1. Proof of Theorem 2.2

There are 11 diagrams in Shulman's definition of symmetric monoidal double category, which we check now for decorated cospan double categories.

Diagram 1 of 11. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \to m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
N_3 is given by $c' \to n_3 \leftarrow d'$ with $y_3 \in F(n_3)$.

$$\begin{tikzcd}
((M_1 \otimes N_1) \odot (M_2 \otimes N_2)) \odot (M_3 \otimes N_3) \arrow[r, "\chi \odot 1"] \arrow[d, "\alpha"'] & ((M_1 \odot M_2) \otimes (N_1 \odot N_2)) \odot (M_3 \otimes N_3) \arrow[d, "\chi"] \\
(M_1 \otimes N_1) \odot ((M_2 \otimes N_2) \odot (M_3 \otimes N_3)) \arrow[d, "1 \odot \chi"'] & ((M_1 \odot M_2) \odot M_3) \otimes ((N_1 \odot N_2) \odot N_3) \arrow[d, "\alpha \otimes \alpha"] \\
(M_1 \otimes N_1) \odot ((M_2 \odot M_3) \otimes (N_2 \odot N_3)) \arrow[r, "\chi"] & (M_1 \odot (M_2 \odot M_3)) \otimes (N_1 \odot (N_2 \odot N_3))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{(m_1+_b m_2)+(n_1+_{b'}n_2),m_3+n_3}(\phi_{m_1+_b m_2,n_1+_{b'}n_2}((F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))),\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+_b m_2)+(n_1+_{b'}n_2))+_{c+c'}(m_3+n_3))$$

$$\phi_{(m_1+_b m_2)+_c m_3,(n_1+_{b'}n_2+_{c'}n_3}((F(\psi\phi_{m_1+_b m_2,m_3}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),x_3),(F(\psi)\phi_{n_1+_{b'}n_2,n_3}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),y_3))) \in F(((m_1+_b m_2)+_c m_3)+((n_1+_{b'}n_2)+_{c'}n_3))$$

$$\phi_{m_1+_b(m_2+_c m_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+_c m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+_b(m_2+_c m_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

(2) Down and then right:

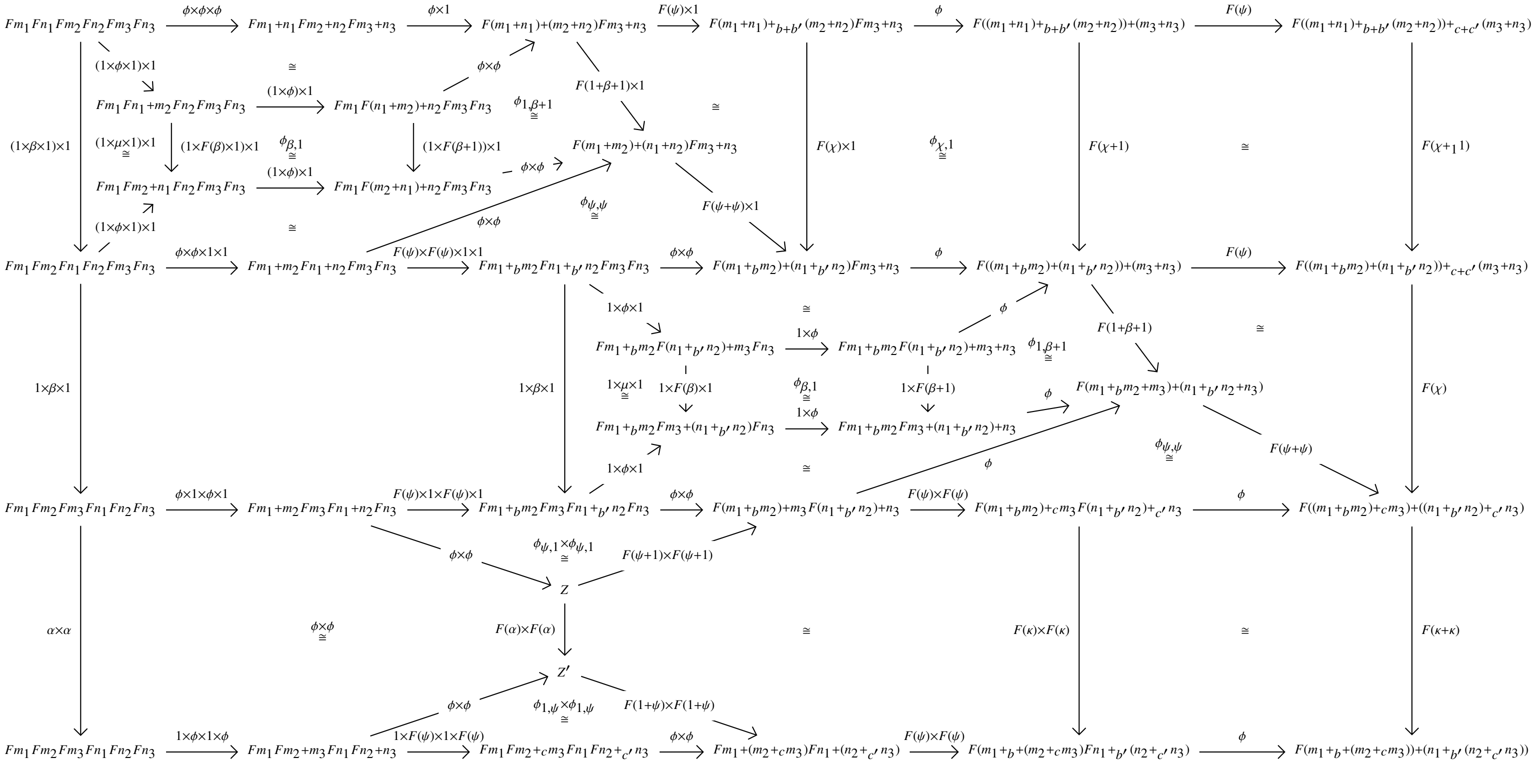
$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3)) \in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{m_1+n_1,(m_2+n_2)+_{c+c'}(m_3+n_3)}((\phi_{m_1,n_1}(x_1,y_1),F(\psi)\phi_{m_2+n_2,m_3+n_3}(\phi_{m_2,n_2}(x_2,y_2),\phi_{m_3,n_3}(x_3,y_3)))) \in F((m_1+n_1)+_{b+b'}((m_2+n_2)+_{c+c'}(m_3+n_3)))$$

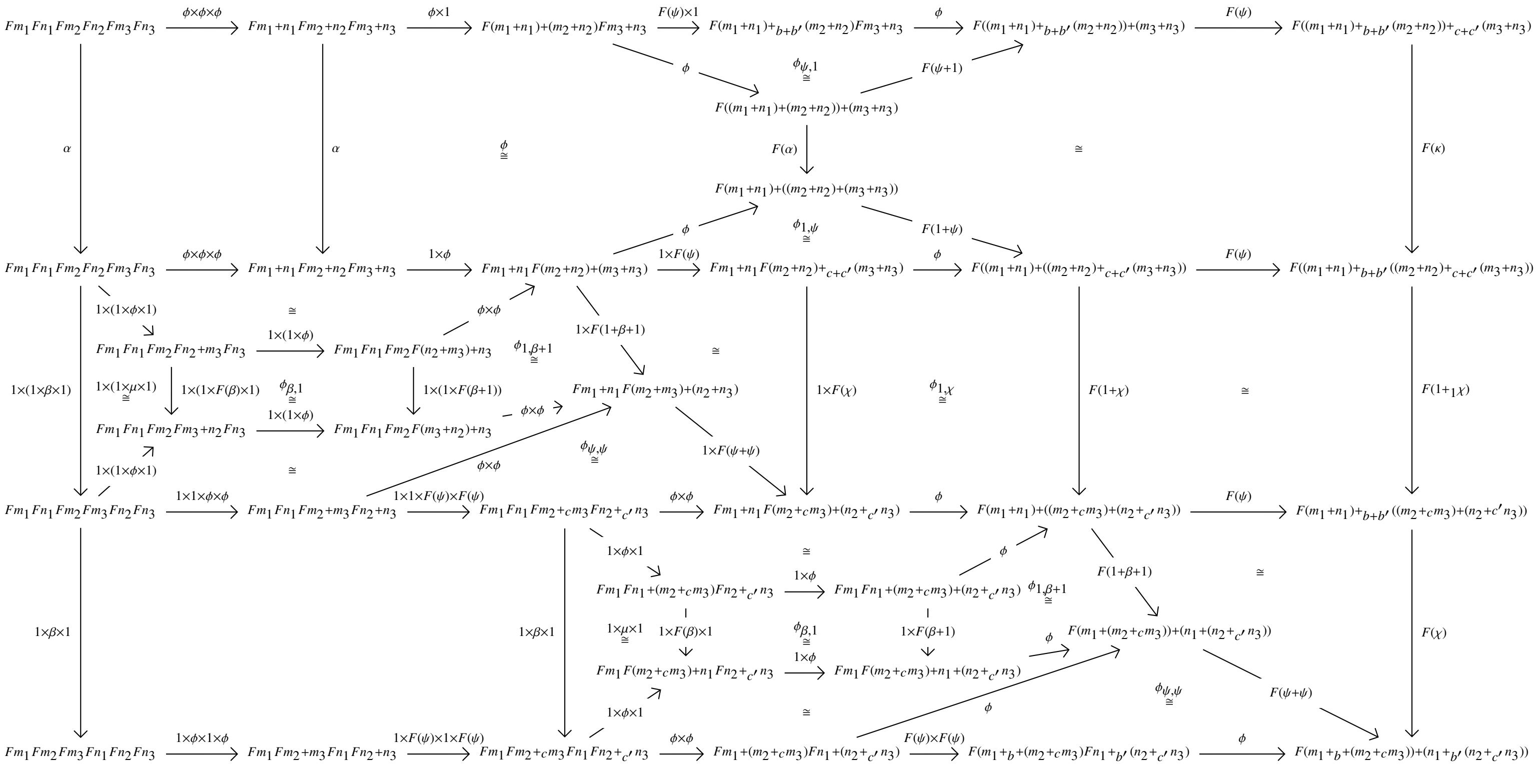
$$F(\psi)\phi_{m_1+n_1,(m_2+cm_3)+(n_2+_{c'}n_3)}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2+cm_3,n_2+_{c'}n_3}((F\psi)\phi_{m_2,m_3}(x_2,x_3),F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+n_1)+_{b+b'}((m_2+_c m_3)+(n_2+_{c'}n_3)))$$

$$\phi_{m_1+_b(m_2+_c m_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+_c m_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3))) \in F((m_1+_b(m_2+_c m_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

Right and then down (omitting morphisms emanating out of 1 on the left due to space restrictions):



Down and then right (omitting morphisms emanating out of 1 on the left due to space restrictions):



Diagrams 2 and 3 of 11. Set up but not solved.

$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} a'$ with $x \in F(m)$.

$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} b'$ with $y \in F(n)$.

