

# Linear versus Network Scheduling: A Critical Path Comparison

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**Abstract:** Linear scheduling methods provide an alternative way of scheduling repetitive projects, to the commonly used network methods. Critical path identification is a major attribute for both methods; therefore, it is very important for practitioners to understand the function of the two methods in this area. The present paper compares the critical path of the recently developed Kallantzis-Lambropoulos repetitive project model against the network scheduling critical path method (CPM), aiming at delving into and pointing out the differences and similarities between them. Initially, the rules for transforming the linear project into an equivalent CPM network are proposed. Then, the rules are applied on a sample linear project. Due to the additional constraint for maintaining resource continuity that the linear method takes into account, the critical paths vary. The constraint is subsequently removed from selected activities and comparison is repeated; the critical paths then coincide. In order to validate the findings and ensure impartiality of results, a random linear project generator is developed. A group of twenty-five random linear projects and their equivalent networks is produced. Their critical paths are analyzed, compared and classified. Conclusions support that the proposed comparison could be beneficial to users of linear scheduling methods, while the random project generator can serve other related research.

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## Introduction

Repetitive projects are quite common in the construction industry. They are divided into two categories (Hegazy and Wassef 2001): Projects that are repetitive due to a uniform repetition of a unit work throughout the project (multiple similar houses, high rise buildings) and projects that are repetitive due to their geometrical layout (highway and pipeline projects). Such projects are usually referred to as repetitive or linear projects (Ammar and Elbeltagi 2001). These projects are scheduled in a way to minimize crew-waiting times and ensure resource continuity (Birrell 1980; Reda 1990). Ensuring work continuity during scheduling provides for an efficient resource utilization strategy (El-Rayes and Moselhi 1998).

During recent years, four methods have been proposed for identifying those activities in a linear schedule that, if their duration changes, the duration of the entire project changes. These are the repetitive scheduling method (RSM) by Harris and Ioannou (1998), the linear scheduling model (LSM) by Harmelink and

Rowings (1998), the critical path linear scheduling method by Ammar and Elbeltagi (2001), and the Kallantzis-Lambropoulos repetitive project model (KLRPM) by Kallantzis and Lambropoulos (2004a). Yang and Ioannou (2004) presented a computerized model that integrates the principles of the RSM method. The sequence of activities described in the previous methods plays the same role as the critical path in the network methods and is crucial for linear methods to be accepted as a viable tool in project planning and control (Harmelink and Rowings 1998). The aforementioned methods use different terms to define this sequence (i.e., controlling activity path, controlling sequence, controlling path, critical path); in the scope of this paper the term critical path is applied.

Validation of results is important, so work has also been done in comparing the critical paths of the aforementioned methods either between them, or with the network critical path method (CPM). Mattila and Park (2003) compared the critical paths of the RSM and the LSM and concluded that their results, for simple activity configurations coincide. Ioannou and Yang (2004) and Kallantzis and Lambropoulos (2004b), in a discussion on the aforementioned paper, showed that the methods differ for some activity configurations.

Yamin and Harmelink (2001) attempted a comparison of the CPM and LSM in many aspects (ease of use, aid in reduction of uncertainty, accuracy in calculations, etc.), including the critical path; the critical path comparison though, was limited to two examples, one of them being a three-activity CPM network transformed into an equivalent linear project. Ammar and Elbeltagi (2001) constructed a precedence network, equivalent to the linear diagram, by assigning to activities finish-to-finish (FF), start-to-start (SS) or both FF and SS relationships depending on the rates of their predecessor(s) and successor(s). The proposed methodology was applied on a sample project, while variable production rates within the same activity were not allowed.

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This paper compares the critical path of KLRPM and CPM aiming at delving into and pointing out the differences and similarities between the linear scheduling method and the network method in terms of critical path. Instead of applying the transformation rules on one or two illustrative examples and comparing the critical paths, a group of 25 multiple-rate random linear projects is examined. This way, not only is the margin for error reduced, but also the possibility of a coincidental or biased result is minimized and statistic conclusions can be drawn with relative safety. In order to produce this group of random linear projects a prototype generator engine, ProjectGenerator (PrjGen), has been developed. Linear projects are automatically converted by PrjGen to equivalent CPM networks.

The paper starts with a brief reference to the KLRPM method. Next, a set of rules is provided for transforming the linear project schedule into an equivalent network one by assigning FF, SS, or finish-to-start (FS) relationships. These rules are then applied on a sample project. Initial comparison shows significant variation in the results, due to the presence of the resource continuity constraint. The resource continuity constraint is subsequently removed from selected activities and critical paths are compared again; in the scope of the later comparison they coincide. The function, input and output data of the random project generator is presented next. Twenty-five random linear projects are generated and transformed into equivalent networks. The critical paths are then compared, first with the resource continuity constraint active and second without it. The results are analyzed and classified in order to draw conclusions.

The proposed comparison can be beneficial to those using or evaluating the linear scheduling method, as coincidence of the results and justification of deviations support the findings of the linear scheduling method and reduce reservations for potential users. Practitioners can take advantage of the results in order to expand the use of the linear scheduling method and increase its acceptance as an integrated scheduling tool. The results can also be valuable to researchers towards expanding the comparison to other areas like total and free float. The limitations and extensions of the proposed method are further discussed. The data used for the random projects is provided in the Appendix.

