market_neutral_strategy

August 9, 2023

SUMMARY

In this project, the market-neutral trading strategy is created using python. Several steps were taken to achieve this task. In summary, the steps taken are;

The data was adequately explored and cleaned.

Rules were put in place to guide the pair selection process. First, DBSCAN clustering algorithm was used to cluster the returns of the stocks. The average volume of the stocks in the training period was used to improve the quality of the clustering process. Then an improved pair selection algorithm which involves multiple criteria (Hurst exponent, half life, zero crossing, cointegration etc) was used to select the best trading pairs.

The allocations of the assets for each pair was done equally since information such as Beta is not available and I do not want to do very complex portfolio optimization for each of the pairs.

The signals for each pair was generated using the z-score with the training data. The z-score in this solution used the 25th and 75th percentiles to compute the lower and upper thresholds for each pair. The strategy framework produced also made available the use of the traditional mean and standard deviation for threshold computation.

Also for the signal generation, feature selection optimization was introduced to help improve the robustness of the signals generated.

On the test data, the optimal feature from the signal generation stage is used to predict the signal using a simple trained machine learning model. Backtesting was carried out and the results of the process are presented.

[ ]:
```
# import libraries

import numpy as np
import pandas as pd
from matplotlib import pyplot as plt
import seaborn as sns

from sklearn import linear_model
from scipy.stats import t, linregress
from scipy import stats

from sklearn.ensemble import RandomForestRegressor
```

```
from numpy.lib.stride_tricks import as_strided


import class_SeriesAnalyser, class_DataProcessor
from sklearn.manifold import TSNE
import matplotlib.cm as cm

from sklearn.cluster import KMeans, DBSCAN
from sklearn.decomposition import PCA
from sklearn import preprocessing



# temporary
# ignore warnings
import warnings
warnings.filterwarnings('ignore')
```

[ ]:
```
technical = pd.read_csv('data/data.csv')
print('Data shape:', technical.shape)
```

```
Data shape: (451113, 4)
```

[ ]:
```
technical.head(5)
```

[ ]:

	ticker	date	last	volume
0	1332 JT	2013-01-04	169.0987	1464100
1	1332 JT	2013-01-07	166.3266	1783500
2	1332 JT	2013-01-08	166.3266	1759800
3	1332 JT	2013-01-09	165.4026	767800
4	1332 JT	2013-01-10	167.2507	1503100

[ ]:
```
technical.tail(5)
```

[ ]:

	ticker	date	last	volume
451108	9984 JT	2021-03-15	10370.0	12041200
451109	9984 JT	2021-03-16	10620.0	11346000
451110	9984 JT	2021-03-17	10400.0	9771000
451111	9984 JT	2021-03-18	10220.0	13941600
451112	9984 JT	2021-03-19	9969.0	20715700

[ ]:
```
technical.describe()
```

[ ]:

	last	volume
count	451113.000000	4.511130e+05
mean	2751.772257	3.469636e+06
std	3988.203423	8.373189e+06
min	63.000000	0.000000e+00

```
25%       1099.145800  7.861000e+05
50%       1901.445100  1.614400e+06
75%       3169.704000  3.595100e+06
max     109550.000000  7.289341e+08
```

[ ]:
```python
# Check for missing values
missing_values = technical.isnull().sum()

# Display the count of missing values for each column
print(missing_values)
```

```
ticker    0
date      0
last      0
volume    0
dtype: int64
```

For better handling of the data, I will use pivot tables to transform the data

[ ]:
```python
# Pivot the DataFrame
pivot_df = technical.pivot_table(index='date', columns='ticker')
```

Then proceed to obtaining the beta tables, price/volume tables

[ ]:
```python
price_df = pivot_df['last']
volume_df = pivot_df['volume']
```

[ ]:
```python
display(price_df.head(3))
display(volume_df.head(3))
```

```
ticker       1332 JT  1333 JT   1334 JT   1605 JT   1721 JT    1801 JT  \
date
2013-01-04  169.0987      NaN  147.8887  970.6780  950.5521  1232.0186
2013-01-07  166.3266      NaN  144.9890  960.1615  924.2187  1236.1949
2013-01-08  166.3266      NaN  144.9890  955.9548  938.6596  1215.3132

ticker       1802 JT   1803 JT  1808 JT   1812 JT  …   9503 JT    9531 JT  \
date                                               …
2013-01-04  413.2189  278.1162      NaN  478.8201  …  847.8471  1727.7597
2013-01-07  411.5426  278.1162      NaN  480.4769  …  804.2336  1744.7820
2013-01-08  407.3517  277.2709      NaN  473.8496  …  795.5109  1761.8042

ticker        9532 JT    9602 JT   9613 JT   9681 JT    9735 JT    9766 JT  \
date
2013-01-04  1332.1473  1367.1194  487.5256  567.3170  3719.2396  1721.7623
2013-01-07  1327.9713  1364.4123  475.2036  594.1617  3732.0499  1695.8510
2013-01-08  1340.4993  1377.0457  470.5604  585.2135  3736.3199  1627.9455

ticker         9983 JT    9984 JT
```

```
date
2013-01-04  20584.8363  1482.3931
2013-01-07  21150.4557  1453.9312
2013-01-08  21243.1802  1472.9058

[3 rows x 248 columns]
```

```
ticker        1332 JT  1333 JT    1334 JT    1605 JT    1721 JT    1801 JT  \
date
2013-01-04  1464100.0      NaN  1553000.0  5312400.0   816600.0  2254000.0
2013-01-07  1783500.0      NaN  1419000.0  3724400.0  1464400.0  3452200.0
2013-01-08  1759800.0      NaN   869000.0  5019600.0   929800.0  3918400.0

ticker        1802 JT    1803 JT  1808 JT    1812 JT  …     9503 JT  \
date                                                  …
2013-01-04  6232000.0  3940000.0      NaN  5406500.0  …  11514100.0
2013-01-07  6580000.0  5511000.0      NaN  6470000.0  …   8334600.0
2013-01-08  6019000.0  5139000.0      NaN  4849000.0  …   6308100.0

ticker        9531 JT    9532 JT   9602 JT    9613 JT    9681 JT   9735 JT  \
date
2013-01-04  2685000.0  1988200.0  482800.0  3148000.0  1114500.0  800500.0
2013-01-07  2401200.0  1495200.0  458000.0  4289000.0  2862000.0  688400.0
2013-01-08  2478000.0  1598800.0  635200.0  4065500.0  1620000.0  760300.0

ticker        9766 JT   9983 JT     9984 JT
date
2013-01-04  1513200.0  587300.0  12023000.0
2013-01-07  1841700.0  980700.0  13892400.0
2013-01-08  2582000.0  616800.0  10749800.0

[3 rows x 248 columns]
```

[ ]:
```
display(price_df.tail(3))
display(volume_df.tail(3))
```

```
ticker      1332 JT  1333 JT  1334 JT  1605 JT  1721 JT  1801 JT  1802 JT  \
date
2021-03-17    561.0   2732.0      NaN    808.0   3400.0   4220.0   1007.0
2021-03-18    565.0   2740.0      NaN    812.0   3455.0   4300.0   1028.0
2021-03-19    567.0   2725.0      NaN    796.0   3455.0   4345.0   1054.0

ticker      1803 JT  1808 JT  1812 JT  …  9503 JT  9531 JT  9532 JT  \
date                                   …
2021-03-17    909.0   1553.0   1558.0  …   1193.0   2450.0   2150.0
2021-03-18    921.0   1605.0   1581.0  …   1199.5   2442.5   2162.0
2021-03-19    926.0   1664.0   1600.0  …   1218.0   2466.0   2167.0

ticker      9602 JT  9613 JT  9681 JT  9735 JT  9766 JT  9983 JT  9984 JT
```

```
date
2021-03-17   4335.0   1764.0      NaN   9680.0   7200.0  96000.0  10400.0
2021-03-18   4340.0   1770.0      NaN   9700.0   7110.0  96930.0  10220.0
2021-03-19   4320.0   1723.0      NaN   9594.0   6850.0  91020.0   9969.0

[3 rows x 248 columns]
ticker        1332 JT   1333 JT  1334 JT     1605 JT   1721 JT    1801 JT  \
date
2021-03-17  1328200.0  169400.0      NaN  10094800.0  530300.0   596900.0
2021-03-18  1421200.0  155300.0      NaN   9579300.0  636100.0   813200.0
2021-03-19  2197500.0  395800.0      NaN  18949400.0  649900.0  1309500.0

ticker        1802 JT    1803 JT     1808 JT    1812 JT  …    9503 JT  \
date                                                     …
2021-03-17  3222500.0  2746000.0   1794500.0  2005300.0  …  3784600.0
2021-03-18  3015600.0  2221500.0   3104200.0  1684000.0  …  3194700.0
2021-03-19  5930300.0  4223300.0  11399100.0  2885800.0  …  7777900.0

ticker        9531 JT    9532 JT   9602 JT    9613 JT  9681 JT   9735 JT  \
date
2021-03-17  1406800.0   964800.0  343900.0  3424800.0      NaN  428200.0
2021-03-18  1647800.0  1222600.0  284500.0  3729700.0      NaN  485600.0
2021-03-19  2931300.0  2216700.0  488200.0  7315700.0      NaN  820300.0

ticker       9766 JT    9983 JT     9984 JT
date
2021-03-17  338500.0   421900.0   9771000.0
2021-03-18  501700.0   678300.0  13941600.0
2021-03-19  870300.0  1542800.0  20715700.0

[3 rows x 248 columns]
```

[ ]:
```
def check_for_NaN(data):
    # Check for NaN values
    nan_values = data.isna().sum()
    # Display the count of NaN values for each column
    print(nan_values)
    return nan_values
```

[ ]:
```
# Check for duplicate values
duplicate_dates = price_df.index.duplicated().sum()
if duplicate_dates > 0:
    print(f'There are {duplicate_dates} duplicate dates in the dataset.')
else:
    print('There are no duplicate dates in the dataset.')
```

```
There are no duplicate dates in the dataset.
```

[ ]:
```python
# Set the threshold for number of NaN values
# threshold = 50
threshold_weight = 0.9

# Remove columns with more than 'threshold' NaN values
filtered_price_df = price_df.dropna(axis=1,␣
↪thresh=threshold_weight*len(price_df))
```

[ ]:
```python
filtered_price_df.describe()
```

[ ]:
```
ticker      1332 JT      1605 JT      1721 JT      1801 JT      1802 JT  \
count   2005.000000  2005.000000  2005.000000  2005.000000  2005.000000
mean     452.057904   991.370424  2088.064198  3496.323099   873.765649
std      161.503418   202.701677   719.025891  1141.600635   238.554496
min      158.934300   481.696500   883.444300  1066.675500   376.339400
25%      335.379900   883.092300  1482.182700  2880.613400   669.593600
50%      480.791700  1001.761300  1878.748000  3523.482500   939.639400
75%      580.191800  1128.023800  2787.294900  4317.753200  1020.506500
max      829.056400  1405.902100  3465.000000  6034.819900  1440.203800

ticker      1803 JT      1812 JT      1925 JT     1928 JT      1963 JT  …  \
count   2005.000000  2005.000000  2005.000000  2005.00000  2005.000000  …
mean     799.472495  1197.523042  2624.096717  1522.67215  1984.930271  …
std      223.414212   400.610475   741.491334   357.30713   661.379057  …
min      238.385300   423.182200  1181.251400   707.72910   716.167100  …
25%      717.317700   894.167600  1895.674500  1243.80950  1537.776400  …
50%      849.465800  1235.738600  2734.549100  1572.16640  1837.213900  …
75%      933.884800  1432.444800  3218.498600  1715.61380  2301.628000  …
max     1269.725700  2304.818500  4131.709100  2357.22680  3816.108500  …

ticker      9502 JT      9503 JT      9531 JT      9532 JT      9602 JT  \
count   2005.000000  2005.000000  2005.000000  2005.000000  2005.000000
mean    1339.960389  1130.087662  2511.699322  1951.431070  3108.727843
std      181.894801   223.517618   299.711989   165.753055   847.150849
min      932.623400   630.651700  1727.759700  1327.971300  1364.412300
25%     1229.000000   973.700000  2312.774500  1845.931700  2472.586600
50%     1325.156100  1114.761900  2521.590100  1975.697400  3077.515800
75%     1477.507800  1271.770700  2687.926300  2064.460600  3754.577100
max     1838.541400  1700.026500  3557.588800  2323.409800  4763.859400

ticker      9613 JT      9735 JT      9766 JT       9983 JT      9984 JT
count   2005.000000  2005.000000  2005.000000   2005.000000  2005.000000
mean    1055.927798  7459.708904  3685.797985  45304.199086  4134.889807
std      276.822370  1590.050279  1412.373827  15558.194492  1407.615313
min      462.167200  3680.808900  1535.915600  20584.836300  1392.738000
25%      834.275600  6289.350700  2199.163300  33676.308200  3323.551200
50%     1079.647200  7668.732000  3765.840300  41434.340400  3921.746900
```

```
75%     1244.142200   8824.419400  4829.847700   54277.472300   4603.677900
max     1770.000000  10470.000000  7460.000000  109550.000000  10635.000000

[8 rows x 208 columns]
```

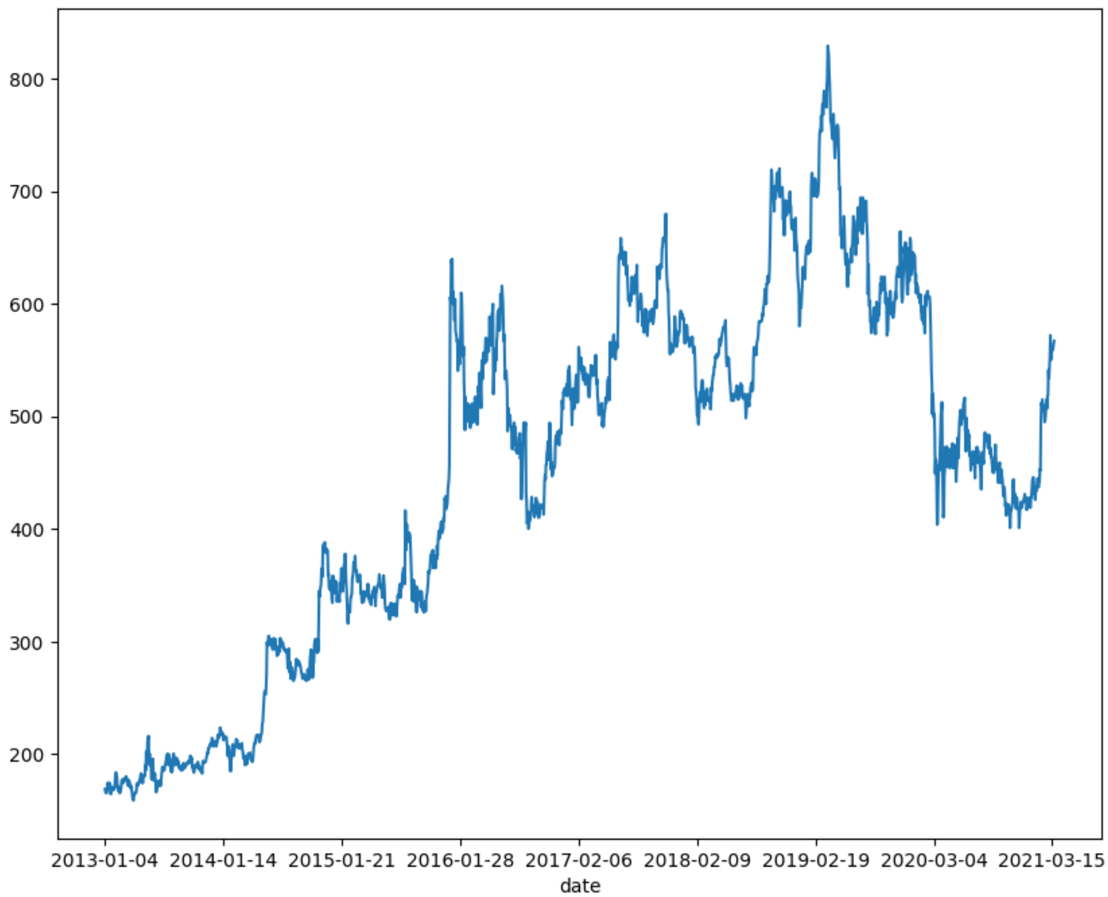
[ ]:
```python
# do same for volume

# Set the threshold for number of NaN values
# threshold = 50
threshold_weight = 0.9

# Remove columns with more than 'threshold' NaN values
filtered_volume_df = volume_df.dropna(axis=1,␣
↪thresh=threshold_weight*len(volume_df))

filtered_volume_df.describe()
```

[ ]:
```
ticker       1332 JT       1605 JT       1721 JT       1801 JT       1802 JT  \
count   2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    2.853706e+06  6.173909e+06  7.255047e+05  1.582422e+06  3.853753e+06
std     2.798325e+06  3.045633e+06  3.661211e+05  1.961919e+06  2.382013e+06
min     5.844000e+05  1.389800e+06  1.735000e+05  2.773000e+05  7.880000e+05
25%     1.676800e+06  4.191400e+06  4.910000e+05  8.175000e+05  2.371700e+06
50%     2.313200e+06  5.465200e+06  6.445000e+05  1.223400e+06  3.239100e+06
75%     3.202500e+06  7.277400e+06  8.520000e+05  1.867200e+06  4.666000e+06
max     6.385470e+07  2.835420e+07  4.233200e+06  5.920440e+07  3.181720e+07

ticker       1803 JT       1812 JT       1925 JT       1928 JT       1963 JT  \
count   2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    3.648841e+06  3.092552e+06  2.138574e+06  3.246529e+06  1.846513e+06
std     2.232276e+06  2.145616e+06  1.066659e+06  1.574956e+06  9.262464e+05
min     1.039900e+06  5.469000e+05  5.700000e+05  8.066000e+05  4.033000e+05
25%     2.210200e+06  1.857500e+06  1.449000e+06  2.197700e+06  1.247300e+06
50%     3.018000e+06  2.575500e+06  1.901700e+06  2.877900e+06  1.649900e+06
75%     4.371000e+06  3.767000e+06  2.541000e+06  3.866000e+06  2.218000e+06
max     2.649300e+07  4.237350e+07  1.582700e+07  2.079500e+07  1.845240e+07

ticker  …       9502 JT       9503 JT       9531 JT       9532 JT  \
count   …  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    …  2.067893e+06  3.308720e+06  1.659184e+06  1.281745e+06
std     …  9.586474e+05  2.789123e+06  7.427532e+05  5.653945e+05
min     …  3.843000e+05  7.763000e+05  3.277000e+05  3.158000e+05
25%     …  1.464300e+06  2.047400e+06  1.145800e+06  9.005000e+05
50%     …  1.859300e+06  2.680100e+06  1.511800e+06  1.147000e+06
75%     …  2.418400e+06  3.727800e+06  1.987600e+06  1.524600e+06
max     …  1.327520e+07  6.585160e+07  6.518000e+06  4.490000e+06
```

```
ticker       9602 JT       9613 JT       9735 JT       9766 JT       9983 JT  \
count   2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    4.858229e+05  3.887022e+06  7.116939e+05  9.776272e+05  7.379879e+05
std     2.797435e+05  1.965328e+06  3.421212e+05  5.884698e+05  3.876526e+05
min     1.342000e+05  8.717000e+05  1.637000e+05  1.616000e+05  1.391000e+05
25%     3.250000e+05  2.709000e+06  4.908000e+05  5.698000e+05  4.914000e+05
50%     4.127000e+05  3.424800e+06  6.351000e+05  8.347000e+05  6.441000e+05
75%     5.530000e+05  4.480000e+06  8.297000e+05  1.207800e+06  8.627000e+05
max     3.105700e+06  3.032350e+07  3.922800e+06  5.766900e+06  4.937300e+06

ticker       9984 JT
count   2.005000e+03
mean    1.763661e+07
std     1.101195e+07
min     4.184400e+06
25%     1.066580e+07
50%     1.444260e+07
75%     2.122020e+07
max     1.547452e+08

[8 rows x 208 columns]
```

[ ]:
```
# before imputing/filling NaN values
fig = plt.figure(1,figsize = (10,8))
filtered_price_df['1332 JT'].plot()
```

[ ]:
```
<AxesSubplot: xlabel='date'>
```



[ ]:
```
filtered_price_df.head(10)
```

[ ]:
```
ticker        1332 JT    1605 JT   1721 JT    1801 JT   1802 JT   1803 JT  \
date
2013-01-04  169.0987   970.6780  950.5521  1232.0186  413.2189  278.1162
2013-01-07  166.3266   960.1615  924.2187  1236.1949  411.5426  278.1162
2013-01-08  166.3266   955.9548  938.6596  1215.3132  407.3517  277.2709
2013-01-09  165.4026   944.3866  945.4553  1240.3712  417.4098  278.9616
2013-01-10  167.2507   942.2833  943.7564  1252.9002  435.0114  284.8789
2013-01-11  170.9468   981.1946  935.2617  1244.5476  435.8496  285.7243
2013-01-15  174.6430  1004.3310  950.5521  1248.7239  430.8206  280.6523
2013-01-16  169.0987   983.2979  944.6059  1211.1369  414.0571  269.6629
2013-01-17  169.0987   986.4528  929.3155  1165.1972  396.4555  263.7455
2013-01-18  174.6430  1000.1244  942.9069  1148.4919  391.4265  263.7455

ticker        1812 JT    1925 JT   1928 JT    1963 JT  …    9502 JT  \
date                                                   …
2013-01-04  478.8201  1214.8674  718.0180  2381.3169  …  1038.6998
```

```
2013-01-07  480.4769  1196.8868  720.2228  2352.1579  …  1009.8470
2013-01-08  473.8496  1181.2514  709.9339  2378.6661  …   992.8748
2013-01-09  482.1337  1203.1409  707.7291  2395.4546  …   987.7831
2013-01-10  490.4178  1225.8122  727.5720  2398.1054  …   997.9665
2013-01-11  490.4178  1210.1768  729.7768  2439.6349  …   984.3887
2013-01-15  488.7610  1262.5553  752.5593  2417.5447  …  1001.3609
2013-01-16  480.4769  1219.5580  738.5958  2403.4070  …   967.4165
2013-01-17  460.5951  1208.6133  727.5720  2356.5760  …   980.9942
2013-01-18  460.5951  1249.2652  745.9450  2327.4170  …   990.3290

ticker       9503 JT    9531 JT    9532 JT    9602 JT   9613 JT    9735 JT  \
date
2013-01-04  847.8471  1727.7597  1332.1473  1367.1194  487.5256  3719.2396
2013-01-07  804.2336  1744.7820  1327.9713  1364.4123  475.2036  3732.0499
2013-01-08  795.5109  1761.8042  1340.4993  1377.0457  470.5604  3736.3199
2013-01-09  780.6823  1766.0598  1336.3233  1440.2130  479.3109  3757.6703
2013-01-10  810.3395  1757.5487  1340.4993  1516.0136  481.9896  3791.8310
2013-01-11  804.2336  1757.5487  1344.6753  1515.1112  485.3827  3834.5318
2013-01-15  804.2336  1800.1043  1373.9074  1516.9160  490.2043  3885.7727
2013-01-16  771.0873  1821.3821  1382.2594  1509.6969  485.9184  3800.3711
2013-01-17  777.1932  1846.9155  1390.6114  1506.0874  472.1677  3791.8310
2013-01-18  792.0218  1846.9155  1378.0834  1501.5754  487.3470  3838.8019

ticker        9766 JT     9983 JT    9984 JT
date
2013-01-04  1721.7623  20584.8363  1482.3931
2013-01-07  1695.8510  21150.4557  1453.9312
2013-01-08  1627.9455  21243.1802  1472.9058
2013-01-09  1612.7561  21159.7281  1442.0720
2013-01-10  1568.9749  20918.6445  1442.0720
2013-01-11  1621.6910  21920.0689  1432.5847
2013-01-15  1617.2236  22086.9730  1439.7002
2013-01-16  1584.1643  21067.0037  1392.7380
2013-01-17  1579.6968  21085.5486  1421.6743
2013-01-18  1596.6732  21725.3475  1432.5847

[10 rows x 208 columns]
```

[ ]:
```
check_for_NaN(filtered_price_df)
```

```
ticker
1332 JT    0
1605 JT    0
1721 JT    0
1801 JT    0
1802 JT    0
          ..
9613 JT    0
```

```
9735 JT    0
9766 JT    0
9983 JT    0
9984 JT    0
Length: 208, dtype: int64
```

[ ]:
```
ticker
1332 JT    0
1605 JT    0
1721 JT    0
1801 JT    0
1802 JT    0
          ..
9613 JT    0
9735 JT    0
9766 JT    0
9983 JT    0
9984 JT    0
Length: 208, dtype: int64
```

There might still be NaN values.

