Optimal Strategy Modeling for Price-Time Biparameter Construction Bidding

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Abstract: In the price-time biparameter construction bidding system, each contractor submits a bid price and construction time to complete the project, which are then aggregated to a total combined bid (TCB) by the client, and the contractor with the lowest TCB is awarded the project. Since bid price can be set as construction cost plus an appropriate markup and construction cost usually depends on construction time, TCB can be expressed as a function of time. By minimizing such a TCB function, the optimal construction time can be obtained, from which the optimal construction cost and bid price and hence the optimal price-time bidding strategy can be sequentially decided. While examining the whole optimal bidding process, this paper focuses on three aspects to enhance the key ideas: discussing the properties of the general and the quadratic time-cost functions, deducing the optimal bidding formulas with quadratic time-cost relationship, and illustrating the procedures for estimating the quadratic time-cost function using a few experience data and the linear regression method. The in-depth examination of the price-time biparameter bidding model in the paper can suggest ideas and methodologies to construction bidding or contractor selection with more criteria.

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Introduction

Since Freidman (1956) published his seminal paper on competitive bidding, there has been continual interest in and regular publication on this topic. Traditionally, almost all construction contracts are procured under the low bid system (Park and Chapin 1992) where, since other decision criteria such as construction time and quality are usually specified in the bidding documents, bid price becomes the dominate or sole criterion used by the client in evaluating the contractors' proposals and awarding the contract. In the low bid system, a contractor's bidding strategy is mainly concerned with setting an appropriate markup level to its construction cost so that its bid price is both competitive among all contractors and profitable to itself. Many statistical models and bidding strategies have been developed to determine the appropriate markup and bid price (Carr 1982; Gates 1967).

The low bid system is most convenient for the client to select the lowest bid and award the contract. But for a participating contractor, how to submit a competitive and profitable bid is not that easy. A profitable bid may be easy to determine: the contractor can estimate the total construction cost according to its own personnel, capital, and technical resources, and then add an appropriate markup such as 10% for profit. But to win the contract, the contractor's bid must be competitive or the lowest possible in relation to all competing contractors' bids. To be competitive, the contractor should know who the competitors are, which may not be a big issue since the contractor should know the major players in the field, if not all. More importantly, the contractor should also figure out what levels of bids the competitors will submit so that he can adjust his bid price to be lower and more competitive, which is difficult and even impossible in the competing marketplace. Hence, it seems that a practical as well as reasonable bidding strategy for a contractor is to set its bid price as low as possible based on its capability, while at the same time compete on other aspects such as construction time and quality in the hope of increasing its bidding competitiveness, although such extra efforts may not directly be recognized in the low bid system.

In fact, bidding strategy modeling has indeed expanded to encompass contracts awarded on a multiple criteria basis since only using bid prices to select contractors has certain problems such as poor project quality and prolonged construction duration (Cheng et al. 2000; Drew and Skitmore 1997; Shen et al. 2004; Singh and Tiong 2005). One of the most natural and important expansions is, apart from the bid price, the inclusion of construction time as an influential criterion for a client to award the contract (Herbsman et al. 1995), since the project's early completion can have a profound contribution to the client's return of investment while the project's delayed delivery may result in loss of business opportunities and potential profits or even create social problems for public projects (Shen et al. 1999). In such cases each contractor is required to submit a bid price as well as a construction time to complete the project. To be competitive, not only should the contractor's submitted bid price be lower, but its proposed construction time should be shorter as well. A popular practice is that, with construction time converted to a monetary term in a proportional manner prespecified by the client to *penalize* late completion or reward early completion of the project, each contractor's proposed bid price and construction time result in a total com-

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bined bid (TCB). The contractor with the lowest TCB is finally awarded the project.

There are also studies using more criteria for construction bidding. For example, (1) Hatush and Skitmore (1997) suggest five criteria to select contractors, including: financial soundness, technical ability, management capability, health and safety, and reputation; (2) Cheng et al. (2000) adopt the factors of management skill, technical ability, and financial status in addition to bid price in contractor selection; and (3) Shen et al. (2004) identify seven competitiveness attributes for contractor assessment in China, including management skill, technical ability, financing ability, organization structure, marketing ability, social influence, and contribution to project objectives. It is noticed that bid price and construction time are clearly measurable and fundamentally influential, while other criteria such as project/proposal quality, technical ability, management capability and financial strength are not only strongly correlated with each other and with the price and time factors, but also difficult to be validly measured. Using many criteria for construction bidding, while promising if elegantly applied, may add considerable uncertainty, complexity, and subjectivity in such areas as developing measurement scales, collecting and validating data, determining assessment formulas, and choosing relative importance weights for the many criteria, and hence may not necessarily produce more transparent and reasonable bidding results. On the other hand, an in-depth understanding of the price-time biparameter bidding model can also suggest ideas and methodologies to construction bidding or contractor selection with more criteria, and hence can be considered an important first step in the areas.

The purpose of this paper is to examine in detail the bidding strategies for price-time biparameter construction contracts from a contractor's viewpoint. After introducing the conventional TCB formula as a linear combination of bid price and construction time and defining bid price as construction cost plus an appropriate markup or profit margin, the paper expresses TCB as a linear function of construction time and cost. These two variables are more directly related to the construction project and the contractor's operations, and typically are interrelated. It is argued that, with respect to a particular construction project and under the contractor's existing and potential capabilities, construction time roughly determines its construction cost, i.e., a contractor's construction cost can usually be expressed as a function of construction time for a project or for a variety of projects (Callahan et al. 1992; Clough et al. 2000; Mubarak 2005; Shen et al. 1999). The paper then studies the general properties of such time-cost functions. The simplest and also most appropriate form of such functions is the second-order polynomial, which is a bowl-type parabola in the time-cost plane and implies that construction cost is first decreasing and then increasing as construction time is lengthened, conforming to the general experience. The properties of such time-cost parabolas are then examined in detail in the paper, together with the deduced formula for the optimal bidding strategy leading to the lowest possible TCB. Illustrative examples are offered on how to fit and analyze the quadratic time-cost function using a few sample data estimated by the contractor for the project to be bid, as well as on how to find the optimal price-time bidding strategy. Certain issues related to previous studies and other bidding models are discussed, and some concluding remarks are finally provided.

TCB Formula and Time-Cost Relationship

TCB Formula

Construction clients have increasingly been developing innovative procurement procedures that call for bids on both bid price and construction time (Herbsman et al. 1995; Shen et al. 1999). In such cases, not only is bid price still very important with lower price more competitive, but construction time is also influential with longer time less competitive. The common principle of these price-time biparameter bidding procedures is that each unit of construction time is assigned a certain monetary value by the client and this unit time value (UTV) will be added into the bid price to *penalize* late completion or reward early completion of the project. This forms the following TCB price for each contractor, with lower TCB perceived to have higher overall competitiveness by the client (Herbsman et al. 1995):

$$\mathrm{TCB} = p + \mathrm{UTV} \times t \tag{1}$$

where p=contractor's bid price; and t=its proposed construction time (usually in days).