## KLRPM Schedule

Critical path determination in the KLRPM is based on time and distance/location constraints, similar to the time and stage buffers as defined by Reda (1990). The minimum time constraint denotes the minimum time buffer between two activities—e.g., the minimum time buffer between concreting and formwork removal allowing for curing of concrete. The minimum distance/location constraint denotes the minimum allowable distance between two activities to allow for the correct or safe execution of work—e.g., the minimum distance between paving and asphalt works in a highway project or the minimum amount of floors between brick-laying and concrete pouring to allow concrete to gain strength in a multistory building. KLRPM places activities in an oriented time–location diagram, where time is depicted in the *Y* axis and distance/location in the *X* axis. Activities in the schedule are positioned the closest possible to each other, providing that time and distance/location constraints between them are not violated. The method consists of two steps: First the potential critical activities are identified and then the critical path is calculated.

In order to identify the potential critical activities, the process starts from the last activity of the project moving through every

activity's driving predecessor until the first activity is reached. This procedure ensures that the longest path in the schedule has been identified. This sequence of activities is called the "critical sequence."

Following the identification of the critical sequence, the critical parts of the activities are determined. Starting from the end of the last activity in the critical sequence, the critical path is found by tracing along the activity until it reaches the point where the constraint with the driving predecessor lies; there, it shifts toward its driving predecessor horizontally or vertically depending on whether it is a distance/location or a time constraint. The procedure continues until the beginning of the first activity is reached. If the constraint with the successor activity lies to the right of the constraint with the predecessor activity, then the part of the activity between the two constraints is called a "critical segment." Based on the aforementioned algorithm, linear activities are grouped into four categories regarding their ability to influence project duration:

- Activities with critical segments that belong to the critical sequence. The constraint of the successor activity lies to the right of the constraint of the predecessor activity. These activities are equivalent to the critical activities in CPM. If their duration increases, total project duration increases;
- Activities with no critical segments that belong to the critical sequence. The constraint of the successor activity lies to the left of the constraint of the predecessor activity. When tracing along the critical path from start to finish, these activities are traversed backwards in time. Even though they affect project duration, they present the paradox of increasing project duration when their progress rate is increased. This paradox was observed by Harris and Ioannou (1998). Although these activities may be considered critical, since they affect project duration, they are classified in a special category as they operate in the opposite way; and
- Activities that have only one critical point. The constraint of the successor activity lies at the same point where constraint of the predecessor activity lies. If this point lies in the beginning or the end of the activity, these activities can proceed in a slower pace than originally planned and just have to start (or finish) on a specific day. This situation is equivalent to CPM activities that have different start float than finish float.
- Activities with no critical segments that do not belong to the critical sequence. These are the nondriving activities in a linear schedule. They proceed in parallel with a driving activity, but at a faster pace.

In the following section, the rules for transforming the linear project into an equivalent network are demonstrated.

## Equivalent CPM Schedule

KLRPM uses the minimum time and minimum distance/location constraints to plot linear activities in the Time-Location Diagram, in the same manner CPM uses the finish-to-start, start-to-start, finish-to-finish, and start-to-finish (SF) relationships to produce the network schedule. In order to calculate the network, the CPM method divides linear activities into discrete parts, thus removing the resource continuity constraint. For example, activity "concreting" in a multihousing project would be divided into "Unit 1 concrete," "Unit 2 concrete," etc. Similarly, in a highway project, activity "excavations" would be divided into "excavations from Station 1 to Station 2," "excavations from Station 2 to Station 3," etc.

The proposed rules map the minimum time and distance/location constraints of KLRPM into FS, SS, and FF relationships of CPM. There is no equivalent of the SF relationship, but it is quite infrequent in linear projects and in network construction.

In repetitive construction projects, a minimum time constraint denotes that two activities cannot approach each other more than a specified amount of time during the whole span of the project. If the two activities diverge, it is logical to assume that the minimum time constraint corresponds to a SS relationship with lag equal to the value of the constraint. On the contrary, if two activities converge, it is logical to assume that the minimum time constraint corresponds to a FF relationship with lag equal to the value of the constraint. Taking under consideration that activities have multiple rates and they may converge and diverge in different parts of the project or that after updating the project two activities that initially converge may diverge (or vice versa), the minimum time constraint is corresponding to a FF–SS relationship with lag equal to the value of the constraint. The proposed correspondence covers the minimum time constraint irrespective to whether the activities diverge or converge. If the activities diverge, the SS relationship will prevail; if they converge, the FF relationship will prevail.

As stated before, the minimum distance/location constraint in a repetitive construction project denotes the minimum stage buffer between two activities. Any delay to the finish of a specific part of the predecessor activity corresponds to a respective delay to the start of the corresponding part of the successor activity. It is logical to assume that the minimum distance/location constraint corresponds to a FS relationship in CPM, regardless whether the activities converge or diverge.

The value of the minimum distance/location constraint denotes which part of the successor activity will be linked to which part of the predecessor activity. For example, in a multiunit construction project, if the minimum distance/location constraint has a value equal to one unit, every unit  $j$  (where  $j=1$  to the number of units) of the successor activity will be linked, with a FS=0 relationship, to the same unit  $j$  of the predecessor activity. If the constraint has a value equal to two units, every unit  $j$  of the successor activity is linked, with a FS=0 relationship, to the unit  $j+1$  of the predecessor activity. Generally, if the constraint has a value equal to  $d$ , then the unit  $j$  of the successor activity will be linked to the unit  $j+d-1$  unit of the predecessor activity.

The above-mentioned transformation rules are applied in the following sample project.

