Markov-Based Optimization Model for Building Facilities Management

Xueqing Zhang, M.ASCE[1]

Abstract: An effective and efficient building facilities management system calls for a systematic performance-based methodology. This paper proposes an integer-programming model based on the Markov decision process: Markov chains are used to model the change of condition index, a consistent scale to measure building performance, and integer programming employed to optimize annual management actions and annual budget allocation, subject to various types of constraints. A wide scope of useful outputs can be obtained from this model. This includes: (1) annual management actions to take; (2) annual budget allocation; (3) the expected condition index (CI) values at the beginning of each year before and after management actions are taken; (4) the expected annual performance levels; and (5) sensitivity analysis and corresponding outputs to different budget scenarios, for all the elements, components, systems and buildings of a building network and for each year over a long time horizon. As a general model, it can be applied in any type of building network.

DOI: 10.1061/(ASCE)0733-9364(2006)132:11(1203)

CE Database subject headings: Assets; Facilities; Deterioration; Optimization; Service life; Stochastic processes; Markov process.

Introduction

Many public and private organizations have a number of buildings (hereinafter referred to as a building network) under their management to serve their business functions. To maintain the building network at a certain required level of performance (which is a composite measure of a number of factors such as health, safety and environmental requirements, type of occupancy, and the organization's business policy), various facilities management actions have been taken periodically. It is a great challenge to manage a building network efficiently and cost effectively in a long time horizon, given: (1) the complexity of the building network (different sizes, ages, civil/mechanical/electrical systems and functional requirements of different buildings in the network); (2) the wide range of risks and uncertainties (for example, fluctuation of labor and material prices, material properties, construction/maintenance quality, loading and usage, climatic and environmental conditions); (3) the different deterioration mechanisms of different systems of the building network) over a long time horizon; and (4) inadequate resources that are a common problem to both public and private sectors.

One main objective in building facilities management is to maximize the overall performance of a building network over a long time horizon by optimizing periodical management actions and the allocation of limited financial budgets to different buildings and their systems and subsystems, subject to various types of constraints. A desirable facilities management model should be able to answer the following questions:

1. How to measure and model the performance of the building network over a long time horizon?
2. How to define and measure the effectiveness of alternative management actions on the buildings and their systems?
3. How to plan the limited resources for short- and long-term works programs and schedule alternative management actions for these programs?
4. How to optimize the distribution of the limited periodical budgets in the building network and the schedules of periodical management actions?
5. How to predict the level of performance of a building network, its buildings and the systems of each building as a result of the optimized distribution and schedules?
6. What are the expected improvements in the performance of the overall building network and the particular building systems of the network; and what are the projected consequences (e.g., serious deterioration and possible failures of part of the building network) due to inadequate resources?

The writer has launched a research, in which an integer-programming model based on the Markov decision process has been developed with an aim to achieve the objective and answer the questions mentioned previously. As a general mathematical model that can be applied in any building network with minor modification, this model has the following characteristics:

1. A building network is classified into a multilevel hierarchical structure according to the UNIFORMAT II elemental classification, which is a widely accepted standard classification of building networks (Charette and Marshall 1999).
2. Performance of the building network, its buildings, systems, components, and elements is represented by a quantitative and consistent condition index (CI).
3. Management actions are taken on the lowest level (the element level) of the hierarchical structure of the building network. These alternative actions are standardized, for example, as replacement, major rehabilitation, minor rehabilitation, and no action.

[1]Assistant Professor, Dept. of Civil Engineering, The Hong Kong University of Science and Technology, Clear Water Bay, Kowloon, Hong Kong. E-mail: zhangxq@ust.hk

Note. Discussion open until April 1, 2007. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on May 25, 2004; approved on February 3, 2006. This paper is part of the ***Journal of Construction Engineering and Management***, Vol. 132, No. 11, November 1, 2006. ©ASCE, ISSN 0733-9364/2006/11-1203–1211/$25.00.

4. Markov chains have been used to model the change of CIs of all elements in the building network. This includes: (1) the transition probabilities of the CIs due to management actions taken; and (2) the transition probabilities for the period between two consecutive time points when management actions are taken, which are due to the deterioration process.
5. Management decisions are made according to the solutions of an integer-programming model based on the Markov decision process, subject to various types of constraints.
6. The performance of an element in year t of a long time horizon is dependent on its CI at the beginning of year t, the effect of the management action taken on it, and its deterioration in year t. The annual performance of the building network, its buildings, systems, and components are derived based on the performance of the elements.

Performance of Building Facilities

Appropriate Time Horizon

Management actions are usually taken periodically to maintain a building network at a certain level of service. Actions taken in a previous period affects the selection and scheduling of actions in the following periods and consequently affect the performance of the building network in the following periods. The effectiveness of these periodical actions should be evaluated in a long time horizon in order to maximize the overall performance of the building network in this long time horizon by optimizing the periodical actions that are subject to different types of constraints.

The expected service life (ESL) of a typical building in the building network may be used as the time horizon. The ESL is the period from the completion of the building to the time when the building cannot provide acceptable service because of physical deterioration, functional obsolescence and/or high operation costs (Hudson et al. 1997). The ESL can be determined from the design life of critical structure components, based on empirical experience, historical data, laboratory and field tests, projected loading and usage, health and safety requirements, and the specific environment where the building network operates (Hudson et al. 1997).