Obviously, the appropriate determination of UTV is crucial for the client to effectively evaluate the contractors' proposals and award the contract, which can only be done by analyzing the financial and social effects of the project's delayed completion. As Herbsman et al. (1995) indicate, UTV is commonly made up of the direct costs (e.g., temporary facilities, moving costs, and alternative solutions) resulting from construction delays, and can also include the associated indirect costs (e.g., job overhead, business opportunity losses, and reduction of potential profits).

Typically, the client can also determine an estimated or target completion time, t_e, based on various factors such as its business development prospect, financial requirements, and local construction industry's capability. If a contractor's proposed completion time t is shorter (longer) than the client's time estimate t_e, then the contractor should be encouraged (discouraged) by reducing (increasing) its bid price p by an amount proportional to $|t-t_e|$. The proportional coefficient is likely to be more effectively determined from this viewpoint, which is just the UTV. The resulting TCB is equal to $p-\mathrm{UTV}\times(t_e-t)$ if $t\leqslant t_e$ and $p+\mathrm{UTV}\times(t-t_e)$ if $t\geqslant t_e$, leading to a universal formula of $\mathrm{TCB}=p+\mathrm{UTV}\times(t-t_e)$ $=p+\mathrm{UTV}\times t-\mathrm{UTV}\times t_e$, which is practically equivalent to Eq. (1) from both the client's and the contractors' perspectives. Based on this decision formula, the contractor with the lowest TCB is awarded the contract, which usually calls for not only a relatively lower bid price but also a relatively shorter construction time since a longer construction time, converted to monetary terms by the multiplication of UTV, implies a higher TCB which lower the contractor's chance of winning the contract.

The contractor's bid price p is typically equal to its estimated construction cost c to complete the project plus a certain markup or profit margin α (e.g., 10%)

$$p = (1 + \alpha)c \tag{2}$$

Then according to Eq. (1), the contractor's TCB is equal to

$$\mathrm{TCB} = (1 + \alpha)c + \mathrm{UTV} \times t \tag{3}$$

Time-Cost Function

Although other objectives (e.g., profit) are also relevant and even more crucial for the contractor's business, to bid for the contract the contractor's most important aim should be to minimize its

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TCB to maximize its chance to win the contract. This seems simple from Eq. (1) or (3) by reducing construction time t and bid price p or construction cost c as much as possible. But in practice the contractor cannot accomplish this since usually c and t are highly related. In general, very short construction time calls for advanced technology and equipment as well as a large managerial staff, many technicians, and construction workers, usually with multiple shifts and overtime work, which results in a very high construction cost or bid price. As construction time becomes longer, the requirements for technology, equipment, and human resources become less demanding and hence the construction cost will go down. To a particular construction company and for every construction project, there is *optimal* construction time when the contractor has the lowest construction cost (Callahan et al. 1992). After this point when construction time becomes much longer, increased salaries and general indirect costs will have a bigger play and result in increased total cost, which will become more and more obvious or serious when construction time becomes extremely longer.

In fact, with respect to a particular construction project and under the contractor's existing and potential capabilities, construction time roughly determines its construction cost, i.e., a contractor's construction cost can usually be expressed as a function of construction time for a project (Callahan et al. 1992; Clough et al. 2000; Mubarak 2005; Shen et al. 1999). Hence, it can be assumed that there is a certain relationship between a contractor's construction time t and cost c with respect to a particular project, explicitly expressed as

$$c = f(t) \tag{4}$$

where $f(t)$=a decreasing function when $t<t_c$ and an increasing function when $t>t_c$; while t_c=*optimal* construction time corresponding to the lowest construction cost $c_{\min}=f(t_c)$. Assuming $f(t)$ is a differentiable function, then it should satisfy: $f'(t)<0$ when $t<t_c$, $f'(t_c)=0$, and $f'(t)>0$ when $t>t_c$.

According to Eqs. (2) and (4), the contractor's bid price p can also be expressed as a function of construction time t

$$p = (1+\alpha)f(t) \tag{5}$$

Then according to Eqs. (3) and (4), the contractor's TCB is finally a function of t

$$\text{TCB} = (1+\alpha)f(t) + \text{UTV} \times t \tag{6}$$

Optimal Bidding Strategy

Now the contractor's objective is to determine an optimal construction time t_{tcb} that leads to the lowest TCB in Eq. (6) and hence maximizes its chance to win the contract. It should be noticed that this optimal construction time t_{tcb} corresponding to the lowest TCB is usually different from the optimal construction time t_c corresponding to the lowest construction cost. Therefore, under decision formula Eq. (6), the contractor's objective is to build its optimal bidding strategy of construction time and bid price, $(t_{\text{tcb}}, p_{\text{tcb}})$, which leads to the minimum TCB and is determined by three steps: (1) finding the relationship between its construction time and cost, i.e., establishing c as an appropriate function of t as in Eq. (4): $c=f(t)$; (2) identifying a reasonable markup level α so that the bid price can be set as in Eq. (2) or (5): $p=(1+\alpha)c=(1+\alpha)f(t)$, which is *assumed* to be a relatively easier job and will not be discussed further in this paper; and (3) minimizing TCB as in Eq. (6) to find the optimal construction time t_{tcb}, which in turn determines the optimal bid price as $p_{\text{tcb}}=(1+\alpha)f(t_{\text{tcb}})$.

The optimal construction time t_{tcb} which minimizes TCB can be found from the necessary equation: $0=d(\text{TCB})/dt=(1+\alpha)f'(t_{\text{tcb}})+\text{UTV}$, i.e.

$$f'(t_{\text{tcb}}) = -\,\text{UTV}/(1+\alpha) < 0 \tag{7}$$

Since $f'(t)<0$ when $t<t_c$, $f'(t_c)=0$, and $f'(t)>0$ when $t>t_c$, it is known that $t_{\text{tcb}}<t_c$, i.e., the optimal construction time t_c corresponding to the lowest construction cost is somewhat too long for minimizing TCB due to the penalty effect of UTV. To balance the effect of UTV, least-cost construction time t_c must be shortened to t_{tcb} to minimize TCB, of course with an increased construction cost $c_{\text{tcb}}=f(t_{\text{tcb}})>f(t_c)=c_{\min}$.

