1. Proof of Theorem 2.2

There are 11 diagrams in Shulman's definition of symmetric monoidal double category, which we check now for decorated cospan double categories.

Diagram 1 of 11. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \to m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
N_3 is given by $c' \to n_3 \leftarrow d'$ with $y_3 \in F(n_3)$.

$$\begin{tikzcd}
((M_1 \otimes N_1) \odot (M_2 \otimes N_2)) \odot (M_3 \otimes N_3) \arrow[r, "\chi \odot 1"] \arrow[d, "\alpha"'] & ((M_1 \odot M_2) \otimes (N_1 \odot N_2)) \odot (M_3 \otimes N_3) \arrow[d, "\chi"] \\
(M_1 \otimes N_1) \odot ((M_2 \otimes N_2) \odot (M_3 \otimes N_3)) \arrow[d, "1 \odot \chi"'] & ((M_1 \odot M_2) \odot M_3) \otimes ((N_1 \odot N_2) \odot N_3) \arrow[d, "\alpha \otimes \alpha"] \\
(M_1 \otimes N_1) \odot ((M_2 \odot M_3) \otimes (N_2 \odot N_3)) \arrow[r, "\chi"] & (M_1 \odot (M_2 \odot M_3)) \otimes (N_1 \odot (N_2 \odot N_3))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{(m_1+_b m_2)+(n_1+_{b'}n_2),m_3+n_3}(\phi_{m_1+_b m_2,n_1+_{b'}n_2}((F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))),\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+_b m_2)+(n_1+_{b'}n_2))+_{c+c'}(m_3+n_3))$$

$$\phi_{(m_1+_b m_2)+_c m_3,(n_1+_{b'}n_2+_{c'}n_3}((F(\psi\phi_{m_1+_b m_2,m_3}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),x_3),(F(\psi)\phi_{n_1+_{b'}n_2,n_3}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),y_3))) \in F(((m_1+_b m_2)+_c m_3)+((n_1+_{b'}n_2)+_{c'}n_3))$$

$$\phi_{m_1+_b(m_2+_c m_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+_c m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+_b(m_2+_c m_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

(2) Down and then right:

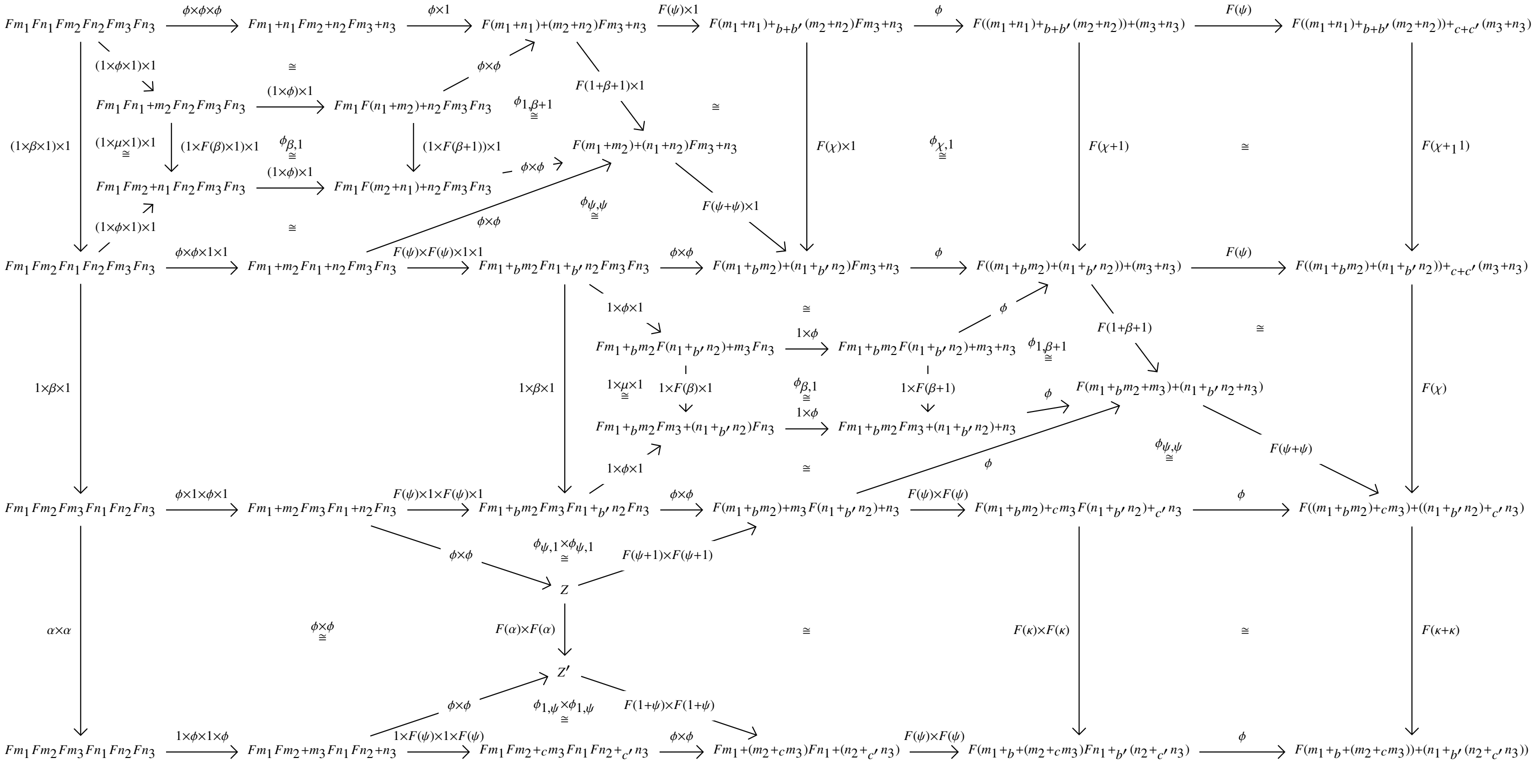
$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{m_1+n_1,(m_2+n_2)+_{c+c'}(m_3+n_3)}((\phi_{m_1,n_1}(x_1,y_1),F(\psi)\phi_{m_2+n_2,m_3+n_3}(\phi_{m_2,n_2}(x_2,y_2),\phi_{m_3,n_3}(x_3,y_3)))) \in F((m_1+n_1)+_{b+b'}((m_2+n_2)+_{c+c'}(m_3+n_3)))$$

