

M_1 is given by $a \rightarrow m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
 M_2 is given by $b \rightarrow m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
 M_3 is given by $c \rightarrow m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
 N_1 is given by $a' \rightarrow n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
 N_2 is given by $b' \rightarrow n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
 N_3 is given by $c' \rightarrow n_3 \leftarrow d'$ with $y_3 \in F(n_3)$.

Decorations:

$$\begin{aligned}
& F(\psi)(\phi_{m_1(n_1+1)} +_{b+b'}(\phi_{m_2(n_2)} +_{m_3+n_3}(F(\psi)\phi_{m_1+n_1, m_2+n_2}(\phi_{m_1, n_1}(x_1, y_1), \phi_{m_2, n_2}(x_2, y_2)), F(\psi)\phi_{m_3, n_3}(x_3, y_3))) \in F(((m_1+n_1) +_{b+b'}(m_2+n_2)) +_{c+c'}(m_3+n_3)) \\
& F(\psi)(\phi_{(m_1+b)m_2} +_{(n_1+b', n_2)}(\phi_{m_1+b, m_2, n_1+n_2}(F(\psi)\phi_{m_1, m_2}(x_1, x_2), F(\psi)\phi_{n_1, n_2}(y_1, y_2))), \phi_{m_3, n_3}(x_3, y_3)) \in F(((m_1+b)m_2) +_{(n_1+b', n_2)}(m_3+n_3)) \\
& \phi_{(m_1+b)m_2} +_{c(m_3, (n_1+b', n_2)+c', n_3)}((F(\psi)\phi_{m_1+b, m_2, n_3}(F(\psi)\phi_{m_1, m_2}(x_1, x_2), x_3), (F(\psi)\phi_{n_1+b', n_2, n_3}(F(\psi)\phi_{n_1, n_2}(y_1, y_2), y_3))) \in F(((m_1+b)m_2) +_{c(m_3)} +_{((n_1+b', n_2)+c', n_3)}) \\
& \phi_{m_1+b(m_2+c)m_3, n_1+b'(n_2+c', n_3)}(F(\psi)\phi_{m_1, m_2+c m_3}((x_1, F(\psi)\phi_{m_2, m_3}(x_2, x_3)), (y_1, F(\psi)\phi_{n_2, n_3}(y_2, y_3)))) \in F((m_1+b(m_2+c m_3)) +_{(n_1+b'(n_2+c', n_3)})
\end{aligned}$$
$$\begin{aligned}
& F(\psi)\phi_{m_1+n_1+b+b'}(m_2+n_2)m_3+n_3)(F(\psi)\phi_{m_1+n_1+m_2+m_3}(\phi_{m_1,j_1}(x_1,y_1),\phi_{m_2,j_2}(x_2,y_2)),F(\psi)\phi_{m_3,j_3}(x_3,y_3)))\in F(((m_1+n_1)+b+b'((m_2+n_2))+c+c'((m_3+n_3))) \\
& F(\psi)\phi_{m_1+n_1,(m_2+n_2))+c+c'((m_3+n_3))}(((\phi_{m_1,j_1}(x_1,y_1),F(\psi)\phi_{m_2+n_2,m_3+n_3}(\phi_{m_2,j_2}(x_2,y_2),\phi_{m_3,j_3}(x_3,y_3)))))\in F((m_1+n_1)+b+b'((m_2+n_2))+c+c'((m_3+n_3))) \\
& F(\psi)\phi_{m_1+n_1,(m_2+cm_3)+(n_2+c',n_3)}(\phi_{m_1,j_1}(x_1,y_1),\phi_{m_2+c,m_3,j_2+c',n_3}((F(\psi)\phi_{m_2,m_3}(x_2,x_3),F(\psi)\phi_{n_2,n_3}(y_2,y_3))))\in F(((m_1+n_1)+b+b'((m_2+c,m_3)+(n_2+c',n_3))) \\
& \phi_{m_1+b(m_2+c,m_3),j_1+b',(n_2+c',n_3)}(F(\psi)\phi_{m_1+m_2+c,m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))))\in F((m_1+b(m_2+c,m_3))+(n_1+b',(n_2+c',n_3)))
\end{aligned}$$

The diagram illustrates the coherence of transformations in a categorical setting, showing various compositions of functors F and natural transformations ϕ , ψ , and χ . The nodes represent expressions involving functors F applied to sums of variables and other functors, with natural transformations ϕ , ψ , and χ mapping between them. The diagram is organized into several rows and columns, with arrows indicating the direction of the transformations. Key nodes include expressions like $Fm_1 Fm_2 Fm_3 Fn_3$, $F(m_1+n_1)+b+m_2 Fm_3+n_3$, and $F((m_1+n_1)+b+b' F(m_2+n_2))+c+c' F(m_3+n_3)$. The diagram illustrates the coherence of these transformations, showing that different paths between the same nodes yield the same result.