Bidding Strategy with Quadratic Time-Cost Function

Quadratic Time-Cost Function

To formulate an optimal bidding strategy composed of construction time t_{tcb} and bid price $p_{\text{tcb}}=(1+\alpha)c_{\text{tcb}}=(1+\alpha)f(t_{\text{tcb}})$ which leads to a minimal TCB, the contractor needs to determine the relationship between c and t, i.e., the functional form of $f(t)$. Since $f(t)$ is first decreasing and, after reaching its minimum, then increasing, an appropriate as well as a simple *approximation* for it is the following quadratic function or second-order polynomial which can possess these properties (it should be noted that higher-order polynomials have complex changing patterns not conforming to the above-mentioned properties of $f(t)$ and hence can hardly be considered):

$$g(t) = a_2t^2 + a_1t + a_0 \tag{8}$$

where $a_2 \neq 0$, a_1, and a_0=*constants*. That is, the *real* cost function $f(t)$ can be written as

$$c = f(t) = g(t) + e(t) \tag{9}$$

where $e(t)$=error term when using $g(t)$ to approximate or represent the real $f(t)$. If $g(t)$ is viewed as the major part or second-order polynomial approximation to $f(t)$, $e(t)$ can also be viewed as the minor or residual part of $f(t)$.

It is noticed that Eq. (9) is a typical curve-fitting or statistical regression problem. With $g(t)=a_2t^2+a_1t+a_0$ as in Eq. (8), Eq. (9) is a parabola-fitting or second-order polynomial regression problem, and $g(t)$ or its three coefficients a_2, a_1, and a_0 can easily be estimated using experience data by means of the popular linear regression technique. If by any appropriate means the contractor has n pairs of reasonable construction time and cost data for a project: $(t_1,c_1),(t_2,c_2),\ldots,(t_n,c_n)$, then according to Eqs. (8) and (9): $c_1=a_2(t_1)^2+a_1t_1+a_0+e(t_1),\ldots\ldots,c_n=a_2(t_n)^2+a_1t_n+a_0+e(t_n)$. Through minimizing the sum of the squared error terms, $\Sigma_{1\leqslant i\leqslant n}[e(t_i)]^2=\Sigma_{1\leqslant i\leqslant n}\{c_i-[a_2(t_i)^2+a_1t_i+a_0]\}^2$, the contractor can get the optimal estimates for a_2, a_1, a_0, and hence for the second-order polynomial $g(t)$, which can be done using linear regression technique available in popular computer packages such as Excel and SPSS.

If $e(t)$ is quite small relative to $g(t)$ or $f(t)$, which can be evaluated by the popular R-squared, R^2, of the above regression problem, it is comfortable to use $g(t)$ to approximate or represent the real $f(t)$, which implies that $g(t)$ should possess the same or main properties of $f(t)$ and, conversely, the analysis results for $g(t)$ should be close to those for $f(t)$ and hence *applicable* to $f(t)$ or the construction contractor. Therefore in the following the

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focus is only on $g(t)$—the contractor's *approximate* time-cost function—which, for convenience while without confusion, will be viewed *as* $f(t)$—the contractor's *real* time-cost function.

Properties of Quadratic Time-Cost Function

Since construction cost $c=f(t)=a_2t^2+a_1t+a_0$ should be positive for all positive t, it is necessary to have $a_2>0$ (otherwise when t is reasonably large, $f(t)$ will be negative) and $a_0 \geqslant 0$ (otherwise when t is reasonably small, $f(t)$ will be negative). Since $f'(t)=2a_2t+a_1$ and the least-cost construction time t_c must be such that $0=f'(t_c)=2a_2t_c+a_1$, it is found that

$$t_c = -a_1/(2a_2) \tag{10}$$

Since t_c is necessarily positive while a_2 is positive, a_1 should be negative. The corresponding least cost is

$$\begin{aligned} c_{\min} &= f(t_c) = a_2[-a_1/(2a_2)]^2 + a_1[-a_1/(2a_2)] + a_0 \\ &= [4a_2a_0 - (a_1)^2]/(4a_2) \end{aligned} \tag{11}$$

Since $c_{\min}$ is also necessarily positive while a_2 is positive, $4a_2a_0 - (a_1)^2$ should be positive or $(a_1)^2 < 4a_2a_0$, which, since $a_1<0$, leads to $a_0>0$ and $a_1>-(4a_2a_0)^{1/2}$. Hence a_1 should satisfy the constraint of $0>a_1>-(4a_2a_0)^{1/2}$.

With Eqs. (10) and (11), the quadratic time-cost function can be clearly written as $c=f(t)=a_2(t-t_c)^2+c_{\min}$, where $c_{\min}$ =contractor's least construction cost and t_c=its least-cost construction time. Further, since $f'(t)=2a_2t+a_1$, from Eq. (7) it is known that: $-\mathrm{UTV}/(1+\alpha)=f'(t_{\mathrm{tcb}})=2a_2t_{\mathrm{tcb}}+a_1$. Hence the optimal construction time leading to the minimal TCB is

$$t_{\mathrm{tcb}} = -[a_1 + \mathrm{UTV}/(1+\alpha)]/(2a_2) \tag{12}$$

Since t_{tcb} is necessarily positive while UTV, α, and a_2 are all positive, $a_1+\mathrm{UTV}/(1+\alpha)$ should be negative, leading to the condition of $a_1<-\mathrm{UTV}/(1+\alpha)<0$. Combined with the previous conditions, the ultimate constraints for the quadratic time-cost function $f(t)$ are

$$a_2>0, \quad a_0>0, \quad -\mathrm{UTV}/(1+\alpha) > a_1 > -(4a_2a_0)^{1/2} \tag{13}$$

Optimal Bidding Strategy

From Eqs. (10) and (12), it is clear that

$$t_{\mathrm{tcb}} = t_c - \mathrm{UTV}/(1+\alpha)/(2a_2) \tag{14}$$

which implies that the least-TCB construction time t_{tcb} is indeed shorter than the least-cost construction time t_c. Obviously, if UTV is higher, implying the value of time is higher or the penalty for delay is greater, then to win the bid, the contractor must be able to complete the project more quickly. That is, the least-TCB construction time t_{tcb} must be much shorter than the least-cost construction time t_c, which implies that, since construction cost is decreasing before time t_c, the contractor will necessarily cost much more than its least possible cost $c_{\min}$.

On the other hand, if α is higher, implying the contractor submits a higher price in order to earn a higher profit margin from undertaking the project, then the least-TCB construction time t_{tcb} will be closer to the least-cost construction time t_c. This implies that the contractor's construction cost will be closer to its least cost $c_{\min}$, making its total profit closer to $\alpha c_{\min}$ which is not that higher. This seems quite reasonable from the viewpoint of the client who proposes the appropriate decision Eq. (1) which, intentionally or unintentionally, can prevent the contractor from earning too high of a profit.