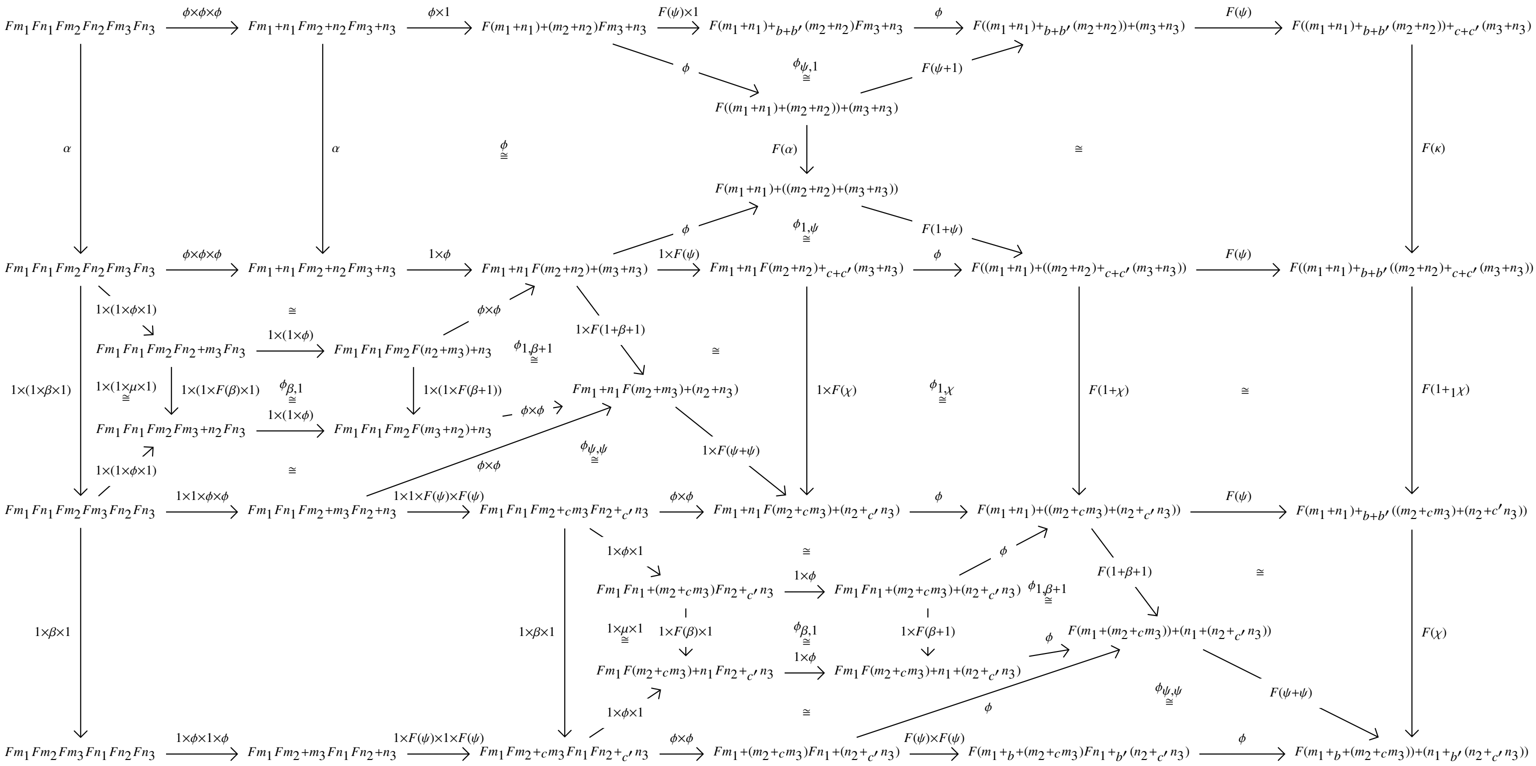
$$F(\psi)\phi_{m_1+n_1,(m_2+_c m_3)+(n_2+_{c'}n_3)}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2+_c m_3,n_2+_{c'}n_3}((F\psi)\phi_{m_2,m_3}(x_2,x_3),F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+n_1)+_{b+b'}((m_2+_c m_3)+(n_2+_{c'}n_3)))$$

$$\phi_{m_1+_b(m_2+_c m_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+_c m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+_b(m_2+_c m_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

Right and then down (omitting morphisms emanating out of 1 on the left due to space restrictions):



Down and then right (omitting morphisms emanating out of 1 on the left due to space restrictions):



Diagrams 2 and 3 of 11. Set up but not solved.

$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} a'$ with $x \in F(m)$.

$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} b'$ with $y \in F(n)$.

$U_a \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{\mathrm{id}} a \xleftarrow{\mathrm{id}} a$ with $\perp_a \in F(a)$.

$a{+}b \to (a{+}b){+}_{a+b}(m{+}n) \leftarrow a'{+}b'$

$F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\perp_{a+b}) \in F((a{+}b){+}_{a+b}(m{+}n))$

$$\begin{tikzcd}
(M\otimes N)\odot(U_{a+b}) \arrow[r, "\mathrm{id}\odot\mu"] \arrow[d, "r"'] & (M\otimes N)\odot(U_a\otimes U_b) \arrow[d, "\chi"] \\
M\otimes N & (M\odot U_a)\otimes(N\odot U_b) \arrow[l, "r\otimes r"']
\end{tikzcd}$$

$a{+}b \to (a{+}b){+}_{a+b}(m{+}n) \leftarrow a'{+}b'$

$F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\phi_{a,b}(\perp_a,\perp_b)) \in F((a{+}b){+}_{a+b}(m{+}n))$

$a{+}b \to m{+}n \leftarrow a'{+}b'$

$\phi_{m,n}(x,y) \in F(m{+}n)$

$a{+}b \to (a{+}_a m){+}(b{+}_b n) \leftarrow a'{+}b'$

$\phi_{a+_a m,b+_b n}(F(\psi)\phi_{a,m}(\perp_a,x),F(\psi)\phi_{b,n}(\perp_b,y)) \in F((a{+}_a m){+}(b{+}_b n))$

Down:

$$\begin{tikzcd}
& F(0)\times(F(m)\times F(n)) \arrow[rr, "1\times\phi_{m,n}"] & & F(0)\times F(m+n) \arrow[r, "F(!_{a+b})\times 1"] \arrow[dr, "\phi_{0,m+n}"'] & F(a+b)\times F(m+n) \arrow[r, "\phi_{a+b,m+n}"] \arrow[d, phantom, "\phi_{!_{a+b},1} \cong"] & F((a+b)+(m+n)) \arrow[r, "F(\psi)"] & F((a+b)+_{a+b}(m+n)) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0\times(x\times y)"] \arrow[dr, bend right, "x\times y"'] & & \lambda_{M\otimes N} \cong & & F(0+(m+n)) \arrow[ur, "F(!_{a+b}+1)"'] \arrow[ddrr, "F(\ell)"'] & \cong & \\
& F(m)\times F(n) \arrow[rrrrr, "\phi_{m,n}"] & & & & & F(m+n)
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
& F(0)\times(F(m)\times F(n)) \arrow[rr, "1\times\phi_{m,n}"] & & F(0)\times F(m+n) \arrow[rr, "F(!_{a+b})\times 1"] & & F(a+b)\times F(m+n) \arrow[r, "\phi_{a+b,m+n}"] & F((a+b)+(m+n)) \arrow[r, "F(\psi)"] & F((a+b)+_{a+b}(m+n)) \arrow[d, "F(\mathrm{id})"] \\
\phi \cong & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[u, "\phi_{0,0}\times 1"] \arrow[rr, "(F(!_a)\times F(!_b))\times 1"] & \phi_{!_a,!_b} \cong & (F(a)\times F(b))\times(F(m)\times F(n)) \arrow[urr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[rr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[d, "1\times\phi\times 1"] \arrow[ddd, bend right, "1\times\beta\times 1"'] & \cong & F(a+b)\times F(m+n) \arrow[r, "\phi_{a+b,m+n}"] & F((a+b)+(m+n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"'] & F((a+b)+_{a+b}(m+n)) \arrow[ddd, "F(\kappa')"] \\
& & 1\times\mu\times 1 \cong & F(a)\times F(b+m)\times F(n) \arrow[r, "1\times\phi"] \arrow[d, "1\times F(\beta)\times 1"'] & F(a)\times F((b+m)+n) \arrow[urr, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[l, phantom, "\phi_{\beta,1} \cong"] & \phi_{1,\beta+1} \cong & & \cong \\
1 \arrow[uuur, bend left, "\phi_0\times(x\times y)"] \arrow[uur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "(\phi_0\times x)\times(\phi_0\times y)"'] \arrow[dddr, bend right, "x\times y"'] & & & F(a)\times F(m+b)\times F(n) \arrow[r, "1\times\phi"] & F(a)\times F((m+b)+n) \arrow[r, "\phi"] & F((a+m)+(b+n)) \arrow[drr, bend left, "F(\psi+\psi)"] & \phi_{\psi,\psi} \cong & \\
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rr, "(F(!_a)\times 1)\times(F(!_b)\times 1)"] \arrow[drr, "\phi_{0,m}\times\phi_{0,n}"'] & & (F(a)\times F(m))\times(F(b)\times F(n)) \arrow[u, "1\times\phi\times 1"] \arrow[rr, "\phi_{a,m}\times\phi_{b,n}"] & \cong & F(a+m)\times F(b+n) \arrow[u, "\phi\times\phi"] \arrow[r, "F(\psi)\times F(\psi)"] & F(a+_a m)\times F(b+_b n) \arrow[r, "\phi_{a+_a m,b+_b n}"] & F((a+_a m)+(b+_b n)) \arrow[dd, "F(\kappa'')"] \\
& \lambda_M\times\lambda_N \cong & & F(0+m)\times F(0+n) \arrow[urr, "F(!_a+1)\times F(!_b+1)"] \arrow[rr, "F(\ell)\times F(\ell)"'] \arrow[u, phantom, "\phi_{!_a,1}\times\phi_{!_b,1} \cong"] & & F(m)\times(n) \arrow[drr, "\phi_{m,n}"'] & \cong & \\
& F(m)\times F(n) \arrow[rrrrrr, "\phi_{m,n}"] & & & & & & F(m+n)
\end{tikzcd}$$