## Sample Project

The following sample project refers to a 5 km gas-pipe relocation consisting of five activities. The activities that comprise the project are excavations (EX), lay pipe (LP), test pipe (TP), backfill (BF), and road reinstatement (RR). They are considered to be full-span with multiple-rates. Data for the linear activities are presented in Table 1. Activity names are shown in the first column of Table 1. The next five columns demonstrate each activity's production rate at that specific kilometer (km), i.e., activity EX has production rate 1/3 km/day for the first 3 km and 1/5 km/day for the last two. Every activity has the previous one as predecessor, except activity EX that has no predecessors. The last two columns show the constraint with the predecessor activity and its value, i.e., activity LP has activity EX as predecessor with minimum time constraint equal to two days, activity TP has activity

**Table 1.** Sample Project Data

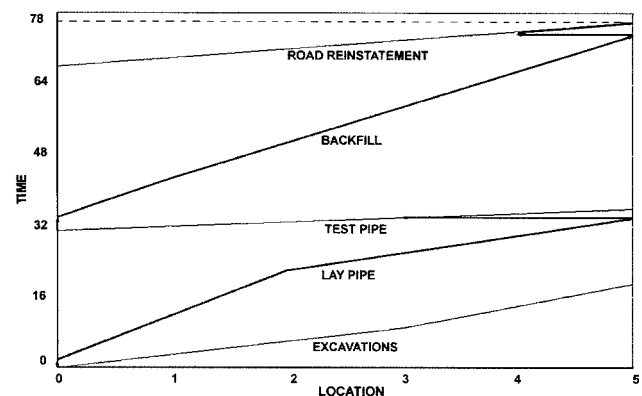
Activity name	Location (km)					Constraint	Value
	1	2	3	4	5		
Excavations (EX)	1/3	1/3	1/3	1/5	1/5		
Lay Pipe (LP)	1/10	1/10	1/4	1/4	1/4	Time	2 days
Test Pipe (TP)	1	1	1	1	1	Distance	2 km
Backfill (BF)	1/9	1/8	1/8	1/8	1/8	Time	3 days
Road reinstatement (RR)	1/2	1/2	1/2	1/2	1/2	Distance	1 km

LP as predecessor with minimum distance/location constraint equal to 2 km, etc.

Initially, the project is scheduled with the linear scheduling method KLRPM and the resource continuity constraint active, according to the procedure described in the relevant section. The steps for identifying the critical path are the following: The critical sequence is found by tracing along driving predecessors from the last to the first activity. The last activity of the project is activity RR. The critical sequence is formed by activities RR, BF, TP, LP, and EX.

According to KLRPM rules, critical path identification procedure starts from the last point of activity "road reinstatement." Activity RR is traced along to the point where the minimum distance with activity BF takes place. There, the critical path shifts horizontally to activity BF. Activity BF is traced along to the point where the minimum time with activity TP takes place. There, the critical path shifts vertically to activity TP. Activity TP is traced along to the point where the minimum distance with activity LP takes place. There, the critical path shifts horizontally to activity LP. Activity LP is traced along to the point where the minimum time with activity EX takes place. There, the critical path shifts vertically to activity EX.

Summarizing, the critical path includes the starting point of activity EX, the whole of activities LP and BF and the last kilometer of activity RR. The part of activity TP between the first and third kilometer has the constraint of the successor activity BF lying to the left of the constraint of the predecessor activity LP. This part of activity TP belongs to the second category of activities, as described in the KLRPM model function earlier, that present the phenomenon of decreasing project overall duration when their duration is increased. Total project time is 78 days. The results are presented in Fig. 1.



**Fig. 1.** Initial KLRPM critical path of sample project

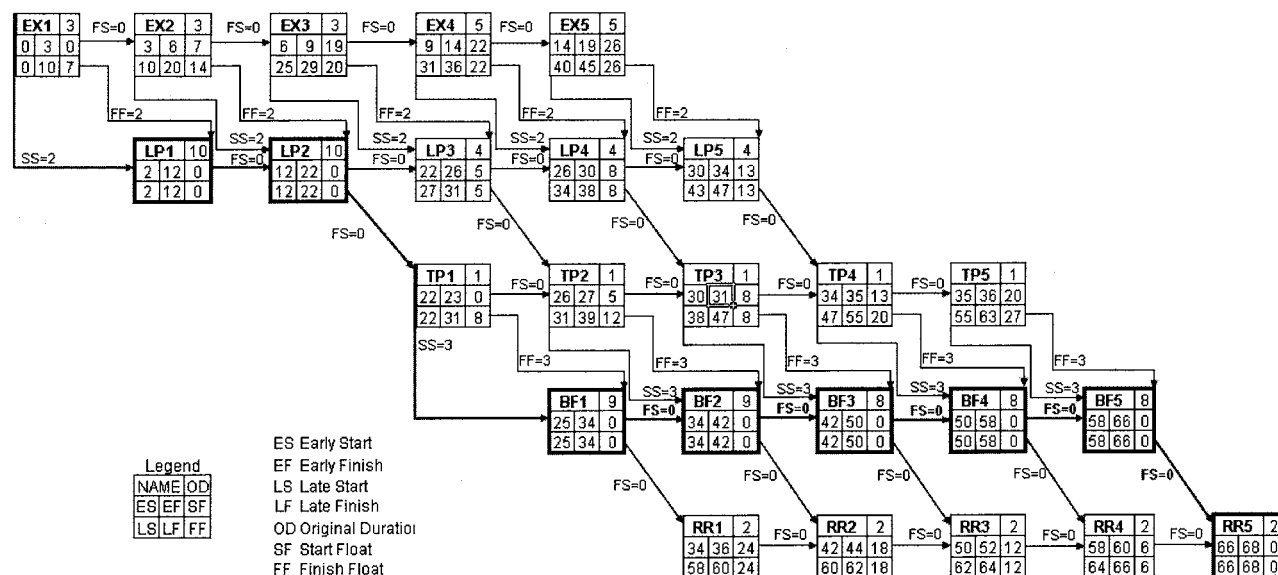


Fig. 2. Solution of equivalent CPM sample project

The linear project is subsequently transformed into the equivalent CPM network of Fig. 2. Activities are broken down into five 1 km parts. Relations are assigned according to the rules mentioned in the previous section. The critical path is calculated.