Based on Eqs. (11) and (12), the construction cost corresponding to construction time t_{tcb} can be calculated as

$$\begin{aligned} c_{\mathrm{tcb}} = f(t_{\mathrm{tcb}}) &= a_2\{-[a_1 + \mathrm{UTV}/(1+\alpha)]/(2a_2)\}^2 + a_1 \\ &\times\{-[a_1 + \mathrm{UTV}/(1+\alpha)]/(2a_2)\} + a_0 = [4a_2a_0 - (a_1)^2]/(4a_2) \\ &+ [\mathrm{UTV}/(1+\alpha)]^2/(4a_2) \equiv c_{\min} + [\mathrm{UTV}/(1+\alpha)]^2/(4a_2) \end{aligned} \tag{15}$$

Clearly, with the least-TCB construction time t_{tcb} shorter than the least-cost time t_c, the corresponding construction cost c_{tcb} is higher than the minimal cost $c_{\min}$, as discussed before.

The minimal TCB realized with construction time t_{tcb} can be calculated as

$$\begin{aligned} \mathrm{TCB}_{\min} &= (1+\alpha)f(t_{\mathrm{tcb}}) + \mathrm{UTV}\times t_{\mathrm{tcb}} = (1+\alpha)\{c_{\min} + [\mathrm{UTV}/ \\ &\times(1+\alpha)]^2/(4a_2)\} + \mathrm{UTV}[t_c - \mathrm{UTV}/(1+\alpha)/(2a_2)] \\ &= (1+\alpha)c_{\min} + \mathrm{UTV}\times t_c - \mathrm{UTV}^2/(1+\alpha)/(4a_2) \\ &\equiv \mathrm{TCB}_c - \mathrm{UTV}^2/(1+\alpha)/(4a_2) \end{aligned} \tag{16}$$

where $\mathrm{TCB}_c=(1+\alpha)c_{\min}+\mathrm{UTV}\times t_c$=TCB corresponding to the least-cost construction time t_c. Clearly, $\mathrm{TCB}_{\min}<\mathrm{TCB}_c$, i.e., although construction cost c_{tcb} with time t_{tcb} is higher than the least-cost $c_{\min}$ with time t_c, more money is saved due to a shorter construction time t_{tcb} than t_c and the effect of UTV, resulting in a smaller and in fact the optimal TCB.

Hence, the contractor can have an optimal bidding strategy composed of construction time t_{tcb} as determined by Eq. (12) or (14) and bid price $p_{\mathrm{tcb}}=(1+\alpha)c_{\mathrm{tcb}}$, where c_{tcb}=contractor's construction cost corresponding to construction time t_{tcb}, as determined by Eq. (15). This optimal strategy, denoted simply as $(p_{\mathrm{tcb}}, t_{\mathrm{tcb}})$, results in the minimal possible TCB given by Eq. (16) the contractor can achieve, and hence gives it the best possible chance to win the contract.

Illustrative Examples

To obtain the above mentioned optimal bidding strategy, it is necessary for the bidding company to estimate its time-cost relationship $c=f(t)=a_2t^2+a_1t+a_0$ with regard to the particular construction project to be bid. In order to estimate the second-order polynomial function $f(t)$, it is only needed to estimate its three coefficients a_2, a_1 and a_0 using some experience data and the linear regression technique as outlined before.

Although other types of data or data from other sources may also be useful, the best way to estimate $f(t)$ is to use the company's *real* data related to the bidding project, which however is impossible since the project has not yet been conducted. But it is possible for the company to estimate some construction time and cost combinations for conducting the project based on its own capacity and experience. In fact, the final construction price and time submitted by every serious contractor to the client is certainly a result of the contractor's estimation and analysis of some feasible construction time and cost combinations, which can be used to estimate the contractor's underlying construction time-cost relationship.

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Using Three Typical Data Points to Decide Optimal Bidding Strategy

Shen et al. (1999) propose estimating the price-time relationship using three *typical* pairs of prices and construction times which, as price and cost are proportionate, can be converted to three pairs of time and cost data (t_1, c_1), (t_2, c_2), and (t_3, c_3) to estimate the time-cost function $f(t)$. The first pair, (t_1, c_1), is composed of the contractor's *shortest* possible construction time t_1 to complete the project to be bid and the corresponding construction cost c_1. The second pair, (t_2, c_2), is the contractor's *most likely* construction time and cost combination to complete the project. The third pair, (t_3, c_3), is composed of the contractor's *lowest* possible construction cost c_3 to complete the project and the corresponding construction time t_3. It is noticed that, in the so-called two-envelop bidding system where contractors compete on bid *price* and proposed construction *quality*, Drew et al. (2002) also use three similar typical sets of price and quality data to estimate the underlying quadratic price-quality function.

Since the second-order polynomial $f(t)$ has three coefficients, it can be exactly estimated using the above three pairs of observations. As performed in Shen et al. (1999), in this special situation it is not necessary to use the regression method to estimate the three coefficients of $f(t)$. It can be done by simply letting the parabola $c = f(t) = a_2 t^2 + a_1 t + a_0$ pass through the three given data points, which results in the following three equations:

$$c_1 = a_2(t_1)^2 + a_1 t_1 + a_0 \tag{17}$$

$$c_2 = a_2(t_2)^2 + a_1 t_2 + a_0 \tag{18}$$

$$c_3 = a_2(t_3)^2 + a_1 t_3 + a_0 \tag{19}$$

From these three equations, it is easy to obtain the exact values of the three coefficients a_2, a_1, and a_0 and hence determine the exact form of $f(t)$.

Suppose that, as given by Shen et al. (1999), with respect to a particular bidding project a participating contractor estimates three possible pairs of construction time and cost according to its own experience and resources as follows: (1) *shortest time* combination: $t_1 = 170$ days and $c_1 = \$5{,}100{,}000$; (2) *most likely* combination: $t_2 = 190$ days and $c_2 = \$4{,}900{,}000$; and (3) *least cost* combination: $t_3 = 210$ days and $c_3 = \$4{,}800{,}000$. Using these three data points, Eqs. (17)–(19) become: $5{,}100{,}000 = 170^2 a_2 + 170 a_1 + a_0$, $4{,}900{,}000 = 190^2 a_2 + 190 a_1 + a_0$, and $4{,}800{,}000 = 210^2 a_2 + 210 a_1 + a_0$, from which the second-order polynomial's coefficients can be easily solved out as: $a_2 = 125$, $a_1 = -55{,}000$, $a_0 = 10{,}837{,}500$. Hence, the contractor's construction time-cost relationship can be determined as the following quadratic function, whose curve is shown in Fig. 1 as a parabola:

$$c = f(t) = 125t^2 - 55{,}000t + 10{,}837{,}500 \tag{20}$$

Suppose further that the client specifies a UTV of \$10, 000/ day and, *just for computational simplicity*, that the contractor's submitted price is equal to its cost: $p = c$, i.e., its profit margin $\alpha = 0$, then the decisive TCB formula becomes

$$\begin{aligned} \text{TCB} &= c + \text{UTV} \times t = 125t^2 - 55{,}000t + 10{,}837{,}500 + 10{,}000t \\ &= 125t^2 - 45{,}000t + 10{,}837{,}500 \end{aligned} \tag{21}$$

which is also a quadratic function and corresponds to a parabola as shown in Fig. 2. The minimum TCB equal to $\text{TCB}_{\min} = 10{,}837{,}500 - 45{,}000^2/(4 \times 125) = \$6{,}787{,}500$, which occurs when construction time equals $t_{\text{tcb}} = 45{,}000/(2 \times 125) = 180$ (days). The corresponding construction cost equals $c_{\text{tcb}} = f(t_{\text{tcb}}) = 125 \times 180^2 - 55{,}000 \times 180 + 10{,}837{,}500 = \$4{,}987{,}500$. Since the contractor's profit margin is assumed to be zero: $\alpha = 0$, its submitted price equals $p_{\text{tcb}} = c_{\text{tcb}} = \$4{,}987{,}500$.

