\cong}"]
\arrow[from=6-7, to=7-7, phantom, "\overset{\phi_{!_0,1}}{\cong}"]
\arrow[from=7-7, to=7-8, phantom, "\cong"]
\end{tikzcd}
$$

$$
\begin{tikzcd}
& F(0)\times F(0) \arrow[r, "F(!_0)\times F(!_0)"] \arrow[dr, "\phi_{0,0}"'] & F(0)\times F(0) \arrow[rr, "\phi_{0,0}"] & & F(0+0) \arrow[r, "F(\psi)"] & F(0+_0 0) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0\times\phi_0"] \arrow[dr, bend right, "\phi_0"'] & & F(0+0) \arrow[urr, "F(!_0+!_0)"] \arrow[drrr, "F(\lambda_0)"] & & & \\
& F(0) \arrow[rrrr, "\mathrm{id}"] & & & & F(0)
\arrow[from=2-1, to=2-3, phantom, "\overset{\lambda_M}{\cong}"]
\arrow[from=1-3, to=2-3, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"]
\arrow[from=1-5, to=3-5, phantom, "\cong"]
\end{tikzcd}
$$

Diagrams 7 and 9 of 11. Should improve this, but seems okay.

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$
$$a \in (\mathsf{A},+,0)$$

$$0+a \to 0+a \leftarrow 0+a$$
$$\phi_{0,a}(\bot_0,\bot_a) \in F(0+a)$$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
& U_a
\end{tikzcd}$$

$$0+a \to 0+a \leftarrow 0+a$$
$$\bot_{0+a} \in F(0+a)$$
$$a \to a \leftarrow a$$
$$\bot_a \in F(a)$$

Right and then down:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, phantom, "\overset{\phi}{\cong}" left, near start] & & F(0{+}a) \arrow[d, "F(\mathrm{id})"] \arrow[dl, phantom, "="] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[ur, "\phi_{0,a}"] \arrow[r, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "F(\lambda)"] \arrow[dl, phantom, "\cong"] \\
& F(0) \arrow[urr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] \arrow[u, phantom, "\overset{\phi}{\cong}" left, near start] & & F(a)
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, "F(\mathrm{id})"] \arrow[drr, phantom, "=", near start] & & F(0{+}a) \arrow[d, "F(\lambda)"] \arrow[dl, phantom, "\cong"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & F(0) \arrow[urr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a)
\end{tikzcd}$$

Removing the lower right $\cong$ which is the same in each diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, phantom, "\overset{\phi}{\cong}" left, near start] & & F(0{+}a) \arrow[d, "F(\mathrm{id})"] \arrow[dl, phantom, "="] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[ur, "\phi_{0,a}"] \arrow[r, "\phi_{0,a}"] & F(0{+}a) \\
& F(0) \arrow[urr, "F(!_{0+a})"] \arrow[u, phantom, "\overset{\phi}{\cong}" left, near start] & &
\end{tikzcd}$$

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, "F(\mathrm{id})"] \arrow[dr, phantom, "="] & & F(0{+}a) \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

The second diagram is then an identity, so our problem reduces to showing that the following diagram is also an identity:

$$
\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"' near end] \arrow[dr, phantom, "\overset{\phi}{\cong}" near end] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[urr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] \arrow[ur, "\phi_{0,a}"] \arrow[ur, phantom, "=" near end, shift left=4] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"'] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & &
\end{tikzcd}
$$

Removing the diagonal $\phi_{0,a}$:

$$
\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"' near end] \arrow[dr, phantom, "\overset{\phi}{\cong}" near end] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[urr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"'] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & &
\end{tikzcd}
$$

This diagram is clearly the same as:

$$
\begin{tikzcd}
 & F(0) \arrow[drr, "F(!_{0+a})"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & & \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"' near end] \arrow[dr, phantom, "\overset{\phi}{\cong}" near end] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"'] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & &
\end{tikzcd}
$$

The two 2-isomorphisms in the top half of the diagram are the inverses of those in the bottom half, when read in a suitable order, and they can be shown to cancel, yielding an identity as desired.

Diagrams 7 and 9 of 11. Set up and solved.

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$
$$a \in (\mathsf{A},+,0)$$

$$0+a \to 0+a \leftarrow 0+a$$

$$\phi_{0,a}(\bot_0,\bot_a) \in F(0+a)$$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
 & U_a
\end{tikzcd}$$

$$0+a \to 0+a \leftarrow 0+a$$

$$\bot_{0+a} \in F(0+a)$$

$$a \to a \leftarrow a$$

$$\bot_a \in F(a)$$

Right and then down:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi\times\phi"] \arrow[d, "\lambda^{-1}"'] \arrow[dr, phantom, "\cong"] & F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[dr, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "1"] & U_0{\times}U_a \arrow[d, "\mu"] \\
1 \arrow[r, "\phi"] \arrow[d, "1"'] & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, "1"] & & F(0{+}a) \arrow[d, "F(\lambda_a)"] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[rr, "F(!_a)"] \arrow[urr, "F(!_{0+a})"] \arrow[ur, phantom, "\overset{F}{\cong}"] & & F(a) & U_a
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi\times\phi"] \arrow[d, "\lambda^{-1}"'] \arrow[dr, phantom, "\cong"] & F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "F(\lambda_a)"] & U_0{\times}U_a \arrow[d, "\lambda_{U_a}"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[urr, "F(!_{0+a})"] \arrow[urr, phantom, shift right=4, "\overset{F}{\cong}"] & & F(a) & U_a
\end{tikzcd}$$

If we remove the regions common to each diagram, and regions that strictly commute, these are clearly equal.

Right and then down:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[dr, "\phi_{0,a}"] & \\
F(0) \arrow[rr, "F(!_{0+a})"] & & F(0{+}a)
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \\
F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

Diagram 10 of 11. Set up and solved (Christina).

$M_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i_1} m_1 \xleftarrow{o_1} b$ with $x_1 \in F(m_1)$.
$M_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i_2} m_2 \xleftarrow{o_2} c$ with $x_2 \in F(m_2)$.
$N_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a' \xrightarrow{i'_1} n_1 \xleftarrow{o'_1} b'$ with $y_1 \in F(n_1)$.
$N_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b' \xrightarrow{i'_2} n_2 \xleftarrow{o'_2} c'$ with $x_2 \in F(n_2)$.

$$
\begin{tikzcd}
\begin{array}{c} a+a'\to(m_1+n_1)+_{b+b'}(m_2+n_2)\leftarrow c+c' \\ \scriptstyle F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2))\in F((m_1+n_1)+_{b+b'}(n_1+n_2)) \end{array} & (M_2\otimes N_2)\odot(M_1\otimes N_1) \arrow[r, "\beta\odot\beta"] \arrow[d, "\chi"'] & (N_2\otimes M_2)\odot(N_1\otimes M_1) \arrow[d, "\chi"] & \begin{array}{c} a'+a\to(n_1+m_1)+_{b'+b}(n_2+m_2)\leftarrow c'+c \\ \scriptstyle F(\psi)\phi_{n_1+m_1,n_2+m_2}(\phi_{n_1,m_1}(y_1,x_1),\phi_{n_2,m_2}(y_2,x_2))\in F((n_1+m_1)+_{b'+b}(n_1+m_2)) \end{array} \\
\begin{array}{c} a+a'\to(m_1+_bm_2)+(n_1+_{b'}n_2)\leftarrow c+c' \\ \scriptstyle \phi_{m_1+_bm_2,n_1+_{b'}m_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))\in F((m_1+_bm_2)+(n_1+_{b'}n_2)) \end{array} & (M_2\odot M_1)\otimes(N_2\odot N_1) \arrow[r, "\beta"] & (N_2\odot N_1)\otimes(M_2\odot M_1) & \begin{array}{c} a'+a\to(n_1+_{b'}n_2)+(m_1+_bm_2)\leftarrow c'+c \\ \scriptstyle \phi_{n_1+_{b'}n_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{m_1,m_2}(x_1,x_2))\in F((n_1+_{b'}n_2)+(m_1+_bm_2)) \end{array}
\end{tikzcd}
$$

Down and then right:

$$
\begin{tikzcd}
 & (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, "1\times\beta\times1"'] \arrow[ddr, phantom, "1\times\mu\times1\ \cong"'] & & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, phantom, "\cong"] & & F((m_1+n_1)+(m_2+n_2)) \arrow[rr, "F(\psi)"] \arrow[ddr, "F(1+\beta+1)"] \arrow[drr, phantom, "\cong"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[ddd, "F(\chi)"] \\
 & & F(m_1)\times F(n_1+m_2)\times F(n_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] \arrow[drr, phantom, "\phi_{\beta,1}\ \cong"] & & F(m_1)\times F((n_1+m_2)+n_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[r, phantom, "\phi_{1,\beta}\ \cong"] & {} & & \\
 & & F(m_1)\times F(m_2+n_1)\times F(n_2) \arrow[rr, "1\times\phi"'] \arrow[dr, phantom, "\cong"] & & F(m_1)\times F((m_2+n_1)+n_2) \arrow[rr, "\phi"] & & F(m_1+((m_2+n_1)+n_2)) \arrow[dr, "F(\psi+\psi)"] & \\
1 \arrow[r, "(x_1\times x_2)\times(y_1\times y_2)"] \arrow[uuur, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[dr, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(m_1)\times F(m_2))\times(F(n_1)\times F(n_2)) \arrow[rr, "\phi_{m_1,m_2}\times\phi_{n_1,n_2}"'] \arrow[ur, "1\times\phi\times1"] \arrow[d, "\beta"] & & F(m_1+m_2)\times F(n_1+n_2) \arrow[rr, "F(\psi)\times F(\psi)"] \arrow[urrr, "\phi_{m_1+m_2,n_1+n_2}"] & & F(m_1+_bm_2)\times F(n_1+_{b'}n_2) \arrow[rr, "\phi_{m_1+_bm_2,n_1+_{b'}n_2}"] \arrow[d, "\beta"] \arrow[u, phantom, "\phi_{\psi,\psi}\ \cong"] \arrow[drr, phantom, "\mu_{m_1+_bm_2,n_1+_{b'}n_2}\ \cong"] & & F((m_1+_bm_2)+(n_1+_{b'}n_2)) \arrow[d, "F(\beta)"] \\
 & F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] & & F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] & & F(n_1+_{b'}n_2)\times F(m_1+_bm_2) \arrow[rr, "\phi_{n_1+_{b'}n_2,m_1+_bm_2}"'] & & F((n_1+_{b'}n_2)+(m_1+_bm_2))
\end{tikzcd}
$$

