1. PROOF OF THEOREM 2.2

There are 11 diagrams in Shulman's definition of symmetric monoidal double category, which we check now for decorated cospan double categories.

Diagram 1 of 11. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \to m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
N_3 is given by $c' \to n_3 \leftarrow d'$ with $y_3 \in F(n_3)$.

$$\begin{tikzcd}
((M_1{\otimes}N_1){\odot}(M_2{\otimes}N_2)){\odot}(M_3{\otimes}N_3) \arrow[r, "\chi\odot 1"] \arrow[d, "\alpha"'] & ((M_1{\odot}M_2){\otimes}(N_1{\odot}N_2)){\odot}(M_3{\otimes}N_3) \arrow[d, "\chi"] \\
(M_1{\otimes}N_1){\odot}((M_2{\otimes}N_2){\odot}(M_3{\otimes}N_3)) \arrow[d, "1\odot\chi"'] & ((M_1{\odot}M_2){\odot}M_3){\otimes}((N_1{\odot}N_2){\odot}N_3) \arrow[d, "\alpha\otimes\alpha"] \\
(M_1{\otimes}N_1){\odot}((M_2{\odot}M_3){\otimes}(N_2{\odot}N_3)) \arrow[r, "\chi"] & (M_1{\odot}(M_2{\odot}M_3)){\otimes}(N_1{\odot}(N_2{\odot}N_3))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3))\in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{(m_1+_bm_2)+(n_1+_{b'}n_2),m_3+n_3}(\phi_{m_1+_bm_2,n_1+_{b'}n_2}((F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))),\phi_{m_3,n_3}(x_3,y_3))\in F(((m_1+_bm_2)+(n_1+_{b'}n_2))+_{c+c'}(m_3+n_3))$$

$$\phi_{(m_1+_bm_2)+cm_3,(n_1+_{b'}n_2+_{c'}n_3}((F(\psi\phi_{m_1+_bm_2,m_3}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),x_3),(F(\psi)\phi_{n_1+_{b'}n_2,n_3}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),y_3)))\in F(((m_1+_bm_2)+_cm_3)+((n_1+_{b'}n_2)+_{c'}n_3))$$

$$\phi_{m_1+_b(m_2+cm_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+cm_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3)))\in F((m_1+_b(m_2+_cm_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

(2) Down and then right:

$$F(\psi)\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),m_3+n_3}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{m_3,n_3}(x_3,y_3))\in F(((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3))$$

$$F(\psi)\phi_{m_1+n_1,(m_2+n_2)+_{c+c'}(m_3+n_3)}((\phi_{m_1,n_1}(x_1,y_1),F(\psi)\phi_{m_2+n_2,m_3+n_3}(\phi_{m_2,n_2}(x_2,y_2),\phi_{m_3,n_3}(x_3,y_3))))\in F((m_1+n_1)+_{b+b'}((m_2+n_2)+_{c+c'}(m_3+n_3)))$$

$$F(\psi)\phi_{m_1+n_1,(m_2+cm_3)+(n_2+_{c'}n_3)}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2+cm_3,n_2+_{c'}n_3}((F\psi)\phi_{m_2,m_3}(x_2,x_3),F(\psi)\phi_{n_2,n_3}(y_2,y_3)))\in F((m_1+n_1)+_{b+b'}((m_2+_cm_3)+(n_2+_{c'}n_3)))$$

$$\phi_{m_1+_b(m_2+cm_3),n_1+_{b'}(n_2+_{c'}n_3)}(F(\psi)\phi_{m_1,m_2+cm_3}((x_1,F(\psi)\phi_{m_2,m_3}(x_2,x_3)),(y_1,F(\psi)\phi_{n_2,n_3}(y_2,y_3)))\in F((m_1+_b(m_2+_cm_3))+(n_1+_{b'}(n_2+_{c'}n_3)))$$

Right and then down (omitting morphisms emanating out of 1 on the left due to space restrictions):

$$\begin{tikzcd}
Fm_1Fn_1Fm_2Fn_2Fm_3Fn_3 \arrow[rr, "\phi\times\phi\times\phi"] \arrow[dr, "(1\times\phi\times1)\times1"] \arrow[dddd, "(1\times\beta\times1)\times1"'] & & Fm_1+n_1Fm_2+n_2Fm_3+n_3 \arrow[rr, "\phi\times1"] \arrow[d, phantom, "\cong"] & & F(m_1+n_1)+(m_2+n_2)Fm_3+n_3 \arrow[rr, "F(\psi)\times1"] \arrow[ddr, "F(1+\beta+1)\times1"] & & F(m_1+n_1)+_{b+b'}(m_2+n_2)Fm_3+n_3 \arrow[rr, "\phi"] \arrow[dddd, "F(\chi)\times1"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2))+(m_3+n_3) \arrow[rr, "F(\psi)"] \arrow[dddd, "F(\chi+1)"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3) \arrow[dddd, "F(\chi+_1 1)"] \\
 & Fm_1Fn_1+m_2Fn_2Fm_3Fn_3 \arrow[rr, "(1\times\phi)\times1"] \arrow[dd, "(1\times F(\beta)\times1)\times1"] & & Fm_1F(n_1+m_2)+n_2Fm_3Fn_3 \arrow[ur, "\phi\times\phi"] \arrow[dd, "(1\times F(\beta+1))\times1"] & \underset{\cong}{\phi_{1,\beta+1}} & \arrow[r, phantom, "\cong"] & & & & & \\
\arrow[r, phantom, "\underset{\cong}{(1\times\mu\times1)\times1}"] & & \underset{\cong}{\phi_{\beta,1}} & & & F(m_1+m_2)+(n_1+n_2)Fm_3+n_3 \arrow[ddr, "F(\psi+\psi)\times1"] & & \underset{\cong}{\phi_{\chi,1}} & & \cong & \\
 & Fm_1Fm_2+n_1Fn_2Fm_3Fn_3 \arrow[rr, "(1\times\phi)\times1"] & \arrow[d, phantom, "\cong"] & Fm_1F(m_2+n_1)+n_2Fm_3Fn_3 \arrow[urr, "\phi\times\phi"] & \underset{\cong}{\phi_{\psi,\psi}} & & & & & & \\
Fm_1Fm_2Fn_1Fn_2Fm_3Fn_3 \arrow[ur, "(1\times\phi\times1)\times1"] \arrow[rr, "\phi\times\phi\times1\times1"] \arrow[dddd, "1\times\beta\times1"'] & & Fm_1+m_2Fn_1+n_2Fm_3Fn_3 \arrow[rr, "F(\psi)\times F(\psi)\times1\times1"] \arrow[uurrr, "\phi\times\phi"'] & & Fm_1+_bm_2Fn_1+_{b'}n_2Fm_3Fn_3 \arrow[rr, "\phi\times\phi"] \arrow[dr, "1\times\phi\times1"] \arrow[dddd, "1\times\beta\times1"'] & & F(m_1+_bm_2)+(n_1+_{b'}n_2)Fm_3+n_3 \arrow[rr, "\phi"] \arrow[d, phantom, "\cong"] & & F((m_1+_bm_2)+(n_1+_{b'}n_2))+(m_3+n_3) \arrow[rr, "F(\psi)"] \arrow[ddr, "F(1+\beta+1)"] & & F((m_1+_bm_2)+(n_1+_{b'}n_2))+_{c+c'}(m_3+n_3) \arrow[dddd, "F(\chi)"] \\
 & & & & & Fm_1+_bm_2F(n_1+_{b'}n_2)+m_3Fn_3 \arrow[rr, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times1"] & & Fm_1+_bm_2F(n_1+_{b'}n_2)+m_3+n_3 \arrow[ur, "\phi"] \arrow[dd, "1\times F(\beta+1)"] & \underset{\cong}{\phi_{1,\beta+1}} & \arrow[r, phantom, "\cong"] & \\
 & & & & \arrow[r, phantom, "\underset{\cong}{1\times\mu\times1}"] & & \underset{\cong}{\phi_{\beta,1}} & & & F(m_1+_bm_2+m_3)+(n_1+_{b'}n_2+n_3) \arrow[ddr, "F(\psi+\psi)"] & \\
 & & & & & Fm_1+_bm_2Fm_3+(n_1+_{b'}n_2)Fn_3 \arrow[rr, "1\times\phi"] & \arrow[d, phantom, "\cong"] & Fm_1+_bm_2Fm_3+(n_1+_{b'}n_2)+n_3 \arrow[urr, "\phi"] & & \underset{\cong}{\phi_{\psi,\psi}} & \\
Fm_1Fm_2Fm_3Fn_1Fn_2Fn_3 \arrow[rr, "\phi\times1\times\phi\times1"] \arrow[ddd, "\alpha\times\alpha"'] & & Fm_1+m_2Fm_3Fn_1+n_2Fn_3 \arrow[rr, "F(\psi)\times1\times F(\psi)\times1"] \arrow[drr, "\phi\times\phi"'] & & Fm_1+_bm_2Fm_3Fn_1+_{b'}n_2Fn_3 \arrow[rr, "\phi\times\phi"] \arrow[ur, "1\times\phi\times1"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}\times\phi_{\psi,1}}"] & & F(m_1+_bm_2)+m_3F(n_1+_{b'}n_2)+n_3 \arrow[rr, "F(\psi)\times F(\psi)"] \arrow[uurrr, "\phi"'] & & F(m_1+_bm_2)+_cm_3F(n_1+_{b'}n_2)+_{c'}n_3 \arrow[rr, "\phi"] \arrow[ddd, "F(\kappa)\times F(\kappa)"'] & & F((m_1+_bm_2)+_cm_3)+((n_1+_{b'}n_2)+_{c'}n_3) \arrow[ddd, "F(\kappa+\kappa)"] \\
 & & \arrow[d, phantom, "\underset{\cong}{\phi\times\phi}"] & & Z \arrow[urr, "F(\psi+1)\times F(\psi+1)"'] \arrow[d, "F(\alpha)\times F(\alpha)"'] & & \arrow[d, phantom, "\cong"] & & & \arrow[d, phantom, "\cong"] & \\
 & & & & Z' \arrow[drr, "F(1+\psi)\times F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}\times\phi_{1,\psi}}"] & & & & & & \\
Fm_1Fm_2Fm_3Fn_1Fn_2Fn_3 \arrow[rr, "1\times\phi\times1\times\phi"] & & Fm_1Fm_2+m_3Fn_1Fn_2+n_3 \arrow[rr, "1\times F(\psi)\times1\times F(\psi)"] \arrow[urr, "\phi\times\phi"] & & Fm_1Fm_2+_cm_3Fn_1Fn_2+_{c'}n_3 \arrow[rr, "\phi\times\phi"] & & Fm_1+(m_2+_cm_3)Fn_1+(n_2+_{c'}n_3) \arrow[rr, "F(\psi)\times F(\psi)"] & & F(m_1+_b+(m_2+_cm_3)Fn_1+_{b'}(n_2+_{c'}n_3) \arrow[rr, "\phi"] & & F(m_1+_b+(m_2+_cm_3))+(n_1+_{b'}(n_2+_{c'}n_3))
\end{tikzcd}$$

Down and then right (omitting morphisms emanating out of 1 on the left due to space restrictions):

