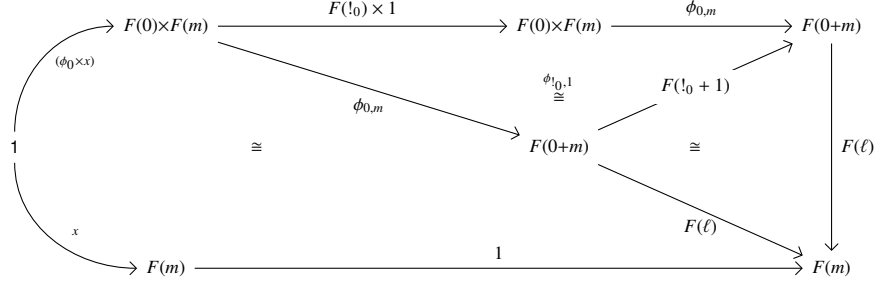


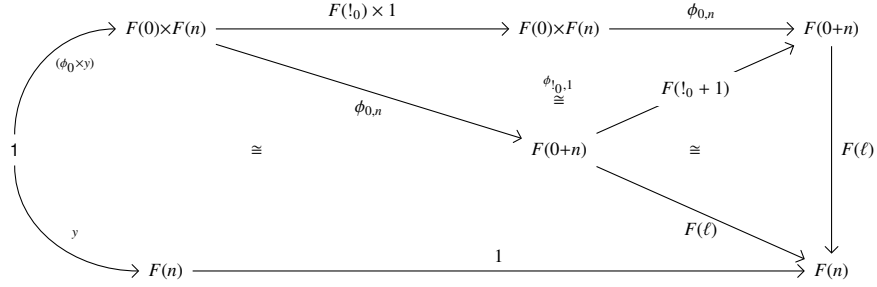
Diagram 1 of 11. Not set up and not solved.

Diagram 4 of 11. Not set up and not solved.

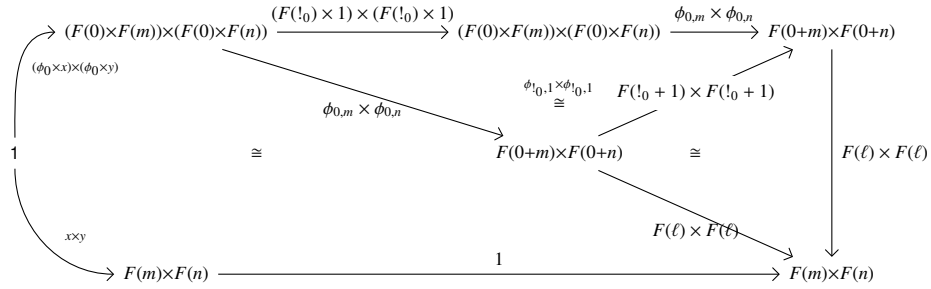
$$\lambda_M: U_0 \otimes M \rightarrow M$$



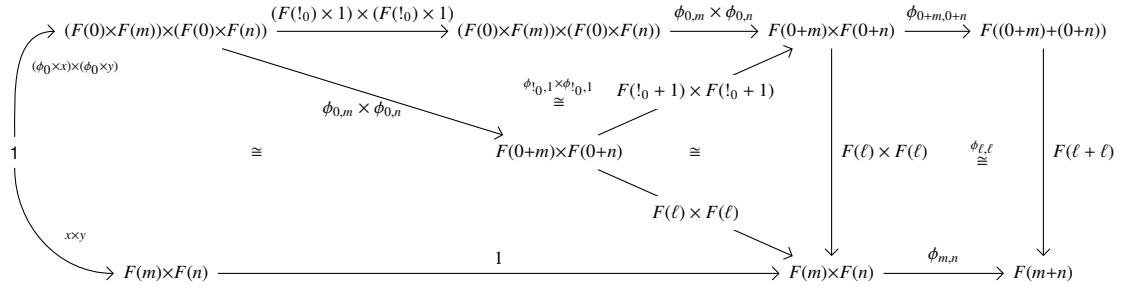
$$\lambda_N: U_0 \otimes N \rightarrow N$$