Right and then down:

$$
\begin{tikzcd}
 & (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[d, "\beta\times\beta"] \arrow[drr, phantom, "\mu_{m_1,n_1}\times\mu_{m_2,n_2}\ \cong"] & & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, "F(\beta)\times F(\beta)"] \arrow[drr, phantom, "\phi_{\beta,\beta}\ \cong"] & & F((m_1+n_1)+(m_2+n_2)) \arrow[rr, "F(\psi)"] \arrow[d, "F(\beta+\beta)"] \arrow[drr, phantom, "\cong"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[d, "F(\beta+_\beta\beta)"] \\
1 \arrow[r, "(y_1\times x_1)\times(y_2\times x_2)"] \arrow[ur, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[dddr, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(n_1)\times F(m_1))\times(F(n_2)\times F(m_2)) \arrow[rr, "\phi_{n_1,m_1}\times\phi_{n_2,m_2}"] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, "1\times\beta\times1"'] \arrow[ddr, phantom, "1\times\mu\times1\ \cong"'] & & F(n_1+m_1)\times F(n_2+m_2) \arrow[rr, "\phi_{n_1+m_1,n_2+m_2}"] \arrow[d, phantom, "\cong"] & & F((n_1+m_1)+(n_2+m_2)) \arrow[rr, "F(\psi)"] \arrow[ddr, "F(1+\beta+1)"] \arrow[drr, phantom, "\cong"] & & F((n_1+m_1)+_{b'+b}(n_2+m_2)) \arrow[ddd, "F(\chi)"] \\
 & & F(n_1)\times F(m_1+n_2)\times F(m_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] \arrow[drr, phantom, "\phi_{\beta,1}\ \cong"] & & F(n_1)\times F((m_1+n_2)+m_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[r, phantom, "\phi_{1,\beta}\ \cong"] & {} & & \\
 & & F(n_1)\times F(n_2+m_1)\times F(m_2) \arrow[rr, "1\times\phi"'] \arrow[dr, phantom, "\cong"] & & F(n_1)\times F((n_2+m_1)+m_2) \arrow[rr, "\phi"] & & F(n_1+((n_2+m_1)+m_2)) \arrow[dr, "F(\psi+\psi)"] & \\
 & F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] \arrow[ur, "1\times\phi\times1"] & & F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] \arrow[urrr, "\phi_{n_1+n_2,m_1+m_2}"] & & F(n_1+_{b'}n_2)\times F(m_1+_bm_2) \arrow[rr, "\phi_{n_1+_{b'}n_2,m_1+_bm_2}"'] \arrow[u, phantom, "\phi_{\psi,\psi}\ \cong"] & & F((n_1+_{b'}n_2)+(m_1+_bm_2))
\end{tikzcd}
$$

Diagram 11 of 11. Set up but not solved.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_{a+b} \arrow[r, "\mu_{a,b}"] \arrow[d, "U_\beta"'] & U_a \otimes U_b \arrow[d, "\beta'"] \\
U_{b+a} \arrow[r, "\mu_{b,a}"] & U_b \otimes U_a
\end{tikzcd}$$

The top two underlying cospans are

$$a + b \to a + b \leftarrow a + b$$

and the bottom two underyling cospans are

$$b + a \to b + a \leftarrow b + a$$

Right and then down:

$$\begin{tikzcd}
& F(0{+}0) \arrow[rr, "F(!_a{+}!_b)"] & & F(a{+}b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi\times\phi"'] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] & F(a){\times}F(b) \arrow[u, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
& F(0){\times}F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
& F(0{+}0) \arrow[rr, "F(!_a{+}!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a{+}b) \arrow[d, "F(\beta)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0{+}0) \arrow[rr, "F(!_b{+}!_a)"] & & F(b{+}a) \arrow[d, "F(\mathrm{id})"] \\
& F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[ur, "\phi_{b,a}"] \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

First we remove the lower left commuting quarter circle in the ‘right and then down’ diagram as well as the upper left commuting quarter circle in the ‘down and then right’ diagram:

$$\begin{tikzcd}
& F(0{+}0) \arrow[rr, "F(!_a{+}!_b)"] & & F(a{+}b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] & F(a){\times}F(b) \arrow[u, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
& F(0){\times}F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

$$
\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
 & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

Next we remove the lower left commuting square with sides β in the first diagram:

$$
\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] & F(a)\times F(b) \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
 & & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
 & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

Next we assume that our pseudofunctor F is normalized, meaning that $F(\mathrm{id})$ is an identitiy, and remove the commuting triangle with sides $\phi_{a,b}$ in the first diagram and the commuting triangle with sides $\phi_{b,a}$ in the second diagram:

$$
\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
 & & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & & F(b+a) \\
 & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}
$$

Next we remove the left quarter circle containing $\overset{\phi}{\cong}$ from each diagram:

$$
\begin{tikzcd}
F(0{+}0) \arrow[rr, "F(!_a+!_b)"] & & F(a{+}b) \arrow[d, "F(\mathrm{id})"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[urr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
 & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
F(0{+}0) \arrow[rr, "F(!_{a+b})"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a{+}b) \arrow[d, "F(\beta)"] \\
F(0{+}0) \arrow[rr, "F(!_{b+a})"] & & F(b{+}a) \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[u, phantom, shift left=8ex, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}
$$

Making each diagram into a rectangle and removing the $F(\text{id})$ from the first diagram:

$$
\begin{tikzcd}
F(0{+}0) \arrow[rrr, "F(!_a+!_b)"] & & & F(a{+}b) \arrow[d, "F(\beta)"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[urrr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[urr, "\phi_{a,b}"] \arrow[r, "\beta"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b{+}a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
F(0{+}0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] & & F(a{+}b) \arrow[dd, "F(\beta)"] \\
F(0{+}0) \arrow[drr, "F(!_b+!_a)"] \arrow[urr, phantom, "\cong"] & & \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"'] \arrow[u, phantom, shift left=6ex, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"'] & F(b{+}a)
\end{tikzcd}
$$

Are these two diagrams the same?

$$
\begin{tikzcd}
F(0) \arrow[rr, "F(!_{a+b})"] \arrow[drr, "F(!_{b+a})"] & & F(a{+}b) \arrow[d, "F(\beta)"] \arrow[d, phantom, shift right=10ex, "\cong"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[u, phantom, shift left=8ex, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
F(c{+}d) \arrow[rr, "F(f+g)"] \arrow[drr, "F(g+f)"] & & F(a{+}b) \arrow[d, "F(\beta)"] \arrow[d, phantom, shift right=10ex, "\cong"] \\
F(c){\times}F(d) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(g)\times F(f)"] \arrow[u, phantom, shift left=8ex, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}
$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[r, "F(\beta)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a)\times F(b) \arrow[u, "\phi_{a,b}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(\beta)"] & F(0+0) \arrow[r, "F(!_b+!_a)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{0,0}}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[r, "F(\beta)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[rr, bend right, "F(!_b)\times F(!_a)"'] & F(a)\times F(b) \arrow[u, "\phi_{a,b}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(\beta)"] & F(0+0) \arrow[r, "F(!_b+!_a)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{0,0}}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

First diagram:

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[r, "F(\beta)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[rr, bend right, "F(!_b)\times F(!_a)"'] & F(a)\times F(b) \arrow[u, "\phi_{a,b}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

Second diagram:

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(\beta)"] \arrow[rrd, "F(!_b+!_a)"] & F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[d, "F(\beta)"] \arrow[ld, phantom, "\cong"] \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"'] \arrow[ru, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"'] & F(b+a)
\end{tikzcd}$$

Diagram 11 of 11. Set up but not solved.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_a \otimes U_b \arrow[r, "\mu_{a,b}"] \arrow[d, "\beta'"'] & U_{a+b} \arrow[d, "U_\beta"] \\
U_b \otimes U_a \arrow[r, "\mu_{b,a}"] & U_{b+a}
\end{tikzcd}$$

Right and then down:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "1\times\phi"] \arrow[rr, bend left=60, "\phi\times\phi"] \arrow[d, "\lambda"'] \arrow[dd, bend right=60, "\lambda"'] & 1{\times}F(0) \arrow[r, "\phi\times 1"] \arrow[d, "\lambda"'] \arrow[r, phantom, shift right=4ex, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\phi_{0,0}"] \arrow[r, phantom, shift right=4ex, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[dr, "\phi_{a,b}"] & F(a{+}b) \arrow[d, "1"] \arrow[dd, bend left=60, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[d, "1"'] & F(0) \arrow[r, "F(\lambda^{-1})"] \arrow[d, "1"'] \arrow[dr, "F(\lambda^{-1})"] & F(0{+}0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, "F(\beta(!_a+!_b))=F((!_b+!_a)\beta)"] \arrow[l, phantom, shift right=2ex, "\Swarrow F_{\lambda^{-1},\beta}"] \arrow[d, phantom, shift left=8ex, "\Nearrow F_{\beta,!_b+!_a}"] & & F(a{+}b) \arrow[d, "F(\beta)"] \arrow[r, phantom, "\overset{F_{1,\beta}}{\Rightarrow}"] \arrow[d, phantom, shift right=6ex, "\Swarrow F_{!_a+!_b,\beta}"] & {} \\
1 \arrow[r, "\phi"] & F(0) \arrow[r, "F(\lambda^{-1})"] & F(0{+}0) \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
1{\times}1 \arrow[rr, "\phi\times\phi"] \arrow[d, "\beta"'] \arrow[dd, bend right=60, "\lambda"'] & & F(0){\times}F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"'] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"] \arrow[r, phantom, shift right=4ex, "\overset{\mu_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \arrow[dd, bend left=60, "F(\beta)"] \\
1{\times}1 \arrow[r, "1\times\phi"] \arrow[d, "\lambda"'] & 1{\times}F(0) \arrow[r, "\phi\times 1"] \arrow[d, "\lambda"'] \arrow[r, phantom, shift right=4ex, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_b)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[r, phantom, shift right=4ex, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[dr, "\phi_{b,a}"] & F(b{+}a) \arrow[d, "1"] \arrow[r, phantom, "\overset{F_{\beta,1}}{\Rightarrow}"] & {} \\
1 \arrow[r, "\phi"] & F(0) \arrow[r, "F(\lambda^{-1})"] & F(0{+}0) \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

Diagram 11 of 11.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_a \otimes U_b \arrow[r, "\mu_{a,b}"] \arrow[d, "\beta"'] & U_{a+b} \arrow[d, "U_\beta"] \\
U_b \otimes U_a \arrow[r, "\mu_{b,a}"] & U_{b+a}
\end{tikzcd}$$

Right and then down:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi_0{\times}\phi_0"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_a){\times}F(!_b)"] \arrow[d, "\phi_{0,0}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[dl, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a{+}b) \arrow[d, "1"] \\
1 \arrow[r, "\phi_0"] \arrow[d, "1"'] & F(0{+}0) \arrow[urr, "F(!_a+!_b)"] \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a{+}b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi_0"] & F(0{+}0) \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi_0{\times}\phi_0"] \arrow[d, "1"'] & F(0){\times}F(0) \arrow[r, "F(!_a){\times}F(!_b)"] \arrow[d, "\beta"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{u_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
1{\times}1 \arrow[r, "\phi_0{\times}\phi_0"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_b){\times}F(!_a)"] \arrow[d, "\phi_{0,0}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[dl, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b{+}a) \arrow[d, "1"] \\
1 \arrow[r, "\phi_0"] & F(0{+}0) \arrow[urr, "F(!_b+!_a)"] \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

As $u\colon F(\beta)\phi \Rightarrow \phi\beta$ is an invertible modification, u satisfies the following equation:

$$\begin{tikzcd}
& F(0){\times}F(0) \arrow[rr, "F(!_a)\times F(!_b)"] \arrow[dl, "\beta"'] \arrow[dr, "\phi_{0,0}"] & & F(a){\times}F(b) \arrow[dr, "\phi_{a,b}"] & \\
F(0){\times}F(0) \arrow[dr, "\phi_{0,0}"'] \arrow[rr, phantom, "u_{0,0}"] & & F(0{+}0) \arrow[rr, "F(!_a+!_b)"] \arrow[dl, "F(\beta)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[dr, phantom, "\cong"] & & F(a{+}b) \arrow[dl, "F(\beta)"] \\
& F(0{+}0) \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a) &
\end{tikzcd}
=
\begin{tikzcd}
& F(0){\times}F(0) \arrow[rr, "F(!_a)\times F(!_b)"] \arrow[dl, "\beta"'] & & F(a){\times}F(b) \arrow[dl, "\beta"'] \arrow[dr, "\phi_{a,b}"] & \\
F(0){\times}F(0) \arrow[rr, "F(!_b)\times F(!_a)"] \arrow[dr, "\phi_{0,0}"'] & & F(b){\times}F(a) \arrow[dr, "\phi_{b,a}"] \arrow[rr, phantom, "u_{a,b}"] & & F(a{+}b) \arrow[dl, "F(\beta)"] \\
& F(0{+}0) \arrow[rr, "F(!_b+!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & & F(b{+}a) &
\end{tikzcd}$$