Standard Building Hierarchical Structure

A systematic approach to facilities management of a building network necessitates a standard classification of building elements. Standard classification facilitates:
1. Reference of a building network, its buildings, systems and elements;
2. Reference of management actions taken on each element of the building network and the corresponding allocation of resources;
3. Data collection, storage, tracking, monitoring, and updating;
4. Data mining and statistical analysis, for example, analysis of the effectiveness and efficiency of alternative management actions on different elements, and cost distribution analysis of these actions;
5. Evaluation and prediction of the performance of different elements over a long time horizon;
6. Life-cycle cost–benefit analysis of alternative management actions; and
7. Reasonable comparison between different buildings in the same network and between different buildings in different networks with an aim to benchmark the best practices.

Table 1. Pavement Serviceability Rating (Based on Alberta Transportation 2001)

Descriptive category	Serviceability
Good	4.1–5.0
Fair+	3.1–4.0
Fair	2.1–3.0
Fair−	1.1–2.0
Poor	0.0–1.0

One widely accepted standard classification is the UNIFORMAT II elemental classification, where elements that are common to most buildings are defined based on their functions regardless of their design specification, materials and/or construction method/technology used (Charette and Marshall 1999). UNIFORMAT II classifies building elements into four levels. Level 1 is the largest element grouping (hereinafter referred to as a system) that identifies major group elements such as the substructure, shell, and interiors. Level 2 subdivides Level 1 (the systems) into group elements (hereinafter referred to as components). For example, the shell is subdivided into three components, the superstructure, exterior closure, and roofing. Level 3 breaks components further into individual elements. For example, the exterior closure is classified into three elements, exterior walls, exterior windows, and exterior doors. Level 4 breaks the individual elements into smaller subelements. For example, the foundation element is divided into four subelements, wall foundations, column foundations, perimeter drainage, and insulation (Charette and Marshall 1999).

Performance Measurement

The CI is often used to measure the performance of infrastructure facilities. It is often a numerical indicator on a continuous scale of $[\alpha,\beta]$, where α represents the worst condition (insufficient facility) and β represents the best condition (new facility). For example, in the present serviceability rating of the pavement of a highway project, $\alpha=0$ and $\beta=5$ (Alberta Transportation 2001). Often, normalized $\alpha=0$ and $\beta=1$ are used. The continuous scale of CI may be converted to several descriptive categories, for example, good, fair+, fair, fair−, and poor as used in the present serviceability rating of pavements that is shown in Table 1.

In-Service Evaluation and Adjustments

A cost effective building facilities management system necessitates a periodical if not continuous in-service recording and evaluation of the input and output data over a long time horizon. Accurate and timely information from in-service evaluation adjusted by a reference to the performance history of building elements is the basis on which to define alternative management actions for different types of elements and to determine a right set of management actions. In-service evaluation involves the following aspects:
1. The CI of each building element at the beginning of each time period before any management action is taken.
2. To what CI value and with what a probability the element will go immediately after a management action is taken at the beginning of this period. This requires evaluating the methods and technologies used to implement the manage-

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211

ment action, assessing the CI of the element immediately after the management action, and the probability with which the element goes to this CI. The actual CI should be checked against the predicted CI obtained from empirical functions in order to adjust and improve these functions.
3. The deterioration of an element in the period between two consecutive management actions. The actual deterioration should be checked against the predicted deterioration and appropriate adjustments made to the empirical deterioration curve of the element. Proper adjustments should also be made to the probabilities with which the element deteriorates to different CIs in a period from a same starting point of CI.
4. The cost related to alternative management actions on an element. The actual cost of an alternative management action should be checked against the predicted cost obtained from empirical functions and corresponding adjustments made to the empirical functions.

Once the CIs of all building elements are determined, the CIs of all components, systems, buildings, and the building network as a whole can be derived as functions of the CIs of the elements, taking into consideration the relative importance of the elements, components, systems, and buildings. This is discussed in the following subsection.

CIs for Building Network

The performance of a building network is dependent on the performance of all the buildings in the network, the performance of a building on the performance of its constituting systems, the performance of a system on the performance of its constituting components, and the performance of a component on the performance of its constituting elements. Therefore, the overall performance of a building network is eventually dependent on the performance of all the building elements.

Composite Element Condition Index

The CI of a building element is a composite measure of a number of key factors such as distress, structural capacity, safety, appearance, obsolescence, and serviceability (Hudson et al. 1997), which can be defined as follows:

$$\mathrm{CI}_t^{bsce} = \sum_{i=1}^{m_{bsce}} w_i^{bsce} v_{ti}^{bsce}$$

$$\sum_{i=1}^{m_{bsce}} w_i^{bsce} = 1$$

$$0 \leqslant v_{ti}^{bsce} \leqslant 1 \quad \text{for } i = 1,2,\ldots,m^{bsce}$$

where CI_t^{bsce}=CI of element e of component c of system s of building b at time t; m_{bsce}=number of factors to be considered in the assessment of CI_t^{bsce}; w_i^{bsce}=weight for factor i; and v_{ti}^{bsce}=value assigned to factor i.