Activity EX1 has start float=0 and finish float  $\neq 0$ ; thus, only the start point of activity EX1 is critical. Activities LP1 and LP2 are critical. Activity TP1 has start float=0 and finish float  $\neq 0$ ; thus, only the start point of activity TP1 is critical. Activities BF1–BF5 are critical. Finally, activity RR5 is critical. Total project time is 68 days.

The two solutions vary significantly both in terms of dates and criticality of activities. This is due to the effect of the resource continuity constraint on linear activity TP. Its successor, BF, starts on the 35th day in the linear schedule, whereas in the CPM equivalent, it starts on the 25th day.

The resource continuity constraint is subsequently removed for linear activity TP, allowing for work interruptions. The linear project critical path is recalculated. The schedule is presented in Fig. 3. The new critical path calculated by KLRPM, includes the starting point of activity EX, the two first kilometers of activity LP (LP1 and LP2), the starting point of TP1, activity BF and the

last kilometer of activity RR. Total project time is now 68 days. The comparison between the relaxed linear project and its equivalent network is presented in Table 2.

The two critical paths and project durations now coincide. It should be pointed out that the influence of the resource continuity constraint makes noncritical activity RR to start on the 58th day

Table 2. KLRPM and CPM Result Comparison for Sample Project

Activity	KLRPM				CPM			
	Start day	Finish day	Critical start	Critical finish	Start day	Finish day	Critical start	Critical finish
EX <sub>1</sub> –EX <sub>3</sub>	0	9	✓		EX <sub>1</sub>	0	3	✓
					EX <sub>2</sub>	3	6	
					EX <sub>3</sub>	6	9	
EX <sub>4</sub> –EX <sub>5</sub>	9	19			EX <sub>4</sub>	9	14	
					EX <sub>5</sub>	14	19	
LP <sub>1</sub> –LP <sub>2</sub>	2	22	✓	✓	LP <sub>1</sub>	2	12	✓
					LP <sub>2</sub>	12	22	✓
LP <sub>3</sub> –LP <sub>5</sub>	22	34			LP <sub>3</sub>	22	26	
					LP <sub>4</sub>	26	30	
					LP <sub>5</sub>	30	34	
TP <sub>1</sub>	22	23	✓		TP <sub>1</sub>	22	23	✓
TP <sub>2</sub>	26	27			TP <sub>2</sub>	26	27	
TP <sub>3</sub>	30	31			TP <sub>3</sub>	30	31	
TP <sub>4</sub> –TP <sub>5</sub>	34	36			TP <sub>4</sub>	34	35	
					TP <sub>5</sub>	35	36	
BF <sub>1</sub> –BF <sub>5</sub>	25	66	✓	✓	BF <sub>1</sub>	25	34	✓
					BF <sub>2</sub>	34	42	✓
					BF <sub>3</sub>	42	50	✓
					BF <sub>4</sub>	50	58	✓
					BF <sub>5</sub>	58	66	✓
RR <sub>1</sub> –RR <sub>4</sub>	58	66			RR <sub>1</sub>	34	36	
					RR <sub>2</sub>	42	44	
					RR <sub>3</sub>	50	52	
					RR <sub>4</sub>	58	60	
RR <sub>5</sub>	66	68	✓	✓	RR <sub>5</sub>	66	68	✓

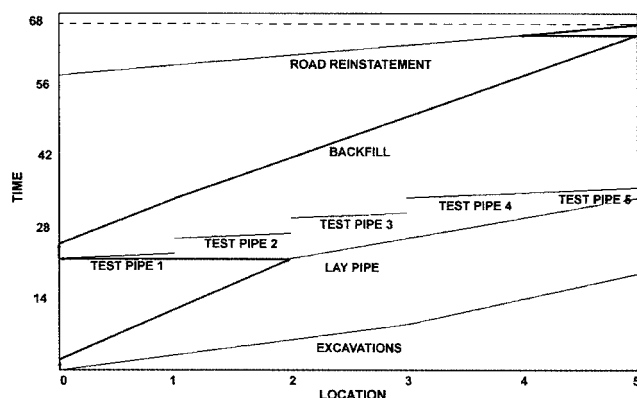


Fig. 3. KLRPM solution for relaxed sample project (resource continuity constraint removed for activity TP)



on the linear schedule, rather than the 34th day on the network schedule. If the resource continuity constraint is also removed for activity RR, the schedules will match completely.

In order to verify the above observations, a group of 25 projects has been tested. For this purpose a generator of random projects was developed. Its input, output and function are presented briefly in the following section.