Next, I will replace any NaN values using forward filling – this is good practice towards productionalization of the code

[ ]:
```
# putting all cleaning steps in a single function
def clean_data(df,threshold_weight=0.9):
    # Remove columns with more than 'threshold' NaN values. A minimum of 90% is␣
↪set for non-NaN values
    filtered_data = df.dropna(axis=1, thresh=threshold_weight*len(df))
    # Perform data imputation for each column
    filtered_data = filtered_data.fillna(method='ffill', axis=0)
    # Remove any duplicate rows - not expected
    filtered_data.drop_duplicates(inplace=True)

    filtered_data.index =  pd.to_datetime(filtered_data.index) # convert date␣
↪to datetime for easy handling

    # some stocks not listed in the first date will still be in the tables, it␣
↪is best to filter them out
    filtered_data = filtered_data.loc[:, filtered_data.iloc[0].notna()] #␣
↪removes '3289 JT' , '6988 JT' , '3863 JT' : not listed in Jan 2013

    return filtered_data
```

[ ]:
```
filtered_price_df = clean_data(price_df)
filtered_volume_df = clean_data(volume_df)
```

```
[ ]: # compare the columns of the cleaned data
print(filtered_volume_df.describe())
```

```
ticker       1332 JT       1605 JT       1721 JT       1801 JT       1802 JT  \
count   2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    2.853706e+06  6.173909e+06  7.255047e+05  1.582422e+06  3.853753e+06
std     2.798325e+06  3.045633e+06  3.661211e+05  1.961919e+06  2.382013e+06
min     5.844000e+05  1.389800e+06  1.735000e+05  2.773000e+05  7.880000e+05
25%     1.676800e+06  4.191400e+06  4.910000e+05  8.175000e+05  2.371700e+06
50%     2.313200e+06  5.465200e+06  6.445000e+05  1.223400e+06  3.239100e+06
75%     3.202500e+06  7.277400e+06  8.520000e+05  1.867200e+06  4.666000e+06
max     6.385470e+07  2.835420e+07  4.233200e+06  5.920440e+07  3.181720e+07

ticker       1803 JT       1812 JT       1925 JT       1928 JT       1963 JT  \
count   2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    3.648841e+06  3.092552e+06  2.138574e+06  3.246529e+06  1.846513e+06
std     2.232276e+06  2.145616e+06  1.066659e+06  1.574956e+06  9.262464e+05
min     1.039900e+06  5.469000e+05  5.700000e+05  8.066000e+05  4.033000e+05
25%     2.210200e+06  1.857500e+06  1.449000e+06  2.197700e+06  1.247300e+06
50%     3.018000e+06  2.575500e+06  1.901700e+06  2.877900e+06  1.649900e+06
75%     4.371000e+06  3.767000e+06  2.541000e+06  3.866000e+06  2.218000e+06
max     2.649300e+07  4.237350e+07  1.582700e+07  2.079500e+07  1.845240e+07

ticker  …       9502 JT       9503 JT       9531 JT       9532 JT  \
count   …  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    …  2.067893e+06  3.308720e+06  1.659184e+06  1.281745e+06
std     …  9.586474e+05  2.789123e+06  7.427532e+05  5.653945e+05
min     …  3.843000e+05  7.763000e+05  3.277000e+05  3.158000e+05
25%     …  1.464300e+06  2.047400e+06  1.145800e+06  9.005000e+05
50%     …  1.859300e+06  2.680100e+06  1.511800e+06  1.147000e+06
75%     …  2.418400e+06  3.727800e+06  1.987600e+06  1.524600e+06
max     …  1.327520e+07  6.585160e+07  6.518000e+06  4.490000e+06

ticker       9602 JT       9613 JT       9735 JT       9766 JT       9983 JT  \
count   2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03  2.005000e+03
mean    4.858229e+05  3.887022e+06  7.116939e+05  9.776272e+05  7.379879e+05
std     2.797435e+05  1.965328e+06  3.421212e+05  5.884698e+05  3.876526e+05
min     1.342000e+05  8.717000e+05  1.637000e+05  1.616000e+05  1.391000e+05
25%     3.250000e+05  2.709000e+06  4.908000e+05  5.698000e+05  4.914000e+05
50%     4.127000e+05  3.424800e+06  6.351000e+05  8.347000e+05  6.441000e+05
75%     5.530000e+05  4.480000e+06  8.297000e+05  1.207800e+06  8.627000e+05
max     3.105700e+06  3.032350e+07  3.922800e+06  5.766900e+06  4.937300e+06

ticker       9984 JT
count   2.005000e+03
mean    1.763661e+07
std     1.101195e+07
min     4.184400e+06
```

```
25%     1.066580e+07
50%     1.444260e+07
75%     2.122020e+07
max     1.547452e+08

[8 rows x 205 columns]
```

The price and volume tables might have different number of stocks,

Next, I will collate the common stocks in all the tables

```
[ ]: # get the column names of the different datasets and collate them
     filtered_price_df_cols = (filtered_price_df.columns).tolist()
     filtered_volume_df_cols = (filtered_volume_df.columns).tolist()
```

```
[ ]: collated_cols = ([x for x in filtered_price_df_cols if all(x in lst for lst in␣
      ↪[filtered_volume_df_cols])])
     collated_cols = sorted(collated_cols)
```

```
[ ]: print(len(collated_cols))
```

```
205
```

After cleaning and filtering, 43 stocks have been removed. We are left with 205 stocks

```
[ ]: # obtain the collated data with the 208 stocks
     collated_price_df = filtered_price_df[collated_cols]
     collated_volume_df = filtered_volume_df[collated_cols]
```

```
[ ]: collated_price_df[collated_price_df.isna().any(axis=1)]
```

```
[ ]: Empty DataFrame
     Columns: [1332 JT, 1605 JT, 1721 JT, 1801 JT, 1802 JT, 1803 JT, 1812 JT, 1925
     JT, 1928 JT, 1963 JT, 2002 JT, 2269 JT, 2282 JT, 2501 JT, 2502 JT, 2503 JT, 2531
     JT, 2768 JT, 2801 JT, 2802 JT, 2871 JT, 2914 JT, 3086 JT, 3099 JT, 3101 JT, 3103
     JT, 3105 JT, 3382 JT, 3401 JT, 3402 JT, 3405 JT, 3407 JT, 3436 JT, 3861 JT, 4004
     JT, 4005 JT, 4021 JT, 4042 JT, 4043 JT, 4061 JT, 4063 JT, 4151 JT, 4183 JT, 4188
     JT, 4208 JT, 4272 JT, 4324 JT, 4452 JT, 4502 JT, 4503 JT, 4506 JT, 4507 JT, 4519
     JT, 4523 JT, 4543 JT, 4568 JT, 4689 JT, 4704 JT, 4901 JT, 4902 JT, 4911 JT, 5020
     JT, 5101 JT, 5108 JT, 5201 JT, 5202 JT, 5214 JT, 5232 JT, 5233 JT, 5301 JT, 5332
     JT, 5333 JT, 5401 JT, 5406 JT, 5411 JT, 5541 JT, 5631 JT, 5703 JT, 5706 JT, 5707
     JT, 5711 JT, 5713 JT, 5714 JT, 5801 JT, 5802 JT, 5803 JT, 5901 JT, 6103 JT, 6113
     JT, 6301 JT, 6302 JT, 6305 JT, 6326 JT, 6361 JT, 6367 JT, 6471 JT, 6472 JT, 6473
     JT, 6479 JT, 6501 JT, …]
     Index: []

     [0 rows x 205 columns]
```

```
[ ]: collated_price_df.to_csv('collated_price_df.csv')
```

Separate the data into training and test sets

training set: 2013, 2014, 2015 data

test set: 2016, 2017, 2018, 2019, 2020, 2021 data

```
[ ]: # divide data into training and testing sets
     # convert index to datetime
     collated_price_df.index = pd.to_datetime(collated_price_df.index)
     collated_volume_df.index = pd.to_datetime(collated_volume_df.index)

     # separate the data
     train_collated_price_df = collated_price_df[(collated_price_df.index).year <=␣
      ↪2015]
     test_collated_price_df = collated_price_df[(collated_price_df.index).year >␣
      ↪2015]


     train_collated_volume_df = collated_volume_df[(collated_volume_df.index).year␣
      ↪<= 2015]
     test_collated_volume_df = collated_volume_df[(collated_volume_df.index).year >␣
      ↪2015]

     # manual check
     display(train_collated_price_df.head(3))
     display(train_collated_price_df.tail(3))


     display(test_collated_price_df.head(3))
     display(test_collated_price_df.tail(3))
```

```
ticker       1332 JT   1605 JT   1721 JT    1801 JT   1802 JT   1803 JT  \
date
2013-01-04  169.0987  970.6780  950.5521  1232.0186  413.2189  278.1162
2013-01-07  166.3266  960.1615  924.2187  1236.1949  411.5426  278.1162
2013-01-08  166.3266  955.9548  938.6596  1215.3132  407.3517  277.2709

ticker       1812 JT    1925 JT   1928 JT    1963 JT  …    9502 JT  \
date                                                  …
2013-01-04  478.8201  1214.8674  718.0180  2381.3169  …  1038.6998
2013-01-07  480.4769  1196.8868  720.2228  2352.1579  …  1009.8470
2013-01-08  473.8496  1181.2514  709.9339  2378.6661  …   992.8748

ticker       9503 JT    9531 JT    9532 JT    9602 JT   9613 JT    9735 JT  \
date
2013-01-04  847.8471  1727.7597  1332.1473  1367.1194  487.5256  3719.2396
2013-01-07  804.2336  1744.7820  1327.9713  1364.4123  475.2036  3732.0499
2013-01-08  795.5109  1761.8042  1340.4993  1377.0457  470.5604  3736.3199
```

```
ticker        9766 JT     9983 JT    9984 JT
date
2013-01-04  1721.7623  20584.8363  1482.3931
2013-01-07  1695.8510  21150.4557  1453.9312
2013-01-08  1627.9455  21243.1802  1472.9058

[3 rows x 205 columns]
ticker       1332 JT    1605 JT    1721 JT    1801 JT   1802 JT   1803 JT  \
date
2015-12-28  602.2811  1029.9669  1546.8918  3446.4557  962.4982  862.6563
2015-12-29  638.8113  1038.7438  1550.4788  3489.9715  978.2053  875.8467
2015-12-30  636.0013  1040.4991  1532.5438  3481.2684  976.4601  870.5705

ticker        1812 JT    1925 JT    1928 JT    1963 JT  …    9502 JT  \
date                                                   …
2015-12-28  1220.1843  2915.5097  1614.3256  1722.4256  …  1426.2970
2015-12-29  1249.4000  2993.1316  1645.0439  1727.0261  …  1434.2014
2015-12-30  1244.2443  2988.0136  1654.3402  1714.1447  …  1460.1101

ticker        9503 JT    9531 JT    9532 JT    9602 JT    9613 JT   9735 JT  \
date
2015-12-28  1252.1446  2484.0534  1916.7884  3101.2568  1085.0907  7411.381
2015-12-29  1259.1227  2512.3221  1935.4888  3133.9508  1092.5612  7497.273
2015-12-30  1273.0791  2561.2314  1951.9629  3138.6214  1098.1641  7529.254

ticker        9766 JT     9983 JT    9984 JT
date
2015-12-28  2642.7538  40539.9639  2964.3586
2015-12-29  2672.5418  40130.6616  2963.3912
2015-12-30  2688.3666  40587.5572  2969.1952

[3 rows x 205 columns]
ticker       1332 JT    1605 JT    1721 JT    1801 JT   1802 JT   1803 JT  \
date
2016-01-04  639.7480  1063.3190  1476.9454  3398.5882  954.6446  852.1039
2016-01-05  600.4077  1067.2686  1481.4291  3381.1819  952.0268  860.8975
2016-01-06  610.7111  1033.0388  1470.6682  3368.1271  958.1351  858.2594

ticker        1812 JT    1925 JT    1928 JT    1963 JT  …    9502 JT  \
date                                                   …
2016-01-04  1211.5915  2896.7440  1613.9214  1678.2608  …  1402.1448
2016-01-05  1206.4358  2890.7731  1603.0083  1664.4593  …  1415.7579
2016-01-06  1201.2801  2892.4791  1598.5623  1635.9363  …  1422.7840

ticker        9503 JT    9531 JT    9532 JT    9602 JT    9613 JT    9735 JT  \
date
2016-01-04  1239.4967  2450.8489  1901.6500  3059.2217  1070.1497  7334.6265
```

```
2016-01-05  1237.7521  2458.0283  1918.1241  3031.1983  1077.6202  7308.1279
2016-01-06  1237.7521  2445.4644  1912.7812  3026.5278  1062.6792  7220.4084

ticker        9766 JT     9983 JT    9984 JT
date
2016-01-04  2602.7262  38693.3443  2898.5807
2016-01-05  2649.2699  38483.9338  2886.9728
2016-01-06  2630.6524  38236.4487  2848.2799

[3 rows x 205 columns]
```

```
ticker      1332 JT  1605 JT  1721 JT  1801 JT  1802 JT  1803 JT  1812 JT  \
date
2021-03-17    561.0    808.0   3400.0   4220.0   1007.0    909.0   1558.0
2021-03-18    565.0    812.0   3455.0   4300.0   1028.0    921.0   1581.0
2021-03-19    567.0    796.0   3455.0   4345.0   1054.0    926.0   1600.0

ticker      1925 JT  1928 JT  1963 JT  …  9502 JT  9503 JT  9531 JT  \
date                                   …
2021-03-17   3387.0   2263.0   1443.0  …   1417.5   1193.0   2450.0
2021-03-18   3465.0   2332.5   1441.0  …   1431.0   1199.5   2442.5
2021-03-19   3466.0   2345.0   1435.0  …   1440.0   1218.0   2466.0

ticker      9532 JT  9602 JT  9613 JT  9735 JT  9766 JT  9983 JT  9984 JT
date
2021-03-17   2150.0   4335.0   1764.0   9680.0   7200.0  96000.0  10400.0
2021-03-18   2162.0   4340.0   1770.0   9700.0   7110.0  96930.0  10220.0
2021-03-19   2167.0   4320.0   1723.0   9594.0   6850.0  91020.0   9969.0

[3 rows x 205 columns]
```

Now, I will proceed with pair selection

[ ]:
```
series_analyser = class_SeriesAnalyser.SeriesAnalyser()
data_processor = class_DataProcessor.DataProcessor()
training_returns = data_processor.get_return_series(train_collated_price_df)
training_returns.head()
```

[ ]:
```
ticker       1332 JT   1605 JT   1721 JT   1801 JT   1802 JT   1803 JT  \
date
2013-01-07 -0.016393 -0.010834 -0.027703  0.003390 -0.004057  0.000000
2013-01-08  0.000000 -0.004381  0.015625 -0.016892 -0.010183 -0.003039
2013-01-09 -0.005555 -0.012101  0.007240  0.020619  0.024691  0.006098
2013-01-10  0.011173 -0.002227 -0.001797  0.010101  0.042169  0.021212
2013-01-11  0.022099  0.041295 -0.009001 -0.006667  0.001927  0.002968

ticker       1812 JT   1925 JT   1928 JT   1963 JT  …   9502 JT   9503 JT  \
date                                               …
```

```
2013-01-07  0.003460 -0.014800  0.003071 -0.012245  … -0.027778 -0.051440
2013-01-08 -0.013793 -0.013063 -0.014286  0.011270  … -0.016807 -0.010846
2013-01-09  0.017483  0.018531 -0.003106  0.007058  … -0.005128 -0.018640
2013-01-10  0.017182  0.018843  0.028037  0.001107  …  0.010309  0.037989
2013-01-11  0.000000 -0.012755  0.003030  0.017318  … -0.013605 -0.007535

ticker       9531 JT   9532 JT   9602 JT   9613 JT   9735 JT   9766 JT  \
date
2013-01-07  0.009852 -0.003135 -0.001980 -0.025275  0.003444 -0.015049
2013-01-08  0.009756  0.009434  0.009259 -0.009771  0.001144 -0.040042
2013-01-09  0.002415 -0.003115  0.045872  0.018596  0.005714 -0.009330
2013-01-10 -0.004819  0.003125  0.052632  0.005589  0.009091 -0.027147
2013-01-11  0.000000  0.003115 -0.000595  0.007040  0.011261  0.033599

ticker       9983 JT   9984 JT
date
2013-01-07  0.027477 -0.019200
2013-01-08  0.004384  0.013051
2013-01-09 -0.003928 -0.020934
2013-01-10 -0.011394  0.000000
2013-01-11  0.047872 -0.006579

[5 rows x 205 columns]
```

[ ]:
```
training_returns[training_returns.isna().any(axis=1)]
```

[ ]:
```
Empty DataFrame
Columns: [1332 JT, 1605 JT, 1721 JT, 1801 JT, 1802 JT, 1803 JT, 1812 JT, 1925
JT, 1928 JT, 1963 JT, 2002 JT, 2269 JT, 2282 JT, 2501 JT, 2502 JT, 2503 JT, 2531
JT, 2768 JT, 2801 JT, 2802 JT, 2871 JT, 2914 JT, 3086 JT, 3099 JT, 3101 JT, 3103
JT, 3105 JT, 3382 JT, 3401 JT, 3402 JT, 3405 JT, 3407 JT, 3436 JT, 3861 JT, 4004
JT, 4005 JT, 4021 JT, 4042 JT, 4043 JT, 4061 JT, 4063 JT, 4151 JT, 4183 JT, 4188
JT, 4208 JT, 4272 JT, 4324 JT, 4452 JT, 4502 JT, 4503 JT, 4506 JT, 4507 JT, 4519
JT, 4523 JT, 4543 JT, 4568 JT, 4689 JT, 4704 JT, 4901 JT, 4902 JT, 4911 JT, 5020
JT, 5101 JT, 5108 JT, 5201 JT, 5202 JT, 5214 JT, 5232 JT, 5233 JT, 5301 JT, 5332
JT, 5333 JT, 5401 JT, 5406 JT, 5411 JT, 5541 JT, 5631 JT, 5703 JT, 5706 JT, 5707
JT, 5711 JT, 5713 JT, 5714 JT, 5801 JT, 5802 JT, 5803 JT, 5901 JT, 6103 JT, 6113
JT, 6301 JT, 6302 JT, 6305 JT, 6326 JT, 6361 JT, 6367 JT, 6471 JT, 6472 JT, 6473
JT, 6479 JT, 6501 JT, …]
Index: []

[0 rows x 205 columns]
```

[ ]:
```
print('Total number of possible pairs: ', len(training_returns.
 ↪columns)*(len(training_returns.columns)-1)/2)
```

```
Total number of possible pairs:  20910.0
```

[ ]:
```
average_volume_info = pd.DataFrame(train_collated_volume_df.mean(axis=0),␣
↪columns=['volume']) # compute average volume of each stock using training␣
↪data
```

[ ]:
```
average_volume_info.head(5)
```

[ ]:
```
                 volume
ticker
1332 JT  2.686977e+06
1605 JT  5.453648e+06
1721 JT  8.709952e+05
1801 JT  2.456847e+06
1802 JT  5.345052e+06
```

[ ]:
```
np.random.seed(42)
N_PRIN_COMPONENTS = 50
pca = PCA(n_components=N_PRIN_COMPONENTS)
pca.fit(training_returns)
```

[ ]: PCA(n_components=50)

[ ]:
```
pca.components_.T.shape
```

[ ]: (205, 50)

[ ]:
```
# adding volume information to improve clustering performance
X = np.hstack(
    (pca.components_.T,
    average_volume_info['volume'][training_returns.columns].values[:,np.
↪newaxis])
)

print(X.shape)
```

```
(205, 51)
```

[ ]:
```
X = preprocessing.StandardScaler().fit_transform(X)
```

[ ]:
```
X.shape
```

[ ]: (205, 51)

[ ]:
```
def cluster_size(counts):
    plt.figure()
    plt.barh(counts.index+1, counts.values)
    #plt.title('Cluster Member Counts')
    plt.yticks(np.arange(1, len(counts)+1, 1))
    plt.xlabel('Stocks within cluster', size=12)
```

```
    plt.ylabel('Cluster Id', size=12)
```

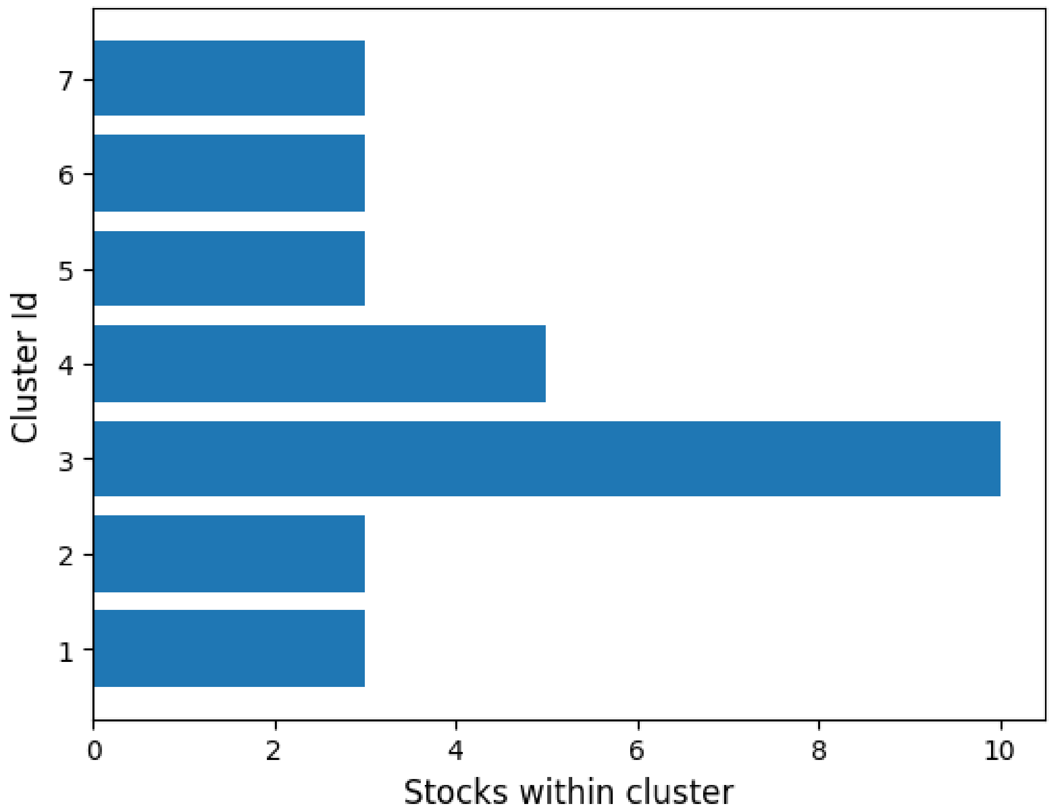
[ ]:
```
clustered_series_all, clustered_series, counts, clf = series_analyser.
 ↪apply_DBSCAN(3.5,
                                                                        ␣
 ↪   3,
                                                                        ␣
 ↪   X,
                                                                        ␣
 ↪   training_returns)
```

```
Clusters discovered: 7
Pairs to evaluate: 70
```

[ ]:
```
plt.figure(1,figsize=(16,12))
cluster_size(counts)


print(counts)
print('Average cluster size: ', np.mean(counts))
```

```
2    10
3     5
0     3
1     3
4     3
5     3
6     3
dtype: int64
Average cluster size:  4.285714285714286
```

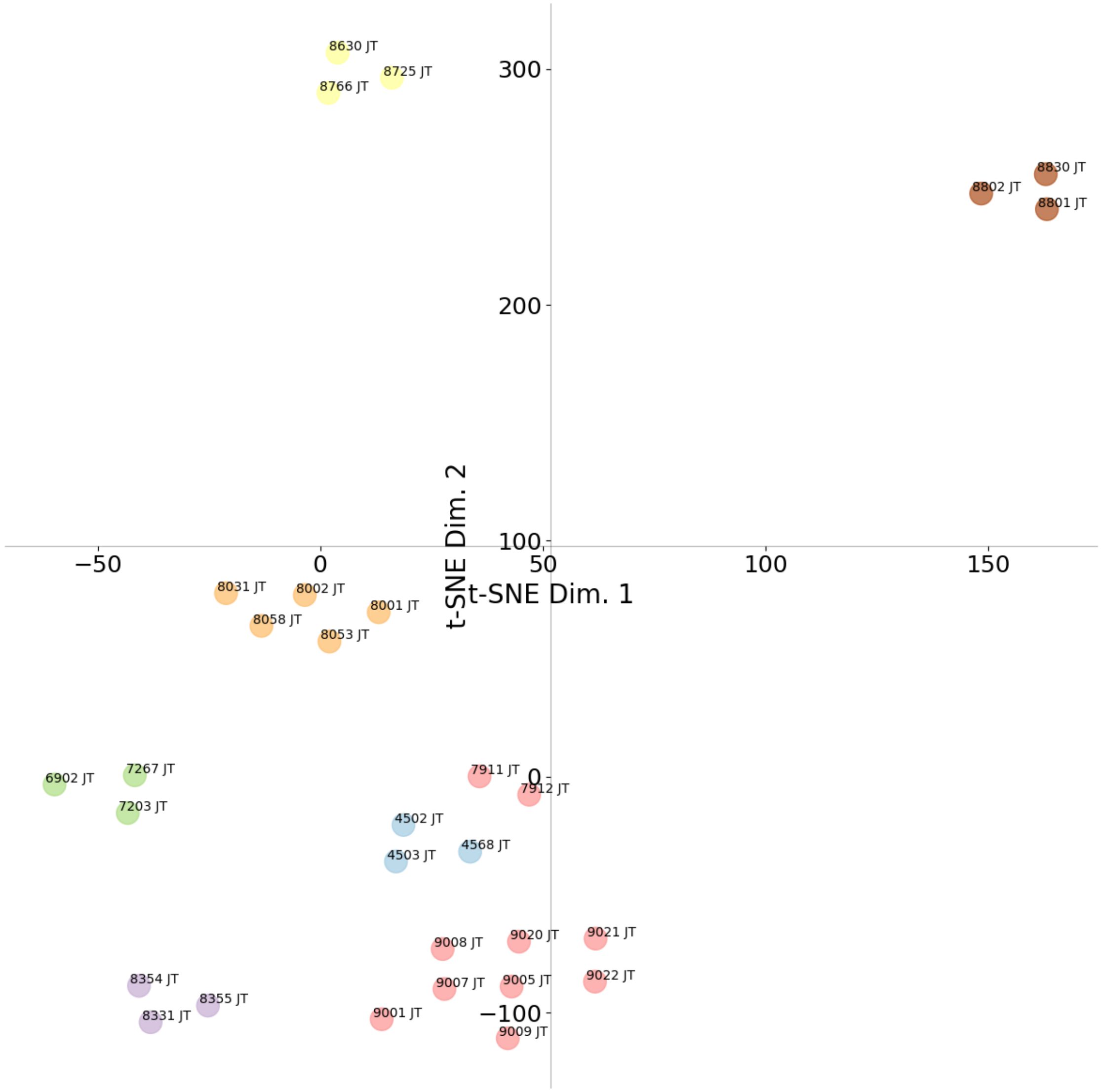
```
<Figure size 1600x1200 with 0 Axes>
```