Component CI

The CI of a component is determined by the following equations:

$$\mathrm{CI}_t^{bsc} = \sum_{e=1}^{m_{bsc}} W_{bsce} \mathrm{CI}_t^{bsce}$$

$$\sum_{e=1}^{m_{bsc}} W_{bsce} = 1$$

where CI_t^{bsc}=CI of component c of system s of building b at time t; m_{bsc}=number of elements included in this component; and W_{bsce}=weight assigned to element e of component c.

System CI

The CI of a system is determined by the following equations:

$$\mathrm{CI}_t^{bs} = \sum_{c=1}^{m_{bs}} W_{bsc} \mathrm{CI}_t^{bsc}$$

$$\sum_{c=1}^{m_{bs}} W_{bsc} = 1$$

where CI_t^{bs}=CI of systems s of building b at time t; m_{bs}=number of components included in system s; and W_{bsc}=weight assigned to component c of system s.

Building CI

The CI of a building is determined by the following equations:

$$\mathrm{CI}_t^{b} = \sum_{s=1}^{m_{b}} W_{bs} \mathrm{CI}_t^{bs}$$

$$\sum_{s=1}^{m_{b}} W_{bs} = 1$$

where CI_t^{b}=CI of building b at time t; m_b=number of systems included in building b; and W_{bs}=weight assigned to system s of building b.

Building Network CI

The CI of the building network at time t, CI_t^N, is determined by the following equation:

$$\mathrm{CI}_t^{N} = \sum_{b=1}^{m} W_b \mathrm{CI}_t^{b} = \sum_{b=1}^{m} \sum_{s=1}^{m_b} \sum_{c=1}^{m_{bs}} \sum_{e=1}^{m_{bsc}} W_b W_{bs} W_{bsc} W_{bsce} \mathrm{CI}_t^{bsce}$$

$$\sum_{b=1}^{m} W_b = 1$$

where m=number of buildings in the network; and W_b=weight assigned to building b.

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211

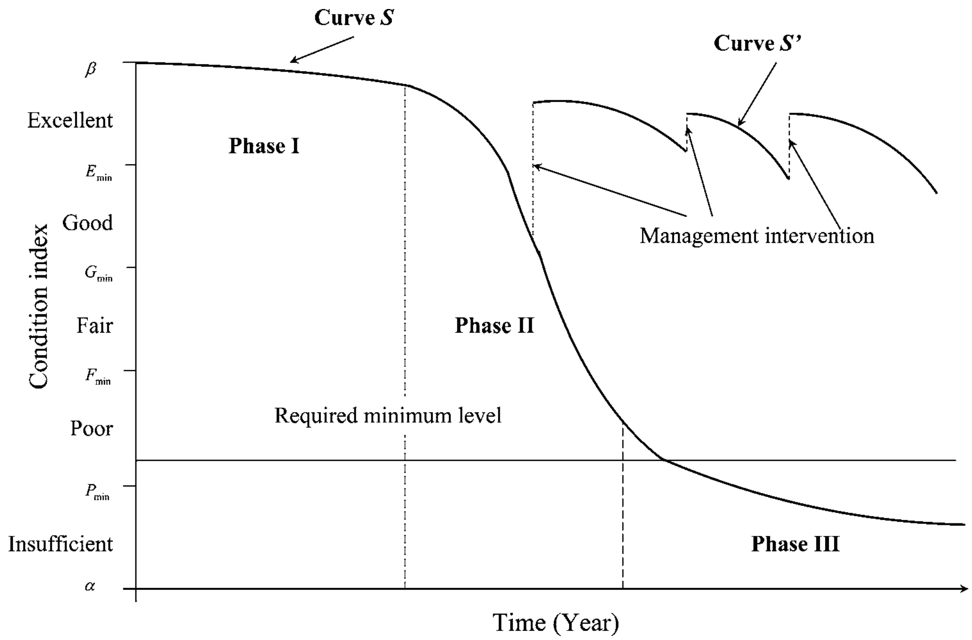

Fig. 1. Performance curve under periodical management actions (based on Hudson et al. 1997)

Markov Decision Process for Building Facilities Management

CI Changing Process

Typical Deterioration Process in Absence of Management Actions

In the absence of any management actions, the deterioration of an element would follow an empirical deterioration process, which is a function of a number of factors such as material properties, construction quality, load, usage, years in service, and environmental conditions. An S-shaped curve is usually used to represent this deterioration process (Hudson et al. 1997). As shown in Fig. 1, at any time point t, the slope of the curve (indicating the rate of deterioration) is less than zero and $\alpha \leqslant \mathrm{CI}_t^{bsce} \leqslant \beta$. This curve is usually characterized by three distinct phases in the element's life cycle. Phase I has a small slope starting with a CI of β when this element is in its best condition. The slope and duration of Phase I significantly influence the overall performance of the element. A smaller slope and longer period indicates better performance while an increasing slope indicates worsening performance and possible early failure of the element. A sharp increase in slope starts in Phase II, where deterioration accelerates to the minimum acceptable level in a relatively short period. This leads to Phase III, a period of decelerating deterioration, where the element is nearly unusable with incipient functional and structural failure (Hudson et al. 1997).