Diagram 4 of 11. Set up but not solved.

M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
P_1 is given by $a'' \to p_1 \leftarrow b''$ with $z_1 \in F(p_1)$.
P_2 is given by $b'' \to p_2 \leftarrow c''$ with $z_2 \in F(p_2)$.

$$\begin{tikzcd}
((M_1 \otimes N_1) \otimes P_1) \odot ((M_2 \otimes N_2) \otimes P_2) \arrow[r, "\alpha \odot \alpha"] \arrow[d, "\chi"'] & (M_1 \otimes (N_1 \otimes P_1)) \odot (M_2 \otimes (N_2 \otimes P_2)) \arrow[d, "\chi"] \\
((M_1 \otimes N_1) \odot (M_2 \otimes N_2)) \otimes (P_1 \odot P_2) \arrow[d, "\chi \otimes 1"'] & (M_1 \odot M_2) \otimes ((N_1 \otimes P_1) \odot (N_2 \otimes P_2)) \arrow[d, "1 \otimes \chi"] \\
((M_1 \odot M_2) \otimes (N_1 \odot N_2)) \otimes (P_1 \odot P_2) \arrow[r, "\alpha"] & (M_1 \odot M_2) \otimes ((N_1 \odot N_2) \otimes (P_1 \odot P_2))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2)) \in F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$F(\psi)\phi_{m_1+(n_1+p_1),m_2+(n_2+p_2)}(\phi_{m_1,n_1+p_1}(x_1,\phi_{n_1,p_1}(y_1,z_1)),\phi_{m_2,n_2+p_2}(x_2,\phi_{n_2,p_2}(y_2,z_2))) \in F((m_1+(n_1+p_1))+_{b+(b'+b'')}(m_2+(n_2+p_2)))$$

$$\phi_{m_1+_b m_2,(n_1+p_1)+_{b'+b''}(n_2+p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)(\phi_{n_1+p_1,n_2+p_2}(\phi_{n_1,p_1}(y_1,z_1),\phi_{n_2,p_2}(y_2,z_2))) \in F((m_1+_b m_2)+((n_1+p_1)+_{b'+b''}(n_2+p_2)))$$

$$\phi_{m_1+_b m_2,(n_1+_{b'} n_2)+(p_1+_{b''} p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'} n_2,p_1+_{b''} p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2))) \in F((m_1+_b m_2)+((n_1+_{b'} n_2)+(p_1+_{b''} p_2)))$$

(2) Down and then right:

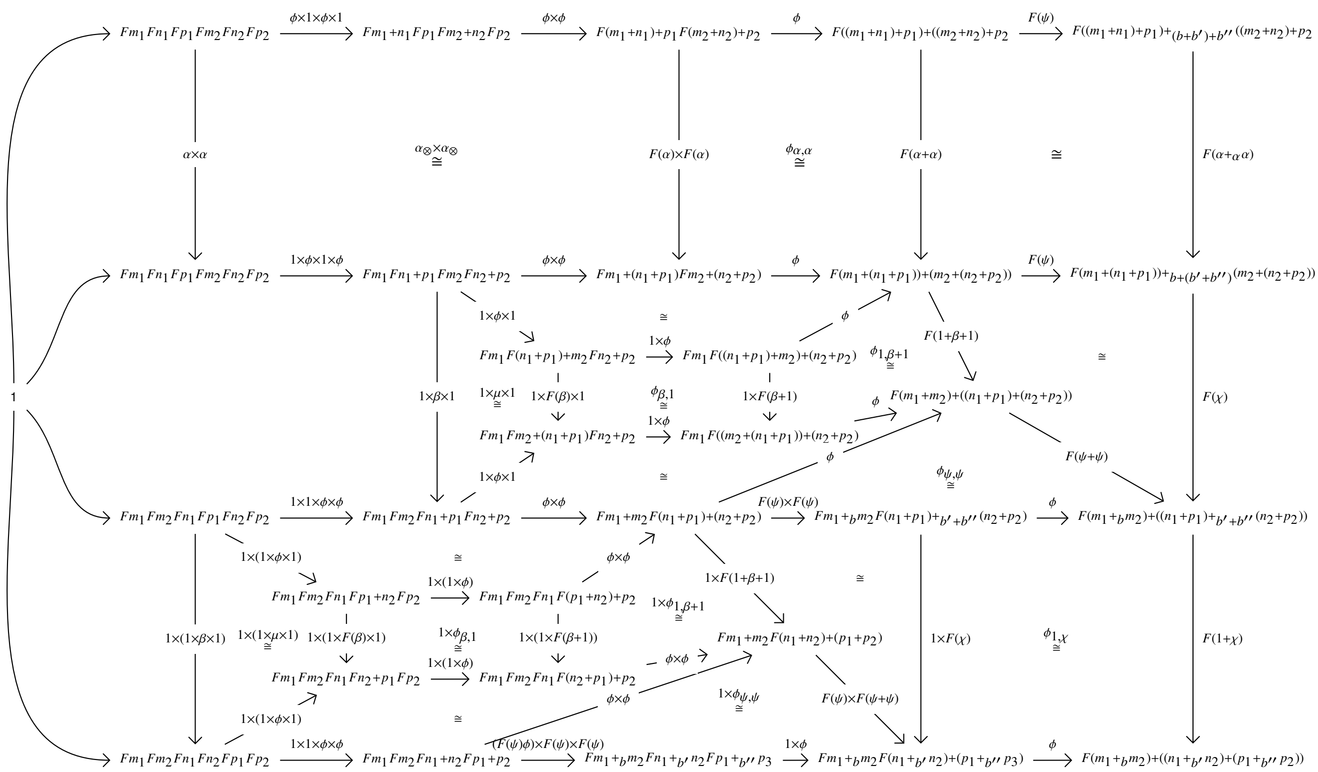
$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2)) \in F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),p_1+_{b''} p_2}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+(p_1+_{b''} p_2))$$