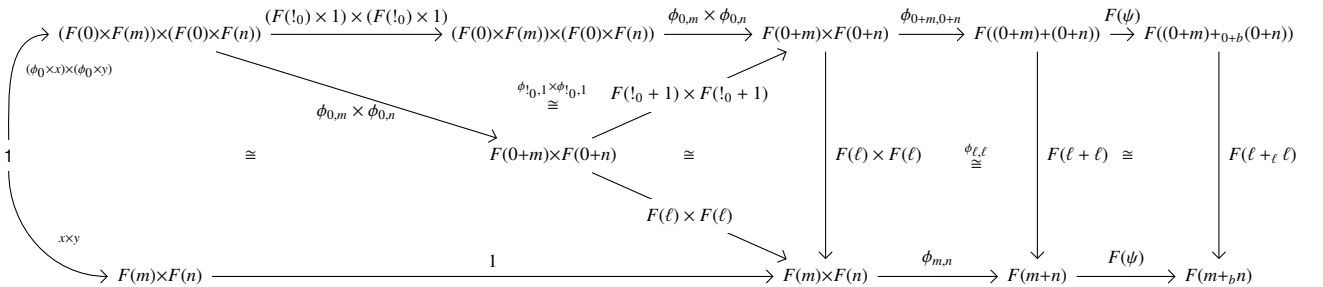
To construct $\lambda_N \odot \lambda_M: (U_0 \otimes N) \odot (U_0 \otimes M) \rightarrow N \odot M$, we first tensor the above two diagrams:



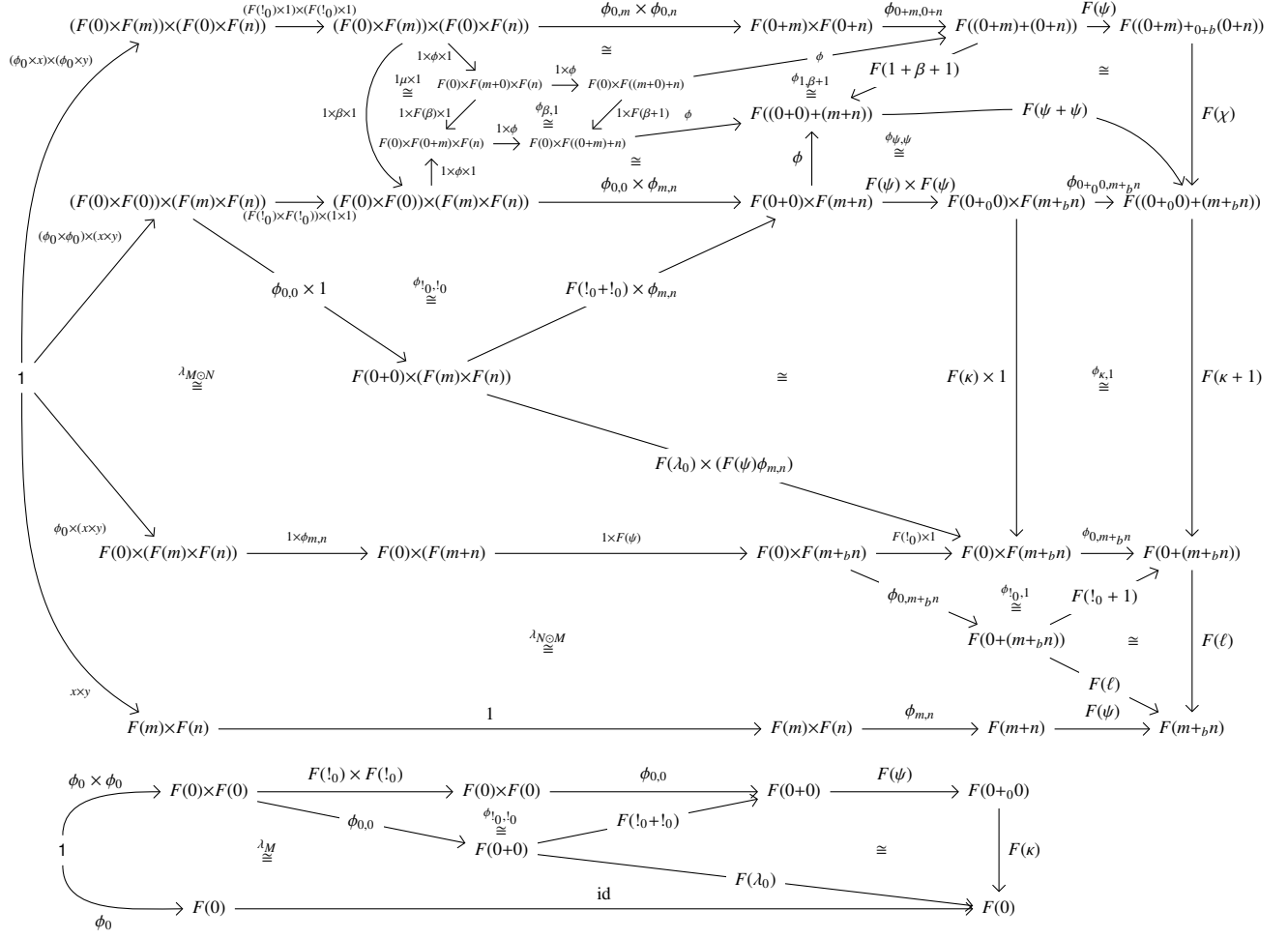
Next, we paste with a square due to pseudonaturality of ϕ :



