Modeling and Simulating Learning Development in Construction

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Abstract: The time required to perform a given process in a repetitive construction environment tends to fall progressively as the same process is repeated for a sufficient number of successions. This paper discusses the aspects and fundamentals of learning development and its impact on the time requirement of repetitive construction processes from a simulation perspective. Common learning curves and their basic parameters and equations are highlighted. Factors that determine the level of learning rates for construction tasks are also investigated. We also introduce a statistically based approach for modeling learning development. Modeling learning phenomena in a simulation experiment is then introduced as applied in MicroCYCLONE (Halpin 1990). An example application is presented reflecting the impact of learning on the time requirement on a high-rise building; the statistically based approach is compared to the currently used deterministic models. This paper presents a simulation-based methodology for incorporating learning development in process simulation modeling and experimentation. In particular, it is suggested that a stochastic learning model be adopted due to the random factors affecting learning in construction. The paper highlights mathematical models often applied in modeling learning development and reviews the factors that contribute to these phenomena. An example application is also presented to illustrate how learning models can be incorporated into a simulation experiment. The effect of learning on the time required to complete a given process, and the significance of using stochastic versus deterministic learning models on various performance measures are also discussed.

Introduction

The impact of learning development is often addressed in the planning of repetitive construction processes (Ashley 1980). The outcome of learning development is a better understanding by labor of the work responsibilities and an improved knowledge of how to perform them. It is a phenomenon that yields an improvement in the productivity of labor and results in an increase in the production of the process. This trend of improvement would eventually converge to zero if the process were repeated for a sufficient time.

The essence of repetition and continuity that evolves in linear construction processes allows for such a phenomenon to occur. The structural process of a high-rise building is a typical example. The process is repeated as the construction phase advances from one floor to another. As learning develops with time, a decrease in the cycle time of involved repetitive tasks takes place. Consequently, this improvement influences the overall duration of the process as well as the project's duration.

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The Economic Commission for Europe ("Effect" 1965) pioneered a study that focused on the impact of learning development on different repetitive construction operations. The report included several proposed learning rates for different construction operations as shown in Table 1. Using the findings of the previous report, Gates and Scarpa (1972, 1978) analyzed the impact of learning development, determining the labor assignment for different repetitive operations. Thomas et al. (1986) investigated the applicability of different learning curve models and concluded that the S-curve model provided the best result. A recent work by Touran et al. (1988) analyzed the level of learning rate in formwork operations and showed that the straight-line model can be used to model the learning development of this type of process.

BASIC ELEMENTS OF LEARNING CURVES

The time required to perform successive iterations of the same process decreases as workers develop learning skills, resulting in a higher production rate as shown in Fig. 1. The curve representing improvement in time can be divided into two general phases, as shown in Fig. 1(*a*).The first phase represents the reduction in time requirement as a result of learning development. The second phase, often referred to as the "plateau phase," reflects the minimum time requirement to perform the process. Two points on the curve are of particular significance: (1) The startup point; and (2) the standard production point.

The startup point represents the starting time required to complete the first cycle of the process, which is influenced by the time required to perform involved tasks. The time needed to perform these tasks is greatly affected by the previous knowledge of participating laborers prior to the start of the process. It is normally expected that with higher previous knowledge of the general process, laborers tend to spend less time performing involved tasks and achieve stabilization in the time required. Work by Cherrington et al. (1987), Baloff (1970), and Hoffmann (1968) addressed the relationship of previous experience and startup time of repetitive processes.

The standard production point is the point that separates the two phases

TABLE 1. Proposed Learning Rates for Selected Construction Process [L = Learning Rate as Defined in (2)][a]

Description (1)	L(%) (2)
Entire structure of ordinary complexity such as high-rise office building and tract housing.	95
Individual construction elements requiring many operations to complete such as carpentry, electrical work, plumbing, erection and fastening of structural units, concreting.	90
Individual construction elements requiring few operations to complete such as masonry, floor and ceiling tile, painting.	85
Construction elements requiring few operations and on-assembly line basis such as field fabrication of trusses, formwork panels, and bar bending.	80
Plant manufacture of building elements such as doors, windows, kitchen cabinets, and prefabricated concrete panels.	90–95

[a]The United Nation (1965) and Gates and Scarpa (1972, 1978).