## Random Project Generator

The presented sample project showed coincidence in the results of the linear and the network scheduling methods. In order to assure no bias, a random project generator was developed. This random project generator became part of a C++ coded software program called PrjGen targeting the following objectives:

- Structured generation of random linear projects with a view to ensuring that the similarity between linear and network scheduling methods is not a matter of coincidence or manipulation; and
- Postprocessing of equivalent network projects by commercial state-of-the-art project management tools (such as the Primavera Project Manager, by Primavera Systems Inc., Penn).

PrjGen incorporates the linear scheduling algorithms described earlier. Structured generation of projects is based on tuning parameters that are important to producing different (appropriately) randomized projects. In particular, PrjGen allows tuning the following parameters relating to the linear scheduling approach:

- Project type in terms of the comprised activities types (e.g., full-span, single-rate, multiple-rate, vertical, horizontal). Different project types can be produced through combining the previous parameters;
- Number of activities with a randomly generated construction project;
- Range of minimum time constraint values;
- Range of minimum distance/location values; and
- Production rates.

As far as the definition and production of predecessor activities is concerned, PrjGen supports two modes:

- A simple mode, where each activity has the previous activity (in the scope of linear scheduling) as predecessor; and
- A more sophisticated mode, randomly generating any predecessor(s) conforming to the introduced linear scheduling discipline.

PrjGen writes all random project parameters (such as activities, predecessor) in appropriately structured files. Additional files reflecting the mapping to the equivalent CPM projects are produced. The latter serve as a basis for post processing results in the Primavera Program. To this end, an interface between the two programs has been implemented. Random linear projects are transformed into equivalent CPM networks in a format recognizable by Primavera Project Planner.

Fig. 4 illustrates in the form of a simplified block diagram, the main components of PrjGen. In producing a random project, the user needs to provide parameters in the form of a file or upon program commencement. The system incorporates a randomizer engine that randomly selects parameters within the allowable ranges. The engine comprises a rich set of validating mechanisms ensuring that the randomized activities comply with the limitations imposed by the KLRPM discipline.

Although PrjGen is clearly oriented to the research purposes of this paper, it can also serve other research tasks relating to construction management. It can be flexibly customized, with little

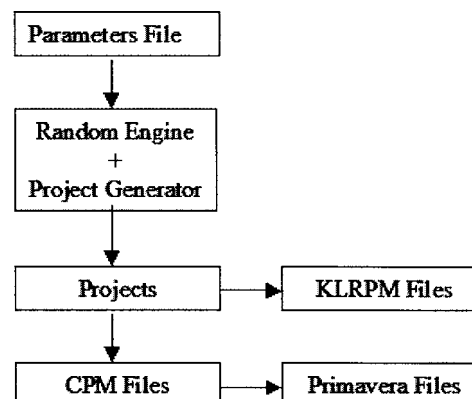


Fig. 4. Simplified block diagram for the PrjGen program

additional effort, to support a project generation subject for other scheduling needs. For the purposes of the current research, PrjGen is used to generate 25 random projects, as demonstrated in the following section.

## Random Project Analysis

Critical path comparison between the linear and the network method showed, in the sample project, that the critical paths vary when linear activities satisfy the resource continuity constraint, while they coincide when the constraint is removed. In order to validate the findings, remove any potential bias and study more comprehensively the differences and similarities between the linear and the network method, a group of 25 random projects is examined. The random linear project generator described in the previous section was used to produce 25 random linear projects. Their general characteristics are presented in Table 3.

The data for each linear project are presented in the Appendix; each project's characteristics are demonstrated on separate tables. Activity name is shown in the first column of each table. The next six columns contain the production rates for each activity in every unit. The final column demonstrates the constraint of every activity with its predecessor, along with its value.

For each random project, the critical path is identified and the early finish is calculated for the following three cases:

- Linear project with resource continuity constraint;
- Equivalent CPM network according to the proposed transformation rules, each linear activity is corresponded to six discrete CPM activities; and
- Initial linear project with resource continuity constraint removed.

Table 3. General Characteristics of Random Projects

Project characteristics	Value range
Project type	15 full-span single rate and 10 full-span multiple rate
Number of activities per project	6 activities
Project dimension	6 units
Predecessor	Previous activity
Minimum time constraint value range	1–4
Minimum distance (buffer) value range	1–2
Production rates	1, 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/9, 1/10