$$\begin{tikzcd}
Fm_1Fn_1Fm_2Fn_2Fm_3Fn_3 \arrow[rr, "\phi\times\phi\times\phi"] \arrow[dddd, "\alpha"'] & & Fm_1+n_1Fm_2+n_2Fm_3+n_3 \arrow[rr, "\phi\times1"] \arrow[dddd, "\alpha"'] & & F(m_1+n_1)+(m_2+n_2)Fm_3+n_3 \arrow[rr, "F(\psi)\times1"] \arrow[drr, "\phi"'] & & F(m_1+n_1)+_{b+b'}(m_2+n_2)Fm_3+n_3 \arrow[rr, "\phi"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}}"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2))+(m_3+n_3) \arrow[rr, "F(\psi)"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2))+_{c+c'}(m_3+n_3) \arrow[dddd, "F(\kappa)"] \\
 & & & & & & F((m_1+n_1)+(m_2+n_2))+(m_3+n_3) \arrow[urr, "F(\psi+1)"'] \arrow[dd, "F(\alpha)"'] & & & & \\
 & & & \arrow[r, phantom, "\underset{\cong}{\phi}"] & & & & & \cong & & \\
 & & & & & & F(m_1+n_1)+((m_2+n_2)+(m_3+n_3)) \arrow[drr, "F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}}"] & & & & \\
Fm_1Fn_1Fm_2Fn_2Fm_3Fn_3 \arrow[rr, "\phi\times\phi\times\phi"] \arrow[dr, "1\times(1\times\phi\times1)"] \arrow[dddd, "1\times(1\times\beta\times1)"'] & & Fm_1+n_1Fm_2+n_2Fm_3+n_3 \arrow[rr, "1\times\phi"] \arrow[d, phantom, "\cong"] & & Fm_1+n_1F(m_2+n_2)+(m_3+n_3) \arrow[rr, "1\times F(\psi)"] \arrow[urr, "\phi"] \arrow[ddr, "1\times F(1+\beta+1)"] & & Fm_1+n_1F(m_2+n_2)+_{c+c'}(m_3+n_3) \arrow[rr, "\phi"] \arrow[dddd, "1\times F(\chi)"] & & F((m_1+n_1)+((m_2+n_2)+_{c+c'}(m_3+n_3))) \arrow[rr, "F(\psi)"] \arrow[dddd, "F(1+\chi)"] & & F((m_1+n_1)+_{b+b'}((m_2+n_2)+_{c+c'}(m_3+n_3))) \arrow[dddd, "F(1+_1\chi)"] \\
 & Fm_1Fn_1Fm_2Fn_2+m_3Fn_3 \arrow[rr, "1\times(1\times\phi)"] \arrow[dd, "1\times(1\times F(\beta)\times1)"] & & Fm_1Fn_1Fm_2F(n_2+m_3)+n_3 \arrow[ur, "\phi\times\phi"] \arrow[dd, "1\times(1\times F(\beta+1))"] & \underset{\cong}{\phi_{1,\beta+1}} & \arrow[r, phantom, "\cong"] & & & & & \\
\arrow[r, phantom, "\underset{\cong}{1\times(1\times\mu\times1)}"] & & \underset{\cong}{\phi_{\beta,1}} & & & Fm_1+n_1F(m_2+m_3)+(n_2+n_3) \arrow[ddr, "1\times F(\psi+\psi)"] & & \underset{\cong}{\phi_{1,\chi}} & & \cong & \\
 & Fm_1Fn_1Fm_2Fm_3+n_2Fn_3 \arrow[rr, "1\times(1\times\phi)"] & \arrow[d, phantom, "\cong"] & Fm_1Fn_1Fm_2F(m_3+n_2)+n_3 \arrow[urr, "\phi\times\phi"] & \underset{\cong}{\phi_{\psi,\psi}} & & & & & & \\
Fm_1Fn_1Fm_2Fm_3Fn_2Fn_3 \arrow[ur, "1\times(1\times\phi\times1)"] \arrow[rr, "1\times1\times\phi\times\phi"] \arrow[dddd, "1\times\beta\times1"'] & & Fm_1Fn_1Fm_2+m_3Fn_2+n_3 \arrow[rr, "1\times1\times F(\psi)\times F(\psi)"] \arrow[uurrr, "\phi\times\phi"'] & & Fm_1Fn_1Fm_2+_cm_3Fn_2+_{c'}n_3 \arrow[rr, "\phi\times\phi"] \arrow[dr, "1\times\phi\times1"] \arrow[dddd, "1\times\beta\times1"'] & & Fm_1+n_1F(m_2+_cm_3)+(n_2+_{c'}n_3) \arrow[rr, "\phi"] \arrow[d, phantom, "\cong"] & & F(m_1+n_1)+((m_2+_cm_3)+(n_2+_{c'}n_3)) \arrow[rr, "F(\psi)"] \arrow[ddr, "F(1+\beta+1)"] & & F(m_1+n_1)+_{b+b'}((m_2+_cm_3)+(n_2+c'n_3)) \arrow[dddd, "F(\chi)"] \\
 & & & & & Fm_1Fn_1+(m_2+_cm_3)Fn_2+_{c'}n_3 \arrow[rr, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times1"] & & Fm_1Fn_1+(m_2+_cm_3)+(n_2+_{c'}n_3) \arrow[ur, "\phi"] \arrow[dd, "1\times F(\beta+1)"] & \underset{\cong}{\phi_{1,\beta+1}} & \arrow[r, phantom, "\cong"] & \\
 & & & & \arrow[r, phantom, "\underset{\cong}{1\times\mu\times1}"] & & \underset{\cong}{\phi_{\beta,1}} & & & F(m_1+(m_2+_cm_3))+(n_1+(n_2+_{c'}n_3)) \arrow[ddr, "F(\psi+\psi)"] & \\
 & & & & & Fm_1F(m_2+_cm_3)+n_1Fn_2+_{c'}n_3 \arrow[rr, "1\times\phi"] & \arrow[d, phantom, "\cong"] & Fm_1F(m_2+_cm_3)+n_1+(n_2+_{c'}n_3) \arrow[urr, "\phi"] & & \underset{\cong}{\phi_{\psi,\psi}} & \\
Fm_1Fm_2Fm_3Fn_1Fn_2Fn_3 \arrow[rr, "1\times\phi\times1\times\phi"] & & Fm_1Fm_2+m_3Fn_1Fn_2+n_3 \arrow[rr, "1\times F(\psi)\times1\times F(\psi)"] & & Fm_1Fm_2+_cm_3Fn_1Fn_2+_{c'}n_3 \arrow[ur, "1\times\phi\times1"] \arrow[rr, "\phi\times\phi"] & & Fm_1+(m_2+_cm_3)Fn_1+(n_2+_{c'}n_3) \arrow[rr, "F(\psi)\times F(\psi)"] \arrow[uurrr, "\phi"'] & & F(m_1+_b+(m_2+_cm_3)Fn_1+_{b'}(n_2+_{c'}n_3) \arrow[rr, "\phi"] & & F(m_1+_b+(m_2+_cm_3))+(n_1+_{b'}(n_2+_{c'}n_3))
\end{tikzcd}$$

Diagrams 2 and 3 of 11. Set up but not solved.

$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} a'$ with $x \in F(m)$.

$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} b'$ with $y \in F(n)$.

$U_a \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{\mathrm{id}} a \xleftarrow{\mathrm{id}} a$ with $\perp_a \in F(a)$.

$$\begin{tikzcd}
\begin{array}{c} a+b\to(a+b)+_{a+b}(m+n)\leftarrow a'+b' \\ F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\perp_{a+b})\in F((a+b)+_{a+b}(m+n)) \end{array} & (M\otimes N)\odot(U_{a+b}) \arrow[r, "\mathrm{id}\odot\mu"] \arrow[d, "r"'] & (M\otimes N)\odot(U_a\otimes U_b) \arrow[d, "\chi"] & \begin{array}{c} a+b\to(a+b)+_{a+b}(m+n)\leftarrow a'+b' \\ F(\psi)\phi_{a+b,m+n}(\phi_{m,n}(x,y),\phi_{a,b}(\perp_a,\perp_b))\in F((a+b)+_{a+b}(m+n)) \end{array} \\
\begin{array}{c} a+b\to m+n\leftarrow a'+b' \\ \phi_{m,n}(x,y)\in F(m+n) \end{array} & M\otimes N & (M\odot U_a)\otimes(N\odot U_b) \arrow[l, "r\otimes r"'] & \begin{array}{c} a+b\to(a+_am)+(b+_bn)\leftarrow a'+b' \\ \phi_{a+_am,b+_bn}(F(\psi)\phi_{a,m}(\perp_a,x),F(\psi)\phi_{b,n}(\perp_b,y))\in F((a+_am)+(b+_bn)) \end{array}
\end{tikzcd}$$

Down:

$$\begin{tikzcd}
& F(0)\times(F(m)\times F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0)\times(F(m+n) \arrow[r, "F(!_{a+b})\times 1"] \arrow[ddr, "\phi_{0,m+n}"'] & F(a+b)\times F(m+n) \arrow[r, "\phi_{a+b,m+n}"] & F((a+b)+(m+n)) \arrow[r, "F(\psi)"] & F((a+b)+_{a+b}(m+n)) \arrow[ddd, "F(\kappa)"] \\
& & & \overset{\phi_{!_{a+b},1}}{\cong} & & \\
1 \arrow[uur, bend left, "\phi_0\times(x\times y)"] \arrow[dr, bend right, "x\times y"'] & & \overset{\lambda_{M\otimes N}}{\cong} & F(0+(m+n)) \arrow[uur, "F(!_{a+b}+1)"'] \arrow[drr, "F(\ell)"] & \cong & \\
& F(m)\times F(n) \arrow[rrrr, "\phi_{m,n}"] & & & & F(m+n)
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
& F(0)\times(F(m)\times F(n)) \arrow[rr, "1\times\phi_{m,n}"] & & F(0)\times F(m+n) \arrow[rr, "F(!_{a+b})\times 1"] & & F(a+b)\times F(m+n) \arrow[r, "\phi_{a+b,m+n}"] & F((a+b)+(m+n)) \arrow[r, "F(\psi)"] & F((a+b)+_{a+b}(m+n)) \arrow[dd, "F(\mathrm{id})"] \\
\overset{\phi}{\cong} & & \overset{\phi_{!_a,!_b}}{\cong} & & & & \cong & \\
& (F(0)\times F(0))\times(F(m)\times F(n)) \arrow[uu, "\phi_{0,0}\times 1"'] \arrow[rr, "(F(!_a)\times F(!_b))\times 1"] & & (F(a)\times F(b))\times(F(m)\times F(n)) \arrow[uurr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[rr, "\phi_{a,b}\times\phi_{m,n}"] \arrow[d, "1\times\phi\times 1"] \arrow[dddd, bend right=60, "1\times\beta\times 1"'] \arrow[dr, phantom, "\cong"] & & F(a+b)\times F(m+n) \arrow[r, "\phi_{a+b,m+n}"] & F((a+b)+(m+n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"] & F((a+b)+_{a+b}(m+n)) \arrow[dddd, "F(\kappa')"] \\
& & \overset{1\times\mu\times 1}{\cong} & F(a)\times F(b+m)\times F(n) \arrow[r, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times 1"'] \arrow[ddr, phantom, "\overset{\phi_{\beta,1}}{\cong}"] & F(a)\times F((b+m)+n) \arrow[dd, "1\times F(\beta+1)"] \arrow[urr, "\phi"] & \overset{\phi_{1,\beta+1}}{\cong} & \cong & \\
1 \arrow[uuuur, bend left, "\phi_0\times(x\times y)"] \arrow[uur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[ddr, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[ddddr, bend right, "x\times y"'] & & & & & F((a+m)+(b+n)) \arrow[ddrr, bend left, "F(\psi+\psi)"] & & \\
& & & F(a)\times F(m+b)\times F(n) \arrow[r, "1\times\phi"] & F(a)\times F((m+b)+n) \arrow[ur, "\phi"] \arrow[d, phantom, "\cong"] & & \overset{\phi_{\psi,\psi}}{\cong} & \\
& (F(0)\times F(m))\times(F(0)\times F(n)) \arrow[rr, "(F(!_a)\times 1)\times(F(!_b)\times 1)"] \arrow[drr, "\phi_{0,m}\times\phi_{0,n}"] & & (F(a)\times F(m))\times(F(b)\times F(n)) \arrow[u, "1\times\phi\times 1"] \arrow[rr, "\phi_{a,m}\times\phi_{b,n}"] \arrow[dr, phantom, "\overset{\phi_{!_a,1}\times\phi_{!_b,1}}{\cong}"] & & F(a+m)\times F(b+n) \arrow[uu, "\phi\times\phi"] \arrow[r, "F(\psi)\times F(\psi)"] & F(a+_am)\times F(b+_bn) \arrow[r, "\phi_{a+_am,b+_bn}"] & F((a+_am)+(b+_bn)) \arrow[dd, "F(\kappa'')"] \\
& \overset{\lambda_M\times\lambda_N}{\cong} & & F(0+m)\times F(0+n) \arrow[urr, "F(!_a+1)\times F(!_b+1)"] \arrow[rr, "F(\ell)\times F(\ell)"'] & & F(m)\times(n) \arrow[drr, "\phi_{m,n}"] & \cong & \\
& F(m)\times F(n) \arrow[rrrrrr, "\phi_{m,n}"] & & & & & & F(m+n)
\end{tikzcd}$$

Diagram 4 of 11. Set up but not solved.