Down and then right (omitting morphisms emanating out of 1 on the left due to space restrictions):

Down and then right:

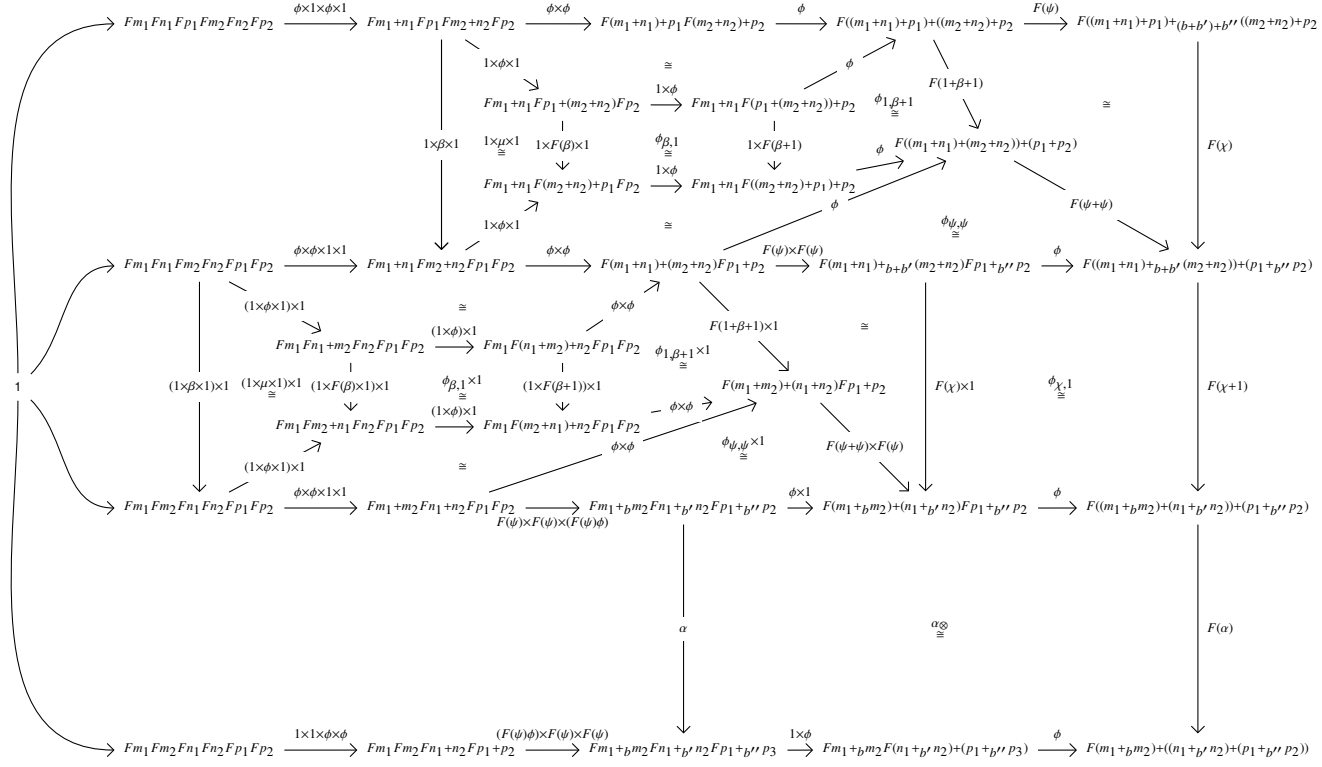


Diagram 5 of 11. Set up but not solved.

$$\begin{array}{ccccc}
 & a, b, c \in (\mathbf{A}, +, 0) & & & \\
 \perp_{(a+b)+c} \in F((a+b)+c) & U_{(a+b)+c} & \xrightarrow{\mu_\beta} & U_{a+(b+c)} & \perp_{a+(b+c)} \in F(a+(b+c)) \\
 & \downarrow \mu & & \downarrow \mu & \\
 \phi_{a+b,c}(\perp_{a+b}, \perp_c) \in F((a+b)+c) & U_{a+b} \otimes U_c & & U_a \otimes U_{b+c} & \phi_{a,b+c}(\perp_a, \perp_{b+c}) \in F(a+(b+c)) \\
 & \downarrow \mu & & \downarrow \mu & \\
 \phi_{a+b,c}(\phi_{a,b}(\perp_a, \perp_b), \perp_c) \in F((a+b)+c) & (U_a \otimes U_b) \otimes U_c & \xrightarrow{\beta'} & U_a \otimes (U_b \otimes U_c) & \phi_{a,b+c}(\perp_a, \phi_{b,c}(\perp_b, \perp_c)) \in F(a+(b+c))
 \end{array}$$

All the left cospans are $(a + b) + c \rightarrow (a + b) + c \leftarrow (a + b) + c$ and all the right cospans are $a + (b + c) \rightarrow a + (b + c) \leftarrow a + (b + c)$.

Right and then down:

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{(a+b)+c})} & F((a+b)+c) & \\
 \phi_0 \curvearrowright & & & & \\
 & F(0) & \xrightarrow{F(!_{a+(b+c)})} & F(a+(b+c)) & \\
 \phi_0 \nearrow & & \cong & & \\
 1 & & F(!_a) \times F(!_{b+c}) & \xrightarrow{\phi_{a,b+c}} & F(a+(b+c)) \\
 \phi_0 \times \phi_0 \nearrow & F(0) \times F(0) & \xrightarrow{F(!_a) \times F(!_{b+c})} & F(a) \times F(b+c) & \downarrow F(\text{id}) \\
 & \uparrow \phi_{0,0} & \uparrow \phi_{!_b, !_c} & \uparrow 1 \times \phi_{b+c} & \\
 & F(0) \times F(0) & \xrightarrow{F(!_a) \times (F(!_b) \times F(!_c))} & F(a) \times (F(b) \times F(c)) & \xrightarrow{1 \times \phi_{b+c}} F(a) \times F(b+c) \xrightarrow{\phi_{a,b+c}} F(a+(b+c)) \\
 \phi_0 \times (\phi_0 \times \phi_0) \nearrow & & & & \\
 & F(0) \times (F(0) \times F(0)) & \xrightarrow{F(!_a) \times (F(!_b) \times F(!_c))} & F(a) \times (F(b) \times F(c)) & \xrightarrow{1 \times \phi_{b+c}} F(a) \times F(b+c) \xrightarrow{\phi_{a,b+c}} F(a+(b+c))
 \end{array}$$