**Table 4.** Random Project Critical Paths

KLRPM initial solution			CPM		KLRPM solution with no continuity constraint	
No.	Early finish	Critical activities	Early finish	Critical activities	Early finish	Critical activities
1	107	A <sub>1</sub> , A <sub>2</sub> , B, C <sub>1</sub> –C <sub>5</sub> , D, <b>E</b> , F	82	A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , c <sub>1</sub> , D <sub>1</sub> , D <sub>2</sub> , D <sub>3</sub> , D <sub>4</sub> , D <sub>5</sub> , D <sub>6</sub> , E <sub>6</sub> , F <sub>6</sub>	82	A <sub>1-2</sub> , B <sub>1</sub> , c <sub>1</sub> , D, <b>E</b> <sub>6</sub> , F <sub>6</sub>
2	148	A, <b>B</b> <sub>3</sub> , <b>B</b> <sub>4</sub> , C, <b>D</b> <sub>2</sub> – <b>D</b> <sub>5</sub> , E <sub>1</sub> , F	104	A <sub>1</sub> , A <sub>2</sub> , A <sub>3</sub> , B <sub>2</sub> , C, D <sub>1</sub> , E <sub>1</sub> , F	104	A <sub>1-3</sub> , B <sub>2</sub> , C, D <sub>1</sub> , E <sub>1</sub> , F
3	119	A, B <sub>6</sub> , C <sub>5-6</sub> , <b>D</b> , e <sub>1</sub> , F	93	A <sub>1</sub> , B <sub>6</sub> , C <sub>5</sub> , D <sub>5</sub> , e <sub>5</sub> , F <sub>5</sub> , F <sub>6</sub>	93	A <sub>1</sub> , B <sub>6</sub> , C <sub>5</sub> , <b>D</b> <sub>5</sub> , e <sub>5</sub> , F <sub>5-6</sub>
4	88	A <sub>1-2</sub> , B, C <sub>3-6</sub> , D, e <sub>6</sub> , F <sub>6</sub>	79	A <sub>1</sub> , A <sub>2</sub> , B <sub>1</sub> , B <sub>2</sub> , c <sub>2</sub> , D, e <sub>6</sub> , F <sub>6</sub>	79	A <sub>1-2</sub> , B <sub>1-2</sub> , c <sub>2</sub> , D, e <sub>6</sub> , F <sub>6</sub>
5	110	a <sub>1</sub> , B <sub>1-2</sub> , C, D <sub>1-4</sub> , E, F <sub>5-6</sub>	93	a <sub>1</sub> , B <sub>1</sub> , B <sub>2</sub> , C, d <sub>5</sub> , E <sub>5-6</sub> , F <sub>5-6</sub>	93	a <sub>1</sub> , B <sub>1-2</sub> , C, d <sub>5</sub> , E <sub>5-6</sub> , F <sub>5-6</sub>
6	143	A, <b>B</b> <sub>2-6</sub> , c <sub>1</sub> , D, <b>E</b> <sub>1-5</sub> , F	83	A <sub>1</sub> , b <sub>1</sub> , c <sub>1</sub> , D <sub>1</sub> , e <sub>1</sub> , F	83	A <sub>1</sub> , b <sub>1</sub> , c <sub>1</sub> , D <sub>1</sub> , e <sub>1</sub> , F
7	116	A, <b>B</b> <sub>3-5</sub> , c <sub>1</sub> , D, <b>E</b> <sub>2-5</sub> , F	90	A, B <sub>2</sub> , c <sub>1</sub> , D, E <sub>6</sub> , F <sub>6</sub>	90	A, B <sub>2</sub> , c <sub>1</sub> , D, E <sub>6</sub> , F <sub>6</sub>
8	118	A <sub>1</sub> , B, C <sub>1-5</sub> , d <sub>1</sub> , E, f <sub>6</sub>	83	A <sub>1</sub> , B <sub>1</sub> , c <sub>1</sub> , d <sub>1</sub> , E, f <sub>6</sub>	83	A <sub>1</sub> , B <sub>1</sub> , c <sub>1</sub> , d <sub>1</sub> , E, f <sub>6</sub>
9	131	A, <b>B</b> <sub>2-6</sub> , C, <b>D</b> , E <sub>1</sub> , F	81	A <sub>1</sub> , b <sub>1</sub> , C, D <sub>6</sub> , E <sub>6</sub> , F <sub>6</sub>	81	A <sub>1</sub> , b <sub>1</sub> , C, <b>D</b> <sub>6</sub> , E <sub>6</sub> , F <sub>6</sub>
10	108	A, <b>B</b> <sub>2-6</sub> , C, d <sub>6</sub> , <b>E</b> , F	83	A <sub>1</sub> , b <sub>1</sub> , C <sub>1</sub> , d <sub>1</sub> , E <sub>1</sub> , F	83	A <sub>1</sub> , b <sub>1</sub> , C <sub>1</sub> , d <sub>1</sub> , <b>E</b> <sub>1</sub> , F
11	77	a <sub>1</sub> , B, c <sub>6</sub> , d <sub>6</sub> , E <sub>6</sub> , F <sub>5-6</sub>	77	a <sub>1</sub> , B, c <sub>6</sub> , d <sub>6</sub> , E <sub>6</sub> , F <sub>5-6</sub>	77	a <sub>1</sub> , B, c <sub>6</sub> , d <sub>6</sub> , E <sub>6</sub> , F <sub>5-6</sub>
12	93	A <sub>1</sub> , b <sub>1</sub> , C, <b>D</b> <sub>2-6</sub> , E, F <sub>6</sub>	73	A <sub>1</sub> , b <sub>1</sub> , C, d <sub>6</sub> , E <sub>6</sub> , F <sub>6</sub>	73	A <sub>1</sub> , b <sub>1</sub> , C, d <sub>6</sub> , E <sub>6</sub> , F <sub>6</sub>