$U_a \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{\mathrm{id}} a \xleftarrow{\mathrm{id}} a$ with $\perp_a \in F(a)$.

$$
\begin{tikzcd}
\begin{array}{c} a{+}b \to (a{+}b)+_{a+b}(m{+}n) \leftarrow a'{+}b' \\ F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\perp_{a+b}) \in F((a{+}b)+_{a+b}(m{+}n)) \end{array} & (M\otimes N)\odot(U_{a+b}) \arrow[r, "\mathrm{id}\odot\mu"] \arrow[d, "r"'] & (M\otimes N)\odot(U_a\otimes U_b) \arrow[d, "\chi"] & \begin{array}{c} a{+}b \to (a{+}b)+_{a+b}(m{+}n) \leftarrow a'{+}b' \\ F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\phi_{a,b}(\perp_a,\perp_b)) \in F((a{+}b)+_{a+b}(m{+}n)) \end{array} \\
\begin{array}{c} a{+}b \to m{+}n \leftarrow a'{+}b' \\ \phi_{m,n}(x,y) \in F(m{+}n) \end{array} & M\otimes N & (M\odot U_a)\otimes(N\odot U_b) \arrow[l, "r\otimes r"'] & \begin{array}{c} a{+}b \to (a+_a m)+(b+_b n) \leftarrow a'{+}b' \\ \phi_{a+_a m,b+_b n}(F(\psi)\phi_{a,m}(\perp_a,x),F(\psi)\phi_{b,n}(\perp_b,y)) \in F((a+_a m)+(b+_b n)) \end{array}
\end{tikzcd}
$$

Down:

$$
\begin{tikzcd}
 & F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m{+}n) \arrow[r, "F(!_{a+b})\times 1"] \arrow[dr, "\phi_{0,m+n}"'] & F(a{+}b)\times F(m{+}n) \arrow[r, "\phi_{a+b,m+n}"] \arrow[d, phantom, "\phi_{!_{a+b},1}\ \cong"] & F((a{+}b)+(m{+}n)) \arrow[r, "F(\psi)"] & F((a{+}b)+_{a+b}(m{+}n)) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0\times(x\times y)"] \arrow[dr, bend right, "x\times y"'] & & \lambda_{M\otimes N}\ \cong & F(0+(m{+}n)) \arrow[ur, "F(!_{a+b}+1)"'] \arrow[drr, "F(\ell)"] & \cong & \\
 & F(m)\times F(n) \arrow[rrrr, "\phi_{m,n}"] & & & & F(m{+}n)
\end{tikzcd}
$$

Right, down and then left:

$$
\begin{tikzcd}
 & F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times F(m{+}n) \arrow[rr, "F(!_{a+b})\times 1"] & & F(a{+}b)\times F(m{+}n) \arrow[r, "\phi_{a+b,m+n}"] & F((a{+}b)+(m{+}n)) \arrow[r, "F(\psi)"] & F((a{+}b)+_{a+b}(m{+}n)) \arrow[d, "F(\mathrm{id})"] \\
 \phi\ \cong & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[u, "\phi_{0,0}\times 1"'] \arrow[r, "(F(!_a)\times F(!_b))\times 1"] \arrow[u, phantom, shift right=12ex, "\phi_{!_a,!_b}\ \cong"] & (F(a)\times F(b))\times(F(m)\times F(n)) \arrow[urr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[rr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[d, "1\times\phi\times 1"'] \arrow[dddd, bend right=60, "1\times\beta\times 1"'] & \cong & F(a{+}b)\times F(m{+}n) \arrow[r, "\phi_{a+b,m+n}"] \arrow[u, phantom, "\cong"] & F((a{+}b)+(m{+}n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"] & F((a{+}b)+_{a+b}(m{+}n)) \arrow[dddd, "F(\kappa')"] \\
 & 1\times\mu\times 1\ \cong & F(a)\times F(b{+}m)\times F(n) \arrow[r, "1\times\phi"] \arrow[r, phantom, shift right=3ex, "\phi_{\beta,1}\ \cong"] \arrow[dd, "1\times F(\beta)\times 1"'] & F(a)\times F((b{+}m)+n) \arrow[urr, "\phi"] \arrow[dd, "1\times F(\beta+1)"] & \phi_{1,\beta+1}\ \cong & & \cong \\
1 \arrow[uuur, bend left, "\phi_0\times(x\times y)"] \arrow[uur, "(\phi_0\times\phi_0)\times(x\times y)"'] \arrow[ddr, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ddddr, bend right, "x\times y"'] & & & & F((a{+}m)+(b{+}n)) \arrow[ddrr, bend left, "F(\psi+\psi)"] & \phi_{\psi,\psi}\ \cong & \\
 & & F(a)\times F(m{+}b)\times F(n) \arrow[r, "1\times\phi"] & F(a)\times F((m{+}b)+n) \arrow[ur, "\phi"] & & & \\
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_a)\times 1)\times(F(!_b)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(a)\times F(m))\times(F(b)\times F(n)) \arrow[u, "1\times\phi\times 1"] \arrow[rr, "\phi_{a,m}\times\phi_{b,n}"] \arrow[u, phantom, shift left=12ex, "\cong"] \arrow[d, phantom, shift left=10ex, "\phi_{!_a,1}\times\phi_{!_b,1}\ \cong"] & & F(a{+}m)\times F(b{+}n) \arrow[uu, "\phi\times\phi"] \arrow[r, "F(\psi)\times F(\psi)"] & F(a+_a m)\times F(b+_b n) \arrow[r, "\phi_{a+_a m,b+_b n}"] & F((a+_a m)+(b+_b n)) \arrow[dd, "F(\kappa'')"] \\
 & \lambda_M\times\lambda_N\ \cong & F(0{+}m)\times F(0{+}n) \arrow[rr, "F(\ell)\times F(\ell)"'] \arrow[urr, "F(!_a+1)\times F(!_b+1)"] & & F(m)\times(n) \arrow[drr, "\phi_{m,n}"] & \cong & \\
 & F(m)\times F(n) \arrow[rrrrr, "\phi_{m,n}"] & & & & & F(m{+}n)
\end{tikzcd}
$$

Diagram 4 of 11. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
P_1 is given by $a'' \to p_1 \leftarrow b''$ with $z_1 \in F(p_1)$.
P_2 is given by $b'' \to p_2 \leftarrow c''$ with $z_2 \in F(p_2)$.

$$
\begin{tikzcd}
((M_1\otimes N_1)\otimes P_1)\odot((M_2\otimes N_2)\otimes P_2) \arrow[r, "\alpha\odot\alpha"] \arrow[d, "\chi"'] & (M_1\otimes(N_1\otimes P_1))\odot(M_2\otimes(N_2\otimes P_2)) \arrow[d, "\chi"] \\
((M_1\otimes N_1)\odot(M_2\otimes N_2))\otimes(P_1\odot P_2) \arrow[d, "\chi\otimes 1"'] & (M_1\odot M_2)\otimes((N_1\otimes P_1)\odot(N_2\otimes P_2)) \arrow[d, "1\otimes\chi"] \\
((M_1\odot M_2)\otimes(N_1\odot N_2))\otimes(P_1\odot P_2) \arrow[r, "\alpha"] & (M_1\odot M_2)\otimes((N_1\odot N_2)\otimes(P_1\odot P_2))
\end{tikzcd}
$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2))\in F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$F(\psi)\phi_{m_1+(n_1+p_1),m_2+(n_2+p_2)}(\phi_{m_1,n_1+p_1}(x_1,\phi_{n_1,p_1}(y_1,z_1)),\phi_{m_2,n_2+p_2}(x_2,\phi_{n_2,p_2}(y_2,z_2)))\in F((m_1+(n_1+p_1))+_{b+(b'+b'')}(m_2+(n_2+p_2)))$$

$$\phi_{m_1+_b m_2,(n_1+p_1)+_{b'+b''}(n_2+p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)(\phi_{n_1+p_1,n_2+p_2}(\phi_{n_1,p_1}(y_1,z_1),\phi_{n_2,p_2}(y_2,z_2)))\in F((m_1+_b m_2)+((n_1+p_1)+_{b'+b''}(n_2+p_2)))$$

$$\phi_{m_1+_b m_2,(n_1+_{b'}n_2)+(p_1+_{b''}p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'}n_2,p_1+_{b''}p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2)))\in F((m_1+_b m_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2)))$$

(2) Down and then right:

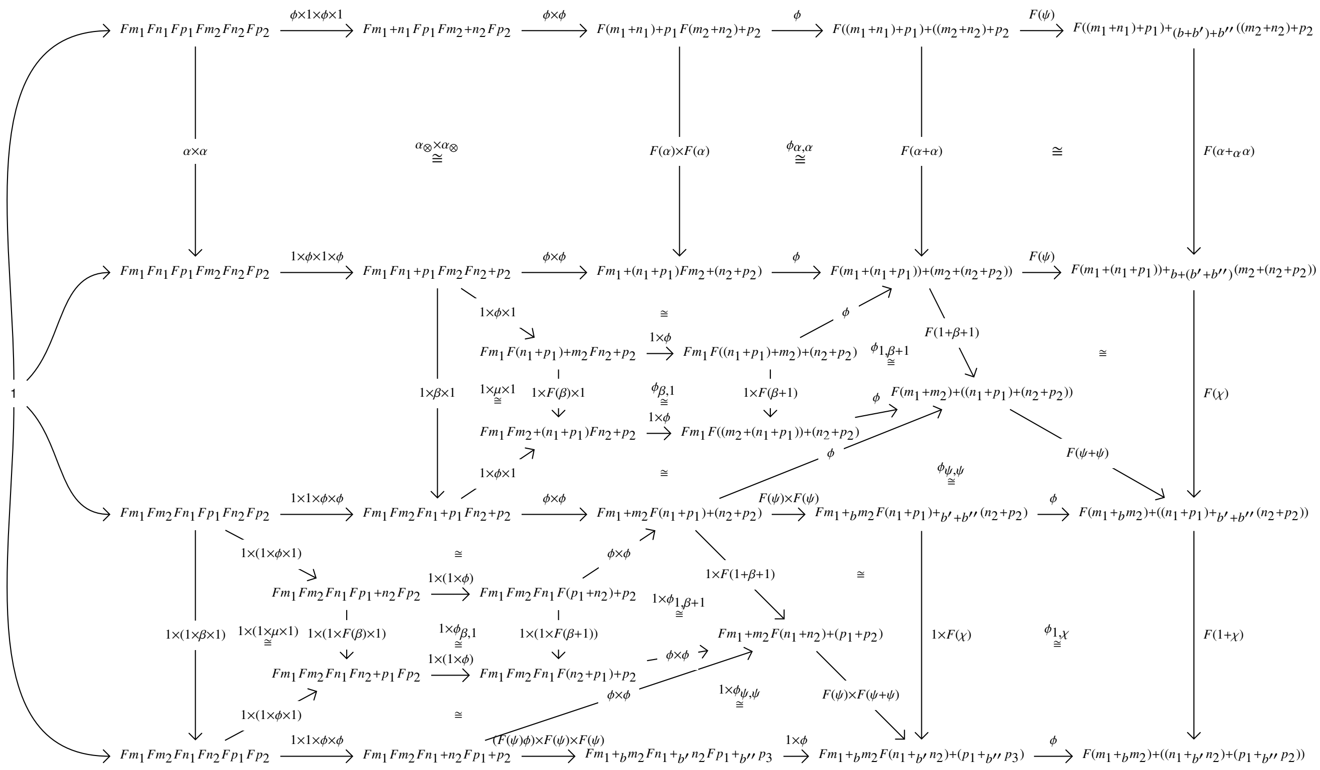
$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2))\in F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),p_1+_{b''}p_2}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2))\in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+(p_1+_{b''}p_2))$$

$$\phi_{(m_1+_b m_2)+(n_1+_{b'}n_2),p_1+_{b''}p_2}(\phi_{m_1+_b m_2,n_1+_{b'}n_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2))\in F(((m_1+_b m_2)+(n_1+_{b'}n_2))+(p_1+_{b''}p_2))$$

$$\phi_{m_1+_b m_2,(n_1+_{b'}n_2)+(p_1+_{b''}p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'}n_2,p_1+_{b''}p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2)))\in F((m_1+_b m_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2)))$$

Right and then down:



Down and then right:

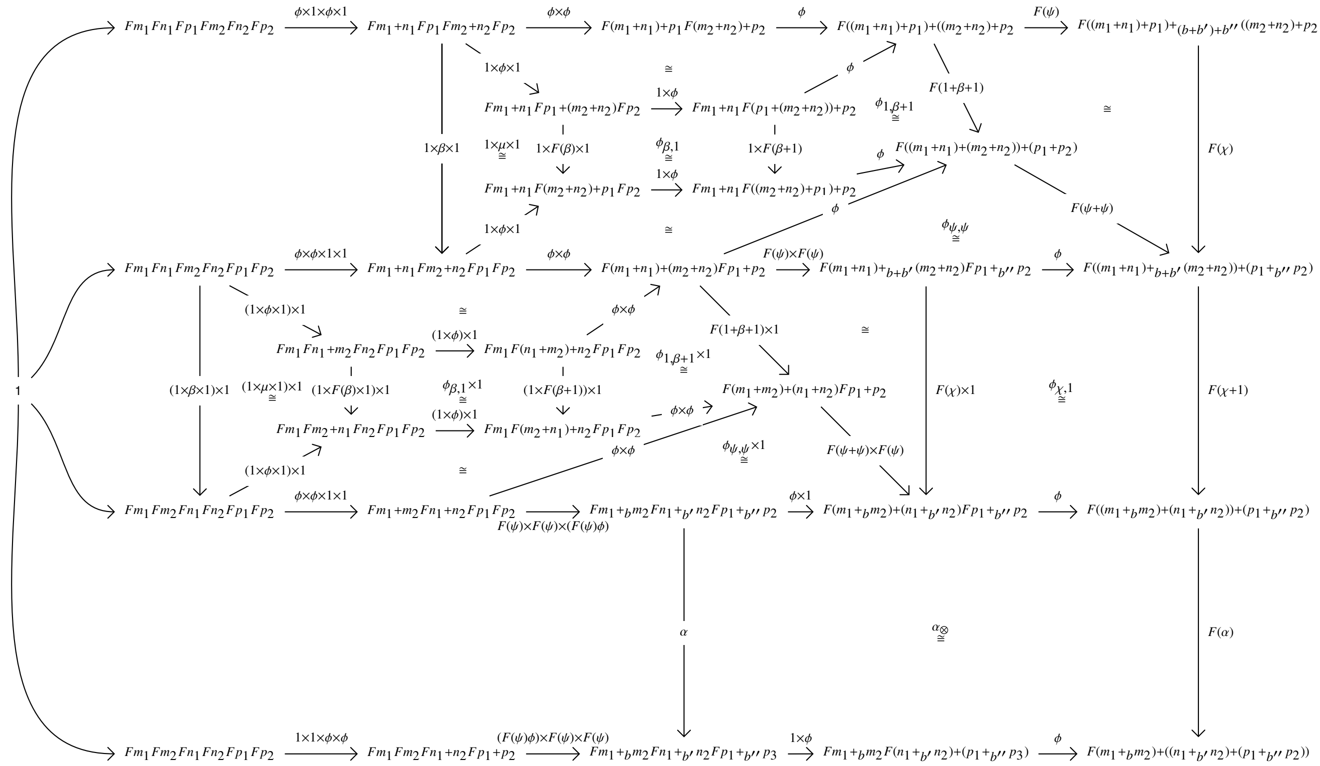


Diagram 5 of 11. Set up but not solved.

$$a, b, c \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
\bot_{(a+b)+c} \in F((a+b)+c) & U_{(a+b)+c} \arrow[r, "\mu_\beta"] \arrow[d, "\mu"] & U_{a+(b+c)} \arrow[d, "\mu"] & \bot_{a+(b+c)} \in F(a+(b+c)) \\
\phi_{a+b,c}(\bot_{a+b}, \bot_c) \in F((a+b)+c) & U_{a+b} \otimes U_c \arrow[d, "\mu"] & U_a \otimes U_{b+c} \arrow[d, "\mu"] & \phi_{a,b+c}(\bot_a, \bot_{b+c}) \in F(a+(b+c)) \\
\phi_{a+b,c}(\phi_{a,b}(\bot_a, \bot_b), \bot_c) \in F((a+b)+c) & (U_a \otimes U_b) \otimes U_c \arrow[r, "\beta'"] & U_a \otimes (U_b \otimes U_c) & \phi_{a,b+c}(\bot_a, \phi_{b,c}(\bot_b, \bot_c)) \in F(a+(b+c))
\end{tikzcd}$$

All the left cospans are $(a + b) + c \to (a + b) + c \leftarrow (a + b) + c$ and all the right cospans are $a + (b + c) \to a + (b + c) \leftarrow a + (b + c)$.

Right and then down:

$$\begin{tikzcd}
& F(0) \arrow[rrrr, "F(!_{(a+b)+c})"] \arrow[rrrrd, phantom, "\cong"] & & & & F((a+b)+c) \arrow[d, "F(\alpha)"] \\
1 \arrow[ur, bend left, "\phi_0"] \arrow[r, "\phi_0"] \arrow[dr, "\phi_0 \times \phi_0"'] \arrow[ddr, bend right, "\phi_0 \times (\phi_0 \times \phi_0)"'] \arrow[rd, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[rrrr, "F(!_{a+(b+c)})"] \arrow[rrrrd, phantom, "\overset{\phi_{!_a, !_{b+c}}}{\cong}"] & & & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
& F(0) \times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a) \times F(!_{b+c})"] \arrow[rrd, phantom, "\overset{\phi_{!_b, !_c}}{\cong}"] & F(a) \times F(b+c) \arrow[rrr, "\phi_{a,b+c}"] & & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
& F(0) \times (F(0) \times F(0)) \arrow[u, "1 \times \phi_{0,0}"] \arrow[rr, "F(!_a) \times (F(!_b) \times F(!_c))"] & & F(a) \times (F(b) \times F(c)) \arrow[ul, "1 \times \phi_{b+c}"'] \arrow[r, "1 \times \phi_{b+c}"] & F(a) \times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
& F(0) \arrow[rrrr, "F(!_{(a+b)+c})"] \arrow[rrrrd, phantom, "\overset{\phi_{!_{a+b}, !_c}}{\cong}"] & & & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
& F(0) \times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_{a+b}) \times F(!_c)"] \arrow[rd, phantom, "\overset{\phi_{!_a, !_b}}{\cong}"] & F(a+b) \times F(c) \arrow[rrr, "\phi_{a+b,c}"] & & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[uur, bend left, "\phi_0"] \arrow[ur, "\phi_0"] \arrow[r, "\phi_0^2 \times \phi_0"'] \arrow[dr, bend right, "\phi_0 \times (\phi_0 \times \phi_0)"'] \arrow[ur, phantom, "\overset{\phi}{\cong}"] & (F(0) \times F(0)) \times F(0) \arrow[u, "\phi_{0,0}"] \arrow[d, "\alpha'"] \arrow[r, "(F(!_a) \times F(!_b)) \times F(!_c)"] & (F(a) \times F(b)) \times F(c) \arrow[u, "\phi_{a,b} \times 1"] \arrow[rr, "\phi_{a,b} \times 1"] \arrow[dr, "\alpha'"] & & F(a+b) \times F(c) \arrow[r, "\phi_{a+b,c}"] \arrow[d, phantom, "\cong"] & F(a+(b+c)) \arrow[d, "F(\alpha)"] \\
& F(0) \times (F(0) \times F(0)) \arrow[rr, "F(!_a) \times (F(!_b) \times F(!_c))"] & & F(a) \times (F(b) \times F(c)) \arrow[r, "1 \times \phi_{b+c}"] & F(a) \times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Diagrams 6 and 8 of 11. Set up but not solved.
$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} c$ with $y \in F(n)$.
$U_0 \in F\mathbb{C}\mathbf{sp}$ is given by $0 \xrightarrow{!} 0 \xleftarrow{!} 0$ with $\perp_0 \in F(0)$.

$$\begin{tikzcd}
\begin{array}{c} 0+a\to(0+m)+_{0+b}(0+n)\leftarrow 0+c \\ (\perp_0\oplus y)\odot(\perp_0\oplus x)\in F((0+m)+_{0+b}(0+n)) \end{array} & (U_0\otimes N)\odot(U_0\otimes M) \arrow[r, "\chi"] \arrow[d, "\lambda_N \odot \lambda_M"'] & (U_0\odot U_0)\otimes(N\odot M) \arrow[d, "\lambda_{U_0}\otimes \mathrm{id}_{N\odot M}"] & \begin{array}{c} 0+a\to(0+_0 0)+(m+_b n)\leftarrow 0+c \\ (\perp_0\odot\perp_0)\oplus(y\odot x)\in F((0+_0 0)+(m+_b n)) \end{array} \\
\begin{array}{c} a\to m+_b n\leftarrow c \\ y\odot x\in F(m+_b n) \end{array} & N\odot M & U_0\otimes(N\odot M) \arrow[l, "\lambda_{N\odot M}"'] & \begin{array}{c} 0+a\to 0+(m+_b n)\leftarrow 0+c \\ \perp_0\oplus(y\odot x)\in F(0+(m+_b n)) \end{array}
\end{tikzcd}$$

Down:

$$\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\phi_{!_0,1}\times\phi_{!_0,1}\ \cong"] & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[dd, "F(\ell+\ell)"] & F((0+m)+_{0+b}(0+n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[rr, phantom, "\lambda_M\times\lambda_N\ \cong"] & & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & \arrow[r, phantom, "\phi_{\ell,\ell}\ \cong"] & \arrow[r, phantom, "\cong"] & {} \\
 & F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rrr, "\phi_{0,m}\times\phi_{0,n}"] \arrow[dr, "1\times\phi\times 1"] \arrow[ddd, bend right, "1\times\beta\times 1"'] & & & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[d, phantom, "\cong"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"] & F((0+m)+_{0+b}(0+n)) \arrow[ddd, "F(\chi)"] \\
 & & \arrow[d, phantom, "1\times\mu\times 1\ \cong"] & F(0)\times F(m+0)\times F(n) \arrow[r, "1\times\phi"] \arrow[dl, "1\times F(\beta)\times 1"'] \arrow[d, phantom, "\phi_{\beta,1}\ \cong"] & F(0)\times F((m+0)+n) \arrow[urr, "\phi"] \arrow[dl, "1\times F(\beta+1)"] & \arrow[d, phantom, "\phi_{1,\beta+1}\ \cong"] & & \\
 & & F(0)\times F(0+m)\times F(n) \arrow[r, "1\times\phi"] & F(0)\times F((0+m)+n) \arrow[rr, "\phi"] \arrow[d, phantom, "\cong"] & & F((0+0)+(m+n)) \arrow[drr, bend left, "F(\psi+\psi)"] \arrow[dr, phantom, "\phi_{\psi,\psi}\ \cong"] & & \\
 & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times 1)"'] \arrow[dr, "\phi_{0,0}\times 1"'] & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[rrr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[u, "1\times\phi\times 1"] \arrow[d, phantom, "\phi_{!_0,!_0}\ \cong"] & & & F(0+0)\times F(m+n) \arrow[r, "F(\psi)\times F(\psi)"] \arrow[u, "\phi"] & F(0+_0 0)\times F(m+_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[dd, "F(\kappa)\times 1"] \arrow[dr, phantom, "\phi_{\kappa,1}\ \cong"] & F((0+_0 0)+(m+_b n)) \arrow[dd, "F(\kappa+1)"] \\
1 \arrow[uuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"'] \arrow[dddr, bend right, "x\times y"'] \arrow[r, phantom, "\lambda_{U_0}\ \cong"] & {} & F(0+0)\times(F(m)\times F(n)) \arrow[urrr, "F(!_0+!_0)\times\phi_{m,n}"'] \arrow[drrrr, "F(\ell)\times(F(\psi)\phi_{m,n})"'] \arrow[rrr, phantom, "\cong"] & & & {} & & \\
 & F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m+n)) \arrow[rrr, "1\times F(\psi)"] \arrow[d, phantom, "\lambda_{N\odot M}\ \cong"] & & & F(0)\times F(m+_b n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m+_b n}"'] & F(0)\times F(m+_b n) \arrow[r, "\phi_{0,m+_b n}"] \arrow[d, phantom, "\phi_{!_0,1}\ \cong"] & F(0+(m+_b n)) \arrow[dd, "F(\ell)"] \\
 & & {} & & & & F(0+(m+_b n)) \arrow[ur, "F(!_0+1)"'] \arrow[dr, "F(\ell)"] \arrow[r, phantom, "\cong"] & {} \\
 & F(m)\times F(n) \arrow[rrrr, "1"] & & & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}$$

(Scratch work) As a check, let us compute the decoration maps $\lambda_N\colon U_0 \otimes N \to N$ and $\lambda_M\colon U_0 \otimes M \to M$ and compose them via $\odot$. The maps λ_N and λ_M are given explicitly in the paper.