Down and then right:

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{(a+b)+c})} & F((a+b)+c) & \\
 \phi_0 \curvearrowright & & & & \\
 & F(0) & \xrightarrow{F(!_{a+(b+c)})} & F(a+(b+c)) & \\
 \phi_0 \nearrow & & \cong & & \\
 1 & & F(!_a) \times F(!_c) & \xrightarrow{\phi_{a+b,c}} & F((a+b)+c) \\
 \phi_0 \times \phi_0 \nearrow & F(0) \times F(0) & \xrightarrow{F(!_a) \times F(!_c)} & F(a+b) \times F(c) & \downarrow F(\text{id}) \\
 & \uparrow \phi_{0,0} & \uparrow \phi_{!_a, !_b} & \uparrow \phi_{a,b} \times 1 & \\
 & F(0) \times F(0) & \xrightarrow{(F(!_a) \times F(!_b)) \times F(!_c)} & (F(a) \times F(b)) \times F(c) & \xrightarrow{\phi_{a,b} \times 1} F(a+b) \times F(c) \xrightarrow{\phi_{a+b,c}} F(a+(b+c)) \\
 \phi_0 \times (\phi_0 \times \phi_0) \nearrow & & & & \\
 & F(0) \times (F(0) \times F(0)) & \xrightarrow{F(!_a) \times (F(!_b) \times F(!_c))} & F(a) \times (F(b) \times F(c)) & \xrightarrow{1 \times \phi_{b+c}} F(a) \times F(b+c) \xrightarrow{\phi_{a,b+c}} F(a+(b+c))
 \end{array}$$

$$\lambda_M: U_0 \otimes M \rightarrow M$$

$$\begin{array}{ccccc}
 & F(0) \times F(m) & \xrightarrow{F(!_0) \times 1} & F(0) \times F(m) & \xrightarrow{\phi_{0,m}} & F(0+m) \\
 & \searrow \phi_{0,m} & & \nearrow \phi_{0,m} & & \nearrow F(\ell) \\
 & & \cong & & & \\
 & & F(0+m) & \xrightarrow{\phi_{!_0,1} \cong} & F(!_0+1) & \xrightarrow{\cong} & F(0+m) \\
 & & & & \searrow F(\ell) & & \searrow F(\ell) \\
 & & & & & & F(m) \\
 & & & & & & \downarrow F(\ell) \\
 & & & & & & F(m) \\
 & & & & & & \downarrow F(\ell) \\
 & & & & & & F(m)
 \end{array}$$

$$\lambda_N: U_0 \otimes N \rightarrow N$$

$$\begin{array}{ccccc}
 & F(0) \times F(n) & \xrightarrow{F(!_0) \times 1} & F(0) \times F(n) & \xrightarrow{\phi_{0,n}} & F(0+n) \\
 & \searrow \phi_{0,n} & & \nearrow \phi_{0,n} & & \nearrow F(\ell) \\
 & & \cong & & & \\
 & & F(0+n) & \xrightarrow{\phi_{!_0,1} \cong} & F(!_0+1) & \xrightarrow{\cong} & F(0+n) \\
 & & & & \searrow F(\ell) & & \searrow F(\ell) \\
 & & & & & & F(n) \\
 & & & & & & \downarrow F(\ell) \\
 & & & & & & F(n) \\
 & & & & & & \downarrow F(\ell) \\
 & & & & & & F(n)
 \end{array}$$

To construct $\lambda_N \odot \lambda_M: (U_0 \otimes N) \odot (U_0 \otimes M) \rightarrow N \odot M$, we first tensor the above two diagrams:

$$\begin{array}{ccccc}
 & (F(0) \times F(m)) \times (F(0) \times F(n)) & \xrightarrow{(F(!_0) \times 1) \times (F(!_0) \times 1)} & (F(0) \times F(m)) \times (F(0) \times F(n)) & \xrightarrow{\phi_{0,m} \times \phi_{0,n}} & F(0+m) \times F(0+n) \\
 & \searrow \phi_{0,m} \times \phi_{0,n} & & \nearrow \phi_{0,m} \times \phi_{0,n} & & \nearrow F(\ell) \times F(\ell) \\
 & & \cong & & & \\
 & & F(0+m) \times F(0+n) & \xrightarrow{\phi_{!_0,1} \times \phi_{!_0,1} \cong} & F(!_0+1) \times F(!_0+1) & \xrightarrow{\cong} & F(0+m) \times F(0+n) \\
 & & & & \searrow F(\ell) \times F(\ell) & & \searrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n) \\
 & & & & & & \downarrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n) \\
 & & & & & & \downarrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n)
 \end{array}$$