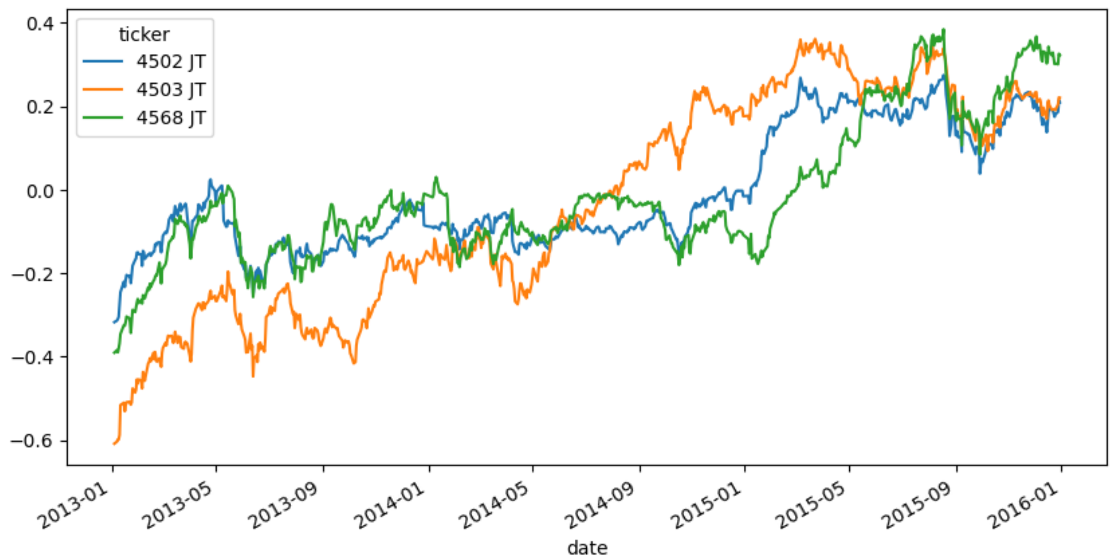

```
[ ]: def plot_TSNE(X, clf, clustered_series_all):
    """
    This function makes use of t-sne to visualize clusters in 2d.
    """

    X_tsne = TSNE(learning_rate=1000, perplexity=25, random_state=1337).
 ↪fit_transform(X)

    # visualization
    fig = plt.figure(1, facecolor='white', figsize=(15,15), frameon=True,␣
 ↪edgecolor='black')
    plt.clf()

    # axis in the middle
    ax = fig.add_subplot(1, 1, 1, alpha=0.9)
    # Move left y-axis and bottim x-axis to centre, passing through (0,0)
    ax.spines['left'].set_position('center')
    ax.spines['left'].set_alpha(0.3)
    ax.spines['bottom'].set_position('center')
    ax.spines['bottom'].set_alpha(0.3)
```

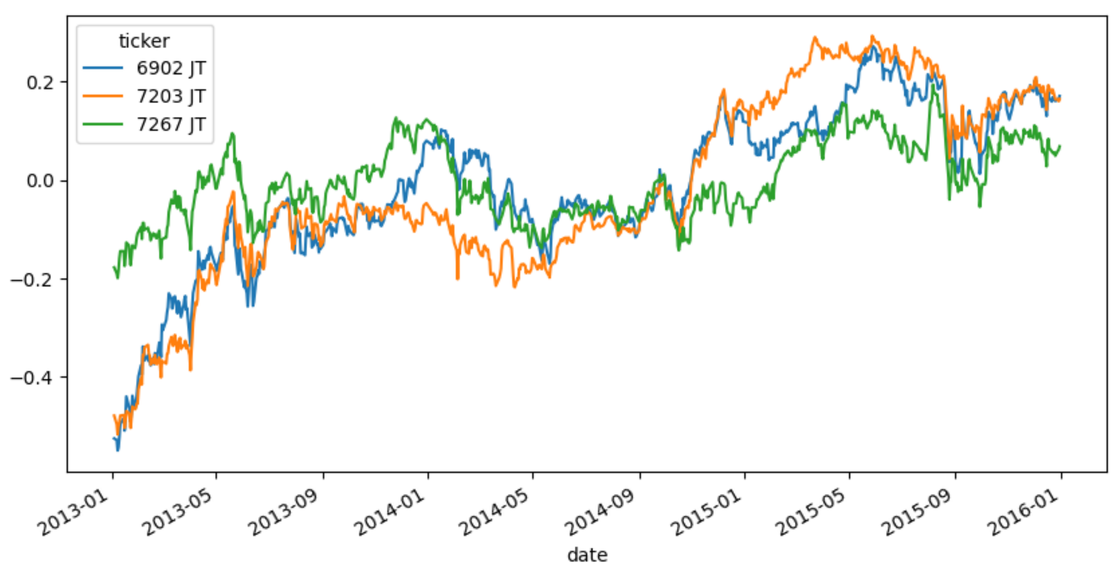
```python
    # Eliminate upper and right axes
    ax.spines['right'].set_color('none')
    ax.spines['top'].set_color('none')
    # Show ticks in the left and lower axes only
    ax.xaxis.set_ticks_position('bottom')
    ax.yaxis.set_ticks_position('left')
    ax.tick_params(which='major', labelsize=18)
    #plt.axis('off')

    # stocks in cluster
    labels = clf.labels_
    x = X_tsne[(labels!=-1), 0]
    y = X_tsne[(labels!=-1), 1]
    tickers = list(clustered_series_all[clustered_series_all != -1].index)
    plt.scatter(
        x,
        y,
        s=300,
        alpha=0.75,
        c=labels[labels!=-1],
        cmap=cm.Paired
    )
    for i, ticker in enumerate(tickers):
        # plt.annotate(ticker, (x[i]-20, y[i]+12), size=15)
        plt.annotate(ticker, (x[i]-2, y[i]+1), size=10)


    # x = np.array(x)
    # y = np.array(y)

    #for i, ticker in enumerate(tickers):
    #    plt.annotate(ticker, (x[i]+20, y[i]+20))#, arrowprops={'arrowstyle':
↪'simple'})

    #plt.title('OPTICS clusters visualized with t-SNE', size=16);
    plt.xlabel('t-SNE Dim. 1', position=(0.92,0), size=20)
    plt.ylabel('t-SNE Dim. 2', position=(0,0.92), size=20)
    # ax.set_xticks(range(-300, 0, 60))
    # ax.set_yticks(range(-300, 0, 60))
    #plt.savefig('DBSCAN_2014_2018_eps0_15.png', bbox_inches='tight',␣
↪pad_inches=0.01)
    plt.savefig('DBSCAN_2013_2015.png', bbox_inches='tight', pad_inches=0.1)
    plt.show()
    # include connections - see quontopian
```

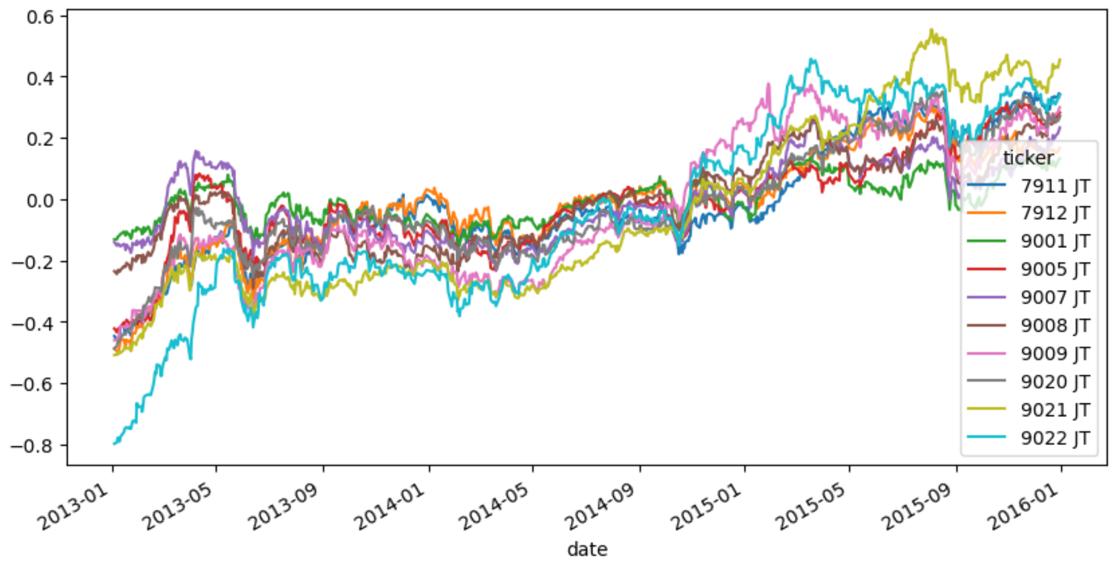
```
plot_TSNE(X,clf, clustered_series_all)
```



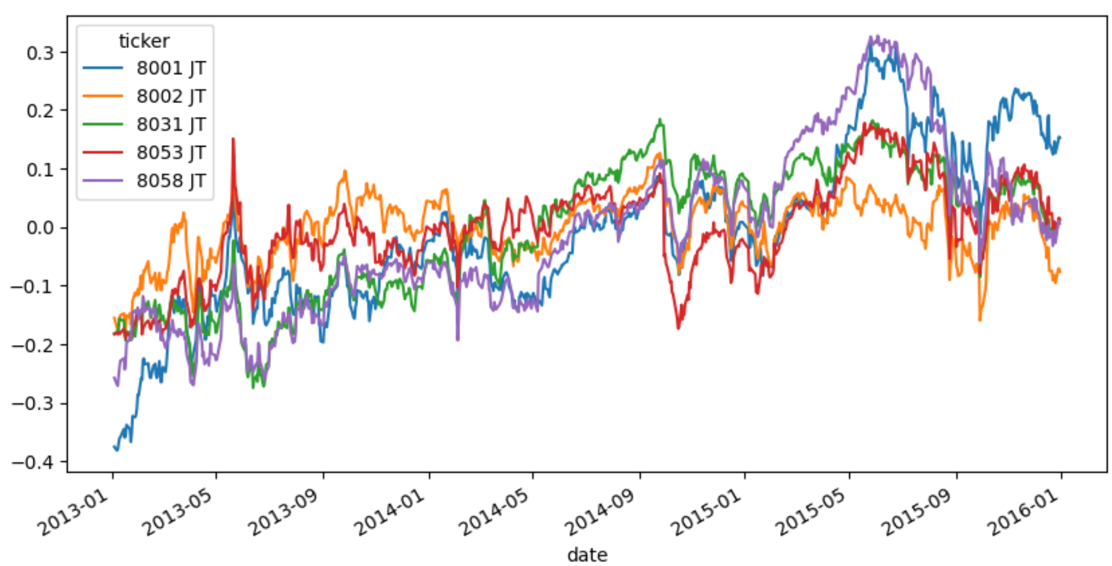
[ ]:
```
for clust in range(len(counts)):
    symbols = list(clustered_series[clustered_series==clust].index)
    means = np.log(train_collated_price_df[symbols].mean())
    series = np.log(train_collated_price_df[symbols]).sub(means)
    series.plot(figsize=(10,5))#title='ETFs Time Series for Cluster %d' %␣
 ↪(clust+1))
    #plt.ylabel('Normalized log prices', size=12)
    #plt.xlabel('Date', size=12)
    plt.savefig('results/clustering_outcome/cluster_{}.png'.
 ↪format(str(clust+1)), bbox_inches='tight', pad_inches=0.1)
```


ticker
4502 JT
4503 JT
4568 JT
0.4
0.2
0.0
−0.2
−0.4
−0.6
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
date

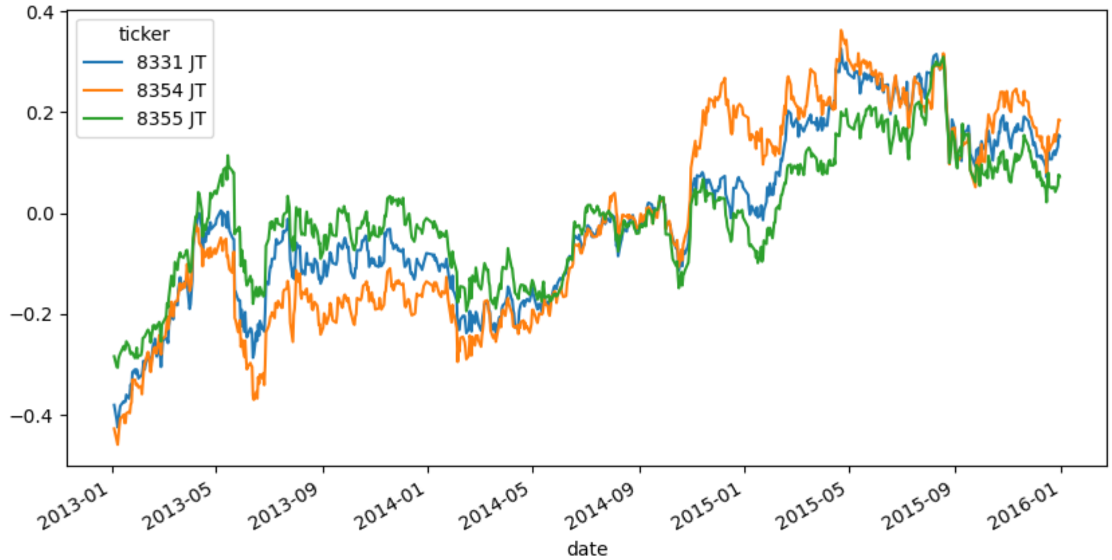

ticker
6902 JT
7203 JT
7267 JT
0.2
0.0
−0.2
−0.4
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
date



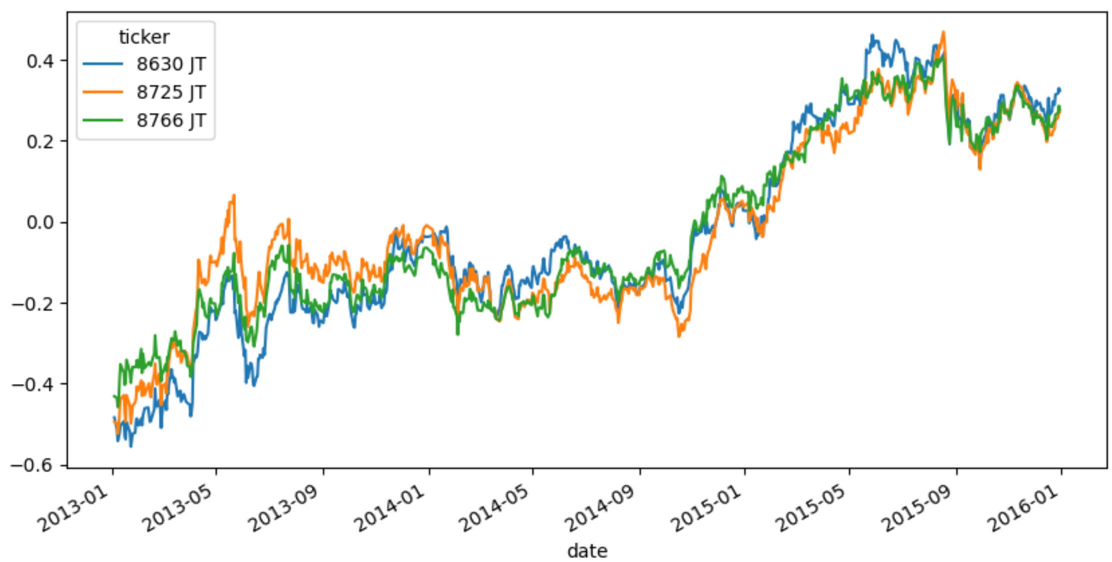
0.6
0.4
0.2
0.0
−0.2
−0.4
−0.6
−0.8
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
date
ticker
7911 JT
7912 JT
9001 JT
9005 JT
9007 JT
9008 JT
9009 JT
9020 JT
9021 JT
9022 JT



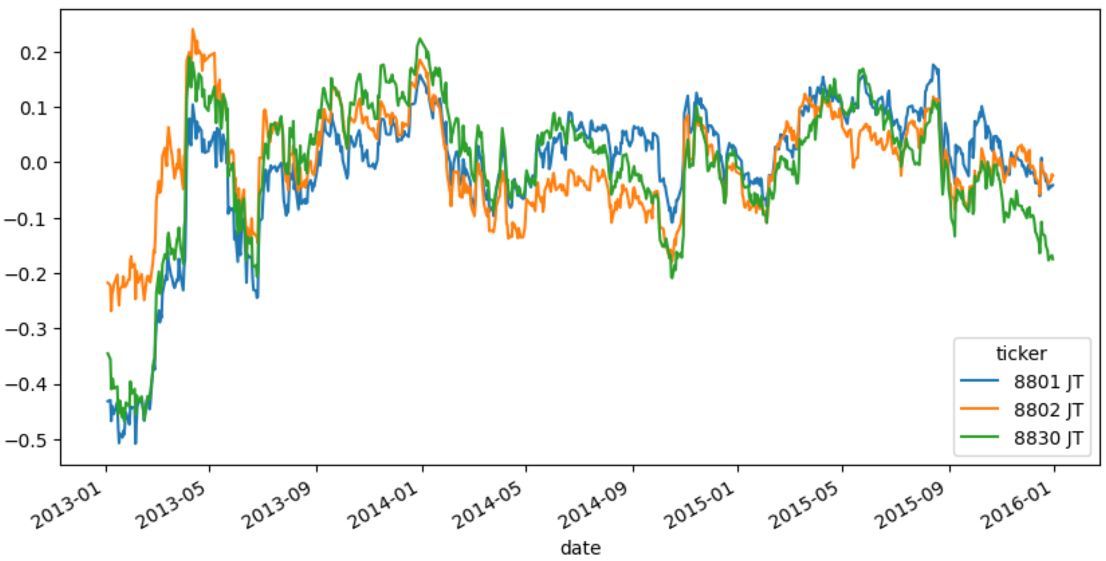
ticker
8001 JT
8002 JT
8031 JT
8053 JT
8058 JT
0.3
0.2
0.1
0.0
−0.1
−0.2
−0.3
−0.4
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
date



ticker
8331 JT
8354 JT
8355 JT
0.4
0.2
0.0
−0.2
−0.4
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
date



ticker
8630 JT
8725 JT
8766 JT
0.4
0.2
0.0
−0.2
−0.4
−0.6
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
date



[ ]:
```
subsample = 1000 # 2500
min_half_life = 1 #78 # number of points in a day
max_half_life = 252 #20000 #~number of points in a year: 78*252

pairs_unsupervised, unique_tickers = series_analyser.
 ↪get_candidate_pairs(clustered_series=clustered_series,
                                                                         ␣
 ↪pricing_df_train=train_collated_price_df,
                                                                         ␣
 ↪pricing_df_test=test_collated_price_df,
                                                                         ␣
 ↪min_half_life=min_half_life,
                                                                         ␣
 ↪max_half_life=max_half_life,
                                                                         ␣
 ↪min_zero_crosings=12,
                                                                          p_value_threshold=0.
 ↪05,
                                                                          hurst_threshold=0.5,
                                                                          subsample=subsample
                                                                          )


all_pairs = []
t_statistic = []
p_value = []
coint_coef = []
```

```
zero_cross = []
half_life = []
hurst_exponent = []
spread = []

pairing_metric = [t_statistic, p_value, coint_coef, zero_cross,␣
↪half_life,hurst_exponent, spread]
pairing_metric_string = ['t_statistic', 'p_value', 'coint_coef', 'zero_cross',␣
↪'half_life', 'hurst_exponent', 'spread']

for pair in pairs_unsupervised:
    all_pairs.append((pair[0],pair[1]))
    for idx, metric in enumerate(pairing_metric):
        metric.append(pair[2][pairing_metric_string[idx]])
    # print(pair[2])

print(all_pairs)
print(hurst_exponent)
```

```
Cluster 7/7Found 17 pairs
The pairs contain 17 unique tickers
Pairs Selection failed stage:  {'cointegration': 53, 'None': 17}
[('6902 JT', '7203 JT'), ('7911 JT', '9001 JT'), ('7911 JT', '9005 JT'), ('7912
JT', '9021 JT'), ('9001 JT', '9005 JT'), ('9001 JT', '9007 JT'), ('9001 JT',
'9008 JT'), ('9001 JT', '9009 JT'), ('9001 JT', '9020 JT'), ('9001 JT', '9021
JT'), ('9001 JT', '9022 JT'), ('9005 JT', '9021 JT'), ('9009 JT', '9022 JT'),
('9020 JT', '9021 JT'), ('8331 JT', '8355 JT'), ('8630 JT', '8766 JT'), ('8725
JT', '8766 JT')]
[0.4309718005136872, 0.3239934705295372, 0.4474111428635595,
0.40243893314841783, 0.357287295667681, 0.2548191929637193, 0.28238815453438304,
0.30862569613252905, 0.31869876919144036, 0.3107271947369549,
0.2598008505974249, 0.3909022075041255, 0.4017056198932035, 0.36736187454357433,
0.2941032783282862, 0.25457063501917987, 0.30963053743445446]
```

[ ]:
```
pairing_metric_df = pd.DataFrame({
    'pairs': all_pairs,
    't_statistic': t_statistic,
    'p_value': p_value,
    'coint_coef': coint_coef,
    'zero_cross': zero_cross,
    'half_life': half_life,
    'hurst_exponent': hurst_exponent,
})



pairing_metric_df
```

[ ]:

```
                pairs  t_statistic   p_value  coint_coef  zero_cross  \
0   (6902 JT, 7203 JT)    -2.954651  0.039352    0.594890          39
1   (7911 JT, 9001 JT)    -3.081635  0.027942    0.473723          39
2   (7911 JT, 9005 JT)    -3.051582  0.030350    0.713315          47
3   (7912 JT, 9021 JT)    -3.176201  0.021404    0.174814          36
4   (9001 JT, 9005 JT)    -2.978553  0.036946    1.013671          43
5   (9001 JT, 9007 JT)    -3.793134  0.002978    0.663768          69
6   (9001 JT, 9008 JT)    -4.156469  0.000780    0.289039          51
7   (9001 JT, 9009 JT)    -3.555290  0.006676    0.287469          39
8   (9001 JT, 9020 JT)    -3.268528  0.016345    0.094821          51
9   (9001 JT, 9021 JT)    -3.669899  0.004557    0.094769          39
10  (9001 JT, 9022 JT)    -3.569096  0.006380    0.033434          53
11  (9005 JT, 9021 JT)    -3.109852  0.025833    0.135167          41
12  (9009 JT, 9022 JT)    -3.102209  0.026390    8.263229          30
13  (9020 JT, 9021 JT)    -3.958953  0.001641    1.001917          60
14  (8331 JT, 8355 JT)    -3.414943  0.010460    1.029639          56
15  (8630 JT, 8766 JT)    -3.718341  0.003862    0.942843          62
16  (8725 JT, 8766 JT)    -3.019312  0.033130    0.719093          45

    half_life  hurst_exponent
0          33        0.430972
1          27        0.323993
2          38        0.447411
3          31        0.402439
4          38        0.357287
5          15        0.254819
6          15        0.282388
7          20        0.308626
8          23        0.318699
9          20        0.310727
10         19        0.259801
11         31        0.390902
12         29        0.401706
13         21        0.367362
14         22        0.294103
15         16        0.254571
16         23        0.309631
```

[ ]:

```python
# I will proceed with using the 7 pairs identified above

# write intermediate function to investigate pairs

# signal evaluation metrics

def compute_hit_rate(signals, actual_returns):
    # Compute the hit rate of the signals
    predicted_directions = np.sign(signals)
```

```
    actual_directions = np.sign(actual_returns)
    num_correct = np.sum(predicted_directions == actual_directions)
    hit_rate = num_correct / len(signals)
    return hit_rate

def compute_profitability(signals, actual_returns):
    # Compute the profitability of the signals
    pnl = signals * actual_returns
    total_profit = np.sum(pnl)
    return total_profit

def compute_risk_reward_ratio(signals, actual_returns):
    # Compute the risk-to-reward ratio of the signals
    positive_pnl = signals * actual_returns
    positive_pnl = positive_pnl[positive_pnl > 0]
    negative_pnl = signals * actual_returns
    negative_pnl = negative_pnl[negative_pnl < 0]

    if len(negative_pnl) > 0:
        risk_reward_ratio = np.abs(np.mean(positive_pnl) / np.
↪mean(negative_pnl))
    else:
        risk_reward_ratio = np.inf

    return risk_reward_ratio


def compute_win_rate(signal, returns):
    """
    Calculates the win rate of the trading signal.

    Args:
    signal (np.ndarray): The trading signal
    returns (np.ndarray): The actual returns

    Returns:
    float: The win rate of the trading signal
    """
    profits = signal * returns
    return np.mean(profits > 0)

def compute_maximum_drawdown(signal, returns):
    """
    Calculates the maximum drawdown of the trading signal.

    Args:
    signal (np.ndarray): The trading signal
```

```
    returns (np.ndarray): The actual returns

    Returns:
    float: The maximum drawdown of the trading signal
    """
    profits = signal * returns
    cum_profits = np.cumsum(profits)
    # remove nan from cum_profits
    cum_profits = [item if isinstance(item, (int, float)) and not np.
 ↪isnan(item) else 0 for item in cum_profits]
    max_drawdown = np.max(np.maximum.accumulate(cum_profits) - cum_profits)
    return max_drawdown
```