$\lambda_M \colon U_0 \otimes M \to M$

$$\begin{tikzcd}
 & F(0)\times F(m) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m}"'] & F(0)\times F(m) \arrow[r, "\phi_{0,m}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+m) \arrow[dd, "F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)"] \arrow[dr, bend right, "x"'] \arrow[r, phantom, "\cong"] & {} & F(0+m) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \\
 & F(m) \arrow[rr, "1"] & & F(m)
\end{tikzcd}$$

$\lambda_N \colon U_0 \otimes N \to N$

$$\begin{tikzcd}
 & F(0)\times F(n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,n}"'] & F(0)\times F(n) \arrow[r, "\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0+n) \arrow[dd, "F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times y)"] \arrow[dr, bend right, "y"'] \arrow[r, phantom, "\cong"] & {} & F(0+n) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \\
 & F(n) \arrow[rr, "1"] & & F(n)
\end{tikzcd}$$

To construct $\lambda_N \odot \lambda_M \colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$, we first tensor the above two diagrams:

$$\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[dd, "F(\ell)\times F(\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[r, phantom, "\cong"] & {} & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \\
 & F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n)
\end{tikzcd}$$

Next, we paste with a square due to pseudonaturality of ϕ:

$$\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] & F((0+m)+(0+n)) \arrow[dd, "F(\ell+\ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[r, phantom, "\cong"] & {} & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \arrow[r, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & {} \\
 & F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n)
\end{tikzcd}$$

Finally, we paste with a square due to pseudonaturality of F to obtain the map $\lambda_N \odot \lambda_M \colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$:

$$\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[dd, "F(\ell+\ell)"] & F((0+m)+_{0+b}(0+n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] \arrow[r, phantom, "\cong"] & {} & F(0+m)\times F(0+n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & {} \arrow[r, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & {} \arrow[r, phantom, "\cong"] & {} \\
 & F(m)\times F(n) \arrow[rr, "1"] & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}$$

Next, we compute the right, down and then left route:

$$
\begin{tikzcd}
 & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] & (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rr, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, "1\times\phi\times 1"] \arrow[ddd, bend right=60, "1\times\beta\times 1"'] & & F(0+m)\times F(0+n) \arrow[r, "\phi_{0+m,0+n}"] & F((0+m)+(0+n)) \arrow[r, "F(\psi)"] \arrow[dl, "F(1+\beta+1)"] & F((0+m)+_{0+b}(0+n)) \arrow[ddd, "F(\chi)"] \\
 & & F(0)\times F(m+0)\times F(n) \arrow[r, "1\times\phi"] \arrow[d, "1\times F(\beta)\times 1"] \arrow[r, phantom, bend left, "\underset{\cong}{1\mu\times 1}"] & F(0)\times F((m+0)+n) \arrow[urr, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[ur, phantom, "\cong"] & & & \\
 & & F(0)\times F(0+m)\times F(n) \arrow[r, "1\times\phi"] & F(0)\times F((0+m)+n) \arrow[r, "\phi"] & F((0+0)+(m+n)) \arrow[rr, "F(\psi+\psi)"] \arrow[u, phantom, "\underset{\cong}{\phi_{1,\beta+1}}"] & & \\
 & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times 1)"'] \arrow[ddr, "\phi_{0,0}\times 1"] & (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[rr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[u, "1\times\phi\times 1"] & & F(0+0)\times F(m+n) \arrow[r, "F(\psi)\times F(\psi)"] \arrow[u, "\phi"] & F(0+_0 0)\times F(m+_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[dd, "F(\kappa)\times 1"] \arrow[u, phantom, "\underset{\cong}{\phi_{\psi,\psi}}"] & F((0+_0 0)+(m+_b n)) \arrow[dd, "F(\kappa+1)"] \arrow[dd, phantom, bend right=60, "\underset{\cong}{\phi_{\kappa,1}}"] \\
1 \arrow[uuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"] \arrow[dddr, bend right, "x\times y"'] \arrow[rr, phantom, "\underset{\cong}{\lambda_{M\odot N}}"] & & & F(0+0)\times(F(m)\times F(n)) \arrow[uur, "F(!_0+!_0)\times\phi_{m,n}"] \arrow[drr, "F(\lambda_0)\times(F(\psi)\phi_{m,n}"] \arrow[u, phantom, "\underset{\cong}{\phi_{!_0,!_0}}"] \arrow[rr, phantom, "\cong"] & & & \\
 & F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m+n) \arrow[rr, "1\times F(\psi)"] & & F(0)\times F(m+_b n) \arrow[r, "F(!_0)\times 1"] \arrow[dr, "\phi_{0,m+_b n}"] & F(0)\times F(m+_b n) \arrow[r, "\phi_{0,m+_b n}"] \arrow[d, phantom, "\underset{\cong}{\phi_{!_0,1}}"] & F(0+(m+_b n)) \arrow[dd, "F(\ell)"] \\
 & & & \underset{\cong}{\lambda_{N\odot M}} & & F(0+(m+_b n)) \arrow[ur, "F(!_0+1)"] \arrow[dr, "F(\ell)"] \arrow[r, phantom, "\cong"] & \\
 & F(m)\times F(n) \arrow[rrr, "1"] & & & F(m)\times F(n) \arrow[r, "\phi_{m,n}"] & F(m+n) \arrow[r, "F(\psi)"] & F(m+_b n)
\end{tikzcd}
$$

$$
\begin{tikzcd}
 & F(0)\times F(0) \arrow[r, "F(!_0)\times F(!_0)"] \arrow[dr, "\phi_{0,0}"] & F(0)\times F(0) \arrow[r, "\phi_{0,0}"] & F(0+0) \arrow[r, "F(\psi)"] & F(0+_0 0) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0\times\phi_0"] \arrow[dr, bend right, "\phi_0"'] \arrow[r, phantom, "\underset{\cong}{\lambda_M}"] & & F(0+0) \arrow[ur, "F(!_0+!_0)"] \arrow[drr, "F(\lambda_0)"] \arrow[u, phantom, "\underset{\cong}{\phi_{!_0,!_0}}"] \arrow[rr, phantom, "\cong"] & & \\
 & F(0) \arrow[rrr, "\mathrm{id}"] & & & F(0)
\end{tikzcd}
$$

Diagrams 7 and 9 of 11. Should improve this, but seems okay.

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$

$$a \in (\mathsf{A},+,0)$$

$0 + a \to 0 + a \leftarrow 0 + a$

$\phi_{0,a}(\perp_0, \perp_a) \in F(0 + a)$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
& U_a
\end{tikzcd}$$

$0 + a \to 0 + a \leftarrow 0 + a$

$\perp_{0+a} \in F(0 + a)$

$a \to a \leftarrow a$

$\perp_a \in F(a)$

Right and then down:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] & & F(0{+}a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, shift left=4, "\overset{\phi}{\cong}"] \arrow[r, phantom, shift right=4, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[r, phantom, shift left=5, "\overset{\phi_{!_0,!_a}}{\cong}"] \arrow[r, phantom, shift right=5, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[ur, "\phi_{0,a}"] \arrow[r, phantom, shift left=5, "="] & F(0{+}a) \arrow[d, "F(\lambda)"] \\
& F(0) \arrow[urr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a) \arrow[ul, phantom, "\cong"]
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] \arrow[ddrr, phantom, shift left=6, "="] & & F(0{+}a) \arrow[dd, "F(\lambda)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & & & \\
& F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] \arrow[uurr, phantom, shift right=6, "\cong"] & & F(a)
\end{tikzcd}$$

Removing the lower right $\cong$ which is the same in each diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] & & F(0{+}a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, shift left=4, "\overset{\phi}{\cong}"] \arrow[r, phantom, shift right=4, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[r, phantom, shift left=5, "\overset{\phi_{!_0,!_a}}{\cong}"] \arrow[r, phantom, shift right=5, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[ur, "\phi_{0,a}"] \arrow[r, phantom, shift left=5, "="] & F(0{+}a) \\
& F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] \arrow[ddrr, phantom, shift left=6, "="] & & F(0{+}a) \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & & & \\
& F(0) \arrow[uurr, "F(!_{0+a})"] & &
\end{tikzcd}$$

The second diagram is then an identity, so our problem reduces to showing that the following diagram is also an identity:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, phantom, shift right=4ex, "\overset{\phi}{\cong}"] & \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0+a) \arrow[d, "F(\mathrm{id})"] \arrow[d, phantom, shift right=6ex, "="] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, phantom, shift right=4ex, "\overset{\phi}{\cong}"] & F(0)\times F(a) \arrow[ur, "\phi_{0,a}"] \arrow[r, "\phi_{0,a}"] \arrow[dl, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

Removing the diagonal $\phi_{0,a}$:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, phantom, shift right=4ex, "\overset{\phi}{\cong}"] & \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, phantom, shift right=4ex, "\overset{\phi}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] \arrow[dl, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

This diagram is clearly the same as:

$$\begin{tikzcd}
 & F(0) \arrow[drr, "F(!_{0+a})"] \arrow[d, phantom, shift right=4ex, "\overset{\phi}{\cong}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & & \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, phantom, shift right=4ex, "\overset{\phi}{\cong}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"'] & &
\end{tikzcd}$$

The two 2-isomorphisms in the top half of the diagram are the inverses of those in the bottom half, when read in a suitable order, and they can be shown to cancel, yielding an identity as desired.

Diagrams 7 and 9 of 11. Set up and solved.

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$
$$a \in (\mathsf{A},+,0)$$

$$0+a \to 0+a \leftarrow 0+a$$
$$\phi_{0,a}(\bot_0,\bot_a) \in F(0+a)$$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
& U_a
\end{tikzcd}$$

$$0+a \to 0+a \leftarrow 0+a$$
$$\bot_{0+a} \in F(0+a)$$
$$a \to a \leftarrow a$$
$$\bot_a \in F(a)$$

Right and then down:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi\times\phi"] \arrow[d, "\lambda^{-1}"'] \arrow[dr, phantom, "\cong"] & F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[drr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[dr, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "1"] & U_0{\times}U_a \arrow[d, "\mu"] \\
1 \arrow[r, "\phi"] \arrow[d, "1"'] & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, "1"] & & F(0{+}a) \arrow[d, "F(\lambda_a)"] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[rru, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] \arrow[rr, phantom, bend left, "\overset{F}{\cong}"] & & F(a) & U_a
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi\times\phi"] \arrow[d, "\lambda^{-1}"'] \arrow[dr, phantom, "\cong"] & F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "F(\lambda_a)"] & U_0{\times}U_a \arrow[d, "\lambda_{U_a}"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[rru, "F(!_{0+a})"] \arrow[rr, "F(!_{0+a})"] \arrow[rr, phantom, bend left, "\overset{F}{\cong}"] & & F(a) & U_a
\end{tikzcd}$$

If we remove the regions common to each diagram, and regions that strictly commute, these are clearly equal.

Right and then down:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[dr, "\phi_{0,a}"] & \\
F(0) \arrow[rr, "F(!_{0+a})"] & & F(0{+}a)
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \\
F(0) \arrow[rru, "F(!_{0+a})"] & &
\end{tikzcd}$$

Diagram 10 of 11. Set up and solved (Christina).