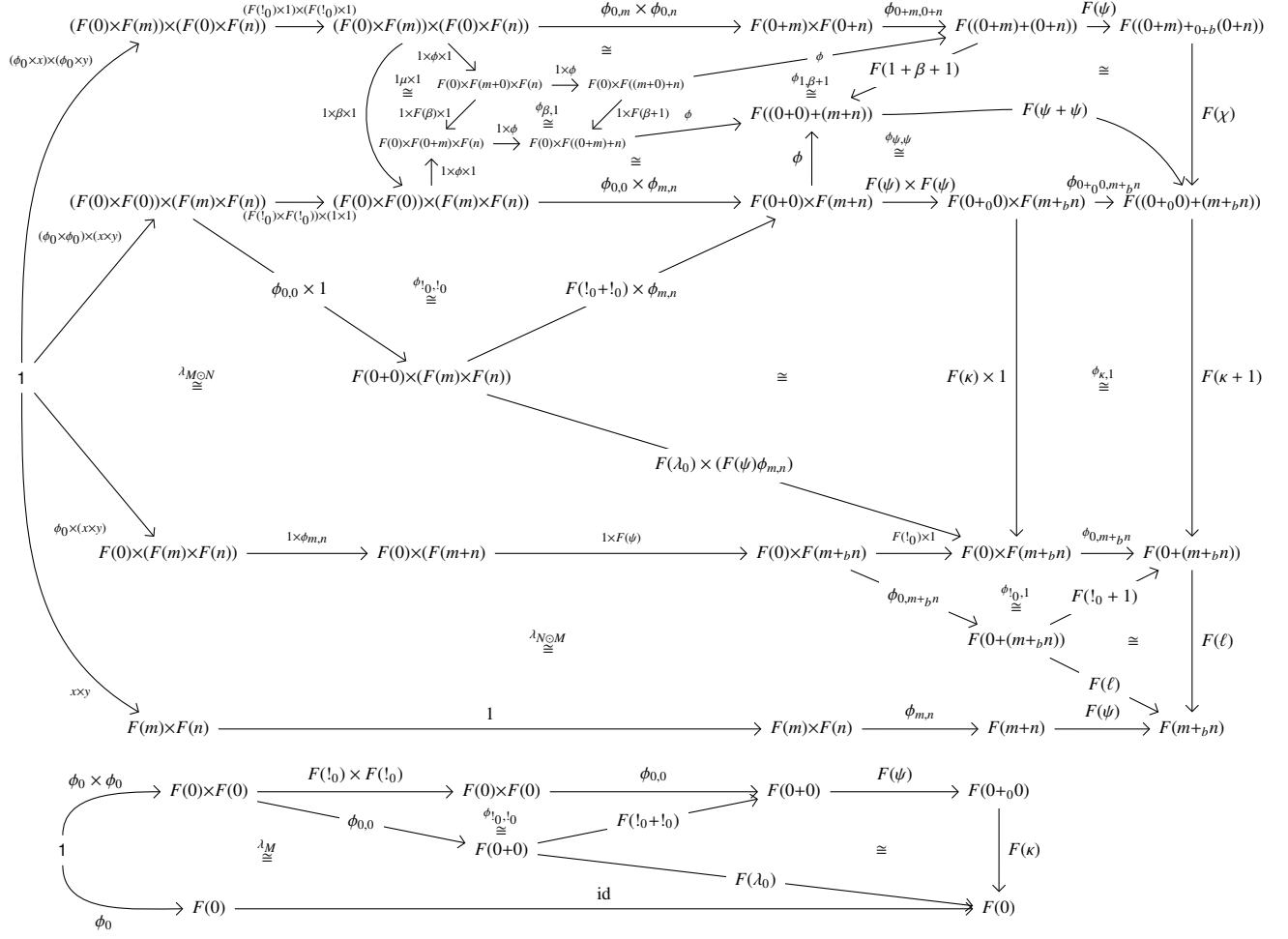
Next, we paste with a square due to pseudonaturality of ϕ :

$$\begin{array}{ccccc}
 & (F(0) \times F(m)) \times (F(0) \times F(n)) & \xrightarrow{(F(!_0) \times 1) \times (F(!_0) \times 1)} & (F(0) \times F(m)) \times (F(0) \times F(n)) & \xrightarrow{\phi_{0,m} \times \phi_{0,n}} & F(0+m) \times F(0+n) & \xrightarrow{\phi_{0+m,0+n}} & F((0+m)+(0+n)) \\
 & \searrow \phi_{0,m} \times \phi_{0,n} & & \nearrow \phi_{0,m} \times \phi_{0,n} & & \nearrow F(\ell) \times F(\ell) & & \nearrow F(\ell + \ell) \\
 & & \cong & & & \\
 & & F(0+m) \times F(0+n) & \xrightarrow{\phi_{!_0,1} \times \phi_{!_0,1} \cong} & F(!_0+1) \times F(!_0+1) & \xrightarrow{\cong} & F(0+m) \times F(0+n) \\
 & & & & \searrow F(\ell) \times F(\ell) & & \searrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n) \\
 & & & & & & \downarrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n) \\
 & & & & & & \downarrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n)
 \end{array}$$

Finally, we paste with a square due to pseudonaturality of F to obtain the map $\lambda_N \odot \lambda_M: (U_0 \otimes N) \odot (U_0 \otimes M) \rightarrow N \odot M$:

$$\begin{array}{ccccc}
 & (F(0) \times F(m)) \times (F(0) \times F(n)) & \xrightarrow{(F(!_0) \times 1) \times (F(!_0) \times 1)} & (F(0) \times F(m)) \times (F(0) \times F(n)) & \xrightarrow{\phi_{0,m} \times \phi_{0,n}} & F(0+m) \times F(0+n) & \xrightarrow{\phi_{0+m,0+n}} & F((0+m)+(0+n)) & \xrightarrow{F(\psi)} & F((0+m)+_{0+b}(0+n)) \\
 & \searrow \phi_{0,m} \times \phi_{0,n} & & \nearrow \phi_{0,m} \times \phi_{0,n} & & \nearrow F(\ell) \times F(\ell) & & \nearrow F(\ell + \ell) & & \nearrow F(\ell +_{\ell} \ell) \\
 & & \cong & & & \\
 & & F(0+m) \times F(0+n) & \xrightarrow{\phi_{!_0,1} \times \phi_{!_0,1} \cong} & F(!_0+1) \times F(!_0+1) & \xrightarrow{\cong} & F(0+m) \times F(0+n) \\
 & & & & \searrow F(\ell) \times F(\ell) & & \searrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n) \\
 & & & & & & \downarrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n) \\
 & & & & & & \downarrow F(\ell) \times F(\ell) \\
 & & & & & & F(m) \times F(n)
 \end{array}$$