Hence, the contractor's optimal bidding strategy for the project is to propose a construction time of $t_{\text{tcb}} = 180$ days and bid price of $p_{\text{tcb}} = \$4{,}987{,}500$, which will result in a minimum possible total combined bid of $\text{TCB}_{\min} = \$6{,}787{,}500$ for the contractor, giving it the highest possible chance to win the bid. This optimal bidding strategy in terms of $(t_{\text{tcb}}, \text{TCB}_{\min})$ is shown as a data point marked by a solid triangle in the TCB parabola in Fig. 2.

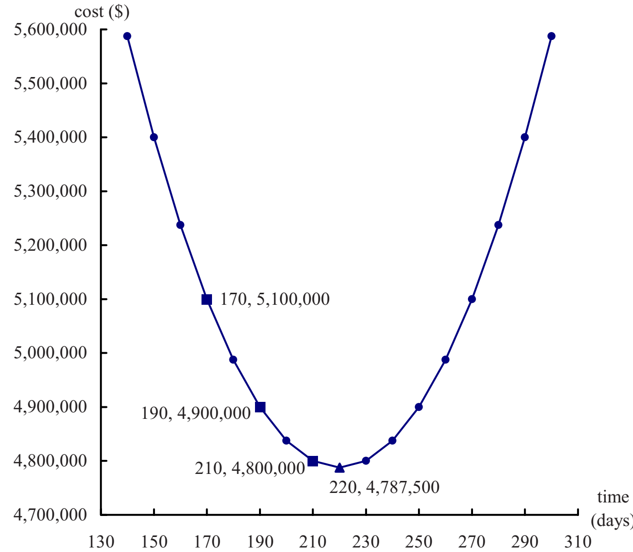

Fig. 1. Three given time-cost data points and fitted parabola

Some Discussions

However, it is necessary to point out that the previous preclaimed lowest construction cost c_3 as suggest by Shen et al. (1999) is

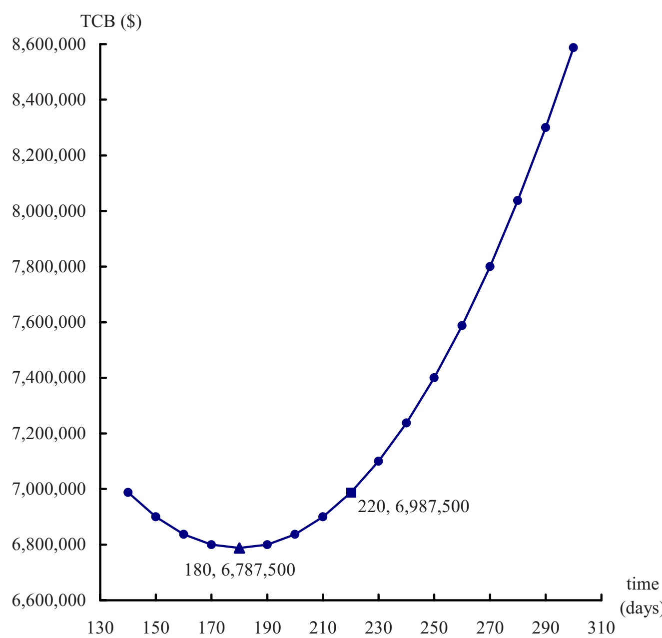

Fig. 2. Time-TCB parabola obtained from three given time-cost data points

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usually not the *true* least cost. This is because the contractor's cost-parabola or cost continuum fitted by the three given data points (t_1, c_1), (t_2, c_2), and (t_3, c_3) marked by solid squares in Fig. 1 will have its true minimum $c_{\min} = [4a_2a_0 - (a_1)^2]/(4a_2)$ as determined by Eq. (11), which by no means can be guaranteed to equal the preclaimed lowest cost c_3. For example, the above illustrative case given by Shen et al. (1999) establishes the contractor's construction time-cost relationship as in Eq. (20), based on which its *true* least cost can be obtained as $c_{\min} = 10{,}837{,}500 - 55{,}000^2/(4 \times 125) = \$4{,}787{,}500$ from Eq. (11), which is lower than the preclaimed lowest cost of $c_3 = \$4{,}800{,}000$. The corresponding least-cost construction time equal to $t_c = 55{,}000/(2 \times 125) = 220$ days, longer than the time of $t_3 = 210$ days corresponding to the preclaimed lowest cost c_3. See Fig. 1 where this *true* least-cost data point $(t_c, c_{\min}) = (220,\ 4{,}787{,}500)$ is shown by a solid triangle, which is below the *preclaimed* least-cost data point $(t_3, c_3) = (210,\ 4{,}800{,}000)$.

It is also clear that the least-TCB data point $(t_{\text{tcb}}, \text{TCB}_{\min}) = (180,\ 6{,}787{,}500)$ in Fig. 2 corresponds to a time-cost data point $(t_{\text{tcb}}, c_{\text{tcb}}) = (180,\ 4{,}987{,}500)$ above the least-cost data point $(t_c, c_{\min}) = (220,\ 4{,}787{,}500)$ as shown in Fig. 1, showing that the least-TCB cost is not the lowest one. Reversely, the least-cost data point $(t_c, c_{\min}) = (220,\ 4{,}787{,}500)$ in Fig. 1 produces, according to Eq. (21), a total combined bid of $\text{TCB}_c = 125 \times 220^2 - 45{,}000 \times 220 + 10{,}837{,}500 = 6{,}987{,}500$, which is higher than $\text{TCB}_{\min} = 6{,}787{,}500$, i.e., the corresponding data point $(t_c, \text{TCB}_c) = (220,\ 6{,}987{,}500)$ marked by a solid square in Fig. 2 is above the least-TCB data point $(t_{\text{tcb}}, \text{TCB}_{\min})$.