13	103	A, <b>B</b> <sub>3-6</sub> , C <sub>1-2</sub> , D, e <sub>6</sub> , f <sub>6</sub>	90	A <sub>1-2</sub> , b <sub>2</sub> , C <sub>1-2</sub> , D, e <sub>6</sub> , f <sub>6</sub>	90	A <sub>1-2</sub> , b <sub>2</sub> , C <sub>1-2</sub> , D, e <sub>6</sub> , f <sub>6</sub>
14	115	a <sub>1</sub> , B, c <sub>6</sub> , <b>D</b> , e <sub>1</sub> , F	80	a <sub>1</sub> , B, c <sub>6</sub> , D <sub>6</sub> , e <sub>6</sub> , F <sub>6</sub>	80	a <sub>1</sub> , B, c <sub>6</sub> , <b>D</b> <sub>6</sub> , e <sub>6</sub> , F <sub>6</sub>
15	138	A, <b>B</b> <sub>1-4</sub> , C, D <sub>5-6</sub> , <b>E</b> <sub>3-5</sub> , F	102	A, b <sub>5</sub> , C <sub>5</sub> , D <sub>4</sub> , E <sub>4</sub> , F <sub>3-6</sub>	102	A, b <sub>5</sub> , C <sub>5</sub> , D <sub>4</sub> , E <sub>4</sub> , F <sub>3-6</sub>
16	112	A, <b>B</b> <sub>1-4</sub> , c <sub>1</sub> , D, e <sub>6</sub> , f <sub>6</sub>	84	A <sub>1-2</sub> , b <sub>1</sub> , c <sub>1</sub> , D, e <sub>6</sub> , f <sub>6</sub>	84	A <sub>1-2</sub> , b <sub>1</sub> , c <sub>1</sub> , D, e <sub>6</sub> , f <sub>6</sub>
17	90	a <sub>1</sub> , B <sub>1</sub> , C, D <sub>6</sub> , <b>E</b> <sub>3-6</sub> , F	85	a <sub>1</sub> , B <sub>1</sub> , C, D <sub>6</sub> , e <sub>6</sub> , F <sub>5-6</sub>	85	a <sub>1</sub> , B <sub>1</sub> , C, D <sub>6</sub> , e <sub>6</sub> , F <sub>5-6</sub>
18	98	A, <b>B</b> , C, <b>D</b> <sub>2-6</sub> , E, f <sub>6</sub>	68	A <sub>1</sub> , B <sub>1</sub> , C, d <sub>6</sub> , E <sub>6</sub> , f <sub>6</sub>	68	A <sub>1</sub> , <b>B</b> <sub>1</sub> , C, d <sub>6</sub> , E <sub>6</sub> , f <sub>6</sub>
19	134	A, <b>B</b> <sub>2-6</sub> , C <sub>1-5</sub> , D <sub>4-6</sub> , <b>E</b> <sub>3-5</sub> , <b>F</b>	107	A <sub>1-5</sub> , b <sub>5</sub> , C <sub>5</sub> , D <sub>4-6</sub> , E <sub>6</sub> , F <sub>5-6</sub>	107	A <sub>1-5</sub> , b <sub>5</sub> , C <sub>5</sub> , D <sub>4-6</sub> , E <sub>6</sub> , F <sub>5-6</sub>
20	137	A, <b>B</b> <sub>1-4</sub> , C, <b>D</b> <sub>3-5</sub> , E, f <sub>6</sub>	102	A, b <sub>5</sub> , C <sub>5</sub> , D <sub>5</sub> , E <sub>4-6</sub> , f <sub>6</sub>	102	A, b <sub>5</sub> , C <sub>5</sub> , D <sub>5</sub> , E <sub>4-6</sub> , f <sub>6</sub>
21	139	A, <b>B</b> <sub>1-4</sub> , C, D <sub>5-6</sub> , <b>E</b> <sub>3-5</sub> , F <sub>3-6</sub>	105	A, b <sub>5</sub> , C <sub>5</sub> , D <sub>4</sub> , e <sub>4</sub> , F <sub>4-6</sub>	105	A, b <sub>5</sub> , C <sub>5</sub> , D <sub>4</sub> , e <sub>4</sub> , F <sub>4-6</sub>
22	81	A <sub>1-5</sub> , B <sub>4-6</sub> , <b>C</b> , D, e <sub>6</sub> , f <sub>6</sub>	76	A <sub>1-2</sub> , b <sub>1</sub> , c <sub>1</sub> , D, e <sub>6</sub> , f <sub>6</sub>	76	A <sub>1-2</sub> , b <sub>1</sub> , c <sub>1</sub> , D, e <sub>6</sub> , f <sub>6</sub>
23	149	A, <b>B</b> <sub>1-4</sub> , c <sub>1</sub> , D, <b>E</b> , F	92	A <sub>1-2</sub> , b <sub>1</sub> , c <sub>1</sub> , D <sub>1</sub> , E <sub>1</sub> , F	92	A <sub>1-2</sub> , b <sub>1</sub> , c <sub>1</sub> , D <sub>1</sub> , <b>E</b> <sub>1</sub> , F
24	104	A <sub>1-3</sub> , <b>B</b> <sub>1-2</sub> , C, <b>D</b> <sub>2-5</sub> , E <sub>1-2</sub> , F <sub>3-6</sub>	84	A <sub>1-3</sub> , B <sub>3-6</sub> , c <sub>6</sub> , D <sub>6</sub> , e <sub>6</sub> , F <sub>6</sub>	84	A <sub>1-3</sub> , B <sub>3-6</sub> , c <sub>6</sub> , D <sub>6</sub> , e <sub>6</sub> , F <sub>6</sub>
25	126	A <sub>1-3</sub> , B <sub>2-6</sub> , C <sub>1-4</sub> , D, <b>E</b> , F	84	A <sub>1-3</sub> , B <sub>2-6</sub> , c <sub>5</sub> , D <sub>5-6</sub> , e <sub>6</sub> , f <sub>6</sub>	84	A <sub>1-3</sub> , B <sub>2-6</sub> , c <sub>5</sub> , D <sub>5-6</sub> , e <sub>6</sub> , f <sub>6</sub>