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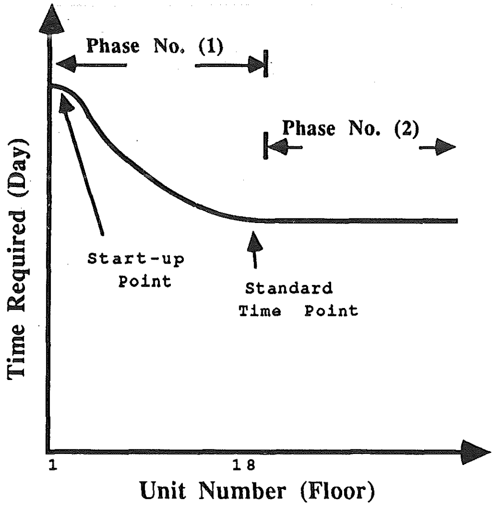
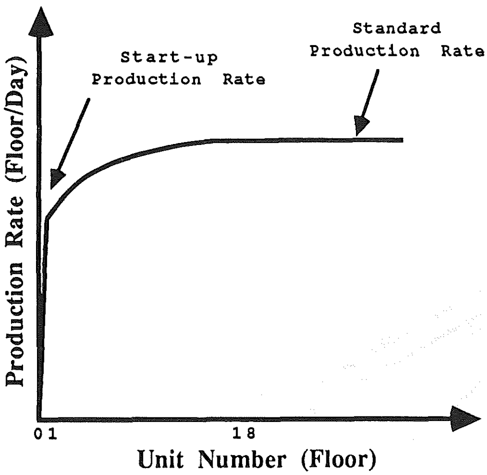


(a)



(b)

FIG. 1. Impact of Learning Phenomena on Time Requirement and Production Rate of Repetitive Processes

and reflects the minimum duration (i.e., standard time) of the process. As the process advances beyond this point, the time required to perform the process remains approximately constant and learning development will show no effect on processing time requirement. Work by Corlett and Morcombe (1970) and previously by Thomopoulos and Lehman (1969) evaluated the relationship between the learning curve of a process and its standard time.

A number of mathematical learning curve models have been introduced since the original work of Wright in 1936. These models were developed based on the best geometric functions that fit collected data. The well-known learning models are shown in Fig. 2. Of these models, the straight-line model is the most commonly used due to the difficulty associated with estimating the parameters of other models. A brief summary of this model is presented in this section. The remainder of the models are summarized in Appendix II for the interested reader. Detailed references and coverage of these models can be found in Spencer (1986), Thomas et al. (1986), Spears (1985), Carlson (1961, 1973), Cochran (1960), and Conway and Schultz (1959).

Straight-Line Model

The straight-line model was first introduced by Wright (1936) and first implemented by the Boeing Company in the aircraft manufacturing industry. The model assumes that the pattern of improvement in processing time due to learning follows a logarithmic scale. The model yields a straight line, as shown in Fig. 2, when plotted on a log-log scale. The straight line model is given by

$$Y_X = aX^n \quad \text{(1)}$$

where

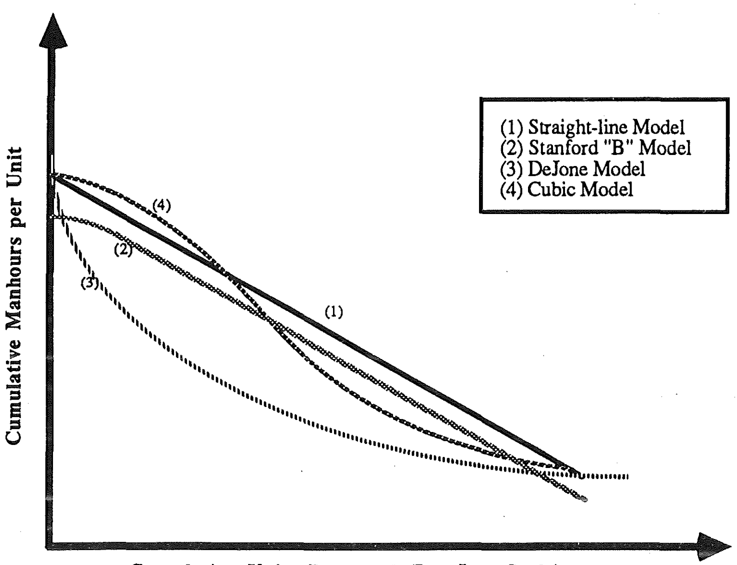


FIG. 2. Typical Learning Models Plotted on Log-Log Scale

$$n = \log_2 L = \frac{\log_{10} L}{\log_{10} 2} \quad (2)$$

and Y_X = the time required for iteration number X, a = the time required to produce the first unit; n = the slope of the logarithmic curve; and L = a measure of the learning rate expressed as a percentage.

The value L is the most significant parameter of the straight-line model variables. Reduction in time due to learning will increase as the value of L decreases, as shown in (1). Fig. 3 shows three different cases of learning rates each corresponding to a given value L. In the case where the time requirement is constant, no learning improvement is taking place, and the value of the learning rate is equal to the slope of the line (zero in this case) subtracted from 100%. Therefore, L = 100%. For L = 90%, the first two floors show an improvement of 10 hr, yielding a slope of 10%, and therefore, a learning rate of 10% subtracted from 100% equals 90%. Thus, as the value of learning rate decreases, a larger learning improvement effect should be anticipated.