Next, we compute the right, down and then left route:



Diagrams 7 and 9 of 11. Set up and solved.

$$\begin{array}{ccc}
 F: (\mathbf{A}, +, 0) \rightarrow (\mathbf{Cat}, \times, 1) & & \\
 a \in (\mathbf{A}, +, 0) & & \\
 0 + a \rightarrow 0 + a \leftarrow 0 + a & \begin{array}{c} U_{0+a} \xrightarrow{\mu} U_0 \otimes U_a \\ \searrow U_{\lambda_a} \quad \downarrow \lambda_{U_a} \\ U_a \end{array} & \begin{array}{c} 0 + a \rightarrow 0 + a \leftarrow 0 + a \\ \phi_{0,a}(\perp_0, \perp_a) \in F(0 + a) \\ a \rightarrow a \leftarrow a \\ \perp_a \in F(a) \end{array} \\
 \perp_{0+a} \in F(0+a) & &
 \end{array}$$

Right and then down:

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \uparrow \phi_{0,0} & & \nearrow \phi_{0,a} & \\
 1 \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_a)} & F(0) \times F(a) & \xrightarrow{\phi_{0,a}} F(0+a) \\
 \phi \searrow & \downarrow \phi_{0,0} & \phi_{!_0, !_a} \cong & \downarrow \phi_{0,a} & \downarrow F(\text{id}) \\
 & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 & & \nearrow F(!_a) & \nearrow F(!_a) & \\
 & & & & \downarrow F(\lambda)
 \end{array}$$

Diagonally:

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \downarrow F(\text{id}) & & \nearrow F(!_a) & \\
 1 \xrightarrow{\phi} & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 \phi \searrow & & & & \\
 & & & & \downarrow F(\lambda)
 \end{array}$$

Removing the lower right \cong which is the same in each diagram:

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \uparrow \phi_{0,0} & & \nearrow \phi_{0,a} & \\
 1 \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_a)} & F(0) \times F(a) & \xrightarrow{\phi_{0,a}} F(0+a) \\
 \phi \searrow & \downarrow \phi_{0,0} & \phi_{!_0, !_a} \cong & \downarrow \phi_{0,a} & \downarrow F(\text{id}) \\
 & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 & & \nearrow F(!_a) & \nearrow F(!_a) & \\
 & & & & \downarrow F(\lambda)
 \end{array}$$

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \downarrow F(\text{id}) & & \nearrow F(!_a) & \\
 1 \xrightarrow{\phi} & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 \phi \searrow & & & & \\
 & & & & \downarrow F(\lambda)
 \end{array}$$

Diagram 10 of 11. Set up and solved (Christina).

$M_1 \in F\mathbf{Csp}$ is given by $a \xrightarrow{i_1} m_1 \xleftarrow{o_1} b$ with $x_1 \in F(m_1)$.

$M_2 \in F\mathbf{Csp}$ is given by $b \xrightarrow{i_2} m_2 \xleftarrow{o_2} c$ with $x_2 \in F(m_2)$.

$N_1 \in F\mathbf{Csp}$ is given by $a' \xrightarrow{i'_1} n_1 \xleftarrow{o'_1} b'$ with $y_1 \in F(n_1)$.

$N_2 \in F\mathbf{Csp}$ is given by $b' \xrightarrow{i'_2} n_2 \xleftarrow{o'_2} c'$ with $x_2 \in F(n_2)$.

$$\begin{array}{ccc}
 a+a' \rightarrow (m_1+n_1)+_{b+b'}(m_2+n_2) \leftarrow c+c' & (M_2 \otimes N_2) \odot (M_1 \otimes N_1) \xrightarrow{\beta \odot \beta} (N_2 \otimes M_2) \odot (N_1 \otimes M_1) & a'+a \rightarrow (n_1+m_1)+_{b'+b}(n_2+m_2) \leftarrow c'+c \\
 F(\psi)\phi_{m_1+n_1, m_2+n_2}(\phi_{m_1, n_1}(x_1, y_1), \phi_{m_2, n_2}(x_2, y_2)) \in F((m_1+n_1)+_{b+b'}(n_1+n_2)) & \downarrow \chi & F(\psi)\phi_{n_1+m_1, n_2+m_2}(\phi_{n_1, m_1}(y_1, x_1), \phi_{n_2, m_2}(y_2, x_2)) \in F((n_1+m_1)+_{b'+b'}(n_1+m_2)) \\
 & (M_2 \odot M_1) \otimes (N_2 \odot N_1) \xrightarrow{\beta} (N_2 \odot N_1) \otimes (M_2 \odot M_1) & \\
 a+a' \rightarrow (m_1+b, m_2)+(n_1+b', n_2) \leftarrow c+c' & & a'+a \rightarrow (n_1+b', n_2)+(m_1+b, m_2) \leftarrow c'+c \\
 \phi_{m_1+b, m_2, n_1+b', n_2}(F(\psi)\phi_{m_1, m_2}(x_1, x_2), F(\psi)\phi_{n_1, n_2}(y_1, y_2)) \in F((m_1+b, m_2)+(n_1+b', n_2)) & & \phi_{n_1+b', n_2}(F(\psi)\phi_{n_1, n_2}(y_1, y_2), F(\psi)\phi_{m_1, m_2}(x_1, x_2)) \in F((n_1+b', n_2)+(m_1+b, m_2))
 \end{array}$$