The right hand side of this equation already appears in the ”down and then right” diagram:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi_0{\times}\phi_0"] \arrow[d, "1"'] & F(0){\times}F(0) \arrow[r, "F(!_a){\times}F(!_b)"] \arrow[d, "\beta"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{u_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
1{\times}1 \arrow[r, "\phi_0{\times}\phi_0"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_b){\times}F(!_a)"] \arrow[d, "\phi_{0,0}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[dl, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b{+}a) \arrow[d, "1"] \\
1 \arrow[r, "\phi_0"] & F(0{+}0) \arrow[urr, "F(!_b+!_a)"] \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

Replacing the red outlined region with the left hand side of the equation satisfied by u, the "down and then right" diagram becomes:

$$\begin{tikzcd}
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "1"] & F(0)\times F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] \arrow[dr, "\phi_{0,0}"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[d, "\phi_{0,0}"] \arrow[r, phantom, "\overset{u_{0,0}}{\cong}"] & F(0+0) \arrow[ur, "F(!_a+!_b)"] \arrow[dl, "F(\beta)"] \arrow[r, phantom, "\cong"] & F(b+a) \arrow[d, "1"] \\
1 \arrow[r, "\phi_0"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] \arrow[urr, "F(!_b+!_a)"] & & F(b+a)
\end{tikzcd}$$

Next, we remove the commuting triangle in the bottom right, which results in:

$$\begin{tikzcd}
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "1"] & F(0)\times F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] \arrow[dr, "\phi_{0,0}"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a+b) \arrow[dd, "F(\beta)"] \\
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[d, "\phi_{0,0}"] \arrow[r, phantom, "\overset{u_{0,0}}{\cong}"] & F(0+0) \arrow[ur, "F(!_a+!_b)"] \arrow[dl, "F(\beta)"] \arrow[r, phantom, "\cong"] & \\
1 \arrow[r, "\phi_0"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] & & F(b+a)
\end{tikzcd}$$

and if we slide the central $F(0+0)$ to the left and add in a commuting triangle to the top right, the diagram becomes:

$$\begin{tikzcd}
1\times 1 \arrow[rr, "\phi_0\times\phi_0"] \arrow[d, "1"] & & F(0)\times F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[dl, "\beta"] \arrow[d, "\phi_{0,0}"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a+b) \arrow[d, "1"] \\
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[dr, "\phi_{0,0}"] \arrow[r, phantom, "\overset{u_{0,0}}{\cong}"] & F(0+0) \arrow[urr, "F(!_a+!_b)"] \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] & \arrow[d, phantom, "\cong"] & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[rr, "\phi_0"] & & F(0+0) \arrow[rr, "F(!_b+!_a)"] & & F(b+a)
\end{tikzcd}$$

Now the right half of this diagram is identical to the right half of the "right and then down" diagram. Removing these identical right halfs, the two diagrams become:

Down and then right:

$$\begin{tikzcd}
1\times 1 \arrow[rr, "\phi_0\times\phi_0"] \arrow[d, "1"] & & F(0)\times F(0) \arrow[dl, "\beta"] \arrow[d, "\phi_{0,0}"] \\
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[dr, "\phi_{0,0}"] \arrow[r, phantom, "\overset{u_{0,0}}{\cong}"] & F(0+0) \arrow[d, "F(\beta)"] \\
1 \arrow[rr, "\phi_0"] & & F(0+0)
\end{tikzcd}$$

Right and then down:

$$\begin{tikzcd}
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[d, "\phi_{0,0}"] \\
1 \arrow[r, "\phi_0"] \arrow[d, "1"] & F(0+0) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi_0"] & F(0+0)
\end{tikzcd}$$

The bottom half of the "right and then down" diagram commutes and removing this bottom half leaves us with:

$$\begin{tikzcd}
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[d, "\phi_{0,0}"] \\
1 \arrow[r, "\phi_0"] & F(0+0)
\end{tikzcd}$$

We now show that the "down and then right" diagram also reduces to this. First we add in a morphism $\phi_{0,0}\colon F(0)\times F(0)\to F(0+0)$ inside of the $u_{0,0}$ isomorphism:

$$\begin{tikzcd}
1\times 1 \arrow[rr, "\phi_0\times\phi_0"] \arrow[d, "1"'] & & F(0)\times F(0) \arrow[dl, "\beta"'] \arrow[d, "\phi_{0,0}"] \\
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[r, "\phi_{0,0}"] \arrow[dr, "\phi_{0,0}"] & F(0+0) \arrow[d, "F(\beta)"] \\
1 \arrow[rr, "\phi_0"] & & F(0+0)
\end{tikzcd}$$

The top half of this diagram commutes as two of the three routes from 1×1 to $F(0+0)$ are identical and the third one swaps two identical objects before combining them via the laxator. Removing this top half yields:

$$\begin{tikzcd}
1\times 1 \arrow[r, "\phi_0\times\phi_0"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F(0)\times F(0) \arrow[r, "\phi_{0,0}"] \arrow[dr, "\phi_{0,0}"] & F(0+0) \arrow[d, "F(\beta)"] \\
1 \arrow[rr, "\phi_0"] & & F(0+0)
\end{tikzcd}$$

Finally, the triangle on the right when whiskered with the morphism $\phi_0\times\phi_0\colon 1\times 1\to F(0)\times F(0)$ also commutes, and removing this triangle leaves us with the isomorphism λ that the other diagram reduced to.

Pentagon axiom. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \to m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
M_4 is given by $d \to m_4 \leftarrow e$ with $x_4 \in F(m_4)$.

$$\begin{tikzcd}
 & & (M_1 \odot M_2) \odot (M_3 \odot M_4) \arrow[drr, "\alpha"] & & \\
((M_1 \odot M_2) \odot M_3) \odot M_4 \arrow[urr, "\alpha"] \arrow[dr, "\alpha \odot 1_{M_4}"] & & & & M_1 \odot (M_2 \odot (M_3 \odot M_4)) \\
 & (M_1 \odot (M_2 \odot M_3)) \odot M_4 \arrow[rr, "\alpha"] & & M_1 \odot ((M_2 \odot M_3) \odot M_4) \arrow[ur, "1_{M_1} \odot \alpha"] &
\end{tikzcd}$$

Upper route:

$$\begin{tikzcd}
Fm_1Fm_2Fm_3Fm_4 \arrow[r,"\phi\times1\times1"] \arrow[ddd,"\alpha"'] & Fm_1{+}m_2Fm_3Fm_4 \arrow[rr,"F(\psi)\times1\times1"] & & Fm_1{+}_bm_2Fm_3Fm_4 \arrow[r,"\phi\times1"] \arrow[ddll,"\alpha"'] & F(m_1{+}_bm_2){+}m_3Fm_4 \arrow[r,"F(\psi)\times1"] \arrow[d,"\phi"] \arrow[dr, phantom, "\overset{\phi_{\psi,1}}{\cong}"] & F(m_1{+}_bm_2){+}_cm_3Fm_4 \arrow[r,"\phi"] & F((m_1{+}_bm_2){+}_cm_3){+}m_4 \arrow[r,"F(\psi)"] & F((m_1{+}_bm_2){+}_cm_3){+}_dm_4 \arrow[ddd,"F(\kappa)"] \\
 & & & \arrow[r, phantom, "\overset{\alpha}{\cong}"] & F((m_1{+}_bm_2){+}m_3){+}m_4 \arrow[urr,"F(\psi+1)"] \arrow[d,"F(\alpha)"] & & \arrow[d, phantom, "\cong"] & \\
 & Fm_1{+}_bm_2Fm_3Fm_4 \arrow[rr,"1\times\phi"] & & Fm_1{+}_bm_2Fm_3{+}m_4 \arrow[r,"\phi"] \arrow[dr,"1\times F(\psi)"] & F(m_1{+}_bm_2){+}(m_3{+}m_4) \arrow[dr,"F(1+\psi)"] \arrow[d, phantom, "\overset{\phi_{1,\psi}}{\cong}"] & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[ur,"F(\psi)\phi\times1"] \arrow[rr,"\phi\times\phi"] \arrow[dr,"1\times F(\psi)\phi"'] \arrow[ddd,"\alpha"'] & & Fm_1{+}m_2Fm_3{+}m_4 \arrow[ur,"F(\psi)\times1"] \arrow[rr,"F(\psi)\times F(\psi)"] \arrow[dr,"1\times F(\psi)"'] & & Fm_1{+}_bm_2Fm_3{+}_dm_4 \arrow[r,"\phi"] \arrow[d, phantom, "\overset{\phi_{\psi,1}}{\cong}"] & F(m_1{+}_bm_2){+}(m_3{+}_dm_4) \arrow[rr,"F(\psi)"] & & F(m_1{+}_bm_2){+}_c(m_3{+}_dm_4) \arrow[ddd,"F(\kappa)"] \\
 & Fm_1Fm_2Fm_3{+}_dm_4 \arrow[rr,"\phi\times1"] \arrow[ddrr,"\alpha"] & & Fm_1{+}m_2Fm_3{+}_dm_4 \arrow[r,"\phi"] \arrow[ur,"F(\psi)\times1"'] & F(m_1{+}m_2){+}(m_3{+}_dm_4) \arrow[ur,"F(\psi+1)"'] \arrow[d,"F(\alpha)"] & & \arrow[d, phantom, "\cong"] & \\
 & & & \arrow[r, phantom, "\overset{\alpha}{\cong}"] & Fm_1{+}(m_2{+}(m_3{+}_dm_4)) \arrow[drr,"F(1+\psi)"] \arrow[dr, phantom, "\overset{\phi_{1,\psi}}{\cong}"] & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r,"1\times1\times\phi"] & Fm_1Fm_2Fm_3{+}m_4 \arrow[rr,"1\times1\times F(\psi)"] & & Fm_1Fm_2Fm_3{+}_dm_4 \arrow[r,"1\times\phi"] & Fm_1Fm_2{+}(m_3{+}_dm_4) \arrow[r,"1\times F(\psi)"] \arrow[u,"\phi"'] & Fm_1Fm_2{+}_c(m_3{+}_dm_4) \arrow[r,"\phi"] & Fm_1{+}(m_2{+}_c(m_3{+}_dm_4)) \arrow[r,"F(\psi)"] & Fm_1{+}_b(m_2{+}_c(m_3{+}_dm_4))
\end{tikzcd}$$