Finally, we paste with a square due to pseudonaturality of F to obtain the map $\lambda_N \odot \lambda_M: (U_0 \otimes N) \odot (U_0 \otimes M) \rightarrow N \odot M$:



Next, we compute the right, down and then left route:



Diagrams 7 and 9 of 11. Set up and solved.

$$\begin{array}{ccc}
 F: (\mathbf{A}, +, 0) \rightarrow (\mathbf{Cat}, \times, 1) & & \\
 a \in (\mathbf{A}, +, 0) & & \\
 0 + a \rightarrow 0 + a \leftarrow 0 + a & \begin{array}{c} U_{0+a} \xrightarrow{\mu} U_0 \otimes U_a \\ \searrow U_{\lambda_a} \quad \downarrow \lambda_{U_a} \\ U_a \end{array} & \begin{array}{c} 0 + a \rightarrow 0 + a \leftarrow 0 + a \\ \phi_{0,a}(\perp_0, \perp_a) \in F(0 + a) \\ a \rightarrow a \leftarrow a \\ \perp_a \in F(a) \end{array} \\
 \perp_{0+a} \in F(0+a) & &
 \end{array}$$

Right and then down:

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \uparrow \phi_{0,0} & & \nearrow \phi_{0,a} & \\
 1 \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_a)} & F(0) \times F(a) & \xrightarrow{\phi_{0,a}} F(0+a) \\
 \phi \searrow & \downarrow \phi_{0,0} & \phi_{!_0, !_a} \cong & \downarrow \phi_{0,a} & \downarrow F(\text{id}) \\
 & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 & & & & \downarrow F(\lambda)
 \end{array}$$

Diagonally:

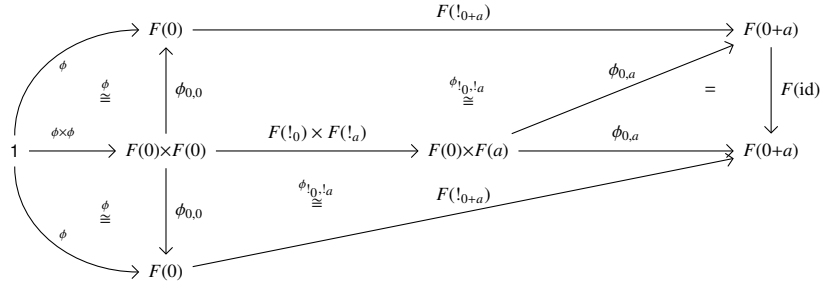
$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \downarrow F(\text{id}) & & \nearrow F(!_{0+a}) & \\
 1 \xrightarrow{\phi} & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 \phi \searrow & & & & \downarrow F(\lambda)
 \end{array}$$

Removing the lower right \cong which is the same in each diagram:

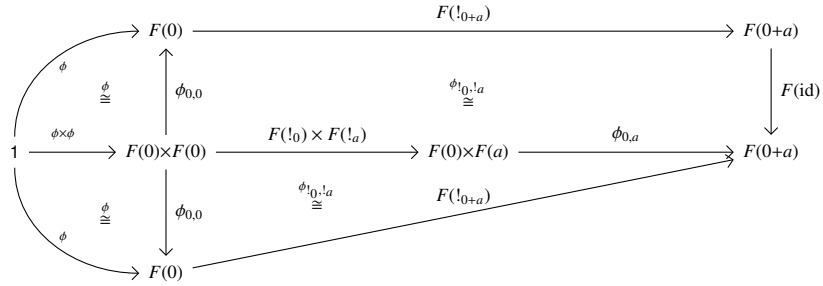
$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \uparrow \phi_{0,0} & & \nearrow \phi_{0,a} & \\
 1 \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_a)} & F(0) \times F(a) & \xrightarrow{\phi_{0,a}} F(0+a) \\
 \phi \searrow & \downarrow \phi_{0,0} & \phi_{!_0, !_a} \cong & \downarrow \phi_{0,a} & \downarrow F(\text{id}) \\
 & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 & & & & \downarrow F(\lambda)
 \end{array}$$

$$\begin{array}{ccccc}
 & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) & \\
 \phi \nearrow & \downarrow F(\text{id}) & & \nearrow F(!_{0+a}) & \\
 1 \xrightarrow{\phi} & F(0) & \xrightarrow{F(!_a)} & F(a) & \\
 \phi \searrow & & & & \downarrow F(\lambda)
 \end{array}$$

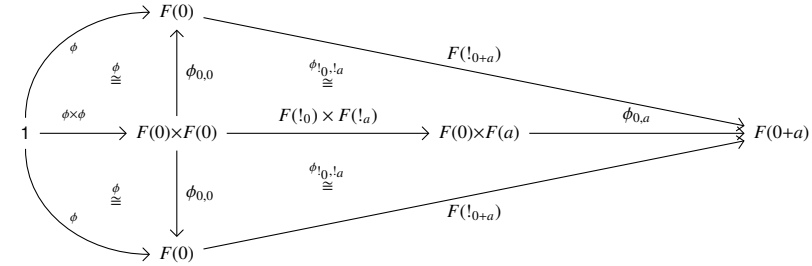
The second diagram is then an identity, so our problem reduces to showing that the following diagram is also an identity:



Removing the diagonal $\phi_{0,a}$:



This diagram is clearly the same as:



The top half of the diagram and the bottom half of the diagram are the same but in reverse order. **I believe these cancel and yield an identity.**

Diagram 11 of 11. Set up but not solved.

$$a, b \in (A, +, 0)$$

$$\begin{array}{ccc} U_{a+b} & \xrightarrow{\mu_{a,b}} & U_a \otimes U_b \\ U_\beta \downarrow & & \downarrow \beta' \\ U_{b+a} & \xrightarrow{\mu_{b,a}} & U_b \otimes U_a \end{array}$$

The top two underlying cospans are

$$a + b \rightarrow a + b \leftarrow a + b$$

and the bottom two underlying cospans are

$$b + a \rightarrow b + a \leftarrow b + a$$

Right and then down:

$$\begin{array}{ccccccc} & & F(0) & \xrightarrow{F(!_{a+b})} & F(a+b) & & \\ & \nearrow \phi & \uparrow \phi_{0,0} & & \nearrow \phi_{a,b} & & \\ 1 & \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_a) \times F(!_b)} & F(a) \times F(b) & \xrightarrow{\phi_{a,b}} & F(a+b) \\ & \searrow \phi \times \phi & \downarrow \beta & & \downarrow \beta & & \downarrow F(\text{id}) \\ & & F(0) \times F(0) & \xrightarrow{F(!_b) \times F(!_a)} & F(b) \times F(a) & \xrightarrow{\phi_{b,a}} & F(b+a) \end{array}$$

$\phi_{!_a, !_b} \cong \mu_{a,b} \cong \phi_{b,a}$

Down and then right:

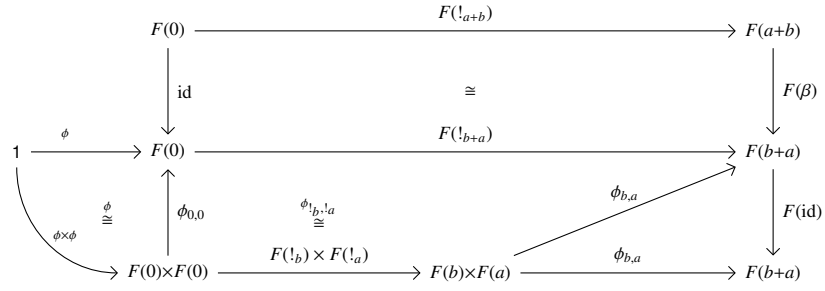
$$\begin{array}{ccccccc} & & F(0) & \xrightarrow{F(!_{a+b})} & F(a+b) & & \\ & \nearrow \phi & \downarrow \text{id} & & \downarrow F(\beta) & & \\ 1 & \xrightarrow{\phi} & F(0) & \xrightarrow{F(!_{b+a})} & F(b+a) & & \\ & \searrow \phi \times \phi & \uparrow \phi_{0,0} & & \nearrow \phi_{b,a} & & \\ & & F(0) \times F(0) & \xrightarrow{F(!_b) \times F(!_a)} & F(b) \times F(a) & \xrightarrow{\phi_{b,a}} & F(b+a) \end{array}$$

$\phi_{!_b, !_a} \cong \mu_{b,a} \cong \phi_{b,a}$

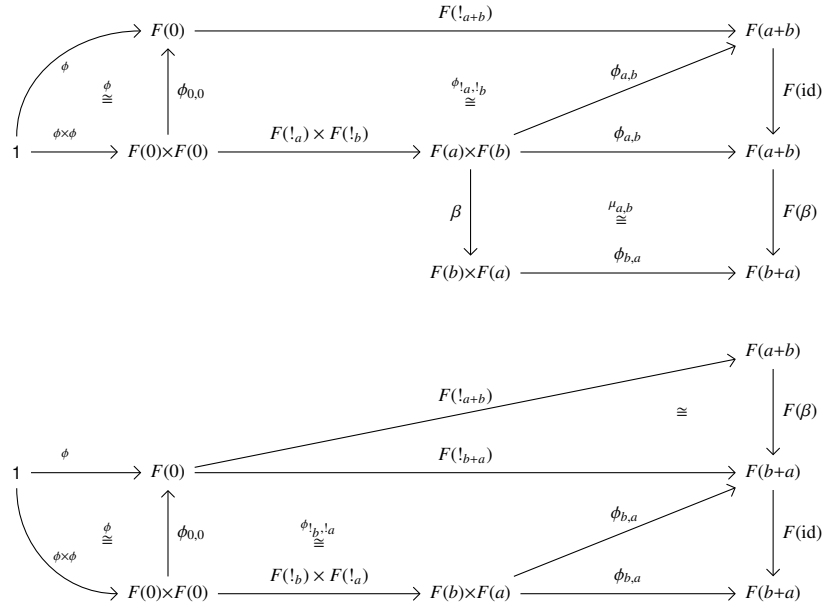
First we remove the lower left commuting quarter circle in the ‘right and then down’ diagram as well as the upper left commuting quarter circle in the ‘down and then right’ diagram:

$$\begin{array}{ccccccc} & & F(0) & \xrightarrow{F(!_{a+b})} & F(a+b) & & \\ & \nearrow \phi & \uparrow \phi_{0,0} & & \nearrow \phi_{a,b} & & \\ 1 & \xrightarrow{\phi \times \phi} & F(0) \times F(0) & \xrightarrow{F(!_a) \times F(!_b)} & F(a) \times F(b) & \xrightarrow{\phi_{a,b}} & F(a+b) \\ & \searrow \phi \times \phi & \downarrow \beta & & \downarrow \beta & & \downarrow F(\text{id}) \\ & & F(0) \times F(0) & \xrightarrow{F(!_b) \times F(!_a)} & F(b) \times F(a) & \xrightarrow{\phi_{b,a}} & F(b+a) \end{array}$$