M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
N_1 is given by $a' \to n_1 \leftarrow b'$ with $y_1 \in F(n_1)$.
N_2 is given by $b' \to n_2 \leftarrow c'$ with $y_2 \in F(n_2)$.
P_1 is given by $a'' \to p_1 \leftarrow b''$ with $z_1 \in F(p_1)$.
P_2 is given by $b'' \to p_2 \leftarrow c''$ with $z_2 \in F(p_2)$.

$$\begin{tikzcd}
((M_1{\otimes}N_1){\otimes}P_1){\odot}((M_2{\otimes}N_2){\otimes}P_2) \arrow[r, "\alpha \odot \alpha"] \arrow[d, "\chi"'] & (M_1{\otimes}(N_1{\otimes}P_1)){\odot}(M_2{\otimes}(N_2{\otimes}P_2)) \arrow[d, "\chi"] \\
((M_1{\otimes}N_1){\odot}(M_2{\otimes}N_2)){\otimes}(P_1{\odot}P_2) \arrow[d, "\chi \otimes 1"'] & (M_1{\odot}M_2){\otimes}((N_1{\otimes}P_1){\odot}(N_2{\otimes}P_2)) \arrow[d, "1 \otimes \chi"] \\
((M_1{\odot}M_2){\otimes}(N_1{\odot}N_2)){\otimes}(P_1{\odot}P_2) \arrow[r, "\alpha"] & (M_1{\odot}M_2){\otimes}((N_1{\odot}N_2){\otimes}(P_1{\odot}P_2))
\end{tikzcd}$$

Decorations:

(1) Right and then down:

$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2)){\in}F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$F(\psi)\phi_{m_1+(n_1+p_1),m_2+(n_2+p_2)}(\phi_{m_1,n_1+p_1}(x_1,\phi_{n_1,p_1}(y_1,z_1)),\phi_{m_2,n_2+p_2}(x_2,\phi_{n_2,p_2}(y_2,z_2))){\in}F((m_1+(n_1+p_1))+_{b+(b'+b'')}(m_2+(n_2+p_2)))$$

$$\phi_{m_1+_bm_2,(n_1+p_1)+_{b'+b''}(n_2+p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)(\phi_{n_1+p_1,n_2+p_2}(\phi_{n_1,p_1}(y_1,z_1),\phi_{n_2,p_2}(y_2,z_2))){\in}F((m_1+_bm_2)+((n_1+p_1)+_{b'+b''}(n_2+p_2)))$$

$$\phi_{m_1+_bm_2,(n_1+_{b'}n_2)+(p_1+_{b''}p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'}n_2,p_1+_{b''}p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2))){\in}F((m_1+_bm_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2)))$$

(2) Down and then right:

$$F(\psi)\phi_{(m_1+n_1)+p_1,(m_2+n_2),p_2}(\phi_{m_1+n_1,p_1}(\phi_{m_1,n_1}(x_1,y_1),z_1),\phi_{m_2+n_2,p_2}(\phi_{m_2,n_2}(x_2,y_2),z_2)){\in}F(((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2))$$

$$\phi_{(m_1+n_1)+_{b+b'}(m_2+n_2),p_1+_{b''}p_2}(F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2)){\in}F(((m_1+n_1)+_{b+b'}(m_2+n_2))+(p_1+_{b''}p_2))$$

$$\phi_{(m_1+_bm_2)+(n_1+_{b'}n_2),p_1+_{b''}p_2}(\phi_{m_1+_bm_2,n_1+_{b'}n_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2)),F(\psi)\phi_{p_1,p_2}(z_1,z_2)){\in}F(((m_1+_bm_2)+(n_1+_{b'}n_2))+(p_1+_{b''}p_2))$$

$$\phi_{m_1+_bm_2,(n_1+_{b'}n_2)+(p_1+_{b''}p_2)}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),\phi_{n_1+_{b'}n_2,p_1+_{b''}p_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{p_1,p_2}(z_1,z_2))){\in}F((m_1+_bm_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2)))$$

Right and then down:

$$\begin{tikzcd}
& Fm_1Fn_1Fp_1Fm_2Fn_2Fp_2 \arrow[rr, "\phi\times1\times\phi\times1"] \arrow[d, "\alpha\times\alpha"] & & Fm_1+n_1Fp_1Fm_2+n_2Fp_2 \arrow[rr, "\phi\times\phi"] \arrow[d, phantom, "\overset{\alpha_\otimes\times\alpha_\otimes}{\cong}"] & & F(m_1+n_1)+p_1F(m_2+n_2)+p_2 \arrow[rr, "\phi"] \arrow[d, "F(\alpha)\times F(\alpha)"] & {} \arrow[d, phantom, "\overset{\phi_{\alpha,\alpha}}{\cong}"] & F((m_1+n_1)+p_1)+((m_2+n_2)+p_2 \arrow[r, "F(\psi)"] \arrow[d, "F(\alpha+\alpha)"] \arrow[dr, phantom, "\cong"] & F((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2 \arrow[d, "F(\alpha+_\alpha\alpha)"] \\
& Fm_1Fn_1Fp_1Fm_2Fn_2Fp_2 \arrow[rr, "1\times\phi\times1\times\phi"] & & Fm_1Fn_1+p_1Fm_2Fn_2+p_2 \arrow[rr, "\phi\times\phi"] \arrow[dr, "1\times\phi\times1"] \arrow[dddd, "1\times\beta\times1"] & {} \arrow[dr, phantom, "\cong"] & Fm_1+(n_1+p_1)Fm_2+(n_2+p_2) \arrow[rr, "\phi"] & & F(m_1+(n_1+p_1))+(m_2+(n_2+p_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] & F(m_1+(n_1+p_1))+_{b+(b'+b'')}(m_2+(n_2+p_2)) \arrow[dddd, "F(\chi)"] \\
& & & & Fm_1F(n_1+p_1)+m_2Fn_2+p_2 \arrow[rr, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times1"] & & Fm_1F((n_1+p_1)+m_2)+(n_2+p_2) \arrow[ur, "\phi"] \arrow[dd, "1\times F(\beta+1)"] \arrow[r, phantom, "\overset{\phi_{1,\beta+1}}{\cong}"] & {} \arrow[r, phantom, "\cong"] & {} \\
1 \arrow[uuur, bend left] \arrow[uur, bend left] \arrow[ddr, bend right] \arrow[ddddddr, bend right] & & & {} \arrow[r, phantom, "\overset{1\times\mu\times1}{\cong}"] & {} & \overset{\phi_{\beta,1}}{\cong} & & F(m_1+m_2)+((n_1+p_1)+(n_2+p_2)) \arrow[ddr, "F(\psi+\psi)"] & \\
& & & & Fm_1Fm_2+(n_1+p_1)Fn_2+p_2 \arrow[rr, "1\times\phi"] \arrow[dr, phantom, "\cong"] & & Fm_1F((m_2+(n_1+p_1))+(n_2+p_2)) \arrow[ur, "\phi"] & \overset{\phi_{\psi,\psi}}{\cong} & \\
& Fm_1Fm_2Fn_1Fp_1Fn_2Fp_2 \arrow[rr, "1\times1\times\phi\times\phi"] \arrow[dr, "1\times(1\times\phi\times1)"] \arrow[dddd, "1\times(1\times\beta\times1)"] & & Fm_1Fm_2Fn_1+p_1Fn_2+p_2 \arrow[rr, "\phi\times\phi"] \arrow[ur, "1\times\phi\times1"] & & Fm_1+m_2F(n_1+p_1)+(n_2+p_2) \arrow[rr, "F(\psi)\times F(\psi)"] \arrow[uurr, "\phi"] \arrow[ddr, "1\times F(1+\beta+1)"] & & Fm_1+_bm_2F(n_1+p_1)+_{b'+b''}(n_2+p_2) \arrow[r, "\phi"] \arrow[dddd, "1\times F(\chi)"] & F(m_1+_bm_2)+((n_1+p_1)+_{b'+b''}(n_2+p_2)) \arrow[dddd, "F(1+\chi)"] \\
& & Fm_1Fm_2Fn_1Fp_1+n_2Fp_2 \arrow[rr, "1\times(1\times\phi)"] \arrow[dd, "1\times(1\times F(\beta)\times1)"] & \cong & Fm_1Fm_2Fn_1F(p_1+n_2)+p_2 \arrow[ur, "\phi\times\phi"] \arrow[dd, "1\times(1\times F(\beta+1))"] & \overset{1\times\phi_{1,\beta+1}}{\cong} & \cong & & \\
& & \overset{1\times(1\times\mu\times1)}{\cong} & \overset{1\times\phi_{\beta,1}}{\cong} & & & Fm_1+m_2F(n_1+n_2)+(p_1+p_2) \arrow[ddr, "F(\psi)\times F(\psi+\psi)"] & {} \arrow[r, phantom, "\overset{\phi_{1,\chi}}{\cong}"] & {} \\
& & Fm_1Fm_2Fn_1Fn_2+p_1Fp_2 \arrow[rr, "1\times(1\times\phi)"] \arrow[dr, phantom, "\cong"] & & Fm_1Fm_2Fn_1F(n_2+p_1)+p_2 \arrow[urr, "\phi\times\phi"] & \overset{1\times\phi_{\psi,\psi}}{\cong} & & & \\
& Fm_1Fm_2Fn_1Fn_2Fp_1Fp_2 \arrow[rr, "1\times1\times\phi\times\phi"] \arrow[ur, "1\times(1\times\phi\times1)"] & & Fm_1Fm_2Fn_1+n_2Fp_1+p_2 \arrow[rr, "(F(\psi)\phi)\times F(\psi)\times F(\psi)"] \arrow[uurrr, "\phi\times\phi"] & & Fm_1+_bm_2Fn_1+_{b'}n_2Fp_1+_{b''}p_3 \arrow[rr, "1\times\phi"] & & Fm_1+_bm_2F(n_1+_{b'}n_2)+(p_1+_{b''}p_3) \arrow[r, "\phi"] & F(m_1+_bm_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2))
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
 & Fm_1Fn_1Fp_1Fm_2Fn_2Fp_2 \arrow[rrr, "\phi\times1\times\phi\times1"] & & & Fm_1+n_1Fp_1Fm_2+n_2Fp_2 \arrow[rrr, "\phi\times\phi"] \arrow[ddrr, "1\times\phi\times1"] \arrow[dddddd, "1\times\beta\times1"] & & & F(m_1+n_1)+p_1F(m_2+n_2)+p_2 \arrow[rrr, "\phi"] \arrow[d, phantom, "\cong"] & & & F((m_1+n_1)+p_1)+((m_2+n_2)+p_2 \arrow[rr, "F(\psi)"] \arrow[ddd, "F(1+\beta+1)"] & & F((m_1+n_1)+p_1)+_{(b+b')+b''}((m_2+n_2)+p_2 \arrow[dddddd, "F(\chi)"] \\
 & & & & & & & & & & & & \\
 & & & & & & Fm_1+n_1Fp_1+(m_2+n_2)Fp_2 \arrow[rr, "1\times\phi"] \arrow[dd, "1\times F(\beta)\times1"] & & Fm_1+n_1F(p_1+(m_2+n_2))+p_2 \arrow[uurr, "\phi"] \arrow[dd, "1\times F(\beta+1)"] & \underset{\cong}{\phi_{1,\beta+1}} & & \cong & \\
 & & & & & \underset{\cong}{1\times\mu\times1} & & \underset{\cong}{\phi_{\beta,1}} & & & F((m_1+n_1)+(m_2+n_2))+(p_1+p_2) \arrow[dddrr, "F(\psi+\psi)"] & & \\
 & & & & & & Fm_1+n_1F(m_2+n_2)+p_1Fp_2 \arrow[rr, "1\times\phi"] & & Fm_1+n_1F((m_2+n_2)+p_1)+p_2 \arrow[urr, "\phi"] & & & & \\
 & & & & & & & \cong & & & \underset{\cong}{\phi_{\psi,\psi}} & & \\
 & Fm_1Fn_1Fm_2Fn_2Fp_1Fp_2 \arrow[rrr, "\phi\times\phi\times1\times1"] \arrow[ddrr, "(1\times\phi\times1)\times1"] \arrow[dddddd, "(1\times\beta\times1)\times1"] & & & Fm_1+n_1Fm_2+n_2Fp_1Fp_2 \arrow[rrr, "\phi\times\phi"] \arrow[uurr, "1\times\phi\times1"] & & & F(m_1+n_1)+(m_2+n_2)Fp_1+p_2 \arrow[rrr, "F(\psi)\times F(\psi)"] \arrow[uuurrr, "\phi"] \arrow[dddr, "F(1+\beta+1)\times1"] & & & F(m_1+n_1)+_{b+b'}(m_2+n_2)Fp_1+_{b''}p_2 \arrow[rr, "\phi"] \arrow[dddddd, "F(\chi)\times1"] & & F((m_1+n_1)+_{b+b'}(m_2+n_2))+(p_1+_{b''}p_2) \arrow[dddddd, "F(\chi+1)"] \\
 & & & & \cong & & & & & \cong & & & \\
 & & & Fm_1Fn_1+m_2Fn_2Fp_1Fp_2 \arrow[rrr, "(1\times\phi)\times1"] \arrow[dd, "(1\times F(\beta)\times1)\times1"] & & & Fm_1F(n_1+m_2)+n_2Fp_1Fp_2 \arrow[uur, "\phi\times\phi"] \arrow[dd, "(1\times F(\beta+1))\times1"] & \underset{\cong}{\phi_{1,\beta+1}\times1} & & & & & \\
1 \arrow[uuuuuuuuur, bend left] \arrow[uuur, bend left] \arrow[dddr, bend right] \arrow[dddddr, bend right] & & \underset{\cong}{(1\times\mu\times1)\times1} & & \underset{\cong}{\phi_{\beta,1}\times1} & & & & F(m_1+m_2)+(n_1+n_2)Fp_1+p_2 \arrow[dddrr, "F(\psi+\psi)\times F(\psi)"] & & & \underset{\cong}{\phi_{\chi,1}} & \\
 & & & Fm_1Fm_2+n_1Fn_2Fp_1Fp_2 \arrow[rrr, "(1\times\phi)\times1"] & & & Fm_1F(m_2+n_1)+n_2Fp_1Fp_2 \arrow[urr, "\phi\times\phi"] & & & & & & \\
 & & & & \cong & & & & \underset{\cong}{\phi_{\psi,\psi}\times1} & & & & \\
 & Fm_1Fm_2Fn_1Fn_2Fp_1Fp_2 \arrow[rrr, "\phi\times\phi\times1\times1"] \arrow[uurr, "(1\times\phi\times1)\times1"] & & & Fm_1+m_2Fn_1+n_2Fp_1Fp_2 \arrow[rrr, swap, "F(\psi)\times F(\psi)\times(F(\psi)\phi)"] \arrow[uuurrrr, "\phi\times\phi"] & & & Fm_1+_bm_2Fn_1+_{b'}n_2Fp_1+_{b''}p_2 \arrow[rrr, "\phi\times1"] \arrow[dd, "\alpha"] & & & F(m_1+_bm_2)+(n_1+_{b'}n_2)Fp_1+_{b''}p_2 \arrow[rr, "\phi"] & & F((m_1+_bm_2)+(n_1+_{b'}n_2))+(p_1+_{b''}p_2) \arrow[dd, "F(\alpha)"] \\
 & & & & & & & & & & \underset{\cong}{\alpha_{\otimes}} & & \\
 & Fm_1Fm_2Fn_1Fn_2Fp_1Fp_2 \arrow[rrr, "1\times1\times\phi\times\phi"] & & & Fm_1Fm_2Fn_1+n_2Fp_1+p_2 \arrow[rrr, "(F(\psi)\phi)\times F(\psi)\times F(\psi)"] & & & Fm_1+_bm_2Fn_1+_{b'}n_2Fp_1+_{b''}p_3 \arrow[rrr, "1\times\phi"] & & & Fm_1+_bm_2F(n_1+_{b'}n_2)+(p_1+_{b''}p_3) \arrow[rr, "\phi"] & & F(m_1+_bm_2)+((n_1+_{b'}n_2)+(p_1+_{b''}p_2))
\end{tikzcd}$$