[ ]:
```
def calculate_rsi(values, period=14):
    """
    Calculates the Relative Strength Index (RSI) for a given list of values.

    Args:
        values (list, np.ndarray): List or numpy array of values.
        period (int): Period for calculating the RSI. Default is 14.

    Returns:
        np.ndarray: Numpy array of RSI values.
    """
    if len(values) < period:
        raise ValueError("Number of values is less than the specified period.")

    deltas = np.diff(values)
    gains = np.where(deltas > 0, deltas, 0)
    losses = -np.where(deltas < 0, deltas, 0)
    avg_gain = np.convolve(gains, np.ones(period) / period, mode='valid')
    avg_loss = np.convolve(losses, np.ones(period) / period, mode='valid')
    rs = avg_gain / avg_loss
    rsi = 100 - (100 / (1 + rs))
    # Pad the resulting array to match the original length
    rsi = np.pad(rsi, (values.size - rsi.size, 0), mode='constant',␣
 ↪constant_values=np.nan)
    rsi = (pd.Series(rsi)).tolist()
    rsi = [item if item is not None else 0 for item in rsi]
    return np.array(rsi)


def calculate_sma(values, period):
    """
    Calculates the Simple Moving Average (SMA) for a given list of values and␣
 ↪period.
```

```
    Args:
        values (list, np.ndarray): List or numpy array of values.
        period (int): Period for which to calculate the SMA.

    Returns:
        np.ndarray: Numpy array of SMA values.
    """
    if len(values) < period:
        raise ValueError("Number of values is less than the specified period.")

    sma_values = np.convolve(values, np.ones((period,))/period, mode='valid')
    sma_values = np.concatenate(([None]*(period-1), sma_values))
    sma_values = [item if item is not None else 0 for item in sma_values]
    return np.array(sma_values)

def calculate_ema(values, period):
    """
    Calculates the Exponential Moving Average (EMA) for a given list of values␣
 ↪and period.

    Args:
        values (list, np.ndarray): List or numpy array of values.
        period (int): Period for which to calculate the EMA.

    Returns:
        np.ndarray: Numpy array of EMA values.
    """
    if len(values) < period:
        raise ValueError("Number of values is less than the specified period.")

    alpha = 2 / (period + 1)
    ema_values = [None]
    for i in range(1, len(values)):
        if ema_values[-1] is None:
            ema = values[i]
        else:
            ema = alpha * values[i] + (1 - alpha) * ema_values[-1]
        ema_values.append(ema)
    ema_values = [item if item is not None else 0 for item in ema_values]
    return np.array(ema_values)
```

[ ]:
```
def zscore_with_zero_exempt(series):
    # we make the healthy assumption that the ratio of the price or moving␣
 ↪averages
    # of two stocks can never be zero
    non_zero_elements = [x for x in series if x != 0]
    mean = np.mean(non_zero_elements)
```

```
    std = np.std(non_zero_elements)

    def zscore_element(x):
        return (x - mean) / std if x != 0 else 0

    return [zscore_element(x) for x in series]

# a = [0, 0, 0, 0, 0, 1, 2, 3, 4]
# result = zscore_with_zero_exempt(a)
# print(result)
# [0, 0, 0, 0, 0, -1.3416407864998738, -0.4472135954999579, 0.4472135954999579,␣
 ↪1.3416407864998738]
```

[ ]:
```
# it is important to know the annualization for metrics computation
# this is the number of trading days in a year, and this value is not always 252

# I will be using the first year in the test data i.e, 2016
first_year_df = collated_price_df[(collated_price_df.index).year < 2017]
first_year_df = first_year_df[(first_year_df.index).year > 2015]

display(first_year_df.head(3))
display(first_year_df.tail(3))

print('Number of trading days in 2016: ', len(first_year_df['1332 JT']))
```

```
ticker        1332 JT    1605 JT    1721 JT    1801 JT    1802 JT    1803 JT  \
date
2016-01-04   639.7480  1063.3190  1476.9454  3398.5882  954.6446   852.1039
2016-01-05   600.4077  1067.2686  1481.4291  3381.1819  952.0268   860.8975
2016-01-06   610.7111  1033.0388  1470.6682  3368.1271  958.1351   858.2594

ticker         1812 JT    1925 JT    1928 JT    1963 JT  …    9502 JT  \
date                                                     …
2016-01-04   1211.5915  2896.7440  1613.9214  1678.2608  …  1402.1448
2016-01-05   1206.4358  2890.7731  1603.0083  1664.4593  …  1415.7579
2016-01-06   1201.2801  2892.4791  1598.5623  1635.9363  …  1422.7840

ticker         9503 JT    9531 JT    9532 JT    9602 JT    9613 JT    9735 JT  \
date
2016-01-04   1239.4967  2450.8489  1901.6500  3059.2217  1070.1497  7334.6265
2016-01-05   1237.7521  2458.0283  1918.1241  3031.1983  1077.6202  7308.1279
2016-01-06   1237.7521  2445.4644  1912.7812  3026.5278  1062.6792  7220.4084

ticker         9766 JT     9983 JT    9984 JT
date
2016-01-04   2602.7262  38693.3443  2898.5807
2016-01-05   2649.2699  38483.9338  2886.9728
2016-01-06   2630.6524  38236.4487  2848.2799
```

```
[3 rows x 205 columns]
```

```
ticker        1332 JT    1605 JT    1721 JT    1801 JT    1802 JT   1803 JT  \
date
2016-12-28  528.3796  1068.9215  1973.1897  3667.0835  1001.0945  968.5973
2016-12-29  517.9635  1057.7216  1958.5191  3644.7777   993.0786  959.6537
2016-12-30  532.1673  1049.2097  1963.1037  3649.2389   994.8599  956.0762

ticker         1812 JT    1925 JT    1928 JT    1963 JT  …    9502 JT  \
date                                                     …
2016-12-28  1428.9196  2809.0271  1639.4673  2016.3163  …  1495.2775
2016-12-29  1420.1317  2807.2725  1626.9268  2015.3737  …  1474.6838
2016-12-30  1421.8893  2803.7634  1626.5088  2002.1766  …  1462.1486

ticker         9503 JT    9531 JT    9532 JT    9602 JT    9613 JT    9735 JT  \
date
2016-12-28  1143.1107  2480.2394  2101.6229  3148.3175  1090.9101  8008.9860
2016-12-29  1116.9426  2458.2010  2070.1874  3119.9968  1077.6525  7952.2770
2016-12-30  1114.7619  2428.3573  2048.3192  3119.9968  1070.0767  7946.6991

ticker         9766 JT     9983 JT    9984 JT
date
2016-12-28  4496.9397  41626.1573  3827.8028
2016-12-29  4388.9756  41039.3315  3765.4512
2016-12-30  4431.2224  40240.8635  3782.5005

[3 rows x 205 columns]
```

```
Number of trading days in 2016:  245
```

[ ]:
```
# strategy performance metrics

APPROX_BDAYS_PER_MONTH = 21
APPROX_BDAYS_PER_YEAR = 245

MONTHS_PER_YEAR = 12
WEEKS_PER_YEAR = 52
QTRS_PER_YEAR = 4

DAILY = 'daily'
WEEKLY = 'weekly'
MONTHLY = 'monthly'
QUARTERLY = 'quarterly'
YEARLY = 'yearly'

ANNUALIZATION_FACTORS = {
    DAILY: APPROX_BDAYS_PER_YEAR,
```

```
    WEEKLY: WEEKS_PER_YEAR,
    MONTHLY: MONTHS_PER_YEAR,
    QUARTERLY: QTRS_PER_YEAR,
    YEARLY: 1
}


# slower numpy
nanmean = np.nanmean
nanstd = np.nanstd
nansum = np.nansum
nanmax = np.nanmax
nanmin = np.nanmin
nanargmax = np.nanargmax
nanargmin = np.nanargmin


def roll(*args, **kwargs): # Calculates a given statistic across a rolling time␣
 ↪period.

    func = kwargs.pop('function')
    window = kwargs.pop('window')
    if len(args) > 2:
        raise ValueError("Cannot pass more than 2 return sets")

    if len(args) == 2:
        if not isinstance(args[0], type(args[1])):
            raise ValueError("The two returns arguments are not the same.")

    if isinstance(args[0], np.ndarray):
        return _roll_ndarray(func, window, *args, **kwargs)
    return _roll_pandas(func, window, *args, **kwargs)


def up(returns, factor_returns, **kwargs): # Calculates a given statistic␣
 ↪filtering only positive factor return periods.

    func = kwargs.pop('function')
    returns = returns[factor_returns > 0]
    factor_returns = factor_returns[factor_returns > 0]
    return func(returns, factor_returns, **kwargs)


def down(returns, factor_returns, **kwargs): # Calculates a given statistic␣
 ↪filtering only negative factor return periods.
```

```
    func = kwargs.pop('function')
    returns = returns[factor_returns < 0]
    factor_returns = factor_returns[factor_returns < 0]
    return func(returns, factor_returns, **kwargs)


def _roll_ndarray(func, window, *args, **kwargs):
    data = []
    for i in range(window, len(args[0]) + 1):
        rets = [s[i-window:i] for s in args]
        data.append(func(*rets, **kwargs))
    return np.array(data)


def _roll_pandas(func, window, *args, **kwargs):
    data = {}
    index_values = []
    for i in range(window, len(args[0]) + 1):
        rets = [s.iloc[i-window:i] for s in args]
        index_value = args[0].index[i - 1]
        index_values.append(index_value)
        data[index_value] = func(*rets, **kwargs)
    return pd.Series(data, index=type(args[0].index)(index_values))


def get_utc_timestamp(dt):

    dt = pd.to_datetime(dt)
    try:
        dt = dt.tz_localize('UTC')
    except TypeError:
        dt = dt.tz_convert('UTC')
    return dt


def rolling_window(array, length, mutable=False):
    if not length:
        raise ValueError("Can't have 0-length window")

    orig_shape = array.shape
    if not orig_shape:
        raise IndexError("Can't restride a scalar.")
    elif orig_shape[0] < length:
        raise IndexError(
            "Can't restride array of shape {shape} with"
            " a window length of {len}".format(
                shape=orig_shape,
```

```
                len=length,
            )
        )

    num_windows = (orig_shape[0] - length + 1)
    new_shape = (num_windows, length) + orig_shape[1:]

    new_strides = (array.strides[0],) + array.strides

    out = as_strided(array, new_shape, new_strides)
    out.setflags(write=mutable)
    return out



def _adjust_returns(returns, adjustment_factor):
    if isinstance(adjustment_factor, (float, int)) and adjustment_factor == 0:
        return returns
    return returns - adjustment_factor


def annualization_factor(period, annualization):
    if annualization is None:
        try:
            factor = ANNUALIZATION_FACTORS[period]
        except KeyError:
            raise ValueError(
                "Period cannot be '{}'. "
                "Can be '{}'.".format(
                    period, "', '".join(ANNUALIZATION_FACTORS.keys())
                )
            )
    else:
        factor = annualization
    return factor


def simple_returns(prices): # Compute simple returns from a timeseries of␣
↪prices.
    if isinstance(prices, (pd.DataFrame, pd.Series)):
        out = prices.pct_change().iloc[1:]
    else:
        # Assume np.ndarray
        out = np.diff(prices, axis=0)
        np.divide(out, prices[:-1], out=out)

    return out
```

```
def cum_returns(returns, starting_value=0, out=None): # Compute cumulative␣
↪returns from simple returns.
    if len(returns) < 1:
        return returns.copy()

    nanmask = np.isnan(returns)
    if np.any(nanmask):
        returns = returns.copy()
        returns[nanmask] = 0

    allocated_output = out is None
    if allocated_output:
        out = np.empty_like(returns)

    np.add(returns, 1, out=out)
    out.cumprod(axis=0, out=out)

    if starting_value == 0:
        np.subtract(out, 1, out=out)
    else:
        np.multiply(out, starting_value, out=out)

    if allocated_output:
        if returns.ndim == 1 and isinstance(returns, pd.Series):
            out = pd.Series(out, index=returns.index)
        elif isinstance(returns, pd.DataFrame):
            out = pd.DataFrame(
                out, index=returns.index, columns=returns.columns,
            )

    return out


def cum_returns_final(returns, starting_value=0): # Compute total returns from␣
↪simple returns.
    if len(returns) == 0:
        return np.nan

    if isinstance(returns, pd.DataFrame):
        result = (returns + 1).prod()
    else:
        result = np.nanprod(returns + 1, axis=0)

    if starting_value == 0:
        result -= 1
```

```python
    else:
        result *= starting_value

    return result


def aggregate_returns(returns, convert_to): # Aggregates returns by week,␣
↪month, or year.


    def cumulate_returns(x):
        return cum_returns(x).iloc[-1]

    if convert_to == WEEKLY:
        grouping = [lambda x: x.year, lambda x: x.isocalendar()[1]]
    elif convert_to == MONTHLY:
        grouping = [lambda x: x.year, lambda x: x.month]
    elif convert_to == QUARTERLY:
        grouping = [lambda x: x.year, lambda x: int(np.ceil(x.month/3.))]
    elif convert_to == YEARLY:
        grouping = [lambda x: x.year]
    else:
        raise ValueError(
            'convert_to must be {}, {} or {}'.format(WEEKLY, MONTHLY, YEARLY)
        )

    return returns.groupby(grouping).apply(cumulate_returns)


def max_drawdown(returns, out=None): # Determines the maximum drawdown of a␣
↪strategy.
    allocated_output = out is None
    if allocated_output:
        out = np.empty(returns.shape[1:])

    returns_1d = returns.ndim == 1

    if len(returns) < 1:
        out[()] = np.nan
        if returns_1d:
            out = out.item()
        return out

    returns_array = np.asanyarray(returns)

    cumulative = np.empty(
        (returns.shape[0] + 1,) + returns.shape[1:],
```

```
        dtype='float64',
    )
    cumulative[0] = start = 100
    cum_returns(returns_array, starting_value=start, out=cumulative[1:])

    max_return = np.fmax.accumulate(cumulative, axis=0)

    nanmin((cumulative - max_return) / max_return, axis=0, out=out)
    if returns_1d:
        out = out.item()
    elif allocated_output and isinstance(returns, pd.DataFrame):
        out = pd.Series(out)

    return out


def annual_return(returns, period=DAILY, annualization=None): #Determines the␣
 ↪mean annual growth rate of returns. This is equivilent
    # to the compound annual growth rate.

    if len(returns) < 1:
        return np.nan

    ann_factor = annualization_factor(period, annualization)
    num_years = len(returns) / ann_factor
    # Pass array to ensure index -1 looks up successfully.
    ending_value = cum_returns_final(returns, starting_value=1)

    return ending_value ** (1 / num_years) - 1


def cagr(returns, period=DAILY, annualization=None):
    return annual_return(returns, period, annualization)


def annual_volatility(returns,
                      period=DAILY,
                      alpha=2.0,
                      annualization=None,
                      out=None):
    # Determines the annual volatility of a strategy.
    allocated_output = out is None
    if allocated_output:
        out = np.empty(returns.shape[1:])

    returns_1d = returns.ndim == 1
```

```
    if len(returns) < 2:
        out[()] = np.nan
        if returns_1d:
            out = out.item()
        return out

    ann_factor = annualization_factor(period, annualization)
    nanstd(returns, ddof=1, axis=0, out=out)
    out = np.multiply(out, ann_factor ** (1.0 / alpha), out=out)
    if returns_1d:
        out = out.item()
    return out


def calmar_ratio(returns, period=DAILY, annualization=None):
    max_dd = max_drawdown(returns=returns)
    if max_dd < 0:
        temp = annual_return(
            returns=returns,
            period=period,
            annualization=annualization
        ) / abs(max_dd)
    else:
        return np.nan

    if np.isinf(temp):
        return np.nan

    return temp



def sharpe_ratio(returns,
                 risk_free=0,
                 period=DAILY,
                 annualization=None,
                 out=None):
    allocated_output = out is None
    if allocated_output:
        out = np.empty(returns.shape[1:])

    return_1d = returns.ndim == 1

    if len(returns) < 2:
        out[()] = np.nan
        if return_1d:
            out = out.item()
```

```
        return out

    returns_risk_adj = np.asanyarray(_adjust_returns(returns, risk_free))
    ann_factor = annualization_factor(period, annualization)

    np.multiply(
        np.divide(
            nanmean(returns_risk_adj, axis=0),
            nanstd(returns_risk_adj, ddof=1, axis=0),
            out=out,
        ),
        np.sqrt(ann_factor),
        out=out,
    )
    if return_1d:
        out = out.item()

    return out


def sortino_ratio(returns,
                  required_return=0,
                  period=DAILY,
                  annualization=None,
                  out=None,
                  _downside_risk=None):

    allocated_output = out is None
    if allocated_output:
        out = np.empty(returns.shape[1:])

    return_1d = returns.ndim == 1

    if len(returns) < 2:
        out[()] = np.nan
        if return_1d:
            out = out.item()
        return out

    adj_returns = np.asanyarray(_adjust_returns(returns, required_return))

    ann_factor = annualization_factor(period, annualization)

    average_annual_return = nanmean(adj_returns, axis=0) * ann_factor
    annualized_downside_risk = (
        _downside_risk
        if _downside_risk is not None else
```

```
        downside_risk(returns, required_return, period, annualization)
    )
    np.divide(average_annual_return, annualized_downside_risk, out=out)
    if return_1d:
        out = out.item()
    elif isinstance(returns, pd.DataFrame):
        out = pd.Series(out)

    return out


def downside_risk(returns,
                  required_return=0,
                  period=DAILY,
                  annualization=None,
                  out=None):
    # Determines the downside deviation below a threshold

    allocated_output = out is None
    if allocated_output:
        out = np.empty(returns.shape[1:])

    returns_1d = returns.ndim == 1

    if len(returns) < 1:
        out[()] = np.nan
        if returns_1d:
            out = out.item()
        return out

    ann_factor = annualization_factor(period, annualization)

    downside_diff = np.clip(
        _adjust_returns(
            np.asanyarray(returns),
            np.asanyarray(required_return),
        ),
        np.NINF,
        0,
    )

    np.square(downside_diff, out=downside_diff)
    nanmean(downside_diff, axis=0, out=out)
    np.sqrt(out, out=out)
    np.multiply(out, np.sqrt(ann_factor), out=out)

    if returns_1d:
```

```
        out = out.item()
    elif isinstance(returns, pd.DataFrame):
        out = pd.Series(out, index=returns.columns)
    return out


SIMPLE_STAT_FUNCS = [
    cum_returns_final,
    annual_return,
    annual_volatility,
    sharpe_ratio,
    calmar_ratio,
    max_drawdown,
    sortino_ratio,
    stats.skew,
    stats.kurtosis,
    cagr
]
```