Change of CI under Periodical Management Actions

Assume that management actions are taken at the beginning of each year and the time for these actions to implement is short such that it can be ignored. Then, if no action is taken at the beginning of some year, an element will remain at the current condition and continues its deterioration in the current year according to curve S in Fig. 1. If any action is taken, the condition of the element will be increased to some higher level immediately after the completion of the action. Then, starting from this higher level, the element deteriorates in current year and follows curve S' in Fig. 1 if the basic deterioration mechanism has not been changed materially by the management action. A new curve needs to be developed if there is a substantial structural or functional improvement due to the management action. The element will deteriorate following the changed mechanism. The element deteriorates to some condition level by the end of the current year (or the beginning of the following year), at which time a new round of management decision is to be made and an action taken on the element. The element either remains in the then current condition or goes to a higher level depending on what action is taken. In the following year, the element deteriorates following the previous deterioration mechanism or a new mechanism starting from the point of condition at the beginning of the following year immediately after a management action is taken. This cycle repeats until the end of the planned long time horizon.

Stochastic CI Changing Process

The discussion in previous sections and Fig. 1 describes a deterministic approach to the deterioration process and the change of CI under periodical management actions, that is, for each element, there is a definite CI value with certainty at any point of time over the planned time horizon. However, in reality, there is a wide range of variability and uncertainty related to the quality and property of materials, construction methods and technology, loading and usage of the facility, and the environment where the facility operates. This has a combined effect on the CI and renders the CI changing process of an element to a stochastic one, mainly including two aspects:

1. The element may go to different conditions with varying probabilities immediately after a management action is taken; and
2. During the current year after a management action is taken, the element may deteriorate to different conditions with varying probabilities by the end of the current year (the beginning of the following year).

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211

Markov Decision Process

A Markov decision process consists of five aspects: decision epochs, states, actions, transition probabilities, and rewards (Puterman 1994), which are discussed in the following subsections.

Decision Epochs and Periods

Decision epochs are the points of time when decisions are made. For the management of building facilities, assume that decisions are made at the beginning of each year of the planned time horizon of N years, and let T denote the set of decision epochs, then $T=\{1,2,\ldots,N\}$.

State and Action Sets

At each decision epoch, each element occupies a state, the CI of the element. Let Ω denote the set of possible states for each element. Then, $\Omega=[\alpha,\beta]$, with α indicating the worst CI and β the best CI. Ω is a continuous set of states, which may be converted to a descriptive and discrete state set Ω' as follows:

$$\Omega' = \{E,G,F,P,I\}$$

$$\mathrm{CI} = \begin{cases} E & E_{\min} \leqslant \mathrm{CI} \leqslant \beta \\ G & G_{\min} \leqslant \mathrm{CI} < E_{\min} \\ F & F_{\min} \leqslant \mathrm{CI} < G_{\min} \\ P & P_{\min} \leqslant \mathrm{CI} < F_{\min} \\ I & \alpha \leqslant \mathrm{CI} < P_{\min} \end{cases}$$

where E=excellent; G=good; F=fair; P=poor; and I=insufficient; $E_{\min}$=minimum numerical value of CI that belongs to category E; $G_{\min}$=minimum numerical value of CI that belongs to category G; $F_{\min}$=minimum numerical value of CI that belongs to category F; and $P_{\min}$=minimum numerical value of CI that belongs to category P.

At the beginning of year t, a management action a is taken on an element that is at state $I_t \in \Omega$. Assuming that there are always four allowable actions (a_1=replacement, a_2=major rehabilitation, a_3=minor rehabilitation, and a_4=no action) no matter at what state an element is, then $A=\{a_1,a_2,a_3,a_4\}$ is the allowable set of management actions. Ω and A do not vary with time t.

Rewards and Transition Probabilities

For an element, as a result of choosing an action $a \in A$ in state $I_t \in \Omega$ at the beginning of year t:

1. The asset manager receives a reward, R_{ta}, that is, the average performance of the element in year t;
2. The element goes to state $J_{ta} \in \Omega$ with a probability $P_{J_{ta}I_{ta}}$ immediately after action a is taken. "No action" does not have any effect on the condition of an element, "replacement" will result in a new element with condition index of β, and "major rehabilitation" and "minor rehabilitation" will increase the condition index to a higher level depending on the current condition of the element.
3. The element is at state $I_{t+1} \in \Omega$ with probability $Q_{I_{t+1}J_{ta}}$($I_{t+1} \leqslant J_{ta}$ due to deterioration in year t) at the beginning of year $t+1$ before any action is taken. The deterioration of the element during year t follows curve S or curve S' in Fig. 1 if its basic deterioration mechanism is not changed materially. Otherwise, it follows the new curve of the changed deterioration mechanism.

The reward R_{ta} is dependent on the current CI of the element, the effects of the management action, and the projected future CI as indicated by its deterioration mechanism. This is calculated using the following equations:

$$R_{ta} = \frac{\sum_{J_{ta}\in\Omega} P_{J_{ta}I_t}\left(J_{ta} + \sum_{I_{t+1}\in\Omega} Q_{I_{t+1}J_{ta}} I_{t+1}\right)}{2}$$

$$\sum_{J_{ta}\in\Omega} P_{J_{ta}I_t} = 1$$

$$\sum_{I_{t+1}\in\Omega} Q_{I_{t+1}J_{ta}} = 1$$

for $t=1,2,\ldots,N$ and $a=1,2,3$, and 4.