Diagram 5 of 11. Set up but not solved.

$$a, b, c \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
\bot_{(a+b)+c} \in F((a+b)+c) & U_{(a+b)+c} \arrow[r, "\mu_\beta"] \arrow[d, "\mu"'] & U_{a+(b+c)} \arrow[d, "\mu"] & \bot_{a+(b+c)} \in F(a+(b+c)) \\
\phi_{a+b,c}(\bot_{a+b},\bot_c) \in F((a+b)+c) & U_{a+b} \otimes U_c \arrow[d, "\mu"'] & U_a \otimes U_{b+c} \arrow[d, "\mu"] & \phi_{a,b+c}(\bot_a,\bot_{b+c}) \in F(a+(b+c)) \\
\phi_{a+b,c}(\phi_{a,b}(\bot_a,\bot_b),\bot_c) \in F((a+b)+c) & (U_a \otimes U_b) \otimes U_c \arrow[r, "\beta'"] & U_a \otimes (U_b \otimes U_c) & \phi_{a,b+c}(\bot_a,\phi_{b,c}(\bot_b,\bot_c)) \in F(a+(b+c))
\end{tikzcd}$$

All the left cospans are $(a+b)+c \to (a+b)+c \leftarrow (a+b)+c$ and all the right cospans are $a+(b+c) \to a+(b+c) \leftarrow a+(b+c)$.

Right and then down:

$$\begin{tikzcd}
& F(0) \arrow[rrr, "F(!_{(a+b)+c})"] \arrow[drrr, phantom, "\cong"] & & & F((a+b)+c) \arrow[d, "F(\alpha)"] \\
& F(0) \arrow[rrr, "F(!_{a+(b+c)})"] \arrow[drrr, phantom, "\overset{\phi_{!_a,!_{b+c}}}{\cong}"] & & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[uur, bend left, "\phi_0"] \arrow[ur, "\phi_0"] \arrow[r, "\phi_0 \times \phi_0"'] \arrow[dr, bend right, "\phi_0\times(\phi_0\times\phi_0)"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[ul, phantom, "\overset{\phi}{\cong}"] \arrow[r, "F(!_a) \times F(!_{b+c})"] & F(a)\times F(b+c) \arrow[rr, "\phi_{a,b+c}"] & & F(a+(b+c)) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times(F(0)\times F(0)) \arrow[u, "1 \times \phi_{0,0}"'] \arrow[ul, phantom, "\overset{\phi}{\cong}"] \arrow[ur, phantom, "\overset{\phi_{!_b!_c}}{\cong}"] \arrow[r, "F(!_a) \times (F(!_b) \times F(!_c))"] & F(a)\times(F(b)\times F(c)) \arrow[u, "1 \times \phi_{b+c}"'] \arrow[r, "1 \times \phi_{b+c}"] & F(a)\times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
& F(0) \arrow[rrr, "F(!_{(a+b)+c})"] \arrow[drrr, phantom, "\overset{\phi_{!_{a+b},!_c}}{\cong}"] & & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[ul, phantom, "\overset{\phi}{\cong}"] \arrow[r, "F(!_{a+b}) \times F(!_c)"] & F(a+b)\times F(c) \arrow[rr, "\phi_{a+b,c}"] & & F((a+b)+c) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[uur, bend left, "\phi_0"] \arrow[ur, "\phi_0"] \arrow[r, "\phi_0^2 \times \phi_0"'] \arrow[dr, bend right, "\phi_0\times(\phi_0\times\phi_0)"'] & (F(0)\times F(0))\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[ul, phantom, "\overset{\phi}{\cong}"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] \arrow[r, "(F(!_a) \times F(!_b)) \times F(!_c)"] \arrow[d, "\alpha'"] & (F(a)\times F(b))\times F(c) \arrow[u, "\phi_{a,b} \times 1"'] \arrow[r, "\phi_{a,b} \times 1"] \arrow[d, "\alpha'"] & F(a+b)\times F(c) \arrow[r, "\phi_{a+b,c}"] \arrow[d, phantom, "\cong"] & F(a+(b+c)) \arrow[d, "F(\alpha)"] \\
& F(0)\times(F(0)\times F(0)) \arrow[r, "F(!_a) \times (F(!_b) \times F(!_c))"] & F(a)\times(F(b)\times F(c)) \arrow[r, "1 \times \phi_{b+c}"] & F(a)\times F(b+c) \arrow[r, "\phi_{a,b+c}"] & F(a+(b+c))
\end{tikzcd}$$

Diagrams 6 and 8 of 11. Set up but not solved.
$M \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i} m \xleftarrow{o} b$ with $x \in F(m)$.
$N \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i'} n \xleftarrow{o'} c$ with $y \in F(n)$.
$U_0 \in F\mathbb{C}\mathbf{sp}$ is given by $0 \xrightarrow{!} 0 \xleftarrow{!} 0$ with $\bot_0 \in F(0)$.

$$\begin{tikzcd}
\begin{array}{c} 0{+}a{\to}(0{+}m){+}_{0+b}(0{+}n){\leftarrow}0{+}c \\ (\bot_0{\oplus}y){\odot}(\bot_0{\oplus}x){\in}F((0{+}m){+}_{0+b}(0{+}n)) \end{array} & (U_0{\otimes}N){\odot}(U_0{\otimes}M) \arrow[r, "\chi"] \arrow[d, "\lambda_N \odot \lambda_M"'] & (U_0{\odot}U_0){\otimes}(N{\odot}M) \arrow[d, "\lambda_{U_0} \otimes \mathrm{id}_{N\odot M}"] & \begin{array}{c} 0{+}a{\to}(0{+}_0 0){+}(m{+}_b n){\leftarrow}0{+}c \\ (\bot_0{\odot}\bot_0){\oplus}(y{\odot}x){\in}F((0{+}_0 0){+}(m{+}_b n)) \end{array} \\
\begin{array}{c} a{\to}m{+}_b n{\leftarrow}c \\ y{\odot}x{\in}F(m{+}_b n) \end{array} & N{\odot}M & U_0{\otimes}(N{\odot}M) \arrow[l, "\lambda_{N\odot M}"'] & \begin{array}{c} 0{+}a{\to}0{+}(m{+}_b n){\leftarrow}0{+}c \\ \bot_0{\oplus}(y{\odot}x){\in}F(0{+}(m{+}_b n)) \end{array}
\end{tikzcd}$$

Down:

$$\begin{tikzcd}
& (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[dr, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] & F(0{+}m){\times}F(0{+}n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] & F((0{+}m){+}(0{+}n)) \arrow[r, "F(\psi)"] \arrow[dd, "F(\ell+\ell)"] & F((0{+}m){+}_{0+b}(0{+}n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[dr, bend right, "x\times y"'] & \overset{\lambda_M\times\lambda_N}{\cong} & F(0{+}m){\times}F(0{+}n) \arrow[ur, "F(!_0+1)\times F(!_0+1)"] \arrow[dr, "F(\ell)\times F(\ell)"'] \arrow[r, phantom, "\cong"] & \arrow[r, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & \arrow[r, phantom, "\cong"] & \\
& F(m){\times}F(n) \arrow[rr, "1"] & & F(m){\times}F(n) \arrow[r, "\phi_{m,n}"] & F(m{+}n) \arrow[r, "F(\psi)"] & F(m{+}_b n)
\end{tikzcd}$$

Right, down and then left:

$$\begin{tikzcd}
& (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "(F(!_0)\times1)\times(F(!_0)\times1)"] & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[rrr, "\phi_{0,m}\times\phi_{0,n}", "\cong"'] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, bend right, "1\times\beta\times1"'] & & & F(0{+}m){\times}F(0{+}n) \arrow[r, "\phi_{0+m,0+n}"] & F((0{+}m){+}(0{+}n)) \arrow[r, "F(\psi)"] \arrow[ddl, "F(1+\beta+1)"'] & F((0{+}m){+}_{0+b}(0{+}n)) \arrow[ddd, "F(\chi)"] \\
& & \overset{1\times\mu\times1}{\cong} & F(0){\times}F(m{+}0){\times}F(n) \arrow[r, "1\times\phi"] \arrow[dl, "1\times F(\beta)\times1"'] \arrow[d, phantom, "\overset{\phi_{\beta,1}}{\cong}"] & F(0){\times}F((m{+}0){+}n) \arrow[dl, "1\times F(\beta+1)"] \arrow[urr, "\phi"] & \overset{\phi_{1,\beta+1}}{\cong} & \cong & \\
& & F(0){\times}F(0{+}m){\times}F(n) \arrow[r, "1\times\phi"] & F(0){\times}F((0{+}m){+}n) \arrow[rr, "\phi"] & \cong & F((0{+}0){+}(m{+}n)) \arrow[drr, bend left, "F(\psi+\psi)"] & \overset{\phi_{\psi,\psi}}{\cong} & \\
& (F(0){\times}F(0)){\times}(F(m){\times}F(n)) \arrow[r, "(F(!_0)\times F(!_0))\times(1\times1)"'] \arrow[ddr, "\phi_{0,0}\times1"] & (F(0){\times}F(0)){\times}(F(m){\times}F(n)) \arrow[rrr, "\phi_{0,0}\times\phi_{m,n}"] \arrow[uu, "1\times\phi\times1"'] & & & F(0{+}0){\times}F(m{+}n) \arrow[r, "F(\psi)\times F(\psi)"] \arrow[uu, "\phi"] & F(0{+}_0 0){\times}F(m{+}_b n) \arrow[r, "\phi_{0+_0 0,m+_b n}"] \arrow[ddd, "F(\kappa)\times1"'] & F((0{+}_0 0){+}(m{+}_b n)) \arrow[ddd, "F(\kappa+1)"] \\
& & & \overset{\phi_{!_0,!_0}}{\cong} & & & & \\
1 \arrow[uuuuur, bend left, "(\phi_0\times x)\times(\phi_0\times y)"] \arrow[uur, "(\phi_0\times\phi_0)\times(x\times y)"] \arrow[dr, "\phi_0\times(x\times y)"'] \arrow[dddr, bend right, "x\times y"'] & \overset{\lambda_{U_0}}{\cong} & F(0{+}0){\times}(F(m){\times}F(n)) \arrow[uurrr, "F(!_0+!_0)\times\phi_{m,n}"'] \arrow[drrrr, "F(\ell)\times(F(\psi)\phi_{m,n})"] & & & \cong & \arrow[r, phantom, "\overset{\phi_{\kappa,1}}{\cong}"] & \\
& F(0){\times}(F(m){\times}F(n)) \arrow[r, "1\times\phi_{m,n}"] & F(0){\times}(F(m{+}n)) \arrow[rrr, "1\times F(\psi)"] & & & F(0){\times}F(m{+}_b n) \arrow[r, "F(!_0)\times1"] \arrow[dr, "\phi_{0,m+_b n}"'] & F(0){\times}F(m{+}_b n) \arrow[r, "\phi_{0,m+_b n}"] \arrow[d, phantom, "\overset{\phi_{!_0,1}}{\cong}"] & F(0{+}(m{+}_b n)) \arrow[dd, "F(\ell)"] \\
& & & \overset{\lambda_{N\odot M}}{\cong} & & & F(0{+}(m{+}_b n)) \arrow[ur, "F(!_0+1)"'] \arrow[dr, "F(\ell)"] \arrow[r, phantom, "\cong"] & \\
& F(m){\times}F(n) \arrow[rrrr, "1"] & & & & F(m){\times}F(n) \arrow[r, "\phi_{m,n}"] & F(m{+}n) \arrow[r, "F(\psi)"] & F(m{+}_b n)
\end{tikzcd}$$

(Scratch work) As a check, let us compute the decoration maps $\lambda_N \colon U_0 \otimes N \to N$ and $\lambda_M \colon U_0 \otimes M \to M$ and compose them via $\odot$. The maps λ_N and λ_M are given explicitly in the paper.