$$\phi_{(m_1+_b m_2)+(n_1+_{b'} n_2),p_1+_{b''} p_2}(\phi_{m_1+_b m_2,n_1+_{b'} n_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2)) \in F(((m_1+_b m_2)+(n_1+_{b'} n_2))+(p_1+_{b''} p_2))$$

$$\phi_{m_1+_b m_2,(n_1+_{b'} n_2)+(p_1+_{b''} p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'} n_2,p_1+_{b''} p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2))) \in F((m_1+_b m_2)+((n_1+_{b'} n_2)+(p_1+_{b''} p_2)))$$

Right and then down:



Down and then right:

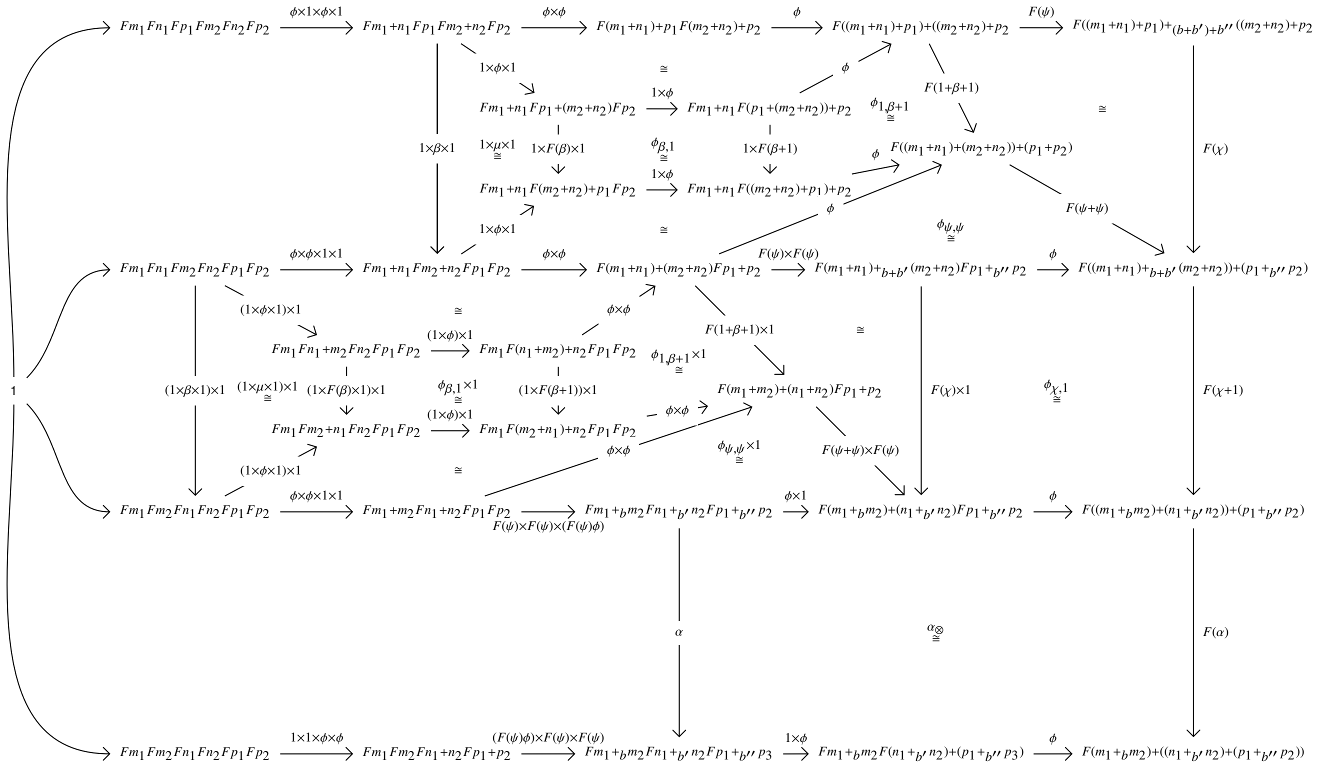


Diagram 5 of 11. Set up but not solved.

$$a, b, c \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
\perp_{(a+b)+c} \in F((a+b)+c) & U_{(a+b)+c} \arrow[r, "\mu_\beta"] \arrow[d, "\mu"] & U_{a+(b+c)} \arrow[d, "\mu"] & \perp_{a+(b+c)} \in F(a+(b+c)) \\
\phi_{a+b,c}(\perp_{a+b}, \perp_c) \in F((a+b)+c) & U_{a+b} \otimes U_c \arrow[d, "\mu"] & U_a \otimes U_{b+c} \arrow[d, "\mu"] & \phi_{a,b+c}(\perp_a, \perp_{b+c}) \in F(a+(b+c)) \\
\phi_{a+b,c}(\phi_{a,b}(\perp_a, \perp_b), \perp_c) \in F((a+b)+c) & (U_a \otimes U_b) \otimes U_c \arrow[r, "\beta'"] & U_a \otimes (U_b \otimes U_c) & \phi_{a,b+c}(\perp_a, \phi_{b,c}(\perp_b, \perp_c)) \in F(a+(b+c))
\end{tikzcd}$$

All the left cospans are $(a+b)+c \to (a+b)+c \leftarrow (a+b)+c$ and all the right cospans are $a+(b+c) \to a+(b+c) \leftarrow a+(b+c)$.
Right and then down:

$$\begin{tikzcd}
 & F(0) \arrow[rrr, "F(!_{(a+b)+c})"] \arrow[drrr, phantom, "\cong"] & & & F((a+b)+c) \arrow[d, "F(\alpha)"] \\
1 \arrow[ur, bend left, "\phi_0"] \arrow[r, "\phi_0"] \arrow[dr, "\phi_0 \times \phi_0"'] \arrow[ddr, bend right, "\phi_0\times(\phi_0\times\phi_0)"'] & F(0) \arrow[rrr, "F(!_{a+(b+c)})"] \arrow[drrr, phantom, "\overset{\phi_{!_a,!_{b+c}}}{\cong}"] & & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
 & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[ul, phantom, "\overset{\phi}{\cong}"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_c}}{\cong}"] \arrow[r, "F(!_a)\times F(!_{b+c})"] & F(a)\times F(b+c) \arrow[rr, "\phi_{a,b+c}"] & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
 & F(0)\times(F(0)\times F(0)) \arrow[u, "1\times\phi_{0,0}"] \arrow[uul, phantom, "\overset{\phi}{\cong}"] \arrow[r, "F(!_a)\times(F(!_b)\times F(!_c))"] & F(a)\times(F(b)\times F(c)) \arrow[u, "1\times\phi_{b+c}"] \arrow[r, "1\times\phi_{b+c}"] & F(a)\times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
 & F(0) \arrow[rrr, "F(!_{(a+b)+c})"] & & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi_0"] \arrow[r, "\phi_0"] \arrow[dr, "\phi_0^2\times\phi_0"'] \arrow[ddr, bend right, "\phi_0\times(\phi_0\times\phi_0)"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[uul, phantom, "\overset{\phi}{\cong}"] \arrow[urrr, phantom, "\overset{\phi_{!_{a+b},!_c}}{\cong}"] \arrow[r, "F(!_{a+b})\times F(!_c)"] & F(a+b)\times F(c) \arrow[rr, "\phi_{a+b,c}"] & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
 & (F(0)\times F(0))\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[ul, phantom, "\overset{\phi}{\cong}"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[d, "\alpha'"] \arrow[r, "(F(!_a)\times F(!_b))\times F(!_c)"] & (F(a)\times F(b))\times F(c) \arrow[u, "\phi_{a,b}\times 1"] \arrow[d, "\alpha'"] \arrow[r, "\phi_{a,b}\times 1"] & F(a+b)\times F(c) \arrow[r, "\phi_{a+b,c}"] \arrow[d, phantom, "\cong"] & F(a+(b+c)) \arrow[d, "F(\alpha)"] \\
 & F(0)\times(F(0)\times F(0)) \arrow[r, "F(!_a)\times(F(!_b)\times F(!_c))"] & F(a)\times(F(b)\times F(c)) \arrow[r, "1\times\phi_{b+c}"] & F(a)\times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Diagrams 6 and 8 of 11. Set up but not solved.

$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} c$ with $y \in F(n)$.
$U_0 \in F\mathbb{C}\mathbf{sp}$ is given by $0 \xrightarrow{!} 0 \xleftarrow{!} 0$ with $\bot_0 \in F(0)$.

$$
\begin{tikzcd}
\substack{0+a\to(0+m)+_{0+b}(0+n)\leftarrow 0+c \\ (\bot_0\oplus y)\odot(\bot_0\oplus x)\in F((0+m)+_{0+b}(0+n))} & (U_0\otimes N)\odot(U_0\otimes M) \arrow[r, "\chi"] \arrow[d, "\lambda_N \odot \lambda_M"'] & (U_0\odot U_0)\otimes(N\odot M) \arrow[d, "\lambda_{U_0}\otimes \mathrm{id}_{N\odot M}"] & \substack{0+a\to(0+_0 0)+(m+_b n)\leftarrow 0+c \\ (\bot_0\odot\bot_0)\oplus(y\odot x)\in F((0+_0 0)+(m+_b n))} \\
\substack{a\to m+_b n\leftarrow c \\ y\odot x\in F(m+_b n)} & N\odot M & U_0\otimes(N\odot M) \arrow[l, "\lambda_{N\odot M}"'] & \substack{0+a\to 0+(m+_b n)\leftarrow 0+c \\ \bot_0\oplus(y\odot x)\in F(0+(m+_b n))}
\end{tikzcd}
$$

Down:

$$
\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[ddr, "\phi_{0,m}\times\phi_{0,n}"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[dd, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dddd, "F(\ell)\times F(\ell)"] \arrow[ddddr, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[dddd, "F(\ell+\ell)"] \arrow[ddddr, phantom, "\cong"] & F((0+m)+_{0+b}(0+n)) \arrow[dddd, "F(\ell+_\ell \ell)"] \\
 & & & & & \\
1 \arrow[uur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ddr, bend right, "x\times y"'] \arrow[rr, phantom, "\overset{\lambda_M\times\lambda_N}{\cong}"] & & F(0+m)\times F(0+n) \arrow[uur, "F(!_0+1)\times F(!_0+1)"] \arrow[ddr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & & & \\
 & & & & & \\
 & F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}
$$

Right, down and then left:

$$
\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rr, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, "1\times\phi\times 1"] \arrow[ddd, bend right, "1\times\beta\times 1"'] & & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"] & F((0+m)+_{0+b}(0+n)) \arrow[ddd, "F(\chi)"] \\
 & & F(0)\times F(m+0)\times F(n) \arrow[r, "1\times\phi"] \arrow[d, "1\times F(\beta)\times 1"] & F(0)\times F((m+0)+n) \arrow[urr, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[r, phantom, "\overset{\phi_{1,\beta+1}}{\cong}"] & {} & {} \arrow[r, phantom, "\cong"] & {} \\
 & & F(0)\times F(0+m)\times F(n) \arrow[r, "1\times\phi"] & F(0)\times F((0+m)+n) \arrow[r, "\phi"] & F((0+0)+(m+n)) \arrow[drr, bend left, "F(\psi+\psi)"] \arrow[dr, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"] & & \\
 & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times 1)"'] \arrow[dr, "\phi_{0,0}\times 1"] & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[u, "1\times\phi\times 1"] \arrow[rr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"] & & F(0+0)\times F(m+n) \arrow[u, "\phi"] \arrow[r, "F(\psi)\times F(\psi)"] & F(0+_0 0)\times F(m+_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[dd, "F(\kappa)\times 1"] \arrow[ddr, phantom, "\overset{\phi_{\kappa,1}}{\cong}"] & F((0+_0 0)+(m+_b n)) \arrow[dd, "F(\kappa+1)"] \\
1 \arrow[uuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"'] \arrow[dddr, bend right, "x\times y"'] \arrow[r, phantom, "\overset{\lambda_{U_0}}{\cong}"] & {} & F(0+0)\times(F(m)\times F(n)) \arrow[urr, "F(!_0+!_0)\times\phi_{m,n}"] \arrow[drrr, "F(\ell)\times(F(\psi)\phi_{m,n})"'] \arrow[rr, phantom, "\cong"] & & {} & & \\
 & F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m+n)) \arrow[rr, "1\times F(\psi)"] \arrow[d, phantom, "\overset{\lambda_{N\odot M}}{\cong}"] & & F(0)\times F(m+_b n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m+_b n}"'] & F(0)\times F(m+_b n) \arrow[r, "\phi_{0,m+_b n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+(m+_b n)) \arrow[dd, "F(\ell)"] \\
 & & {} & & & F(0+(m+_b n)) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"] \arrow[r, phantom, "\cong"] & {} \\
 & F(m)\times F(n) \arrow[rrr, "1"] & & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}
$$

(Scratch work) As a check, let us compute the decoration maps $\lambda_N \colon U_0 \otimes N \to N$ and $\lambda_M \colon U_0 \otimes M \to M$ and compose them via $\odot$. The maps λ_N and λ_M are given explicitly in the paper.