Factors Determining Learning Rates

Considering the preceding discussion, the effect of learning development is typically expressed as a function of the learning rate L. This parameter

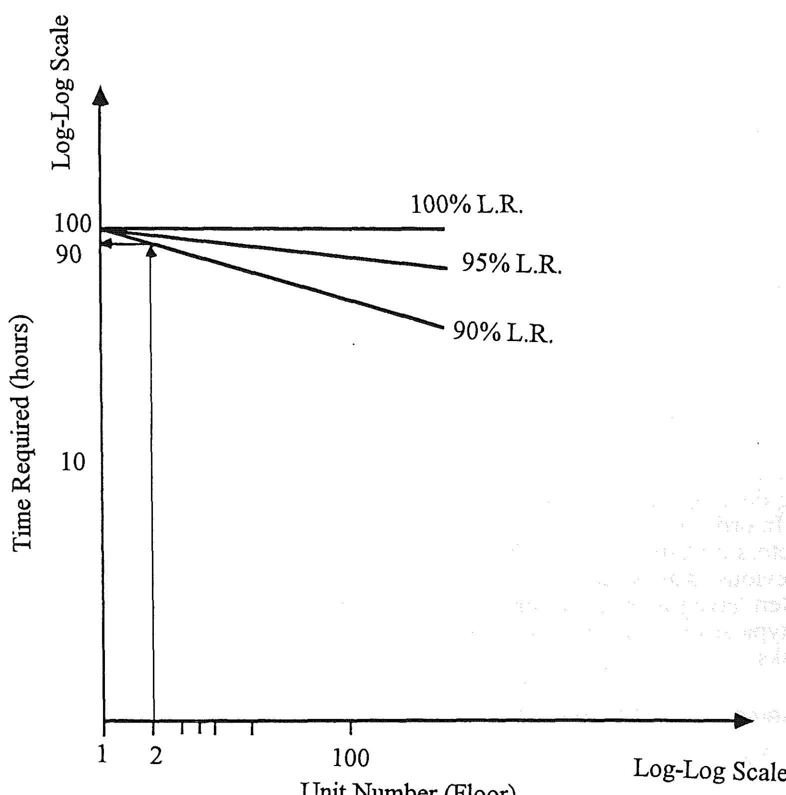


FIG. 3. Learning Curves of Straight-Line Model with Different Learning Rates

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is usually assumed to be constant, as it is addressed by the straight-line and other learning models. Other models (e.g., the cubic model) assume that the value of the learning rate changes due to the influence of other factors, yet the value of the learning rate follows a specified pattern in determining the impact of learning development. Based on the specified level of the learning rate, the basic equation of each model can then be used to predict the impact of learning development.

In practice, the value of the learning rate assigned to a particular task infers the speed of buildup in work familiarity for the involved labor. Similar to labor productivity, learning rate is greatly influenced by various factors, as summarized in Fig. 4. Among the listed factors, Tanner (1985) found that characteristics of the task itself have the greatest impact. Subfactors, e.g., complexity, newness, hazard, danger, and boredom, dictate the characteristics of the task. Each of these subfactors is also determined based on various parameters. The level of hazard associated with a given task can be measured based on the level of dust, radiation, and pressure encountered, for example.

Another essential factor that affects the level of learning rate is the skill of management on job sites. Planning strategies, incentive programs, safety precautions, and availability of required materials and tools are management-related subfactors. An effective planning system and a knowledgeable management team can induce a climate in which learning will be relatively rapid. Adequate management supervision, motivation, and an incentive program will further aid in achieving the accomplishment of this objective. Inadequate planning can result in efficiency and errors in the completed tasks which, when corrected, will give an unsatisfactory rate of learning.

Morale level, possessed skills and cognitive skills are subfactors that determine the characteristics of labor on site. A high morale level reflects a positive environment that encourages learning development. Labor that possesses good skills and mental efforts will result in a quick learning development. The absence of these subfactors yields a negative influence on learning improvement. Such subfactors will have a direct influence on the duration of tasks for which labor crews are responsible.

Other factors that influence the learning rate are a function of the project characteristics such as the weather conditions, altitude of work, accessibility of project site, equipment breakdown, interruption (e.g., accidents and holidays), and project location. With an increase in height of the working area (e.g., high-rise buildings), fear of heights will have an influence on the productivity of labor, as well as on the enhancement of learning skills. Like management, the influence of these external subfactors will affect the learning development of all crews involved in the whole project.

In order to specify a learning rate for each task, analysis of these various factors and subfactors collectively must be taken into account. Among all previous factors, characteristics of the task itself, as indicated previously, often have the greatest impact. The work by Touran et al. (1988) provides a typical example for measuring the learning rate of typical construction tasks.