Optimization Model Based on Markov Decision Process

Methodology of Optimization Model

From the discussion in the previous sections, it is known that the slope (the deterioration rate) and duration of Phase I of the deterioration curve significantly influence the overall performance of the element. The smaller the slope and the longer the duration of Phase I, the better the performance of the element. The performance of the building network is dependent on the performance of all the building elements. The actual performance curve of a building network can be derived from the actual performance of all the building elements. The objective of building facilities management is to maximize the overall performance of the building network. Therefore, annual management actions on all building elements should be scheduled in such a way that the slope of the actual performance curve of the building network is minimized over the planned time horizon.

Different costs are required for different management actions on different building elements. While "no action" spends no money, increasing costs are needed for "minor rehabilitation," "major rehabilitation," and "replacement." If there were unlimited financial resources and no other constraints, then taking "replacement" action for all elements to keep them always new would maximize the overall performance of the building network. However, this is not the case in reality. For example, there is always a budget constraint (usually on an annual basis). Therefore, the aim of the optimization model is to maximize the overall performance of the building network over the planned time horizon by optimizing the set of annual management actions on all building elements, subject to various constraints, such as annual budget and the minimum performance requirement for a building, system, component, or element. This requires that the limited resources be allocated to alternative management actions cost effectively, that is, to maximize the performance per unit of funding spent. Generally, this requires that management actions be taken to avoid a higher rate of deterioration over the planned time horizon, taking into consideration the relative importance of the buildings, systems, components, and elements.

An integer-programming model based on the Markov decision process has been developed based on the methodology discussed above. Details of this model are presented in the following subsections.

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211

Decision Variables

1. Y_{tIa}^{bsce}=a binary variable; $Y_{tIa}^{bsce}=1$ if action a is taken when element e of component c of system s of building b is currently at a CI of I at the beginning of year t; $Y_{tIa}^{bsce}=0$ if action a is not taken.
2. b=building index; $b=1,2,\ldots,m$, where m is the total number of buildings in the building network.
3. s=system index; $s=1,2,\ldots,m_b$, where m_b is the total number of systems in building b.
4. c=component index; $c=1,2,\ldots,m_{bs}$, where m_{bs} is the total number of components in system s of building b.
5. e=element index; $e=1,2,\ldots,m_{bsc}$, where m_{bsc} is the total number of elements in component c of system s of building b.
6. I=condition index of element e of component c of system s of building b at the beginning of year t before action a is taken, $I \in \Omega$.
7. a=an action to be taken; $a=1$ (replacement), 2 (major rehabilitation), 3 (minor rehabilitation), and 4 (no action).

Objective Function

$$\text{Max } Z_t = \frac{1}{2}\sum_{b=1}^{m}\sum_{s=1}^{m_b}\sum_{c=1}^{m_{bs}}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4}\sum_{J_{ta}^{bsce}\in\Omega} W_b W_{bs} W_{bsc} W_{bsce} Y_{tIa}^{bsce} P_{J_{ta}I_t}^{bsce} \times\left(\sum_{J_{ta}^{bsce}\in\Omega} J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}\in\Omega} I_{t+1}^{bsce} Q_{I_{t+1}J_{ta}}^{bsce}\right)$$

where Z_t=weighted overall performance level of the building network in year t; W_b=weight of building b; $\sum_{b=1}^{m} W_b=1$; W_{bs}=weight of system s of building b; $\sum_{s=1}^{m_b} W_{bs}=1$, for $b=1,2,\ldots,m$; W_{bsc}=weight of component c of system s of building b; $\sum_{c=1}^{m_{bs}} W_{bsc}=1$, for $b=1,2,\ldots,m$; and $s=1,2,\ldots,m_{bs}$; W_{bsce}=weight of element e of component c of system s of building b; $\sum_{c=1}^{m_{bsc}} W_{bsce}=1$, for $b=1,2,\ldots,m$; $s=1,2,\ldots,m_{bs}$; and $c=1,2,\ldots,m_{bsc}$; J_{ta}^{bsce}=condition index of element e of component c of system s of building b at the beginning of year t immediately after action a is taken, when the element is at condition I_t^{bsce} at the beginning of year t before any action is taken; $P_{J_{ta}I_t}^{bsce}$=probability of element e of component c of system s of building b to goes to condition J_{ta}^{bsce} immediately after action a is taken when it is in condition I_t^{bsce} at the beginning of year t, $\sum_{J_{ta}^{bsce}\in\Omega} P_{J_{ta}I_t}^{bsce}=1$, for $a=1,2,3$ and 4; and $Q_{I_{t+1}J_{ta}}^{bsce}$=probability of element e of component c of system s of building b to goes to condition I_{t+1}^{bsce} at the beginning of year $t+1$ before any action is taken when it is in condition J_{ta}^{bsce} at the beginning of year t; $\sum_{I_{t+1}^{bsce}\in\Omega} Q_{I_{t+1}J_{ta}}^{bsce}=1$, for $a=1,2,3$ and 4.