$\lambda_M\colon U_0 \otimes M \to M$

$$\begin{tikzcd}
 & F(0){\times}F(m) \arrow[r, "F(!_0)\times 1"] \arrow[rd, "\phi_{0,m}"'] & F(0){\times}F(m) \arrow[r, "\phi_{0,m}"] & F(0{+}m) \arrow[dd, "F(\ell)"] \\
1 \arrow[ru, bend left, "(\phi_0{\times}x)"] \arrow[rd, bend right, "x"'] & \cong & F(0{+}m) \arrow[ru, "F(!_0+1)"] \arrow[rd, "F(\ell)"'] \arrow[u, phantom, "\overset{\phi_{!_0,1}}{\cong}"] \arrow[r, phantom, "\cong"] & \\
 & F(m) \arrow[rr, "1"] & & F(m)
\end{tikzcd}$$

$\lambda_N\colon U_0 \otimes N \to N$

$$\begin{tikzcd}
 & F(0){\times}F(n) \arrow[r, "F(!_0)\times 1"] \arrow[rd, "\phi_{0,n}"'] & F(0){\times}F(n) \arrow[r, "\phi_{0,n}"] & F(0{+}n) \arrow[dd, "F(\ell)"] \\
1 \arrow[ru, bend left, "(\phi_0{\times}y)"] \arrow[rd, bend right, "y"'] & \cong & F(0{+}n) \arrow[ru, "F(!_0+1)"] \arrow[rd, "F(\ell)"'] \arrow[u, phantom, "\overset{\phi_{!_0,1}}{\cong}"] \arrow[r, phantom, "\cong"] & \\
 & F(n) \arrow[rr, "1"] & & F(n)
\end{tikzcd}$$

To construct $\lambda_N \odot \lambda_M\colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$, we first tensor the above two diagrams:

$$\begin{tikzcd}
 & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[rd, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] & F(0{+}m){\times}F(0{+}n) \arrow[dd, "F(\ell)\times F(\ell)"] \\
1 \arrow[ru, bend left, "(\phi_0{\times}x){\times}(\phi_0{\times}y)"] \arrow[rd, bend right, "x{\times}y"'] & \cong & F(0{+}m){\times}F(0{+}n) \arrow[ru, "F(!_0+1)\times F(!_0+1)"] \arrow[rd, "F(\ell)\times F(\ell)"'] \arrow[u, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] \arrow[r, phantom, "\cong"] & \\
 & F(m){\times}F(n) \arrow[rr, "1"] & & F(m){\times}F(n)
\end{tikzcd}$$

Next, we paste with a square due to pseudonaturality of ϕ:

$$\begin{tikzcd}
 & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[rd, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] & F(0{+}m){\times}F(0{+}n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] \arrow[rdd, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0{+}m){+}(0{+}n)) \arrow[dd, "F(\ell+\ell)"] \\
1 \arrow[ru, bend left, "(\phi_0{\times}x){\times}(\phi_0{\times}y)"] \arrow[rd, bend right, "x{\times}y"'] & \cong & F(0{+}m){\times}F(0{+}n) \arrow[ru, "F(!_0+1)\times F(!_0+1)"] \arrow[rd, "F(\ell)\times F(\ell)"'] \arrow[u, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] \arrow[r, phantom, "\cong"] & & \\
 & F(m){\times}F(n) \arrow[rr, "1"] & & F(m){\times}F(n) \arrow[r, "\phi_{m,n}"] & F(m{+}n)
\end{tikzcd}$$

Finally, we paste with a square due to pseudonaturality of F to obtain the map $\lambda_N \odot \lambda_M\colon (U_0 \otimes N) \odot (U_0 \otimes M) \to N \odot M$:

$$\begin{tikzcd}
 & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "(F(!_0)\times 1)\times(F(!_0)\times 1)"] \arrow[rd, "\phi_{0,m}\times\phi_{0,n}"'] & (F(0){\times}F(m)){\times}(F(0){\times}F(n)) \arrow[r, "\phi_{0,m}\times\phi_{0,n}"] & F(0{+}m){\times}F(0{+}n) \arrow[r, "\phi_{0+m,0+n}"] \arrow[dd, "F(\ell)\times F(\ell)"] \arrow[rdd, phantom, "\overset{\phi_{\ell,\ell}}{\cong}"] & F((0{+}m){+}(0{+}n)) \arrow[r, "F(\psi)"] \arrow[dd, "F(\ell+\ell)"] \arrow[rdd, phantom, "\cong"] & F((0{+}m){+}_{0+b}(0{+}n)) \arrow[dd, "F(\ell+_\ell \ell)"] \\
1 \arrow[ru, bend left, "(\phi_0{\times}x){\times}(\phi_0{\times}y)"] \arrow[rd, bend right, "x{\times}y"'] & \cong & F(0{+}m){\times}F(0{+}n) \arrow[ru, "F(!_0+1)\times F(!_0+1)"] \arrow[rd, "F(\ell)\times F(\ell)"'] \arrow[u, phantom, "\overset{\phi_{!_0,1}\times\phi_{!_0,1}}{\cong}"] \arrow[r, phantom, "\cong"] & & & \\
 & F(m){\times}F(n) \arrow[rr, "1"] & & F(m){\times}F(n) \arrow[r, "\phi_{m,n}"] & F(m{+}n) \arrow[r, "F(\psi)"] & F(m{+}_b n)
\end{tikzcd}$$

Next, we compute the right, down and then left route:

$$\begin{tikzcd}
& (F(0)\times F(m))\times(F(0)\times F(n)) & (F(0)\times F(m))\times(F(0)\times F(n)) & & & F(0+m)\times F(0+n) & F((0+m)+(0+n)) & F((0+m)+_{0+b}(0+n)) \\
& & & F(0)\times F(m+0)\times F(n) & F(0)\times F((m+0)+n) & & & \\
& & & F(0)\times F(0+m)\times F(n) & F(0)\times F((0+m)+n) & F((0+0)+(m+n)) & & \\
& (F(0)\times F(0))\times(F(m)\times F(n)) & (F(0)\times F(0))\times(F(m)\times F(n)) & & & F(0+0)\times F(m+n) & F(0+_0 0)\times F(m+_b n) & F((0+_0 0)+(m+_b n)) \\
1 & & & F(0+0)\times(F(m)\times F(n)) & & & & \\
& F(0)\times(F(m)\times F(n)) & F(0)\times(F(m+n) & & & F(0)\times F(m+_b n) & F(0)\times F(m+_b n) & F(0+(m+_b n)) \\
& & & & & & F(0+(m+_b n)) & \\
& F(m)\times F(n) & & & & F(m)\times F(n) & F(m+n) & F(m+_b n)
\arrow[from=1-2, to=1-3, "(F(!_0)\times1)\times(F(!_0)\times1)"]
\arrow[from=1-3, to=1-6, "\phi_{0,m}\times\phi_{0,n}"]
\arrow[from=1-6, to=1-7, "\phi_{0+m,0+n}"]
\arrow[from=1-7, to=1-8, "F(\psi)"]
\arrow[from=1-8, to=4-8, "F(\chi)"]
\arrow[from=1-3, to=2-4, "1\times\phi\times1"]
\arrow[from=1-3, to=4-3, bend right, "1\times\beta\times1"']
\arrow[from=2-4, to=2-5, "1\times\phi"]
\arrow[from=2-4, to=3-4, "1\times F(\beta)\times1"']
\arrow[from=2-5, to=3-5, "1\times F(\beta+1)"]
\arrow[from=3-4, to=3-5, "1\times\phi"]
\arrow[from=4-3, to=3-4, "1\times\phi\times1"']
\arrow[from=2-5, to=1-7, "\phi"]
\arrow[from=3-5, to=3-6, "\phi"]
\arrow[from=1-7, to=3-6, "F(1+\beta+1)"]
\arrow[from=3-6, to=4-8, bend left, "F(\psi+\psi)"]
\arrow[from=4-6, to=3-6, "\phi"]
\arrow[from=4-2, to=4-3, "(F(!_0)\times F(!_0))\times(1\times1)"']
\arrow[from=4-3, to=4-6, "\phi_{0,0}\times\phi_{m,n}"]
\arrow[from=4-6, to=4-7, "F(\psi)\times F(\psi)"]
\arrow[from=4-7, to=4-8, "\phi_{0+_0 0,m+_b n}"]
\arrow[from=4-7, to=6-7, "F(\kappa)\times1"']
\arrow[from=4-8, to=6-8, "F(\kappa+1)"]
\arrow[from=4-2, to=5-4, "\phi_{0,0}\times1"]
\arrow[from=5-4, to=4-6, "F(!_0+!_0)\times\phi_{m,n}"]
\arrow[from=5-4, to=6-7, "F(\lambda_0)\times(F(\psi)\phi_{m,n})"]
\arrow[from=5-1, to=1-2, bend left, "(\phi_0\times x)\times(\phi_0\times y)"]
\arrow[from=5-1, to=4-2, "(\phi_0\times\phi_0)\times(x\times y)"]
\arrow[from=5-1, to=6-2, "\phi_0\times(x\times y)"']
\arrow[from=5-1, to=8-2, bend right, "x\times y"']
\arrow[from=6-2, to=6-3, "1\times\phi_{m,n}"]
\arrow[from=6-3, to=6-6, "1\times F(\psi)"]
\arrow[from=6-6, to=6-7, "F(!_0)\times1"]
\arrow[from=6-7, to=6-8, "\phi_{0,m+_b n}"]
\arrow[from=6-6, to=7-7, "\phi_{0,m+_b n}"]
\arrow[from=7-7, to=6-8, "F(!_0+1)"]
\arrow[from=7-7, to=8-8, "F(\ell)"]
\arrow[from=6-8, to=8-8, "F(\ell)"]
\arrow[from=8-2, to=8-6, "1"]
\arrow[from=8-6, to=8-7, "\phi_{m,n}"]
\arrow[from=8-7, to=8-8, "F(\psi)"]
\arrow[from=1-3, to=2-5, phantom, "\cong"]
\arrow[from=1-3, to=3-4, phantom, "\overset{1\mu\times1}{\cong}"]
\arrow[from=2-4, to=3-5, phantom, "\overset{\phi_{\beta,1}}{\cong}"]
\arrow[from=3-5, to=4-3, phantom, "\cong"]
\arrow[from=2-5, to=3-6, phantom, "\overset{\phi_{1,\beta+1}}{\cong}"]
\arrow[from=1-7, to=4-8, phantom, "\cong"]
\arrow[from=3-6, to=4-7, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"]
\arrow[from=4-3, to=5-4, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"]
\arrow[from=4-2, to=6-2, phantom, "\overset{\lambda_{M\odot N}}{\cong}"]
\arrow[from=4-6, to=6-6, phantom, "\cong"]
\arrow[from=4-7, to=6-8, phantom, "\overset{\phi_{\kappa,1}}{\cong}"]
\arrow[from=6-7, to=7-7, phantom, "\overset{\phi_{!_0,1}}{\cong}"]
\arrow[from=6-3, to=8-6, phantom, "\overset{\lambda_{N\odot M}}{\cong}"]
\arrow[from=6-8, to=8-7, phantom, "\cong"]
\end{tikzcd}$$

$$\begin{tikzcd}
& F(0)\times F(0) & & F(0)\times F(0) & & F(0+0) & F(0+_0 0) \\
1 & & & F(0+0) & & & \\
& F(0) & & & & & F(0)
\arrow[from=2-1, to=1-2, bend left, "\phi_0\times\phi_0"]
\arrow[from=2-1, to=3-2, bend right, "\phi_0"']
\arrow[from=1-2, to=1-4, "F(!_0)\times F(!_0)"]
\arrow[from=1-4, to=1-6, "\phi_{0,0}"]
\arrow[from=1-6, to=1-7, "F(\psi)"]
\arrow[from=1-7, to=3-7, "F(\kappa)"]
\arrow[from=1-2, to=2-4, "\phi_{0,0}"]
\arrow[from=2-4, to=1-6, "F(!_0+!_0)"]
\arrow[from=2-4, to=3-7, "F(\lambda_0)"]
\arrow[from=3-2, to=3-7, "\mathrm{id}"]
\arrow[from=1-2, to=3-2, phantom, "\overset{\lambda_M}{\cong}"]
\arrow[from=1-4, to=2-4, phantom, "\overset{\phi_{!_0,!_0}}{\cong}"]
\arrow[from=1-6, to=3-7, phantom, "\cong"]
\end{tikzcd}$$

Diagrams 7 and 9 of 11. Should improve this, but seems okay.