Variability of Learning Rates

The learning rate of each construction task is a function of various factors. These factors often vary during the life of the project due to normal construction processes. Changes in the progress of construction (e.g., planning changes, equipment breakdowns, interruptions, temperature, and weather)

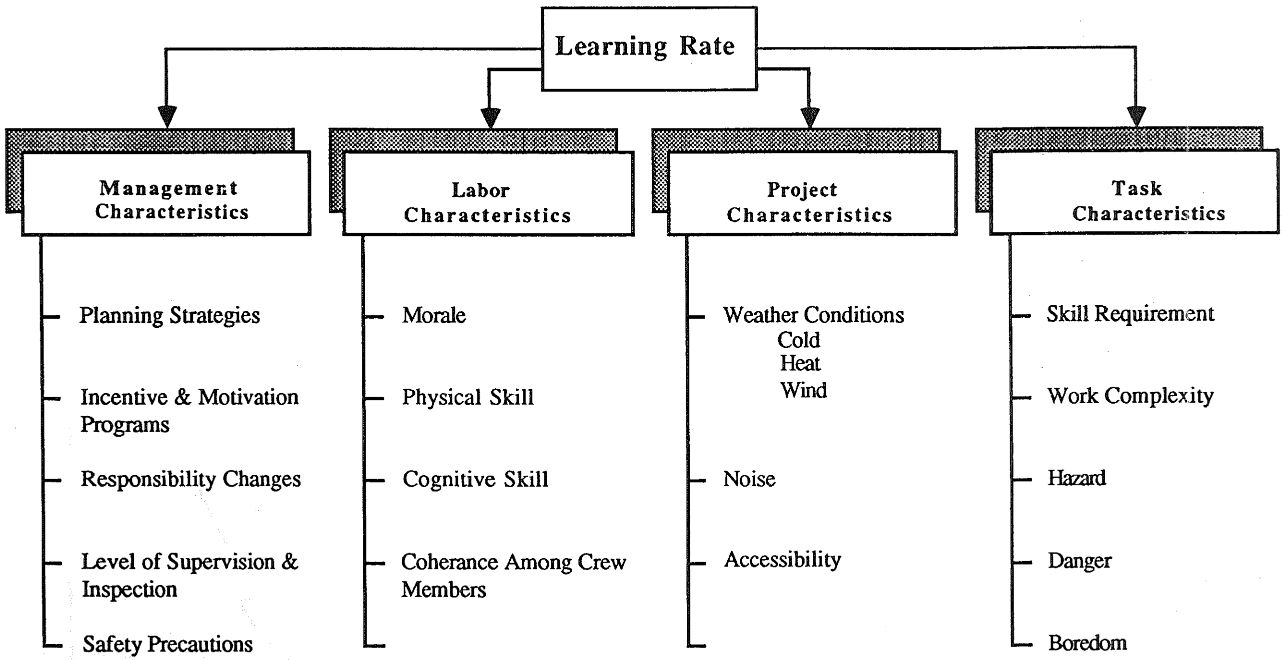

FIG. 4. Major Factors Determining Learning Rates

have an inconstant influence on the previous factors noted, as well as on the value of the learning rate, as schematically shown in Fig. 5. The specified value of learning rates are expected to fluctuate randomly according to an underlying random process from one time event to another. It is therefore suggested that learning rates be modeled as random variables rather than deterministic ones, as shown in Fig. 6.

Consider first the example of work changes. Researchers in labor psychology and in the manufacturing industry have indicated that the accumulated skills of labor can decrease time to achieve stability in the production by laborers [see Spears (1985), Bricston (1978), and Haldham (1970)]. Similarly, interruption or breakdowns due to either failure of inspection, labor strike, or long holidays can impact the accumulated learning skills of laborers. The impact of interruption is related to the time interval involved. The longer the time period, the higher will be the impact.

Other essential and interrelated factors are weather and noise. The variation of weather conditions can slow down or improve learning development. These influences (i.e., weather and noise) fluctuate over time, creating a random influence on learning improvement.

Due to the inconsistent effect of these factors, learning rates actually behave randomly and can be better viewed as random processes. This ne-