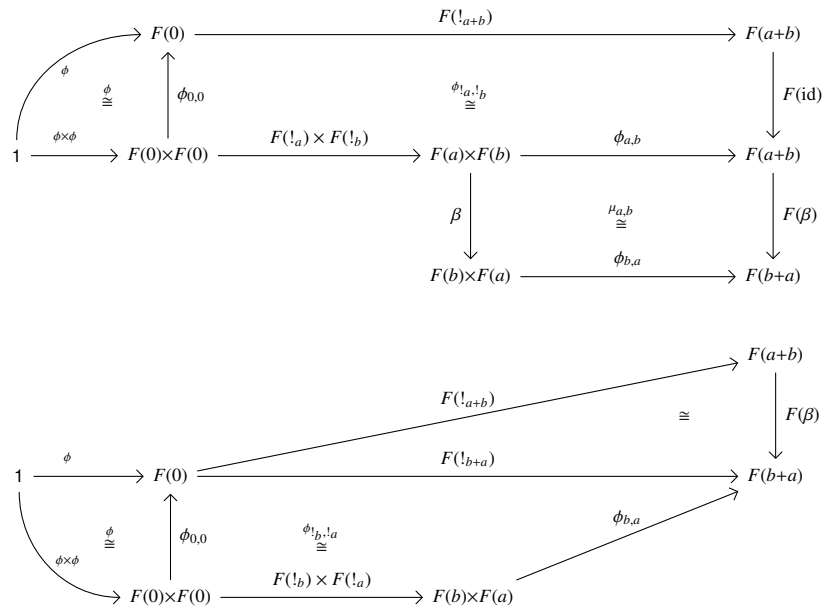
$\phi_{!_a, !_b} \cong \mu_{a,b} \cong \phi_{b,a}$



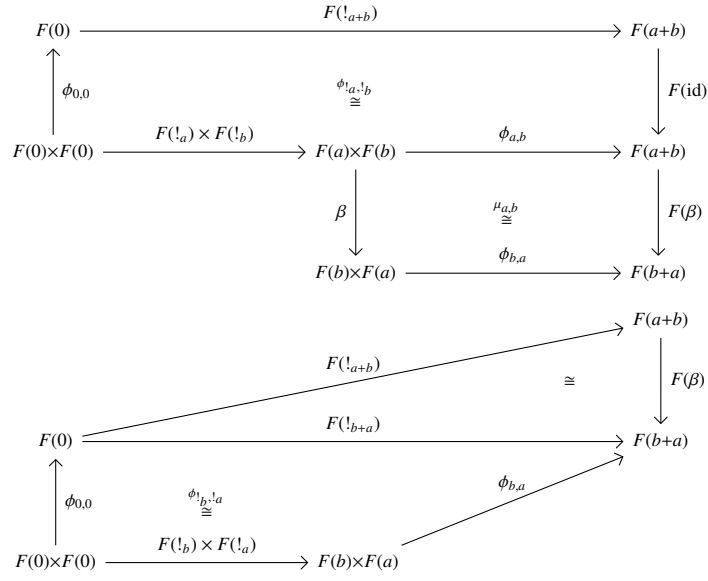
Next we remove the lower left commuting square with sides β in the first diagram and remove the $\text{id}: F(0) \rightarrow F(0)$ morphism in the second diagram:



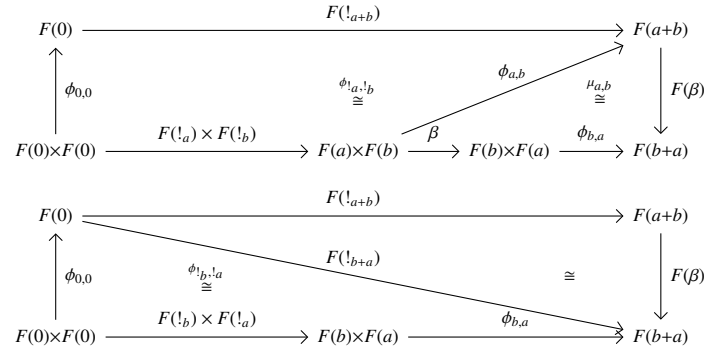
Next we assume that our pseudofunctor F is normalized, meaning that $F(\text{id})$ is an identity, and remove the commuting triangle with sides $\phi_{a,b}$ in the first diagram and the commuting triangle with sides $\phi_{b,a}$ in the second diagram:



Next we remove the left quarter circle containing \cong from each diagram:



Making each diagram into a rectangle and removing the $F(\text{id})$ from the first diagram:



Are these two diagrams the same?