[ ]:

```
class Backtesting:
    def __init__(self, data, asset1,␣
↪asset2,allocation,signal_effectiveness_metric='hit_rate',use_percentile=False):
↪
        self.data = data # data = (train_data, test_data)
        self.train_data = data['train_data']
        self.test_data = data['test_data']
        # self.whole_data = data['whole_data']

        self.asset1 = asset1
        self.asset2 = asset2
        self.allocation = allocation
        self.signal_effectiveness_metric = signal_effectiveness_metric

        self.portfolio = pd.DataFrame()
        self.portfolio['asset1'] = self.test_data[self.asset1]
        self.portfolio['asset2'] = self.test_data[self.asset2]

        self.use_percentile = use_percentile

    def extract_features(self,data):
        # metric is a dataframe containing all the metrics at each time step
        sma_5_ratio = calculate_sma(data['asset1'], period=5) /␣
↪calculate_sma(data['asset2'], period=5)
        sma_10_ratio = calculate_sma(data['asset1'], period=10) /␣
↪calculate_sma(data['asset2'], period=10)
        sma_30_ratio = calculate_sma(data['asset1'], period=30) /␣
↪calculate_sma(data['asset2'], period=30)
```

```
        sma_50_ratio = calculate_sma(data['asset1'], period=50) /␣
↪calculate_sma(data['asset2'], period=50)
        ema_5_ratio = calculate_ema(data['asset1'], period=5) /␣
↪calculate_ema(data['asset2'], period=5)
        ema_10_ratio = calculate_ema(data['asset1'], period=10) /␣
↪calculate_ema(data['asset2'], period=10)
        ema_30_ratio = calculate_ema(data['asset1'], period=30) /␣
↪calculate_ema(data['asset2'], period=30)
        ema_50_ratio = calculate_ema(data['asset1'], period=50) /␣
↪calculate_ema(data['asset2'], period=50)
        ema_95_ratio = calculate_ema(data['asset1'], period=95) /␣
↪calculate_ema(data['asset2'], period=95)
        rsi_14_ratio = calculate_rsi(data['asset1'], period=14) /␣
↪calculate_rsi(data['asset1'], period=14)

        # clean the extracts
        sma_5_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in sma_5_ratio]
        sma_10_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in sma_10_ratio]
        sma_30_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in sma_30_ratio]
        sma_50_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in sma_50_ratio]
        ema_5_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in ema_5_ratio]
        ema_10_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in ema_10_ratio]
        ema_30_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in ema_30_ratio]
        ema_50_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in ema_50_ratio]
        ema_95_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in ema_95_ratio]
        rsi_14_ratio = [item if isinstance(item, (int, float)) and not np.
↪isnan(item) else 0 for item in rsi_14_ratio]

        data['price_ratio'] = data['asset1']/data['asset2']
        data['sma_5_ratio'] =  sma_5_ratio
        data['sma_10_ratio'] =  sma_10_ratio
        data['sma_30_ratio'] = sma_30_ratio
        data['sma_50_ratio'] =  sma_50_ratio
        data['ema_5_ratio'] =  ema_5_ratio
        data['ema_10_ratio'] =  ema_10_ratio
        data['ema_30_ratio'] = ema_30_ratio
        data['ema_50_ratio'] =  ema_50_ratio
```

```
        data['ema_95_ratio'] = ema_95_ratio
        data['rsi_14_ratio'] = rsi_14_ratio

        return data

    def estimate_signal_feature_importance(self,feature_name):
        # access the feature from self.feature
        feature_data = self.features[feature_name]
        # get z score of feature to create signal
        feature_zscore = zscore_with_zero_exempt(feature_data)
        feature_zscore_without_zeroes = [y for y in feature_zscore if y != 0]
        # create signal based on the z score
        if self.use_percentile == True:
            z_upper_limit = np.percentile(feature_zscore_without_zeroes, 75) #␣
↪single value that can be shared for test data
            z_lower_limit = np.percentile(feature_zscore_without_zeroes, 25)
        else:
            z_upper_limit = np.mean(feature_zscore_without_zeroes) + np.
↪std(feature_zscore_without_zeroes) # single value that can be shared for␣
↪test data
            z_lower_limit = np.mean(feature_zscore_without_zeroes) - np.
↪std(feature_zscore_without_zeroes)

        signals1 = np.select([feature_zscore> \
                                              z_upper_limit, feature_zscore <␣
↪z_lower_limit], [-1, 1], default=0)

        actual_return1 = self.features['asset1'].pct_change()

        # also evaluate for signal2 and compute average of the two as the total␣
↪signal metric
        signals2 = -signals1
        actual_return2 = self.features['asset2'].pct_change()


        # evaluate the performance
        computed_signal_metric = 0
        if self.signal_effectiveness_metric == 'hit_rate':
            hit_rate_1 = compute_hit_rate(signals1, actual_return1)
            hit_rate_2 = compute_hit_rate(signals2, actual_return2)
            computed_signal_metric = (hit_rate_1 + hit_rate_2) / 2

        elif self.signal_effectiveness_metric == 'profitability':
            profitability_1 = compute_profitability(signals1, actual_return1)
            profitability_2 = compute_profitability(signals2, actual_return2)
            computed_signal_metric = (profitability_1 + profitability_2) / 2
```

```
        elif self.signal_effectiveness_metric == 'risk_reward':
            risk_reward_ratio_1 = compute_risk_reward_ratio(signals1,␣
↪actual_return1)
            risk_reward_ratio_2 = compute_risk_reward_ratio(signals2,␣
↪actual_return2)
            computed_signal_metric = (risk_reward_ratio_1 +␣
↪risk_reward_ratio_2) / 2

        elif self.signal_effectiveness_metric == 'maximum_drawdown':
            maximum_drawdown_1 = compute_maximum_drawdown(signals1,␣
↪actual_return1)
            maximum_drawdown_2 = compute_maximum_drawdown(signals2,␣
↪actual_return2)
            computed_signal_metric = (maximum_drawdown_1 + maximum_drawdown_2) /
↪ 2

        elif self.signal_effectiveness_metric == 'win_rate':
            win_rate_1 = compute_win_rate(signals1, actual_return1)
            win_rate_2 = compute_win_rate(signals2, actual_return2)
            computed_signal_metric = (win_rate_1 + win_rate_2) / 2

        return computed_signal_metric,signals1



    def generate_trading_signal(self):
        # create a dataframe for trading signals

        # signals should be made on training data

        self.signals = pd.DataFrame()
        self.signals['asset1'] = self.train_data[self.asset1]
        self.signals['asset2'] = self.train_data[self.asset2]

        # try to optimize signal generation here
        features = pd.DataFrame()
        features['asset1'] = self.signals['asset1']
        features['asset2'] = self.signals['asset2']
        features = self.extract_features(features)
        # self.features = features.drop(columns=['asset1', 'asset2'])
        self.features = features
        # print(features.head(10))

        # next, be able to select the feature that optimizes the chosen␣
↪objective
        metric_dict = {}
```

```
        signal1_dict = {}
        ratio_features = [x for x in self.features.columns if x not in␣
↪['asset1', 'asset2']]


        for ratio in ratio_features:
            importance,signal1 = self.estimate_signal_feature_importance(ratio)
            metric_dict[ratio] = importance
            signal1_dict[ratio] = signal1


        metric_dict = {key: value for key, value in metric_dict.items() if not␣
↪np.isinf(value)}
        metric_dict = {key: value for key, value in metric_dict.items() if␣
↪value != 0.0}




        print(metric_dict)

        # select the feature with the highest importance
        self.optimal_feature = max(metric_dict, key=metric_dict.get)

        print(self.optimal_feature)

        # fetch the ratio from the features dataframe
        ratios = self.features[self.optimal_feature]



        # ratios = self.signals.asset1 / self.signals.asset2
        self.train_input = ratios



        # build model based on the ratio and the z scores

        # Create the Random Forest Regressor model
        self.model = RandomForestRegressor(n_estimators=100, random_state=42)

        # print(list(signal1_dict[self.optimal_feature])) # debug

        # calculate z-scores
        self.signals['z'] = zscore_with_zero_exempt(ratios)
```

```
        train_x = self.train_input
        train_y = self.signals['z']



        # clean train_x and train_y just before training
        train_x = [x for x in train_x if x != 0]
        train_y = [y for y in train_y if y != 0]

        # print(train_y)

        # define upper and lower threshold
        if self.use_percentile == True:
            self.signals['z upper limit'] = np.percentile(train_y, 75) # single␣
↪value that can be shared for test data
            self.signals['z lower limit'] = np.percentile(train_y, 25) #␣
↪train_y is signals['z'] without the zeroes
        else:
            self.signals['z upper limit'] = np.mean(train_y) + np.std(train_y)␣
↪# single value that can be shared for test data
            self.signals['z lower limit'] = np.mean(train_y) - np.std(train_y)␣
↪# train_y is signals['z'] without the zeroes


        # Train the model on the training data
        train_x = np.array(train_x)
        train_y = np.array(train_y)

        train_x = train_x.reshape(-1,1)
        train_y = train_y.reshape(-1,1)
        self.model.fit(train_x, train_y)


        # print(self.signals['z'])# ignore the zeroes in the limits computation

        # create signal - short if z-score is greater than upper limit else long
        self.signals['signals1'] = 0
        self.signals['signals1'] = np.select([self.signals['z'] > \
                                    self.signals['z upper limit'], self.
↪signals['z'] < self.signals['z lower limit']], [-1, 1], default=0)

        # we take the first order difference to obtain portfolio position in␣
↪that stock
        self.signals['positions1'] = self.signals['signals1'].diff()
        self.signals['signals2'] = -self.signals['signals1']
        self.signals['positions2'] = self.signals['signals2'].diff()
```

```
        # print(list(self.signals['signals1']))
        # print(self.signals.head(10))
        # print(self.signals.tail(10))

    def plot_signals(self,chart_name = ""):
        # visualize trading signals and position
        # use signals for training data
        # use positions for testing data

        fig=plt.figure(figsize=(14,6))
        bx = fig.add_subplot(111)
        bx2 = bx.twinx()

        #plot two different assets
        l1, = bx.plot(self.signals['asset1'], c='#4abdac')
        l2, = bx2.plot(self.signals['asset2'], c='#907163')
        u1, = bx.plot(self.signals['asset1'][self.signals['positions1'] == 1],␣
↪lw=0, marker='^', markersize=8, c='g',alpha=0.7)
        d1, = bx.plot(self.signals['asset1'][self.signals['positions1'] == -1],␣
↪lw=0,marker='v',markersize=8, c='r',alpha=0.7)
        u2, = bx2.plot(self.signals['asset2'][self.signals['positions2'] == 1],␣
↪lw=0,marker=2,markersize=9, c='g',alpha=0.9, markeredgewidth=3)
        d2, = bx2.plot(self.signals['asset2'][self.signals['positions2'] ==␣
↪-1], lw=0,marker=3,markersize=9, c='r',alpha=0.9,markeredgewidth=3)
        bx.set_ylabel(self.asset1,)
        bx2.set_ylabel(self.asset2, rotation=270)
        bx.yaxis.labelpad=15
        bx2.yaxis.labelpad=15
        bx.set_xlabel('Date')
        bx.xaxis.labelpad=15
        plt.legend([l1,l2,u1,d1,u2,d2], [self.asset1, self.asset2,'LONG {}'.
↪format(self.asset1),
                'SHORT {}'.format(self.asset1),
                'LONG {}'.format(self.asset2),
                'SHORT {}'.format(self.asset2)], loc ='best')
        plt.title('Pair Trading')
        plt.xlabel('Date')
        plt.grid(True)

        plt.tight_layout()
        if chart_name == '':
            plt.savefig('results/signal/'+str(self.asset1)+'_'+str(self.
↪asset2)+'_signal_chart', dpi=300)
        else:
            plt.savefig(chart_name, dpi=300)
```

```
    def evaluate_signals(self):
        # calculate actual return for both stocks
        self.signals['actual_return1'] = self.signals['asset1'].pct_change()
        self.signals['actual_return2'] = self.signals['asset2'].pct_change()

        print("=================== Evaluation of signal for "+str(self.asset1)␣
↪+ "    =================")
        hit_rate = compute_hit_rate(self.signals['signals1'], self.
↪signals['actual_return1'])
        profitability = compute_profitability(self.signals['signals1'], self.
↪signals['actual_return1'])
        risk_reward_ratio = compute_risk_reward_ratio(self.signals['signals1'],␣
↪self.signals['actual_return1'])

        maximum_drawdown = compute_maximum_drawdown(self.signals['signals1'],␣
↪self.signals['actual_return1'])
        win_rate = compute_win_rate(self.signals['signals1'], self.
↪signals['actual_return1'])

        print("Hit Rate:", hit_rate)
        print("Profitability:", profitability)
        print("Risk-to-Reward Ratio:", risk_reward_ratio)

        print("Maximum Drawdown:", maximum_drawdown)
        print("Win Rate:", win_rate)

        print('\n')

        print("=================== Evaluation of signal for "+str(self.asset2)␣
↪+ "    =================")
        hit_rate = compute_hit_rate(self.signals['signals2'], self.
↪signals['actual_return2'])
        profitability = compute_profitability(self.signals['signals2'], self.
↪signals['actual_return2'])
        risk_reward_ratio = compute_risk_reward_ratio(self.signals['signals2'],␣
↪self.signals['actual_return2'])

        maximum_drawdown = compute_maximum_drawdown(self.signals['signals2'],␣
↪self.signals['actual_return2'])
        win_rate = compute_win_rate(self.signals['signals2'], self.
↪signals['actual_return2'])

        print("Hit Rate:", hit_rate)
        print("Profitability:", profitability)
        print("Risk-to-Reward Ratio:", risk_reward_ratio)
```

```
        print("Maximum Drawdown:", maximum_drawdown)
        print("Win Rate:", win_rate)
        print('\n')



    def backtest(self):

        # use the signal model to create the signal on the test set
        # show another set of plots

        # need to adjust self.signals['positions1'] , self.
↪signals['positions2']
        # also need to set self.signals['asset1'] and self.signals['asset2'] to␣
↪the test set

        # combine both training and testing sets, then extract features on the␣
↪whole data
        # then extract the test set again

        # create a data frame with the prices of the two stocks
        new_features = pd.DataFrame()
        new_features['asset1'] = list(self.train_data[self.asset1]) + list(self.
↪test_data[self.asset1])
        new_features['asset2'] = list(self.train_data[self.asset2]) + list(self.
↪test_data[self.asset2])

        # extract features
        new_features = self.extract_features(new_features)

        # obtain the data of the optimal feature
        whole_optimal_feature_data = new_features[self.optimal_feature]
        self.test_input = whole_optimal_feature_data[-len(self.test_data[self.
↪asset1]):]


        # Make predictions on the test data
        test_x = np.array(self.test_input)
        test_x = test_x.reshape(-1, 1)
        test_y = self.model.predict(test_x)

        self.z_score_pred = test_y.tolist()

        # print(self.z_score_pred)
```

```
        self.portfolio['z'] = self.z_score_pred # this is where I will make␣
↪predictions using the model trained with the training data

        self.portfolio['z upper limit'] = (self.signals['z upper limit'])[0] #␣
↪setting the limit to any value in the upper limit column
        self.portfolio['z lower limit'] = (self.signals['z lower limit'])[0]

        # create signal - short if z-score is greater than upper limit else long
        self.portfolio['signals1'] = 0
        self.portfolio['signals1'] = np.select([self.portfolio['z'] > \
                                         self.portfolio['z upper limit'], self.
↪portfolio['z'] < self.portfolio['z lower limit']], [-1, 1], default=0)

        # we take the first order difference to obtain portfolio position in␣
↪that stock
        self.portfolio['positions1'] = self.portfolio['signals1'].diff()
        self.portfolio['signals2'] = -self.portfolio['signals1']
        self.portfolio['positions2'] = self.portfolio['signals2'].diff()

        # print(self.portfolio.head(10))

        # self.portfolio.to_csv('test_portfolio.csv')

        # initial capital to calculate the actual pnl
        initial_capital = self.allocation / 2

        # shares to buy for each position
        positions1 = initial_capital// max(self.portfolio['asset1'])
        positions2 = initial_capital// max(self.portfolio['asset2'])

        # since there are two assets, we calculate each asset Pnl
        # separately and in the end we aggregate them into one portfolio


        self.portfolio['holdings1'] = self.portfolio['positions1'].cumsum() *␣
↪self.portfolio['asset1'] * positions1
        self.portfolio['cash1'] = initial_capital - (self.
↪portfolio['positions1'] * self.portfolio['asset1'] * positions1).cumsum()
        self.portfolio['total asset1'] = self.portfolio['holdings1'] + self.
↪portfolio['cash1']
        self.portfolio['return1'] = self.portfolio['total asset1'].pct_change()

        # pnl for the 2nd asset
        self.portfolio['holdings2'] = self.portfolio['positions2'].cumsum() *␣
↪self.portfolio['asset2'] * positions2
```

```
        self.portfolio['cash2'] = initial_capital - (self.
↪portfolio['positions2'] * self.portfolio['asset2'] * positions2).cumsum()
        self.portfolio['total asset2'] = self.portfolio['holdings2'] + self.
↪portfolio['cash2']
        self.portfolio['return2'] = self.portfolio['total asset2'].pct_change()


        # total pnl and z-score
        self.portfolio['total asset'] = self.portfolio['total asset1'] + self.
↪portfolio['total asset2']
        self.portfolio = self.portfolio.dropna()



    def plot_portfolio_performance(self, chart_name = ""):
        # plot the asset value change of the portfolio and pnl along with␣
↪z-score
        # first plot the signal with the test set
        # then provide the backtesting results
        # if possible, have both plots in one figure


        fig=plt.figure(figsize=(14,6))
        ax = fig.add_subplot(111)
        ax2 = ax.twinx()

        #plot two different assets
        l1, = ax.plot(self.portfolio['asset1'], c='#4abdac')
        l2, = ax2.plot(self.portfolio['asset2'], c='#907163')
        u1, = ax.plot(self.portfolio['asset1'][self.portfolio['positions1'] ==␣
↪1], lw=0, marker='^', markersize=8, c='g',alpha=0.7)
        d1, = ax.plot(self.portfolio['asset1'][self.portfolio['positions1'] ==␣
↪-1], lw=0,marker='v',markersize=8, c='r',alpha=0.7)
        u2, = ax2.plot(self.portfolio['asset2'][self.portfolio['positions2'] ==␣
↪1], lw=0,marker=2,markersize=9, c='g',alpha=0.9, markeredgewidth=3)
        d2, = ax2.plot(self.portfolio['asset2'][self.portfolio['positions2'] ==␣
↪-1], lw=0,marker=3,markersize=9, c='r',alpha=0.9,markeredgewidth=3)
        ax.set_ylabel(self.asset1,)
        ax2.set_ylabel(self.asset2, rotation=270)
        ax.yaxis.labelpad=15
        ax2.yaxis.labelpad=15
        ax.set_xlabel('Date')
        ax.xaxis.labelpad=15
        ax.legend([l1,l2,u1,d1,u2,d2], [self.asset1, self.asset2,'LONG {}'.
↪format(self.asset1),
                'SHORT {}'.format(self.asset1),
```

```
                'LONG {}'.format(self.asset2),
                'SHORT {}'.format(self.asset2)], loc ='best')
        plt.title('Pair Trading')
        plt.grid(True)


        if chart_name == '':
            plt.savefig('results/backtesting/'+str(self.asset1)+'_'+str(self.
↪asset2)+'_portfolio_signal_chart', dpi=300)
        else:
            plt.savefig(chart_name, dpi=300)


        # Plot the second plot
        fig2 = plt.figure(figsize=(14,6),)
        ax3 = fig2.add_subplot(111)
        ax4 = ax3.twinx()
        l3, = ax3.plot(self.portfolio['total asset'], c='g')
        l4, = ax4.plot(self.portfolio['z'], c='black', alpha=0.3)
        b2 = ax4.fill_between(self.portfolio.index,self.portfolio['z upper␣
↪limit'],\
                        self.portfolio['z lower limit'], \
                        alpha=0.2,color='#ffb48f')
        ax3.set_ylabel('Asset Value')
        ax4.set_ylabel('Z Statistics',rotation=270)
        ax3.yaxis.labelpad=15
        ax4.yaxis.labelpad=15
        ax3.set_xlabel('Date')
        ax3.xaxis.labelpad=15
        plt.title('Portfolio Performance with Profit and Loss')

        # ax3.legend([l2,b,l1],['Z Statistics',
        #                       'Z Statistics +-1 Sigma',
        #                       'Total Portfolio Value'],loc='upper left')

        # Show both plots
        # plt.show()

        # end here␣
↪----------------------------------------------------------------

        # fig = plt.figure(figsize=(14,6),)
        # ax = fig.add_subplot(111)
        # ax2 = ax.twinx()
        # l1, = ax.plot(self.portfolio['total asset'], c='g')
        # l2, = ax2.plot(self.portfolio['z'], c='black', alpha=0.3)
```

```
        # b = ax2.fill_between(self.portfolio.index,self.portfolio['z upper␣
↪limit'],\
        #                  self.portfolio['z lower limit'], \
        #                  alpha=0.2,color='#ffb48f')
        # ax.set_ylabel('Asset Value')
        # ax2.set_ylabel('Z Statistics',rotation=270)
        # ax.yaxis.labelpad=15
        # ax2.yaxis.labelpad=15
        # ax.set_xlabel('Date')
        # ax.xaxis.labelpad=15
        # plt.title('Portfolio Performance with Profit and Loss')
        # plt.legend([l2,b,l1],['Z Statistics',
        #                      'Z Statistics +-1 Sigma',
        #                      'Total Portfolio Value'],loc='upper left')

        if chart_name == '':
            plt.savefig('results/backtesting/'+str(self.asset1)+'_'+str(self.
↪asset2)+'_portfolio_perf_chart', dpi=300)
        else:
            plt.savefig(chart_name, dpi=300)

    def obtain_performance(self):
        # calculate CAGR
        final_portfolio = self.portfolio['total asset'].iloc[-1]
        # delta = (self.portfolio.index[-1] - self.portfolio.index[0]).days
        # print('Number of days = ', delta)


        total_returns = self.portfolio['total asset'].pct_change()
        self.total_returns = total_returns
        self.total_assets_value = final_portfolio

        mdd = max_drawdown(total_returns) # I will need to convert to␣
↪percentage myself
        shp_rat = sharpe_ratio(total_returns, annualization = 252)
        ann_vol = annual_volatility(total_returns)
        sortino = sortino_ratio(total_returns)
        cagr_estimate = cagr(total_returns)


        print("=======Performance Metrics ("+str(self.asset1)+'_'+str(self.
↪asset2)+'_pair)'+"=======")
        print('Compound Annual Growth Rate: {0: .4%}'.format(cagr_estimate))
        print('Maximum Drawdown: {0: .4%}'.format(mdd))
        print('Annual Volatility: {0: .4%}'.format(ann_vol))
        print('Sharpe Ratio: ', shp_rat)
        print('Sortino Ratio: ', sortino)
```

```
        print('\n')
        print('\n')
```

```
[ ]: # loop through all_pairs

# use the beta information to compute portfolio weights

Total_capital = 1000000

for idx, pair in enumerate(all_pairs):
    stock1 = pair[0]
    stock2 = pair[1]

    pair_capital = Total_capital / len(all_pairs) # equal distribution of␣
↪capital among pairs
    filtered_closing_data = {'train_data' : train_collated_price_df,␣
↪'test_data' : test_collated_price_df}

    model = Backtesting(filtered_closing_data, stock1, stock2,allocation =␣
↪pair_capital,signal_effectiveness_metric='risk_reward',use_percentile=True)
    model.generate_trading_signal()
    model.evaluate_signals()
    model.plot_signals()
    model.backtest()
    model.plot_portfolio_performance()
    model.obtain_performance()


    # collate total portfolio value
    if idx == 0: # first iteration
        total_portfolio = model.portfolio['total asset']
    else:
        total_portfolio += model.portfolio['total asset']


# compute overall porfolio performance and give metrics

final_portfolio = total_portfolio.iloc[-1]

total_returns = total_portfolio.pct_change()
total_assets_value = final_portfolio

mdd = max_drawdown(total_returns) # I will need to convert to percentage myself
shp_rat = sharpe_ratio(total_returns, annualization = 245) # 245 for this data
ann_vol = annual_volatility(total_returns)
```

```
sortino = sortino_ratio(total_returns)
cagr_estimate = cagr(total_returns)


print("=====================  Overall Performance Metrics ␣
↪=======================")
print('Final portfolio value: {0:.2f}'.format(total_assets_value))
print('Compound Annual Growth Rate: {0: .4%}'.format(cagr_estimate))
print('Maximum Drawdown: {0: .4%}'.format(mdd))
print('Annual Volatility: {0: .4%}'.format(ann_vol))
print('Sharpe Ratio: ', shp_rat)
print('Sortino Ratio: ', sortino)
print('\n')
print('\n')
```

```
{'price_ratio': 1.0061916314083328, 'sma_5_ratio': 1.0604521683219073,
'sma_10_ratio': 1.0465787252806342, 'sma_30_ratio': 1.0281756292198412,
'sma_50_ratio': 1.0392128015793913, 'ema_5_ratio': 1.0332023116806845,
'ema_10_ratio': 1.046093447533166, 'ema_30_ratio': 1.0394700590626618,
'ema_50_ratio': 1.0348809174131988, 'ema_95_ratio': 1.0259382354154378}
sma_5_ratio
=================== Evaluation of signal for 6902 JT    =================
Hit Rate: 0.24965893587994542
Profitability: 0.14755635091729613
Risk-to-Reward Ratio: 1.0482285907263607
Maximum Drawdown: 0.4173720447679574
Win Rate: 0.24556616643929058


=================== Evaluation of signal for 7203 JT    =================
Hit Rate: 0.25102319236016374
Profitability: 0.16364209978427813
Risk-to-Reward Ratio: 1.0726757459174538
Maximum Drawdown: 0.2694225858625581
Win Rate: 0.24556616643929058


=======Performance Metrics (6902 JT_7203 JT_pair)=======
Compound Annual Growth Rate:  2.7025%
Maximum Drawdown: -8.4167%
Annual Volatility:  4.5277%
Sharpe Ratio:  0.6207746273635493
Sortino Ratio:  0.8886148070321043




{'price_ratio': 1.0338723059799562, 'sma_5_ratio': 1.0297230749051258,
```

```
'sma_10_ratio': 1.0281762127654872, 'sma_30_ratio': 1.0144379066877103,
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'ema_10_ratio': 1.0339861285467764, 'ema_30_ratio': 1.021838094292567,
'ema_50_ratio': 1.017493215102113, 'ema_95_ratio': 1.0155544403902963}
ema_10_ratio
=================== Evaluation of signal for 7911 JT    =================
Hit Rate: 0.2646657571623465
Profitability: 0.25073335615022463
Risk-to-Reward Ratio: 1.0161642047541228
Maximum Drawdown: 0.25111018475319347
Win Rate: 0.24965893587994542


=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.2551159618008186
Profitability: -0.11726868580472527
Risk-to-Reward Ratio: 1.05180805233943
Maximum Drawdown: 0.30151203304048235
Win Rate: 0.22237380627557982


=======Performance Metrics (7911 JT_9001 JT_pair)=======
Compound Annual Growth Rate:  5.0440%
Maximum Drawdown: -16.9158%
Annual Volatility:  11.8050%
Sharpe Ratio:  0.483093497529044
Sortino Ratio:  0.695147885030451




{'price_ratio': 0.9617480579595556, 'sma_5_ratio': 1.0282390212634849,
'sma_10_ratio': 1.025974936599201, 'sma_30_ratio': 1.0099985810431822,
'sma_50_ratio': 0.9597323396062423, 'ema_5_ratio': 1.0075278317500476,
'ema_10_ratio': 1.038981517707041, 'ema_30_ratio': 1.0176941809034954,
'ema_50_ratio': 1.0032210512395467, 'ema_95_ratio': 0.9547715902123641}
ema_10_ratio
=================== Evaluation of signal for 7911 JT    =================
Hit Rate: 0.27967257844474763
Profitability: 0.19318073423590398
Risk-to-Reward Ratio: 0.9442457227075586
Maximum Drawdown: 0.25111018475319347
Win Rate: 0.2592087312414734


=================== Evaluation of signal for 9005 JT    =================
Hit Rate: 0.2551159618008186
Profitability: 0.08075468682055764
```

```
Risk-to-Reward Ratio: 1.1337173127065232
Maximum Drawdown: 0.28953164394526354
Win Rate: 0.22783083219645292


=======Performance Metrics (7911 JT_9005 JT_pair)=======
Compound Annual Growth Rate:  4.4513%
Maximum Drawdown: -17.1033%
Annual Volatility:  11.2523%
Sharpe Ratio:  0.4498485732099169
Sortino Ratio:  0.6496067880190538




{'price_ratio': 0.9215260683190782, 'sma_5_ratio': 0.9670927062607875,
'sma_10_ratio': 0.9697324463311944, 'sma_30_ratio': 0.972399055964676,
'sma_50_ratio': 0.9386440772476088, 'ema_5_ratio': 0.9480276400204609,
'ema_10_ratio': 0.9639573945602737, 'ema_30_ratio': 0.981862295377423,
'ema_50_ratio': 0.9329710290187867, 'ema_95_ratio': 0.9258777540164382}
ema_30_ratio
=================== Evaluation of signal for 7912 JT    =================
Hit Rate: 0.2592087312414734
Profitability: 0.1208464871900169
Risk-to-Reward Ratio: 0.9597276476044881
Maximum Drawdown: 0.19766425992303338
Win Rate: 0.2551159618008186


=================== Evaluation of signal for 9021 JT    =================
Hit Rate: 0.24965893587994542
Profitability: -0.05689184227212707
Risk-to-Reward Ratio: 1.003996943150358
Maximum Drawdown: 0.24747223774151483
Win Rate: 0.24147339699863574


=======Performance Metrics (7912 JT_9021 JT_pair)=======
Compound Annual Growth Rate:  1.9841%
Maximum Drawdown: -8.4534%
Annual Volatility:  4.0893%
Sharpe Ratio:  0.5083115647404642
Sortino Ratio:  0.8064758342251398




{'price_ratio': 1.0318713832294284, 'sma_5_ratio': 1.054987906219237,
```

```
'sma_10_ratio': 1.0789118642826747, 'sma_30_ratio': 1.076042975909862,
'sma_50_ratio': 0.9998344541356241, 'ema_5_ratio': 1.050815056866833,
'ema_10_ratio': 1.0706132725701494, 'ema_30_ratio': 1.073000060383125,
'ema_50_ratio': 1.0650512228230045, 'ema_95_ratio': 1.0333929690436146}
sma_10_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.2728512960436562
Profitability: 0.09011513861940623
Risk-to-Reward Ratio: 1.0079705067967233
Maximum Drawdown: 0.2093500630796875
Win Rate: 0.2373806275579809


=================== Evaluation of signal for 9005 JT    =================
Hit Rate: 0.25102319236016374
Profitability: 0.21227630243403606
Risk-to-Reward Ratio: 1.1498532217686261
Maximum Drawdown: 0.34592378360841625
Win Rate: 0.22783083219645292


=======Performance Metrics (9001 JT_9005 JT_pair)=======
Compound Annual Growth Rate:  0.8367%
Maximum Drawdown: -18.7078%
Annual Volatility:  8.4054%
Sharpe Ratio:  0.14325484866119456
Sortino Ratio:  0.19931007502433853




{'price_ratio': 0.9887370052127582, 'sma_5_ratio': 1.0531023155237507,
'sma_10_ratio': 1.0438298235645176, 'sma_30_ratio': 1.028596418661827,
'sma_50_ratio': 1.0493698453676983, 'ema_5_ratio': 1.0391802176075964,
'ema_10_ratio': 1.0444626631439364, 'ema_30_ratio': 1.0333773164020037,
'ema_50_ratio': 1.05895594379157, 'ema_95_ratio': 1.085287398655287}
ema_95_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.24147339699863574
Profitability: -0.1283473790167977
Risk-to-Reward Ratio: 1.0830083252496712
Maximum Drawdown: 0.29815454522973417
Win Rate: 0.21418826739427013


=================== Evaluation of signal for 9007 JT    =================
Hit Rate: 0.2660300136425648
Profitability: 0.33509744325263546
```

```
Risk-to-Reward Ratio: 1.0875664720609026
Maximum Drawdown: 0.1761394893479038
Win Rate: 0.25375170532060026


=======Performance Metrics (9001 JT_9007 JT_pair)=======
Compound Annual Growth Rate:  3.7517%
Maximum Drawdown: -16.2715%
Annual Volatility:  8.2745%
Sharpe Ratio:  0.4936490752721409
Sortino Ratio:  0.7343348557946108




{'price_ratio': 0.9798830741860125, 'sma_5_ratio': 1.0305651902513557,
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'ema_50_ratio': 1.0683437214158733, 'ema_95_ratio': 1.0947380922514596}
ema_95_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.28240109140518416
Profitability: 0.026253102348855806
Risk-to-Reward Ratio: 0.9903496069435813
Maximum Drawdown: 0.26599690528209285
Win Rate: 0.24147339699863574


=================== Evaluation of signal for 9008 JT    =================
Hit Rate: 0.24693042291950887
Profitability: 0.11344175170461357
Risk-to-Reward Ratio: 1.1991265775593378
Maximum Drawdown: 0.25578534617116655
Win Rate: 0.22237380627557982


=======Performance Metrics (9001 JT_9008 JT_pair)=======
Compound Annual Growth Rate:  0.3554%
Maximum Drawdown: -8.6835%
Annual Volatility:  2.4604%
Sharpe Ratio:  0.1588213497731524
Sortino Ratio:  0.22692189502154597




{'price_ratio': 0.9748261237502811, 'sma_5_ratio': 1.0303192070594558,
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sma_5_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.24965893587994542
Profitability: 0.043177328870359966
Risk-to-Reward Ratio: 1.0799326266943696
Maximum Drawdown: 0.21051093858518122
Win Rate: 0.22373806275579808


=================== Evaluation of signal for 9009 JT    =================
Hit Rate: 0.27012278308321963
Profitability: 0.29770617326914306
Risk-to-Reward Ratio: 0.9807057874245421
Maximum Drawdown: 0.23920550092195936
Win Rate: 0.25648021828103684


=======Performance Metrics (9001 JT_9009 JT_pair)=======
Compound Annual Growth Rate:  1.1541%
Maximum Drawdown: -9.3058%
Annual Volatility:  5.1406%
Sharpe Ratio:  0.25269475168405237
Sortino Ratio:  0.34025161053179737




{'price_ratio': 1.0010333288968087, 'sma_5_ratio': 1.0345458516684458,
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ema_50_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.2592087312414734
Profitability: 0.032617103215418
Risk-to-Reward Ratio: 1.1076282215549724
Maximum Drawdown: 0.21878555472704975
Win Rate: 0.22510231923601637


=================== Evaluation of signal for 9020 JT    =================
Hit Rate: 0.2646657571623465
Profitability: 0.2550385990052586
```

```
Risk-to-Reward Ratio: 1.0002920680188692
Maximum Drawdown: 0.289077848361164
Win Rate: 0.25648021828103684


=======Performance Metrics (9001 JT_9020 JT_pair)=======
Compound Annual Growth Rate: -1.0299%
Maximum Drawdown: -29.5167%
Annual Volatility:  11.4370%
Sharpe Ratio:  -0.03367065535039943
Sortino Ratio:  -0.04592521038320267




{'price_ratio': 0.9802134159824969, 'sma_5_ratio': 1.0109028148249912,
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ema_30_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.26330150068212826
Profitability: -0.03943043086446063
Risk-to-Reward Ratio: 1.0342033924259326
Maximum Drawdown: 0.2220557700275435
Win Rate: 0.2291950886766712


=================== Evaluation of signal for 9021 JT    =================
Hit Rate: 0.24965893587994542
Profitability: 0.03158028708732852
Risk-to-Reward Ratio: 1.0475789548490322
Maximum Drawdown: 0.4036470942253596
Win Rate: 0.24147339699863574


=======Performance Metrics (9001 JT_9021 JT_pair)=======
Compound Annual Growth Rate:  1.9400%
Maximum Drawdown: -11.9065%
Annual Volatility:  4.5498%
Sharpe Ratio:  0.45160320314169716
Sortino Ratio:  0.7490720908490306




{'price_ratio': 1.0080723578165267, 'sma_5_ratio': 1.0578133465155823,
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ema_50_ratio
=================== Evaluation of signal for 9001 JT    =================
Hit Rate: 0.26193724420190995
Profitability: -0.1358371308735613
Risk-to-Reward Ratio: 1.0175351176107656
Maximum Drawdown: 0.3026166718781659
Win Rate: 0.22646657571623466


=================== Evaluation of signal for 9022 JT    =================
Hit Rate: 0.2673942701227831
Profitability: 0.7749456198430593
Risk-to-Reward Ratio: 1.2022662254961047
Maximum Drawdown: 0.2474511622459804
Win Rate: 0.2578444747612551


=======Performance Metrics (9001 JT_9022 JT_pair)=======
Compound Annual Growth Rate:  2.2465%
Maximum Drawdown: -5.2122%
Annual Volatility:  2.9236%
Sharpe Ratio:  0.786064700362153
Sortino Ratio:  1.349066460341659




{'price_ratio': 0.9666260352981592, 'sma_5_ratio': 1.028749237761786,
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sma_30_ratio
=================== Evaluation of signal for 9005 JT    =================
Hit Rate: 0.2673942701227831
Profitability: 0.3610060280368549
Risk-to-Reward Ratio: 1.100016632507636
Maximum Drawdown: 0.251895854718163
Win Rate: 0.23874488403819918


=================== Evaluation of signal for 9021 JT    =================
Hit Rate: 0.24010914051841747
Profitability: -0.12883980649337567
```

```
Risk-to-Reward Ratio: 0.972946051314056
Maximum Drawdown: 0.4047556357962355
Win Rate: 0.23192360163710776


=======Performance Metrics (9005 JT_9021 JT_pair)=======
Compound Annual Growth Rate:  2.0771%
Maximum Drawdown: -4.2722%
Annual Volatility:  3.3981%
Sharpe Ratio:  0.6313041954469241
Sortino Ratio:  0.9211761008553473




{'price_ratio': 1.0487346113014502, 'sma_5_ratio': 1.0943497965599285,
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sma_10_ratio
=================== Evaluation of signal for 9009 JT    =================
Hit Rate: 0.2605729877216917
Profitability: -0.38973489704862396
Risk-to-Reward Ratio: 0.8811803660147578
Maximum Drawdown: 0.626632292585282
Win Rate: 0.24010914051841747


=================== Evaluation of signal for 9022 JT    =================
Hit Rate: 0.2755798090040928
Profitability: 1.1916595952524673
Risk-to-Reward Ratio: 1.3531354626881116
Maximum Drawdown: 0.23618448397183012
Win Rate: 0.2646657571623465


=======Performance Metrics (9009 JT_9022 JT_pair)=======
Compound Annual Growth Rate: -0.5473%
Maximum Drawdown: -13.7744%
Annual Volatility:  5.4135%
Sharpe Ratio:  -0.07548348077721911
Sortino Ratio:  -0.10716471325099898




{'price_ratio': 0.9446836092443671, 'sma_5_ratio': 0.9938817626913453,
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ema_95_ratio
=================== Evaluation of signal for 9020 JT    =================
Hit Rate: 0.2660300136425648
Profitability: 0.2075649648155189
Risk-to-Reward Ratio: 1.0132849271729305
Maximum Drawdown: 0.2142291634451594
Win Rate: 0.2551159618008186


=================== Evaluation of signal for 9021 JT    =================
Hit Rate: 0.24556616643929058
Profitability: -0.18045045175760255
Risk-to-Reward Ratio: 0.994585658979374
Maximum Drawdown: 0.3285441682670982
Win Rate: 0.23601637107776263


=======Performance Metrics (9020 JT_9021 JT_pair)=======
Compound Annual Growth Rate:  0.0000%
Maximum Drawdown:  0.0000%
Annual Volatility:  0.0000%
Sharpe Ratio:  nan
Sortino Ratio:  nan




{'price_ratio': 0.9080284299029402, 'sma_5_ratio': 0.9452244308512727,
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ema_95_ratio
=================== Evaluation of signal for 8331 JT    =================
Hit Rate: 0.27148703956343795
Profitability: 0.2561797842923861
Risk-to-Reward Ratio: 0.9936401674723749
Maximum Drawdown: 0.3903635611985743
Win Rate: 0.2551159618008186


=================== Evaluation of signal for 8355 JT    =================
Hit Rate: 0.2605729877216917
Profitability: 0.024222184350931153
```

```
Risk-to-Reward Ratio: 0.9812941180084722
Maximum Drawdown: 0.3088794729095924
Win Rate: 0.24693042291950887


=======Performance Metrics (8331 JT_8355 JT_pair)=======
Compound Annual Growth Rate:  2.0012%
Maximum Drawdown: -3.3579%
Annual Volatility:  2.5101%
Sharpe Ratio:  0.8139298856408202
Sortino Ratio:  1.3504351307940998




{'price_ratio': 0.9413654887251532, 'sma_5_ratio': 1.0117592199026761,
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sma_30_ratio
=================== Evaluation of signal for 8630 JT    =================
Hit Rate: 0.2551159618008186
Profitability: 0.7687359610659454
Risk-to-Reward Ratio: 1.214779924252699
Maximum Drawdown: 0.22320988677730602
Win Rate: 0.24829467939972716


=================== Evaluation of signal for 8766 JT    =================
Hit Rate: 0.23328785811732605
Profitability: -0.41335211078897416
Risk-to-Reward Ratio: 0.9471306198418502
Maximum Drawdown: 0.5779918217625938
Win Rate: 0.22783083219645292


=======Performance Metrics (8630 JT_8766 JT_pair)=======
Compound Annual Growth Rate:  1.6497%
Maximum Drawdown: -11.9862%
Annual Volatility:  6.0291%
Sharpe Ratio:  0.30598160607334646
Sortino Ratio:  0.44010198931442607




{'price_ratio': 0.9625814285912294, 'sma_5_ratio': 1.0311336361244732,
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'ema_50_ratio': 0.974401021167888, 'ema_95_ratio': 1.0209158697254943}
sma_10_ratio
=================== Evaluation of signal for 8725 JT    =================
Hit Rate: 0.26330150068212826
Profitability: 0.7555632130614133
Risk-to-Reward Ratio: 1.1081572937713795
Maximum Drawdown: 0.2526161517207888
Win Rate: 0.2605729877216917


=================== Evaluation of signal for 8766 JT    =================
Hit Rate: 0.23192360163710776
Profitability: -0.42572004075942926
Risk-to-Reward Ratio: 0.9648207636297436
Maximum Drawdown: 0.523821007966769
Win Rate: 0.2305593451568895


=======Performance Metrics (8725 JT_8766 JT_pair)=======
Compound Annual Growth Rate:  1.3117%
Maximum Drawdown: -8.8925%
Annual Volatility:  4.8372%
Sharpe Ratio:  0.29795033980961855
Sortino Ratio:  0.4268425602333351




======================  Overall Performance Metrics  ========================
Final portfolio value: 1097596.52
Compound Annual Growth Rate:  1.8127%
Maximum Drawdown: -4.3362%
Annual Volatility:  2.6030%
Sharpe Ratio:  0.7037255498602681
Sortino Ratio:  1.0234233836419095
```
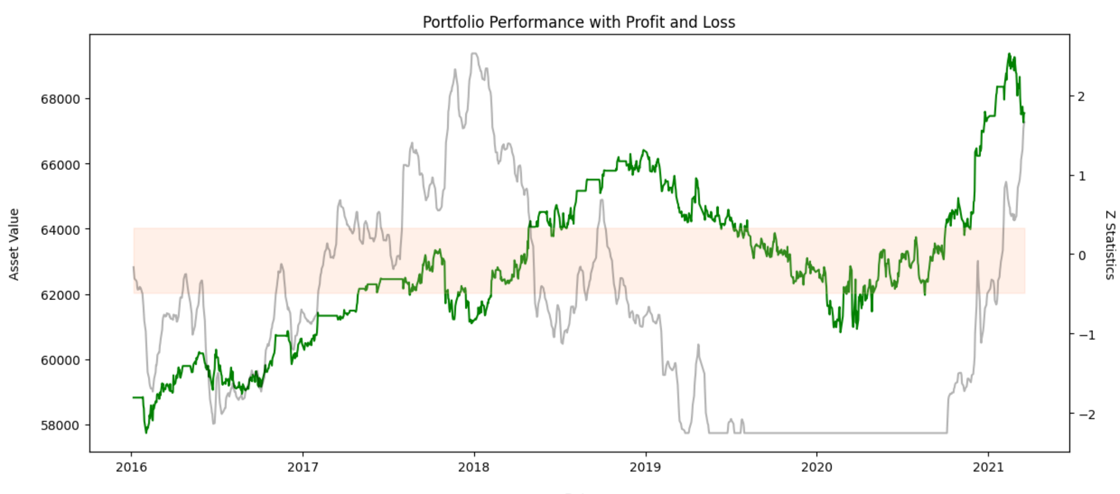
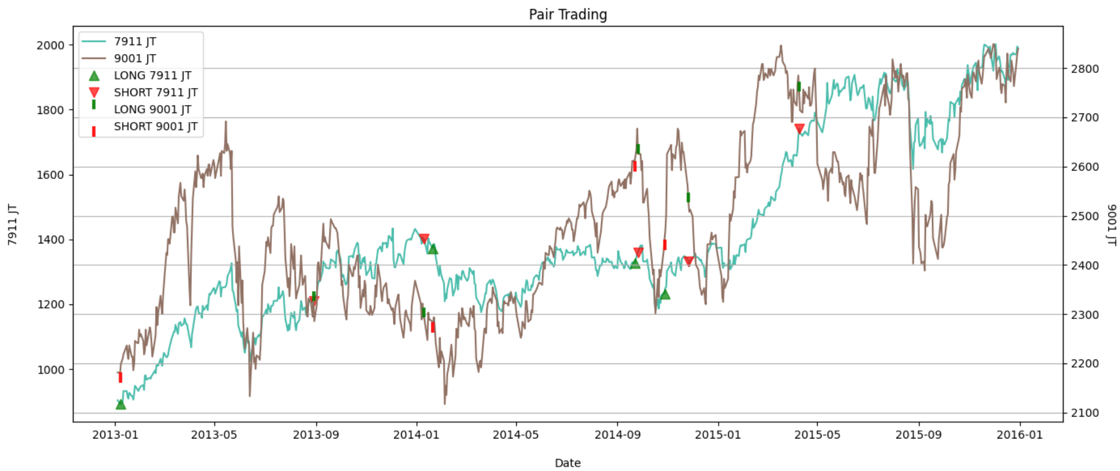
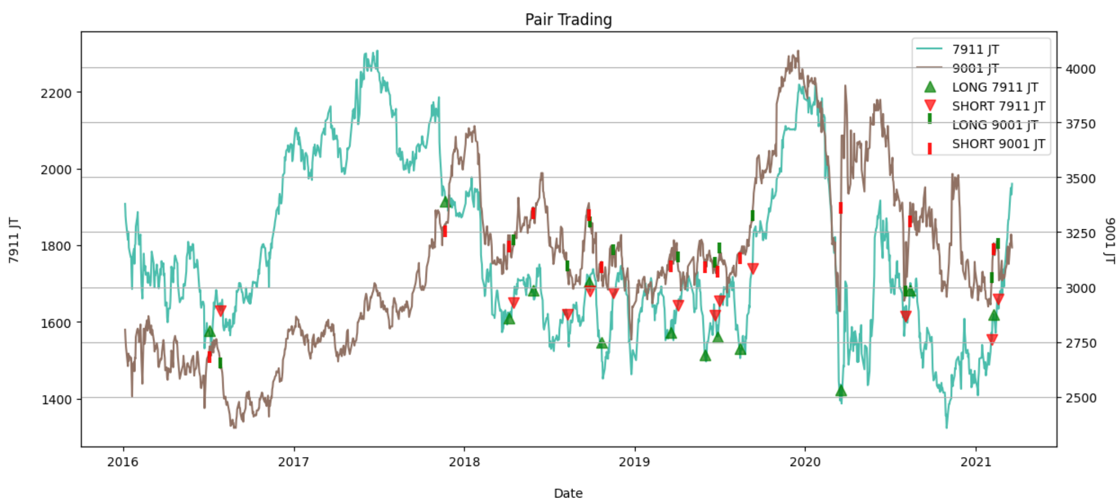