$M_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i_1} m_1 \xleftarrow{o_1} b$ with $x_1 \in F(m_1)$.

$M_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i_2} m_2 \xleftarrow{o_2} c$ with $x_2 \in F(m_2)$.

$N_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a' \xrightarrow{i'_1} n_1 \xleftarrow{o'_1} b'$ with $y_1 \in F(n_1)$.

$N_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b' \xrightarrow{i'_2} n_2 \xleftarrow{o'_2} c'$ with $x_2 \in F(n_2)$.

$$
\begin{tikzcd}
\begin{array}{c} a+a'\to(m_1+n_1)+_{b+b'}(m_2+n_2)\leftarrow c+c' \\ F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2))\in F((m_1+n_1)+_{b+b'}(n_1+n_2)) \end{array} & (M_2\otimes N_2)\odot(M_1\otimes N_1) \arrow[r, "\beta\odot\beta"] \arrow[d, "\chi"'] & (N_2\otimes M_2)\odot(N_1\otimes M_1) \arrow[d, "\chi"] & \begin{array}{c} a'+a\to(n_1+m_1)+_{b'+b}(n_2+m_2)\leftarrow c'+c \\ F(\psi)\phi_{n_1+m_1,n_2+m_2}(\phi_{n_1,m_1}(y_1,x_1),\phi_{n_2,m_2}(y_2,x_2))\in F((n_1+m_1)+_{b'+b}(n_1+m_2)) \end{array} \\
\begin{array}{c} a+a'\to(m_1+_b m_2)+(n_1+_{b'}n_2)\leftarrow c+c' \\ \phi_{m_1+_b m_2,n_1+_{b'}m_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))\in F((m_1+_b m_2)+(n_1+_{b'}n_2)) \end{array} & (M_2\odot M_1)\otimes(N_2\odot N_1) \arrow[r, "\beta"] & (N_2\odot N_1)\otimes(M_2\odot M_1) & \begin{array}{c} a'+a\to(n_1+_{b'}n_2)+(m_1+_b m_2)\leftarrow c'+c \\ \phi_{n_1+_{b'}n_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{m_1,m_2}(x_1,x_2))\in F((n_1+_{b'}n_2)+(m_1+_b m_2)) \end{array}
\end{tikzcd}
$$

Down and then right:

$$
\begin{tikzcd}[column sep=small]
& (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, "1\times\beta\times1"'] & & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, phantom, "\cong"] & & F((m_1+n_1)+(m_2+n_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[ddd, "F(\chi)"] \arrow[ddl, phantom, "\cong"] \\
& & F(m_1)\times F(n_1+m_2)\times F(n_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] \arrow[rrd, phantom, "\overset{\phi_{\beta,1}}{\cong}"] \arrow[dl, phantom, "\overset{1\times\mu\times1}{\cong}"] & & F(m_1)\times F((n_1+m_2)+n_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[r, phantom, "\overset{\phi_{1,\beta}}{\cong}"] & & \\
& & F(m_1)\times F(m_2+n_1)\times F(n_2) \arrow[rr, "1\times\phi"'] & \cong & F(m_1)\times F((m_2+n_1)+n_2) \arrow[r, "\phi"] & F(m_1+((m_2+n_1)+n_2)) \arrow[dr, "F(\psi+\psi)"] \arrow[d, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"] & \\
1 \arrow[r, "(x_1\times x_2)\times(y_1\times y_2)"] \arrow[ruuu, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[rd, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(m_1)\times F(m_2))\times(F(n_1)\times F(n_2)) \arrow[rr, "\phi_{m_1,m_2}\times\phi_{n_1,n_2}"'] \arrow[ur, "1\times\phi\times1"] \arrow[d, "\beta"'] & & F(m_1+m_2)\times F(n_1+n_2) \arrow[rru, "\phi_{m_1+m_2,n_1+n_2}"] \arrow[rr, "F(\psi)\times F(\psi)"] & & F(m_1+_b m_2)\times F(n_1+_{b'}n_2) \arrow[r, "\phi_{m_1+_b m_2,n_1+_{b'}n_2}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{\mu_{m_1+_b m_2,n_1+_{b'}n_2}}{\cong}"] & F((m_1+_b m_2)+(n_1+_{b'}n_2)) \arrow[d, "F(\beta)"] \\
& F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] & & F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] & & F(n_1+_{b'}n_2)\times F(m_1+_b m_2) \arrow[r, "\phi_{n_1+_{b'}n_2,m_1+_b m_2}"'] & F((n_1+_{b'}n_2)+(m_1+_b m_2))
\end{tikzcd}
$$

Right and then down:

$$
\begin{tikzcd}[column sep=small]
& (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[d, "\beta\times\beta"'] \arrow[drr, phantom, "\overset{\mu_{m_1,n_1}\times\mu_{m_2,n_2}}{\cong}"] & & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, "F(\beta)\times F(\beta)"] \arrow[drr, phantom, "\overset{\phi_{\beta,\beta}}{\cong}"] & & F((m_1+n_1)+(m_2+n_2)) \arrow[r, "F(\psi)"] \arrow[d, "F(\beta+\beta)"] \arrow[dr, phantom, "\cong"] & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[d, "F(\beta+_\beta\beta)"] \\
1 \arrow[r, "(y_1\times x_1)\times(y_2\times x_2)"] \arrow[ru, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[rddd, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(n_1)\times F(m_1))\times(F(n_2)\times F(m_2)) \arrow[rr, "\phi_{n_1,m_1}\times\phi_{n_2,m_2}"] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, "1\times\beta\times1"'] & & F(n_1+m_1)\times F(n_2+m_2) \arrow[rr, "\phi_{n_1+m_1,n_2+m_2}"] \arrow[d, phantom, "\cong"] & & F((n_1+m_1)+(n_2+m_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] & F((n_1+m_1)+_{b'+b}(n_2+m_2)) \arrow[ddd, "F(\chi)"] \arrow[ddl, phantom, "\cong"] \\
& & F(n_1)\times F(m_1+n_2)\times F(m_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] \arrow[rrd, phantom, "\overset{\phi_{\beta,1}}{\cong}"] \arrow[dl, phantom, "\overset{1\times\mu\times1}{\cong}"] & & F(n_1)\times F((m_1+n_2)+m_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[r, phantom, "\overset{\phi_{1,\beta}}{\cong}"] & & \\
& & F(n_1)\times F(n_2+m_1)\times F(m_2) \arrow[rr, "1\times\phi"'] & \cong & F(n_1)\times F((n_2+m_1)+m_2) \arrow[r, "\phi"] & F(n_1+((n_2+m_1)+m_2)) \arrow[dr, "F(\psi+\psi)"] \arrow[d, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"] & \\
& F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] \arrow[ur, "1\times\phi\times1"] & & F(n_1+n_2)\times F(m_1+m_2) \arrow[rru, "\phi_{n_1+n_2,m_1+m_2}"] \arrow[rr, "F(\psi)\times F(\psi)"'] & & F(n_1+_{b'}n_2)\times F(m_1+_b m_2) \arrow[r, "\phi_{n_1+_{b'}n_2,m_1+_b m_2}"'] & F((n_1+_{b'}n_2)+(m_1+_b m_2))
\end{tikzcd}
$$

Diagram 11 of 11. Set up but not solved.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_{a+b} \arrow[r, "\mu_{a,b}"] \arrow[d, "U_\beta"'] & U_a \otimes U_b \arrow[d, "\beta'"] \\
U_{b+a} \arrow[r, "\mu_{b,a}"] & U_b \otimes U_a
\end{tikzcd}$$

The top two underlying cospans are

$$a + b \rightarrow a + b \leftarrow a + b$$

and the bottom two underyling cospans are

$$b + a \rightarrow b + a \leftarrow b + a$$

Right and then down:

$$\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi\times\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] & F(a)\times F(b) \arrow[u, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
 & F(0)\times F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
 & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[u, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] \arrow[ur, "\phi_{b,a}"] \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

First we remove the lower left commuting quarter circle in the ‘right and then down’ diagram as well as the upper left commuting quarter circle in the ‘down and then right’ diagram:

$$\begin{tikzcd}
 & F(0+0) \arrow[rr, "F(!_a+!_b)"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] & F(a)\times F(b) \arrow[u, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
 & F(0)\times F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$
\begin{tikzcd}
& F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

Next we remove the lower left commuting square with sides β in the first diagram:

$$
\begin{tikzcd}
& F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[ur, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
& F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

Next we assume that our pseudofunctor F is normalized, meaning that $F(\mathrm{id})$ is an identitiy, and remove the commuting triangle with sides $\phi_{a,b}$ in the first diagram and the commuting triangle with sides $\phi_{b,a}$ in the second diagram:

$$
\begin{tikzcd}
& F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
& F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0+0) \arrow[rr, "F(!_b+!_a)"] & & F(b+a) \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}
$$

Next we remove the left quarter circle containing $\overset{\phi}{\cong}$ from each diagram:

$$
\begin{tikzcd}
F(0+0) \arrow[rr, "F(!_a+!_b)"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
F(0+0) \arrow[rr, "F(!_{a+b})"] \arrow[d, "F(\beta)"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
F(0+0) \arrow[rr, "F(!_{b+a})"] & & F(b+a) \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}
$$

Making each diagram into a rectangle and removing the $F(\mathrm{id})$ from the first diagram:

$$
\begin{tikzcd}
F(0+0) \arrow[rrr, "F(!_a+!_b)"] & & & F(a+b) \arrow[d, "F(\beta)"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\beta"] \arrow[urr, "\phi_{a,b}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
F(0+0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] & & F(a+b) \arrow[dd, "F(\beta)"] \\
F(0+0) \arrow[drr, "F(!_b+!_a)"] & & \cong \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"'] \arrow[u, phantom, shift left=4, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"'] & F(b+a)
\end{tikzcd}
$$

Are these two diagrams the same?

$$
\begin{tikzcd}
F(0) \arrow[rr, "F(!_{a+b})"] \arrow[drr, "F(!_{b+a})"] & & F(a+b) \arrow[d, "F(\beta)"] \arrow[d, phantom, shift right=6, "\cong"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[u, phantom, shift left=10, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$
\begin{tikzcd}
F(c+d) \arrow[rr, "F(f+g)"] \arrow[drr, "F(g+f)"] & & F(a+b) \arrow[d, "F(\beta)"] \arrow[d, phantom, shift right=6, "\cong"] \\
F(c){\times}F(d) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(g)\times F(f)"] \arrow[u, phantom, shift left=10, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}
$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[r, "F(\beta)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a)\times F(b) \arrow[u, "\phi_{a,b}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(\beta)"] & F(0+0) \arrow[r, "F(!_b+!_a)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{0,0}}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[r, "F(\beta)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[rr, bend right, "F(!_b)\times F(!_a)"'] & F(a)\times F(b) \arrow[u, "\phi_{a,b}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(\beta)"] & F(0+0) \arrow[r, "F(!_b+!_a)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{0,0}}{\cong}"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

First diagram:

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[r, "F(\beta)"] & F(b+a) \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[rr, bend right, "F(!_b)\times F(!_a)"'] & F(a)\times F(b) \arrow[u, "\phi_{a,b}"] \arrow[r, "\beta"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b)\times F(a) \arrow[u, "\phi_{b,a}"']
\end{tikzcd}$$

Second diagram:

$$\begin{tikzcd}
F(0+0) \arrow[r, "F(\beta)"] \arrow[rrd, "F(!_b+!_a)"] & F(0+0) \arrow[r, "F(!_a+!_b)"] & F(a+b) \arrow[d, "F(\beta)"] \arrow[dl, phantom, "\cong"] \\
F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"'] \arrow[u, phantom, shift left=6, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"'] & F(b+a)
\end{tikzcd}$$