Lower route:

$$
\begin{tikzcd}
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "\phi\times1\times1"] \arrow[ddd, "\alpha\times1"'] & Fm_1{+}m_2Fm_3Fm_4 \arrow[r, "F(\psi)\times1\times1"] \arrow[dr, "\phi\times1"] \arrow[ddd, phantom, "\underset{\cong}{\alpha\times1}"] & Fm_1{+_b}m_2Fm_3Fm_4 \arrow[r, "\phi\times1"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}\times1}"] & F(m_1{+_b}m_2){+}m_3Fm_4 \arrow[r, "F(\psi)\times1"] \arrow[ddd, phantom, "\cong"] & F(m_1{+_b}m_2){+_c}m_3Fm_4 \arrow[r, "\phi"] \arrow[ddd, "F(\kappa)\times1"'] \arrow[dddr, phantom, "\underset{\cong}{\phi_{\kappa,1}}"] & F((m_1{+_b}m_2){+_c}m_3){+}m_4 \arrow[r, "F(\psi)"] \arrow[ddd, "F(\kappa+1)"'] \arrow[dddr, phantom, "\cong"] & F((m_1{+_b}m_2){+_c}m_3){+_d}m_4 \arrow[ddd, "F(\kappa+_1 1)"] \\
 & & F(m_1{+}m_2){+}m_3Fm_4 \arrow[ur, "F(\psi+1)\times1"] \arrow[d, "F(\alpha)\times1"'] & & & & \\
 & & Fm_1{+}(m_2{+}m_3)Fm_4 \arrow[dr, "F(1+\psi)\times1"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}\times1}"] & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times\phi\times1"] \arrow[ddd, "1\times\alpha\times1"'] & Fm_1Fm_2{+}m_3Fm_4 \arrow[r, "1\times F(\psi)\times1"] \arrow[ur, "\phi\times1"] & Fm_1Fm_2{+_c}m_3Fm_4 \arrow[r, "\phi\times1"] \arrow[ddd, "\alpha"'] & Fm_1{+}(m_2{+_c}m_3)Fm_4 \arrow[r, "F(\psi)\times1"] \arrow[dr, "\phi"] \arrow[ddd, phantom, "\underset{\cong}{\alpha}"] & Fm_1{+_b}(m_2{+_c}m_3)Fm_4 \arrow[r, "\phi"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}}"] & F(m_1{+_b}(m_2{+_c}m_3)){+}m_4 \arrow[r, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & F(m_1{+_b}(m_2{+_c}m_3)){+_d}m_4 \arrow[ddd, "F(\kappa)"] \\
 & & & & F(m_1{+}(m_2{+_c}m_3)){+}m_4 \arrow[ur, "F(\psi+1)"] \arrow[d, "F(\alpha)"'] & & \\
 & & & & Fm_1{+}((m_2{+_c}m_3){+}m_4) \arrow[dr, "F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}}"] & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times\phi\times1"] \arrow[ddd, "1\times\alpha"'] & Fm_1Fm_2{+}m_3Fm_4 \arrow[r, "1\times F(\psi)\times1"] \arrow[dr, "1\times\phi"] \arrow[ddd, phantom, "\underset{\cong}{1\times\alpha}"] & Fm_1Fm_2{+_c}m_3Fm_4 \arrow[r, "1\times\phi"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{\psi,1}}"] & Fm_1F(m_2{+_c}m_3){+}m_4 \arrow[r, "1\times F(\psi)"] \arrow[ur, "\phi"] \arrow[ddd, phantom, "\cong"] & Fm_1F(m_2{+_c}m_3){+_d}m_4 \arrow[r, "\phi"] \arrow[ddd, "1\times F(\kappa)"'] \arrow[dddr, phantom, "\underset{\cong}{\phi_{1,\kappa}}"] & Fm_1{+}((m_2{+_c}m_3){+_d}m_4) \arrow[r, "F(\psi)"] \arrow[ddd, "F(1+\kappa)"'] \arrow[dddr, phantom, "\cong"] & Fm_1{+_b}((m_2{+_c}m_3){+_d}m_4) \arrow[ddd, "F(1+_1\kappa)"] \\
 & & Fm_1F(m_2{+}m_3){+}m_4 \arrow[ur, "1\times F(\psi+1)"] \arrow[d, "1\times F(\alpha)"'] & & & & \\
 & & Fm_1Fm_2{+}(m_3{+}m_4) \arrow[dr, "1\times F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{1,\psi}}"] & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times1\times\phi"] & Fm_1Fm_2Fm_3{+}m_4 \arrow[r, "1\times1\times F(\psi)"] \arrow[ur, "1\times\phi"] & Fm_1Fm_2Fm_3{+_d}m_4 \arrow[r, "1\times\phi"] & Fm_1Fm_2{+}(m_3{+_d}m_4) \arrow[r, "1\times F(\psi)"] & Fm_1Fm_2{+_c}(m_3{+_d}m_4) \arrow[r, "\phi"] & Fm_1{+}(m_2{+_c}(m_3{+_d}m_4)) \arrow[r, "F(\psi)"] & Fm_1{+_b}(m_2{+_c}(m_3{+_d}m_4))
\end{tikzcd}
$$

Triangle identity. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
U_b is given by $b \to b \leftarrow b$ with $\perp_b \in F(b)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.

$$\begin{tikzcd}
& M_1 \odot M_2 & \\
(M_1 \odot U_b) \odot M_2 \arrow[ur, "\rho \odot 1_{M_2}"] \arrow[rr, "\alpha"] & & M_1 \odot (U_b \odot M_2) \arrow[ul, "1_{M_1} \odot \lambda"']
\end{tikzcd}$$

Up:

$$\begin{tikzcd}
1 \arrow[r, "(x_1\times\phi_0)\times x_2"] \arrow[ddr, "x_1\times x_2"'] & Fm_1F0Fm_2 \arrow[r, "(1\times F(!_b))\times 1"] \arrow[dr, "\phi\times 1"] \arrow[d, phantom, "\overset{\rho\times 1}{\cong}"] & Fm_1FbFm_2 \arrow[r, "\phi_{m_1,b}\times 1"] \arrow[d, phantom, "\overset{\phi_{1,!_b}\times 1}{\cong}"] & Fm_1{+}bFm_2 \arrow[r, "F(\psi)\times 1"] \arrow[d, phantom, "\cong"] & Fm_1{+}_bbFm_2 \arrow[r, "\phi_{m_1+_bb,m_2}"] \arrow[ddlll, "F(\kappa)\times 1"] \arrow[dd, phantom, "\overset{\phi_{\kappa,1}}{\cong}"] & F(m_1{+}_b){+}m_2 \arrow[r, "F(\psi)"] \arrow[ddl, "F(\kappa+1)"] \arrow[dd, phantom, "\cong"] & F(m_1{+}_bb){+}_bm_2 \arrow[dd, "F(\kappa+_1 1)"] \\
& & Fm_1{+}0{\times}Fm_2 \arrow[ur, "F(1+!_b)\times 1"] \arrow[dl, "F(r)\times 1"] & & & & \\
& Fm_1Fm_2 \arrow[rrr, "\phi"] & & & Fm_1{+}m_2 \arrow[rr, "F(\psi)"] & & Fm_1{+}_bm_2
\end{tikzcd}$$

Across and then up:

$$\begin{tikzcd}
& Fm_1F0Fm_2 \arrow[r, "(1\times F(!_b))\times 1"] & Fm_1FbFm_2 \arrow[r, "\phi_{m_1,b}\times 1"] \arrow[ddd, "\alpha"] \arrow[ddd, phantom, shift left=14ex, "\overset{\alpha}{\cong}"] & Fm_1{+}bFm_2 \arrow[r, "F(\psi)\times 1"] \arrow[dr, "\phi"] & Fm_1{+}_bbFm_2 \arrow[r, "\phi_{m_1+_bb,m_2}"] \arrow[d, phantom, "\overset{\phi_{\psi,1}}{\cong}"] & F(m_1{+}_bb){+}m_2 \arrow[r, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & F(m_1{+}_bb){+}_bm_2 \arrow[ddd, "F(\kappa)"] \\
& & & & F(m_1{+}b){+}m_2 \arrow[ur, "F(\psi+1)"] \arrow[d, "F(\alpha)"] & & \\
& & & & Fm_1{+}(b{+}m_2) \arrow[dr, "F(1+\psi)"] \arrow[d, phantom, "\overset{\phi_{1,\psi}}{\cong}"] & & \\
1 \arrow[uuur, "(x_1\times\phi_0)\times x_2"] \arrow[r, "x_1\times(\phi_0\times x_2)"] \arrow[ddr, "x_1\times x_2"'] & Fm_1F0Fm_2 \arrow[r, "1\times(F(!_b)\times 1)"] \arrow[dr, "1\times\phi"] \arrow[d, phantom, "\overset{1\times\lambda}{\cong}"] & Fm_1FbFm_2 \arrow[r, "1\times\phi_{b,m_2}"] \arrow[d, phantom, "\overset{1\times\phi_{!_b,1}}{\cong}"] & Fm_1Fb{+}m_2 \arrow[r, "1\times F(\psi)"] \arrow[ur, "\phi"] \arrow[d, phantom, "\cong"] & Fm_1Fb{+}_bm_2 \arrow[r, "\phi_{m_1,b+_bm_2}"] \arrow[ddlll, "1\times F(\kappa)"] \arrow[dd, phantom, "\overset{\phi_{1,\kappa}}{\cong}"] & Fm_1{+}(b{+}_bm_2) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\kappa)"] \arrow[dd, phantom, "\cong"] & Fm_1{+}_b(b{+}_bm_2) \arrow[dd, "F(1+_1\kappa)"] \\
& & Fm_1{\times}F0{+}m_2 \arrow[ur, "1\times F(!_b+1)"] \arrow[dl, "1\times F(\ell)"] & & & & \\
& Fm_1Fm_2 \arrow[rrr, "\phi"] & & & Fm_1{+}m_2 \arrow[rr, "F(\psi)"] & & Fm_1{+}_bm_2
\end{tikzcd}$$

3. Proof of Theorem 2.3

Diagram 1 of 7. Set up but not solved.

$$\begin{tikzcd}
(\mathbb{H}(M_1)\otimes\mathbb{H}(M_2))\otimes\mathbb{H}(M_3) \arrow[rr, "\alpha'"] \arrow[d, "\mathbb{H}_{M_1,M_2}\otimes 1"'] & & \mathbb{H}(M_1)\otimes(\mathbb{H}(M_2)\otimes\mathbb{H}(M_3)) \arrow[d, "1\otimes\mathbb{H}_{M_2\otimes M_3}"] \\
\mathbb{H}(M_1\otimes M_2)\otimes\mathbb{H}(M_3) \arrow[d, "\mathbb{H}_{M_1\otimes M_2,M_3}"'] & & \mathbb{H}(M_1)\otimes\mathbb{H}(M_2\otimes M_3) \arrow[d, "\mathbb{H}_{M_1,M_2\otimes M_3}"] \\
\mathbb{H}((M_1\otimes M_2)\otimes M_3) \arrow[rr, "\mathbb{H}(\alpha)"] & & \mathbb{H}(M_1\otimes(M_2\otimes M_3))
\end{tikzcd}$$

M_1 is given by $a_1 \xrightarrow{i_1} m_1 \xleftarrow{o_1} b_1$ with $x_1 \in F(m_1)$.
M_2 is given by $a_2 \xrightarrow{i_2} m_2 \xleftarrow{o_2} b_2$ with $x_2 \in F(m_2)$.
M_3 is given by $a_3 \xrightarrow{i_3} m_3 \xleftarrow{o_3} b_3$ with $x_3 \in F(m_3)$.
$\mathbb{H}(M_1)$ is given by $H(a_1) \xrightarrow{H(i_1)} H(m_1) \xleftarrow{H(o_1)} H(b_1)$ with $\bar{x_1} = \theta_m E(x_1)\phi_0 \in F'(H(m_1))$.
$\mathbb{H}(M_2)$ is given by $H(a_2) \xrightarrow{H(i_2)} H(m_2) \xleftarrow{H(o_2)} H(b_2)$ with $\bar{x_2} = \theta_m E(x_2)\phi_0 \in F'(H(m_2))$.
$\mathbb{H}(M_3)$ is given by $H(a_3) \xrightarrow{H(i_3)} H(m_3) \xleftarrow{H(o_3)} H(b_3)$ with $\bar{x_3} = \theta_m E(x_3)\phi_0 \in F'(H(m_3))$.
The decoration on $M_1 \otimes M_2$ is given by

$$x_1 \oplus x_2 \colon = 1 \cong 1\times 1 \xrightarrow{x_1\times x_2} F(m_1)\times F(m_2) \xrightarrow{\phi_{m_1,m_2}} F(m_1+m_2).$$

The decoration on $\mathbb{H}(M_1 \otimes M_2)$ is given by

$$x_1 \bar{\oplus} x_2 = \theta_{m_1+m_2} E(x_1\oplus x_2)\phi_0 = 1 \cong 1\times 1 \xrightarrow{\phi_0} E(1\times 1) \xrightarrow{E(x_1\times x_2)} E(F(m_1)\times F(m_2)) \xrightarrow{E(\phi_{m_1,m_2})} E(F(m_1+m_2)) \xrightarrow{\theta_{m_1+m_2}} F'(H(m_1+m_2)).$$