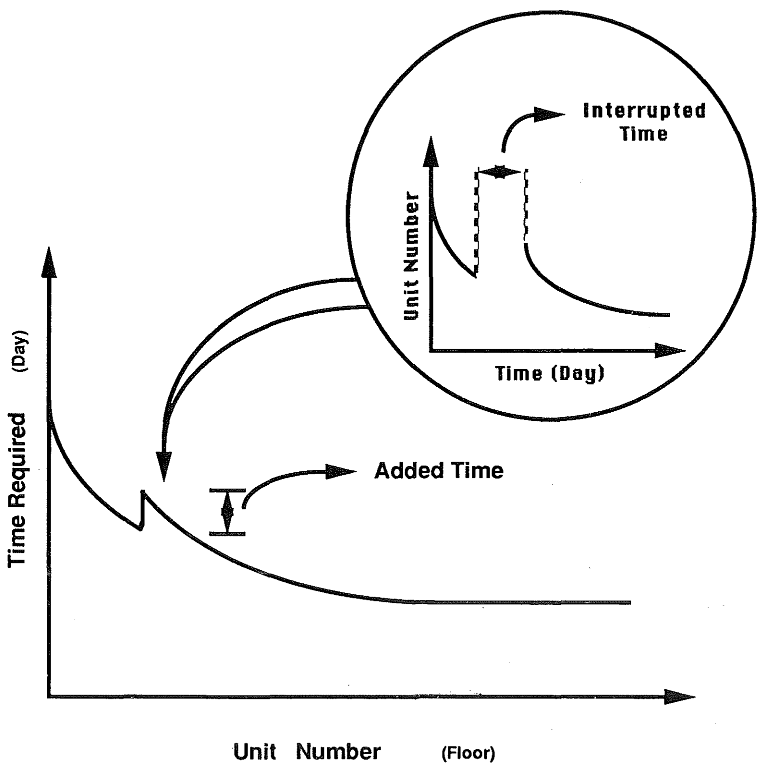

FIG. 5. Schematic Representation of Effect of Interruptions on Learning Development

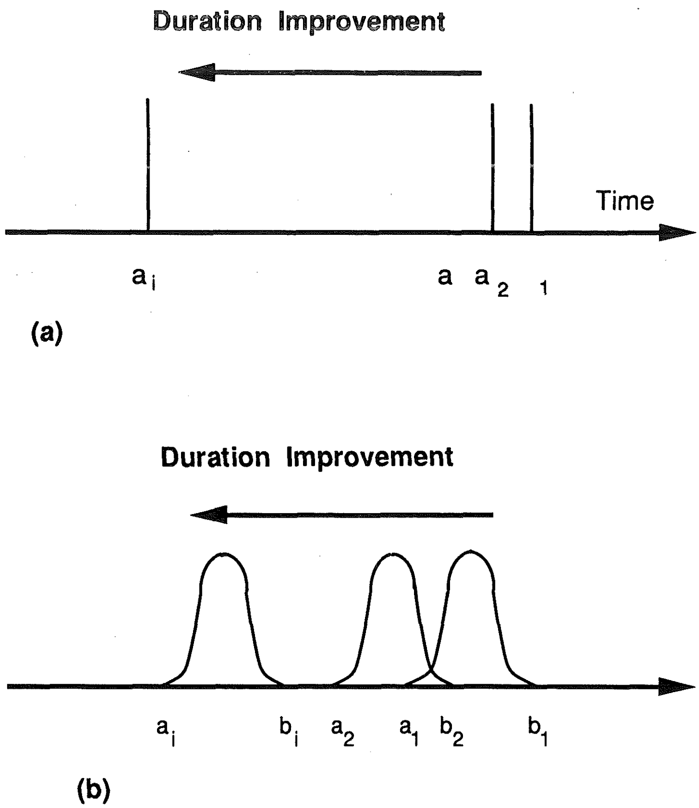

FIG. 6. Schematic Representation of Models: (*a*) Deterministic Duration Parameters; (*b*) Stochastic Duration Parameters

cessitates modeling the learning development as a region bounded by two curves rather than one deterministic curve, as shown in Fig. 7.

The improvement in time as the number of units increases is represented by the shaded region shown in Fig. 7, in contrast with the deterministic curve of Fig. 2. In this illustration, learning development is bounded between the rates 90% and 100% to a most likely learning rate of 95%. The model shown in Fig. 7 suggests that the rate at which improvement in time takes place as the number of units increases may vary between the set boundaries according to the underlying random process being modeled. To illustrate this point, the time required to complete the first floor unit as shown in Fig. 7 is 100 hr. At the 10th floor, the rate of improvement can be anywhere between 90% and 100%, yielding a time requirement for this floor to be between 75 and 100 hr depending on the sampled value of the learning rate.

In a simulation experiment, the same analogy used in this illustration can be applied. When a duration is required for a given work task, say the 10th unit in this simple example, a learning rate is sampled. In this case, a triangular distribution is chosen to model the underlying process because the minimum, most likely, and maximum are the only values available. Therefore, L triangular (90, 95, 100) is the statistical model for the learning