Diagram 11 of 11. Set up but not solved.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_a \otimes U_b \arrow[r, "\mu_{a,b}"] \arrow[d, "\beta'"'] & U_{a+b} \arrow[d, "U_\beta"] \\
U_b \otimes U_a \arrow[r, "\mu_{b,a}"] & U_{b+a}
\end{tikzcd}$$

Right and then down:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "1\times\phi"] \arrow[d, "\lambda"'] \arrow[rr, bend left, "\phi\times\phi"] \arrow[dd, bend right=60, "\lambda"'] & 1{\times}F(0) \arrow[r, "\phi\times 1"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[dr, "\phi_{a,b}"] & F(a{+}b) \arrow[d, "1"] \arrow[dd, bend left=60, "F(\beta)"] \arrow[dd, phantom, shift left=6ex, "\overset{F_{1,\beta}}{\Rightarrow}"] \\
1 \arrow[r, "\phi"] \arrow[d, "1"'] & F(0) \arrow[r, "F(\lambda^{-1})"] \arrow[d, "1"'] \arrow[dr, "F(\lambda^{-1})"] \arrow[dr, phantom, shift left=2ex, "\overset{F_{\lambda^{-1},\beta}}{\Swarrow}"] & F(0{+}0) \arrow[rr, "F(!_a+!_b)"] \arrow[d, "F(\beta)"] \arrow[drr, "F(\beta(!_a+!_b))=F((!_b+!_a)\beta)"] \arrow[dr, phantom, "\overset{F_{\beta,!_b+!_a}}{\Nearrow}"] & \arrow[r, phantom, "\overset{F_{!_a+!_b,\beta}}{\Swarrow}"] & F(a{+}b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[r, "F(\lambda^{-1})"] & F(0{+}0) \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
1{\times}1 \arrow[rr, "\phi\times\phi"] \arrow[d, "\beta"'] \arrow[dd, bend right=60, "\lambda"'] & & F(0){\times}F(0) \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"'] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a{+}b) \arrow[d, "F(\beta)"] \arrow[dd, bend left=60, "F(\beta)"] \arrow[dd, phantom, shift left=6ex, "\overset{F_{\beta,1}}{\Rightarrow}"] \\
1{\times}1 \arrow[r, "1\times\phi"] \arrow[d, "\lambda"'] & 1{\times}F(0) \arrow[r, "\phi\times 1"] \arrow[d, "\lambda"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[r, "F(!_b)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[dr, "\phi_{b,a}"] & F(b{+}a) \arrow[d, "1"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[r, "F(\lambda^{-1})"] & F(0{+}0) \arrow[rr, "F(!_b+!_a)"] & & F(b{+}a)
\end{tikzcd}$$

2. Proof of Theorem 2.1

Pentagon axiom. Set up but not solved.

M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \to m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
M_4 is given by $d \to m_4 \leftarrow e$ with $x_4 \in F(m_4)$.

$$
\begin{tikzcd}
 & (M_1 \odot M_2) \odot (M_3 \odot M_4) \arrow[dr, "\alpha"] & \\
((M_1 \odot M_2) \odot M_3) \odot M_4 \arrow[ur, "\alpha"] \arrow[d, "\alpha \odot 1_{M_4}"] & & M_1 \odot (M_2 \odot (M_3 \odot M_4)) \\
(M_1 \odot (M_2 \odot M_3)) \odot M_4 \arrow[rr, "\alpha"] & & M_1 \odot ((M_2 \odot M_3) \odot M_4) \arrow[u, "1_{M_1} \odot \alpha"]
\end{tikzcd}
$$

Upper route:

$$
\begin{tikzcd}
Fm_1Fm_2Fm_3Fm_4 \arrow[rr, "\phi\times1\times1"] \arrow[ddd, "\alpha"] & & Fm_1{+}m_2Fm_3Fm_4 \arrow[rr, "F(\psi)\times1\times1"] & & Fm_1{+}_bm_2Fm_3Fm_4 \arrow[rr, "\phi\times1"] \arrow[ddll, "\alpha"] \arrow[dd, phantom, "\overset{\alpha}{\cong}"] & & F(m_1{+}_bm_2){+}m_3Fm_4 \arrow[rr, "F(\psi)\times1"] \arrow[d, "\phi"] \arrow[dr, phantom, "\overset{\phi_{\psi,1}}{\cong}"] & & F(m_1{+}_bm_2){+}_cm_3Fm_4 \arrow[rr, "\phi"] & & F((m_1{+}_bm_2){+}_cm_3){+}m_4 \arrow[rr, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & & F((m_1{+}_bm_2){+}_cm_3){+}_dm_4 \arrow[ddd, "F(\kappa)"] \\
 & & & & & & F((m_1{+}_bm_2){+}m_3){+}m_4 \arrow[urrrr, "F(\psi+1)"] \arrow[d, "F(\alpha)"] & & & & & & \\
 & & Fm_1{+}_bm_2Fm_3Fm_4 \arrow[rr, "1\times\phi"] & & Fm_1{+}_bm_2Fm_3{+}m_4 \arrow[rr, "\phi"] \arrow[drr, "1\times F(\psi)"] & & F(m_1{+}_bm_2){+}(m_3{+}m_4) \arrow[drr, "F(1+\psi)"] \arrow[d, phantom, "\overset{\phi_{1,\psi}}{\cong}"] & & & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[urr, "F(\psi)\phi\times1"] \arrow[rrr, "\phi\times\phi"] \arrow[drr, "1\times F(\psi)\phi"] \arrow[ddd, "\alpha"] & & & Fm_1{+}m_2Fm_3{+}m_4 \arrow[ur, "F(\psi)\times1"] \arrow[rrr, "F(\psi)\times F(\psi)"] \arrow[dr, "1\times F(\psi)"] & & & Fm_1{+}_bm_2Fm_3{+}_dm_4 \arrow[rr, "\phi"] \arrow[d, phantom, "\overset{\phi_{\psi,1}}{\cong}"] & & F(m_1{+}_bm_2){+}(m_3{+}_dm_4) \arrow[rrrr, "F(\psi)"] & & & & F(m_1{+}_bm_2){+}_c(m_3{+}_dm_4) \arrow[ddd, "F(\kappa)"] \\
 & & Fm_1Fm_2Fm_3{+}_dm_4 \arrow[rr, "\phi\times1"] \arrow[ddrr, "\alpha"] & & Fm_1{+}m_2Fm_3{+}_dm_4 \arrow[rr, "\phi"] \arrow[urr, "F(\psi)\times1"] \arrow[dd, phantom, "\overset{\alpha}{\cong}"] & & F(m_1{+}m_2){+}(m_3{+}_dm_4) \arrow[urr, "F(\psi+1)"] \arrow[d, "F(\alpha)"] & & & & & & \\
 & & & & & & Fm_1{+}(m_2{+}(m_3{+}_dm_4)) \arrow[drrrr, "F(1+\psi)"] \arrow[dr, phantom, "\overset{\phi_{1,\psi}}{\cong}"] & & & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[rr, "1\times1\times\phi"] & & Fm_1Fm_2Fm_3{+}m_4 \arrow[rr, "1\times1\times F(\psi)"] & & Fm_1Fm_2Fm_3{+}_dm_4 \arrow[rr, "1\times\phi"] & & Fm_1Fm_2{+}(m_3{+}_dm_4) \arrow[u, "\phi"] \arrow[rr, "1\times F(\psi)"] & & Fm_1Fm_2{+}_c(m_3{+}_dm_4) \arrow[rr, "\phi"] & & Fm_1{+}(m_2{+}_c(m_3{+}_dm_4)) \arrow[rr, "F(\psi)"] \arrow[uuu, phantom, "\cong"] & & Fm_1{+}_b(m_2{+}_c(m_3{+}_dm_4))
\end{tikzcd}
$$

Lower route:

$$
\begin{tikzcd}[column sep=small]
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "\phi\times 1\times 1"] \arrow[ddd, "\alpha\times 1"'] & Fm_1{+}m_2Fm_3Fm_4 \arrow[r, "F(\psi)\times 1\times 1"] \arrow[dr, "\phi\times 1"] \arrow[ddd, phantom, "\underset{\cong}{\alpha\times 1}"] & Fm_1{+_b}m_2Fm_3Fm_4 \arrow[r, "\phi\times 1"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}\times 1}"] & F(m_1{+_b}m_2){+}m_3Fm_4 \arrow[r, "F(\psi)\times 1"] \arrow[ddd, phantom, "\cong"] & F(m_1{+_b}m_2){+_c}m_3Fm_4 \arrow[r, "\phi"] \arrow[ddd, "F(\kappa)\times 1"'] \arrow[dddr, phantom, "\underset{\cong}{\phi_{\kappa,1}}"] & F((m_1{+_b}m_2){+_c}m_3){+}m_4 \arrow[r, "F(\psi)"] \arrow[ddd, "F(\kappa+1)"'] \arrow[dddr, phantom, "\cong"] & F((m_1{+_b}m_2){+_c}m_3){+_d}m_4 \arrow[ddd, "F(\kappa+_1 1)"] \\
 & & F(m_1{+}m_2){+}m_3Fm_4 \arrow[ur, "F(\psi+1)\times 1"'] \arrow[d, "F(\alpha)\times 1"'] & & & & \\
 & & Fm_1{+}(m_2{+}m_3)Fm_4 \arrow[dr, "F(1+\psi)\times 1"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}\times 1}"] & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times\phi\times 1"] \arrow[ddd, "1\times\alpha\times 1"'] & Fm_1Fm_2{+}m_3Fm_4 \arrow[r, "1\times F(\psi)\times 1"] \arrow[ur, "\phi\times 1"] & Fm_1Fm_2{+_c}m_3Fm_4 \arrow[r, "\phi\times 1"] \arrow[ddd, "\alpha"'] & Fm_1{+}(m_2{+_c}m_3)Fm_4 \arrow[r, "F(\psi)\times 1"] \arrow[dr, "\phi"] \arrow[ddd, phantom, "\underset{\cong}{\alpha}"] & Fm_1{+_b}(m_2{+_c}m_3)Fm_4 \arrow[r, "\phi"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}}"] & F(m_1{+_b}(m_2{+_c}m_3)){+}m_4 \arrow[r, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & F(m_1{+_b}(m_2{+_c}m_3)){+_d}m_4 \arrow[ddd, "F(\kappa)"] \\
 & & & & F(m_1{+}(m_2{+_c}m_3)){+}m_4 \arrow[ur, "F(\psi+1)"'] \arrow[d, "F(\alpha)"'] & & \\
 & & & & Fm_1{+}((m_2{+_c}m_3){+}m_4) \arrow[dr, "F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}}"] & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times\phi\times 1"] \arrow[ddd, "1\times\alpha"'] & Fm_1Fm_2{+}m_3Fm_4 \arrow[r, "1\times F(\psi)\times 1"] \arrow[dr, "1\times\phi"] \arrow[ddd, phantom, "\underset{\cong}{1\times\alpha}"] & Fm_1Fm_2{+_c}m_3Fm_4 \arrow[r, "1\times\phi"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{\psi,1}}"] & Fm_1F(m_2{+_c}m_3){+}m_4 \arrow[r, "1\times F(\psi)"] \arrow[ur, "\phi"] \arrow[ddd, phantom, "\cong"] & Fm_1F(m_2{+_c}m_3){+_d}m_4 \arrow[r, "\phi"] \arrow[ddd, "1\times F(\kappa)"'] \arrow[dddr, phantom, "\underset{\cong}{\phi_{1,\kappa}}"] & Fm_1{+}((m_2{+_c}m_3){+_d}m_4) \arrow[r, "F(\psi)"] \arrow[ddd, "F(1+\kappa)"'] \arrow[dddr, phantom, "\cong"] & Fm_1{+_b}((m_2{+_c}m_3){+_d}m_4) \arrow[ddd, "F(1+_1\kappa)"] \\
 & & Fm_1F(m_2{+}m_3){+}m_4 \arrow[ur, "1\times F(\psi+1)"'] \arrow[d, "1\times F(\alpha)"'] & & & & \\
 & & Fm_1Fm_2{+}(m_3{+}m_4) \arrow[dr, "1\times F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{1,\psi}}"] & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times 1\times\phi"] & Fm_1Fm_2Fm_3{+}m_4 \arrow[r, "1\times 1\times F(\psi)"] \arrow[ur, "1\times\phi"] & Fm_1Fm_2Fm_3{+_d}m_4 \arrow[r, "1\times\phi"] & Fm_1Fm_2{+}(m_3{+_d}m_4) \arrow[r, "1\times F(\psi)"] & Fm_1Fm_2{+_c}(m_3{+_d}m_4) \arrow[r, "\phi"] & Fm_1{+}(m_2{+_c}(m_3{+_d}m_4)) \arrow[r, "F(\psi)"] & Fm_1{+_b}(m_2{+_c}(m_3{+_d}m_4))
\end{tikzcd}
$$