$\lambda_M \colon U_0 \otimes M \to M$

$$
\begin{tikzcd}
& F(0)\times F(m) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m}"'] & F(0)\times F(m) \arrow[r, "\phi_{0,m}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+m) \arrow[dd, "F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)"] \arrow[dr, bend right, "x"'] \arrow[rr, phantom, "\cong"] & & F(0+m) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \\
& F(m) \arrow[rr, "1"] & & F(m)
\end{tikzcd}
$$

$\lambda_N \colon U_0 \otimes N \to N$

$$
\begin{tikzcd}
& F(0)\times F(n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,n}"'] & F(0)\times F(n) \arrow[r, "\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+n) \arrow[dd, "F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times y)"] \arrow[dr, bend right, "y"'] \arrow[rr, phantom, "\cong"] & & F(0+n) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \\
& F(n) \arrow[rr, "1"] & & F(n)
\end{tikzcd}
$$

To construct $\lambda_N \odot \lambda_M \colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$, we first tensor the above two diagrams:

$$
\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n)
\end{tikzcd}
$$

Next, we paste with a square due to pseudonaturality of ϕ:

$$
\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \arrow[r, "\phi_{0+m,0+n}"] \arrow[ddr, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0+m)+(0+n)) \arrow[dd, "F(\ell+\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & {} & \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n)
\end{tikzcd}
$$

Finally, we paste with a square due to pseudonaturality of F to obtain the map $\lambda_N \odot \lambda_M \colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$:

$$
\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \arrow[r, "\phi_{0+m,0+n}"] \arrow[ddr, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0+m)+(0+n)) \arrow[dd, "F(\ell+\ell)"] \arrow[r, "F(\psi)"] \arrow[ddr, phantom, "\cong"] & F((0+m)+_{0+b}(0+n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & {} & & \\
& F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}
$$

Next, we compute the right, down and then left route:

$$
\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rrr, "\phi_{0,m}\times\phi_{0,n}"] \arrow[dr, "1\times\phi\times 1"] \arrow[dddd, bend right, "1\times\beta\times 1"'] & & & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"] \arrow[ddr, phantom, "\cong"] & F((0+m)+_{0+b}(0+n)) \arrow[dddd, "F(\chi)"] \\
& & & F(0)\times F(m+0)\times F(n) \arrow[r, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times 1"'] \arrow[l, phantom, "1\mu\times 1 \cong"] \arrow[dr, phantom, "\phi_{\beta,1} \cong"] & F(0)\times F((m+0)+n) \arrow[urr, "\phi"] \arrow[dd, "1\times F(\beta+1)"] \arrow[u, phantom, "\cong"] & & & \\
& & & & & F((0+0)+(m+n)) \arrow[ddrr, bend left, "F(\psi+\psi)"] \arrow[u, phantom, "\phi_{1,\beta+1} \cong"] \arrow[dr, phantom, "\phi_{\psi,\psi} \cong"] & & \\
& & & F(0)\times F(0+m)\times F(n) \arrow[r, "1\times\phi"] & F(0)\times F((0+m)+n) \arrow[ur, "\phi"] \arrow[d, phantom, "\cong"] & & & \\
& (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times 1)"'] \arrow[dr, "\phi_{0,0}\times 1"'] & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[rrr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[ur, "1\times\phi\times 1"'] & & & F(0+0)\times F(m+n) \arrow[r, "F(\psi)\times F(\psi)"] \arrow[uu, "\phi"] \arrow[d, phantom, "\cong"] & F(0+_0 0)\times F(m+_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[dd, "F(\kappa)\times 1"] \arrow[ddr, phantom, "\phi_{\kappa,1} \cong"] & F((0+_0 0)+(m+_b n)) \arrow[dd, "F(\kappa+1)"] \\
1 \arrow[uuuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"'] \arrow[dddr, bend right, "x\times y"'] \arrow[r, phantom, "\lambda_{M\odot N} \cong"] & & F(0+0)\times(F(m)\times F(n)) \arrow[urrr, "F(!_0+!_0)\times\phi_{m,n}"'] \arrow[drrrr, "F(\lambda_0)\times(F(\psi)\phi_{m,n}"] \arrow[u, phantom, "\phi_{!_0,!_0} \cong"] & & & & & \\
& F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m+n) \arrow[rrr, "1\times F(\psi)"] \arrow[dd, phantom, "\lambda_{N\odot M} \cong"] & & & F(0)\times F(m+_b n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m+_b n}"'] & F(0)\times F(m+_b n) \arrow[r, "\phi_{0,m+_b n}"] \arrow[d, phantom, "\phi_{!_0,1} \cong"] & F(0+(m+_b n)) \arrow[dd, "F(\ell)"] \\
& & & & & & F(0+(m+_b n)) \arrow[ur, "F(!_0+1)"'] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & \\
& F(m)\times F(n) \arrow[rrrr, "1"] & & & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}
$$

$$
\begin{tikzcd}
& F(0)\times F(0) \arrow[r, "F(!_0)\times F(!_0)"] \arrow[dr, "\phi_{0,0}"'] & F(0)\times F(0) \arrow[r, "\phi_{0,0}"] \arrow[d, phantom, "\phi_{!_0,!_0} \cong"] & F(0+0) \arrow[r, "F(\psi)"] \arrow[d, phantom, "\cong"] & F(0+_0 0) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0\times\phi_0"] \arrow[dr, bend right, "\phi_0"'] \arrow[r, phantom, "\lambda_M \cong"] & & F(0+0) \arrow[ur, "F(!_0+!_0)"'] \arrow[drr, "F(\lambda_0)"] & & \\
& F(0) \arrow[rrr, "\mathrm{id}"] & & & F(0)
\end{tikzcd}
$$

Diagrams 7 and 9 of 11. Should improve this, but seems okay.

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$
$$a \in (\mathsf{A},+,0)$$

$$0+a \to 0+a \leftarrow 0+a$$
$$\phi_{0,a}(\bot_0,\bot_a) \in F(0+a)$$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
 & U_a
\end{tikzcd}$$

$$0+a \to 0+a \leftarrow 0+a$$
$$\bot_{0+a} \in F(0+a)$$
$$a \to a \leftarrow a$$
$$\bot_a \in F(a)$$

Right and then down:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, "\phi\times\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[ur, "\phi_{0,a}"] \arrow[r, "\phi_{0,a}"] \arrow[ur, phantom, "="', near end] & F(0+a) \arrow[d, "F(\lambda)"] \\
 & F(0) \arrow[urr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] \arrow[u, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"', near end] & & F(a) \arrow[ul, phantom, "\cong"]
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] \arrow[ddrr, phantom, "=", near start] & & F(0+a) \arrow[dd, "F(\lambda)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & & & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a) \arrow[uull, phantom, "\cong", near start]
\end{tikzcd}$$

Removing the lower right $\cong$ which is the same in each diagram:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, "\phi\times\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[ur, "\phi_{0,a}"] \arrow[r, "\phi_{0,a}"] \arrow[ur, phantom, "="', near end] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] \arrow[u, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"', near end] & &
\end{tikzcd}$$

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] \arrow[ddrr, phantom, "=", near start] & & F(0+a) \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & & & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] & &
\end{tikzcd}$$

The second diagram is then an identity, so our problem reduces to showing that the following diagram is also an identity:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[drr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[dr, phantom, "\overset{\phi}{\cong}"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] \arrow[ur, "\phi_{0,a}"] \arrow[ur, phantom, "="] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

Removing the diagonal $\phi_{0,a}$:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[drr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[dr, phantom, "\overset{\phi}{\cong}"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

This diagram is clearly the same as:

$$\begin{tikzcd}
 & F(0) \arrow[drr, "F(!_{0+a})"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & & \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[dr, phantom, "\overset{\phi}{\cong}"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"'] & &
\end{tikzcd}$$

The two 2-isomorphisms in the top half of the diagram are the inverses of those in the bottom half, when read in a suitable order, and they can be shown to cancel, yielding an identity as desired.

Diagram 10 of 11. Set up and solved (Christina).

$M_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i_1} m_1 \xleftarrow{o_1} b$ with $x_1 \in F(m_1)$.
$M_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i_2} m_2 \xleftarrow{o_2} c$ with $x_2 \in F(m_2)$.
$N_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a' \xrightarrow{i'_1} n_1 \xleftarrow{o'_1} b'$ with $y_1 \in F(n_1)$.
$N_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b' \xrightarrow{i'_2} n_2 \xleftarrow{o'_2} c'$ with $x_2 \in F(n_2)$.

$$
\begin{tikzcd}
\substack{a+a'\to(m_1+n_1)+_{b+b'}(m_2+n_2)\leftarrow c+c' \\ F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2))\in F((m_1+n_1)+_{b+b'}(n_1+n_2))} & (M_2\otimes N_2)\odot(M_1\otimes N_1) \arrow[r, "\beta\odot\beta"] \arrow[d, "\chi"'] & (N_2\otimes M_2)\odot(N_1\otimes M_1) \arrow[d, "\chi"] & \substack{a'+a\to(n_1+m_1)+_{b'+b}(n_2+m_2)\leftarrow c'+c \\ F(\psi)\phi_{n_1+m_1,n_2+m_2}(\phi_{n_1,m_1}(y_1,x_1),\phi_{n_2,m_2}(y_2,x_2))\in F((n_1+m_1)+_{b'+b}(n_1+m_2))} \\
\substack{a+a'\to(m_1+_b m_2)+(n_1+_{b'}n_2)\leftarrow c+c' \\ \phi_{m_1+_b m_2,n_1+_{b'}m_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))\in F((m_1+_b m_2)+(n_1+_{b'}n_2))} & (M_2\odot M_1)\otimes(N_2\odot N_1) \arrow[r, "\beta"] & (N_2\odot N_1)\otimes(M_2\odot M_1) & \substack{a'+a\to(n_1+_{b'}n_2)+(m_1+_b m_2)\leftarrow c'+c \\ \phi_{n_1+_{b'}n_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{m_1,m_2}(x_1,x_2))\in F((n_1+_{b'}n_2)+(m_1+_b m_2))}
\end{tikzcd}
$$

Down and then right:

$$
\begin{tikzcd}
 & (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[ddd, "1\times\beta\times1"'] \arrow[dr, "1\times\phi\times1"] & & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, phantom, "\cong"] & & F((m_1+n_1)+(m_2+n_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[ddd, "F(\chi)"] \arrow[dd, phantom, "\cong"'] \\
 & \arrow[d, phantom, "\substack{1\times\mu\times1\\ \cong}"] & F(m_1)\times F(n_1+m_2)\times F(n_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] & \arrow[d, phantom, "\substack{\phi_{\beta,1}\\ \cong}"] & F(m_1)\times F((n_1+m_2)+n_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] & \arrow[l, phantom, "\substack{\phi_{1,\beta}\\ \cong}"] & \\
 & & F(m_1)\times F(m_2+n_1)\times F(n_2) \arrow[rr, "1\times\phi"'] & \arrow[d, phantom, "\cong"] & F(m_1)\times F((m_2+n_1)+n_2) \arrow[r, "\phi"] & F(m_1+((m_2+n_1)+n_2)) \arrow[dr, "F(\psi+\psi)"] \arrow[d, phantom, "\substack{\phi_{\psi,\psi}\\ \cong}"] & \\
1 \arrow[r, "(x_1\times x_2)\times(y_1\times y_2)"] \arrow[uuur, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[dr, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(m_1)\times F(m_2))\times(F(n_1)\times F(n_2)) \arrow[rr, "\phi_{m_1,m_2}\times\phi_{n_1,n_2}"'] \arrow[ur, "1\times\phi\times1"] \arrow[d, "\beta"] & & F(m_1+m_2)\times F(n_1+n_2) \arrow[rr, "F(\psi)\times F(\psi)"] \arrow[urr, "\phi_{m_1+m_2,n_1+n_2}"] & & F(m_1+_b m_2)\times F(n_1+_{b'}n_2) \arrow[r, "\phi_{m_1+_b m_2,n_1+_{b'}n_2}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\substack{\mu_{m_1+_b m_2,n_1+_{b'}n_2}\\ \cong}"] & F((m_1+_b m_2)+(n_1+_{b'}n_2)) \arrow[d, "F(\beta)"] \\
 & F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] & & F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] & & F(n_1+_{b'}n_2)\times F(m_1+_b m_2) \arrow[r, "\phi_{n_1+_{b'}n_2,m_1+_b m_2}"'] & F((n_1+_{b'}n_2)+(m_1+_b m_2))
\end{tikzcd}
$$

Right and then down:

$$
\begin{tikzcd}
 & (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[d, "\beta\times\beta"'] & \arrow[d, phantom, "\substack{\mu_{m_1,n_1}\times\mu_{m_2,n_2}\\ \cong}"] & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, "F(\beta)\times F(\beta)"] & \arrow[d, phantom, "\substack{\phi_{\beta,\beta}\\ \cong}"] & F((m_1+n_1)+(m_2+n_2)) \arrow[r, "F(\psi)"] \arrow[d, "F(\beta+\beta)"] \arrow[dr, phantom, "\cong"] & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[d, "F(\beta+_\beta\beta)"] \\
1 \arrow[r, "(y_1\times x_1)\times(y_2\times x_2)"] \arrow[ur, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[dddr, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(n_1)\times F(m_1))\times(F(n_2)\times F(m_2)) \arrow[rr, "\phi_{n_1,m_1}\times\phi_{n_2,m_2}"] \arrow[ddd, "1\times\beta\times1"'] \arrow[dr, "1\times\phi\times1"] & & F(n_1+m_1)\times F(n_2+m_2) \arrow[rr, "\phi_{n_1+m_1,n_2+m_2}"] \arrow[d, phantom, "\cong"] & & F((n_1+m_1)+(n_2+m_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] & F((n_1+m_1)+_{b'+b}(n_2+m_2)) \arrow[ddd, "F(\chi)"] \arrow[dd, phantom, "\cong"'] \\
 & \arrow[d, phantom, "\substack{1\times\mu\times1\\ \cong}"] & F(n_1)\times F(m_1+n_2)\times F(m_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] & \arrow[d, phantom, "\substack{\phi_{\beta,1}\\ \cong}"] & F(n_1)\times F((m_1+n_2)+m_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] & \arrow[l, phantom, "\substack{\phi_{1,\beta}\\ \cong}"] & \\
 & & F(n_1)\times F(n_2+m_1)\times F(m_2) \arrow[rr, "1\times\phi"'] & \arrow[d, phantom, "\cong"] & F(n_1)\times F((n_2+m_1)+m_2) \arrow[r, "\phi"] & F(n_1+((n_2+m_1)+m_2)) \arrow[dr, "F(\psi+\psi)"] \arrow[d, phantom, "\substack{\phi_{\psi,\psi}\\ \cong}"] & \\
 & F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] \arrow[ur, "1\times\phi\times1"] & & F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] \arrow[urr, "\phi_{n_1+n_2,m_1+m_2}"] & & F(n_1+_{b'}n_2)\times F(m_1+_b m_2) \arrow[r, "\phi_{n_1+_{b'}n_2,m_1+_b m_2}"'] & F((n_1+_{b'}n_2)+(m_1+_b m_2))
\end{tikzcd}
$$