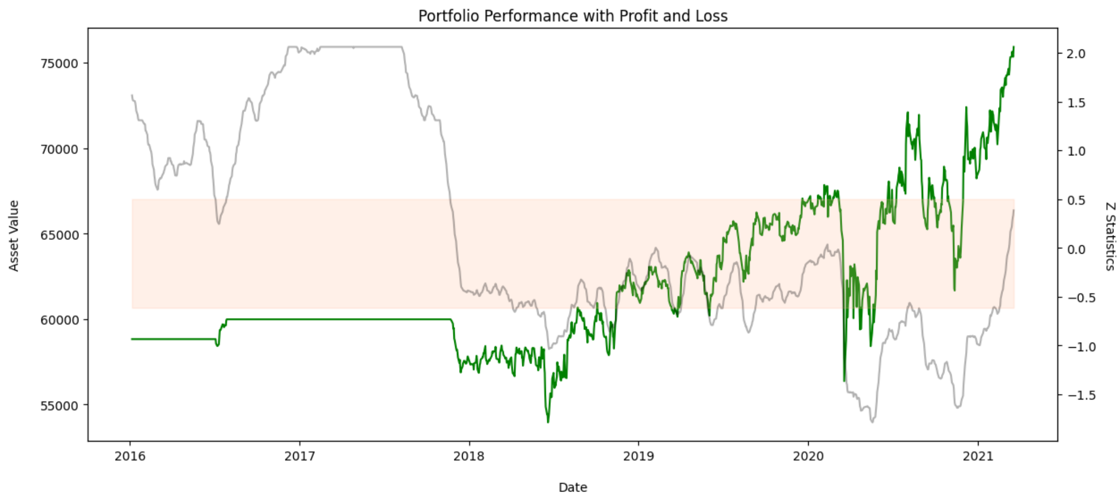
Pair Trading
6902 JT
7203 JT
LONG 6902 JT
SHORT 6902 JT
LONG 7203 JT
SHORT 7203 JT
Date


Pair Trading
6902 JT
7203 JT
LONG 6902 JT
SHORT 6902 JT
LONG 7203 JT
SHORT 7203 JT
Date


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date


Pair Trading
7911 JT
9001 JT
LONG 7911 JT
SHORT 7911 JT
LONG 9001 JT
SHORT 9001 JT
Date


Pair Trading
7911 JT
9001 JT
LONG 7911 JT
SHORT 7911 JT
LONG 9001 JT
SHORT 9001 JT
Date


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date
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70000
65000
60000
55000
2016
2017
2018
2019
2020
2021
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5


Pair Trading
7911 JT
9005 JT
LONG 7911 JT
SHORT 7911 JT
LONG 9005 JT
SHORT 9005 JT
Date
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2013-05
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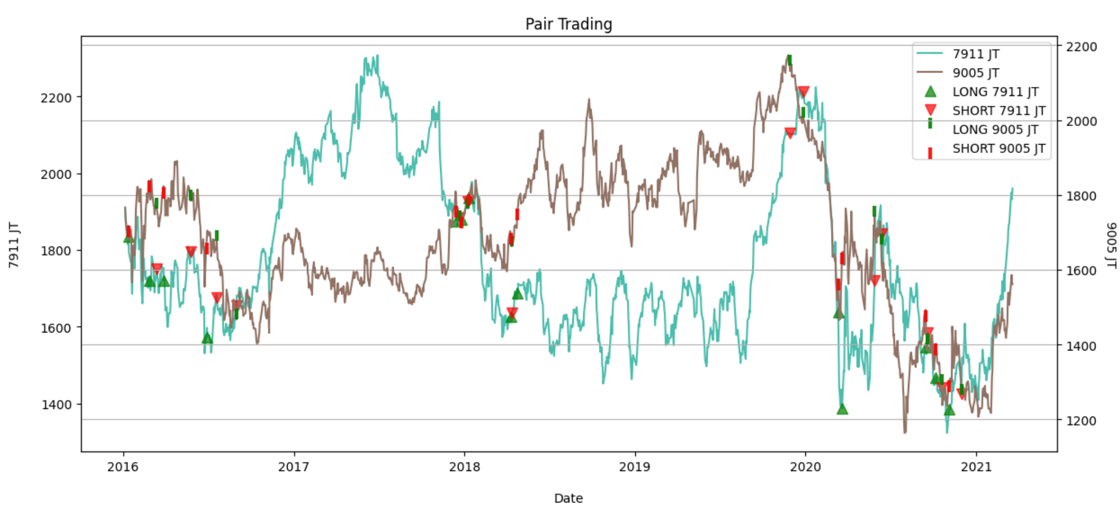
2014-09
2015-01
2015-05
2015-09
2016-01


Pair Trading
7911 JT
9005 JT
LONG 7911 JT
SHORT 7911 JT
LONG 9005 JT
SHORT 9005 JT
Date
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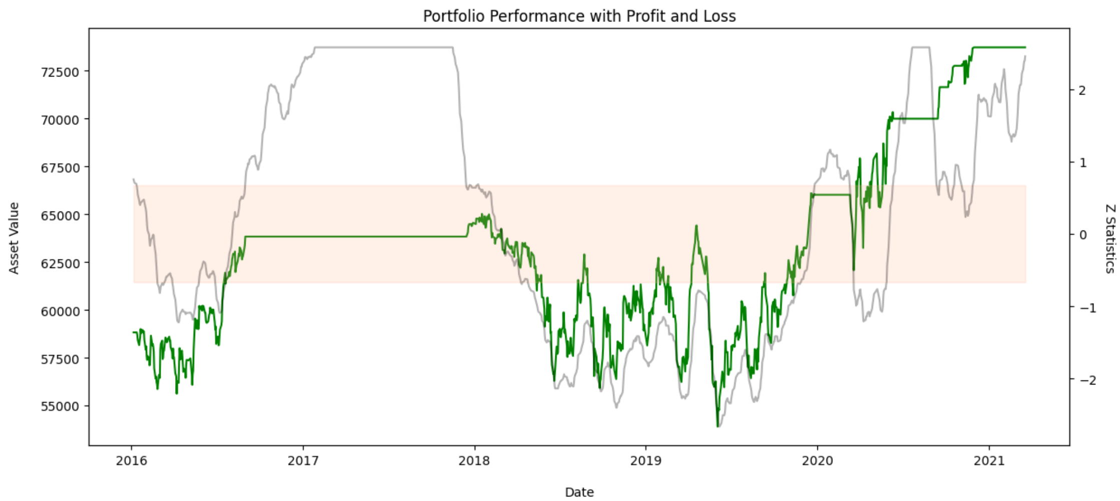
2017
2018
2019
2020
2021


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date
72500
70000
67500
65000
62500
60000
57500
55000
2016
2017
2018
2019
2020
2021
2
1
0
−1
−2


Pair Trading
7912 JT
9021 JT
LONG 7912 JT
SHORT 7912 JT
LONG 9021 JT
SHORT 9021 JT
Date
2400
2200
2000
1800
1600
1400
1200
8000
7000
6000
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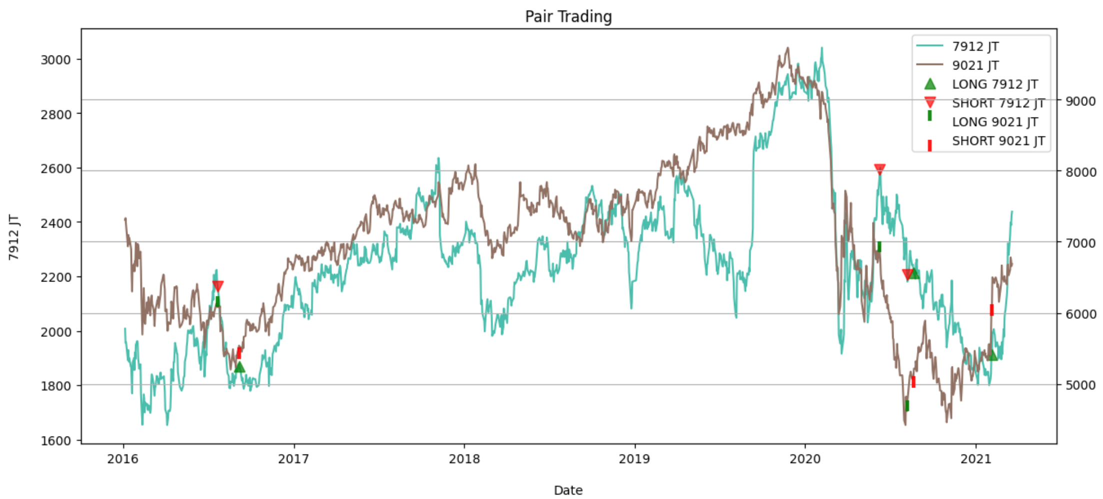
2015-05
2015-09
2016-01