Some useful maps

Given $a \in (A, +, 0)$, the map $U_{\lambda_a}: U_{0+a} \rightarrow U_a$ is given by:

$$\begin{array}{ccccc}
 \phi & \xrightarrow{\quad} & F(0) & \xrightarrow{F(!_{0+a})} & F(0+a) \\
 \downarrow 1 & & \downarrow F(\text{id}) & \searrow F(!_{0+a}) & \downarrow F(\lambda_a) \\
 \phi & \xrightarrow{\quad} & F(0) & \xrightarrow{F(!_a)} & F(a)
 \end{array}$$

\cong

where the \cong is given by pseudonaturality of F : we have a unique map in $!_a: 0 \rightarrow a$ in A but also a map $\lambda_a \circ !_a: 0 \rightarrow a$ where λ_a is the left unitor of $(A, +, 0)$, and so $F(!_a) = F(\lambda_a \circ !_a) \cong F(\lambda_a)F(!_a)$.

The left unitor $\lambda'_{U_a}: U_0 \otimes U_a \rightarrow U_a$ is given by:

$$\begin{array}{ccccc}
 \phi \times \phi & \xrightarrow{\quad} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_a)} & F(0) \times F(a) & \xrightarrow{\phi_{0,a}} & F(0+a) \\
 \downarrow 1 & & \downarrow \phi_{0,0} & \searrow \phi_{!_0, !_a} & \searrow F(!_a) & \searrow F(!_a) & \downarrow F(\lambda_a) \\
 \phi & \xrightarrow{\quad} & F(0) & \xrightarrow{F(!_a)} & F(a)
 \end{array}$$

\cong

where the \cong in the lower right is the same as the one in the first diagram.

For an arbitrary M , the left unitor $\lambda'_M: U_0 \otimes M \rightarrow M$ is given by:

$$\begin{array}{ccccc}
 \phi_0 \times x & \xrightarrow{\quad} & F(0) \times F(m) & \xrightarrow{F(!_0) \times 1} & F(0) \times F(m) & \xrightarrow{\phi_{0,m}} & F(0+m) \\
 \downarrow 1 & & \downarrow \phi_{0,m} & \searrow \phi_{!_0, !_1} & \searrow F(!_0 + 1) & \searrow F(\lambda_m) & \downarrow F(\lambda_m) \\
 x & \xrightarrow{\quad} & F(m) & \xrightarrow{\text{id}} & F(m)
 \end{array}$$

\cong

For an arbitrary M given by $a \rightarrow (m, x) \leftarrow b$, the map $\lambda_M: U_b \odot M \rightarrow M$ is given by:

$$\begin{array}{ccccc}
 x \times \phi_0 & \xrightarrow{\quad} & F(m) \times F(0) & \xrightarrow{1 \times F(!_b)} & F(m) \times F(b) & \xrightarrow{\phi_{m,b}} & F(m+b) & \xrightarrow{F(\psi)} & F(m+_b b) \\
 \downarrow 1 & & \downarrow \phi_{m,0} & \searrow \phi_{!_1, !_b} & \searrow F(1+_b) & \searrow F(\lambda_m) & \searrow F(\lambda_m) & \searrow F(\lambda_m) & \downarrow F(\kappa) \\
 x & \xrightarrow{\quad} & F(m) & \xrightarrow{\text{id}} & F(m)
 \end{array}$$

\cong

In particular, if $M = U_0$ above, then the map $\lambda_{U_0}: U_0 \odot U_0 \rightarrow U_0$ is given by:

$$\begin{array}{ccccc}
 \phi_0 \times \phi_0 & \xrightarrow{\quad} & F(0) \times F(0) & \xrightarrow{F(!_0) \times F(!_0)} & F(0) \times F(0) & \xrightarrow{\phi_{0,0}} & F(0+0) & \xrightarrow{F(\psi)} & F(0+_0 0) \\
 \downarrow 1 & & \downarrow \phi_{0,0} & \searrow \phi_{!_0, !_0} & \searrow F(!_0 + !_0) & \searrow F(\lambda_0) & \searrow F(\lambda_0) & \searrow F(\lambda_0) & \downarrow F(\kappa) \\
 \phi_0 & \xrightarrow{\quad} & F(0) & \xrightarrow{\text{id}} & F(0)
 \end{array}$$

\cong