Triangle identity. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
U_b is given by $b \to b \leftarrow b$ with $\perp_b \in F(b)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.

$$\begin{tikzcd}
 & M_1 \odot M_2 & \\
(M_1 \odot U_b) \odot M_2 \arrow[ur, "\rho \odot 1_{M_2}"] \arrow[rr, "\alpha"] & & M_1 \odot (U_b \odot M_2) \arrow[ul, "1_{M_1} \odot \lambda"']
\end{tikzcd}$$

Up:

$$\begin{tikzcd}[column sep=small]
1 \arrow[rr, "(x_1\times\phi_0)\times x_2"] \arrow[ddrr, "x_1\times x_2"'] & & Fm_1F0Fm_2 \arrow[rr, "(1\times F(!_b))\times 1"] \arrow[drr, "\phi\times 1"] \arrow[d, phantom, "\overset{\rho\times 1}{\cong}"] & & Fm_1FbFm_2 \arrow[rr, "\phi_{m_1,b}\times 1"] \arrow[d, phantom, "\overset{\phi_{1,!_b}\times 1}{\cong}"] & & Fm_1{+}bFm_2 \arrow[rr, "F(\psi)\times 1"] \arrow[d, phantom, "\cong"] & & Fm_1{+_b}bFm_2 \arrow[rr, "\phi_{m_1+_bb,m_2}"] \arrow[ddllllll, "F(\kappa)\times 1"] & & F(m_1{+_b}){+}m_2 \arrow[rr, "F(\psi)"] \arrow[ddlll, "F(\kappa+1)"] \arrow[dr, phantom, "\cong"] & & F(m_1{+_b}b){+_b}m_2 \arrow[dd, "F(\kappa+_1 1)"] \\
 & & & & Fm_1{+}0{\times}Fm_2 \arrow[uurr, "F(1+!_b)\times 1"] \arrow[dll, "F(r)\times 1"] & & & \arrow[d, phantom, "\overset{\phi_{\kappa,1}}{\cong}"] & & & & & \\
 & & Fm_1Fm_2 \arrow[rrrrr, "\phi"] & & & & & Fm_1{+}m_2 \arrow[rrrrr, "F(\psi)"] & & & & & Fm_1{+_b}m_2
\end{tikzcd}$$

Across and then up:

$$\begin{tikzcd}[column sep=small]
 & & Fm_1F0Fm_2 \arrow[rr, "(1\times F(!_b))\times 1"] & & Fm_1FbFm_2 \arrow[rr, "\phi_{m_1,b}\times 1"] \arrow[ddd, "\alpha"] & & Fm_1{+}bFm_2 \arrow[rr, "F(\psi)\times 1"] \arrow[drr, "\phi"] \arrow[ddd, phantom, "\overset{\alpha}{\cong}"] & & Fm_1{+_b}bFm_2 \arrow[rr, "\phi_{m_1+_bb,m_2}"] \arrow[d, phantom, "\overset{\phi_{\psi,1}}{\cong}"] & & F(m_1{+_b}b){+}m_2 \arrow[rr, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & & F(m_1{+_b}b){+_b}m_2 \arrow[ddd, "F(\kappa)"] \\
 & & & & & & & & F(m_1{+}b){+}m_2 \arrow[urr, "F(\psi+1)"] \arrow[d, "F(\alpha)"] & & & & \\
 & & & & & & & & Fm_1{+}(b{+}m_2) \arrow[drr, "F(1+\psi)"] \arrow[d, phantom, "\overset{\phi_{1,\psi}}{\cong}"] & & & & \\
1 \arrow[uuurr, "(x_1\times\phi_0)\times x_2"] \arrow[rr, "x_1\times(\phi_0\times x_2)"] \arrow[ddrr, "x_1\times x_2"'] & & Fm_1F0Fm_2 \arrow[rr, "1\times(F(!_b)\times 1)"] \arrow[drr, "1\times\phi"] \arrow[d, phantom, "\overset{1\times\lambda}{\cong}"] & & Fm_1FbFm_2 \arrow[rr, "1\times\phi_{b,m_2}"] \arrow[d, phantom, "\overset{1\times\phi_{!_b,1}}{\cong}"] & & Fm_1Fb{+}m_2 \arrow[rr, "1\times F(\psi)"'] \arrow[urr, "\phi"] \arrow[d, phantom, "\cong"] & & Fm_1Fb{+_b}m_2 \arrow[rr, "\phi_{m_1,b+_bm_2}"] \arrow[ddllllll, "1\times F(\kappa)"] & & Fm_1{+}(b{+_b}m_2) \arrow[rr, "F(\psi)"] \arrow[ddlll, "F(1+\kappa)"] \arrow[dr, phantom, "\cong"] & & Fm_1{+_b}(b{+_b}m_2) \arrow[dd, "F(1+_1\kappa)"] \\
 & & & & Fm_1{\times}F0{+}m_2 \arrow[urr, "1\times F(!_b+1)"] \arrow[dll, "1\times F(\ell)"] & & & \arrow[d, phantom, "\overset{\phi_{1,\kappa}}{\cong}"] & & & & & \\
 & & Fm_1Fm_2 \arrow[rrrrr, "\phi"] & & & & & Fm_1{+}m_2 \arrow[rrrrr, "F(\psi)"] & & & & & Fm_1{+_b}m_2
\end{tikzcd}$$

3. Proof of Theorem 2.3

Diagram 1 of 7. Set up but not solved.

$$\begin{tikzcd}
(\mathbb{H}(M_1)\otimes\mathbb{H}(M_2))\otimes\mathbb{H}(M_3) \arrow[rr, "\alpha'"] \arrow[d, "\mathbb{H}_{M_1,M_2}\otimes 1"'] & & \mathbb{H}(M_1)\otimes(\mathbb{H}(M_2)\otimes\mathbb{H}(M_3)) \arrow[d, "1\otimes\mathbb{H}_{M_2\otimes M_3}"] \\
\mathbb{H}(M_1\otimes M_2)\otimes\mathbb{H}(M_3) \arrow[d, "\mathbb{H}_{M_1\otimes M_2,M_3}"'] & & \mathbb{H}(M_1)\otimes\mathbb{H}(M_2\otimes M_3) \arrow[d, "\mathbb{H}_{M_1,M_2\otimes M_3}"] \\
\mathbb{H}((M_1\otimes M_2)\otimes M_3) \arrow[rr, "\mathbb{H}(\alpha)"] & & \mathbb{H}(M_1\otimes(M_2\otimes M_3))
\end{tikzcd}$$

M_1 is given by $a_1 \xrightarrow{i_1} m_1 \xleftarrow{o_1} b_1$ with $x_1 \in F(m_1)$.
M_2 is given by $a_2 \xrightarrow{i_2} m_2 \xleftarrow{o_2} b_2$ with $x_2 \in F(m_2)$.
M_3 is given by $a_3 \xrightarrow{i_3} m_3 \xleftarrow{o_3} b_3$ with $x_3 \in F(m_3)$.
$\mathbb{H}(M_1)$ is given by $H(a_1) \xrightarrow{H(i_1)} H(m_1) \xleftarrow{H(o_1)} H(b_1)$ with $\bar{x_1} = \theta_m E(x_1)\phi_0 \in F'(H(m_1))$.
$\mathbb{H}(M_2)$ is given by $H(a_2) \xrightarrow{H(i_2)} H(m_2) \xleftarrow{H(o_2)} H(b_2)$ with $\bar{x_2} = \theta_m E(x_2)\phi_0 \in F'(H(m_2))$.
$\mathbb{H}(M_3)$ is given by $H(a_3) \xrightarrow{H(i_3)} H(m_3) \xleftarrow{H(o_3)} H(b_3)$ with $\bar{x_3} = \theta_m E(x_3)\phi_0 \in F'(H(m_3))$.
The decoration on $M_1 \otimes M_2$ is given by

$$x_1 \oplus x_2 \colon = 1 \cong 1\times 1 \xrightarrow{x_1\times x_2} F(m_1)\times F(m_2) \xrightarrow{\phi_{m_1,m_2}} F(m_1+m_2).$$

The decoration on $\mathbb{H}(M_1 \otimes M_2)$ is given by

$$x_1 \bar{\oplus} x_2 = \theta_{m_1+m_2} E(x_1 \oplus x_2)\phi_0 = 1 \cong 1\times 1 \xrightarrow{\phi_0} E(1\times 1) \xrightarrow{E(x_1\times x_2)} E(F(m_1)\times F(m_2)) \xrightarrow{E(\phi_{m_1,m_2})} E(F(m_1+m_2)) \xrightarrow{\theta_{m_1+m_2}} F'(H(m_1+m_2)).$$

The decoration on $\mathbb{H}(M_1) \otimes \mathbb{H}(M_2)$ is given by

$$\bar{x_1} \oplus \bar{x_2} = 1 \cong 1\times 1 \xrightarrow{\phi_0\times\phi_0} E(1)\times E(1) \xrightarrow{E(x_1)\times E(x_2)} E(F(m_1))\times E(F(m_2)) \xrightarrow{\theta_{m_1}\times\theta_{m_2}} F'(H(m_1))\times F'(H(m_2)) \xrightarrow{\phi'_{H(m_1),H(m_2)}{}'} F'(H(m_1)+H(m_2)).$$

The natural isomorphism $\mathbb{H}_\otimes \colon \mathbb{H}(M_1) \otimes \mathbb{H}(M_2) \to \mathbb{H}(M_1 \otimes M_2)$ is given by:

$$\begin{tikzcd}
 & & E(1)\times 1 \arrow[r, "1\times\phi_0"] \arrow[d, "\rho"'] \arrow[dr, phantom, "\phi \atop \cong"] & E(1)\times E(1) \arrow[r, "E(x_1)\times E(x_2)"] \arrow[d, "E_{1,1}"] \arrow[dr, phantom, "E_{x_1,x_2} \atop \cong"] & E(F(m_1))\times E(F(m_2)) \arrow[r, "\theta_{m_1}\times\theta_{m_2}"] \arrow[d, "E_{F(m_1),F(m_2)}"] & F'(H(m_1))\times F'(H(m_2)) \arrow[r, "\phi'_{H(m_1),H(m_2)}"] & F'(H(m_1)+H(m_2)) \arrow[d, "F'(H_{m_1,m_2})"] \\
1 \arrow[r, "\phi_0"] & E(1) \arrow[ur, "\rho^{-1}"] \arrow[r, "1"'] & E(1) \arrow[r, "E(\rho^{-1})"] & E(1\times 1) \arrow[r, "E(x_1\times x_2)"] & E(F(m_1)\times F(m_2)) \arrow[r, "E(\phi_{m_1,m_2})"] & E(F(m_1+m_2)) \arrow[r, "\theta_{m_1+m_2}"] & F'(H(m_1+m_2))
\end{tikzcd}$$