The decoration on $\mathbb{H}(M_1) \otimes \mathbb{H}(M_2)$ is given by

$$\bar{x_1}\oplus\bar{x_2} = 1 \cong 1\times 1 \xrightarrow{\phi_0\times\phi_0} E(1)\times E(1) \xrightarrow{E(x_1)\times E(x_2)} E(F(m_1))\times E(F(m_2)) \xrightarrow{\theta_{m_1}\times\theta_{m_2}} F'(H(m_1))\times F'(H(m_2)) \xrightarrow{\phi'_{H(m_1),H(m_2)}} F'(H(m_1)+H(m_2)).$$

The natural isomorphism $\mathbb{H}_\otimes \colon \mathbb{H}(M_1) \otimes \mathbb{H}(M_2) \to \mathbb{H}(M_1 \otimes M_2)$ is given by:

$$\begin{tikzcd}[column sep=small]
 & & E(1)\times 1 \arrow[r, "1\times\phi_0"] \arrow[d, "\rho"'] \arrow[dr, phantom, "\underset{\cong}{\phi}"] & E(1)\times E(1) \arrow[r, "E(x_1)\times E(x_2)"] \arrow[d, "E_{1,1}"] \arrow[dr, phantom, "\underset{\cong}{E_{x_1,x_2}}"] & E(F(m_1))\times E(F(m_2)) \arrow[r, "\theta_{m_1}\times\theta_{m_2}"] \arrow[d, "E_{F(m_1),F(m_2)}"] & F'(H(m_1))\times F'(H(m_2)) \arrow[r, "\phi'_{H(m_1),H(m_2)}"] & F'(H(m_1)+H(m_2)) \arrow[d, "F'(H_{m_1,m_2})"] \\
1 \arrow[r, "\phi_0"] & E(1) \arrow[ur, "\rho^{-1}"] \arrow[r, "1"'] & E(1) \arrow[r, "E(\rho^{-1})"] & E(1\times 1) \arrow[r, "E(x_1\times x_2)"] & E(F(m_1)\times F(m_2)) \arrow[r, "E(\phi_{m_1,m_2})"] & E(F(m_1+m_2)) \arrow[r, "\theta_{m_1+m_2}"] & F'(H(m_1+m_2))
\end{tikzcd}$$

Regarding the commuting pentagon, 1) the monoidal natural isomorphism $\theta\colon EF \to F'H$ is a strict monoidal natural isomorphism (not pseudo), and 2) the laxator of the composite $F'H$ utilizes the laxators of both F' and H.
Right and then down:

$$\begin{tikzcd}[column sep=tiny]
 & (E(1)\times E(1))\times E(1) \arrow[r, "(E(x_1)\times E(x_2))\times E(x_3)"] \arrow[d, "\alpha"] & (E(F(m_1))\times E(F(m_2)))\times E(F(m_3)) \arrow[r, "(\theta_{m_1}\times\theta_{m_2})\times\theta_{m_3}"] & (F'(H(m_1))\times F'(H(m_2)))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2)}\times 1"] \arrow[d, "\alpha"] & F'(H(m_1)+H(m_2))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1)+H(m_2),H(m_3)}"] \arrow[d, phantom, "\underset{\cong}{\phi'_{H(m_1),H(m_2),H(m_3)}}"] & F'((H(m_1)+H(m_2))+H(m_3)) \arrow[d, "F'(\alpha)"] \\
 & E(1)\times((E(1)\times E(1)) \arrow[r, "E(x_1)\times(E(x_2)\times E(x_3))"] \arrow[d, "1\times E_{1,1}"] \arrow[dr, phantom, "\underset{\cong}{1\times E_{x_2,x_3}}"] & E(F(m_1))\times(E(F(m_2))\times E(F(m_3))) \arrow[r, "\theta_{m_1}\times(\theta_{m_2}\times\theta_{m_3})"] \arrow[d, "1\times E_{F(m_2),F(m_3)}"] & F'(H(m_1))\times(F'(H(m_2))\times F'(H(m_3))) \arrow[r, "1\times\phi'_{H(m_2),H(m_3)}"] & F'(H(m_1))\times F'(H(m_2)+H(m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2)+H(m_3)}"] \arrow[d, "1\times F'(H_{m_2,m_3})"] \arrow[dr, phantom, "\underset{\cong}{\phi'_{1,H_{m_2,m_3}}}"] & F'(H(m_1)+(H(m_2)+H(m_3))) \arrow[d, "F'(1+H_{m_2,m_3})"] \\
1 \arrow[uur, "(\phi_0\times\phi_0)\times\phi_0"] \arrow[ur, "\phi_0\times(\phi_0\times\phi_0)"] \arrow[r, "\phi_0\times\phi_0"] \arrow[dr, "\phi_0"'] & E(1)\times E(1\times 1) \arrow[r, "E(x_1)\times E(x_2\times x_3)"] \arrow[d, "E_{1,1\times 1}"] \arrow[dr, phantom, "\underset{\cong}{E_{x_1,x_2\times x_3}}"] & E(F(m_1))\times E(F(m_2)\times F(m_3)) \arrow[r, "1\times E(\phi_{m_2,m_3})"] \arrow[d, "E_{F(m_1),F(m_2)\times F(m_3)}"] \arrow[dr, phantom, "\underset{\cong}{E_{1,\phi_{m_2,m_3}}}"] & E(F(m_1))\times E(F(m_2+m_3)) \arrow[r, "\theta_{m_1}\times\theta_{m_2+m_3}"] \arrow[d, "E_{F(m_1),F(m_2+m_3)}"] & F'(H(m_1))\times F'(H(m_2+m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2+m_3)}"] & F'(H(m_1)+H(m_2+m_3)) \arrow[d, "F'(H_{m_1,m_2+m_3})"] \\
 & E(1\times(1\times 1)) \arrow[r, "E(x_1\times(x_2\times x_3))"] & E(F(m_1)\times(F(m_2)\times F(m_3))) \arrow[r, "E(1\times\phi_{m_2,m_2})"] & E(F(m_1)\times F(m_2+m_3)) \arrow[r, "E(\phi_{m_1,m_2+m_3})"] & E(F(m_1+(m_2+m_3))) \arrow[r, "\theta_{m_1+(m_2+m_3)}"] & F'(H(m_1+(m_2+m_3)))
\end{tikzcd}$$

(I'm being a little reckless on the left with the ϕs)
Down and then right:

$$
\begin{tikzcd}
 & (E(1)\times E(1))\times E(1) \arrow[r, "(E(x_1)\times E(x_2))\times E(x_3)"] \arrow[d, "E_{1,1}\times 1"] \arrow[r, phantom, shift right=6ex, "\underset{\cong}{E_{x_1,x_2}\times 1}"] & (E(F(m_1))\times E(F(m_2)))\times E(F(m_3)) \arrow[r, "(\theta_{m_1}\times\theta_{m_2})\times\theta_{m_3}"] \arrow[d, "E_{F(m_1),F(m_2)}\times 1"] & (F'(H(m_1))\times F'(H(m_2)))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2)}\times 1"] & F'(H(m_1)+H(m_2))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1)+H(m_2),H(m_3)}"] \arrow[d, "F'(H_{m_1,m_2})\times 1"] \arrow[r, phantom, shift right=6ex, "\underset{\cong}{\phi'_{H_{m_1,m_2},1}}"] & F'((H(m_1)+H(m_2))+H(m_3)) \arrow[d, "F'(H_{m_1,m_2}+1)"] \\
1 \arrow[ur, "(\phi_0\times\phi_0)\times\phi_0"] \arrow[r, "\phi_0\times\phi_0"] \arrow[dr, "\phi_0"] \arrow[ddr, "\phi_0"] & E(1\times 1)\times E(1) \arrow[r, "E(x_1\times x_2)\times E(x_3)"] \arrow[d, "E_{1\times 1,1}"] \arrow[r, phantom, shift right=6ex, "\underset{\cong}{E_{x_1\times x_2,x_3}}"] & E(F(m_1)\times F(m_2))\times E(F(m_3)) \arrow[r, "E(\phi_{m_1,m_2})\times 1"] \arrow[d, "E_{F(m_1)\times F(m_2),F(m_3)}"] \arrow[r, phantom, shift right=6ex, "\underset{\cong}{E_{\phi_{m_1,m_2},1}}"] & E(F(m_1+m_2))\times E(F(m_3)) \arrow[r, "\theta_{m_1+m_2}\times\theta_{m_3}"] \arrow[d, "E_{F(m_1+m_2),F(m_3)}"] & F'(H(m_1+m_2))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1+m_2),H(m_3)}"] & F'(H(m_1+m_2)+H(m_3)) \arrow[d, "F'(H_{m_1+m_2,m_3})"] \\
 & E((1\times 1)\times 1) \arrow[r, "E((x_1\times x_2)\times x_3)"] \arrow[d, "E(\alpha)"] \arrow[r, phantom, shift right=6ex, "\underset{\cong}{E(\alpha_{x_1,x_2,x_3})}"] & E((F(m_1)\times F(m_2))\times F(m_3)) \arrow[r, "E(\phi_{m_1,m_2}\times 1)"] \arrow[d, "E(\alpha)"] & E(F(m_1+m_2)\times F(m_3)) \arrow[r, "E(\phi_{m_1+m_2,m_3})"] \arrow[d, phantom, "\underset{\cong}{E(\phi_{m_1,m_2,m_3})}"] & E(F((m_1+m_2)+m_3)) \arrow[r, "\theta_{(m_1+m_2)+m_3}"] \arrow[d, "E(F(\alpha))"] & F'(H((m_1+m_2)+m_3)) \arrow[d, "F'(H(\alpha))"] \\
 & E(1\times(1\times 1)) \arrow[r, "E(x_1\times(x_2\times x_3))"] & E(F(m_1)\times(F(m_2)\times F(m_3))) \arrow[r, "E(1\times\phi_{m_2,m_2})"] & E(F(m_1)\times F(m_2+m_3)) \arrow[r, "E(\phi_{m_1,m_2+m_3})"] & E(F(m_1+(m_2+m_3))) \arrow[r, "\theta_{m_1+(m_2+m_3)}"] & F'(H(m_1+(m_2+m_3)))
\end{tikzcd}
$$

Diagram 2 of 7. Set up but not solved.

The natural isomorphism $\mathbb{H}_U \colon 1_{F'\mathbb{C}\mathbf{sp}} \to \mathbb{H}(1_{F\mathbb{C}\mathbf{sp}})$ is given by:

$$\begin{tikzcd}
 & F'(0') \arrow[rrr, "F'(!_{0'})"] \arrow[rr, "F'(\phi'''_0)"'] & & & F'(0') \arrow[d, "F'(\phi'''_0)"] \arrow[dll, phantom, "\cong"] \\
1 \arrow[ur, "\phi'_0"] \arrow[dr, "\phi''_0"'] & & & F'(H(0)) \arrow[r, "F'(H(!_0))"] & F'(H(0)) \\
 & E(1) \arrow[r, "E(\phi_0)"'] & E(F(0)) \arrow[ur, "\theta_0"] \arrow[r, "E(F(!_0))"'] & E(F(0)) \arrow[ur, "\theta_0"'] &
\end{tikzcd}$$

$$\begin{tikzcd}
1_{F'\mathbb{C}\mathbf{sp}} \otimes \mathbb{H}(M) \arrow[r, "\mathbb{H}_U \otimes 1"] \arrow[d, "\ell"'] & \mathbb{H}(1_{F\mathbb{C}\mathbf{sp}}) \otimes \mathbb{H}(M) \arrow[d, "H_{1_{F\mathbb{C}\mathbf{sp}},M}"] \\
\mathbb{H}(M) & \mathbb{H}(1_{F\mathbb{C}\mathbf{sp}} \otimes M) \arrow[l, "\mathbb{H}(\ell')"']
\end{tikzcd}$$

M is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.