Down and then right:

$$\begin{array}{c}
 \begin{array}{c}
 (x_1 \times y_1) \times (x_2 \times y_2) \\
 \downarrow 1 \times \beta \times 1 \\
 (x_1 \times x_2) \times (y_1 \times y_2) \\
 \downarrow \beta \\
 y_1 \times y_2 \times x_1 \times x_2
 \end{array}
 \end{array}$$

Diagram illustrating the "Down and then right" path in the proof. The diagram shows a complex network of nodes and arrows representing the derivation of the final result from the initial expressions. The nodes are arranged in a grid-like structure, with arrows indicating the sequence of operations and transformations. The final result is reached by following the path from the top-left node, down to the bottom-left node, and then right to the bottom-right node.

Right and then down:

$$\begin{array}{c}
 \begin{array}{c}
 (x_1 \times y_1) \times (x_2 \times y_2) \\
 \downarrow \beta \times \beta \\
 (y_1 \times x_1) \times (y_2 \times x_2) \\
 \downarrow 1 \times \beta \times 1 \\
 y_1 \times y_2 \times x_1 \times x_2
 \end{array}
 \end{array}$$

Diagram illustrating the "Right and then down" path in the proof. The diagram shows a complex network of nodes and arrows representing the derivation of the final result from the initial expressions. The nodes are arranged in a grid-like structure, with arrows indicating the sequence of operations and transformations. The final result is reached by following the path from the top-left node, right to the top-right node, and then down to the bottom-right node.

Diagram 11 of 11. Set up but not solved.

$$a, b \in (A, +, 0)$$

$$\begin{array}{ccc} U_{a+b} & \xrightarrow{\mu_{a,b}} & U_a \otimes U_b \\ U_\beta \downarrow & & \downarrow \beta' \\ U_{b+a} & \xrightarrow{\mu_{b,a}} & U_b \otimes U_a \end{array}$$

The top two underlying cospans are

$$a + b \rightarrow a + b \leftarrow a + b$$

and the bottom two underlying cospans are

$$b + a \rightarrow b + a \leftarrow b + a$$

Right and then down:

$$\begin{array}{ccccccc} & & F(0) & \xrightarrow{F(!_{a+b})} & F(a+b) & & \\ & \nearrow \phi & \uparrow \phi_{0,0} & & \nearrow \phi_{a,b} & & \\ 1 & \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_a) \times F(!_b)} & F(a) \times F(b) & \xrightarrow{\phi_{a,b}} & F(a+b) \\ & \searrow \phi \times \phi & \downarrow \beta & & \downarrow \beta & & \downarrow F(\text{id}) \\ & & F(0) \times F(0) & \xrightarrow{F(!_b) \times F(!_a)} & F(b) \times F(a) & \xrightarrow{\phi_{b,a}} & F(b+a) \end{array}$$

$\phi_{!_a, !_b} \cong \mu_{a,b} \cong \phi_{b,a}$

Down and then right:

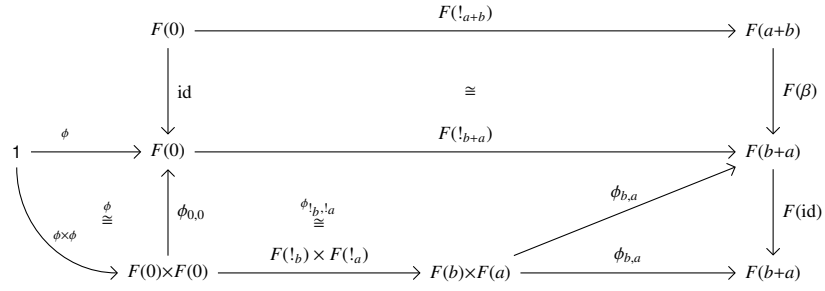
$$\begin{array}{ccccccc} & & F(0) & \xrightarrow{F(!_{a+b})} & F(a+b) & & \\ & \nearrow \phi & \downarrow \text{id} & & \downarrow F(\beta) & & \\ 1 & \xrightarrow{\phi} & F(0) & \xrightarrow{F(!_{b+a})} & F(b+a) & & \\ & \searrow \phi \times \phi & \uparrow \phi_{0,0} & & \nearrow \phi_{b,a} & & \\ & & F(0) \times F(0) & \xrightarrow{F(!_b) \times F(!_a)} & F(b) \times F(a) & \xrightarrow{\phi_{b,a}} & F(b+a) \end{array}$$