Regarding the commuting pentagon, 1) the monoidal natural isomorphism $\theta\colon EF \to F'H$ is a strict monoidal natural isomorphism (not pseudo), and 2) the laxator of the composite $F'H$ utilizes the laxators of both F' and H.
Right and then down:

$$\begin{tikzcd}
 & (E(1)\times E(1))\times E(1) \arrow[r, "(E(x_1)\times E(x_2))\times E(x_3)"] \arrow[d, "\alpha"] & (E(F(m_1))\times E(F(m_2)))\times E(F(m_3)) \arrow[r, "(\theta_{m_1}\times\theta_{m_2})\times\theta_{m_3}"] & (F'(H(m_1))\times F'(H(m_2)))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2)}\times 1"] \arrow[d, "\alpha"] \arrow[dr, phantom, "\phi'_{H(m_1),H(m_2),H(m_3)} \atop \cong"] & F'(H(m_1)+H(m_2))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1)+H(m_2),H(m_3)}"] & F'((H(m_1)+H(m_2))+H(m_3)) \arrow[d, "F'(\alpha)"] \\
 & E(1)\times((E(1)\times E(1)) \arrow[r, "E(x_1)\times(E(x_2)\times E(x_3))"] \arrow[d, "1\times E_{1,1}"] \arrow[dr, phantom, "1\times E_{x_2,x_3} \atop \cong"] & E(F(m_1))\times(E(F(m_2))\times E(F(m_3))) \arrow[r, "\theta_{m_1}\times(\theta_{m_2}\times\theta_{m_3})"] \arrow[d, "1\times E_{F(m_2),F(m_3)}"] & F'(H(m_1))\times(F'(H(m_2))\times F'(H(m_3))) \arrow[r, "1\times\phi'_{H(m_2),H(m_3)}"] & F'(H(m_1))\times F'(H(m_2)+H(m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2)+H(m_3)}"] \arrow[d, "1\times F'(H_{m_2,m_3})"] \arrow[dr, phantom, "\phi'_{1,H_{m_2,m_3}} \atop \cong"] & F'(H(m_1)+(H(m_2)+H(m_3))) \arrow[d, "F'(1+H_{m_2,m_3})"] \\
1 \arrow[uur, "(\phi_0\times\phi_0)\times\phi_0"] \arrow[ur, "\phi_0\times(\phi_0\times\phi_0)"] \arrow[r, "\phi_0\times\phi_0"] \arrow[dr, "\phi_0"'] & E(1)\times E(1\times 1) \arrow[r, "E(x_1)\times E(x_2\times x_3)"] \arrow[d, "E_{1,1\times 1}"] \arrow[dr, phantom, "E_{x_1,x_2\times x_3} \atop \cong"] & E(F(m_1))\times E(F(m_2)\times F(m_3)) \arrow[r, "1\times E(\phi_{m_2,m_3})"] \arrow[d, "E_{F(m_1),F(m_2)\times F(m_3)}"] \arrow[dr, phantom, "E_{1,\phi_{m_2,m_3}} \atop \cong"] & E(F(m_1))\times E(F(m_2+m_3)) \arrow[r, "\theta_{m_1}\times\theta_{m_2+m_3}"] \arrow[d, "E_{F(m_1),F(m_2+m_3)}"] & F'(H(m_1))\times F'(H(m_2+m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2+m_3)}"] & F'(H(m_1)+H(m_2+m_3)) \arrow[d, "F'(H_{m_1,m_2+m_3})"] \\
 & E(1\times(1\times 1)) \arrow[r, "E(x_1\times(x_2\times x_3))"] & E(F(m_1)\times(F(m_2)\times F(m_3))) \arrow[r, "E(1\times\phi_{m_2,m_2})"] & E(F(m_1)\times F(m_2+m_3)) \arrow[r, "E(\phi_{m_1,m_2+m_3})"] & E(F(m_1+(m_2+m_3))) \arrow[r, "\theta_{m_1+(m_2+m_3)}"] & F'(H(m_1+(m_2+m_3)))
\end{tikzcd}$$

(I'm being a little reckless on the left with the ϕs)
Down and then right:

$$
\begin{tikzcd}[column sep=small]
 & (E(1)\times E(1))\times E(1) \arrow[r, "(E(x_1)\times E(x_2))\times E(x_3)"] \arrow[d, "E_{1,1}\times 1"'] \arrow[dr, phantom, "\underset{\cong}{E_{x_1,x_2}\times 1}"] & (E(F(m_1))\times E(F(m_2)))\times E(F(m_3)) \arrow[r, "(\theta_{m_1}\times\theta_{m_2})\times\theta_{m_3}"] \arrow[d, "E_{F(m_1),F(m_2)}\times 1"] & (F'(H(m_1))\times F'(H(m_2)))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1),H(m_2)}\times 1"] & F'(H(m_1)+H(m_2))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1)+H(m_2),H(m_3)}"] \arrow[d, "F'(H_{m_1,m_2})\times 1"] \arrow[dr, phantom, "\underset{\cong}{\phi'_{H_{m_1,m_2},1}}"] & F'((H(m_1)+H(m_2))+H(m_3)) \arrow[d, "F'(H_{m_1,m_2}+1)"] \\
 & E(1\times 1)\times E(1) \arrow[r, "E(x_1\times x_2)\times E(x_3)"] \arrow[d, "E_{1\times 1,1}"'] \arrow[dr, phantom, "\underset{\cong}{E_{x_1\times x_2,x_3}}"] & E(F(m_1)\times F(m_2))\times E(F(m_3)) \arrow[r, "E(\phi_{m_1,m_2})\times 1"] \arrow[d, "E_{F(m_1)\times F(m_2),F(m_3)}"'] \arrow[dr, phantom, "\underset{\cong}{E_{\phi_{m_1,m_2},1}}"] & E(F(m_1+m_2))\times E(F(m_3)) \arrow[r, "\theta_{m_1+m_2}\times\theta_{m_3}"] \arrow[d, "E_{F(m_1+m_2),F(m_3)}"] & F'(H(m_1+m_2))\times F'(H(m_3)) \arrow[r, "\phi'_{H(m_1+m_2),H(m_3)}"] & F'(H(m_1+m_2)+H(m_3)) \arrow[d, "F'(H_{m_1+m_2,m_3})"] \\
1 \arrow[uur, "(\phi_0\times\phi_0)\times\phi_0"] \arrow[ur, "\phi_0\times\phi_0"] \arrow[r, "\phi_0"] \arrow[dr, "\phi_0"'] & E((1\times 1)\times 1) \arrow[r, "E((x_1\times x_2)\times x_3)"] \arrow[d, "E(\alpha)"'] \arrow[dr, phantom, "\underset{\cong}{E(\alpha_{x_1,x_2,x_3})}"] & E((F(m_1)\times F(m_2))\times F(m_3)) \arrow[r, "E(\phi_{m_1,m_2}\times 1)"] \arrow[d, "E(\alpha)"] & E(F(m_1+m_2)\times F(m_3)) \arrow[r, "E(\phi_{m_1+m_2,m_3})"] \arrow[d, phantom, "\underset{\cong}{E(\phi_{m_1,m_2,m_3})}"] & E(F((m_1+m_2)+m_3)) \arrow[r, "\theta_{(m_1+m_2)+m_3}"] \arrow[d, "E(F(\alpha))"] & F'(H((m_1+m_2)+m_3)) \arrow[d, "F'(H(\alpha))"] \\
 & E(1\times(1\times 1)) \arrow[r, "E(x_1\times(x_2\times x_3))"] & E(F(m_1)\times(F(m_2)\times F(m_3))) \arrow[r, "E(1\times\phi_{m_2,m_2})"] & E(F(m_1)\times F(m_2+m_3)) \arrow[r, "E(\phi_{m_1,m_2+m_3})"] & E(F(m_1+(m_2+m_3))) \arrow[r, "\theta_{m_1+(m_2+m_3)}"] & F'(H(m_1+(m_2+m_3)))
\end{tikzcd}
$$

4. Some useful maps

Given $a \in (\mathsf{A}, +, 0)$, the map $U_{\lambda_a} : U_{0+a} \to U_a$ is given by:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] & & F(0{+}a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] & & \cong & \\
& F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"'] & & F(a)
\end{tikzcd}$$

where the $\cong$ is given by pseudonaturality of F: we have a unique map in $!_a : 0 \to a$ in A but also a map $\lambda_a \circ !_{0+a} : 0 \to a$ where λ_a is the left unitor of $(\mathsf{A}, +, 0)$, and so $F(!_a) = F(\lambda_a \circ !_{0+a}) \cong F(\lambda_a) F(!_{0+a})$.

The left unitor $\lambda'_{U_a} : U_0 \otimes U_a \to U_a$ is given by:

$$\begin{tikzcd}
& F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[dd, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0{+}a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi \times \phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "\overset{\phi}{\cong}"] & & {} & \cong \\
& F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"'] & & F(a)
\end{tikzcd}$$

where the $\cong$ in the lower right is the same as the one in the first diagram.

For an arbitrary M, the left unitor $\lambda'_M : U_0 \otimes M \to M$ is given by:

$$\begin{tikzcd}
& F(0){\times}F(m) \arrow[r, "F(!_0) \times 1"] \arrow[dr, "\phi_{0,m}"'] & F(0){\times}F(m) \arrow[r, "\phi_{0,m}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0{+}m) \arrow[dd, "F(\lambda_m)"] \\
1 \arrow[ur, bend left, "\phi_0 \times x"] \arrow[dr, bend right, "x"'] \arrow[r, phantom, "\overset{\lambda_M}{\cong}"] & {} & F(0{+}m) \arrow[ur, "F(!_0 + 1)"] \arrow[dr, "F(\lambda_m)"'] \arrow[r, phantom, "\cong"] & {} \\
& F(m) \arrow[rr, "\mathrm{id}"] & & F(m)
\end{tikzcd}$$

For an arbitrary M given by $a \to (m, x) \leftarrow b$, the map $\lambda_M : U_b \odot M \to M$ is given by:

$$\begin{tikzcd}
& F(m){\times}F(0) \arrow[r, "1 \times F(!_b)"] \arrow[dr, "\phi_{m,0}"'] & F(m){\times}F(b) \arrow[r, "\phi_{m,b}"] \arrow[d, phantom, "\overset{\phi_{1,!_b}}{\cong}"] & F(m{+}b) \arrow[r, "F(\psi)"] & F(m{+_b}b) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "x \times \phi_0"] \arrow[dr, bend right, "x"'] \arrow[r, phantom, "\overset{\lambda_M}{\cong}"] & {} & F(m{+}0) \arrow[ur, "F(1+!_b)"] \arrow[drr, "F(\lambda_m)"'] & \cong & \\
& F(m) \arrow[rrr, "\mathrm{id}"] & & & F(m)
\end{tikzcd}$$

In particular, if $M = U_0$ above, then the map $\lambda_{U_0} : U_0 \odot U_0 \to U_0$ is given by:

$$\begin{tikzcd}
& F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_0)"] \arrow[dr, "\phi_{0,0}"'] & F(0){\times}F(0) \arrow[r, "\phi_{0,0}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"] & F(0{+}0) \arrow[r, "F(\psi)"] & F(0{+_0}0) \arrow[dd, "F(\kappa)"] \\
1 \arrow[ur, bend left, "\phi_0 \times \phi_0"] \arrow[dr, bend right, "\phi_0"'] \arrow[r, phantom, "\overset{\lambda_M}{\cong}"] & {} & F(0{+}0) \arrow[ur, "F(!_0+!_0)"] \arrow[drr, "F(\lambda_0)"'] & \cong & \\
& F(0) \arrow[rrr, "\mathrm{id}"] & & & F(0)
\end{tikzcd}$$