CRF - 3
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Kriging with Mixed Kernel Learning (MiKL) for Complex Aerospace Problems

Rhea Patricia Liem[1], Kehinde Sikirulai Oyetunde[1], Pramudita Satria Palar[2], Koji Shimoyama[3]

[1] The Hong Kong University of Science and Technology, Hong Kong SAR
[2] Institut Teknologi Bandung West Java, 40132, Indonesia
[3] Institute of Fluid Science, Tohoku University, Sendai 980-8577, Japan

1. Introduction

- Most aerospace problems are complex: non-linear, expensive, noisy, multidimensional and **heterogeneous**.
- Surrogate models such as Kriging are used in deriving cheap approximations to solve such problems.
- Kriging is a nonparametric interpolation technique that approximates the relationship between input, $\boldsymbol{x}$ and the output, $\mathbf{y}$ of a black-box function.
- We propose a novel method based on Kriging to address these problems.
- Our method (**MiKL**) selects the best kernel for each variable through the simultaneous optimization of the hyperparameters and a weight matrix using Cross Validation Error as the objective function.

2. Kriging

Kriging models the output of a black-box function as a realization of a stochastic process.

$$Y(\boldsymbol{x}) = \mu + Z(\boldsymbol{x})$$

μ is the global model and Z ($\mathbf{x}$) is the stochastic component.

For a simple kriging model with the constant trend μ, the prediction mean, m and variance s^2, at a point $\mathbf{x}$ are;

$$m(\boldsymbol{x}) = \mu + \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} (\boldsymbol{y} - \boldsymbol{1}\mu)$$
$$s^2(\boldsymbol{x}) = \hat{\sigma}^2\left(1 - \boldsymbol{r}(\boldsymbol{x})^T \boldsymbol{R}^{-1} \boldsymbol{r}(\boldsymbol{x})\right)$$

where,

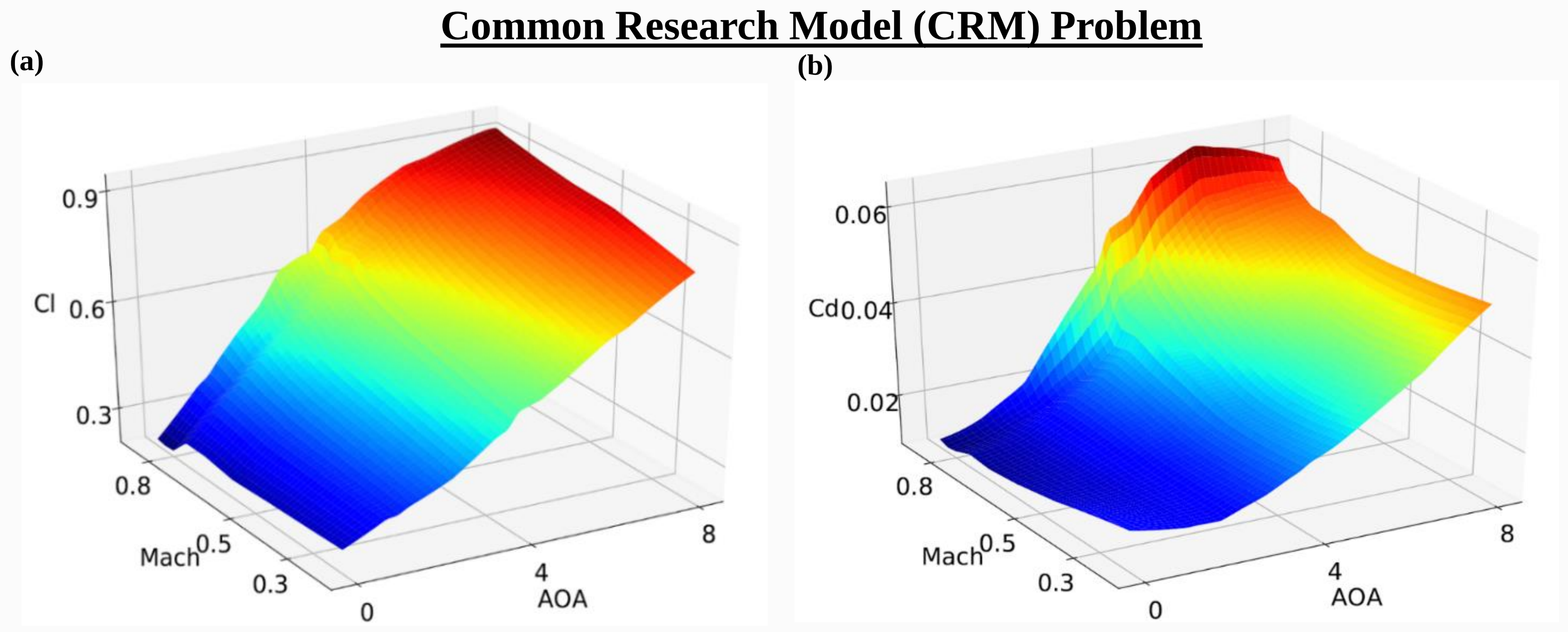
$$\boldsymbol{R} = \prod_{d=1}^{N_d} R\left(\theta^{(d)}, {h_{ij}}^{(d)}\right)$$