In fact, before the second-order polynomial $f(t)$ or the contractor's cost continuum is completely estimated using sample observations, the contractor's least construction cost is unknown. Therefore, the contractor just needs to estimate some feasible combinations of construction time and cost to approximate its full construction time and cost relationship, and there is no need to preclaim these combinations as *optimal* or *extreme* ones. Of course, in order for the estimated time-cost relationship to be representative, the sample combinations are better to cover a wider range of possible data or to be more widely distributed. In this connection, the above three pairs of construction time and cost data as suggested by Shen et al. (1999) are not very appropriate or sufficient to estimate a representative construction time and cost relationship for the contractor due to two reasons. First, their *names* may not be appropriate. Since *optimal* time and cost are not easy to be preidentified, the three typical sets of construction time and cost combinations for estimating the contractor's underlying construction time-cost relationship are better renamed as: a short-time combination (t_1, c_1), a likely time-cost combination (t_2, c_2), and a low-cost combination (t_3, c_3). Second, and more importantly, the three data points are all in the left or decreasing part of the parabola-type curve of the contractor's time-cost relationship since with time increased from $t_1 = 170$ to $t_2 = 190$ and then to $t_3 = 210$, the cost decreases from $c_1 = 5{,}100{,}000$ to $c_2 = 4{,}900{,}000$ and then to $c_3 = 4{,}800{,}000$. With no data points in the right or increasing part of the time-cost parabola, it is suspicious that the parabola estimated from the three given data points can well represent the contractor's true time-cost relationship.

Using More Data Points to Decide Optimal Bidding Strategy

Obviously, if the contractor's true time-cost relationship can be represented by a parabola-type curve, a better way to approximate this second-order polynomial is first to estimate some experience data points which are roughly symmetrically distributed around the curve's decreasing and increasing parts, and then to fit the curve using these well-designed sample data by means of the statistical regression technique. It is suggested to estimate the experience time-cost data from a short time point to a long time point, roughly in an equal time pace. Although more data points are preferred to build a more accurate time-cost relationship for the contractor, 4–10 data points seem to be appropriate since estimating too many experience data may be too time consuming and expensive.

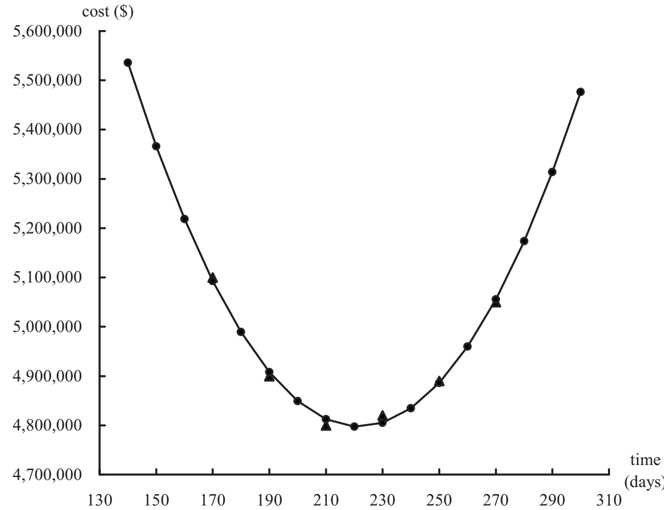

Fig. 3. Six given time-cost data points and fitted parabola

Taking the above case as an example, three data points (t_1, c_1), (t_2, c_2), and (t_3, c_3) have already been estimated around the parabola's left or decreasing part with an equal time interval of $t_2 - t_1 = t_3 - t_2 = 20$ days. Then it is quite natural and also desirable to estimate three more data points (t_4, c_4), (t_5, c_5), and (t_6, c_6) with the same time interval of 20, which implies that $t_4 = 230$, $t_5 = 250$, and $t_6 = 270$. So we just need to estimate the contractor's construction costs corresponding to these three new construction times, and ideally these three new time-cost data points should be around the parabola's right or increasing branch. Suppose that, corresponding to the three new construction times of $t_4 = 230$, $t_5 = 250$, and $t_6 = 270$ days, the three new construction costs are estimated to be $c_4 = \$4{,}820{,}000$, $c_5 = \$4{,}890{,}000$, and $c_6 = \$5{,}050{,}000$. Since $c_3 < c_4 < c_5 < c_6$, the three new data points are around the parabola's right or increasing part, just as hoped.

Using the six data points to fit a quadratic function $c = f(t) = a_2t^2 + a_1t + a_0$ with the help of the regression technique, the results are as follows: $a_2 = 110.7143$, $a_1 = -49{,}085.7143$, $a_0 = 10{,}237{,}785.7$, and $R^2 = 0.993$. Hence the contractor's time-cost relationship is estimated as the following second-order polynomial (see Fig. 3):

$$c = f(t) = 110.7143t^2 - 49{,}085.7143t + 10{,}237{,}785.7 \quad (22)$$

Here, $R^2 = 0.993$ means that the cost-parabola Eq. (22) fits very closely to the six data points as marked by solid triangles in Fig. 3 ($R^2 = 1$ implies a perfect fit as in the previous case of three data points).

However, if the three new data points are ignored and just the previous three data points (t_1, c_1), (t_2, c_2), and (t_3, c_3) are used to estimate the time-cost relationship, then parabola (20) as shown in Fig. 4 is obtained, which clearly does not fit all six experience data points as well as parabola (22) estimated from all six data

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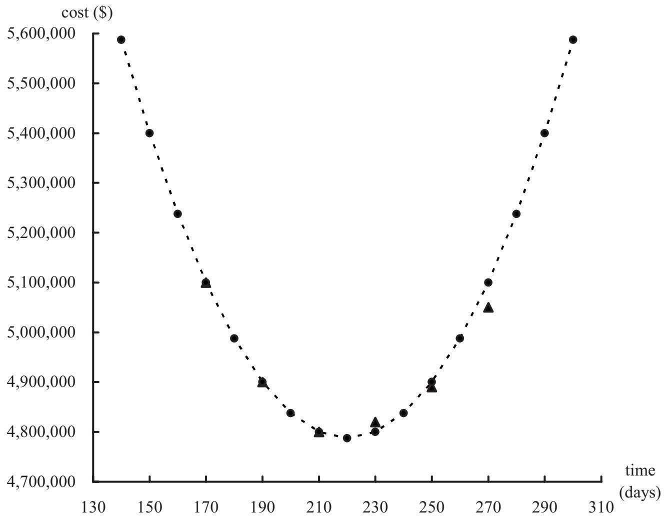

Fig. 4. Six given time-cost data points and parabola fitted from only first three data points

points. Fig. 5 further shows the differences of the two fitted parabolas, i.e., the cost-parabola fitted by the previous three data points could have either overestimated or underestimated the construction cost compared with the cost-parabola fitted by all six data points. This simple comparison shows the advantage of using more experience data and hence a contractor should, if feasible, use more experience data to more accurately derive its underlying construction time-cost relationship.