Diagram 11 of 11. Set up but not solved.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_{a+b} \arrow[r, "\mu_{a,b}"] \arrow[d, "U_\beta"'] & U_a \otimes U_b \arrow[d, "\beta'"] \\
U_{b+a} \arrow[r, "\mu_{b,a}"] & U_b \otimes U_a
\end{tikzcd}$$

The top two underlying cospans are

$$a + b \rightarrow a + b \leftarrow a + b$$

and the bottom two underyling cospans are

$$b + a \rightarrow b + a \leftarrow b + a$$

Right and then down:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{a+b})"] & & F(a{+}b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[r, phantom, shift left=4, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] \arrow[r, phantom, shift left=8, "\underset{\cong}{\phi_{!_a,!_b}}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[ur, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\underset{\cong}{\mu_{a,b}}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
 & F(0){\times}F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{a+b})"] \arrow[d, "\mathrm{id}"] \arrow[drr, phantom, "\cong"] & & F(a{+}b) \arrow[d, "F(\beta)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[rr, "F(!_{b+a})"] \arrow[dr, phantom, "\underset{\cong}{\phi_{!_b,!_a}}"] & & F(b{+}a) \arrow[d, "F(\mathrm{id})"] \\
 & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_b)\times F(!_a)"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

First we remove the lower left commuting quarter circle in the ‘right and then down’ diagram as well as the upper left commuting quarter circle in the ‘down and then right’ diagram:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{a+b})"] & & F(a{+}b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[r, phantom, shift left=4, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] \arrow[r, phantom, shift left=8, "\underset{\cong}{\phi_{!_a,!_b}}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[ur, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\underset{\cong}{\mu_{a,b}}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
 & F(0){\times}F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[d, "\mathrm{id}"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[rr, "F(!_{b+a})"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Next we remove the lower left commuting square with sides β in the first diagram and remove the $\mathrm{id}\colon F(0) \to F(0)$ morphism in the second diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] & F(a)\times F(b) \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
& & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[urr, "F(!_{a+b})"] \arrow[rr, "F(!_{b+a})"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & \arrow[ur, phantom, "\cong"] & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Next we assume that our pseudofunctor F is normalized, meaning that $F(\mathrm{id})$ is an identitiy, and remove the commuting triangle with sides $\phi_{a,b}$ in the first diagram and the commuting triangle with sides $\phi_{b,a}$ in the second diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
& & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[urr, "F(!_{a+b})"] \arrow[rr, "F(!_{b+a})"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & \arrow[ur, phantom, "\cong"] & F(b+a) \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}$$

Next we remove the left quarter circle containing $\overset{\phi}{\cong}$ from each diagram:

$$\begin{tikzcd}
F(0) \arrow[rr, "F(!_{a+b})"] & & F(a{+}b) \arrow[d, "F(\mathrm{id})"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
 & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

$$\begin{tikzcd}
 & & F(a{+}b) \arrow[d, "F(\beta)"] \\
F(0) \arrow[urr, "F(!_{a+b})"] \arrow[rr, "F(!_{b+a})"] \arrow[ur, phantom, "\cong"] & & F(b{+}a) \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}$$

Making each diagram into a rectangle and removing the $F(\mathrm{id})$ from the first diagram:

$$\begin{tikzcd}
F(0) \arrow[rrr, "F(!_{a+b})"] & & & F(a{+}b) \arrow[d, "F(\beta)"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[urr, "\phi_{a,b}"] \arrow[r, "\beta"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b{+}a)
\end{tikzcd}$$

$$\begin{tikzcd}
F(0) \arrow[rr, "F(!_{a+b})"] \arrow[drr, "F(!_{b+a})"] & & F(a{+}b) \arrow[d, "F(\beta)"] \arrow[d, phantom, "\cong"'] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b{+}a)
\end{tikzcd}$$

Are these two diagrams the same?

Some useful maps

Given $a \in (\mathsf{A}, +, 0)$, the map $U_{\lambda_a} \colon U_{0+a} \to U_a$ is given by:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] & & F(0+a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] & & \cong & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"'] & & F(a)
\end{tikzcd}$$

where the $\cong$ is given by pseudonaturality of F: we have a unique map in $!_a \colon 0 \to a$ in A but also a map $\lambda_a \circ !_{0+a} \colon 0 \to a$ where λ_a is the left unitor of $(\mathsf{A}, +, 0)$, and so $F(!_a) = F(\lambda_a \circ !_{0+a}) \cong F(\lambda_a) F(!_{0+a})$.

The left unitor $\lambda'_{U_a} \colon U_0 \otimes U_a \to U_a$ is given by:

$$\begin{tikzcd}
 & F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[dd, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi \times \phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "\overset{\phi}{\cong}"] & & \overset{\phi_{!_0,!_a}}{\cong} \qquad \cong & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"'] & & F(a)
\end{tikzcd}$$

where the $\cong$ in the lower right is the same as the one in the first diagram.

For an arbitrary M, the left unitor $\lambda'_M \colon U_0 \otimes M \to M$ is given by:

$$\begin{tikzcd}
 & F(0){\times}F(m) \arrow[r, "F(!_0) \times 1"] \arrow[dr, "\phi_{0,m}"] & F(0){\times}F(m) \arrow[r, "\phi_{0,m}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+m) \arrow[dd, "F(\lambda_m)"] \\
1 \arrow[ur, bend left, "\phi_0 \times x"] \arrow[dr, bend right, "x"'] \arrow[r, phantom, "\overset{\lambda_M}{\cong}"] & & F(0+m) \arrow[ur, "F(!_0 + 1)"] \arrow[dr, "F(\lambda_m)"] \arrow[r, phantom, "\cong"] & \\
 & F(m) \arrow[rr, "\mathrm{id}"] & & F(m)
\end{tikzcd}$$

For an arbitrary M given by $a \to (m, x) \leftarrow b$, the map $\lambda_M \colon U_b \odot M \to M$ is given by:

$$\begin{tikzcd}
 & F(m){\times}F(0) \arrow[r, "1 \times F(!_b)"] \arrow[dr, "\phi_{m,0}"] & F(m){\times}F(b) \arrow[r, "\phi_{m,b}"] \arrow[d, phantom, "\overset{\phi_{1,!_b}}{\cong}"] & F(m+b) \arrow[r, "F(\psi)"] & F(m{+_b}b) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "x \times \phi_0"] \arrow[dr, bend right, "x"'] \arrow[r, phantom, "\overset{\lambda_M}{\cong}"] & & F(m+0) \arrow[ur, "F(1+!_b)"] \arrow[drr, "F(\lambda_m)"] & \cong & \\
 & F(m) \arrow[rrr, "\mathrm{id}"] & & & F(m)
\end{tikzcd}$$

In particular, if $M = U_0$ above, then the map $\lambda_{U_0} \colon U_0 \odot U_0 \to U_0$ is given by:

$$\begin{tikzcd}
 & F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_0)"] \arrow[dr, "\phi_{0,0}"] & F(0){\times}F(0) \arrow[r, "\phi_{0,0}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"] & F(0+0) \arrow[r, "F(\psi)"] & F(0{+_0}0) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0 \times \phi_0"] \arrow[dr, bend right, "\phi_0"'] \arrow[r, phantom, "\overset{\lambda_M}{\cong}"] & & F(0+0) \arrow[ur, "F(!_0+!_0)"] \arrow[drr, "F(\lambda_0)"] & \cong & \\
 & F(0) \arrow[rrr, "\mathrm{id}"] & & & F(0)
\end{tikzcd}$$