Pair Trading
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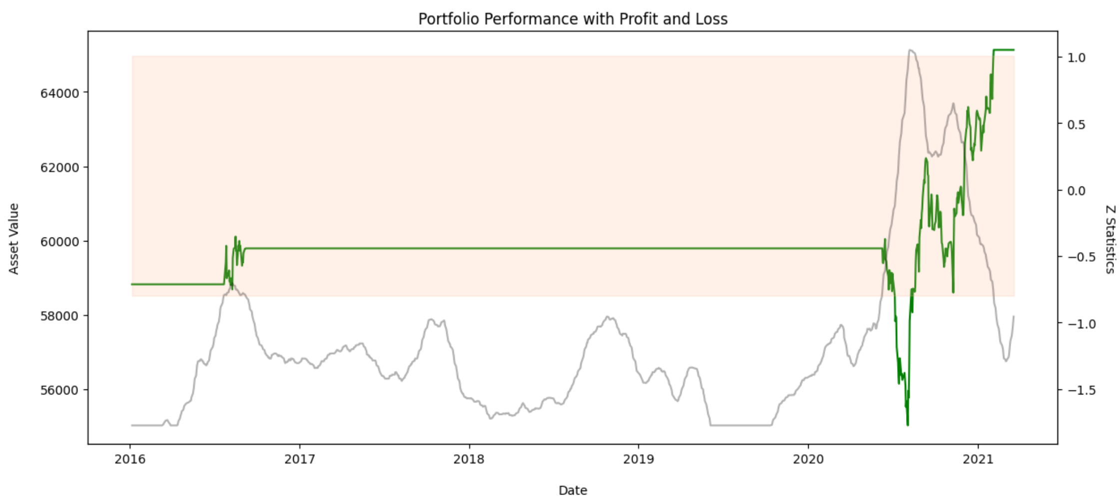
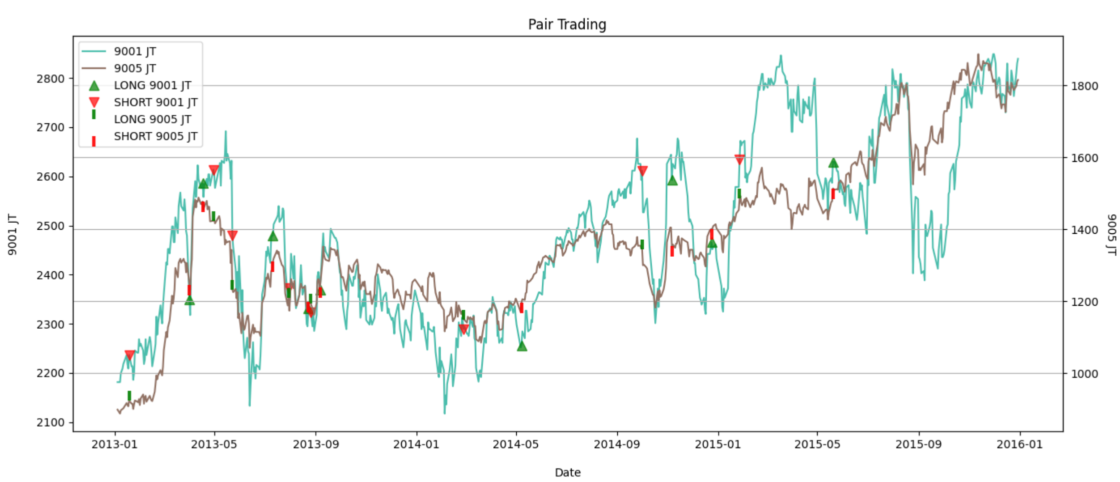
9021 JT
LONG 7912 JT
SHORT 7912 JT
LONG 9021 JT
SHORT 9021 JT
Date


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date


Pair Trading
9001 JT
9005 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9005 JT
SHORT 9005 JT
Date


Pair Trading
9001 JT
9005 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9005 JT
SHORT 9005 JT
4000
3750
3500
3250
3000
2750
2500
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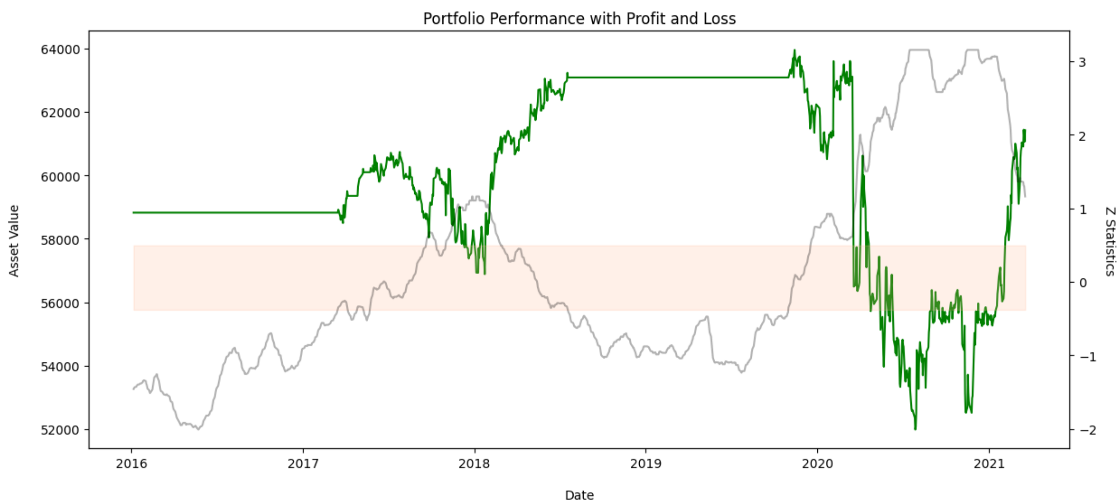
1600
1400
1200
2016
2017
2018
2019
2020
2021
Date


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
64000
62000
60000
58000
56000
54000
52000
3
2
1
0
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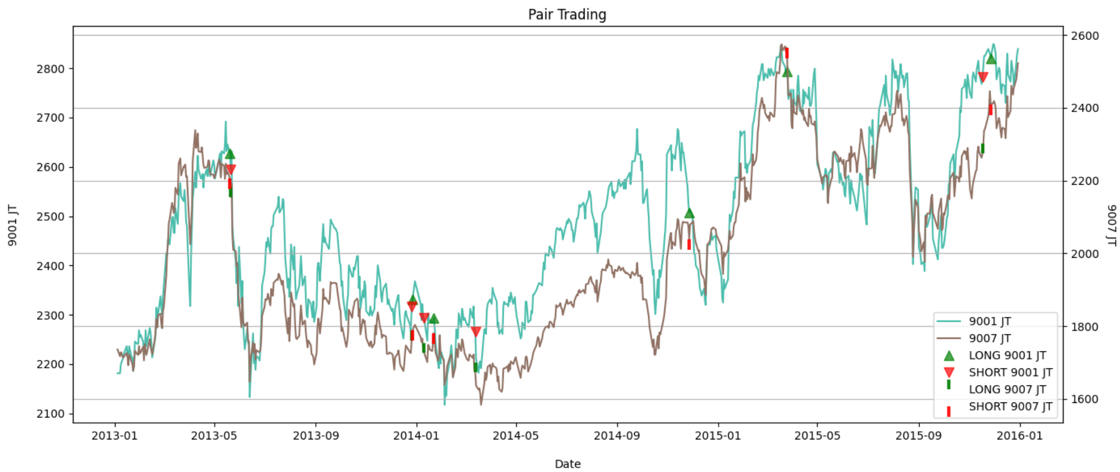
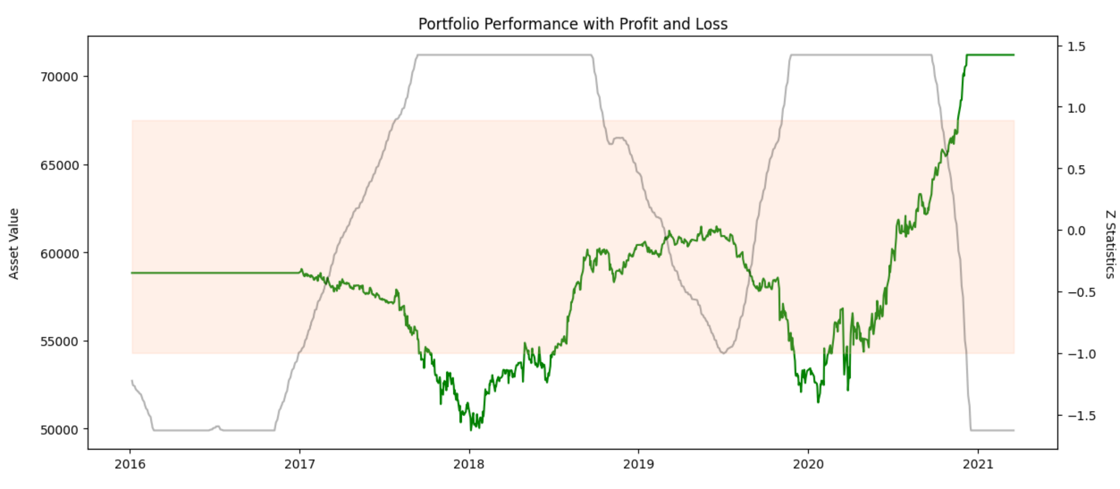
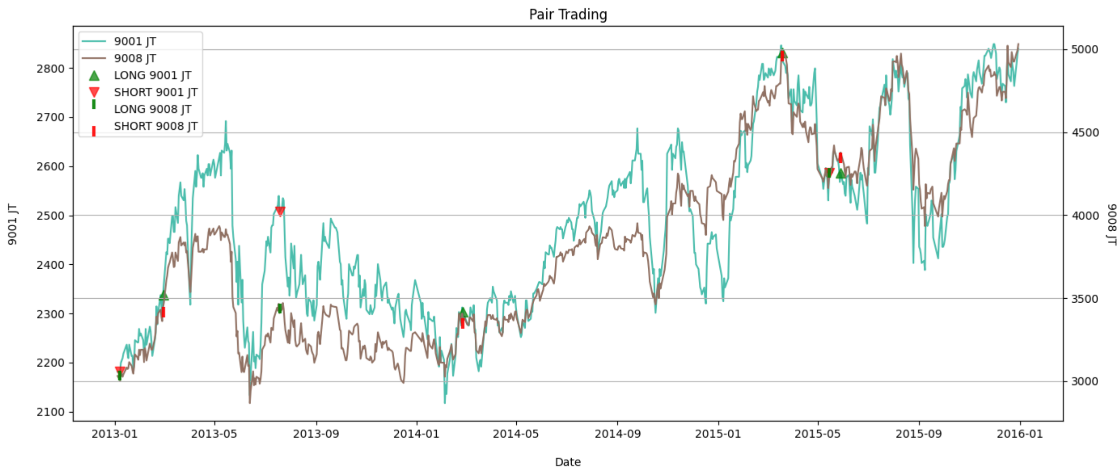
2018
2019
2020
2021
Date

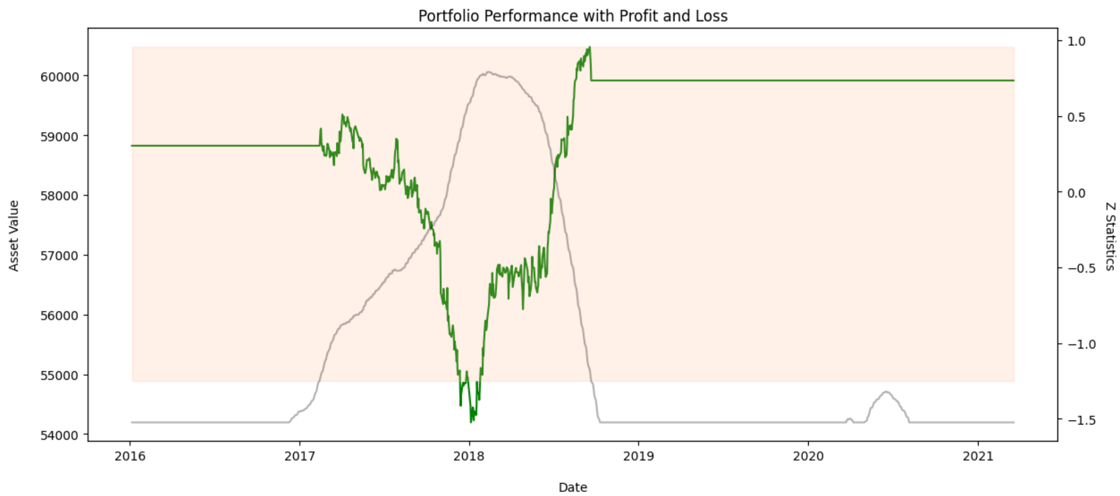

Pair Trading
9001 JT
9007 JT
Date
LONG 9001 JT
SHORT 9001 JT
LONG 9007 JT
SHORT 9007 JT


Pair Trading
9001 JT
9007 JT
Date
LONG 9001 JT
SHORT 9001 JT
LONG 9007 JT
SHORT 9007 JT


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date

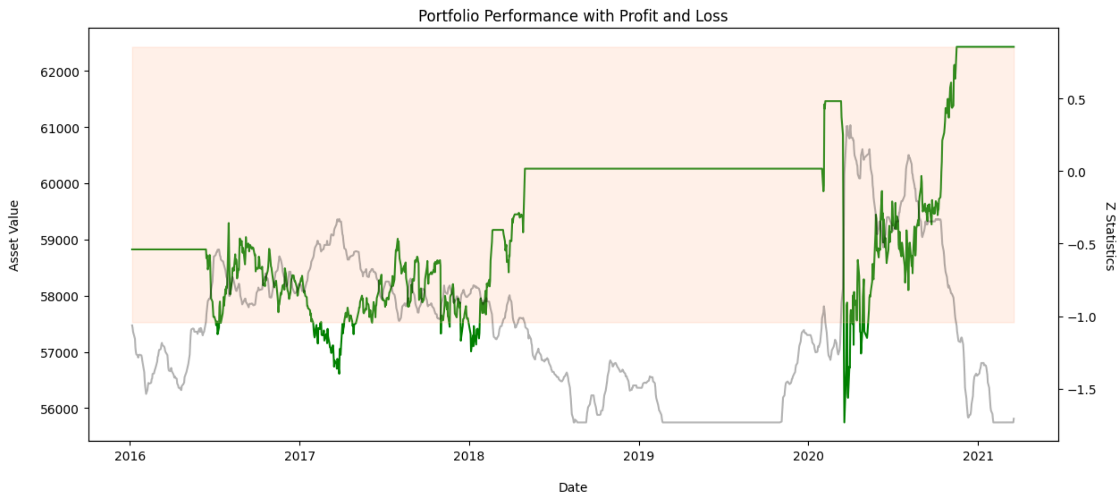

Pair Trading
9001 JT
9008 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9008 JT
SHORT 9008 JT
Date


Pair Trading
9001 JT
9008 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9008 JT
SHORT 9008 JT
Date

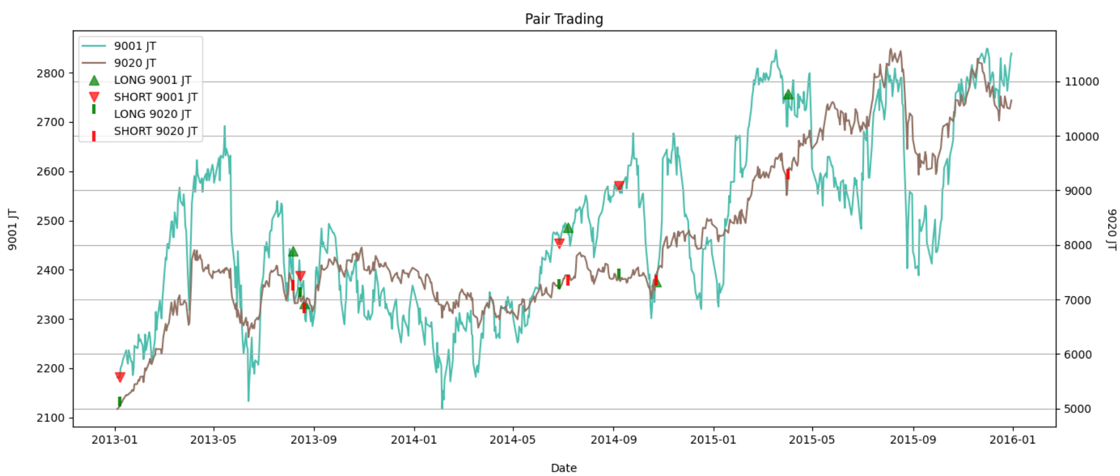

Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date


Pair Trading
9001 JT
9009 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9009 JT
SHORT 9009 JT
Date

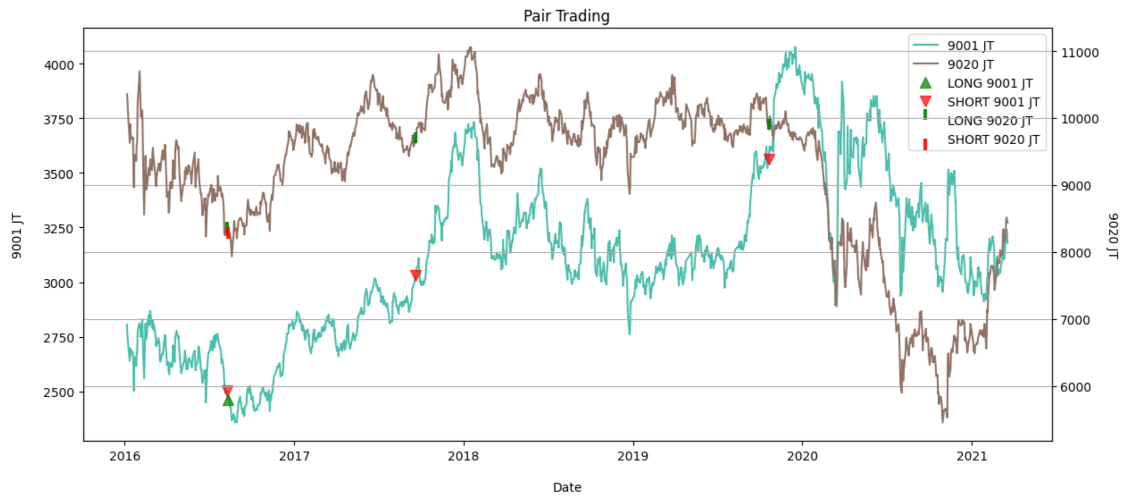

Pair Trading
9001 JT
9009 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9009 JT
SHORT 9009 JT
Date

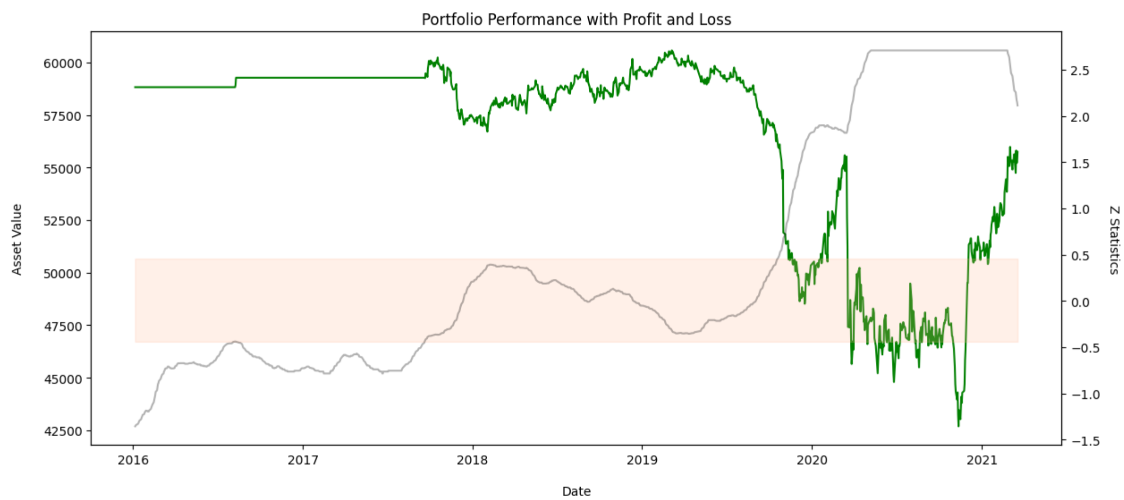


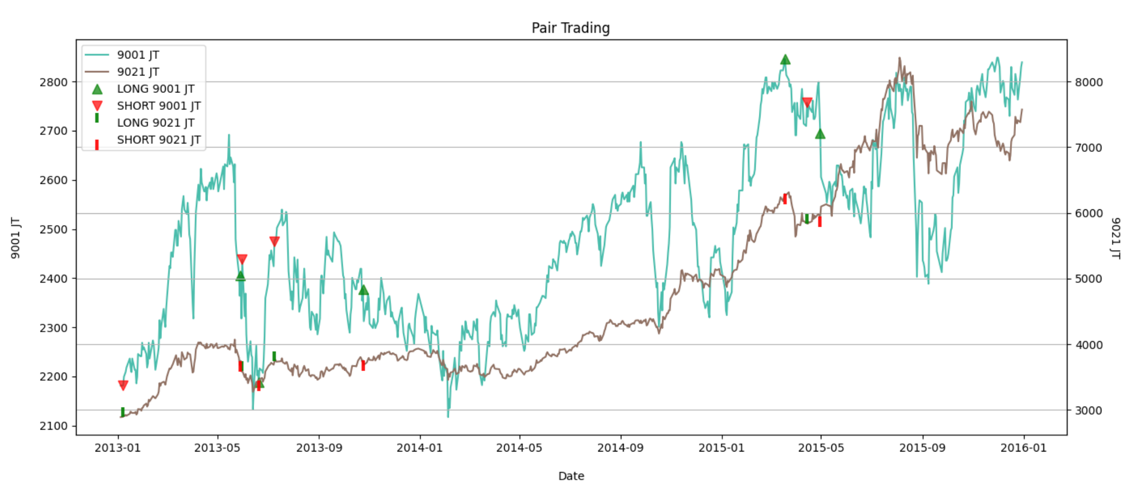
Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date
62000
61000
60000
59000
58000
57000
56000
0.5
0.0
−0.5
−1.0
−1.5
2016
2017
2018
2019
2020
2021



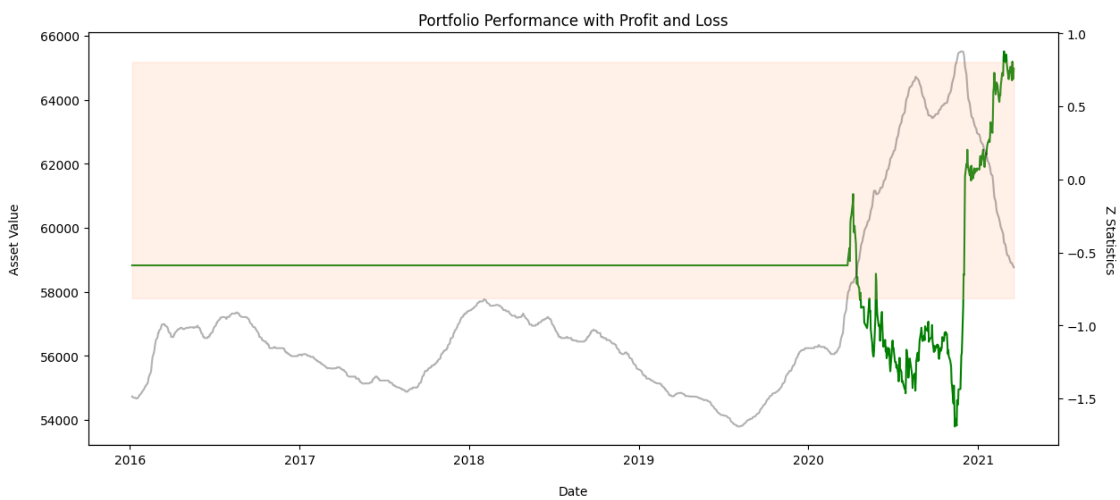
Pair Trading
9001 JT
9020 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9020 JT
SHORT 9020 JT
Date
2800
2700
2600
2500
2400
2300
2200
2100
11000
10000
9000
8000
7000
6000
5000
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01

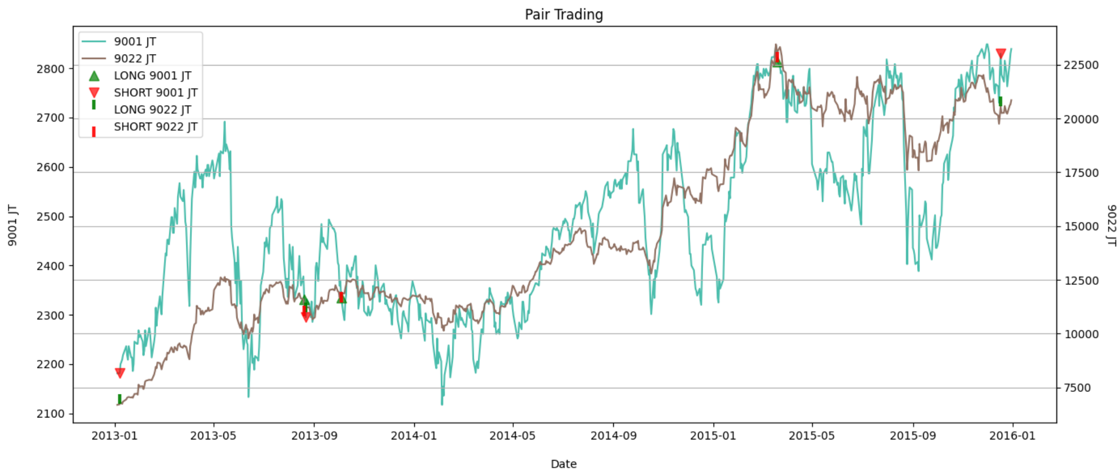

Pair Trading
9001 JT
9020 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9020 JT
SHORT 9020 JT
Date
9001 JT
9020 JT