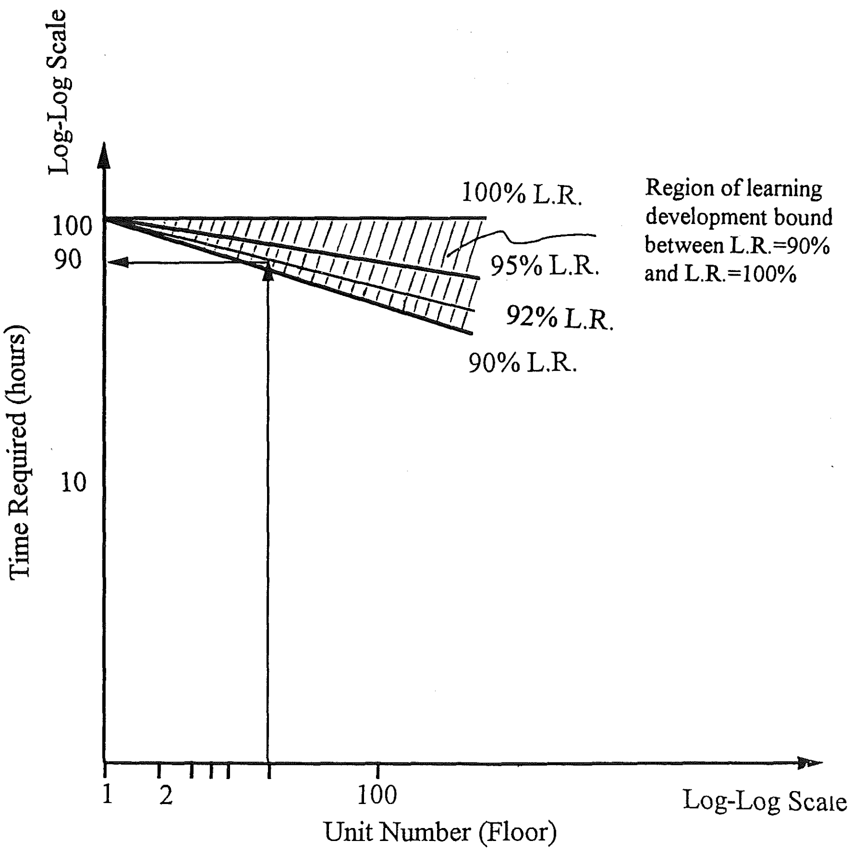

FIG. 7. Stochastic Model for Learning Development

rate. A uniform random number on the interval (0, 1) is generated and transformed into the required triangular distribution with the given parameters yielding a learning rate of 92%, for example. Graphically, a new line corresponding to the 92% learning rate is plotted on Fig. 7. Mapping a vertical line from the 10th floor point to the 92% line and then drawing a horizontal line to the vertical axis yield the required duration of the task. In this illustration, the value is 78 h. Although the illustration was graphical, in a simulation experiment the same analogy is employed, but numerical methods rather than graphing techniques are utilized. It should be noted at this point that the learning rate is always applied to the duration of the previous unit. Thus, an increase in duration is not possible; this agrees with reality.

This method was implemented in MicroCYCLONE according to the algorithm given in Appendix III. In theory, any distribution can be used to model the underlying random process driving the learning rate fluctuations. The choice of the triangular distribution in this paper is rather arbitrary for purposes of illustration. When a strong data base is available, a more flexible distribution, e.g., the β-family, can be used to accurately model the underlying process using a representative sample.

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The stochastic-based learning development methods presented in this section better reflect the true nature of the problem and allow for more flexibility in dealing with the learning phenomena. The advantages of the method are highlighted in the example application in the following section.

EXAMPLE APPLICATION: STRUCTURAL PROCESS FOR HIGH-RISE BUILDING

To demonstrate the potential of the proposed methodology as well as the impact of learning on the output of a construction process, the method is applied to the simulation of the structural process of the Peachtree Plaza Hotel as modeled by Halpin (1976). The building structure of the hotel involved the construction of 63 repetitive floors that were identical and required the same construction processes. Each floor is a circular slab with a diameter of 116 feet. A floor consists of 10 sections, defined by shear walls radiating from the center of the slab. Every slab has a center core and a walkway at the center with shear wall boundaries. A detailed discussion of the process is presented in Halpin and Woodhead (1976). A CYCLONE model of the process is shown in Fig. 8.

In the context of this analysis, three general cases were examined. The first case is a simulation experiment where there has been no impact for learning development from one floor to the other. The second case involves

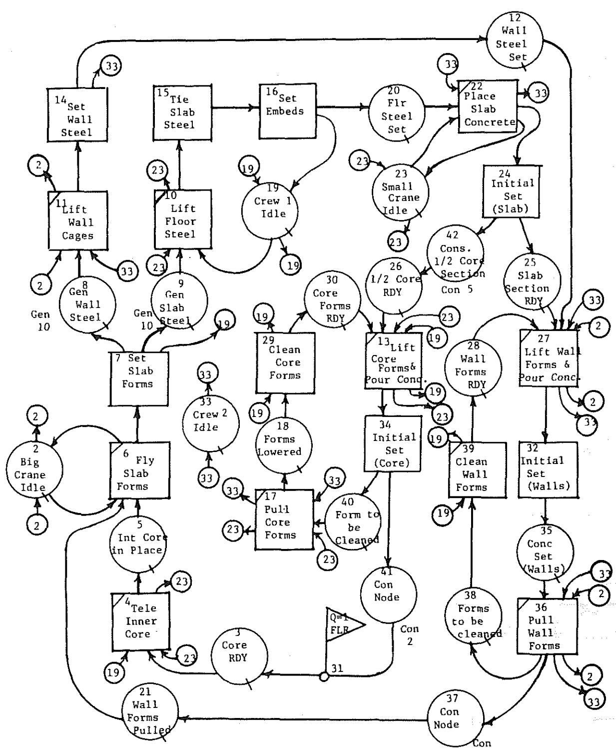

FIG. 8. CYCLONE Model of Structural Process

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an experiment with a learning rate of 95% applied to all labor-intensive tasks. This value was chosen based on data given in Table 1. The third case considers the impact of learning with a variable learning rate, which is modeled by a triangular distribution with the parameters (90%, 95%, 100%).