Assume as before that UTV=$\$10{,}000/\text{day}$ and profit margin $\alpha=0$, then the optimal construction time leading to the minimum TCB is given by Eq. (12) as $t_{\text{tcb}}=-[a_1+\text{UTV}/(1+\alpha)]/(2a_2)=-(-49{,}085.7143+10{,}000)/(2\times 110.7143)\approx 177$ days. The corresponding construction cost or bid price in this example can be calculated from Eq. (22) as $c_{\text{tcb}}=p_{\text{tcb}}=110.7143\times 177^2-49{,}085.7143\times 177+10{,}237{,}785.7\approx \$5{,}018{,}182$. That is, the contractor's optimal bidding strategy is to submit a bid price of $p_{\text{tcb}}=\$5{,}018{,}182$ with a construction time of $t_{\text{tcb}}=177$ days, which results in the minimum $\text{TCB}:\text{TCB}_{\min}=c_{\text{tcb}}+\text{UTV}\times t_{\text{tcb}}=5{,}018{,}182+10{,}000\times 177=\$6{,}788{,}182$.

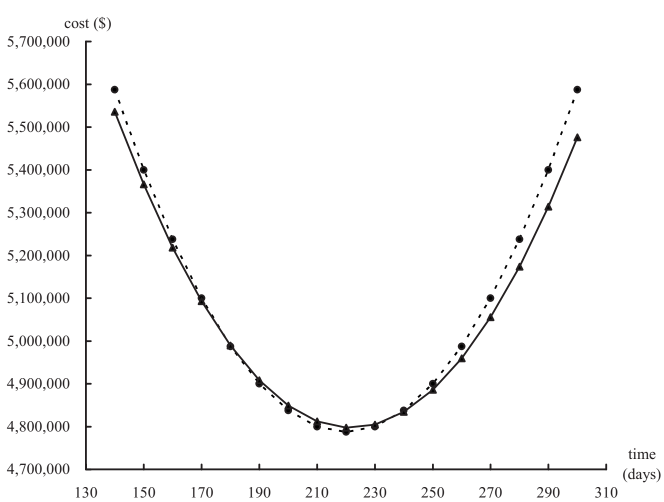

Fig. 5. Two parabolas fitted from all six data points and from only first three data points

It should be noticed that, with the estimated $c=f(t)$ as in Eq. (22), the contractor's least cost can be calculated from Eq. (11) as $c_{\min}=a_0-(a_1)^2/(4a_2)=10{,}237{,}785.7-49{,}085.7143^2/(4\times 110.7143)\approx \$4{,}797{,}188$ which, according to Eq. (10), occurs with the construction time equal to $t_c=-a_1/(2a_2)=49{,}085.7143/(2\times 110.7143)\approx 222$ days, and leads to a total combined bid of $\text{TCB}_c=c_{\min}+\text{UTV}\times t_c=4{,}797{,}188+10{,}000\times 222=\$7{,}017{,}188$. Clearly, the construction cost $c_{\text{tcb}}=\$5{,}018{,}182$ in the contractor's optimal bidding strategy is higher than its least possible cost $c_{\min}=\$4{,}797{,}188$ since the construction time is significantly shortened to $t_{\text{tcb}}=177$ days from $t_c=222$ days, which, due to the balancing effect of UTV on construction time, reduces the TCB from $\text{TCB}_c=\$7{,}017{,}188$ to the most competitive $\text{TCB}_{\min}=\$6{,}788{,}182$.

Further Discussions

In the above optimal bidding strategy, the construction time of $t_{\text{tcb}}=177$ days is significantly shorter than that of $t_c=222$ days corresponding to the least construction cost of $c_{\min}=\$4{,}797{,}188$. It should be noted that this speeding up is accompanied by a higher construction cost of $c_{\text{tcb}}=\$5{,}018{,}182$ and is achievable based on the model or the contractor's construction time-cost relationship. However, there may be concerns that the contractor cannot finish the project on schedule, resulting in losses to the client. There is also another possibility that the contractor finishes the project earlier than schedule, which should be rewarded but is not reflected in the price-time bidding model. To deal with these issues, a price-time with incentive/disincentive (I/D) bidding model can be applied (Herbsman et al. 1995). Taking the above case as an example, this new model views the construction time of $t_{\text{tcb}}=177$ days originally proposed by the contractor as the target time. If the contractor finishes the project 1 day *later* than the target time, it will be charged by the client a disincentive of \$10, 000 (which generally is the UTV); if the contractor finishes the project 1 day *earlier* than the target time, it will be rewarded by the client as an incentive of \$10, 000. In this way, risks faced by both parties can be reduced and better bidding strategies can be found. For example, the contractor can subtract \$100, 000 (I/D for 10 days) from the above optimal bid price of \$5, 018, 182 and propose a construction time of 187 days and bid price of \$4, 918, 182, which results in the same TCB of \$6, 788, 182. If the contractor finishes the project in 177 days, it will receive \$100, 000 incentives for early completion, which makes its profit unchanged but with less risk and hence forms an improved bidding strategy.

Another issue is that the client may announce an estimated or target completion time, t_e. If the client just rewards early completion and penalizes late completion, the price-time bidding model still applies with the reward/penalty reflected by the UTV (or the I/D in the price-time with the I/D model). However, if the client does *not* accept late completion, a contractor *must* propose a construction time t equal to or shorter than the client's time estimate t_e, which forms a bidding constraint. In this case, the bidding model becomes a *constrained* optimization problem: minimizing TCB subject to $t\leqslant t_e$, whose solution or the resulting optimal price-time bidding strategy may be different than that obtained previously from the unconstrained optimization problem: minimizing TCB. Taking the above quadratic model as an example where $c=f(t)\equiv a_2t^2+a_1t+a_0$, $p=(1+\alpha)f(t)$, $\text{TCB}=p+\text{UTV}\times t=(1+\alpha)f(t)+\text{UTV}\times t$, the original *unconstrained* optimal construction time is given by Eq. (12) as $t_{\text{tcb}}=-[a_1+\text{UTV}/(1+\alpha)]/(2a_2)$, which corresponds to a construction cost of $c_{\text{tcb}}=f(t_{\text{tcb}})$ as given in Eq. (15), a bid price of $p_{\text{tcb}}=(1+\alpha)f(t_{\text{tcb}})$, and

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a minimum TCB of $\mathrm{TCB}_{\min}=(1+\alpha)f(t_{\mathrm{tcb}})+\mathrm{UTV}\times t_{\mathrm{tcb}}$ as given in Eq. (16). If this *unconstrained* optimal $t_{\mathrm{tcb}}\leqslant t_e$, then t_{tcb} is still optimal in the *constrained* case, $p_{\mathrm{tcb}}=(1+\alpha)f(t_{\mathrm{tcb}})$ is still the optimal bid price, and $\mathrm{TCB}_{\min}=(1+\alpha)f(t_{\mathrm{tcb}})+\mathrm{UTV}\times t_{\mathrm{tcb}}$ is still the minimum TCB. However, if the *unconstrained* optimal $t_{\mathrm{tcb}}>t_e$, then t_{tcb} is not feasible and t_e will be the new optimal construction time in the *constrained* case, $p_e=(1+\alpha)f(t_e)$ will be the new optimal bid price, and $\mathrm{TCB}_e=(1+\alpha)f(t_e)+\mathrm{UTV}\times t_e$ will be the new minimum TCB.