$\mathbb{H}(M)$ is given by $H(a) \xrightarrow{H(i)} H(m) \xleftarrow{H(o)} H(b)$ with $\bar{x} = \theta_m E(x)\phi_0 \in F'(H(m))$.

$1_{F\mathbb{C}\mathbf{sp}}$ is given by $0 \xrightarrow{1} 0 \xleftarrow{1} 0$ with $1 \xrightarrow{\phi_0} F(0) \xrightarrow{F(!_0)} F(0) \in F(0)$.

$\mathbb{H}(1_{F\mathbb{C}\mathbf{sp}})$ is given by $H(0) \xrightarrow{1} H(0) \xleftarrow{1} H(0)$ with $1 \xrightarrow{\phi_0} E(1) \xrightarrow{E(\phi_0)} E(F(0)) \xrightarrow{E(F(!_0))} E(F(0)) \xrightarrow{\theta_0} F'(H(0))$.

$1_{F'\mathbb{C}\mathbf{sp}}$ is given by $0' \xrightarrow{1} 0' \xleftarrow{1} 0'$ with $1 \xrightarrow{\phi'_0} F'(0') \xrightarrow{F'(!_{0'})} F'(0')$ where $0'$ is the monoidal unit of $(\mathsf{A}', +, 0')$.

Down:

$$\begin{tikzcd}
 & 1{\times}E(1) \arrow[r, "\phi'_0{\times}E(x)"] & F'(0'){\times}E(F(m)) \arrow[r, "F'(!_{0'}){\times}1"] \arrow[dr, "1{\times}\theta_m"'] & F'(0'){\times}E(F(m)) \arrow[r, "1{\times}\theta_m"] \arrow[dr, phantom, "\overset{\phi'_{!_{0'},1}}{\cong}"] & F'(0'){\times}F'(H(m)) \arrow[r, "\phi'_{0',H(m)}"] & F'(0'{+}H(m)) \arrow[ddd, "F'(\ell)"] \\
1 \arrow[ur, "1{\times}\phi''_0"] \arrow[dddr, "\phi''_0"'] & & & F'(0'){\times}F'(H(m)) \arrow[r, "\phi'_{0',H(m)}"] \arrow[dr, phantom, "\overset{\lambda}{\cong}"] & F'(0'{+}H(m)) \arrow[ur, "F'(!_{0'}{+}1)"] \arrow[ddr, "F'(\ell)"] \arrow[r, phantom, "\cong"] & \\
 & & & 1{\times}E(F(m)) \arrow[u, "\phi'_0{\times}\theta_m"] \arrow[r, "1{\times}\theta_m"] & 1{\times}F'(H(m)) \arrow[dr, "\lambda"] & \\
 & E(1) \arrow[rr, "E(x)"] & & E(F(m)) \arrow[u, "\lambda^{-1}"] \arrow[rr, "\theta_m"] & & F'(H(m))
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
 & 1{\times}E(1) \arrow[r, "\phi'_0{\times}E(x)"] \arrow[dd, "\phi''_0{\times}1"] & F'(0'){\times}E(F(m)) \arrow[rr, "F'(!_{0'}){\times}\theta_m"] \arrow[dr, "F'(\phi'''){\times}1"] & & F'(0'){\times}F'(H(m)) \arrow[r, "\phi'_{0',H(m)}"] \arrow[dd, "F'(\phi'''){\times}1"] \arrow[ddr, phantom, "\overset{\phi'_{\phi''',1}}{\cong}"] & F'(0'{+}H(m)) \arrow[dd, "F'(\phi'''{+}1)"] \\
 & & & F'(H(0)){\times}E(F(m)) \arrow[dr, "F'(H(!_0)){\times}\theta_m"] \arrow[ur, phantom, "\cong"] & & \\
 & E(1){\times}E(1) \arrow[r, "E(\phi){\times}E(x)"] \arrow[dd, "E_{1,1}"] \arrow[ddr, phantom, "\overset{E_{\phi,x}}{\cong}"] & E(F(0)){\times}E(F(m)) \arrow[r, "E(F(!_0)){\times}1"'] \arrow[ur, "\theta_0{\times}1"] \arrow[dd, "E_{F(0),F(m)}"] \arrow[ddr, phantom, "\overset{E_{F(!_0),1}}{\cong}"] & E(F(0)){\times}E(F(m)) \arrow[r, "\theta_0{\times}\theta_m"'] \arrow[dd, "E_{F(0),F(m)}"] & F'(H(0)){\times}F'(H(m)) \arrow[r, "\phi'_{H(0),H(m)}"] & F'(H(0){+}H(m)) \arrow[dd, "F'(H_{0,m})"] \\
1 \arrow[uuur, "1{\times}\phi''_0"] \arrow[ur, "\phi''_0{\times}\phi''_0"] \arrow[dr, "\phi''_0"] \arrow[dddr, "\phi''_0"'] & & & & & \\
 & E(1{\times}1) \arrow[r, "E(\phi{\times}x)"] \arrow[dr, "E(1{\times}x)"'] \arrow[dd, "E(\lambda)"] \arrow[ddrr, phantom, "\cong"] & E(F(0){\times}F(m)) \arrow[r, "E(F(!_0){\times}1)"] \arrow[dr, "E(\phi_{0,m})"'] & E(F(0){\times}F(m)) \arrow[r, "E(\phi_{0,m})"] \arrow[d, phantom, "\overset{E(\phi_{!_0,1})}{\cong}"] & E(F(0{+}m)) \arrow[r, "\theta_{0+m}"] \arrow[ddl, "E(F((\ell))"] & F'(H(0{+}m)) \arrow[dd, "F'(H(\ell))"] \\
 & & E(1{\times}F(m)) \arrow[u, "E(\phi{\times}1)"'] \arrow[dr, "E(\lambda)"'] \arrow[r, phantom, "E(\overset{\lambda}{\cong})"] & E(F(0{+}m)) \arrow[ur, "E(F(!_0{+}1))"] \arrow[d, "E(F(\ell))"] \arrow[r, phantom, "\cong"] & & \\
 & E(1) \arrow[rr, "E(x)"] & & E(F(m)) \arrow[rr, "\theta_m"] & & F'(H(m))
\end{tikzcd}$$

Diagram 3 of 7. Set up but not solved.

$$\begin{tikzcd}
\mathbb{H}(M)\otimes\mathbb{H}(N) \arrow[r, "\beta"] \arrow[d, "\mathbb{H}_{M,N}"'] & \mathbb{H}(N)\otimes\mathbb{H}(M) \arrow[d, "\mathbb{H}_{N,M}"] \\
\mathbb{H}(M\otimes N) \arrow[r, "\mathbb{H}(\beta)"] & \mathbb{H}(N\otimes M)
\end{tikzcd}$$

M is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
$\mathbb{H}(M)$ is given by $H(a) \xrightarrow{H(i)} H(m) \xleftarrow{H(o)} H(b)$ with $\bar{x} = \theta_m E(x)\phi_0 \in F'(H(m))$.
N is given by $a' \xrightarrow{i'} n \xleftarrow{o'} b'$ with $y \in F(n)$.
$\mathbb{H}(N)$ is given by $H(a') \xrightarrow{H(i')} H(n) \xleftarrow{H(o')} H(b')$ with $\bar{y} = \theta_n E(y)\phi_0 \in F'(H(n))$.
$M \otimes N$ has decoration $1 \xrightarrow{x\times y} F(m)\times F(n) \xrightarrow{\phi_{m,n}} F(m+n)$. $\mathbb{H}(M\otimes N)$ has decoration

$$1 \xrightarrow{\phi''_0} E(1) \xrightarrow{E(x\times y)} E(F(m)\times F(n)) \xrightarrow{E(\phi_{m,n})} E(F(m+n)) \xrightarrow{\theta_{m+n}} F'(H(m+n)).$$

$\mathbb{H}(M)\otimes\mathbb{H}(N)$ has decoration

$$1 \xrightarrow{\phi''_0\times\phi''_0} E(1)\times E(1) \xrightarrow{E(x)\times E(y)} E(F(m))\times E(F(n)) \xrightarrow{\theta_m\times\theta_n} F'(H(m))\times F'(H(n)) \xrightarrow{\phi'_{H(m),H(n)}} F'(H(m)+H(n)).$$

Right and then down:

$$\begin{tikzcd}
 & E(1)\times E(1) \arrow[r, "E(x)\times E(y)"] \arrow[d, "\beta"] & E(F(m))\times E(F(n)) \arrow[r, "\theta_m\times\theta_n"] \arrow[d, "\beta"] & F'(H(m))\times F'(H(n)) \arrow[r, "\phi'_{H(m),H(n)}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\mu'_{H(m),H(n)} \cong"] & F'(H(m)+H(n)) \arrow[d, "F'(\beta)"] \\
1 \arrow[ur, "\phi''_0\times\phi''_0"] \arrow[r, "\phi''_0\times\phi''_0"] \arrow[dr, "\phi''_0"'] & E(1)\times E(1) \arrow[r, "E(y)\times E(x)"] \arrow[d, "E_{1,1}"] \arrow[dr, phantom, "\phi''_{y,x} \cong"] & E(F(n))\times E(F(m)) \arrow[r, "\theta_n\times\theta_m"] \arrow[d, "E_{F(n),F(m)}"] & F'(H(n))\times F'(H(m)) \arrow[r, "\phi'_{H(n),H(m)}"] & F'(H(n)+H(m)) \arrow[d, "F'(H_{n,m})"] \\
 & E(1\times 1) \arrow[r, "E(y\times x)"] & E(F(n)\times F(m)) \arrow[r, "E(\phi_{n,m})"] & E(F(n+m)) \arrow[r, "\theta_{n+m}"] & F'(H(n+m))
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
 & E(1)\times E(1) \arrow[r, "E(x)\times E(y)"] \arrow[d, "E_{1,1}"] \arrow[dr, phantom, "\phi''_{x,y} \cong"] & E(F(m))\times E(F(n)) \arrow[r, "\theta_m\times\theta_n"] \arrow[d, "E_{F(m),F(n)}"] & F'(H(m))\times F'(H(n)) \arrow[r, "\phi'_{H(m),H(n)}"] & F'(H(m)+H(n)) \arrow[d, "F'(H_{m,n})"] \\
1 \arrow[ur, "\phi''_0\times\phi''_0"] \arrow[r, "\phi''_0"] \arrow[dr, "\phi''_0"'] & E(1\times 1) \arrow[r, "E(x\times y)"] \arrow[d, "E(\beta)"] \arrow[dr, phantom, "\cong"] & E(F(m)\times F(n)) \arrow[r, "E(\phi_{m,n})"] \arrow[d, "E(\beta)"] \arrow[dr, phantom, "E(\mu_{m,n}) \cong"] & E(F(m+n)) \arrow[r, "\theta_{m+n}"] \arrow[d, "E(F(\beta))"] & F'(H(m+n)) \arrow[d, "F'(H(\beta))"] \\
 & E(1\times 1) \arrow[r, "E(y\times x)"] & E(F(n)\times F(m)) \arrow[r, "E(\phi_{n,m})"] & E(F(n+m)) \arrow[r, "\theta_{n+m}"] & F'(H(n+m))
\end{tikzcd}$$

Diagram 4 of 7. Set up but not solved.