The straight-line learning model was used in this method for purposes of illustration. A similar approach can be used for other models. The triangular distribution was selected as a stochastic distribution since it provides a good estimate when the most likely value can be ascertained along with a maximum and a minimum value (Pritsker 1986). It was also assumed that the standard time requirement of each labor-intensive task will not be less than 85% of the initial estimated time of that task. For example, if the starting time for a crew to perform the forming of the first slab is 3 days (i.e., 30 hr, 10 hr/day), then as this task is repeated for enough cycles, the minimum time (i.e., standard time) to repeat this task is $30 \times 0.85 = 25.5$ hr.

For each of the previously mentioned cases, 12 different independent simulation runs were conducted using different initial seed numbers. In each run different initial seed numbers for all tasks were selected. Common random numbers were used across the three cases to facilitate comparison of the results. Fig. 9 shows the estimated mean time requirement of each floor for the three different cases as estimated from the 12 simulation runs conducted for each experiment.

For a learning rate of 100% (i.e., no impact for learning), the estimated mean time required to complete each of the 63 floors were approximately equal. The values ranged between 7.54 days/floor and 7.81 days/floor, with a difference of 0.27 days (2.7 hr). Variation in the estimated mean time requirement from one floor to another was due to the variability in time duration of the involved tasks. For this experiment, the values of the startup time and the standard production time were equal.

In the second and third cases, the estimated time requirement curves for both cases were descending. The influence of learning development was clear in the first 15 floors; however, the effect of learning then started to

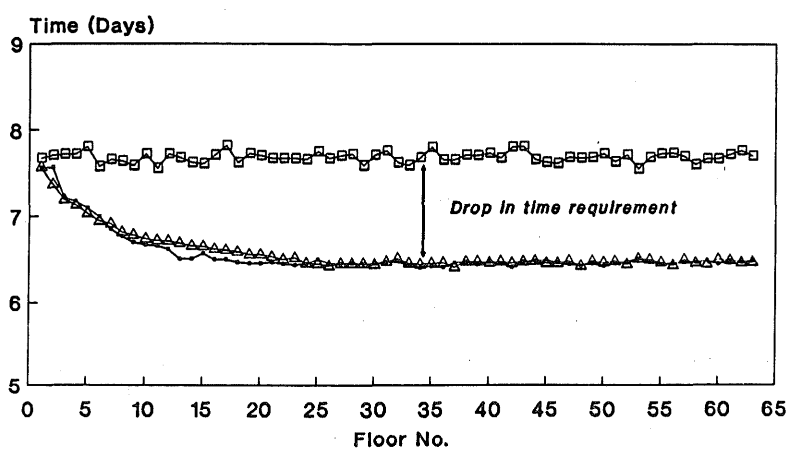

FIG. 9. Impact of Learning Development on Time Requirement Using Different Learning Rates

vanish. The estimated mean values of time requirements at the startup point in both cases were 7.56 days/floor for $L = 95\%$ and 7.57 days/floor for L = triangular (90%, 95%, 100%). The estimated mean times at the standard production point were 6.45 days/floor and 6.53 days/floor for the second and third cases, respectively. The mean values of both cases are very close, yielding very close and overlapping curves. This is to be expected since the most likely value selected for the triangular distribution of the third case equals the deterministic value of the second case. The same cannot be inferred about the levels of variability in the results, which should be clearly reflected when confidence intervals are constructed around the estimate of the mean.

When a confidence level of 95% was constructed, the results of both cases [i.e., $L = 95\%$ versus L = Triangular (90%, 95%, 100%)], it was evident that there was a significant difference in the size of the output variability of both cases. Fig. 10 shows envelopes of variability covered by the upper and lower curves for each of these cases using a 95% confidence level. For a constant learning rate of 95%, values of the upper bound and the lower bound ranged between 6.38 days and 6.52 days, after the standard production point, which revealed a diminutive variability. However, the magnitude of this variability was greater with the variable learning rate and ranged between 5.8 days and 7.4 days after reaching the standard production point. The reason was due to the variability in the value of learning rates, which added to the variability in the output of the process.

Finally, considering the effect of learning development versus not considering it yields substantially differing estimates of the duration of the process, as well as of the overall project. In the case of $L = 95\%$ or L = triangular (90%, 95%, 100%), the mean time requirement to complete all 63 floors was found to be approximately equal to 415 days. However, assigning an L value of 100% reflected a time requirement of 481 days, which corresponds to a 16% increase in the time estimate. This indicates that when the effect of learning is not considered, the duration of the processes is often overestimated.