Constraints

Budget Constraints

1. Network budget constraint

$$\sum_{b=1}^{m}\sum_{s=1}^{m_b}\sum_{c=1}^{m_{bs}}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce} \leqslant B_t^N$$

where C_{tIa}^{bsce}=cost corresponding to action a when element e of component c of system s of building b is at condition I in the beginning of year t; and B_t^N=total budget available in year t.

2. Building budget constraints

$$\sum_{s=1}^{m_b}\sum_{c=1}^{m_{bs}}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce} \leqslant B_t^b \quad \text{for } b=1,2,\ldots,m$$

where B_t^b=maximum amount of money that can be used for building b in year t.

Minimum Acceptable Performance Constraints

Minimum performance levels may be required for the building network, individual buildings, systems, components, and elements, depending on their relative importance, health, safety, and environment requirements, and the economics.

1. Minimum performance requirement for an element

$$\frac{1}{2}\sum_{a=1}^{4}\sum_{J_{ta}^{bsce}\in\Omega} P_{J_{ta}I_t}^{bsce}\left(J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}\in\Omega} I_{t+1}^{bsce} Q_{I_{t+1}J_{ta}}^{bsce}\right) Y_{tIa}^{bsce} \geq M_{bsce}$$

for $b=1,2,\ldots,m$; $s=1,2,\ldots,m_b$; $c=1,2,\ldots,m_{bs}$; and $e=1,2,\ldots,m_{bsc}$ where M_{bsce}=required minimum level of performance allowable for element e of component c of system s of building b.

2. Minimum performance requirement for a component

$$\frac{1}{2}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4}\sum_{J_{ta}^{bsce}\in\Omega} W_{bsce} P_{J_{ta}I_t}^{bsce} \times\left(J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}\in\Omega} I_{t+1}^{bsce} Q_{I_{t+1}J_{ta}}^{bsce}\right) Y_{tIa}^{bsce} \geq M_{bsc}$$

for $b=1,2,\ldots, m$; $s=1,2,\ldots,m_b$; and $c=1,2,\ldots,m_{bs}$ where M_{bsc}=required minimum performance level for component c of system s of building b.

3. Minimum performance requirement for a system

$$\frac{1}{2}\sum_{c=1}^{m_{bs}}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4}\sum_{J_{ta}^{bsce}\in\Omega} W_{bsc} W_{bsce} P_{J_{ta}I_t}^{bsce} \times\left(J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}\in\Omega} I_{t+1}^{bsce} Q_{I_{t+1}J_{ta}}^{bsce}\right) Y_{tIa}^{bsce} \geq M_{bs}$$

for $b=1,2,\ldots,m$ and $s=1,2,\ldots,m_b$ where M_{bs}=required minimum performance level for system s of building b.

4. Minimum performance requirement for a building

$$\frac{1}{2}\sum_{s=1}^{m_b}\sum_{c=1}^{m_{bs}}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4}\sum_{J_{ta}^{bsce}\in\Omega} W_{bs} W_{bsc} W_{bsce} P_{J_{ta}I_t}^{bsce} \times\left(J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}\in\Omega} I_{t+1}^{bsce} Q_{I_{t+1}J_{ta}}^{bsce}\right) Y_{tIa}^{bsce} \geq M_b$$

for $b=1,2,\ldots,m$ where M_b=required minimum performance level for building b.

5. Minimum performance requirement for the building network

$$\frac{1}{2}\sum_{b=1}^{m}\sum_{s=1}^{m_b}\sum_{c=1}^{m_{bs}}\sum_{e=1}^{m_{bsc}}\sum_{a=1}^{4}\sum_{J_{ta}^{bsce}\in\Omega} W_b W_{bs} W_{bsc} W_{bsce} P_{J_{ta}I_t}^{bsce} \times\left(J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}\in\Omega} I_{t+1}^{bsce} Q_{I_{t+1}J_{ta}}^{bsce}\right) Y_{tIa}^{bsce} \geq M_N$$

where M_N=required minimum performance level for the building network.

Only One Action Actually Taken for an Element

$$\sum_{a=1}^{4} Y_{tIa}^{bsce} = 1$$

for $b=1,2,\ldots,m$; $s=1,2,\ldots,m_b$; $c=1,2,\ldots,m_{bs}$; $e=1,2,\ldots,m_{bsc}$; and $I \in \Omega$.

Binary Constraints

Y_{tIa}^{bsce} is binary for $b=1,2,\ldots,m$; $s=1,2,\ldots,m_b$; $c=1,2,\ldots,m_{bs}$; $e=1,2,\ldots,m_{bsc}$; $I \in \Omega$; and $a=1,2,3,4$.

Other Constraints

Other constraints may be added when necessary, for example, in addition to the budget constraints for the building network and individual buildings, there may be constraints for different systems of different buildings in the network.