$$
\begin{tikzcd}
(\mathbb{H}(M)\odot\mathbb{H}(N))\odot\mathbb{H}(P) \arrow[r, "\alpha'"] \arrow[d, "\odot_{M,N}\odot 1"'] & \mathbb{H}(M)\odot(\mathbb{H}(N)\odot\mathbb{H}(P)) \arrow[d, "1\odot\odot_{N\odot P}"] \\
\mathbb{H}(M\odot N)\odot\mathbb{H}(P) \arrow[d, "\odot_{M\odot N,P}"'] & \mathbb{H}(M)\odot\mathbb{H}(N\odot P) \arrow[d, "\odot_{M,N\odot P}"] \\
\mathbb{H}((M\odot N)\odot P) \arrow[r, "\mathbb{H}(\alpha)"] & \mathbb{H}(M\odot(N\odot P))
\end{tikzcd}
$$

M is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
N is given by $b \xrightarrow{i'} n \xleftarrow{o'} c$ with $y \in F(n)$.
P is given by $c \xrightarrow{i''} p \xleftarrow{o''} d$ with $z \in F(p)$.
$\mathbb{H}(M)$ is given by $H(a) \xrightarrow{H(i)} H(m) \xleftarrow{H(o)} H(b)$ with $\bar{x} = \theta_m E(x)\phi_0 \in F'(H(m))$.
$\mathbb{H}(N)$ is given by $H(b) \xrightarrow{H(i')} H(n) \xleftarrow{H(o')} H(c)$ with $\bar{y} = \theta_n E(y)\phi_0 \in F'(H(n))$.
$\mathbb{H}(P)$ is given by $H(c) \xrightarrow{H(i'')} H(p) \xleftarrow{H(o'')} H(d)$ with $\bar{z} = \theta_p E(z)\phi_0 \in F'(H(p))$.
The map $\odot_{M,N}\colon \mathbb{H}(M)\odot\mathbb{H}(N) \to \mathbb{H}(M\odot N)$ is given by:

$$
\begin{tikzcd}[column sep=small]
 & & E(1)\times 1 \arrow[r, "1\times\phi_0"] \arrow[dd, "\rho"] & E(1)\times E(1) \arrow[r, "E(x)\times E(y)"] \arrow[dd, "E_{1,1}"] & E(F(m))\times E(F(n)) \arrow[r, "\theta_m\times\theta_n"] \arrow[dd, "E_{F(m),F(n)}"] & F'(H(m))\times F'(H(n)) \arrow[rr, "\phi'_{H(m),H(n)}"] & & F'(H(m)+H(n)) \arrow[rr, "F'(\psi)"] \arrow[dl, "F'(H_{m,n})"] & & F'(H(m)+_{H(b)}H(n)) \arrow[dd, "F'(\kappa)"] \\
1 \arrow[r, "\phi_0"] & E(1) \arrow[ur, "\rho^{-1}"] \arrow[dr, "1"'] & \arrow[r, phantom, "\underset{\cong}{\phi}"] & \arrow[r, phantom, "\underset{\cong}{E_{x_1,x_2}}"] & & & F'(H(m+n)) \arrow[drrr, "F'(H(\psi))"] & & \cong & \\
 & & E(1) \arrow[r, "E(\rho^{-1})"] & E(1\times 1) \arrow[r, "E(x\times y)"] & E(F(m)\times F(n)) \arrow[r, "E(\phi_{m,n})"] & E(F(m+n)) \arrow[ur, "\theta_{m+n}"] \arrow[rr, "E(F(\psi))"] & & E(F(m+_b n)) \arrow[rr, "\theta_{m+_b n}"] & & F'(H(m+_b n))
\end{tikzcd}
$$

Right and then down:

$$
\begin{tikzcd}[column sep=tiny]
(E(1)\times E(1))\times E(1) \arrow[rr, "(E(x)\times E(y))\times E(z)"] \arrow[ddd, "\alpha"] & & (E(F(m))\times E(F(n)))E(F(p)) \arrow[rr, "(\theta_m\times\theta_n)\times\theta_p"] & & (F'(H(m))\times F'(H(n)))F('H(p)) \arrow[rr, "\phi'\times 1"] \arrow[ddd, "\alpha"] & & F'(H(m)+H(n))\times F'(H(p)) \arrow[rr, "F'(\psi)\times 1"] \arrow[drr, "\phi'"] & & F'(H(m)+_{H(b)}H(n))\times F'(H(p)) \arrow[rr, "\phi'"] \arrow[d, phantom, "\underset{\cong}{\phi'_{\psi,1}}"] & & F'((H(m)+_{H(b)}H(n))+H(p)) \arrow[rr, "F'(\psi)"] & & F'((H(m)+_{H(b)}H(n))+_{H(c)}H(p)) \arrow[ddd, "F'(\kappa)"] \\
 & & & & & & \arrow[d, phantom, "\underset{\cong}{\phi'}"] & & F'((H(m)+H(n))+H(p)) \arrow[urr, "F'(\psi+1)"'] \arrow[d, "F'(\alpha)"] & & \arrow[d, phantom, "\cong"] & & \\
 & & & & & & & & F'(H(m)+(H(n)+H(p))) \arrow[drr, "F'(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi'_{1,\psi}}"] & & & & \\
E(1)\times(E(1)\times E(1)) \arrow[rr, "E(x)\times(E(y)E(z))"] \arrow[dd, "1\times E_{1,1}"] & & E(F(m))\times(E(F(n))\times E(F(p))) \arrow[rr, "\theta_m\times(\theta_n\times\theta p)"] \arrow[dd, "1\times E_{F(n),F(p)}"] & & F'(H(m))\times(F'(H(n))\times F'(H(p))) \arrow[rr, "1\times\phi'"] & & F'(H(m))\times F'(H(n)+H(p)) \arrow[rr, "1\times F'(\psi)"] \arrow[urr, "\phi'"] \arrow[dl, "1\times F'(H_{n,p})"] & & F'(H(m))\times F'(H(n)+_{H(c)}H(p)) \arrow[rr, "\phi'"] \arrow[dd, "1\times F'(\kappa)"] & & F'(H(m)+(H(n)+_{H(c)}H(p))) \arrow[rr, "F'(\psi)"] \arrow[dd, "F'(1+\kappa)"] & & F'(H(m)+_{H(b)}(H(n)+_{H(c)}H(p))) \arrow[dd, "F'(1+_1\kappa)"] \\
\arrow[rr, phantom, "\underset{\cong}{1\times\phi''_{y,z}}"] & & & & & F'(H(m))\times F'(H(n+p)) \arrow[drrr, "1\times F'(H(\psi))"] & \arrow[rr, phantom, "\cong"] & & \arrow[rr, phantom, "\underset{\cong}{\phi'_{1,\kappa}}"] & & \arrow[rr, phantom, "\cong"] & & \\
E(1)\times(E(1\times 1)) \arrow[rr, "E(x)\times E(y\times z)"] \arrow[dd, "E_{1,1\times 1}"] & & E(F(m))\times E(F(n)\times F(p)) \arrow[rr, "1\times E(\phi)"] \arrow[dd, "E_{F(m),F(n)\times F(p)}"] & & E(F(m))\times E(F(n+p)) \arrow[rr, "1\times E(F(\psi))"] \arrow[ur, "\theta_m\times\theta_{n+p}"] \arrow[dd, "E_{F(m),F(n+p)}"] & & E(F(m))\times E(F(n+_c p)) \arrow[rr, "\theta_m\times\theta_{n+_c p}"] \arrow[dd, "E_{F(m),F(n+_c p)}"] & & F'(H(m))\times F'(H(n+_c p)) \arrow[rr, "\phi'"] & & F'(H(m)+H(n+_c p)) \arrow[rr, "F'(\psi)"] \arrow[dl, "F'(H_{m,n+_c p})"] & & F'(H(m)+_{H(b)}H(n+_c p)) \arrow[dd, "F'(\kappa)"] \\
\arrow[rr, phantom, "\underset{\cong}{\phi''_{x,y\times z}}"] & & \arrow[rr, phantom, "\underset{\cong}{\phi''_{1,\phi}}"] & & \arrow[rr, phantom, "\underset{\cong}{\phi''_{1,F(\psi)}}"] & & & & & F'(H(m+(n+_c p))) \arrow[drrr, "F'(H(\psi))"] & \arrow[rr, phantom, "\cong"] & & \\
E(1\times(1\times 1)) \arrow[rr, "E(x\times(y\times z))"] & & E(F(m)\times(F(n)\times F(p))) \arrow[rr, "E(1\times\phi)"] & & E(F(m)\times F(n+p)) \arrow[rr, "E(1\times F(\psi))"] & & E(F(m)\times F(n+_c p)) \arrow[rr, "E(\phi)"] & & E(F(m+(n+_c p))) \arrow[rr, "E(F(\psi))"] \arrow[ur, "\theta_{m+(n+_c p)}"] & & E(F(m+_b+(n+_c p))) \arrow[rr, "\theta_{m+_b(n+_c p)}"] & & F'(H(m+_b+(n+_c p)))
\end{tikzcd}
$$

Down and then right:

$$
\begin{tikzcd}[column sep=tiny]
(E(1)\times E(1))\times E(1) \arrow[rr, "(E(x)\times E(y))\times E(z)"] \arrow[dd, "E_{1,1}\times 1"] \arrow[ddrr, phantom, "\underset{\cong}{\phi''_{x\times y,z}}"] & & (E(F(m))\times E(F(n)))E(F(p)) \arrow[rr, "(\theta_m\times\theta_n)\times\theta_p"] \arrow[dd, "E_{F(m),F(n)}\times 1"] & & (F'(H(m))\times F'(H(n)))F('H(p)) \arrow[rr, "\phi'\times 1"] & & F'(H(m)+H(n))\times F'(H(p)) \arrow[rr, "F'(\psi)\times 1"] \arrow[dl, "F'(H_{m,n})\times 1"] \arrow[ddrr, phantom, "\cong"] & & F'(H(m)+_{H(b)}H(n))\times F'(H(p)) \arrow[rr, "\phi'"] \arrow[dd, "F'(\kappa)\times 1"] \arrow[ddrr, phantom, "\underset{\cong}{\phi'_{\kappa,1}}"] & & F'((H(m)+_{H(b)}H(n))+H(p)) \arrow[rr, "F'(\psi)"] \arrow[dd, "F'(\kappa+1)"] \arrow[ddrr, phantom, "\cong"] & & F'((H(m)+_{H(b)}H(n))+_{H(c)}H(p)) \arrow[dd, "F'(\kappa+_1 1)"] \\
& & & & & F'(H(m+n))\times F'(H(p)) \arrow[drrr, "F'(\psi)\times 1"] & & & & & & & \\
E(1\times 1)\times E(1) \arrow[rr, "E(x\times y)\times E(z)"] \arrow[dd, "E_{1\times 1,1}"] \arrow[ddrr, phantom, "\underset{\cong}{\phi''_{x\times y,z}}"] & & E(F(m)\times F(n))\times E(F(p)) \arrow[rr, "E(\phi)\times 1"] \arrow[dd, "E_{F(m)\times F(n),F(p)}"] \arrow[ddrr, phantom, "\underset{\cong}{\phi''_{\phi,1}}"] & & E(F(m+n))\times E(F(p)) \arrow[rr, "E(F(\psi))\times 1"] \arrow[ur, "\theta_{m+n}\times\theta_p"] \arrow[dd, "E_{F(m+n),F(p)}"] \arrow[ddrr, phantom, "\underset{\cong}{\phi''_{F(\psi),1}}"] & & E(F(m+_b n))\times E(F(p)) \arrow[rr, "\theta_{m+_b n}\times\theta_p"] \arrow[dd, "E_{F(m+_b n),F(p)}"] & & F'(H(m+_b n))\times F'(H(p)) \arrow[rr, "\phi'"] & & F'(H(m+_b n)+H(p)) \arrow[rr, "F'(\psi)"] \arrow[dl, "F'(H_{m+_b n,p})"] \arrow[ddrr, phantom, "\cong"] & & F'(H(m+_b n)+_{H(c)}H(p)) \arrow[dd, "F'(\kappa)"] \\
& & & & & & & & & F'(H((m+_b n)+p)) \arrow[drrr, "F'(H(\psi))"] & & & \\
E((1\times 1)\times 1) \arrow[rr, "E((x\times y)\times z)"] \arrow[ddd, "E(\alpha)"] \arrow[dddrr, phantom, "\cong"] & & E((F(m)\times F(n))\times F(p)) \arrow[rr, "E(\phi\times 1)"] \arrow[ddd, "E(\alpha)"] & & E(F(m+n)\times F(p)) \arrow[rr, "E(F(\psi)\times 1)"] \arrow[drr, "E(\phi)"] \arrow[ddd, phantom, "\underset{\cong}{E(\phi)}"] & & E(F(m+_b n)\times F(p)) \arrow[rr, "E(\phi)"] \arrow[d, phantom, "\underset{\cong}{E(\phi_{\psi,1})}"] & & E(F((m+_b n)+p)) \arrow[rr, "E(F(\psi))"] \arrow[ur, "\theta_{(m+_b n)+p}"] \arrow[ddd, phantom, "E(\cong)"] & & E(F((m+_b n)+_c p)) \arrow[rr, "\theta_{(m+_b n)+_c p}"] \arrow[ddd, "E(F(\kappa))"] & & F'(H((m+_b n)+_c p)) \arrow[ddd, "F'(H(\kappa))"] \\
& & & & & & E(F((m+n)+p)) \arrow[urr, "E(F(\psi+1))"] \arrow[d, "E(F(\alpha))"] & & & & & & \\
& & & & & & E(F(m+(n+p))) \arrow[drr, "E(F(1+\psi))"] \arrow[d, phantom, "\underset{\cong}{E(\phi_{1,\psi})}"] & & & & & & \\
E(1\times(1\times 1)) \arrow[rr, "E(x\times(y\times z))"] & & E(F(m)\times(F(n)\times F(p))) \arrow[rr, "E(1\times\phi)"] & & E(F(m)\times F(n+p)) \arrow[rr, "E(1\times F(\psi))"] \arrow[urr, "E(\phi)"] & & E(F(m)\times F(n+_c p)) \arrow[rr, "E(\phi)"] & & E(F(m+(n+_c p))) \arrow[rr, "E(F(\psi))"] & & E(F(m+_b+(n+_c p))) \arrow[rr, "\theta_{m+_b(n+_c p)}"] & & F'(H(m+_b+(n+_c p)))
\end{tikzcd}
$$