3. MiKL Framework

Get the training data $\{x_1, y_1, \dots, x_i, y_i\}$ → Get predefined list of kernels → Weight optimization using SLSQP → Select the best kernel-variable → Final optimization using constrNMPy → Complete model training

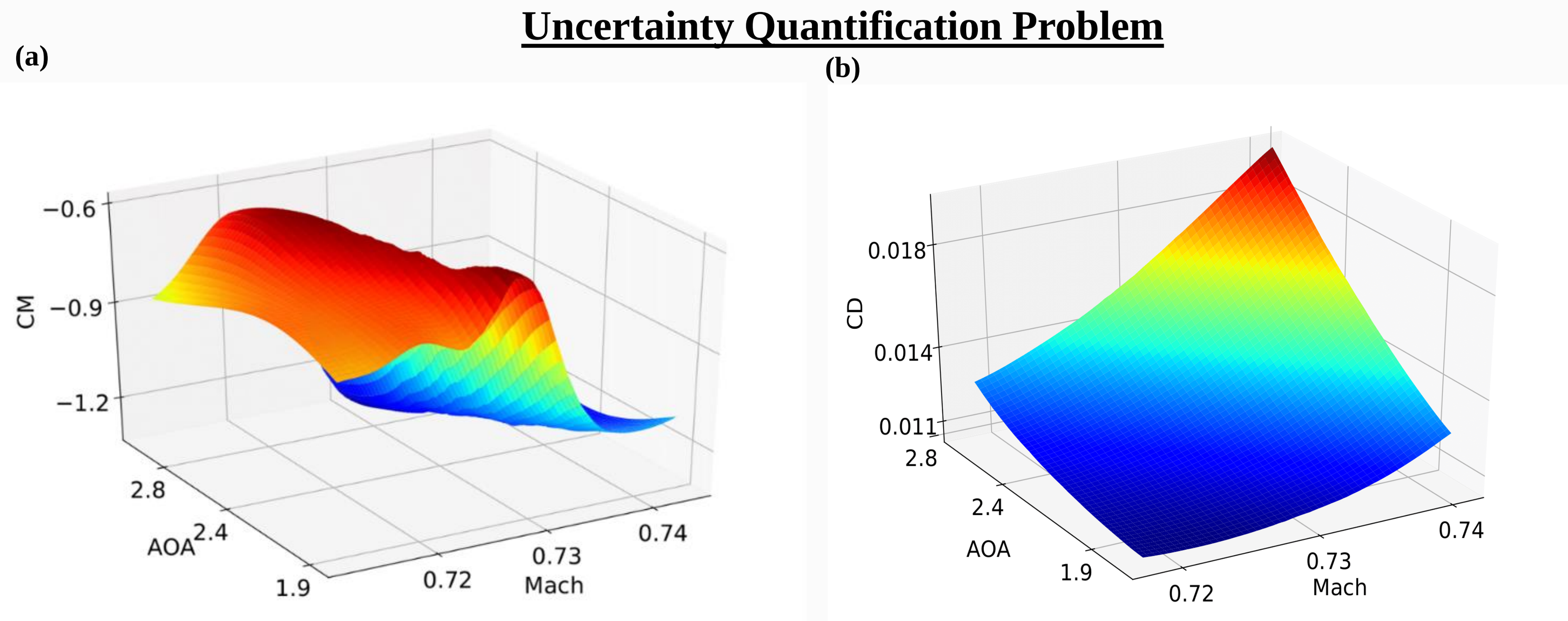
4. Test Cases

Common Research Model (CRM) Problem



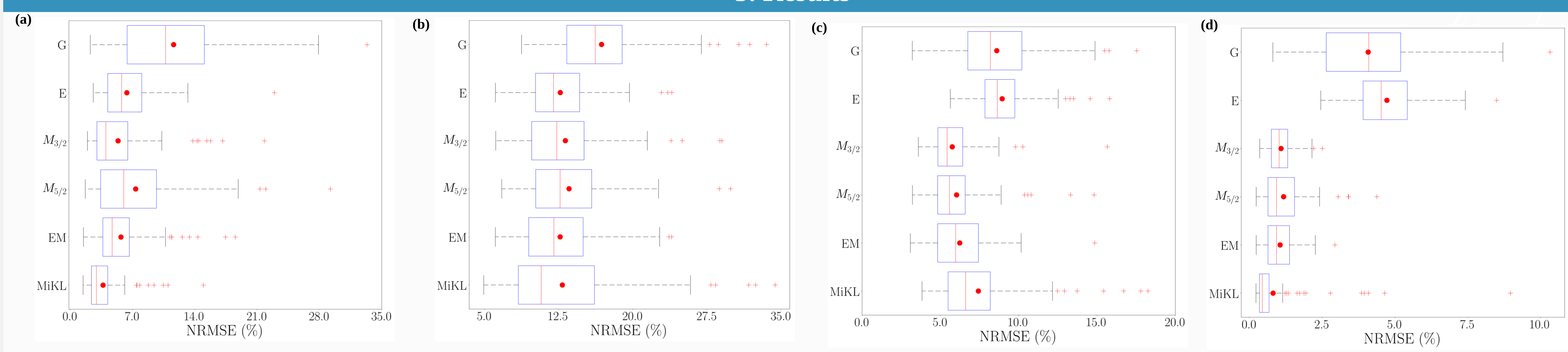
Approximated response surfaces of the aerodynamic coefficients in a CRM problem (a) Lift Coefficient, and (b) Drag Coefficient

Uncertainty Quantification Problem



Approximated response surfaces of the aerodynamic coefficients in an Uncertainty Quantification (UQ) problem (a) Pitching Moment Coefficient, and (b) Drag Coefficient

5. Results



Performance comparison of our proposed approach with the Gaussian, Exponential, Matérn $^3/_2$ and Matérn $^5/_2$ single kernels and Ensemble methods when benchmarked with (a) CRM-C_L (b) CRM-C_D (c) UQ-C_M (d) UQ-C_D problems

6. Summary

- The heterogeneity in complex problems can be captured with the proposed method.
- Preliminary results obtained using our method are promising.
- Computational complexity remains an issue.

7. Future Work

- Detect underlying patterns in function profiles to assign kernels to variables without having to use computationally costly or bias-prone methods.
- Implement sparse kernel approach within the framework.

SELECTED REFERENCES

[1] Palar P. and Shimoyama K., “Kriging with Composite Kernel Learning for Surrogate Modeling in Computer Experiments” Proceedings of the AIAA SciTech Forum, San Diego, CA, 2019.
[2] Melkumyan A., and Ramos F., “Multi-kernel Gaussian Processes” Proceedings of the 22nd International Joint Conference on Artificial Intelligence, 2011.

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