Inputs of Optimization Model

A current and complete database for all constituents of the building network is the first step toward an effective and efficient building facilities management system. This database should include general description for each building (e.g., type, location, and year of construction) and detailed information of each building (e.g., a complete list of systems/components/elements and their relative importance). In summary, the database should include the following information, which are the inputs that are necessary in running the optimization model: m=number of buildings in the building network; m_b=number of systems in building b; m_{bs}=number of components in system s of building b; m_{bsc}=number of elements in component c of system s of building b; B_t^N=total budget available for the building network in year t; B_t^b=maximum amount of money that can be assigned to building b in year t; I_t^{bsce}=CI of element e of component c of system s of building b at the beginning of year t before any management action is taken; C_{tIa}^{bsce}=cost corresponding to action a when element e of component c of system s of building b is in condition I_t^{bsce} at the beginning of year t; J_{ta}^{bsce}=CI of element e of component c of system s of building b at the beginning of year t immediately after action a is taken; $P_{J_{ta}I_t}^{bsce}$=probability of element e of component c of system s of building b to goes to condition J_{ta}^{bsce} immediately after action a is taken when it is in condition I_t^{bsce} at the beginning of year t; $Q_{I_{t+1}J_{ta}}^{bsce}$=probability of element e of component c of system s of building b to goes to condition I_{t+1a}^{bsce} at the beginning of year $t+1$ before any action is taken when it is in condition J_{ta}^{bsce} at the beginning of year t; W_b=weight of building b; W_{bs}=weight of system s of building b; W_{bsc}=weight of component c of system s of building b; W_{bsce}=weight of element e of component c of system s of building b; M_{bsce}=minimum required performance level for element e of component c of system s of building b; M_{bsc}=minimum required performance level for component c of system s of building b; M_{bs}=minimum required performance level for system s of building b; M_b=minimum required performance level for building b; and M_N=minimum required performance level for the building network.

Outputs of Optimization Model

The following information can be obtained based on the solutions of the optimization model discussed above:

Annual Actions to Take

If $Y_{tIa}^{bsce}=1$, action a ($a=1$ replacement; 2 major rehabilitation; 3 minor rehabilitation; and 4 no action) is taken when element e of component c of system s of building b is in condition I at the beginning of year t. If $Y_{tIa}^{bsce}=0$, action a is not taken.

Annual Budget Allocation

1. Budget allocated to an element: Let C_t^{bsce} be the budget allocated to element e of component c of system s of building b in the beginning of year t, then

$$C_t^{bsce} = \sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce}$$

for $b=1,2,\ldots,m$; $s=1,\ldots,m_b$; $c=1,2,\ldots,m_{bs}$; $e=1,2,\ldots,m_{bsc}$; and $t=1,\ldots,N$.

2. Budget allocated to a component: Let C_t^{bsc} be the budget allocated to component c of system s of building b in the beginning of year t, then

$$C_t^{bsc} = \sum_{e=1}^{m_{bsc}} C_t^{bsce} = \sum_{e=1}^{m_{bsc}} \sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce}$$

for $b=1,\ldots,m$; $s=1,\ldots,m_b$; $c=1,2,\ldots,m_{bs}$; and $t=1,\ldots,N$.

3. Budget allocated to a system: Let C_t^{bs} be the budget allocated to system s of building b in the beginning of year t, then

$$C_t^{bs} = \sum_{c=1}^{m_{bs}} C_t^{bsc} = \sum_{c=1}^{m_{bs}} \sum_{e=1}^{m_{bsc}} \sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce}$$

for $b=1,\ldots,m$; $s=1,\ldots,m_b$; and $t=1,\ldots,N$.

4. Budget allocated to a building: Let C_t^b be the budget allocated to building b in the beginning of year t, then

$$C_t^b = \sum_{s=1}^{m_b} C_t^{bs} = \sum_{s=1}^{m_b} \sum_{c=1}^{m_{bs}} \sum_{e=1}^{m_{bsc}} \sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce}$$

for $b=1,\ldots,m$ and $t=1,\ldots,N$.

5. Total amount of money spent on the building network: Let C_t^N be the total amount of money spent on the building network in the beginning of year t, then

$$C_t^N = \sum_{b=1}^{m} C_t^b = \sum_{b=1}^{m} \sum_{s=1}^{m_b} \sum_{c=1}^{m_{bs}} \sum_{e=1}^{m_{bsc}} \sum_{a=1}^{4} C_{tIa}^{bsce} Y_{tIa}^{bsce}$$

for $t=1,2,\ldots,N$.

Expected Values of CIs

Let (1) EI_t^{bsce}, EI_t^{bsc}, EI_t^{bs}, EI_t^b, EI_t^N be the expected values of the CIs of element e of component c of system s of building b, component c of system s of building b, system s of building b, building b, and the building network in year t before management actions are taken, respectively; and (2) EJ_t^{bsce}, EJ_t^{bsc}, EJ_t^{bs}, EJ_t^b, EJ_t^N be the corresponding expected values of CIs immediately after management actions are taken, and suppose that the CIs of all the building elements in the beginning of year one of the time horizon N are known, that is, I_1^{bsce} is known, then

$$EI_1^{bsce} = I_1^{bsce}$$

$$EI_1^{bsc} = \sum_{e=1}^{m_{bsc}} W_{bsce} EI_1^{bsce}$$

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211

$$EI_1^{bs} = \sum_{c=1}^{m_{bs}} W_{bsc} EI_1^{bsc}$$

$$EI_1^{b} = \sum_{s=1}^{m_{b}} W_{bs} EI_1^{bs}$$

$$EI_1^{N} = \sum_{b=1}^{m} W_{b} EI_1^{b}$$

$$EJ_t^{bsce} = \sum_{a=1}^{4} J_{ta}^{bsce} P_{J_{ta}I_t}^{bsce} Y_{tIa}^{bsce}$$