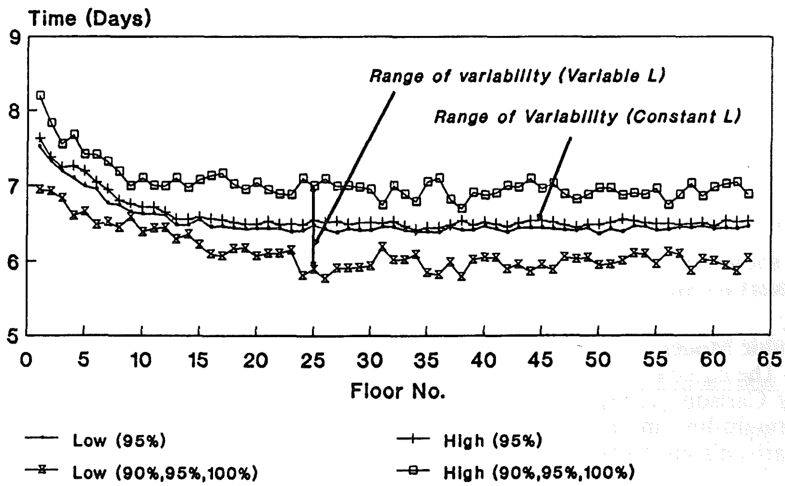

FIG. 10. Differences between Deterministic and Variable Learning Rates (with 95% Confidence on Estimated Mean)

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Conclusion

Various factors influence the time requirement of repetitive construction processes. The time requirement curve does not have a constant slope or a predetermined shape. It is often composed of scattered points that are better modeled by an envelope rather than by a single deterministic curve. This paper demonstrated how a variable learning rate can be incorporated into a simulation experiment by viewing the problem as a random process. The variability of the performance measure in a simulation experiment greatly increases with this approach, yielding a more accurate representation of the real situation. It was also demonstrated that the exclusion of learning development from a simulation experiment can easily lead to overestimates in the project duration.

Appendix I. Brief Review of Commonly Used Learning Models

Stanford *B* Model

The Stanford *B* model is an enhanced version of the straight-line model. This model was introduced by the Stanford Research Institute in 1946. Garg and Milliman (1961) described the Boeing Company's finding that the Stanford *B* model was better at describing the impact of learning development than the straight-line model. This model argues that improvement in labor performance due to previous learning will be smaller at the beginning of the project. However, the trend of this improvement will eventually increase to approximate the same pattern as the straight-line curve. The model is given by

$$Y_X = a(X + b)^n \quad (3)$$

where b = a factor describing the level of experience that was acquired prior to the start of the work. When the variable b is assigned a zero value, (3) will yield comparable results to the straight-line model.

DeJone Model

The DeJone model (1957) is another modification of the straight-line model. It assumes that work organization and perfecting the use of machinery and tools have a great influence on the magnitude of learning development. This modeling was achieved by introducing another parameter m that defines the incompressibility in the time required due to the influence of previous factors. The DeJone model is given by

$$Y_X = a[m + (1 - m) * X^n] \quad (4)$$

It should be noted that when the value of m is set to zero, the equation will revert to the basic equation of the straight-line model.

Cubic Model

The cubic model, which is often referred to as the *S* model, was proposed by Carlson (1973). Carlson indicated that a further enhancement of the straight-line model can be achieved using a curve with multiple slopes. Carlson's model is given by

$$\log Y_X = a + n_1 \log X + c(\log X)^2 + d(\log X)^3 \quad (5)$$

where

$$n_1 = \log_2 L_1 = \frac{\log_{10} L_1}{\log_{10} 2} \quad \text{(6)}$$

and a = the initial time required to perform the task; n_1 = the slope of the initial logarithmic phase; and c and d = constant coefficients that are estimated using the basic equation of the model and another data point along the curve. In addition to the models presented in this section, the literature includes other learning curve models. Carl (1946) proposed a model of learning development using an S type function that differs from the one suggested by Carlson. Guibert (1945) presented a complicated multiparameter function with various restrictive assumptions. Levy (1965) introduced a mathematical model that accounts for learning development in the case of implementing new processes where previous labor experience is minimal. Pegels (1969) proposed an exponential model that includes the impact of startup time. Spears (1985) derived a function with four constants that can reveal aspects of learning and transfer of knowledge. Asher (1956), Carlson (1961), Tanner (1985), Spencer (1986), and Thomas et al. (1986) include a further discussion of these models and a summary of other learning-curve models.

Appendix II. Algorithm for Duration Adjustment with Learning Development

Duration is required for a given task associated with learning improvement at the realization X. The following algorithm is executed:

1. Update realization number X: $X = X + 1$.
2. If $X = 1$, save initial duration in a_1.
3. If learning rate L is deterministic go to step 5.
4. Sample the value of L according to the statistical model specified.
5. Apply straight-line model (or any other model as desired) to get the duration for this particular iteration: $Y_X = aX^n$; ($n = \log_2 L$).
6. Check against threshold value specified Y_{min}: If $Y_X < Y_{min}$ then set $Y_X = Y_{min}$.
7. Update duration record for given task, and continue simulation.

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