24/12/2020

$$
\begin{tikzcd}
U^{FCsp}_{a+b} \arrow[r, equal] \arrow[d, "u^{FCsp}"'] & U^{FCsp}_{a+b} \arrow[d, equal, "E_U"] \\
U^{FCsp}_a \otimes^{FCsp} U^{FCsp}_b \arrow[d, equal, "E_U \otimes E_U"'] \arrow[r, phantom, "\neq"] & E(U^{{}_L Csp(\int F)}_{a+b}) \arrow[d, "E(u)"] \\
E(U_a^{Csp(\int F)}) \otimes E(U_b^{Csp(\int F)}) \arrow[r, "\phi"'] & E(U_a \otimes U_b^{Csp(\int F)})
\end{tikzcd}
$$

$E : {}_L Csp(\int F) \longrightarrow FCsp$

IN THE END, ONLY USE THAT THE TARGET DECORATION IS PART OF AN INITIAL OBJECT, SO ANY ISO INVOLVING IT IS UNIQUE

LHS

$$
\begin{tikzcd}
1 \arrow[r, "\phi_0"] \arrow[d, "\phi_0 \times \phi_0"'] \arrow[dr, phantom, "\cong"] & F(0) \arrow[r, "F(!_{a+b})"] \arrow[dr, phantom, "\overset{\phi_{!a,!b}}{\cong}"] & F(a+b) \arrow[d, equal] & \\
F(0) \times F(0) \arrow[r, "F(!_a)\times F(!_b)"'] \arrow[ur, "\phi_{0,0}"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "F(\iota_a)\times F(\iota_b)"'] \arrow[dr, phantom, "\overset{\phi_{\iota_a,\iota_b}}{\cong}"] & F(a+b) \arrow[d, "F(\iota_a+\iota_b)"] \arrow[r, equal] \arrow[dr, phantom, "\cong"] & F(a+b) \\
& F(a+b)\times F(a+b) \arrow[r, "\phi_{a+b}"'] & F(a+b+a+b) \arrow[ur, "F(\nabla)"'] &
\end{tikzcd}
$$

RHS All equalities, and then the canonical isomorphism between $\perp_{a+b}$ and $\perp_a + \perp_b$ in the fibre $F(a+b)$

i.e.

$$(a+b, \perp_{a+b}) \cong (a+b, \perp_a + \perp_b) \text{ as colimits in } \textstyle\int F.$$

Could try and write down, but prefer trying universal argument as follows.

Sketch: the target decoration is by def. of tensor in ${}_L Csp(\int F)$

$$\underbrace{(a, \perp_a) + (b, \perp_b)}_{\text{in } \int F} = (a+b, \underbrace{F(\iota_a)\perp_a + F(\iota_b)\perp_b}_{\text{in } F(a+b)}) \text{ a colimit (sum).}$$

In fact, I think we can now use an easier trick: by (19) we know that $\perp_a, \perp_b$ are the initial objects in $F(a)$, $F(b)$!! We chose them that way. Since reindexing functors preserve them,

$$\left.\begin{array}{l} F(\iota_a)(\perp_a) \text{ initial in } F(a+b) \\ F(\iota_b)(\perp_b) \text{ initial in } F(a+b) \end{array}\right\} F(\iota_a)\perp_a + F(\iota_b)\perp_b \text{ is initial in } F(a+b) !!!$$

Since initial objects are unique up to unique iso (there is no cocone under them), $\circledast$ necessarily commutes!!!