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$

$$a \in (\mathsf{A},+,0)$$

$$0+a \to 0+a \leftarrow 0+a$$

$$\phi_{0,a}(\bot_0,\bot_a) \in F(0+a)$$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
 & U_a
\end{tikzcd}$$

$$0+a \to 0+a \leftarrow 0+a$$

$$\bot_{0+a} \in F(0+a)$$

$$a \to a \leftarrow a$$

$$\bot_a \in F(a)$$

Right and then down:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0{+}a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"'] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[ur, "\phi_{0,a}"] \arrow[u, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] \arrow[ur, phantom, "="', shift right=4ex] \arrow[dr, phantom, "\cong", shift left=4ex] & F(0{+}a) \arrow[d, "F(\lambda)"] \\
 & F(0) \arrow[urr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a)
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] \arrow[ddrr, phantom, "=", near start] & & F(0{+}a) \arrow[dd, "F(\lambda)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & {} & & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] \arrow[uurr, phantom, "\cong"', near end] & & F(a)
\end{tikzcd}$$

Removing the lower right ≅ which is the same in each diagram:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0{+}a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"'] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0){\times}F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[ur, "\phi_{0,a}"] \arrow[u, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] \arrow[ur, phantom, "="', shift right=4ex] & F(0{+}a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] \arrow[ddrr, phantom, "=", near start] & & F(0{+}a) \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] \arrow[r, phantom, "="] & {} & & \\
 & F(0) \arrow[uurr, "F(!_{0+a})"] & &
\end{tikzcd}$$

The second diagram is then an identity, so our problem reduces to showing that the following diagram is also an identity:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] & F(0)\times F(a) \arrow[ur, "\phi_{0,a}"] \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\arrow[from=2-1, to=1-2, phantom, shift right=3ex, "\overset{\phi}{\cong}"]
\arrow[from=2-1, to=3-2, phantom, shift left=3ex, "\overset{\phi}{\cong}"]
\arrow[from=2-2, to=1-4, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"]
\arrow[from=2-2, to=3-3, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"]
\arrow[from=2-3, to=1-4, phantom, shift left=4ex, "="]
\end{tikzcd}$$

Removing the diagonal $\phi_{0,a}$:

$$\begin{tikzcd}
 & F(0) \arrow[rr, "F(!_{0+a})"] & & F(0+a) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"] & &
\arrow[from=2-1, to=1-2, phantom, shift right=3ex, "\overset{\phi}{\cong}"]
\arrow[from=2-1, to=3-2, phantom, shift left=3ex, "\overset{\phi}{\cong}"]
\arrow[from=2-2, to=1-4, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"]
\arrow[from=2-2, to=3-3, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"]
\end{tikzcd}$$

This diagram is clearly the same as:

$$\begin{tikzcd}
 & F(0) \arrow[drr, "F(!_{0+a})"] & & \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi"'] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"'] \arrow[d, "\phi_{0,0}"] \arrow[r, "F(!_0)\times F(!_a)"] & F(0)\times F(a) \arrow[r, "\phi_{0,a}"] & F(0+a) \\
 & F(0) \arrow[urr, "F(!_{0+a})"'] & &
\arrow[from=2-1, to=1-2, phantom, shift right=3ex, "\overset{\phi}{\cong}"]
\arrow[from=2-1, to=3-2, phantom, shift left=3ex, "\overset{\phi}{\cong}"]
\arrow[from=1-2, to=2-3, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"]
\arrow[from=3-2, to=2-3, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"]
\end{tikzcd}$$

The two 2-isomorphisms in the top half of the diagram are the inverses of those in the bottom half, when read in a suitable order, and they can be shown to cancel, yielding an identity as desired.

Diagrams 7 and 9 of 11. Set up and solved. (Second attempt - Dr Baez's scratch work in TeX form)

$$F\colon (\mathsf{A},+,0) \to (\mathbf{Cat},\times,1)$$

$$a \in (\mathsf{A},+,0)$$

$0 + a \to 0 + a \leftarrow 0 + a$

$\phi_{0,a}(\bot_0, \bot_a) \in F(0 + a)$

$$\begin{tikzcd}
U_0 \otimes U_a \arrow[r, "\mu"] \arrow[dr, "\lambda_{U_a}"'] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
& U_a
\end{tikzcd}$$

$0 + a \to 0 + a \leftarrow 0 + a$

$\bot_{0+a} \in F(0 + a)$

$a \to a \leftarrow a$

$\bot_a \in F(a)$

Right and then down:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi \times \phi"] \arrow[d, "\lambda^{-1}"'] \arrow[dr, phantom, "\cong"] & F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[d, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[dr, "\phi_{0,a}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0{+}a) \arrow[d, "1"] & U_0{\times}U_a \arrow[d, "\mu"] \\
1 \arrow[r, "\phi"] \arrow[d, "1"'] & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, "1"] & & F(0{+}a) \arrow[d, "F(\lambda_a)"] \arrow[dl, phantom, "\overset{F}{\cong}"] & U_{0+a} \arrow[d, "U_{\lambda_a}"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[rr, "F(!_a)"] \arrow[urr, "F(!_{0+a})"] & & F(a) & U_a
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
1{\times}1 \arrow[r, "\phi \times \phi"] \arrow[d, "\lambda^{-1}"'] \arrow[dr, phantom, "\cong"] & F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "F(\lambda_a)"] \arrow[dl, phantom, "\overset{F}{\cong}"] & U_0{\times}U_a \arrow[d, "\lambda_{U_a}"] \\
1 \arrow[r, "\phi"] & F(0) \arrow[rr, "F(!_{0+a})"] \arrow[urr, "F(!_{0+a})"] & & F(a) & U_a
\end{tikzcd}$$

First we remove the regions common to each diagram. This consists of the laxator triangles ($\overset{F}{\cong}$) in the lower right of each diagram as well as the two squares on the left in the ”right and then down” diagram together with the single square on the left in the ”diagonally” diagram.
Right and then down:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[d, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[dr, "\phi_{0,a}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0{+}a) \arrow[d, "1"] \\
F(0) \arrow[rr, "F(!_{0+a})"] \arrow[d, "1"] & & F(0{+}a) \\
F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0) \times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \\
F(0) \arrow[urr, "F(!_{0+a})"] & &
\end{tikzcd}$$

Then we remove the bottom commuting triangle of the first diagram and add in a commuting triangle to the second diagram, resulting in the following two rectangles which represent equal 2-morphisms:

Right and then down:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] \arrow[drr, "\phi_{0,a}"] \arrow[d, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & & F(0{+}a) \arrow[d, "1"] \\
F(0) \arrow[rrr, "F(!_{0+a})"] & {} & & F(0{+}a)
\end{tikzcd}$$

Diagonally:

$$\begin{tikzcd}
F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[d, "\phi_{0,0}"] \arrow[dr, phantom, "\overset{\phi_{!_0,!_a}}{\cong}"] & F(0){\times}F(a) \arrow[r, "\phi_{0,a}"] & F(0{+}a) \arrow[d, "1"] \\
F(0) \arrow[rr, "F(!_{0+a})"] \arrow[urr, "F(!_{0+a})"] & {} & F(0{+}a)
\end{tikzcd}$$

We also could have achieved two equal 2-morphisms given by $\overset{\phi_{!_0,!_a}}{\cong}$ by removing the two commuting regions of the first diagram after the first step.

Diagram 10 of 11. Set up and solved (Christina).

$M_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a \xrightarrow{i_1} m_1 \xleftarrow{o_1} b$ with $x_1 \in F(m_1)$.
$M_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b \xrightarrow{i_2} m_2 \xleftarrow{o_2} c$ with $x_2 \in F(m_2)$.
$N_1 \in F\mathbb{C}\mathbf{sp}$ is given by $a' \xrightarrow{i'_1} n_1 \xleftarrow{o'_1} b'$ with $y_1 \in F(n_1)$.
$N_2 \in F\mathbb{C}\mathbf{sp}$ is given by $b' \xrightarrow{i'_2} n_2 \xleftarrow{o'_2} c'$ with $x_2 \in F(n_2)$.

$$\begin{tikzcd}
\substack{a+a'\to(m_1+n_1)+_{b+b'}(m_2+n_2)\leftarrow c+c' \\ F(\psi)\phi_{m_1+n_1,m_2+n_2}(\phi_{m_1,n_1}(x_1,y_1),\phi_{m_2,n_2}(x_2,y_2))\in F((m_1+n_1)+_{b+b'}(n_1+n_2))} & (M_2\otimes N_2)\odot(M_1\otimes N_1) \arrow[r, "\beta\odot\beta"] \arrow[d, "\chi"'] & (N_2\otimes M_2)\odot(N_1\otimes M_1) \arrow[d, "\chi"] & \substack{a'+a\to(n_1+m_1)+_{b'+b}(n_2+m_2)\leftarrow c'+c \\ F(\psi)\phi_{n_1+m_1,n_2+m_2}(\phi_{n_1,m_1}(y_1,x_1),\phi_{n_2,m_2}(y_2,x_2))\in F((n_1+m_1)+_{b'+b}(n_1+m_2))} \\
\substack{a+a'\to(m_1+_b m_2)+(n_1+_{b'}n_2)\leftarrow c+c' \\ \phi_{m_1+_b m_2,n_1+_{b'}m_2}(F(\psi)\phi_{m_1,m_2}(x_1,x_2),F(\psi)\phi_{n_1,n_2}(y_1,y_2))\in F((m_1+_b m_2)+(n_1+_{b'}n_2))} & (M_2\odot M_1)\otimes(N_2\odot N_1) \arrow[r, "\beta"] & (N_2\odot N_1)\otimes(M_2\odot M_1) & \substack{a'+a\to(n_1+_{b'}n_2)+(m_1+_b m_2)\leftarrow c'+c \\ \phi_{n_1+_{b'}n_2}(F(\psi)\phi_{n_1,n_2}(y_1,y_2),F(\psi)\phi_{m_1,m_2}(x_1,x_2))\in F((n_1+_{b'}n_2)+(m_1+_b m_2))}
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}[column sep=small]
& (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, "1\times\beta\times1"'] && F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, phantom, "\cong"] && F((m_1+n_1)+(m_2+n_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] \arrow[dr, phantom, "\cong"] & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[ddd, "F(\chi)"] \\
& {} \arrow[d, phantom, "\overset{1\times\mu\times1}{\cong}"] & F(m_1)\times F(n_1+m_2)\times F(n_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] & {} \arrow[d, phantom, "\overset{\phi_{\beta,1}}{\cong}"] & F(m_1)\times F((n_1+m_2)+n_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[dr, phantom, "\overset{\phi_{1,\beta}}{\cong}"] & & \\
& {} & F(m_1)\times F(m_2+n_1)\times F(n_2) \arrow[rr, "1\times\phi"'] & {} \arrow[d, phantom, "\cong"] & F(m_1)\times F((m_2+n_1)+n_2) \arrow[r, "\phi"] & F(m_1+((m_2+n_1)+n_2)) \arrow[dr, "F(\psi+\psi)"] \arrow[d, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"] & \\
1 \arrow[r, "(x_1\times x_2)\times(y_1\times y_2)"] \arrow[uuur, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[dr, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(m_1)\times F(m_2))\times(F(n_1)\times F(n_2)) \arrow[rr, "\phi_{m_1,m_2}\times\phi_{n_1,n_2}"'] \arrow[ur, "1\times\phi\times1"] \arrow[d, "\beta"] && F(m_1+m_2)\times F(n_1+n_2) \arrow[rr, "F(\psi)\times F(\psi)"] \arrow[urr, "\phi_{m_1+m_2,n_1+n_2}"] && F(m_1+_b m_2)\times F(n_1+_{b'}n_2) \arrow[r, "\phi_{m_1+_b m_2,n_1+_{b'}n_2}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{\mu_{m_1+_b m_2,n_1+_{b'}n_2}}{\cong}"] & F((m_1+_b m_2)+(n_1+_{b'}n_2)) \arrow[d, "F(\beta)"] \\
& F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] && F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] && F(n_1+_{b'}n_2)\times F(m_1+_b m_2) \arrow[r, "\phi_{n_1+_{b'}n_2,m_1+_b m_2}"'] & F((n_1+_{b'}n_2)+(m_1+_b m_2))
\end{tikzcd}$$