Diagram 5 of 7. Not set up and not solved.

$$\begin{tikzcd}
U_{\mathbb{H}(b)} \odot \mathbb{H}(M) \arrow[r, "\mathbb{H}_U \odot 1"] \arrow[d, "\ell"'] & \mathbb{H}(U_b) \odot \mathbb{H}(M) \arrow[d, "\odot_{U_b,M}"] \\
\mathbb{H}(M) & \mathbb{H}(U_b \odot M) \arrow[l, "\mathbb{H}(\ell)"']
\end{tikzcd}$$

M is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
U_b is given by $b \xrightarrow{1} b \xleftarrow{1} b$ with $\perp_b \in F(b)$.
$\mathbb{H}(M)$ is given by $H(a) \xrightarrow{H(i)} H(m) \xleftarrow{H(o)} H(b)$ with $\bar{x} = \theta_m E(x)\phi_0'' \in F'(H(m))$.
$U_{\mathbb{H}(b)}$ is given by $H(b) \xrightarrow{1} H(b) \xleftarrow{1} H(b)$ with $\perp_{\mathbb{H}(b)} \in F'(H(b))$.
$\mathbb{H}(U_b)$ is given by $H(b) \xrightarrow{1} H(b) \xleftarrow{1} H(b)$ with $\bar{\perp}_b = \theta_b E(\perp_b)\phi_0'' \in F'(H(b))$.
The map $\mathbb{H}_U \colon U_{\mathbb{H}(b)} \to \mathbb{H}(U_b)$ is given by:

$$\begin{tikzcd}
 & F'(0') \arrow[rrr, "F'(!_{H(b)})"] \arrow[dr, "F'(\phi_0''')"] & \arrow[r, phantom, "\cong"] & & F'(H(b)) \arrow[dd, "1"] \\
1 \arrow[ur, "\phi_0'"] \arrow[dr, "\phi_0''"'] & & F'(H(0)) \arrow[urr, "F'(H(!_b))"] & & \\
 & E(1) \arrow[r, "E(\phi_0)"] & E(F(0)) \arrow[u, "\theta_0"] \arrow[r, "E(F(!_b))"] & E(F(b)) \arrow[r, "\theta_b"] & F'(H(b))
\end{tikzcd}$$

The left commuting region is a naturality triangle of units for a (strict) monoidal natural isomorphism and the right commuting region is an ordinary naturality square for a (strict) monoidal natural isomorphism.
Down:

$$\begin{tikzcd}
 & F'(0') \times E(1) \arrow[r, "1 \times E(x)"] & F'(0') \times E(F(m)) \arrow[r, "1 \times \theta_m"] & F'(0') \times F'(H(m)) \arrow[r, "F'(!_{H(b)}) \times 1"] \arrow[dr, "\phi'_{0',H(m)}"] & F'(H(b)) \times F'(H(m)) \arrow[r, "\phi'_{H(b),H(m)}"] \arrow[d, phantom, "\phi'_{!_{H(b)},1} \cong"] & F'(H(b)+H(m)) \arrow[r, "F'(\psi)"] \arrow[dr, phantom, "\cong"] & F'(H(b)+_{H(b)}H(m)) \arrow[dd, "F'(\kappa)"] \\
1 \arrow[ur, "\phi_0' \times \phi_0''"] \arrow[dr, "\phi_0''"'] & & \arrow[r, phantom, "\lambda \cong"] & & F'(0'+H(m)) \arrow[ur, "F'(!_{H(b)}+1)"] \arrow[drr, "F(\lambda)"] & & \\
 & E(1) \arrow[rr, "E(x)"] & & E(F(m)) \arrow[rrr, "\theta_m"] & & & F'(H(m))
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
 & F'(0')\times E(1) \arrow[r, "1\times E(x)"] \arrow[dr, "F'(\phi'''_0)\times E(x)"] & F'(0')\times E(F(m)) \arrow[r, "1\times\theta_m"] \arrow[d, phantom, "\cong"] & F'(0')\times F'(H(m)) \arrow[r, "F'(!_{H(b)})\times 1"] & F'(H(b))\times F'(H(m)) \arrow[r, "\phi'_{H(b),H(m)}"] \arrow[dd, "1"] & F'(H(b)+H(m)) \arrow[r, "F'(\psi)"] \arrow[dd, "1"] & F'(H(b)+_{H(b)}H(m)) \arrow[dd, "1"] \\
 & & F'(H(0))\times E(F(m)) \arrow[urr, "F'(H(!_b))\times\theta_m"] & & & & \\
 & E(1)\times E(1) \arrow[r, "E(\phi)\times E(x)"] & E(F(0))\times E(F(m)) \arrow[r, "E(F(!_b))\times 1"] \arrow[u, "\theta_0\times 1"] & E(F(b))\times E(F(m)) \arrow[r, "\theta_b\times\theta_m"] & F'(H(b))\times F'(H(m)) \arrow[r, "\phi'_{H(b),H(m)}"] & F'(H(b)+H(m)) \arrow[r, "F'(\psi)"] & F'(H(b)+_{H(b)}H(m)) \\
1 \arrow[uuur, "\phi'_0\times\phi''_0"] \arrow[ur, "\phi''_0\times\phi''_0"] \arrow[dr] \arrow[ddr] & & & & & & \\
 & C1 \arrow[r] & C2 \arrow[r] & C3 \arrow[r] & C4 \arrow[r] & C5 \arrow[r] & C6 \\
 & D1 \arrow[r] & D2 \arrow[r] & D3 \arrow[r] & D4 \arrow[r] & D5 \arrow[r] & D6
\end{tikzcd}$$

Diagram 6 of 7. Not set up and not solved.

Diagram 7 of 7. Not set up and not solved.

4. Some useful maps

Given $a \in (\mathsf{A}, +, 0)$, the map $U_{\lambda_a} \colon U_{0+a} \to U_a$ is given by:

$$
\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] & & F(0+a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] & & \cong & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a)
\end{tikzcd}
$$

where the $\cong$ is given by pseudonaturality of F: we have a unique map in $!_a \colon 0 \to a$ in A but also a map $\lambda_a \circ !_{0+a} \colon 0 \to a$ where λ_a is the left unitor of $(\mathsf{A}, +, 0)$, and so $F(!_a) = F(\lambda_a \circ !_{0+a}) \cong F(\lambda_a) F(!_{0+a})$.

The left unitor $\lambda'_{U_a} \colon U_0 \otimes U_a \to U_a$ is given by:

$$
\begin{tikzcd}
 & F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[dd, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0+a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi \times \phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "\overset{\phi}{\cong}"] & & {} & \cong \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a)
\end{tikzcd}
$$

where the $\cong$ in the lower right is the same as the one in the first diagram.

For an arbitrary M, the left unitor $\lambda'_M \colon U_0 \otimes M \to M$ is given by:

$$
\begin{tikzcd}
 & F(0){\times}F(m) \arrow[r, "F(!_0) \times 1"] \arrow[dr, "\phi_{0,m}"] & F(0){\times}F(m) \arrow[r, "\phi_{0,m}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+m) \arrow[dd, "F(\lambda_m)"] \\
1 \arrow[ur, bend left, "\phi_0 \times x"] \arrow[dr, bend right, "x"'] & \overset{\lambda_M}{\cong} & F(0+m) \arrow[ur, "F(!_0 + 1)"] \arrow[dr, "F(\lambda_m)"] & \cong \\
 & F(m) \arrow[rr, "\mathrm{id}"] & & F(m)
\end{tikzcd}
$$

For an arbitrary M given by $a \rightarrow (m, x) \leftarrow b$, the map $\lambda_M \colon U_b \odot M \to M$ is given by:

$$
\begin{tikzcd}
 & F(m){\times}F(0) \arrow[r, "1 \times F(!_b)"] \arrow[dr, "\phi_{m,0}"] & F(m){\times}F(b) \arrow[r, "\phi_{m,b}"] \arrow[d, phantom, "\overset{\phi_{1,!_b}}{\cong}"] & F(m+b) \arrow[r, "F(\psi)"] & F(m+_b b) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "x \times \phi_0"] \arrow[dr, bend right, "x"'] & \overset{\lambda_M}{\cong} & F(m+0) \arrow[ur, "F(1+!_b)"] \arrow[drr, "F(\lambda_m)"] & \cong & \\
 & F(m) \arrow[rrr, "\mathrm{id}"] & & & F(m)
\end{tikzcd}
$$

In particular, if $M = U_0$ above, then the map $\lambda_{U_0} \colon U_0 \odot U_0 \to U_0$ is given by:

$$
\begin{tikzcd}
 & F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_0)"] \arrow[dr, "\phi_{0,0}"] & F(0){\times}F(0) \arrow[r, "\phi_{0,0}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"] & F(0+0) \arrow[r, "F(\psi)"] & F(0+_0 0) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0 \times \phi_0"] \arrow[dr, bend right, "\phi_0"'] & \overset{\lambda_M}{\cong} & F(0+0) \arrow[ur, "F(!_0+!_0)"] \arrow[drr, "F(\lambda_0)"] & \cong & \\
 & F(0) \arrow[rrr, "\mathrm{id}"] & & & F(0)
\end{tikzcd}
$$