$$EJ_t^{bsc} = \sum_{e=1}^{m_{bsc}} W_{bsce} EJ_t^{bsce}$$

$$EJ_t^{bs} = \sum_{c=1}^{m_{bs}} W_{bsc} EI_t^{bsc}$$

$$EJ_t^{b} = \sum_{s=1}^{m_{b}} W_{bs} EJ_t^{bs}$$

$$EJ_t^{N} = \sum_{b=1}^{m} W_{b} EJ_t^{b}$$

$$EI_{t+1}^{bsce} = \sum_{a=1}^{4} \sum_{I_{t+1}^{bsce} \in \Omega} Q_{I_{t+1}J_{ta}}^{bsce} I_{t+1}^{bsce}$$

$$EI_{t+1}^{bsc} = \sum_{e=1}^{m_{bsc}} W_{bsce} EI_{t+1}^{bsce}$$

$$EI_{t+1}^{bs} = \sum_{c=1}^{m_{bs}} W_{bsc} EI_{t+1}^{bsc}$$

$$EI_{t+1}^{b} = \sum_{s=1}^{m_{b}} W_{bs} EI_{t+1}^{bs}$$

$$EI_{t+1}^{N} = \sum_{b=1}^{m} W_{b} EI_{t+1}^{b}$$

Expected Performance Levels

Let Z_t^{bsce}, Z_t^{bsc}, Z_t^{bs}, Z_t^{b}, Z_t^{N} be the expected performance levels of an element, component, system, building, and the building network in year t of the time horizon, respectively, then

$$Z_t^{bsce} = \frac{1}{2} \sum_{a=1}^{4} \sum_{J_{ta}^{bsce} \in \Omega} P_{J_{ta}I_t}^{bsce} Y_{tIa}^{bsce} \left(J_{ta}^{bsce} + \sum_{I_{t+1}^{bsce}} Q_{I_{t+1}J_{ta}}^{bsce} I_{t+1}^{bsce} \right)$$

$$Z_t^{bsc} = \sum_{e=1}^{m_{bsc}} Z_t^{bsce}$$

$$Z_t^{bs} = \sum_{c=1}^{m_{bs}} Z_t^{bsc}$$

$$Z_t^{b} = \sum_{s=1}^{m_{b}} Z_t^{bs}$$

$$Z_t^{N} = \sum_{b=1}^{m} Z_t^{b}$$

Different Scenarios and Sensitivity Analysis

Please note that the outputs discussed in the above are corresponding to a particular scenario, that is, based on predetermined values of a set of variables, such as C_{tIa}^{bsce}, B_t^N, $P_{J_{ta}I_t}^{bsce}$, and $Q_{I_{t+1}J_{ta}}^{bsce}$. As there are many uncertainties in the determination of these values, different scenarios can be explored by using different ranges of these variables. This is quite useful in the sensitivity analysis of some important variables such as the B_t^N.

Conclusions

An integer-programming model based on the Markov decision process has been developed, in which the performance of a building network is measured by the CI that is modeled by Markov chains. This mathematical model optimizes periodical management actions and consequently the allocation of limited periodical resources toward achieving the maximum overall performance of the building network that is subject to various types of constraints.

The proposed optimization model requires substantial input data, which is often derived from in-service evaluation, historical data, and expert opinions. Accuracy of inputs is critical to the successful application of this optimization model. The predicted deterioration process, the predicted change of the CI due to a management action, the predicted costs corresponding to alternative management actions, and the predicted transition probabilities regarding the change of the CI should be compared with the actual data obtained from in-service evaluation during the planning time horizon, and appropriate modifications and adjustments made to improve future predictions.

A wide scope of useful outputs for building facilities management can be obtained from the optimization model, including:

1. Annual actions to take for all the elements of the building network;
2. Annual budget allocation to different buildings and to different systems, components, and elements of each building;
3. The expected CI values for all the elements, components, systems, and buildings of the building network at the beginning of each year over a predetermined long time horizon before any management actions are taken, and the corresponding expected CI values immediately after management actions are taken;
4. The expected annual performance levels of all the elements, components, systems, and buildings of the building network for each year over the predetermined long time horizon; and
5. Sensitivity analysis and corresponding outputs for different budget scenarios.

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211

References

Alberta Transportation. (2001). *Final Rep. for Business Concept Definition*, Edmonton, Canada.

Charette, R. P., and Marshall, H. E. (1999). UNIFORMAT II elemental classification for building specifications, cost estimating, and cost analysis, National Technical Information Service, Technology Administration, U.S. Department of Commerce, Springfield, Va.

Hudson, W. R., Haas, R., and Uddin, W. (1997). *Infrastructure management: Design, construction, maintenance, rehabilitation, and renovation*, McGraw-Hill, New York.

Puterman, M. L. (1994). *Markov decision processes: Discrete stochastic dynamic programming*, Wiley, New York.

J. Constr. Eng. Manage., 2006, 132(11): 1203-1211