Right and then down:

$$\begin{tikzcd}[column sep=small]
& (F(m_1)\times F(n_1))\times(F(m_2)\times F(n_2)) \arrow[rr, "\phi_{m_1,n_1}\times\phi_{m_2,n_2}"] \arrow[d, "\beta\times\beta"] & {} \arrow[d, phantom, "\overset{\mu_{m_1,n_1}\times\mu_{m_2,n_2}}{\cong}"] & F(m_1+n_1)\times F(m_2+n_2) \arrow[rr, "\phi_{m_1+n_1,m_2+n_2}"] \arrow[d, "F(\beta)\times F(\beta)"] & {} \arrow[d, phantom, "\overset{\phi_{\beta,\beta}}{\cong}"] & F((m_1+n_1)+(m_2+n_2)) \arrow[r, "F(\psi)"] \arrow[d, "F(\beta+\beta)"] \arrow[dr, phantom, "\cong"] & F((m_1+n_1)+_{b+b'}(m_2+n_2)) \arrow[d, "F(\beta+_\beta\beta)"] \\
1 \arrow[r, "(y_1\times x_1)\times(y_2\times x_2)"] \arrow[ur, bend left, "(x_1\times y_1)\times(x_2\times y_2)"] \arrow[dddr, bend right, "y_1\times y_2\times x_1\times x_2"'] & (F(n_1)\times F(m_1))\times(F(n_2)\times F(m_2)) \arrow[rr, "\phi_{n_1,m_1}\times\phi_{n_2,m_2}"] \arrow[dr, "1\times\phi\times1"] \arrow[ddd, "1\times\beta\times1"'] && F(n_1+m_1)\times F(n_2+m_2) \arrow[rr, "\phi_{n_1+m_1,n_2+m_2}"] \arrow[d, phantom, "\cong"] && F((n_1+m_1)+(n_2+m_2)) \arrow[r, "F(\psi)"] \arrow[dd, "F(1+\beta+1)"] \arrow[dr, phantom, "\cong"] & F((n_1+m_1)+_{b'+b}(n_2+m_2)) \arrow[ddd, "F(\chi)"] \\
& {} \arrow[d, phantom, "\overset{1\times\mu\times1}{\cong}"] & F(n_1)\times F(m_1+n_2)\times F(m_2) \arrow[rr, "1\times\phi"] \arrow[d, "1\times F(\beta)\times1"'] & {} \arrow[d, phantom, "\overset{\phi_{\beta,1}}{\cong}"] & F(n_1)\times F((m_1+n_2)+m_2) \arrow[ur, "\phi"] \arrow[d, "1\times F(\beta+1)"] \arrow[dr, phantom, "\overset{\phi_{1,\beta}}{\cong}"] & & \\
& {} & F(n_1)\times F(n_2+m_1)\times F(m_2) \arrow[rr, "1\times\phi"'] & {} \arrow[d, phantom, "\cong"] & F(n_1)\times F((n_2+m_1)+m_2) \arrow[r, "\phi"] & F(n_1+((n_2+m_1)+m_2)) \arrow[dr, "F(\psi+\psi)"] \arrow[d, phantom, "\overset{\phi_{\psi,\psi}}{\cong}"] & \\
& F(n_1)\times F(n_2)\times F(m_1)\times F(m_2) \arrow[rr, "\phi_{n_1,n_2}\times\phi_{m_1,m_2}"'] \arrow[ur, "1\times\phi\times1"] && F(n_1+n_2)\times F(m_1+m_2) \arrow[rr, "F(\psi)\times F(\psi)"'] \arrow[urr, "\phi_{n_1+n_2,m_1+m_2}"] && F(n_1+_{b'}n_2)\times F(m_1+_b m_2) \arrow[r, "\phi_{n_1+_{b'}n_2,m_1+_b m_2}"'] & F((n_1+_{b'}n_2)+(m_1+_b m_2))
\end{tikzcd}$$

Diagram 11 of 11. Set up but not solved.

$$a, b \in (\mathsf{A}, +, 0)$$

$$\begin{tikzcd}
U_{a+b} \arrow[r, "\mu_{a,b}"] \arrow[d, "U_\beta"'] & U_a \otimes U_b \arrow[d, "\beta'"] \\
U_{b+a} \arrow[r, "\mu_{b,a}"] & U_b \otimes U_a
\end{tikzcd}$$

The top two underlying cospans are

$$a + b \rightarrow a + b \leftarrow a + b$$

and the bottom two underyling cospans are

$$b + a \rightarrow b + a \leftarrow b + a$$

Right and then down:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"'] \arrow[r, "\phi\times\phi"] \arrow[dr, bend right, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] & F(a)\times F(b) \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& F(0)\times F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Down and then right:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[d, "\mathrm{id}"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[rr, "F(!_{b+a})"] \arrow[dr, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

First we remove the lower left commuting quarter circle in the ‘right and then down’ diagram as well as the upper left commuting quarter circle in the ‘down and then right’ diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[dr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"'] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[d, "\beta"] & F(a)\times F(b) \arrow[ur, "\phi_{a,b}"] \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& F(0)\times F(0) \arrow[r, "F(!_b)\times F(!_a)"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] \arrow[d, "\mathrm{id}"] \arrow[drr, phantom, "\cong"] & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[rr, "F(!_{b+a})"] & & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Next we remove the lower left commuting square with sides β in the first diagram and remove the $\mathrm{id}\colon F(0) \to F(0)$ morphism in the second diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[ur, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
& & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[urr, "F(!_{a+b})"] \arrow[rr, "F(!_{b+a})"] & \arrow[ur, phantom, "\cong"] & F(b+a) \arrow[d, "F(\mathrm{id})"] \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Next we assume that our pseudofunctor F is normalized, meaning that $F(\mathrm{id})$ is an identitiy, and remove the commuting triangle with sides $\phi_{a,b}$ in the first diagram and the commuting triangle with sides $\phi_{b,a}$ in the second diagram:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{a+b})"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[ur, phantom, "\overset{\phi}{\cong}"] \arrow[r, "\phi\times\phi"] & F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a)\times F(!_b)"] \arrow[ur, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a)\times F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& & F(b)\times F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
& & & F(a+b) \arrow[d, "F(\beta)"] \\
1 \arrow[r, "\phi"] \arrow[dr, bend right, "\phi\times\phi"'] \arrow[dr, phantom, "\overset{\phi}{\cong}"] & F(0) \arrow[urr, "F(!_{a+b})"] \arrow[rr, "F(!_{b+a})"] & \arrow[ur, phantom, "\cong"] & F(b+a) \\
& F(0)\times F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b)\times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b)\times F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}$$

Next we remove the left quarter circle containing $\overset{\phi}{\cong}$ from each diagram:

$$\begin{tikzcd}
F(0) \arrow[rr, "F(!_{a+b})"] & & F(a+b) \arrow[d, "F(\mathrm{id})"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a) \times F(!_b)"] \arrow[urr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[r, "\phi_{a,b}"] \arrow[d, "\beta"'] \arrow[dr, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(a+b) \arrow[d, "F(\beta)"] \\
& F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
& & F(a+b) \arrow[d, "F(\beta)"] \\
F(0) \arrow[urr, "F(!_{a+b})"] \arrow[rr, "F(!_{b+a})"] \arrow[urr, phantom, "\cong" near end] & & F(b+a) \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b) \times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[ur, "\phi_{b,a}"] &
\end{tikzcd}$$

Making each diagram into a rectangle and removing the $F(\mathrm{id})$ from the first diagram:

$$\begin{tikzcd}
F(0) \arrow[rrr, "F(!_{a+b})"] & & & F(a+b) \arrow[d, "F(\beta)"] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_a) \times F(!_b)"] \arrow[urr, phantom, "\overset{\phi_{!_a,!_b}}{\cong}"] & F(a){\times}F(b) \arrow[urr, "\phi_{a,b}"] \arrow[r, "\beta"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] \arrow[ur, phantom, "\overset{\mu_{a,b}}{\cong}"] & F(b+a)
\end{tikzcd}$$

$$\begin{tikzcd}
F(0) \arrow[rr, "F(!_{a+b})"] \arrow[drr, "F(!_{b+a})"] & & F(a+b) \arrow[d, "F(\beta)"] \arrow[d, phantom, "\cong"', shift right=6ex] \\
F(0){\times}F(0) \arrow[u, "\phi_{0,0}"] \arrow[r, "F(!_b) \times F(!_a)"] \arrow[ur, phantom, "\overset{\phi_{!_b,!_a}}{\cong}"] & F(b){\times}F(a) \arrow[r, "\phi_{b,a}"] & F(b+a)
\end{tikzcd}$$

Are these two diagrams the same?

Pentagon axiom. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.
M_3 is given by $c \to m_3 \leftarrow d$ with $x_3 \in F(m_3)$.
M_4 is given by $d \to m_4 \leftarrow e$ with $x_4 \in F(m_4)$.

$$\begin{tikzcd}
 & & (M_1 \odot M_2) \odot (M_3 \odot M_4) \arrow[drr, "\alpha"] & & \\
((M_1 \odot M_2) \odot M_3) \odot M_4 \arrow[urr, "\alpha"] \arrow[dr, "\alpha \odot 1_{M_4}"] & & & & M_1 \odot (M_2 \odot (M_3 \odot M_4)) \\
 & (M_1 \odot (M_2 \odot M_3)) \odot M_4 \arrow[rr, "\alpha"] & & M_1 \odot ((M_2 \odot M_3) \odot M_4) \arrow[ur, "1_{M_1} \odot \alpha"] &
\end{tikzcd}$$

Upper route:

$$\begin{tikzcd}
Fm_1Fm_2Fm_3Fm_4 \arrow[rr, "\phi\times1\times1"] \arrow[ddd, "\alpha"'] & & Fm_1+m_2Fm_3Fm_4 \arrow[rr, "F(\psi)\times1\times1"] & & Fm_1+_bm_2Fm_3Fm_4 \arrow[rr, "\phi\times1"] \arrow[ddll, "\alpha"] & & F(m_1+_bm_2)+m_3Fm_4 \arrow[rr, "F(\psi)\times1"] \arrow[d, "\phi"] & & F(m_1+_bm_2)+_cm_3Fm_4 \arrow[rr, "\phi"] & & F((m_1+_bm_2)+_cm_3)+m_4 \arrow[rr, "F(\psi)"] & & F((m_1+_bm_2)+_cm_3)+_dm_4 \arrow[ddd, "F(\kappa)"] \\
 & & & & \underset{\cong}{\alpha} & & F((m_1+_bm_2)+m_3)+m_4 \arrow[d, "F(\alpha)"] \arrow[urrrr, "F(\psi+1)"] & \underset{\cong}{\phi_{\psi,1}} & & & \cong & & \\
 & & Fm_1+_bm_2Fm_3Fm_4 \arrow[rr, "1\times\phi"] & & Fm_1+_bm_2Fm_3+m_4 \arrow[rr, "\phi"] \arrow[drr, "1\times F(\psi)"] & & F(m_1+_bm_2)+(m_3+m_4) \arrow[drrr, "F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}}"] & & & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[urr, "F(\psi)\phi\times1"] \arrow[rrr, "\phi\times\phi"] \arrow[drr, "1\times F(\psi)\phi"'] \arrow[ddd, "\alpha"'] & & & Fm_1+m_2Fm_3+m_4 \arrow[ur, "F(\psi)\times1"] \arrow[rrr, "F(\psi)\times F(\psi)"] \arrow[dr, "1\times F(\psi)"'] & & & Fm_1+_bm_2Fm_3+_dm_4 \arrow[rrr, "\phi"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}}"] & & & F(m_1+_bm_2)+(m_3+_dm_4) \arrow[rrr, "F(\psi)"] & & & F(m_1+_bm_2)+_c(m_3+_dm_4) \arrow[ddd, "F(\kappa)"] \\
 & & Fm_1Fm_2Fm_3+_dm_4 \arrow[rr, "\phi\times1"] \arrow[ddrr, "\alpha"] & & Fm_1+m_2Fm_3+_dm_4 \arrow[rr, "\phi"] \arrow[urr, "F(\psi)\times1"'] & & F(m_1+m_2)+(m_3+_dm_4) \arrow[d, "F(\alpha)"] \arrow[urrr, "F(\psi+1)"'] & & & & & & \\
 & & & & \underset{\cong}{\alpha} & & Fm_1+(m_2+(m_3+_dm_4)) \arrow[drrrr, "F(1+\psi)"] & \underset{\cong}{\phi_{1,\psi}} & & & \cong & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[rr, "1\times1\times\phi"] & & Fm_1Fm_2Fm_3+m_4 \arrow[rr, "1\times1\times F(\psi)"] & & Fm_1Fm_2Fm_3+_dm_4 \arrow[rr, "1\times\phi"] & & Fm_1Fm_2+(m_3+_dm_4) \arrow[u, "\phi"'] \arrow[rr, "1\times F(\psi)"] & & Fm_1Fm_2+_c(m_3+_dm_4) \arrow[rr, "\phi"] & & Fm_1+(m_2+_c(m_3+_dm_4)) \arrow[rr, "F(\psi)"] & & Fm_1+_b(m_2+_c(m_3+_dm_4))
\end{tikzcd}$$