Summary and Concluding Remarks

Under the price-time biparameter bidding system where the client predetermines a linear weighting formula to calculate TCB, a target bidder's objective is simply to get the lowest possible TCB according to its own capability because this maximizes its chance of winning the contract. Usually, a contractor can design a few sets of feasible bid prices and construction times which can then be converted to a few TCBs. Only one bid price and construction time combination will result in the "lowest" TCB, which is the "optimal" bid price and construction time combination that could be submitted to the client. However, these "optimal" TCB, bid price, and construction time may not be truly optimal since, without knowing other feasible TCBs, bid prices, and construction times or without knowing the TCB continuum, such preclaims of optimality are not convincing. To resolve this problem, Shen et al. (1999) offer contractors a simple approach to identify their true optimum price-time bidding strategies. The approach assumes a quadratic functional relationship between a contractor's bid price and construction time, and requires the contractor to develop a TCB continuum through estimating the said function based on three "typical" sets of bid price and construction time combinations.

The present paper continues the work of and brings improvements over Shen et al. (1999) in certain aspects. First, construction cost (rather than bid price) and its relationship to construction time are directly studied, since they are more directly related to the construction project and the contractor's operations. Second, the general and quadratic construction time-cost functions are examined in detail, and the formulas for the optimal bidding strategy leading to the lowest TCB are deduced, which can help contractors better understand the constraints in their operations and the conditions in their optimal bids. Third, it is noted that the three "typical" sets of construction time and price or cost combinations suggested by Shen et al. (1999) are not very appropriate to estimate the time-cost parabola since they are all in the left or decreasing part of the parabola. It is thus suggested that, whenever feasible, a few more time-cost data points in the right or increasing part of the parabola should be added to estimate the parabola more accurately. The illustrative example provided in the paper confirms this observation. Finally, this paper's detailed examinations of the price-time biparameter bidding model can suggest ideas for construction bidding or contractor selection with more criteria (Cheng et al. 2000; Hatush and Skitmore 1997; Shen et al. 2004). In multicriteria construction bidding, such matters as developing valid measurements and choosing sensible weights for the many criteria certainly add extra complexity, and it may also be difficult to determine an appropriate decision formula to evaluate the competing contractors' strengths and weaknesses. All these could be better accomplished by analyzing the interrelationships among the criteria using the contractors' experience or estimated data, which apparently is a natural extension to the current paper's major scheme.

However, certain "external" constraints should be satisfied to effectively apply the proposed optimal bidding model, including a high degree of confidence from both the client and the contractor in relevant bidding documents as well as a low potential of geotechnical or environmental unknowns and third party interference to affect the project's progress (Shen et al. 1999). There are also some "internal" limitations in this research. A noticeable one is that, due to the difficulty in obtaining contractors' real data, the current paper only uses artificial data to build and explain the optimal bidding model, making it a theoretical one which may lack enough reliability and usefulness. Hence, as a suggestion for future research in forming a contractor's optimal bidding strategy for a construction project, practical data should be secured and applied whenever possible. This not only includes the contractor's *estimated* time-cost data related to the bidding project as done in this paper, but can also be other types of data such as the contractor's historical bidding data and other contractors' previous bidding data for similar projects. With practical data at hand, which makes extensive numerical analyses and various comparisons feasible, the methodology given in this paper could help the contractor reliably estimate its time-cost function and form its optimal bidding strategy. Another limitation of this paper is that the TCB is calculated from the contractor's perspective only, but in practice the client also needs a clear methodology to validate, adjust, or even recalculate the contractors' TCBs for awarding the project, which is an important future research topic for which the approach proposed in this paper could be useful.

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References

Callahan, M. T., Quackenbush, D. G., and Rowings, J. E. (1992). *Construction project scheduling*, McGraw-Hill, New York.

Carr, R. I. (1982). "General bidding model." *J. Constr. Div.*, 108(4), 639–650.

Cheng, W. L., Li, H., and Love, P. (2000). "Establishment of critical success factors for construction partnering." *J. Manage. Eng.*, 16(2), 84–92.

Clough, R. H., Sears, G. A., and Sears, S. K. (2000). *Construction project management*, 4th Ed., Wiley, New York.

Drew, D. S., Shen, L. Y., and Zou, P. X. W. (2002). "Developing an optimal bidding strategy in two-envelope fee bidding." *Constr. Manage. Econom.*, 20(7), 611–620.

Drew, D. S., and Skitmore, R. M. (1997). "The effect of contract type and contract size on competitiveness in bidding." *Constr. Manage. Econom.*, 15(5), 469–489.

Friedman, L. (1956). "A competitive-bidding strategy." *Oper. Res.*, 4(1), 104–102.

Gates, M. (1967). "Bidding strategies and probabilities." *J. Constr. Div.*, 93(1), 75–107.

Hatush, Z., and Skitmore, M. (1997). "Criteria for contractor selection." *Constr. Manage. Econom.*, 15(1), 19–38.

Herbsman, Z. J., Chen, W. T., and Epstein, W. C. (1995). "Time is money: innovative contracting methods in highway construction." *J. Constr. Eng. Manage.*, 121(3), 273–281.

J. Constr. Eng. Manage., 2009, 135(4): 298-306

Mubarak, S. A. (2005). *Construction project scheduling and control*, Pearson Prentice-Hall, Upper Saddle River, N.J.

Park, W. R., and Chapin, W. B. (1992). *Construction bidding*, 2nd Ed., Wiley, New York.

Shen, L. Y., Drew, D. S., and Zhang, Z. H. (1999). "Optimal bid model for price-time biparameter construction contracts." *J. Constr. Eng. Manage.*, 125(3), 204–209.

Shen, L. Y., Li, Q. M., Drew, D., and Shen, Q. P. (2004). "Awarding construction contracts on multicriteria basis in China." *J. Constr. Eng. Manage.*, 130(3), 385–393.

Singh, D., and Tiong, R. L. K. (2005). "A fuzzy decision framework for contractor selection." *J. Constr. Eng. Manage.*, 131(1), 62–70.

J. Constr. Eng. Manage., 2009, 135(4): 298-306