$\phi_{!_b, !_a} \cong \mu_{b,a} \cong \phi_{b,a}$

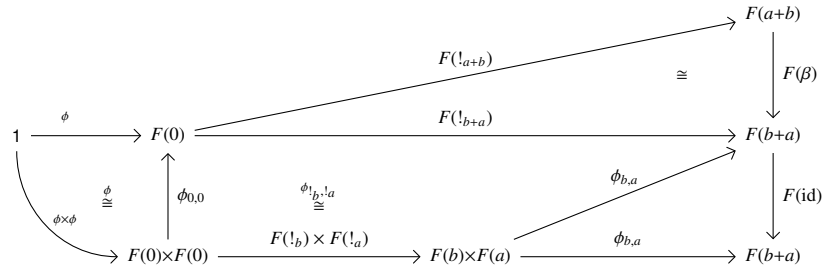
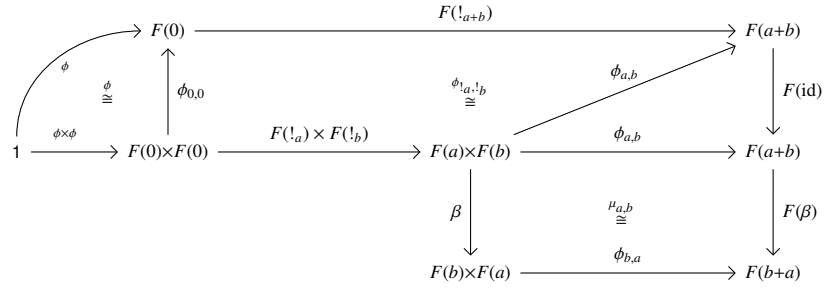
First we remove the lower left commuting quarter circle in the ‘right and then down’ diagram as well as the upper left commuting quarter circle in the ‘down and then right’ diagram:

$$\begin{array}{ccccccc} & & F(0) & \xrightarrow{F(!_{a+b})} & F(a+b) & & \\ & \nearrow \phi & \uparrow \phi_{0,0} & & \nearrow \phi_{a,b} & & \\ 1 & \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_a) \times F(!_b)} & F(a) \times F(b) & \xrightarrow{\phi_{a,b}} & F(a+b) \\ & \searrow \phi \times \phi & \downarrow \beta & & \downarrow \beta & & \downarrow F(\text{id}) \\ & & F(0) \times F(0) & \xrightarrow{F(!_b) \times F(!_a)} & F(b) \times F(a) & \xrightarrow{\phi_{b,a}} & F(b+a) \end{array}$$

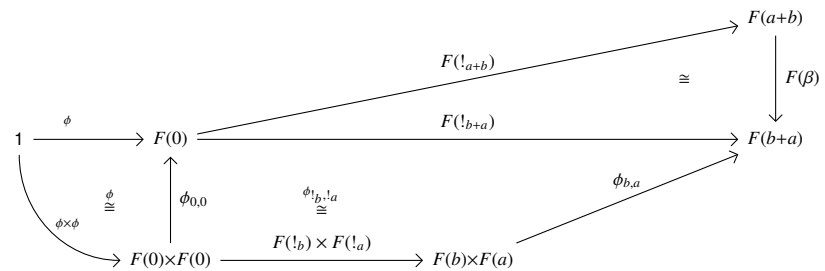
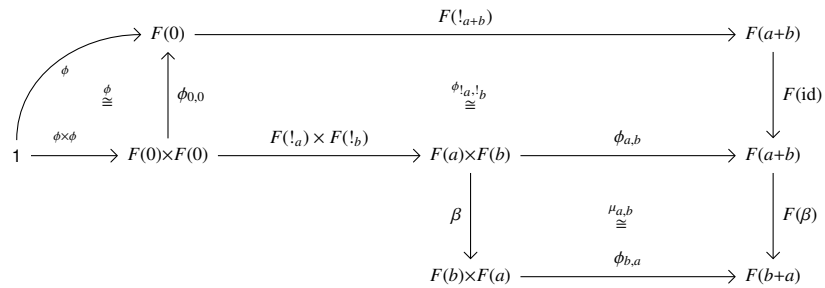
$\phi_{!_a, !_b} \cong \mu_{a,b} \cong \phi_{b,a}$



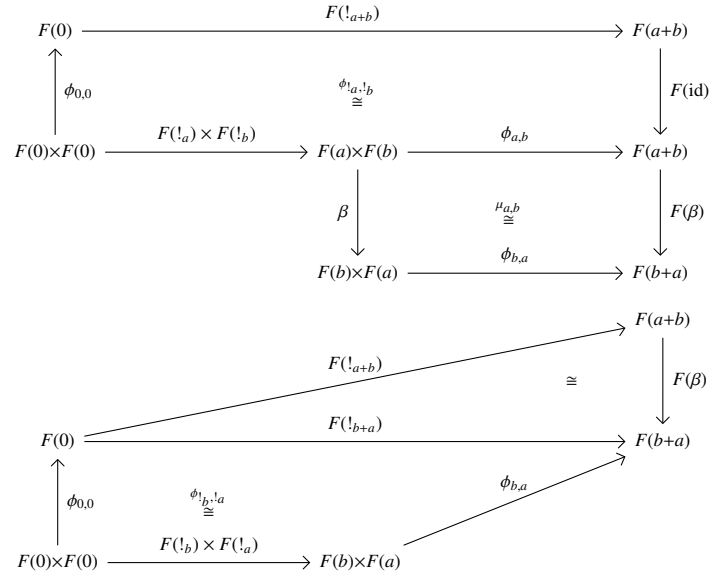
Next we remove the lower left commuting square with sides β in the first diagram and remove the $\text{id}: F(0) \rightarrow F(0)$ morphism in the second diagram:



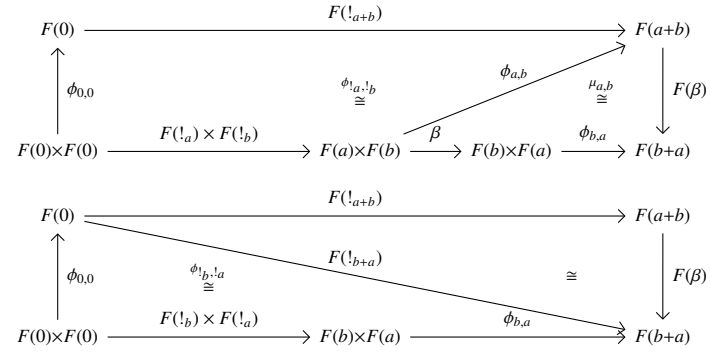
Next we assume that our pseudofunctor F is normalized, meaning that $F(\text{id})$ is an identity, and remove the commuting triangle with sides $\phi_{a,b}$ in the first diagram and the commuting triangle with sides $\phi_{b,a}$ in the second diagram:



Next we remove the left quarter circle containing \cong from each diagram:



Making each diagram into a rectangle and removing the $F(\text{id})$ from the first diagram:



Are these two diagrams the same?

Some useful maps

Given $a \in (\mathbf{A}, +, 0)$, the map $U_{\lambda_a}: U_{0+a} \rightarrow U_a$ is given by:

$$\begin{array}{ccccc}
 \phi & \xrightarrow{\quad} & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) \\
 \downarrow 1 & & \downarrow F(\text{id}) & \searrow F(!_{0+a}) & \downarrow F(\lambda_a) \\
 \phi & \xrightarrow{\quad} & F(0) & \xrightarrow{F(!_a)} & F(a)
 \end{array}
 \quad \cong$$

where the \cong is given by pseudonaturality of F : we have a unique map in $!_a: 0 \rightarrow a$ in \mathbf{A} but also a map $\lambda_a \circ !_a: 0 \rightarrow a$ where λ_a is the left unitor of $(\mathbf{A}, +, 0)$, and so $F(!_a) = F(\lambda_a \circ !_a) \cong F(\lambda_a)F(!_a)$.

The left unitor $\lambda'_{U_a}: U_0 \otimes U_a \rightarrow U_a$ is given by:

$$\begin{array}{ccccc}
 \phi \times \phi & \xrightarrow{\quad} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_a)} & F(0) \times F(a) & \xrightarrow{\phi_{0,a}} & F(0+a) \\
 \downarrow 1 & & \downarrow \phi_{0,0} & \searrow \phi_{!_0, !_a} & \searrow F(!_a) & \searrow F(!_a) & \downarrow F(\lambda_a) \\
 \phi & \xrightarrow{\quad} & F(0) & \xrightarrow{F(!_a)} & F(a) & & F(a)
 \end{array}
 \quad \cong$$

where the \cong in the lower right is the same as the one in the first diagram.

For an arbitrary M , the left unitor $\lambda'_M: U_0 \otimes M \rightarrow M$ is given by:

$$\begin{array}{ccccc}
 \phi_0 \times x & \xrightarrow{\quad} & F(0) \times F(m) & \xrightarrow{F(!_0) \times 1} & F(0) \times F(m) & \xrightarrow{\phi_{0,m}} & F(0+m) \\
 \downarrow 1 & & \downarrow \phi_{0,m} & \searrow \phi_{!_0, !_1} & \searrow F(!_0 + 1) & \searrow F(\lambda_m) & \downarrow F(\lambda_m) \\
 x & \xrightarrow{\quad} & F(m) & \xrightarrow{\text{id}} & F(m) & & F(m)
 \end{array}
 \quad \cong$$

For an arbitrary M given by $a \rightarrow (m, x) \leftarrow b$, the map $\lambda_M: U_b \odot M \rightarrow M$ is given by:

$$\begin{array}{ccccccc}
 x \times \phi_0 & \xrightarrow{\quad} & F(m) \times F(0) & \xrightarrow{1 \times F(!_b)} & F(m) \times F(b) & \xrightarrow{\phi_{m,b}} & F(m+b) & \xrightarrow{F(\psi)} & F(m+_b b) \\
 \downarrow 1 & & \downarrow \phi_{m,0} & \searrow \phi_{!_1, !_b} & \searrow F(1+_b) & \searrow F(\lambda_m) & \searrow F(\lambda_m) & \searrow F(\kappa) & \downarrow F(\kappa) \\
 x & \xrightarrow{\quad} & F(m) & \xrightarrow{\text{id}} & F(m) & & F(m) & & F(m)
 \end{array}
 \quad \cong$$

In particular, if $M = U_0$ above, then the map $\lambda_{U_0}: U_0 \odot U_0 \rightarrow U_0$ is given by:

$$\begin{array}{ccccccc}
 \phi_0 \times \phi_0 & \xrightarrow{\quad} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_0)} & F(0) \times F(0) & \xrightarrow{\phi_{0,0}} & F(0+0) & \xrightarrow{F(\psi)} & F(0+_0 0) \\
 \downarrow 1 & & \downarrow \phi_{0,0} & \searrow \phi_{!_0, !_0} & \searrow F(!_0 + !_0) & \searrow F(\lambda_0) & \searrow F(\lambda_0) & \searrow F(\kappa) & \downarrow F(\kappa) \\
 \phi_0 & \xrightarrow{\quad} & F(0) & \xrightarrow{\text{id}} & F(0) & & F(0) & & F(0)
 \end{array}
 \quad \cong$$