Lower route:

$$\begin{tikzcd}
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "\phi\times1\times1"] \arrow[ddd, "\alpha\times1"'] & Fm_1+m_2Fm_3Fm_4 \arrow[r, "F(\psi)\times1\times1"] \arrow[dr, "\phi\times1"] \arrow[ddd, phantom, "\underset{\cong}{\alpha\times1}"] & Fm_1+_bm_2Fm_3Fm_4 \arrow[r, "\phi\times1"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}\times1}"] & F(m_1+_bm_2)+m_3Fm_4 \arrow[r, "F(\psi)\times1"] \arrow[ddd, phantom, "\cong"] & F(m_1+_bm_2)+_cm_3Fm_4 \arrow[r, "\phi"] \arrow[ddd, "F(\kappa)\times1"'] \arrow[rddd, phantom, "\underset{\cong}{\phi_{\kappa,1}}"] & F((m_1+_bm_2)+_cm_3)+m_4 \arrow[r, "F(\psi)"] \arrow[ddd, "F(\kappa+1)"'] \arrow[rddd, phantom, "\cong"] & F((m_1+_bm_2)+_cm_3)+_dm_4 \arrow[ddd, "F(\kappa+_1 1)"] \\
& & F(m_1+m_2)+m_3Fm_4 \arrow[ur, "F(\psi+1)\times1"] \arrow[d, "F(\alpha)\times1"'] & & & & \\
& & Fm_1+(m_2+m_3)Fm_4 \arrow[dr, "F(1+\psi)\times1"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}\times1}"] & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times\phi\times1"] \arrow[ddd, "1\times\alpha\times1"'] & Fm_1Fm_2+m_3Fm_4 \arrow[r, "1\times F(\psi)\times1"] \arrow[ur, "\phi\times1"] & Fm_1Fm_2+_cm_3Fm_4 \arrow[r, "\phi\times1"] \arrow[ddd, "\alpha"'] & Fm_1+(m_2+_cm_3)Fm_4 \arrow[r, "F(\psi)\times1"] \arrow[dr, "\phi"] \arrow[ddd, phantom, "\underset{\cong}{\alpha}"] & Fm_1+_b(m_2+_cm_3)Fm_4 \arrow[r, "\phi"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}}"] & F(m_1+_b(m_2+_cm_3))+m_4 \arrow[r, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & F(m_1+_b(m_2+_cm_3))+_dm_4 \arrow[ddd, "F(\kappa)"] \\
& & & & F(m_1+(m_2+_cm_3))+m_4 \arrow[ur, "F(\psi+1)"] \arrow[d, "F(\alpha)"'] & & \\
& & & & Fm_1+((m_2+_cm_3)+m_4) \arrow[dr, "F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}}"] & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times\phi\times1"] \arrow[ddd, "1\times\alpha"'] & Fm_1Fm_2+m_3Fm_4 \arrow[r, "1\times F(\psi)\times1"] \arrow[dr, "1\times\phi"] \arrow[ddd, phantom, "\underset{\cong}{1\times\alpha}"] & Fm_1Fm_2+_cm_3Fm_4 \arrow[r, "1\times\phi"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{\psi,1}}"] & Fm_1F(m_2+_cm_3)+m_4 \arrow[r, "1\times F(\psi)"] \arrow[ur, "\phi"] \arrow[ddd, phantom, "\cong"] & Fm_1F(m_2+_cm_3)+_dm_4 \arrow[r, "\phi"] \arrow[ddd, "1\times F(\kappa)"'] \arrow[rddd, phantom, "\underset{\cong}{\phi_{1,\kappa}}"] & Fm_1+((m_2+_cm_3)+_dm_4) \arrow[r, "F(\psi)"] \arrow[ddd, "F(1+\kappa)"'] \arrow[rddd, phantom, "\cong"] & Fm_1+_b((m_2+_cm_3)+_dm_4) \arrow[ddd, "F(1+_1\kappa)"] \\
& & Fm_1F(m_2+m_3)+m_4 \arrow[ur, "1\times F(\psi+1)"] \arrow[d, "1\times F(\alpha)"'] & & & & \\
& & Fm_1Fm_2+(m_3+m_4) \arrow[dr, "1\times F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{1,\psi}}"] & & & & \\
Fm_1Fm_2Fm_3Fm_4 \arrow[r, "1\times1\times\phi"] & Fm_1Fm_2Fm_3+m_4 \arrow[r, "1\times1\times F(\psi)"] \arrow[ur, "1\times\phi"] & Fm_1Fm_2Fm_3+_dm_4 \arrow[r, "1\times\phi"] & Fm_1Fm_2+(m_3+_dm_4) \arrow[r, "1\times F(\psi)"] & Fm_1Fm_2+_c(m_3+_dm_4) \arrow[r, "\phi"] & Fm_1+(m_2+_c(m_3+_dm_4)) \arrow[r, "F(\psi)"] & Fm_1+_b(m_2+_c(m_3+_dm_4))
\end{tikzcd}$$

Triangle identity. Set up but not solved.
M_1 is given by $a \to m_1 \leftarrow b$ with $x_1 \in F(m_1)$.
U_b is given by $b \to b \leftarrow b$ with $\bot_b \in F(b)$.
M_2 is given by $b \to m_2 \leftarrow c$ with $x_2 \in F(m_2)$.

$$\begin{tikzcd}
 & M_1 \odot M_2 & \\
(M_1 \odot U_b) \odot M_2 \arrow[ur, "\rho \odot 1_{M_2}"] \arrow[rr, "\alpha"] & & M_1 \odot (U_b \odot M_2) \arrow[ul, "1_{M_1} \odot \lambda"']
\end{tikzcd}$$

Up:

$$\begin{tikzcd}[column sep=small]
1 \arrow[r, "(x_1\times\phi_0)\times x_2"] \arrow[ddr, "x_1\times x_2"'] & Fm_1F0Fm_2 \arrow[r, "(1\times F(!_b))\times 1"] \arrow[dr, "\phi\times 1"] \arrow[dd, phantom, "\underset{\cong}{\rho\times 1}"] & Fm_1FbFm_2 \arrow[r, "\phi_{m_1,b}\times 1"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,!_b}\times 1}"] & Fm_1{+}bFm_2 \arrow[rr, "F(\psi)\times 1"] \arrow[d, phantom, "\cong"] & & Fm_1{+_b}bFm_2 \arrow[r, "\phi_{m_1+_bb,m_2}"] \arrow[ddllll, "F(\kappa)\times 1"] & F(m_1{+_b}){+}m_2 \arrow[r, "F(\psi)"] \arrow[ddll, "F(\kappa+1)"] \arrow[dd, phantom, "\cong"] & F(m_1{+_b}b){+_b}m_2 \arrow[dd, "F(\kappa+_1 1)"] \\
 & & Fm_1{+}0{\times}Fm_2 \arrow[ur, "F(1+!_b)\times 1"] \arrow[dl, "F(r)\times 1"] & & & & & \\
 & Fm_1Fm_2 \arrow[rrr, "\phi"] & & & Fm_1{+}m_2 \arrow[u, phantom, "\underset{\cong}{\phi_{\kappa,1}}"] \arrow[rrr, "F(\psi)"] & & & Fm_1{+_b}m_2
\end{tikzcd}$$

Across and then up:

$$\begin{tikzcd}[column sep=small]
 & Fm_1F0Fm_2 \arrow[r, "(1\times F(!_b))\times 1"] & Fm_1FbFm_2 \arrow[r, "\phi_{m_1,b}\times 1"] \arrow[ddd, "\alpha"] & Fm_1{+}bFm_2 \arrow[rr, "F(\psi)\times 1"] \arrow[drr, "\phi"] \arrow[ddd, phantom, "\underset{\cong}{\alpha}"] & & Fm_1{+_b}bFm_2 \arrow[r, "\phi_{m_1+_bb,m_2}"] \arrow[d, phantom, "\underset{\cong}{\phi_{\psi,1}}"] & F(m_1{+_b}b){+}m_2 \arrow[r, "F(\psi)"] \arrow[ddd, phantom, "\cong"] & F(m_1{+_b}b){+_b}m_2 \arrow[ddd, "F(\kappa)"] \\
 & & & & & F(m_1{+}b){+}m_2 \arrow[ur, "F(\psi+1)"] \arrow[d, "F(\alpha)"] & & \\
 & & & & & Fm_1{+}(b{+}m_2) \arrow[dr, "F(1+\psi)"] \arrow[d, phantom, "\underset{\cong}{\phi_{1,\psi}}"] & & \\
1 \arrow[uuur, "(x_1\times\phi_0)\times x_2"] \arrow[r, "x_1\times(\phi_0\times x_2)"] \arrow[ddr, "x_1\times x_2"'] & Fm_1F0Fm_2 \arrow[r, "1\times(F(!_b)\times 1)"] \arrow[dr, "1\times\phi"] \arrow[dd, phantom, "\underset{\cong}{1\times\lambda}"] & Fm_1FbFm_2 \arrow[r, "1\times\phi_{b,m_2}"] \arrow[d, phantom, "\underset{\cong}{1\times\phi_{!_b,1}}"] & Fm_1Fb{+}m_2 \arrow[urr, "\phi"] \arrow[rr, "1\times F(\psi)"] \arrow[d, phantom, "\cong"] & & Fm_1Fb{+_b}m_2 \arrow[r, "\phi_{m_1,b+_bm_2}"] \arrow[ddllll, "1\times F(\kappa)"] & Fm_1{+}(b{+_b}m_2) \arrow[r, "F(\psi)"] \arrow[ddll, "F(1+\kappa)"] \arrow[dd, phantom, "\cong"] & Fm_1{+_b}(b{+_b}m_2) \arrow[dd, "F(1+_1\kappa)"] \\
 & & Fm_1{\times}F0{+}m_2 \arrow[ur, "1\times F(!_b+1)"] \arrow[dl, "1\times F(\ell)"] & & & & & \\
 & Fm_1Fm_2 \arrow[rrr, "\phi"] & & & Fm_1{+}m_2 \arrow[u, phantom, "\underset{\cong}{\phi_{1,\kappa}}"] \arrow[rrr, "F(\psi)"] & & & Fm_1{+_b}m_2
\end{tikzcd}$$

3. SOME USEFUL MAPS

Given $a \in (\mathsf{A}, +, 0)$, the map $U_{\lambda_a} \colon U_{0+a} \to U_a$ is given by:

$$\begin{tikzcd}
& F(0) \arrow[rr, "F(!_{0+a})"] \arrow[dd, "F(\mathrm{id})"] & & F(0{+}a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[ur, bend left, "\phi"] \arrow[dr, bend right, "\phi"'] & & \cong & \\
& F(0) \arrow[uurr, "F(!_{0+a})"] \arrow[rr, "F(!_a)"] & & F(a)
\end{tikzcd}$$

where the $\cong$ is given by pseudonaturality of F: we have a unique map in $!_a \colon 0 \to a$ in A but also a map $\lambda_a \circ !_{0+a} \colon 0 \to a$ where λ_a is the left unitor of $(\mathsf{A}, +, 0)$, and so $F(!_a) = F(\lambda_a \circ !_{0+a}) \cong F(\lambda_a)F(!_{0+a})$.

The left unitor $\lambda'_{U_a} \colon U_0 \otimes U_a \to U_a$ is given by:

$$\begin{tikzcd}
& & F(0){\times}F(0) \arrow[r, "F(!_0)\times F(!_a)"] \arrow[dd, "\phi_{0,0}"] & F(0){\times}F(a) \arrow[rr, "\phi_{0,a}"] & & F(0{+}a) \arrow[dd, "F(\lambda_a)"] \\
1 \arrow[urr, bend left, "\phi\times\phi"] \arrow[drr, bend right, "\phi"'] & \overset{\phi}{\cong} & & \overset{\phi_{!_0,!_a}}{\cong} & \cong & \\
& & F(0) \arrow[uurrr, "F(!_{0+a})"] \arrow[rrr, "F(!_a)"] & & & F(a)
\end{tikzcd}$$

where the $\cong$ in the lower right is the same as the one in the first diagram.

For an arbitrary M, the left unitor $\lambda'_M \colon U_0 \otimes M \to M$ is given by:

$$\begin{tikzcd}
& F(0){\times}F(m) \arrow[rr, "F(!_0)\times 1"] \arrow[ddrr, "\phi_{0,m}"] & & F(0){\times}F(m) \arrow[rr, "\phi_{0,m}"] & & F(0{+}m) \arrow[dddd, "F(\lambda_m)"] \\
& & & \overset{\phi_{!_0,1}}{\cong} & & \\
1 \arrow[uur, bend left, "\phi_0\times x"] \arrow[ddr, bend right, "x"'] & & \overset{\lambda_M}{\cong} & F(0{+}m) \arrow[uurr, "F(!_0+1)"'] \arrow[ddrr, "F(\lambda_m)"'] & \cong & \\
& & & & & \\
& F(m) \arrow[rrrr, "\mathrm{id}"] & & & & F(m)
\end{tikzcd}$$

For an arbitrary M given by $a \to (m, x) \leftarrow b$, the map $\lambda_M \colon U_b \odot M \to M$ is given by:

$$\begin{tikzcd}
& F(m){\times}F(0) \arrow[rr, "1\times F(!_b)"] \arrow[ddrr, "\phi_{m,0}"] & & F(m){\times}F(b) \arrow[r, "\phi_{m,b}"] & F(m{+}b) \arrow[rr, "F(\psi)"] & & F(m{+}_b b) \arrow[dddd, "F(\kappa)"] \\
& & & \overset{\phi_{1,!_b}}{\cong} & & & \\
1 \arrow[uur, bend left, "x\times\phi_0"] \arrow[ddr, bend right, "x"'] & & \overset{\lambda_M}{\cong} & F(m{+}0) \arrow[uur, "F(1+!_b)"'] \arrow[ddrrr, "F(\lambda_m)"'] & & \cong & \\
& & & & & & \\
& F(m) \arrow[rrrrr, "\mathrm{id}"] & & & & & F(m)
\end{tikzcd}$$

In particular, if $M = U_0$ above, then the map $\lambda_{U_0} \colon U_0 \odot U_0 \to U_0$ is given by:

$$\begin{tikzcd}
& F(0){\times}F(0) \arrow[rr, "F(!_0)\times F(!_0)"] \arrow[ddrr, "\phi_{0,0}"] & & F(0){\times}F(0) \arrow[r, "\phi_{0,0}"] & F(0{+}0) \arrow[rr, "F(\psi)"] & & F(0{+}_0 0) \arrow[dddd, "F(\kappa)"] \\
& & & \overset{\phi_{!_0,!_0}}{\cong} & & & \\
1 \arrow[uur, bend left, "\phi_0\times\phi_0"] \arrow[ddr, bend right, "\phi_0"'] & & \overset{\lambda_M}{\cong} & F(0{+}0) \arrow[uur, "F(!_0+!_0)"'] \arrow[ddrrr, "F(\lambda_0)"'] & & \cong & \\
& & & & & & \\
& F(0) \arrow[rrrrr, "\mathrm{id}"] & & & & & F(0)
\end{tikzcd}$$