4/1/2021

$$
\begin{tikzcd}
U_{0+a} \arrow[r, "\text{under (17)}"] \arrow[dr] & U_0 \otimes U_a \arrow[d, "\text{above (12)}"] \\
& U_a
\end{tikzcd}
$$

$$
\begin{tikzcd}
1 \arrow[r, "\phi_0"] \arrow[dr, "\phi_0"'] & F(0) \arrow[r, "F(!_{0+a})"] \arrow[d, equal] \arrow[dr, phantom, "\cong"] & F(0+a) \arrow[d, "\wr"] \\
& F(0) \arrow[r, "F(!_a)"'] & F(a)
\end{tikzcd}
$$

Indeed, this iso is formed using pseudofunctoriality of F. We can break it further as Kenny did, or we can leave as is, for the time being.

Now, let's compute the upper composite.

$$
\begin{tikzcd}
1 \arrow[rr, "\phi_0"] \arrow[dr, "\phi_0"] \arrow[drr, "\phi_0 \times \phi_0"'] \arrow[ddr, "\phi_0"'] & & F(0) \arrow[r, "F(!_{0+a})"] & F(0+a) \arrow[dr, equal] & \\
& F(0) \arrow[ur, equal] \arrow[r, "\phi_0 \times 1"] \arrow[dr, phantom, "\wr"] & F(0) \times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[u, phantom, shift left=4, "J_0 \cong"] \arrow[r, "F(!_0) \times F(!_a)"'] \arrow[ur, phantom, "\phi_{!_0,!_a} \cong"] & F(0) \times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \arrow[d, "\wr"] \\
& F(0) \arrow[rrr, "F(!_a)"'] & & & F(a) \arrow[ul, "\phi_0 \times 1"] \arrow[ul, phantom, shift right=4, "J_a \cong"]
\end{tikzcd}
$$

$!_{0+a} \cong !_0 + !_a$

where J is (33), namely

$$
\begin{tikzcd}
F(0) \arrow[r, "\phi_0 \times 1"] \arrow[dr] \arrow[dr, phantom, shift left=3, "J_0 \cong"] & F(0) \times F(0) \arrow[d, "\phi_{0,0}"] \\
& F(0)
\end{tikzcd}
$$

suppressing coherence isos

and

$$
\begin{tikzcd}
F(a) \arrow[r, "\phi_0 \times 1"] \arrow[dr] \arrow[dr, phantom, shift left=3, "J_a \cong"] & F(0) \times F(a) \arrow[d, "\phi_{0,a}"] \\
& F(0+a)
\end{tikzcd}
$$

Now, here is the modification axiom for J:

$$
\begin{tikzcd}
F(0) \arrow[rr, "F(!_a)"] \arrow[dd, bend right] \arrow[d, "\phi_0 \times 1"] \arrow[dd, phantom, shift right=3, "J_0 \cong"] & & F(a) \arrow[d, "\phi_0 \times 1"] \\
F(0) \times F(0) \arrow[d, "\phi_{0,0}"] \arrow[rr, "\overset{1}{\boxed{F(!_0)}} \times F(!_a)"] \arrow[urr, phantom, "\wr"] & & F(0) \times F(a) \arrow[d, "\phi_{0,a}"] \\
F(0+0) \arrow[rr, "F(!_{0+a})"'] \arrow[urr, phantom, "\cong \phi_{!_0,!_a}"] & & F(0+a)
\end{tikzcd}
=
\begin{tikzcd}
F(0) \arrow[r, "F(!_a)"] \arrow[dd, bend right] \arrow[ddr, phantom, "\cong"] & F(a) \arrow[d, "\phi_0 \times 1"] \arrow[dd, bend right] \arrow[dd, phantom, shift right=2, "J_a \cong"] \\
& F(0) \times F(a) \arrow[d, "\phi_{0,a}"] \\
F(0+0) \arrow[r, "F(!_{0+a})"'] & F(0+a)
\end{tikzcd}
$$

CAN PASTE ITS INVERSE TO THE LEFT HAND SIDE!