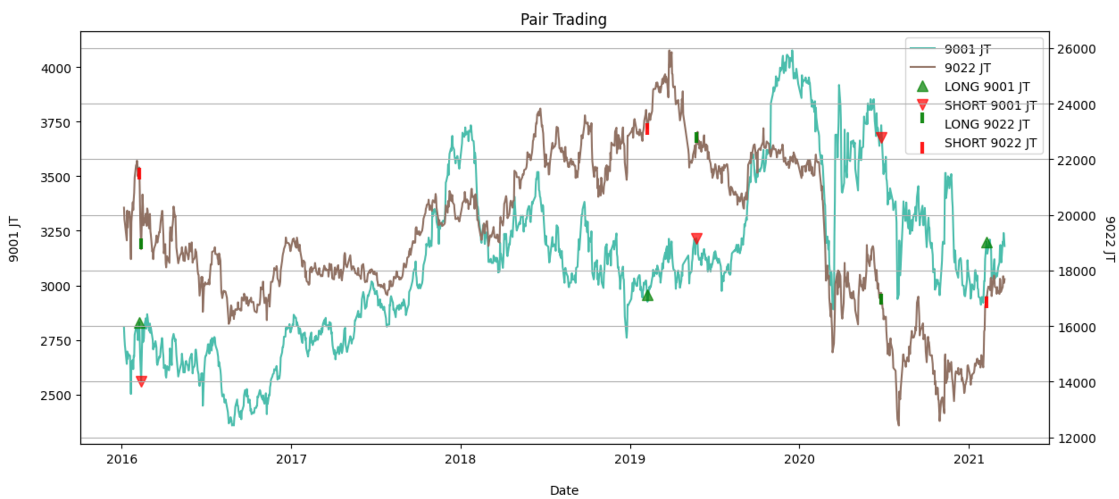

Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date

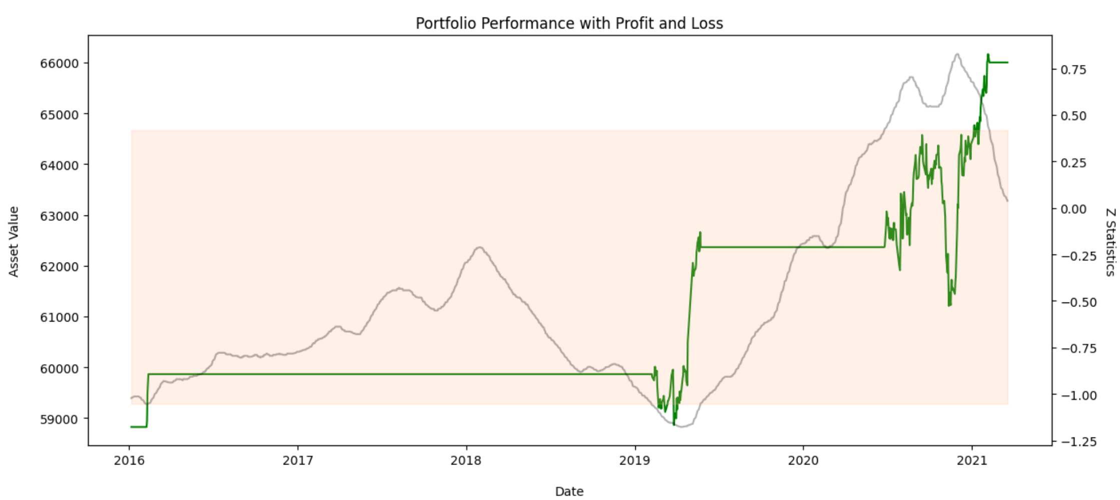

Pair Trading
9001 JT
9021 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9021 JT
SHORT 9021 JT
Date
9001 JT
9021 JT


Pair Trading
9001 JT
9021 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9021 JT
SHORT 9021 JT
4000
3750
3500
3250
3000
2750
2500
9000
8000
7000
6000
5000
2016
2017
2018
2019
2020
2021
Date

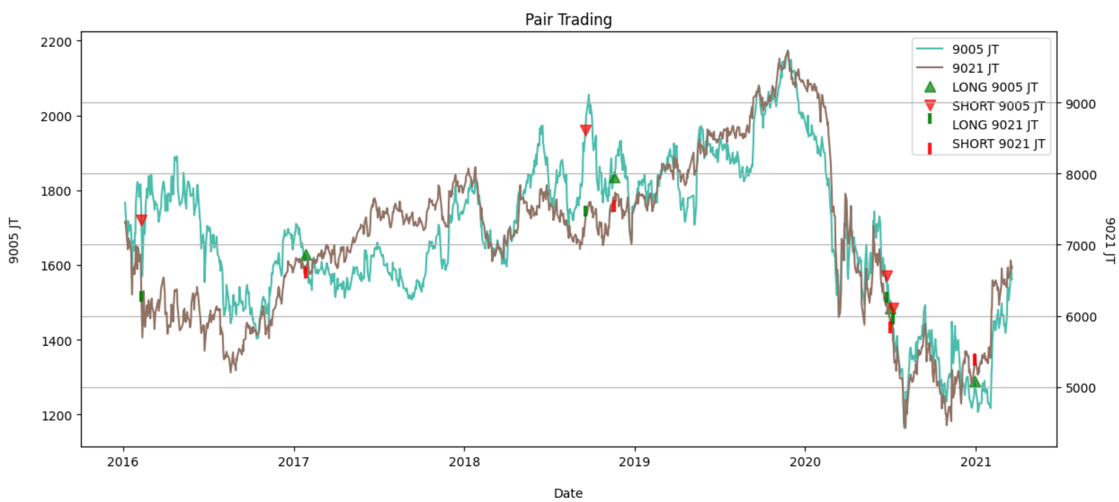

Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
66000
64000
62000
60000
58000
56000
54000
1.0
0.5
0.0
−0.5
−1.0
−1.5
2016
2017
2018
2019
2020
2021
Date

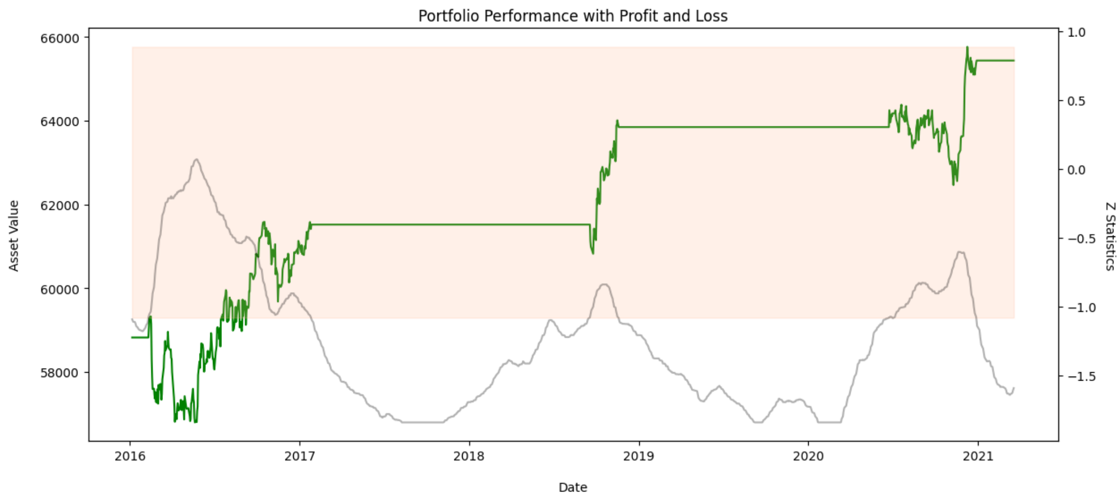

Pair Trading
9001 JT
9022 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9022 JT
SHORT 9022 JT
Date


Pair Trading
9001 JT
9022 JT
LONG 9001 JT
SHORT 9001 JT
LONG 9022 JT
SHORT 9022 JT
Date

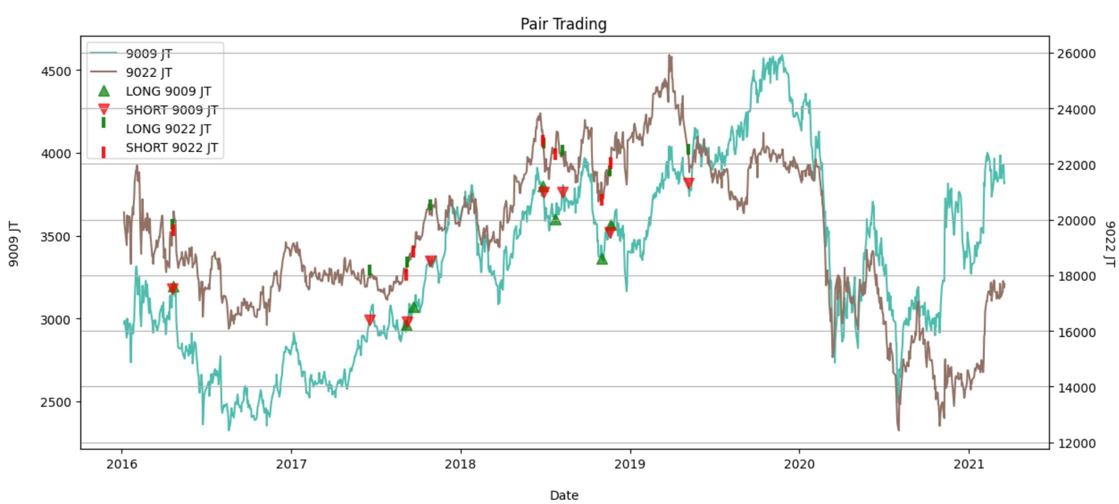

Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date

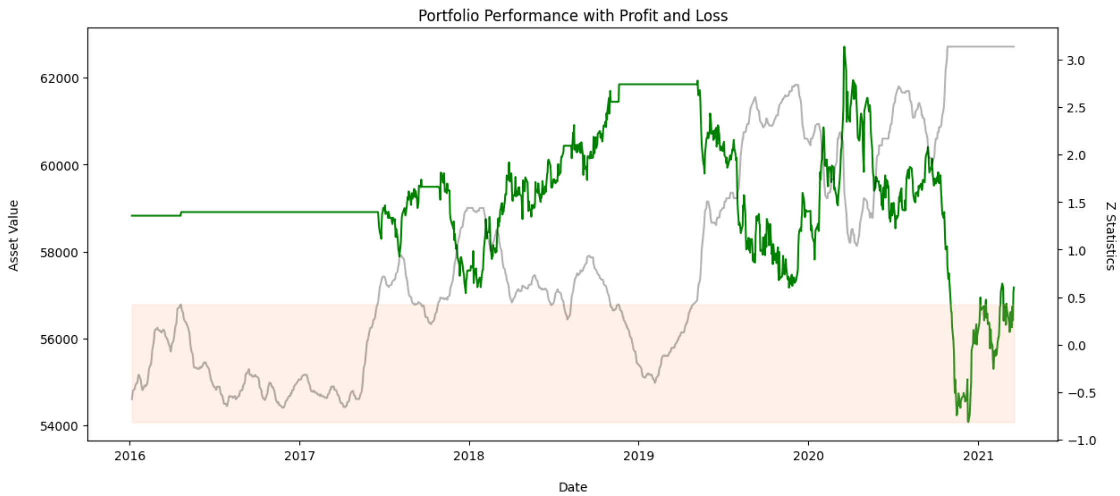

Pair Trading
9005 JT
9021 JT
LONG 9005 JT
SHORT 9005 JT
LONG 9021 JT
SHORT 9021 JT
Date


Pair Trading
9005 JT
9021 JT
LONG 9005 JT
SHORT 9005 JT
LONG 9021 JT
SHORT 9021 JT
Date

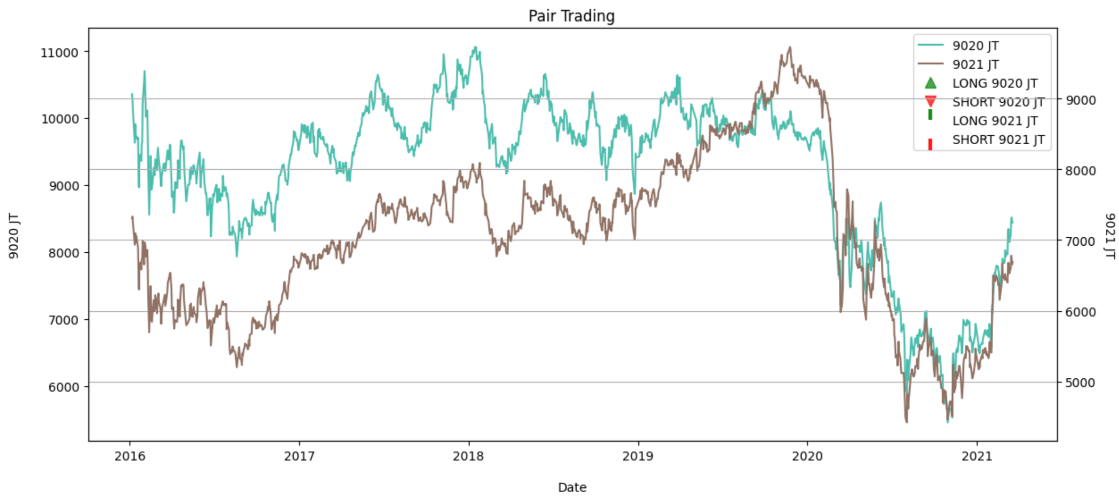

Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date
66000
64000
62000
60000
58000
1.0
0.5
0.0
−0.5
−1.0
−1.5
2016
2017
2018
2019
2020
2021


Pair Trading
9009 JT
9022 JT
LONG 9009 JT
SHORT 9009 JT
LONG 9022 JT
SHORT 9022 JT
Date
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01

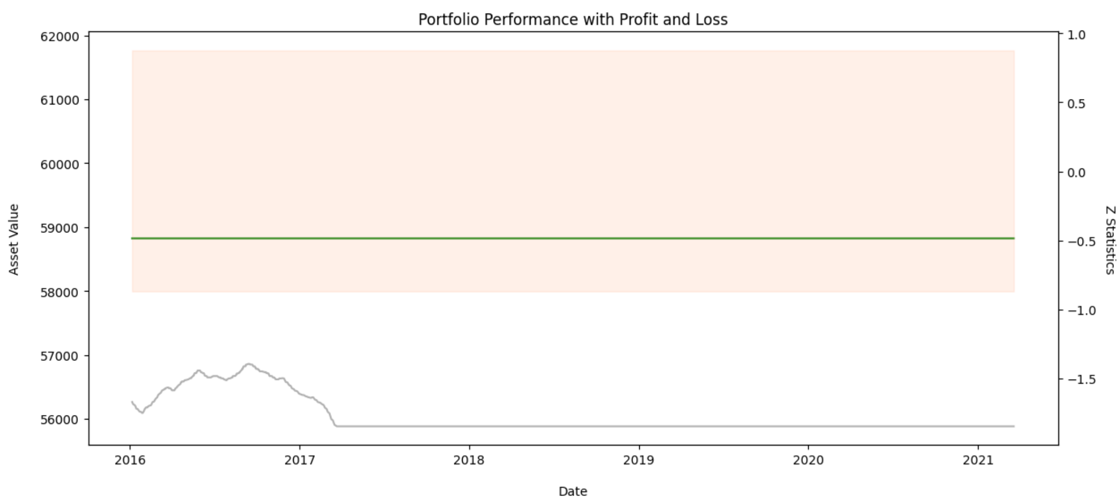

Pair Trading
9009 JT
9022 JT
LONG 9009 JT
SHORT 9009 JT
LONG 9022 JT
SHORT 9022 JT
Date
2016
2017
2018
2019
2020
2021


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date
62000
60000
58000
56000
54000
3.0
2.5
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
2016
2017
2018
2019
2020
2021

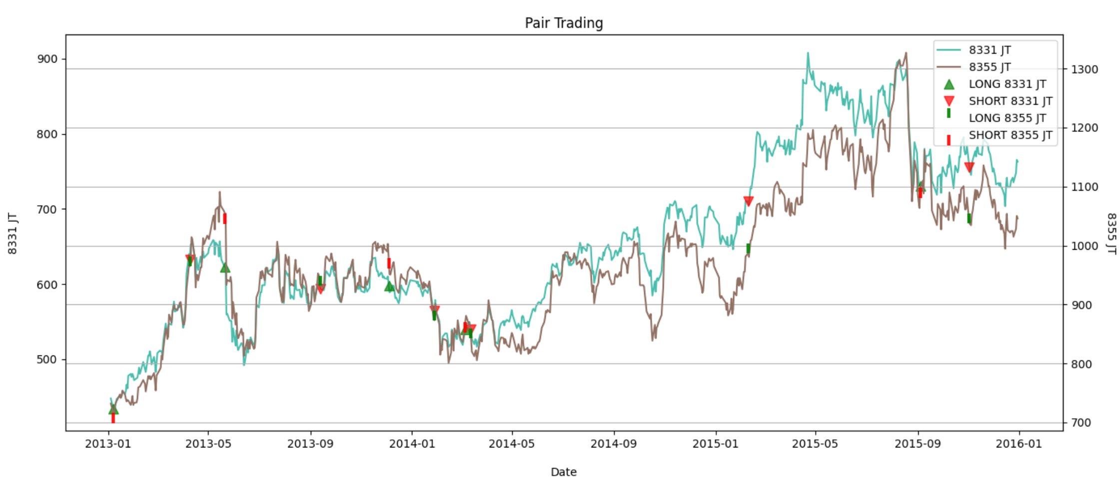

Pair Trading
9020 JT
9021 JT
LONG 9020 JT
SHORT 9020 JT
LONG 9021 JT
SHORT 9021 JT
11000
10000
9000
8000
7000
6000
5000
8000
7000
6000
5000
4000
3000
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01
Date



Pair Trading
9020 JT
9021 JT
LONG 9020 JT
SHORT 9020 JT
LONG 9021 JT
SHORT 9021 JT
Date

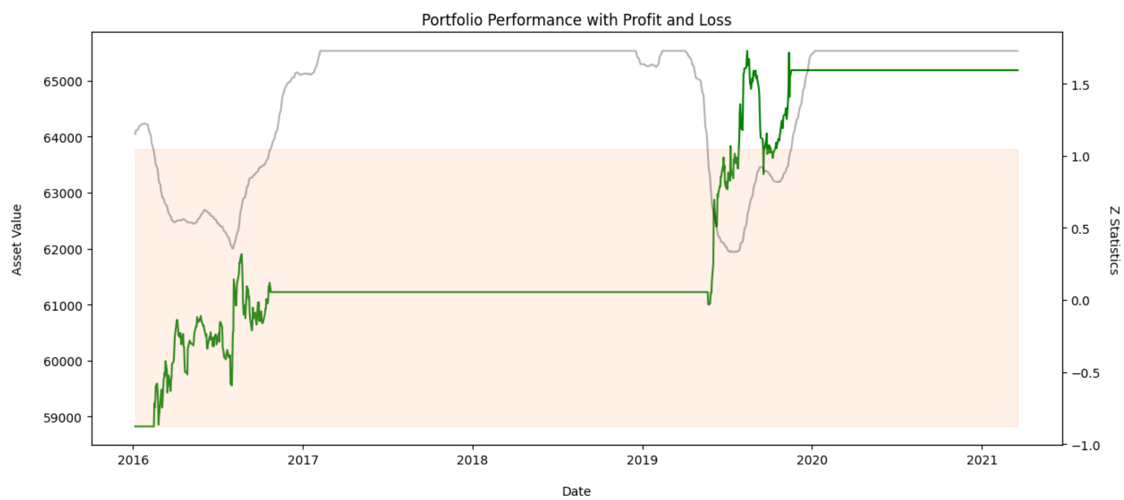


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date



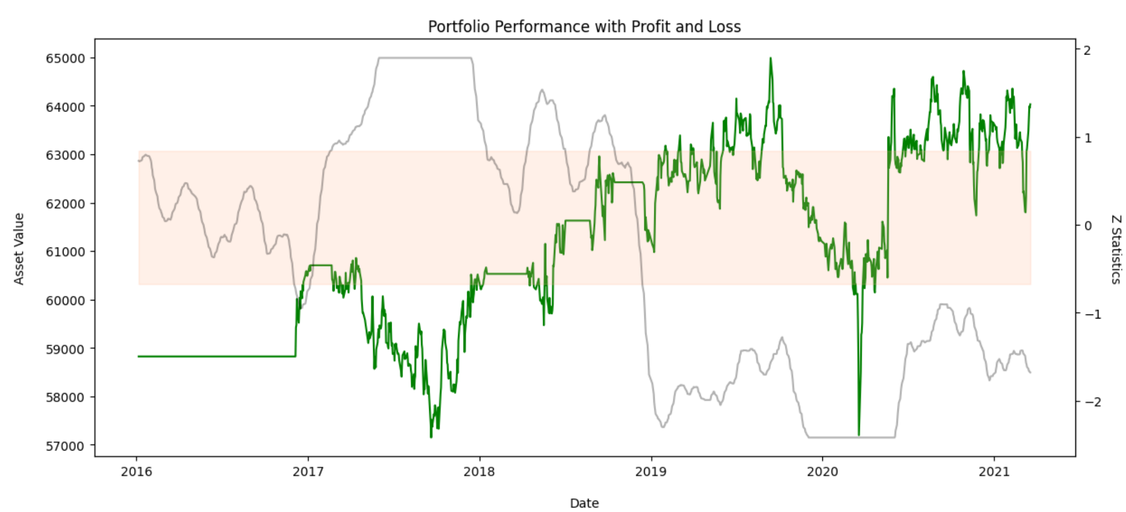
Pair Trading
8331 JT
8355 JT
LONG 8331 JT
SHORT 8331 JT
LONG 8355 JT
SHORT 8355 JT
Date


Pair Trading
8331 JT
8355 JT
LONG 8331 JT
SHORT 8331 JT
LONG 8355 JT
SHORT 8355 JT
900
800
700
600
500
400
1100
1000
900
800
700
600
2016
2017
2018
2019
2020
2021
Date
8331 JT
8355 JT

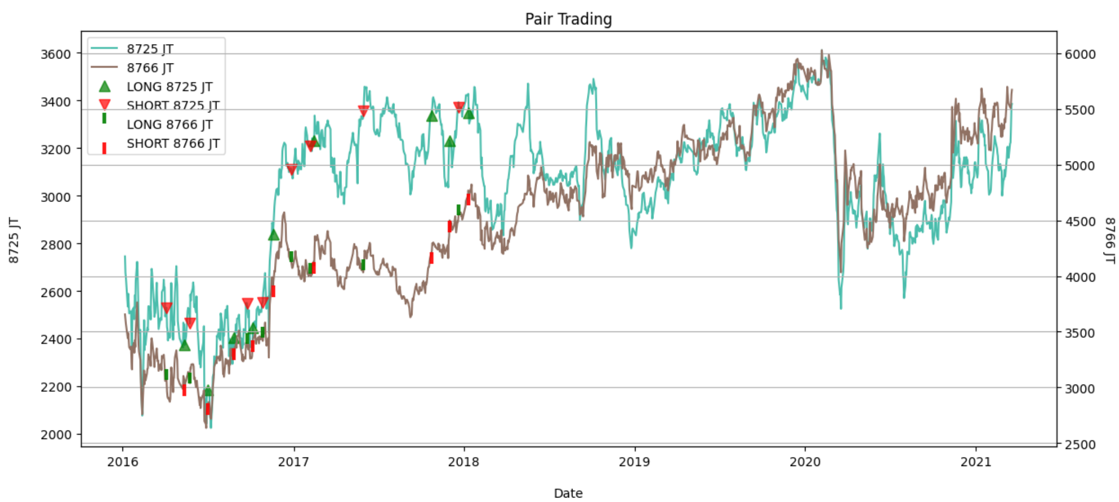

Portfolio Performance with Profit and Loss
65000
64000
63000
62000
61000
60000
59000
Asset Value
1.5
1.0
0.5
0.0
−0.5
−1.0
Z Statistics
2016
2017
2018
2019
2020
2021
Date

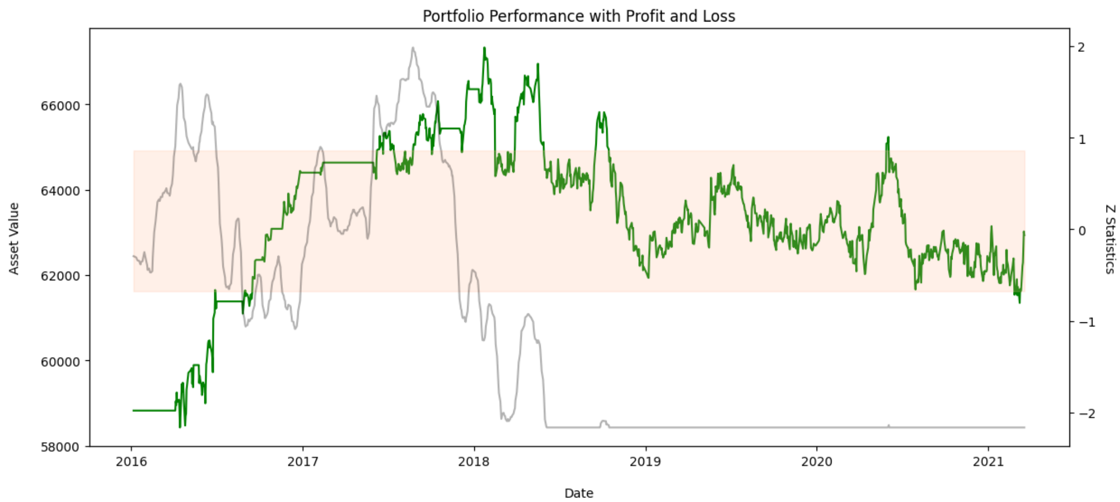

Pair Trading
8630 JT
8766 JT
LONG 8630 JT
SHORT 8630 JT
LONG 8766 JT
SHORT 8766 JT
Date


Pair Trading
8630 JT
8766 JT
LONG 8630 JT
SHORT 8630 JT
LONG 8766 JT
SHORT 8766 JT
Date


Portfolio Performance with Profit and Loss
Asset Value
Z Statistics
Date


Pair Trading
8725 JT
8766 JT
LONG 8725 JT
SHORT 8725 JT
LONG 8766 JT
SHORT 8766 JT
Date
2013-01
2013-05
2013-09
2014-01
2014-05
2014-09
2015-01
2015-05
2015-09
2016-01


Pair Trading
8725 JT
8766 JT
LONG 8725 JT
SHORT 8725 JT
LONG 8766 JT
SHORT 8766 JT
Date
2016
2017
2018
2019
2020
2021



[ ]:
```
print("======================  Overall Performance Metrics ␣
↪========================")
print('Final portfolio value: {0:.2f}'.format(total_assets_value))
print('Compound Annual Growth Rate: {0: .4%}'.format(cagr_estimate))
print('Maximum Drawdown: {0: .4%}'.format(mdd))
print('Annual Volatility: {0: .4%}'.format(ann_vol))
print('Sharpe Ratio: ', shp_rat)
print('Sortino Ratio: ', sortino)
print('\n')
print('\n')
```

```
======================  Overall Performance Metrics  ========================
Final portfolio value: 1097596.52
Compound Annual Growth Rate:  1.8127%
Maximum Drawdown: -4.3362%
Annual Volatility:  2.6030%
Sharpe Ratio:  0.7037255498602681
Sortino Ratio:  1.0234233836419095
```

[ ]:

[ ]: