BÀI 2. GIỚI HẠN HÀM SỐ

- CHƯƠNG 3. GIỚI HẠN. HÀM SỐ LIÊN TỤC
- |FanPage: **Nguyễn Bảo Vương**

PHẦN B. BÀI TẬP TỰ LUẬN (PHÂN DẠNG)

Dạng 1. Giới hạn tại 1 điểm

Câu 1. **(SGK-CTST 11-Tập 1)** Tìm các giới hạn sau:

a) $\lim\limits_{x\to 3}\left(2x^2-x\right)$;

b) $\lim\limits_{x\to -1}\dfrac{x^2+2x+1}{x+1}$.

Lời giải:

a) $\lim\limits_{x\to 3}\left(2x^2-x\right)=2.3^2-3=15$

b) $\lim\limits_{x\to -1}\dfrac{x^2+2x+1}{x+1}=\lim\limits_{x\to -1}\dfrac{(x+1)^2}{x+1}=\lim\limits_{x\to -1}(x+1)=-1+1=0$

Câu 2. **(SGK-CTST 11-Tập 1)** Tìm các giới hạn sau:

a) $\lim\limits_{x\to -2}\left(x^2+5x-2\right)$;

b) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}$.

Lời giải:

a) $\lim\limits_{x\to -2}\left(x^2+5x-2\right)=\lim\limits_{x\to -2}x^2+\lim\limits_{x\to -2}(5x)-\lim\limits_{x\to -2}2=(-2)^2+5\lim\limits_{x\to -2}x-2=4+5.(-2)-2=-8$

b) $\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}=\lim\limits_{x\to 1}\dfrac{(x+1)(x-1)}{x-1}=\lim\limits_{x\to 1}(x+1)=\lim\limits_{x\to 1}x+\lim\limits_{x\to 1}1=1+1=2$

Câu 3. **(SGK-CTST 11-Tập 1)** Tìm các giới hạn sau:

a) $\lim\limits_{x\to -2}\left(x^2-7x+4\right)$;

b) $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-9}$

c) $\lim\limits_{x\to 1}\dfrac{3-\sqrt{x+8}}{x-1}$

Lời giải:

a) $\lim\limits_{x\to -2}\left(x^2-7x+4\right)=\lim\limits_{x\to -2}x^2-7\cdot\lim\limits_{x\to -2}x+\lim\limits_{x\to -2}4=(-2)^2-7\cdot(-2)+4=22$

b) $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-9}=\lim\limits_{x\to 3}\dfrac{x-3}{(x-3)(x+3)}=\lim\limits_{x\to 3}\dfrac{1}{x+3}=\dfrac{1}{3+3}=\dfrac{1}{6}$

c) $\lim\limits_{x\to 1}\dfrac{3-\sqrt{x+8}}{x-1}=\lim\limits_{x\to 1}\dfrac{(3-\sqrt{x+8})(3+\sqrt{x+8})}{(x-1)(3+\sqrt{x+8})}=\lim\limits_{x\to 1}\dfrac{9-x-8}{(x-1)(3+\sqrt{x+8})}$

$=\lim\limits_{x\to 1}\dfrac{1-x}{(x-1)(3+\sqrt{x+8})}=\lim\limits_{x\to 1}\dfrac{-1}{3+\sqrt{x+8}}=\dfrac{-1}{3+\sqrt{1+8}}=\dfrac{-1}{6}$

Câu 4. **(SGK-CTST 11-Tập 1)** Cho hàm số $f(x)=\begin{cases}-x^2 & \text{khi } x<1\\ x & \text{khi } x\ge 1\end{cases}$.

Tìm các giới hạn $\lim\limits_{x\to 1^+}f(x);\lim\limits_{x\to 1^-}f(x);\lim\limits_{x\to 1}f(x)$ (nếu có).

Lời giải:

$\lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} x = 1$; $\lim_{x\to 1^-} f(x) = \lim_{x\to 1^-}\left(-x^2\right) = -1$

Do $\lim_{x\to 1^+} f(x) \neq \lim_{x\to 1^-} f(x)$ nên không tồn tại $\lim_{x\to 1} f(x)$

Câu 5. Tính giới hạn

a. $\mathop{\mathrm{Lim}}_{x\to -2}\frac{(3x+1)(2-3x)}{x+1}$ b. $\mathop{\mathrm{Lim}}_{x\to 0}\frac{1+x+x^2+x^3}{1+x}$

c. $\mathop{\mathrm{Lim}}_{x\to -1}\frac{\sqrt{3x^2+1}-x}{x-1}$

d. $\mathop{\mathrm{Lim}}_{x\to 1}\sqrt{\frac{5x-1}{2x+7}}$ e. $\mathop{\mathrm{Lim}}_{x\to 2}\frac{\sqrt{x^2-x+1}}{x-1}$

f. $\mathop{\mathrm{Lim}}_{x\to 1}\frac{\sqrt{x+8}-3}{x-2}$

Lời giải

a. $\mathop{\mathrm{Lim}}_{x\to -2}\frac{(3x+1)(2-3x)}{x+1} = 40$

b. $\mathop{\mathrm{Lim}}_{x\to 0}\frac{1+x+x^2+x^3}{1+x} = 1$

c. $\mathop{\mathrm{Lim}}_{x\to -1}\frac{\sqrt{3x^2+1}-x}{x-1} = \frac{-3}{2}$

d. $\mathop{\mathrm{Lim}}_{x\to 1}\sqrt{\frac{5x-1}{2x+7}} = \sqrt{\frac{5-1}{2+7}} = \frac{2}{3}$

e. $\mathop{\mathrm{Lim}}_{x\to 2}\frac{\sqrt{x^2-x+1}}{x-1} = \frac{\sqrt{4-2+1}}{2-1} = \sqrt{3}$

f. $\mathop{\mathrm{Lim}}_{x\to 1}\frac{\sqrt{x+8}-3}{x-2} = \frac{\sqrt{1+8}-3}{1-2} = 0$

Câu 6. Tính giới hạn

a. $\mathop{\mathrm{Lim}}_{x\to -3}\left|\frac{(x+1)(2-x)}{x-1}\right|$ b. $\mathop{\mathrm{Lim}}_{x\to 1}\frac{\sqrt{2x^2+x-1}}{|x+1|}$

Lời giải

a. $\mathop{\mathrm{Lim}}_{x\to -3}\left|\frac{(x+1)(2-x)}{x-1}\right| = \left|\frac{(-3+1)(2+3)}{-3-1}\right| = \frac{5}{2}$

b. $\mathop{\mathrm{Lim}}_{x\to 1}\frac{\sqrt{2x^2+x-1}}{|x+1|} = \frac{\sqrt{2+1-1}}{|1+1|} = \frac{\sqrt{2}}{2}$

Câu 7. Tính giới hạn

a. $\mathop{\mathrm{Lim}}_{x\to -2}\frac{x^4-16}{x^3+2x^2}$ b. $\mathop{\mathrm{Lim}}_{x\to -4}\frac{x^2+3x-4}{x^2+4x}$

c. $\mathop{\mathrm{Lim}}_{x\to 1}\frac{x^3-1}{x(x+5)-6}$

d. $\lim\limits_{x\to -5}\dfrac{x^2+2x-15}{x+5}$ e. $\lim\limits_{x\to 1}\dfrac{x+x^2+...+x^n-n}{x-1}$

Lời giải

a. $\lim\limits_{x\to -2}\dfrac{x^4-16}{x^3+2x^2}=\lim\limits_{x\to -2}\dfrac{\left(x^2-4\right)\left(x^2+4\right)}{x^2\left(x+2\right)}=\lim\limits_{x\to -2}\dfrac{\left(x-2\right)\left(x^2+4\right)}{x^2}=-8$

b. $\lim\limits_{x\to -4}\dfrac{x^2+3x-4}{x^2+4x}=\lim\limits_{x\to -4}\dfrac{\left(x-1\right)\left(x+4\right)}{x\left(x+4\right)}=\lim\limits_{x\to -4}\dfrac{x-1}{x}=\dfrac{5}{4}$

c. $\lim\limits_{x\to 1}\dfrac{x^3-1}{x\left(x+5\right)-6}=\lim\limits_{x\to 1}\dfrac{\left(x-1\right)\left(x^2+x+1\right)}{\left(x-1\right)\left(x+6\right)}=\lim\limits_{x\to 1}\dfrac{x^2+x+1}{x+6}=\dfrac{3}{7}$

d. $\lim\limits_{x\to -5}\dfrac{x^2+2x-15}{x+5}=\lim\limits_{x\to -5}\dfrac{\left(x+5\right)\left(x-3\right)}{x+5}=\lim\limits_{x\to -5}\left(x-3\right)=-8$

e. $\lim\limits_{x\to 1}\dfrac{x+x^2+...+x^n-n}{x-1}=\lim\limits_{x\to 1}\dfrac{x-1+x^2-1+...+x^n-1}{x-1}$

$=\lim\limits_{x\to 1}\left(1+\dfrac{x^2-1}{x-1}+...+\dfrac{x^n-1}{x-1}\right)=\lim\limits_{x\to 1}\left(1+x+1+...+x^{n-1}+x^{n-2}+1\right)$

$=\lim\limits_{x\to 1}\left(\underbrace{1+...+1}_{n}+\underbrace{x+...+x}_{n-1}+...+\underbrace{x^{n-2}+x^{n-2}}_{2}+x^{n-1}\right)=\left(n+n-1+n-2+...+1\right)=\dfrac{n}{2}\left(n+1\right)$

Câu 8. Tính giới hạn

a. $\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{4x}$ b. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+7}-2}{x-1}$

Lời giải

a.

$\lim\limits_{x\to 0}\dfrac{\sqrt{4+x}-2}{4x}=\lim\limits_{x\to 0}\dfrac{\left(\sqrt{4+x}-2\right)\left(\sqrt{4+x}+2\right)}{4x.\left(\sqrt{4+x}+2\right)}=\lim\limits_{x\to 0}\dfrac{4+x-4}{4x.\left(\sqrt{4+x}+2\right)}=\lim\limits_{x\to 0}\dfrac{1}{4.\left(\sqrt{4+x}+2\right)}=\dfrac{1}{16}$

b. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+7}-2}{x-1}=\lim\limits_{x\to 1}\dfrac{\left(\sqrt[3]{x+7}-2\right)\left(\sqrt[3]{\left(x+7\right)^2}+2\sqrt[3]{x+7}+4\right)}{\left(x-1\right).\left(\sqrt[3]{\left(x+7\right)^2}+2\sqrt[3]{x+7}+4\right)}$

$=Lim_{x\to 1}\dfrac{x+7-2^3}{\left(x-1\right).\left(\sqrt[3]{\left(x+7\right)^2}+2\sqrt[3]{x+7}+4\right)}==Lim_{x\to 1}\dfrac{1}{\left(\sqrt[3]{\left(x+7\right)^2}+2\sqrt[3]{x+7}+4\right)}=\dfrac{1}{12}$

Câu 9. Tính giới hạn

a. $\lim\limits_{x\to 2}\dfrac{\sqrt{2x+5}-3}{\sqrt{x+2}-2}$ b. $\lim\limits_{x\to 1}\dfrac{x^3-\sqrt{3x-2}}{x-1}$

Lời giải

a. $\lim\limits_{x\to 2}\dfrac{\sqrt{2x+5}-3}{\sqrt{x+2}-2}=\lim\limits_{x\to 2}\dfrac{\left(\sqrt{2x+5}-3\right)\left(\sqrt{2x+5}+3\right)\left(\sqrt{x+2}+2\right)}{\left(\sqrt{x+2}-2\right)\left(\sqrt{2x+5}+3\right)\left(\sqrt{x+2}+2\right)}$

$$=\lim_{x\to 2}\frac{(2x+5-9)\left(\sqrt{x+2}+2\right)}{(x+2-4)\left(\sqrt{2x+5}+3\right)}=\lim_{x\to 2}\frac{2\left(\sqrt{x+2}+2\right)}{\sqrt{2x+5}+3}=\frac{4}{3}$$

b. $$\lim_{x\to 1}\frac{x^3-\sqrt{3x-2}}{x-1}=\lim_{x\to 1}\frac{\left(x^3-1\right)-\left(\sqrt{3x-2}-1\right)}{x-1}=\lim_{x\to 1}\left[\frac{x^3-1}{x-1}-\frac{\sqrt{3x-2}-1}{x-1}\right]$$

$$=\lim_{x\to 1}\left[x^2+x+1-\frac{3x-2-1}{(x-1)\left(\sqrt{3x-2}+1\right)}\right]=\lim_{x\to 1}\left[x^2+x+1-\frac{3}{\sqrt{3x-2}+1}\right]=3-\frac{3}{2}=\frac{3}{2}$$

Câu 10. Tính giới hạn

a. $\lim\limits_{x\to -1}\dfrac{\sqrt[4]{x+2}-1}{\sqrt[3]{x+2}-1}$ b. $\lim\limits_{x\to 1}\dfrac{\sqrt[3]{x+7}-\sqrt{x+3}}{x-1}$

Lời giải

a. Đặt $t=\sqrt[12]{x+2}\Rightarrow x=t^{12}-2$ khi đó $x\to -1$ thì $t\to 1$. Do đó:

$$\lim_{x\to -1}\frac{\sqrt[4]{x+2}-1}{\sqrt[3]{x+2}-1}=\lim_{x\to 1}\frac{t^3-1}{t^4-1}=\lim_{x\to 1}\frac{(t-1)\left(t^2+t+1\right)}{(t-1)(t+1)\left(t^2+1\right)}=\lim_{x\to 1}\frac{t^2+t+1}{(t+1)\left(t^2+1\right)}=\frac{3}{4}$$

b. $$\lim_{x\to 1}\frac{\sqrt[3]{x+7}-\sqrt{x+3}}{x-1}=\lim_{x\to 1}\frac{\left(\sqrt[3]{x+7}-2\right)-\left(\sqrt{x+3}-2\right)}{x-1}=\lim_{x\to 1}\left[\frac{\sqrt[3]{x+7}-2}{x-1}-\frac{\sqrt{x+3}-2}{x-1}\right]$$

$$=\lim_{x\to 1}\left[\frac{x+7-2^3}{(x-1)\left(\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4\right)}-\frac{1}{\sqrt{x+3}+2}\right]$$

$$=\lim_{x\to 1}\left[\frac{1}{\sqrt[3]{(x+7)^2}+2\sqrt[3]{x+7}+4}-\frac{1}{\sqrt{x+3}+2}\right]=\frac{1}{12}-\frac{1}{4}=-\frac{1}{6}$$

Câu 11. Tính giới hạn

a. $\lim\limits_{x\to -1}\dfrac{2x^2+3x+1}{x^2-1}$ b. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+2x}-1}{3x}$

Lời giải

a. $$\lim_{x\to -1}\frac{2x^2+3x+1}{x^2-1}=\lim_{x\to -1}\frac{2(x+1)\left(x+\frac{1}{2}\right)}{(x+1)(x-1)}=\lim_{x\to -1}\frac{2x+1}{x-1}=\frac{1}{2}$$

b. $$\lim_{x\to 0}\frac{\sqrt{1+2x}-1}{3x}=\lim_{x\to 0}\frac{\left(\sqrt{1+2x}-1\right)\left(\sqrt{1+2x}+1\right)}{3x\left(\sqrt{1+2x}+1\right)}=\lim_{x\to 0}\frac{2x}{3x\left(\sqrt{1+2x}+1\right)}=\lim_{x\to 0}\frac{2}{3\left(\sqrt{1+2x}+1\right)}=\frac{1}{3}$$

Dạng 2. Giới hạn của hàm số tại vô cực

Câu 12. **(SGK-CTST 11-Tập 1)** Tìm các giới hạn sau:

a) $\lim\limits_{x\to+\infty}\dfrac{1-3x^2}{x^2+2x}$;

b) $\lim\limits_{x\to-\infty}\dfrac{2}{x+1}$.

Lời giải:

a) $\lim\limits_{x\to+\infty}\dfrac{1-3x^2}{x^2+2x}=\lim\limits_{x\to+\infty}\dfrac{1-3x^2}{x^2+2x}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{1}{x^2}-3}{1+\dfrac{2}{x}}=\dfrac{\lim\limits_{x\to+\infty}\left(\dfrac{1}{x^2}-3\right)}{\lim\limits_{x\to+\infty}\left(1+\dfrac{2}{x}\right)}=\dfrac{\lim\limits_{x\to+\infty}\dfrac{1}{x^2}-3}{1+\lim\limits_{x\to+\infty}\dfrac{2}{x}}=\dfrac{0-3}{1+0}=-3$

b) $\lim\limits_{x\to-\infty}\dfrac{2}{x+1}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{2}{x}}{1+\dfrac{1}{x}}=\dfrac{\lim\limits_{x\to-\infty}\dfrac{2}{x}}{\lim\limits_{x\to-\infty}\left(1+\dfrac{1}{x}\right)}=\dfrac{\lim\limits_{x\to-\infty}\dfrac{2}{x}}{1+\lim\limits_{x\to-\infty}\dfrac{1}{x}}=\dfrac{0}{1+0}=0$

Câu 13. (SGK-CTST 11-Tập 1) Một cái hồ đang chứa $200m^3$ nước mặn với nồng độ muối $10kg/m^3$. Người ta ngọt hoá nước trong hồ bằng cách bơm nước ngọt vào hồ với tốc độ $2m^3$ / phút.

a) Viết biểu thức $C(t)$ biểu thị nồng độ muối trong hồ sau t phút kể từ khi bắt đầu bơm.

b) Tìm giới hạn $\lim\limits_{t\to+\infty}C(t)$ và giải thích ý nghĩa.

Lời giải:

a) $C(t)=\dfrac{10.200}{200+2t}=\dfrac{1000}{100+t}$

b) $\lim\limits_{x\to+\infty}C(t)=\lim\limits_{x\to+\infty}\dfrac{2000}{100+2t}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{1000}{t}}{\dfrac{100}{t}+1}=\dfrac{\lim\limits_{x\to+\infty}\dfrac{1000}{t}}{\lim\limits_{x\to+\infty}\left(\dfrac{100}{t}+1\right)}$

$=\dfrac{\lim\limits_{x\to+\infty}\dfrac{1000}{t}}{\lim\limits_{x\to+\infty}\dfrac{100}{t}+1}=\dfrac{0}{0+1}=0$

Vậy khi t càng lớn và tiến tới $+\infty$ thì nồng độ muối tiến tới 0

Câu 14. (SGK-CTST 11-Tập 1) Tìm các giới hạn sau:

a) $\lim\limits_{x\to3^-}\dfrac{2x}{x-3}$;

b) $\lim\limits_{x\to+\infty}(3x-1)$.

Lời giải:

a) $\lim\limits_{x\to3^-}\dfrac{2x}{x-3}$

Ta có: $\lim\limits_{x\to3^-}2x=2.3=6;\ \lim\limits_{x\to3^-}\dfrac{1}{x-3}=-\infty$

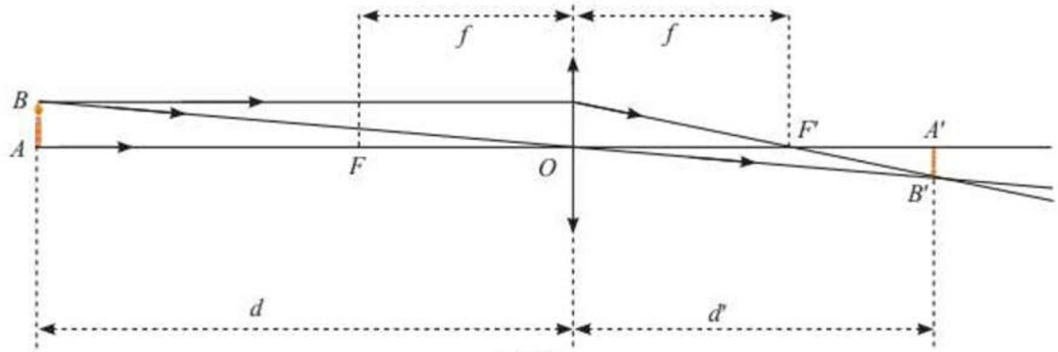
Do đó: $\lim\limits_{x\to3^-}\dfrac{2x}{x-3}=\lim\limits_{x\to3^-}\left[2x\cdot\dfrac{1}{x-3}\right]=-\infty$

b) Ta có: $\lim\limits_{x\to+\infty}x=+\infty$

$\lim\limits_{x\to+\infty}\left(3-\dfrac{1}{x}\right)=3-\lim\limits_{x\to+\infty}\dfrac{1}{x}=3-0=3$

Do đó: $\lim\limits_{x\to+\infty}(3x-1)=\lim\limits_{x\to+\infty}\left[x\left(3-\dfrac{1}{x}\right)\right]=+\infty$

Câu 15. **(SGK-CTST 11-Tập 1)** Một thấu kính hội tụ có tiêu cự là $f>0$ không đổi. Gọi d và d' lần lượt là khoảng cách từ vật thật và ảnh của nó tới quang tâm O của thấu kính (Hình 5).



Hình 5

Ta có công thức: $\dfrac{1}{d}+\dfrac{1}{d'}=\dfrac{1}{f}$ hay $d'=\dfrac{df}{d-f}$.

Xét hàm số $g(d)=\dfrac{df}{d-f}$. Tìm các giới hạn sau đây và giải thích ý nghĩa.

a) $\lim\limits_{d\to f^+} g(d)$;

b) $\lim\limits_{d\to +\infty} g(d)$.

Lời giải:

a) Ta có: $\lim\limits_{d\to f^+} df=f^2>0$; $\lim\limits_{d\to f^+}\dfrac{1}{d-f}=+\infty$

Suy ra: $\lim\limits_{d\to f^+} g(d)=\lim\limits_{d\to f^+}\dfrac{df}{d-f}=\lim\limits_{d\to f^+}\left[df\cdot\dfrac{1}{d-f}\right]=+\infty$

Vậy khi vật tiến gần tới tiêu điểm thì ảnh càng lớn và tiến tới $+\infty$

b) $\lim\limits_{d\to +\infty} g(d)=\lim\limits_{d\to +\infty}\dfrac{df}{d-f}=\lim\limits_{d\to +\infty}\dfrac{f}{1-\dfrac{f}{d}}=\dfrac{f}{1-0}=f$

Vậy khi vật ở rất xa, tiến tới $+\infty$ thì ảnh của vật nằm trên tiêu điểm

Câu 16. **(SGK-CTST 11-Tập 1)** Trong hồ có chứa 6000 lít nước ngọt. Người ta bơm nước biển có nồng độ muối là 30 gam/lít vào hồ với tốc độ 15 lít/phút.

a) Chứng tỏ rằng nồng độ muối của nước trong hồ sau t phút kể từ khi bắt đầu bơm là $C(t)=\dfrac{30t}{400+t}$ (gam/lít).

b) Nồng độ muối trong hồ như thế nào nếu $t\to+\infty$.

Lời giải:

a) Sau thời gian t, số lít nước bơm vào hồ là: $15t$ (lít)

Trong $15t$ lít nước biển có lượng muối: $30.15t=450t$ (gam)

Nồng độ muối trong hồ sau thời gian t phút: $C(t)=\dfrac{450t}{6000+15t}=\dfrac{30t}{400+t}$

b) $\lim\limits_{x\to+\infty} C(t)=\lim\limits_{x\to+\infty}\dfrac{30t}{400+t}=\lim\limits_{x\to+\infty}\dfrac{30}{\dfrac{400}{t}+1}=\dfrac{30}{0+1}=30$

Câu 17. Tính giới hạn

a. $\lim\limits_{x\to-\infty}\dfrac{x^2+x+1}{2x^3+2x+5}$ b. $\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^2-x+1}}{5x^2-1}$

c. $\lim\limits_{x\to+\infty}\dfrac{x\sqrt{x}+\sqrt[3]{x^2}+2}{x\sqrt{x^3}+1}$ d. $\lim\limits_{x\to+\infty}\dfrac{2x^2+1}{x^3-3x^2+2}$

Lời giải

a. $\lim\limits_{x\to-\infty}\dfrac{x^2+x+1}{2x^3+2x+5}=\lim\limits_{x\to-\infty}\dfrac{x^2\left(1+\dfrac{1}{x}+\dfrac{1}{x^2}\right)}{x^3\left(2+\dfrac{2}{x^2}+\dfrac{5}{x^3}\right)}=\lim\limits_{x\to-\infty}\dfrac{\left(1+\dfrac{1}{x}+\dfrac{1}{x^2}\right)}{x\left(2+\dfrac{2}{x^2}+\dfrac{5}{x^3}\right)}=0$

b. $\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{x^2-x+1}}{5x^2-1}=\lim\limits_{x\to+\infty}\dfrac{\sqrt[3]{\dfrac{1}{x^4}-\dfrac{1}{x^5}+\dfrac{1}{x^6}}}{5-\dfrac{1}{x^2}}=0$

c. $\lim\limits_{x\to+\infty}\dfrac{x\sqrt{x}+\sqrt[3]{x^2}+2}{x\sqrt{x^3}+1}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{x\sqrt{x}+\sqrt[3]{x^2}+2}{x\sqrt{x^3}}}{\dfrac{x\sqrt{x^3}+1}{x\sqrt{x^3}}}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{1}{x}+\dfrac{\sqrt[3]{x^2}}{x\sqrt{x^3}}+\dfrac{2}{x\sqrt{x^3}}}{\sqrt{1+\dfrac{1}{x^3}}}=0$

d. $\lim\limits_{x\to+\infty}\dfrac{2x^2+1}{x^3-3x^2+2}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{2}{x}+\dfrac{1}{x^3}}{1-\dfrac{3}{x}+\dfrac{2}{x^3}}=0$

Câu 18. Tính giới hạn

a. $\lim\limits_{x\to+\infty}\sqrt{\dfrac{x^3+1}{2x^3+5}}$ b. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-x+1}}{2x+1}$ c. $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^3-1}}{\sqrt{2x^2+1}}$

d. $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^6+x^4+x^2+1}}{\sqrt{2x^2+1}}$ e. $\lim\limits_{x\to-\infty}\dfrac{x-\sqrt{2x^2+1}}{2x+3\sqrt{x^2+1}}$

Lời giải

a. $\lim\limits_{x\to+\infty}\sqrt{\dfrac{x^3+1}{2x^3+5}}=\lim\limits_{x\to+\infty}\sqrt{x^3\dfrac{1+\dfrac{1}{x^3}}{2+\dfrac{5}{x^3}}}=+\infty$

b. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2-x+1}}{2x+1}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2\left(1-\dfrac{1}{x}+\dfrac{1}{x^2}\right)}}{x\left(2+\dfrac{1}{x}\right)}=\lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{\left(1-\dfrac{1}{x}+\dfrac{1}{x^2}\right)}}{x\left(2+\dfrac{1}{x}\right)}=-\dfrac{1}{2}$

c. $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^3-1}}{\sqrt{2x^2+1}}=\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^3\left(1-\dfrac{1}{x^3}\right)}}{\sqrt{x^2\left(2+\dfrac{1}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{x.\sqrt[3]{\left(1-\dfrac{1}{x^3}\right)}}{|x|\sqrt{\left(2+\dfrac{1}{x^2}\right)}}=-\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{\left(1-\dfrac{1}{x^3}\right)}}{\sqrt{\left(2+\dfrac{1}{x^2}\right)}}=\dfrac{-\sqrt{2}}{2}$

d. $\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^6+x^4+x^2+1}}{\sqrt{2x^2+1}}=\lim\limits_{x\to-\infty}\dfrac{\sqrt[3]{x^6\left(1+\dfrac{1}{x^2}+\dfrac{1}{x^4}+\dfrac{1}{x^6}\right)}}{\sqrt{x^2\left(2+\dfrac{1}{x^2}\right)}}=\lim\limits_{x\to-\infty}\dfrac{x^2.\sqrt[3]{\left(1+\dfrac{1}{x^2}+\dfrac{1}{x^4}+\dfrac{1}{x^6}\right)}}{|x|.\sqrt{\left(2+\dfrac{1}{x^2}\right)}}=+\infty$

e. $\lim\limits_{x\to-\infty}\dfrac{x-\sqrt{2x^2+1}}{2x+3\sqrt{x^2+1}}=\lim\limits_{x\to-\infty}\dfrac{x-|x|\sqrt{2+\dfrac{1}{x^2}}}{2x+3|x|\sqrt{1+\dfrac{1}{x^2}}}=\dfrac{1+\sqrt{2}}{-1}=-1-\sqrt{2}$

Câu 19. Tính giới hạn

a. $\lim\limits_{x\to+\infty}\dfrac{2x^2-3x+1}{-2+3x-4x^2}$

b. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x}+3x}{\sqrt{4x^2+1}-x+3}$

Lời giải

a. $\lim\limits_{x\to+\infty}\dfrac{2x^2-3x+1}{-2+3x-4x^2}=\lim\limits_{x\to+\infty}\dfrac{2-\dfrac{3}{x}+\dfrac{1}{x^2}}{\dfrac{-2}{x^2}+\dfrac{3}{x}-4}=\dfrac{-1}{2}$

b. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x}+3x}{\sqrt{4x^2+1}-x+3}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2.\left(1+\dfrac{2}{x}\right)}+3x}{\sqrt{x^2.\left(4+\dfrac{1}{x^2}\right)}-x+3}$

$=\lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{1+\dfrac{2}{x}}+3x}{|x|\sqrt{4+\dfrac{1}{x^2}}-x+3}=\lim\limits_{x\to-\infty}\dfrac{-x.\sqrt{1+\dfrac{2}{x}}+3x}{-x.\sqrt{4+\dfrac{1}{x^2}}-x+3}=\lim\limits_{x\to-\infty}\dfrac{-\sqrt{1+\dfrac{2}{x}}+3}{-\sqrt{4+\dfrac{1}{x^2}}-1+\dfrac{3}{x}}=\dfrac{-2}{3}$

Câu 20. Tính giới hạn

a. $\lim\limits_{x\to-\infty}\dfrac{3x+5}{2x^2+1}$

b. $\lim\limits_{x\to+\infty}\dfrac{-4x^2+1}{2-x}$

Lời giải

a. $\lim\limits_{x\to-\infty}\dfrac{3x+5}{2x^2+1}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{3}{x}+\dfrac{5}{x^2}}{2+\dfrac{1}{x^2}}=0$

b. $\lim\limits_{x\to+\infty}\dfrac{-4x^2+1}{2-x}=\lim\limits_{x\to+\infty}\dfrac{-4+\dfrac{1}{x^2}}{\dfrac{2}{x^2}-\dfrac{1}{x}}=+\infty$

Vì $\lim\limits_{x\to+\infty}\left(-4+\dfrac{1}{x^2}\right)<0$ và $\lim\limits_{x\to+\infty}\left(\dfrac{2}{x^2}-\dfrac{1}{x}\right)=0;\quad \dfrac{2}{x^2}-\dfrac{1}{x}<0;\quad \forall x>2$

Câu 21. Tính giới hạn

a. $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-x+2}+2x\right)$ b. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+2x-3}-x\right)$

Lời giải

a. $\lim\limits_{x\to-\infty}\left(\sqrt{4x^2-x+2}+2x\right)=\lim\limits_{x\to-\infty}\dfrac{4x^2-x+2-4x^2}{\sqrt{4x^2-x+2}-2x}=\lim\limits_{x\to-\infty}\dfrac{2-x}{\sqrt{4x^2-x+2}-2x}$

$=\lim\limits_{x\to-\infty}\dfrac{2-x}{-x.\sqrt{4-\frac{1}{x}+\frac{2}{x^2}}-2x}=\lim\limits_{x\to-\infty}\dfrac{\frac{2}{x}-1}{-\sqrt{4-\frac{1}{x}+\frac{2}{x^2}}-2x}=\dfrac{1}{4}$

b. $\lim\limits_{x\to+\infty}\left(\sqrt{x^2+2x-3}-x\right)=\lim\limits_{x\to+\infty}\dfrac{2x-3}{\sqrt{x^2+2x-3}+x}=\lim\limits_{x\to+\infty}\dfrac{2-\frac{3}{x}}{\sqrt{1+\frac{2}{x}-\frac{3}{x^2}}+1}=1$

Câu 22. Tính giới hạn

a. $\lim\limits_{x\to-\infty}\left(-x^3+x^2-x+1\right)$ b. $\lim\limits_{x\to-\infty}\left(2x-\sqrt{4x^2+2x-1}\right)$

Lời giải

a. $\lim\limits_{x\to-\infty}\left(-x^3+x^2-x+1\right)=\lim\limits_{x\to-\infty}\left[x^3\left(-1+\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}\right)\right]=+\infty$

Vì $\lim\limits_{x\to-\infty}x^3=-\infty$; $\lim\limits_{x\to-\infty}\left(-x^3+x^2-x+1\right)=\lim\limits_{x\to-\infty}\left(-1+\frac{1}{x}-\frac{1}{x^2}+\frac{1}{x^3}\right)=-1<0$

b. $\lim\limits_{x\to-\infty}\left(2x-\sqrt{4x^2+2x-1}\right)=\lim\limits_{x\to-\infty}\left(2x-|x|\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}\right)=\lim\limits_{x\to-\infty}\left(2x+x\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}\right)$

$=\lim\limits_{x\to-\infty}\left[x\left(2+\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}\right)\right]=-\infty$

Vì $\lim\limits_{x\to-\infty}\left(2+\sqrt{4+\frac{2}{x}-\frac{1}{x^2}}\right)=4>0$; $\lim\limits_{x\to-\infty}x=-\infty$

Câu 23. Tính giới hạn

a. $\lim\limits_{x\to\infty}\dfrac{2x^2-x+3}{x^2+1}$ b. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+1}}{x}$

Lời giải

a. $\lim\limits_{x\to\infty}\dfrac{2x^2-x+3}{x^2+1}=\lim\limits_{x\to\infty}\dfrac{2-\frac{1}{x}+\frac{3}{x^2}}{1+\frac{1}{x^2}}=2$

b. $\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+1}}{x}=\lim\limits_{x\to+\infty}\dfrac{x.\sqrt{1+\dfrac{1}{x^2}}}{x}=\lim\limits_{x\to+\infty}\sqrt{1+\dfrac{1}{x^2}}=1$

Dạng 3. Giới hạn một bên

Câu 24. **(SGK-CTST 11-Tập 1)** Cho hàm số $f(x)=\begin{cases}1-2x & \text{khi } x\le -1\\ x^2+2 & \text{khi } x>-1\end{cases}$

Tìm các giới hạn $\lim\limits_{x\to-1^+}f(x)$, $\lim\limits_{x\to-1^-}f(x)$ và $\lim\limits_{x\to-1}f(x)$ (nếu có).

Lời giải:

Giả sử (x_n) là dãy số bất kì, $x_n<-1$ và $x_n\to-1$. Khi đó $f(x_n)=1-2x_n$ nên $\lim f(x_n)=\lim(1-2x_n)=1-2\cdot(-1)=3$

Vậy $\lim\limits_{x\to-1^-}f(x)=3$

Giả sử (x_n) là dãy số bất kì, $x_n>-1$ và $x_n\to-1$. Khi đó $f(x_n)=x_n^2+2$ nên $\lim f(x_n)=\lim(x_n^2+2)=(-1)^2+2=3$

Vậy $\lim\limits_{x\to-1^+}f(x)=3$

Suy ra $\lim\limits_{x\to-1}f(x)=3$

Câu 25. **(SGK-CTST 11-Tập 1)** Tìm các giới hạn sau:

a) $\lim\limits_{x\to-1^+}\dfrac{1}{x+1}$;

b) $\lim\limits_{x\to-\infty}(1-x^2)$;

c) $\lim\limits_{x\to3^-}\dfrac{x}{3-x}$.

Lời giải:

a) $\lim\limits_{x\to-1^+}\dfrac{1}{x+1}=+\infty$

b) $\lim\limits_{x\to-\infty}(1-x^2)=\lim\limits_{x\to-\infty}\left[x^2\cdot\left(\dfrac{1}{x^2}-1\right)\right]=\lim\limits_{x\to-\infty}x^2\cdot\lim\limits_{x\to-\infty}\left(\dfrac{1}{x^2}-1\right)=(+\infty)\cdot(0-1)=-\infty$

c) $\lim\limits_{x\to3^-}\dfrac{x}{3-x}=\lim\limits_{x\to3^-}x.\lim\limits_{x\to3^-}\dfrac{1}{3-x}=+\infty$

Câu 26. Tìm giới hạn

a. $\lim\limits_{x\to3^+}\dfrac{1+3x-2x^2}{x-3}$ b. $\lim\limits_{x\to2^+}\dfrac{\sqrt{x^2-4}}{x-2}$ c. $\lim\limits_{x\to2^+}\dfrac{2x+1}{x-2}$

d. $\lim\limits_{x\to2^-}\dfrac{2x+1}{x-2}$ e. $\lim\limits_{x\to3^+}\dfrac{3x+4}{3-x}$ f. $\lim\limits_{x\to3^-}\left(\sqrt{3-x}+x\right)$

Lời giải

a. $\lim\limits_{x\to3^+}\dfrac{1+3x-2x^2}{x-3}=-\infty$

b. $\lim\limits_{x\to2^+}\dfrac{\sqrt{x^2-4}}{x-2}=\lim\limits_{x\to2^+}\dfrac{\sqrt{(x-2)(x+2)}}{x-2}=\lim\limits_{x\to2^+}\dfrac{\sqrt{x+2}}{\sqrt{x-2}}=+\infty$

c. $\lim\limits_{x\to2^+}\dfrac{2x+1}{x-2}=+\infty$

d. $\lim\limits_{x\to 2^-}\dfrac{2x+1}{x-2}=-\infty$

e. $\lim\limits_{x\to 3^+}\dfrac{3x+4}{3-x}=-\infty$

f. $\lim\limits_{x\to 3^-}\left(\sqrt{3-x}+x\right)=3$

Câu 27. Tìm giới hạn

a. $\lim\limits_{x\to 2^-}\dfrac{|2-x|}{2x^2-5x+2}$ b. $\lim\limits_{x\to 3^-}\dfrac{|3-x|}{3-x}$ c. $\lim\limits_{x\to -2^+}\dfrac{\sqrt{x^2+4x+4}}{x+2}$

Lời giải

a. $\lim\limits_{x\to 2^-}\dfrac{|2-x|}{2x^2-5x+2}=\lim\limits_{x\to 2^-}\dfrac{2-x}{(x-2)(2x-1)}=\lim\limits_{x\to 2^-}\dfrac{-1}{2x-1}=\dfrac{-1}{3}$

b. $\lim\limits_{x\to 3^-}\dfrac{|3-x|}{3-x}=\lim\limits_{x\to 3^-}\dfrac{3-x}{3-x}=1$

c. $\lim\limits_{x\to -2^+}\dfrac{\sqrt{x^2+4x+4}}{x+2}=\lim\limits_{x\to -2^+}\dfrac{|x+2|}{x+2}=\lim\limits_{x\to -2^+}\dfrac{x+2}{x+2}=1$

Câu 28. Tìm giới hạn

a. $\lim\limits_{x\to 4^-}(4-x)\sqrt{\dfrac{2x+1}{x^3-64}}$ b. $\lim\limits_{x\to +\infty}(2x-1)\sqrt{\dfrac{x^2+1}{x^4+3x+1}}$

Lời giải

a. $\lim\limits_{x\to 4^-}(4-x)\sqrt{\dfrac{2x+1}{x^3-64}}=\lim\limits_{x\to 4^-}\sqrt{\dfrac{(2x+1)(4-x)^2}{(x-4)(x^2+4x+16)}}=\lim\limits_{x\to 4^-}\sqrt{\dfrac{(2x+1)(4-x)}{(x^2+4x+16)}}=0$

b. $\lim\limits_{x\to +\infty}(2x-1)\sqrt{\dfrac{x^2+1}{x^4+3x+1}}=\lim\limits_{x\to +\infty}\sqrt{\dfrac{(x^2+1)(2x-1)^2}{x^4+3x+1}}=\lim\limits_{x\to +\infty}\sqrt{\dfrac{\left(1+\dfrac{1}{x^2}\right)\left(2-\dfrac{1}{x}\right)^2}{1+\dfrac{3}{x^3}+\dfrac{1}{x^4}}}=2$

Bài toán chứng minh sự tồn tại của giới hạn tại 1 điểm.

Nếu $\lim\limits_{x\to x_0^-}f(x)=\lim\limits_{x\to x_0^+}f(x)=L$ thì tồng tại $\lim\limits_{x\to x_0}f(x)=L$.

Câu 29. Tìm giới hạn của các hàm số sau:

a) $f(x)=\begin{cases}\dfrac{x^2-3x+2}{x^2-1} & khi\ x>1\\ -\dfrac{x}{2} & khi\ x\le 1\end{cases}$ tại $x=1$.

b) $f(x)=\begin{cases}\dfrac{1-\sqrt{\cos 2x}}{\sin^2 x} & khi\ x\ge 0\\ \cos x & khi\ x<0\end{cases}$ tại $x=0$

c) $f(x)=\begin{cases}x^2-2x+3 & khi\ x\le 2\\ 4x-3 & khi\ x>2\end{cases}$ tại $x=2$

Lời giải

a) Có $\lim\limits_{x\to 1^-}f(x)=\lim\limits_{x\to 1^-}\left(-\dfrac{x}{2}\right)=-\dfrac{1}{2}$

$\lim\limits_{x\to 1^+}f(x)=\lim\limits_{x\to 1^+}\dfrac{x^2-3x+2}{x^2-1}=\lim\limits_{x\to 1^+}\dfrac{(x-1)(x-2)}{(x-1)(x+1)}=\lim\limits_{x\to 1^+}\dfrac{x-2}{x+1}=-\dfrac{1}{2}$.

$\Rightarrow \lim\limits_{x\to 1^-} f(x) = \lim\limits_{x\to 1^+} f(x) = -\dfrac{1}{2} \Rightarrow \lim\limits_{x\to 1} f(x) = -\dfrac{1}{2}$

b) Có

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \frac{1-\sqrt{\cos 2x}}{\sin^2 x} = \lim_{x\to 0^+} \frac{1-\cos 2x}{\left(1+\sqrt{\cos 2x}\right)\sin^2 x} = \lim_{x\to 0^+} \frac{2\sin^2 x}{\left(1+\sqrt{\cos 2x}\right)\sin^2 x} = \lim_{x\to 0^+} \frac{2}{\left(1+\sqrt{\cos 2x}\right)} = 1$$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (\cos x) = 1$

$\Rightarrow \lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^+} f(x) = 1 \Rightarrow \lim\limits_{x\to 0} f(x) = 1$

c) Có $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \left(x^2 - 2x + 3\right) = 3$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (4x-3) = 5$

$\Rightarrow \lim\limits_{x\to 2^-} f(x) \neq \lim\limits_{x\to 2^+} f(x)$

Vậy không tồn tại giới hạn của hàm số $f(x)$ tại $x=2$.

Câu 30. Tìm m để các hàm số có giới hạn tại:

a) $f(x) = \begin{cases} \dfrac{\sqrt{1+x^2}-1}{\sqrt[3]{1-x}-1} & khi\ x<0 \\ m+\dfrac{1}{2} & khi\ x\ge 0 \end{cases}$ tại $x=0$

b) $f(x) = \begin{cases} x+m & khi\ x<0 \\ \dfrac{x^2+100x+3}{x+3} & khi\ x\ge 0 \end{cases}$ tại $x=0$

c) $f(x) = \begin{cases} \dfrac{\sqrt[3]{3x+2}-2}{x-2} & khi\ x>2 \\ mx-\dfrac{1}{4} & khi\ x\le 2 \end{cases}$ tại $x=2$

Lời giải

a) Có

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{\sqrt{1+x^2}-1}{\sqrt[3]{1-x}-1} = \lim_{x\to 0^-} \frac{x^2\left(\sqrt[3]{(1-x)^2}+\sqrt[3]{1-x}+1\right)}{\left(\sqrt{1+x^2}+1\right)(-x)} = \lim_{x\to 0^-} \frac{-x\left(\sqrt[3]{(1-x)^2}+\sqrt[3]{1-x}+1\right)}{\sqrt{1+x^2}+1} = 0$$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \left(m+\dfrac{1}{2}\right) = m+\dfrac{1}{2}$

Để tồn tại giới hạn của $f(x)$ tại $x=0$ thì $m+\dfrac{1}{2}=0 \Leftrightarrow m=-\dfrac{1}{2}$.

b) Có $\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x+m) = m$

$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} \dfrac{x^2+100x+3}{x+3} = 1$

Để tồn tại giới hạn của $f(x)$ tại $x=0$ thì $m=1$.

c) Có $\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \dfrac{\sqrt[3]{3x+2}-2}{x-2} = \lim\limits_{x\to 2^+} \dfrac{3x+2-8}{(x-2)\left(\sqrt[3]{(3x+2)^2}+2\sqrt[3]{3x+2}+2^2\right)} = \dfrac{1}{4}$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \left(mx+\dfrac{1}{4}\right) = 2m+\dfrac{1}{4}$

Để tồn tại giới hạn của $f(x)$ tại $x=0$ thì $2m+\frac{1}{4}=\frac{1}{4}\Leftrightarrow m=0$

Câu 31. Tìm giá trị của $a;b;c$ để $\lim\limits_{x\to 1}\frac{\sqrt{ax+b}+cx}{x^3-2x^2+x}=-\frac{1}{2}$.

Lời giải

Ta có

$$\lim_{x\to 1}\frac{\sqrt{ax+b}+cx}{x^3-2x^2+x}=-\frac{1}{2}\Leftrightarrow \lim_{x\to 1}\frac{-(cx^2-ax-b)}{x(x-1)^2\left(\sqrt{ax+b}-cx\right)}=-\frac{1}{2}\quad (*)$$

Để xảy ra (*) thì điều kiện cần là

$$\begin{cases}-(c^2x^2-ax-b)=k(\mathrm{x}-1)^2 \quad (\mathrm{k}\neq 0)\\ \sqrt{a.1+b}-c\neq 0\\ \dfrac{k}{\sqrt{a.1+b}-c}=-\dfrac{1}{2}\end{cases}\Leftrightarrow\begin{cases}k<0\\ a=-2k,\ b=k\\ \sqrt{a+b}-c\neq 0\\ \left[\begin{array}{l}\begin{cases}c=\sqrt{-k}\\ \dfrac{k}{\sqrt{-2k+k}-\sqrt{-k}}=-\dfrac{1}{2}\ (\text{PTVN})\end{cases}\\ \begin{cases}c=-\sqrt{-k}\\ \dfrac{k}{\sqrt{-2k+k}+\sqrt{-k}}=-\dfrac{1}{2}\Rightarrow k=-1\end{cases}\end{array}\right.\end{cases}\Rightarrow\begin{cases}a=-2k=2\\ b=k=-1\\ c=\sqrt{-k}=1\end{cases}$$

Thử lại: với $a=2,b=-1,c=1$ thỏa mãn yêu cầu bài toán.

Dạng 4. Một vài quy tắc tính giới hạn vô cực

Câu 32. Tính giới hạn

a. $\lim\limits_{x\to-\infty}\left(-2x^3+2x\sqrt{x}-x+1\right)$

b. $\lim\limits_{x\to+\infty}\left(x^4\sqrt{x}+2x\sqrt[3]{x}-2\right)$

c. $\lim\limits_{x\to-\infty}\sqrt[3]{3x^2+\frac{2x\sqrt{x}}{5}-4}$

d. $\lim\limits_{x\to-\infty}\frac{-4}{\sqrt{x^3-4x+3}}$

e. $\lim\limits_{x\to-\infty}\frac{\sqrt{2x^2-x+1}}{x-\sqrt{x}}$

Lời giải

a. $\lim\limits_{x\to-\infty}\left(-2x^3+2x\sqrt{x}-x+1\right)=\lim\limits_{x\to-\infty}x^3\left(-2+\frac{2}{x\sqrt{x}}-\frac{1}{x^2}+\frac{1}{x^3}\right)=+\infty$

b. $\lim\limits_{x\to+\infty}\left(x^4\sqrt{x}+2x\sqrt[3]{x}-2\right)=\lim\limits_{x\to+\infty}x^4\sqrt{x}\left(1+\frac{2}{x^3\sqrt[6]{x}}-\frac{2}{x^4\sqrt{x}}\right)=+\infty$

c. $\lim\limits_{x\to-\infty}\sqrt[3]{3x^2+\frac{2x\sqrt{x}}{5}-4}=\lim\limits_{x\to-\infty}\sqrt[3]{x^2(3+\frac{2}{5\sqrt{x}}-\frac{4}{x^2})}=+\infty$

d. $\lim\limits_{x\to-\infty}\frac{-4}{\sqrt{x^3-4x+3}}=\lim\limits_{x\to-\infty}\frac{-4}{\sqrt{x^3(1-\frac{4}{x^2}+\frac{3}{x^3})}}=0$

e. $\lim\limits_{x\to-\infty}\dfrac{\sqrt{2x^2-x+1}}{x-\sqrt{x}}=\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2(2-\frac{1}{x}+\frac{1}{x^2})}}{x(1-\frac{1}{\sqrt{x}})}=\lim\limits_{x\to-\infty}\dfrac{|x|\sqrt{2-\frac{1}{x}+\frac{1}{x^2}}}{x(1-\frac{1}{\sqrt{x}})}=\lim\limits_{x\to-\infty}-\dfrac{\sqrt{2-\frac{1}{x}+\frac{1}{x^2}}}{(1-\frac{1}{\sqrt{x}})}=-\sqrt{2}$

Câu 33. Tìm giới hạn

a. $\lim\limits_{x\to-\frac{3}{2}}\dfrac{x(x-1)}{(2x+3)^2}$

b. $\lim\limits_{x\to4}\dfrac{x\sqrt{5x+2}}{(x-4)^2(x+11)}$

c. $\lim\limits_{x\to-1}\left[\dfrac{x-1}{(x+1)(2x^2-x-3)}\right]$

d. $\lim\limits_{x\to0}\left(\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{2}{x^4}\right)$

e. $\lim\limits_{x\to-1}\dfrac{2x+1}{x^2-3x+4}$

Lời giải

a. $\left.\begin{array}{l}\lim\limits_{x\to-\frac{3}{2}}x(x-1)=\dfrac{15}{4}\\ \lim\limits_{x\to-\frac{3}{2}}\dfrac{1}{(2x+3)^2}=+\infty\end{array}\right\}\Rightarrow\lim\limits_{x\to-\frac{3}{2}}\dfrac{x(x-1)}{(2x+3)^2}=+\infty$

b. $\left.\begin{array}{l}\lim\limits_{x\to4}\dfrac{x\sqrt{5x+2}}{x+11}=\dfrac{4\sqrt{22}}{15}\\ \lim\limits_{x\to4}\dfrac{1}{(x-4)^2}=+\infty\end{array}\right\}\Rightarrow\lim\limits_{x\to4}\dfrac{x\sqrt{5x+2}}{(x-4)^2(x+11)}=+\infty$

c. $\lim\limits_{x\to-1}\left[\dfrac{x-1}{(x+1)(2x^2-x-3)}\right]=\lim\limits_{x\to-1}\left[\dfrac{x-1}{(x+1)^2(2x-3)}\right]=+\infty$

d. $\lim\limits_{x\to0}\left(\dfrac{1}{x}+\dfrac{1}{x^2}+\dfrac{2}{x^4}\right)=\lim\limits_{x\to0}\dfrac{1}{x^4}\left(x^3+x^2+2\right)=+\infty$

e. $\lim\limits_{x\to-1}\dfrac{2x+1}{x^2-3x+4}=\dfrac{-2+1}{1+3+4}=\dfrac{-1}{8}$

Câu 34. Tìm giới hạn

a. $\lim\limits_{x\to-\infty}\sqrt[3]{\dfrac{x^2+5}{6x^2-3x+2}}$ b. $\lim\limits_{x\to-2}\dfrac{x^4-16}{x^2+6x+8}$ c. $\lim\limits_{x\to3}\dfrac{x^4-27x}{2x^2-3x-9}$

Lời giải

a. $\lim\limits_{x\to-\infty}\sqrt[3]{\dfrac{x^2+5}{6x^2-3x+2}}=\lim\limits_{x\to-\infty}\sqrt[3]{\dfrac{1+\frac{5}{x^2}}{6-\frac{3}{x}+\frac{2}{x^2}}}=\sqrt[3]{\dfrac{1}{6}}$

b. $\lim\limits_{x\to-2}\dfrac{x^4-16}{x^2+6x+8}=\lim\limits_{x\to-2}\dfrac{(x^2-4)(x^2+4)}{(x+2)(x+4)}=\lim\limits_{x\to-2}\dfrac{(x-2)(x^2+4)}{x+4}=-16$

c. $\lim\limits_{x\to3}\dfrac{x^4-27x}{2x^2-3x-9}=\lim\limits_{x\to3}\dfrac{x(x^3-27)}{(x-3)(2x+3)}=\lim\limits_{x\to3}\dfrac{x(x-3)(x^2+3x+9)}{(x-3)(2x+3)}=\lim\limits_{x\to3}\dfrac{x(x^2+3x+9)}{2x+3}=9$

Câu 35. Tìm giới hạn

a. $\lim\limits_{x\to-\infty}\dfrac{3x^3-5x-6}{1-4x^3+x^2}$ b. $\lim\limits_{x\to-\infty}\dfrac{(3x^2+8)(2x+1)}{5-4x^3}$

Lời giải

a. $\lim\limits_{x\to-\infty}\dfrac{3x^3-5x-6}{1-4x^3+x^2}=\lim\limits_{x\to-\infty}\dfrac{3-\dfrac{5}{x^2}-\dfrac{6}{x^3}}{\dfrac{1}{x^2}-4+\dfrac{1}{x}}=\dfrac{-3}{4}$

b. $\lim\limits_{x\to-\infty}\dfrac{(3x^2+8)(2x+1)}{5-4x^3}=\lim\limits_{x\to-\infty}\dfrac{(3+\dfrac{8}{x^2})(2+\dfrac{1}{x})}{\dfrac{5}{x^3}-4}=\dfrac{6}{-4}=\dfrac{-3}{2}$

Câu 36. Tìm giới hạn

a. $\lim\limits_{x\to+\infty}\dfrac{-5x+7}{3-2x}$ b. $\lim\limits_{x\to-\infty}\dfrac{7}{2x-1}$

Lời giải

a. $\lim\limits_{x\to+\infty}\dfrac{-5x+7}{3-2x}=\lim\limits_{x\to+\infty}\dfrac{-5+\dfrac{7}{x}}{\dfrac{3}{x}-2}=\dfrac{5}{2}$

b. $\lim\limits_{x\to-\infty}\dfrac{7}{2x-1}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{7}{x}}{2-\dfrac{1}{x}}=\dfrac{0}{2}=0$

Câu 37. TÌm giới hạn

a. $\lim\limits_{x\to+\infty}\dfrac{-2x^4-x+7}{1+5x^5}$ b. $\lim\limits_{x\to-\infty}\dfrac{4x^2+3x-6}{2x+3}$

Lời giải

a. $\lim\limits_{x\to+\infty}\dfrac{-2x^4-x+7}{1+5x^5}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{-2}{x}-\dfrac{1}{x^4}+\dfrac{7}{x^5}}{\dfrac{1}{x^5}+5}=\dfrac{0}{5}=0$

b. $\lim\limits_{x\to-\infty}\dfrac{4x^2+3x-6}{2x+3}=\lim\limits_{x\to-\infty}\dfrac{4+\dfrac{3}{x}-\dfrac{6}{x^2}}{\dfrac{2}{x}+\dfrac{3}{x^2}}=\dfrac{4}{0}=-\infty$

Câu 38. Tìm giới hạn

a. $\lim\limits_{x\to+\infty}\dfrac{x\sqrt{x}+1}{3x^2+2x+7}$ b. $\lim\limits_{x\to-\infty}\dfrac{x-2x^2+8}{5x^2+4}$

Lời giải

a. $\lim\limits_{x\to+\infty}\dfrac{x\sqrt{x}+1}{3x^2+2x+7}=\lim\limits_{x\to+\infty}\dfrac{\dfrac{1}{\sqrt{x}}+\dfrac{1}{x^2}}{3+\dfrac{2}{x}+\dfrac{7}{x^3}}=\dfrac{0}{3}=0$

b. $\lim\limits_{x\to-\infty}\dfrac{x-2x^2+8}{5x^2+4}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{1}{x}-2+\dfrac{8}{x^2}}{5+\dfrac{4}{x^2}}=\dfrac{-2}{5}$

Câu 39. Tìm giới hạn

a. $\lim\limits_{x\to-\infty}\dfrac{3x^2-5}{4-x}$ b. $\lim\limits_{x\to-\infty}\dfrac{3+x-2x^3}{3-2x+5x^3}$

Lời giải

a. $\lim\limits_{x\to-\infty}\dfrac{3x^2-5}{4-x}=\lim\limits_{x\to-\infty}\dfrac{3-\dfrac{5}{x^2}}{\dfrac{4}{x^2}-\dfrac{1}{x}}=\dfrac{3}{0}=+\infty$

b. $\lim\limits_{x\to-\infty}\dfrac{3+x-2x^3}{3-2x+5x^3}=\lim\limits_{x\to-\infty}\dfrac{\dfrac{3}{x^3}+\dfrac{1}{x^2}-2}{\dfrac{3}{x^3}-\dfrac{2}{x^2}+5}=\dfrac{-2}{5}$

Câu 40. Tìm giới hạn

a. $\lim\limits_{x\to-\infty}(2-3x+5x^2)$ b. $\lim\limits_{x\to+\infty}(7x^4-4x+2)$

Lời giải

a. $\lim\limits_{x\to-\infty}(2-3x+5x^2)=\lim\limits_{x\to-\infty}x^2\left(\dfrac{2}{x^2}-\dfrac{3}{x}+5\right)=+\infty$

b. $\lim\limits_{x\to+\infty}(7x^4-4x+2)=\lim\limits_{x\to+\infty}x^4(7-\dfrac{4}{x^3}+\dfrac{2}{x^4})=+\infty$

Câu 41. Tìm giới hạn

a. $\lim\limits_{x\to-2}\dfrac{4+5x}{(-x-2)^2}$ b. $\lim\limits_{x\to-\infty}\dfrac{3x^2+4x-5}{x+3}$

c. $\lim\limits_{x\to+\infty}(1-8x^3-x^2)$ d. $\lim\limits_{x\to-\infty}(6x^5-x+2)$

Giải

a. $\lim\limits_{x\to-2}\dfrac{4+5x}{(-x-2)^2}=\dfrac{4+5.(-2)}{(2-2)^2}=\dfrac{-6}{0}=-\infty$

b. $\lim\limits_{x\to-\infty}\dfrac{3x^2+4x-5}{x+3}=\lim\limits_{x\to-\infty}\dfrac{3+\dfrac{4}{x}-\dfrac{5}{x^2}}{\dfrac{1}{x}+\dfrac{3}{x^2}}=\dfrac{3}{0}=-\infty$

c. $\lim\limits_{x\to+\infty}(1-8x^3-x^2)=\lim\limits_{x\to+\infty}x^3\left(\dfrac{1}{x^3}-8-\dfrac{1}{x}\right)=-\infty$

d. $\lim_{x\to-\infty}(6x^5-x+2)=\lim_{x\to-\infty}x^5\left(6-\frac{1}{x^4}+\frac{2}{x^5}\right)=-\infty$

Dạng 5. Giới hạn vô định

Câu 42. **(SGK-CTST 11-Tập 1)** Tìm các giới hạn sau:

a) $\lim_{x\to+\infty}\frac{4x+3}{2x}$

b) $\lim_{x\to-\infty}\frac{2}{3x+1}$;

c) $\lim_{x\to+\infty}\frac{\sqrt{x^2+1}}{x+1}$.

Lời giải:

a) $\lim_{x\to+\infty}\frac{4x+3}{2x}=\lim_{x\to+\infty}\frac{4+\frac{3}{x}}{2}=\frac{4+0}{2}=2$

b) $\lim_{x\to-\infty}\frac{2}{3x+1}=\lim_{x\to-\infty}\frac{\frac{2}{x}}{3+\frac{1}{x}}=\frac{0}{3+0}=0$

c) $\lim_{x\to+\infty}\frac{\sqrt{x^2+1}}{x+1}=\lim_{x\to+\infty}\frac{\sqrt{1+\frac{1}{x^2}}}{1+\frac{1}{x}}=\frac{\sqrt{1+0}}{1+0}=1$

Câu 43. Tìm các giới hạn sau:

a. $\lim_{x\to2}\frac{x^2-3x+2}{x^2+x-6}$ b. $\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}$ c. $\lim_{x\to-1}\frac{x^5+1}{x^3+1}$

d. $\lim_{x\to2}\frac{x^3+3x^2-9x-2}{x^3-x-6}$ e. $\lim_{x\to1}\frac{x+x^2+...+x^n-n}{x-1}$

Lời giải

a. $\lim_{x\to2}\frac{x^2-3x+2}{x^2+x-6}=\lim_{x\to2}\frac{(x-2)(x-1)}{(x-2)(x+3)}=\lim_{x\to2}\frac{x-1}{x+3}=\frac{1}{5}$

b. $\lim_{x\to-2}\frac{x^3+3x^2+2x}{x^2-x-6}=\lim_{x\to-2}\frac{x(x+2)(x+1)}{(x+2)(x-3)}=-\frac{2}{5}$

c. $\lim_{x\to-1}\frac{x^5+1}{x^3+1}=\lim_{x\to-1}\frac{(x+1)(x^4-x^3+x^2-x+1)}{(x+1)(x^2-x+1)}=\frac{5}{3}$

d. $\lim_{x\to2}\frac{x^3+3x^2-9x-2}{x^3-x-6}=\lim_{x\to2}\frac{(x-2)(x^2+5x+1)}{(x-2)(x^2+2x+3)}=\frac{15}{11}$

e. $\lim_{x\to1}\frac{x-1+x^2-1+...+x^n-1}{x-1}=\lim_{x\to1}\underbrace{1+...+1}_{nso'1}+\underbrace{x+...+x}_{n-1so'x}+...+x^{n-1}=n+n-1+...+2+1=\frac{n(n+1)}{2}$

Câu 44. Tìm các giới hạn sau:

a. $\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}$ b. $\lim_{x\to 2}\frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}$ c. $\lim_{x\to 0}\frac{x}{\sqrt{1+x}-1}$

Lời giải

a. $\lim_{x\to 2}\frac{\sqrt{x^2+5}-3}{x-2}=\lim_{x\to 2}\frac{x^2+5-9}{\left(\sqrt{x^2+5}+3\right)(x-2)}=\lim_{x\to 2}\frac{x+2}{\sqrt{x^2+5}+3}=\frac{2}{3}$

b. $\lim_{x\to 2}\frac{x-\sqrt{x+2}}{\sqrt{4x+1}-3}=\lim_{x\to 2}\frac{(x+1)(x-2)\left(\sqrt{4x+1}+3\right)}{\left(\sqrt{x+2}+x\right)4(x-2)}=\frac{9}{8}$

c. $\lim_{x\to 0}\frac{x}{\sqrt{1+x}-1}=\lim_{x\to 0}\frac{x\left(\sqrt{1+x}+1\right)}{x}=\lim_{x\to 0}\sqrt{1+x}+1=2$

Câu 45. Tìm các giới hạn sau:

a. $\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}$ b. $\lim_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}$ c. $\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}$

d. $\lim_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}$

Lời giải

a. $\lim_{x\to 0}\frac{1-\sqrt[3]{1-x}}{3x}=\lim_{x\to 0}\frac{1-(1-x)}{3x\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}=\lim_{x\to 0}\frac{1}{3\left(1+\sqrt[3]{1-x}+\sqrt[3]{(1-x)^2}\right)}=\frac{1}{9}$

b. $\lim_{x\to -1}\frac{\sqrt[3]{x}+1}{\sqrt{x^2+3}-2}=\lim_{x\to -1}\frac{(x+1)\left(\sqrt{x^2+3}+2\right)}{\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)\left(x^2-1\right)}=\lim_{x\to -1}\frac{\left(\sqrt{x^2+3}+2\right)}{\left(\sqrt[3]{x^2}-\sqrt[3]{x}+1\right)(x-1)}=-\frac{2}{3}$

c. $\lim_{x\to 1}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+1}{(x-1)^2}=\lim_{x\to 1}\frac{\left(\sqrt[3]{x}-1\right)^2}{\left(\sqrt[3]{x^3}-1\right)^2}=\lim_{x\to 1}\frac{1}{\left(\sqrt[3]{x^2}+\sqrt[3]{x}+1\right)^2}=\frac{1}{9}$

Câu 46. Tìm các giới hạn sau:

a. $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt{1-x}}{x}$ b. $\lim_{x\to 0}\frac{\sqrt{3x+4}-\sqrt[3]{8+5x}}{x}$

c. $\lim_{x\to 2}\frac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}$ d. $\lim_{x\to 0}\frac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}$.

Lời giải

a. $\lim_{x\to 0}\frac{\sqrt[3]{1+x}-\sqrt{1-x}}{x}=\lim_{x\to 0}\frac{\sqrt[3]{1+x}-1+1-\sqrt{1-x}}{x}=\frac{5}{6}$.

b. $\lim_{x\to 0}\frac{\sqrt{3x+4}-\sqrt[3]{8+5x}}{x}=\lim_{x\to 0}\frac{\sqrt{3x+4}-2+2-\sqrt[3]{8+5x}}{x}=\frac{1}{3}$.

c. $\lim_{x\to 2}\frac{\sqrt[3]{8x+11}-\sqrt{x+7}}{x^2-3x+2}=\lim_{x\to 2}\frac{\sqrt[3]{8x+11}-3+3-\sqrt{x+7}}{(x-1)(x-2)}$.

$$=\lim_{x\to 2}\left[\frac{8}{(x-1)\left(\sqrt[3]{(8x+11)^2}+3\sqrt[3]{8x+11}+9\right)}-\frac{1}{(x-1)\left(3+\sqrt{x+7}\right)}\right]=\frac{8}{27}-\frac{1}{6}=\frac{7}{54}.$$

d. $\lim_{x\to 0}\frac{\sqrt{1+4x}-\sqrt[3]{1+6x}}{x^2}=\lim_{x\to 0}\frac{\sqrt{1+4x}-(2x+1)+(2x+1)-\sqrt[3]{1+6x}}{x^2}$

$$=\lim_{x\to 0}\left[\frac{-4}{\left(\sqrt{1+4x}+(2x+1)\right)}+\frac{8x+12}{\left((2x+1)^2+(2x+1)\sqrt[3]{1+6x}+\sqrt[3]{(1+6x)^2}\right)}\right]=\frac{-4}{2}+\frac{12}{3}=2.$$

Câu 47. Tìm giới hạn

a. $\lim\limits_{x\to+\infty}\frac{x^2+1}{2x^2-x+1}$ b. $\lim\limits_{x\to\pm\infty}\frac{2x^2-x+1}{x-1}$ c. $\lim\limits_{x\to\pm\infty}\frac{2x^2+1}{x^3-3x^2+2}$

Lời giải

a. $\lim\limits_{x\to+\infty}\frac{x^2+1}{2x^2-x+1}=\lim\limits_{x\to+\infty}\frac{1+\frac{1}{x^2}}{2-\frac{1}{x}+\frac{1}{x^2}}=\frac{1}{2}$.

b. $\lim\limits_{x\to\pm\infty}\frac{2x^2-x+1}{x-1}=\lim\limits_{x\to\pm\infty}\frac{2x-1+\frac{1}{x}}{1-\frac{1}{x}}=\pm\infty$.

c. $\lim\limits_{x\to\pm\infty}\frac{2x^2+1}{x^3-3x^2+2}=\lim\limits_{x\to\pm\infty}\frac{2+\frac{1}{x^2}}{x-3+\frac{2}{x^2}}=0$

Câu 48. Tìm giới hạn

a. $\lim\limits_{x\to-\infty}\frac{\sqrt[3]{x^6-x^4+1}+\sqrt{x^6+1}}{2x-1}$ b. $\lim\limits_{x\to+\infty}\frac{x\sqrt{x}+1}{x^2+x+1}$

c. $\lim\limits_{x\to-\infty}\frac{\sqrt{x^2+2x+2}-x}{\sqrt{9x^2+1}-x+2}$ d. $\lim\limits_{x\to\pm\infty}\frac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}+2-x}$

Lời giải

a. $\lim\limits_{x\to-\infty}\frac{\sqrt[3]{x^6-x^4+1}+\sqrt{x^6+1}}{2x-1}=\lim\limits_{x\to-\infty}\frac{x^2\sqrt[3]{1-\frac{1}{x^2}+\frac{1}{x^6}}+|x^3|\sqrt{1+\frac{1}{x^6}}}{x\left(2-\frac{1}{x}\right)}$

$$=\lim_{x\to-\infty}\frac{x\sqrt[3]{1-\frac{1}{x^2}+\frac{1}{x^6}}-x^2\sqrt{1+\frac{1}{x^6}}}{\left(2-\frac{1}{x}\right)}=-\infty$$

b. $\lim\limits_{x\to+\infty}\frac{x\sqrt{x}+1}{x^2+x+1}=\lim\limits_{x\to+\infty}\frac{\sqrt{\frac{1}{x}+\frac{1}{x^2}}}{1+\frac{1}{x}+\frac{1}{x^2}}=0$

c.

$$\lim_{x\to-\infty}\frac{\frac{2}{x}-1-\sqrt{1+\frac{2}{x}}}{-\sqrt{9+\frac{1}{x^2}}-1+\frac{2}{x}}=\frac{1}{2}$$

$$\lim_{x\to-\infty}\frac{\sqrt{x^2+2x}+2-x}{\sqrt{9x^2+1}-x+2}=\lim_{x\to-\infty}\frac{|x|\sqrt{1+\frac{2}{x}}+x\left(\frac{2}{x}-1\right)}{|x|\sqrt{9+\frac{1}{x^2}}+x\left(-1+\frac{2}{x}\right)}=\lim_{x\to-\infty}\frac{-x\sqrt{1+\frac{2}{x}}+x\left(\frac{2}{x}-1\right)}{-x\sqrt{9+\frac{1}{x^2}}+x\left(-1+\frac{2}{x}\right)}$$

d. $\lim\limits_{x\to\pm\infty}\dfrac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}+2-x}$

$$+\lim\limits_{x\to+\infty}\dfrac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}+2-x}=\lim\limits_{x\to+\infty}\dfrac{\sqrt{1+\dfrac{2}{x}+\dfrac{3}{x^2}}+4+\dfrac{1}{x^2}}{\sqrt{4+\dfrac{1}{x^2}}+\dfrac{2}{x}-1}=5$$

$$+\lim\limits_{x\to-\infty}\dfrac{\sqrt{x^2+2x+3}+4x+1}{\sqrt{4x^2+1}+2-x}=\lim\limits_{x\to-\infty}\dfrac{-\sqrt{1+\dfrac{2}{x}+\dfrac{3}{x^2}}+4+\dfrac{1}{x^2}}{-\sqrt{4+\dfrac{1}{x^2}}+\dfrac{2}{x}-1}=-1$$

Câu 49. Tìm giới hạn

a. $\lim\limits_{x\to+\infty}\dfrac{(x-1)\sqrt{x^2+x\sqrt{x}+1}}{\sqrt{x^2+1}+2x}$

b. $\lim\limits_{x\to-\infty}\dfrac{(x-1)(2x-1)(3x-1)(4x-1)(5x-1)}{(4x+5)^5}$

c. $\lim\limits_{x\to-\infty}\dfrac{(x-1)\sqrt{x^2+x\sqrt{x}+1}}{\sqrt{x^2+1}+2x}$

d. $\lim\limits_{x\to+\infty}x^2\left(\sqrt[3]{x^3+1}-x\right)$

Lời giải

a. $$\lim\limits_{x\to-\infty}\dfrac{(x-1)\sqrt{x^2+x\sqrt{x}+1}}{\sqrt{x^2+1}+2x}=\lim\limits_{x\to-\infty}\dfrac{(x-1)|x|\sqrt{1+\sqrt{\dfrac{1}{x}+\dfrac{1}{x^2}}}}{|x|\sqrt{1+\dfrac{1}{x^2}}+2x}=\lim\limits_{x\to-\infty}\dfrac{(x-1)\sqrt{1+\sqrt{\dfrac{1}{x}+\dfrac{1}{x^2}}}}{\sqrt{1+\dfrac{1}{x^2}}+2}=+\infty$$

b. $$\lim\limits_{x\to-\infty}\dfrac{(x-1)(2x-1)(3x-1)(4x-1)(5x-1)}{(4x+5)^5}=\lim\limits_{x\to-\infty}\dfrac{\left(1-\dfrac{1}{x}\right)\left(2-\dfrac{1}{x}\right)\left(3-\dfrac{1}{x}\right)\left(4-\dfrac{1}{x}\right)\left(5-\dfrac{1}{x}\right)}{\left(4+\dfrac{5}{x}\right)^5}=\dfrac{15}{128}$$

c. $$\lim\limits_{x\to-\infty}\dfrac{(x-1)\sqrt{x^2+x\sqrt{x}+1}}{\sqrt{x^2+1}+2x}=\lim\limits_{x\to-\infty}\dfrac{(x-1)|x|\sqrt{1+\sqrt{\dfrac{1}{x}+\dfrac{1}{x^2}}}}{|x|\sqrt{1+\dfrac{1}{x^2}}+2x}=\lim\limits_{x\to-\infty}\dfrac{-(x-1)\sqrt{1+\sqrt{\dfrac{1}{x}+\dfrac{1}{x^2}}}}{-\sqrt{1+\dfrac{1}{x^2}}+2}=+\infty$$

d. $$\lim\limits_{x\to+\infty}x^2\left(\sqrt[3]{x^3+1}-x\right)=\lim\limits_{x\to+\infty}\dfrac{x^2\left(\sqrt[3]{x^3+1}-x\right)\left(\left(\sqrt[3]{x^3+1}\right)^2+x.\left(\sqrt[3]{x^3+1}-x\right)+x^2\right)}{\left(\left(\sqrt[3]{x^3+1}\right)^2+x.\left(\sqrt[3]{x^3+1}-x\right)+x^2\right)}$$

$$=\lim\limits_{x\to+\infty}\dfrac{x^2}{\left(\left(\sqrt[3]{x^3+1}\right)^2+x.\left(\sqrt[3]{x^3+1}-x\right)+x^2\right)}=\lim\limits_{x\to+\infty}\dfrac{1}{\left(\sqrt[3]{1+\dfrac{1}{x^3}}\right)^2+\sqrt[3]{1+\dfrac{1}{x^3}}+1}=\dfrac{1}{3}$$

Câu 50. Tìm giới hạn

a. $\lim\limits_{x\to-\infty}\dfrac{|2x|^3-|x|+1}{4|x^3|+x^2+1}$ b. $\lim\limits_{x\to-\infty}\dfrac{\left|x\sqrt{x^2+3}+1\right|}{|x^2-1|+x}$

Lời giải

a. $\lim_{x\to-\infty}\frac{|2x|^3-|x|+1}{4|x^3|+x^2+1}=\lim_{x\to-\infty}\frac{8-\frac{1}{|x|^2}+\frac{1}{|x|^3}}{4+\frac{1}{|x|}+\frac{1}{|x|^3}}=2$

b. $\lim_{x\to-\infty}\frac{\left|x\sqrt{x^2+3}+1\right|}{|x^2-1|+x}=\lim_{x\to-\infty}\frac{\left|x|x|\sqrt{1+\frac{3}{x^2}}+1\right|}{|x^2-1|+x}=\lim_{x\to-\infty}\frac{\left|\frac{x}{|x|}\sqrt{1+\frac{3}{x^2}}+\frac{1}{x^2}\right|}{\left|1-\frac{1}{x^2}\right|+\frac{1}{x}}=1$

Dạng 6. Giới hạn của hàm lượng giác

Câu 51. Tìm giới hạn

a. $\lim_{x\to0}\frac{\sin x}{3x}$ b. $\lim_{x\to0}\frac{\sin 2x}{\sin\frac{x}{2}}$

Lời giải

a. $\lim_{x\to0}\frac{\sin x}{3x}=\frac{1}{3}\lim_{x\to0}\frac{\sin x}{x}=\frac{1}{3}$

b. $\lim_{x\to0}\frac{\sin 2x}{\sin\frac{x}{2}}=\lim_{x\to0}\frac{\frac{\sin 2x}{2x}}{\frac{\sin\frac{x}{2}}{\frac{x}{2}}}.\frac{2x}{\frac{x}{2}}=\lim_{x\to0}\frac{\frac{\sin 2x}{2x}}{\frac{\sin\frac{x}{2}}{\frac{x}{2}}}.4=4.$

Câu 52. Tìm giới hạn

a. $\lim_{x\to0}\frac{1-\cos 4x}{2x^2}$ b. $\lim_{x\to0}\frac{\sin^2 x}{2x^2}$

c. $\lim_{x\to0}\frac{1-\cos 3x}{1-\cos 5x}$ d. $\lim_{x\to0}\frac{3-\cos x-\cos 2x-\cos 3x}{1--\cos x}$

Lời giải

a. $\lim_{x\to0}\frac{1-\cos 4x}{2x^2}=\lim_{x\to0}\frac{2\sin^2 2x}{2x^2}=\lim_{x\to0}4.\left(\frac{\sin 2x}{2x}\right)^2=4$.

b. $\lim_{x\to0}\frac{\sin^2 x}{2x^2}=\lim_{x\to0}\frac{1}{2}\left(\frac{\sin x}{x}\right)^2=\frac{1}{2}$.

c. $\lim_{x\to0}25\frac{1-\cos 3x}{1-\cos 5x}=\lim_{x\to0}\frac{\frac{1-\cos2\frac{3x}{2}}{2}}{\frac{1-\cos2\frac{5x}{2}}{2}}=\lim_{x\to0}\frac{\sin^2\frac{3x}{2}}{\sin^2\frac{5x}{2}}=\lim_{x\to0}\left(\frac{\sin^2\frac{3x}{2}\left(\frac{5x}{2}\right)^2}{\left(\frac{3x}{2}\right)^2\sin^2\frac{5x}{2}}.\frac{9}{25}\right)=\frac{9}{25}$.

d. $\lim_{x\to0}\frac{3-\cos x-\cos 2x-\cos 3x}{1--\cos x}=\lim_{x\to0}\left(1+\frac{1-\cos 2x}{1-\cos x}+\frac{1-\cos 3x}{1-\cos x}\right)$

$=1+\lim_{x\to0}\frac{\sin^2 x}{\sin^2\frac{x}{2}}+\lim_{x\to0}\frac{\sin^2\frac{3x}{2}}{\sin^2\frac{x}{2}}=1+\lim_{x\to0}\left(\frac{\sin^2 x}{x^2}.\frac{\left(\frac{x}{2}\right)^2}{\sin^2\frac{x}{2}}.4\right)+\lim_{x\to0}\left(\frac{\sin^2\frac{3x}{2}}{\left(\frac{3x}{2}\right)^2}.\frac{\left(\frac{x}{2}\right)^2}{\sin^2\frac{x}{2}}.9\right)$

$=\mathbf{1+4+9=14}$

Câu 53. Tìm giới hạn

a. $\lim\limits_{x\to\frac{\pi}{3}}\dfrac{\sin\left(x-\dfrac{\pi}{3}\right)}{1-2\cos x}$ b. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}$

Lời giải

a. Đặt $t = x = \dfrac{\pi}{3}$

$$\Rightarrow \lim\limits_{x\to\frac{\pi}{3}}\frac{\sin\left(x-\dfrac{\pi}{3}\right)}{1-2\cos x}=\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin t}{1-2\cos\left(t+\dfrac{\pi}{3}\right)}=\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin t}{2\sin^2\left(\dfrac{t}{2}+\dfrac{\pi}{6}\right)}=\lim\limits_{x\to\frac{\pi}{3}}\frac{\sin t}{t}.\frac{t}{2\sin^2\left(\dfrac{t}{2}+\dfrac{\pi}{6}\right)}=0$$

b. $$\lim\limits_{x\to 0}\frac{\sqrt{1+\sin^2 x}-\cos x}{\sin^2 x}=\lim\limits_{x\to 0}\frac{1+\sin^2 x-\cos^2 x}{\left(\sqrt{1+\sin^2 x}+\cos x\right)\sin^2 x}=\lim\limits_{x\to 0}\frac{2\sin^2 x}{\left(\sqrt{1+\sin^2 x}+\cos x\right)\sin^2 x}$$

$$=\lim\limits_{x\to 0}\frac{2}{\left(\sqrt{1+\sin^2 x}+\cos x\right)}=1$$