The critical path and early finish dates for each case are presented in Table 4. Critical activities are marked with capital letters, the subscript referring to the respective part of the activity (i.e., A<sub>1</sub>, B<sub>2-5</sub>, and C). Activities with either critical start float or critical finish float are marked with small letters (i.e., c<sub>1</sub>). Float calculations have been performed manually, as Primavera Project Planner cannot calculate start and finish float simultaneously. Finally, linear activities that belong to the second category group as defined by KLRPM are marked with bold capital letters (i.e. **D**, **E**<sub>1-3</sub>).

The results show that the initial solutions by KLRPM and CPM vary significantly in 24 out of 25 cases. The early finish of the projects, as calculated by KLRPM, is significantly larger by an average of 32% and a maximum of 72%, whereas there is no situation where it is smaller. Moreover, critical paths are also completely different. This is due to the effect of the resource continuity constraint. The only situation where project early finish and critical path coincide is Project 11. It can be observed from Table 4 that Project 11 is the only project that does not contain a linear activity belonging to the second category group of KLRPM as analyzed in the relevant section.

All such activities are identified in the 25 projects and the resource continuity constraint is removed; the early finish of the KLRPM and CPM projects coincide now in 25 out of the 25 cases. Considering them critical (which they are as explained in

the section explaining the KLRPM function), critical paths coincide also in 25 out of the 25 cases. The above analysis indicates that the presence of such activities, subject to the resource continuity constraint, is responsible for the critical path and early finish differences between the linear and the network schedule.

Other researchers have also observed that the resource continuity constraint may lead to longer project finish times (El-Rayes and Moselhi 2001). Removing the resource continuity constraint from the second category of KLRPM activities leads to shorter project durations and can be used to accelerate linear projects without increasing production rates (Kallantzis and Lambropoulos 2005; Yang and Ioannou 2004).

## Conclusions

This paper presented a critical path comparison between the linear scheduling method KLRPM and network scheduling method CPM. In order to compare the two methods, the linear schedule is converted to an equivalent network according to a set of rules. Results showed that the equivalent linear projects produce different critical paths and longer durations when compared to their network equivalents. However, when the resource continuity constraint is removed, project execution times and critical paths coincide.

The analysis of the 25 random projects showed that the critical path differences between the linear and network methods emanate from specific activities quite common in linear projects. These activities have the relation with the successor activity earlier in time than the relation with the predecessor activity. They present the paradox of increasing total project duration when their duration is shortened. When the resource continuity constraint is active, they force successor activities to start later in time, thus presenting longer durations than their network equivalents, where activities are divided into segments. When the constraint is removed, project durations coincide. The examination of a group of random projects, instead of just one or two, has minimized the margin for error. In the situation where only Project 11 was used as a sample project, it would have led to erroneous conclusions.

In conclusion, the critical path identified by the linear scheduling method differs from its network equivalent only by the effect of the resource continuity constraint. This finding assists in reducing reservations in the use of linear scheduling methods, since differences in project finish times and critical path between them and their network equivalents are adequately justified. Prac-

tioners can leverage this comparison to expand the use of linear scheduling methods and increase their acceptance as scheduling tools. Further, as the differences between the critical paths of the linear and the network scheduling methods have been established, the proposed method can serve as a robust base in order to expand the comparison in the area of total and free float.

The developed software PrjGen can be used, with minor transformations, to produce more complex random linear projects for research purposes. Moreover, it can assist practitioners by transforming linear projects into equivalent networks in format-ready importable by commercial scheduling software P3, thus reducing time allocated for the specific task. Finally, the output files of PrjGen can be modified to be imported by other commercial scheduling software.

The proposed transformation rules have been tested in full-span linear activities. Additional work is required in order to confirm the findings in linear projects containing other kinds of linear activities, such as bar or partial. Moreover, float comparison is necessary for a complete study of the correspondence between network and linear scheduling methods.

## Appendix

Activity	1. units/rates						constr. value
	1	2	3	4	5	6	
A			1/2				
B			1/8				Dist.
C			1/4				Dist.
D			1/10				Time
E			1/4				Time
F			1/5				Time

Activity	2. units/rates						constr. value
	1	2	3	4	5	6	
A			1/8				
B			1/2				Dist.
C			1/9				Dist.
D			1/4				Dist.
E			1/5				Dist.
F			1/10				Dist.

Activity	3. units/rates						constr. value
	1	2	3	4	5	6	
A			1/10				
B			1/8				Dist.
C			1/5				Dist.
D			1/3				Time
E			1/5				Time
F			1/9				Time

Activity	4. units/rates						constr. value
	1	2	3	4	5	6	
A			1				
B			1/5				Dist.
C			1/3				Time
D			1/10				Dist.
E			1/8				Time
F			1/3				Dist.

Activity	5. units/rates						constr. value
	1	2	3	4	5	6	
A			1/8				
B			1/9				Time
C			1/10				Dist.
D			1				Dist.
E			1/5				Time
F			1/3				Dist.

Activity	6. units/rates						constr. value
	1	2	3	4	5	6	
A			1/8				
B			1				Time
C			1/3				Dist.
D			1/8				Time
E			1/3				Dist.
F			1/10				Time

Activity	7. units/rates						constr. value
	1	2	3	4	5	6	
A			1/9				
B			1/5				Dist.
C			1/8				Dist.
D			1/10				Time
E			1				Dist.
F			1/3				Dist.

Activity	8. units/rates						constr. value
	1	2	3	4	5	6	
A			1/5				
B			1/8				Dist.
C			1				Dist.
D			1/3				Time
E			1/10				Time
F			1				Time

Activity	9. units/rates						constr. value
	1	2	3	4	5	6	
A			1/8				
B			1				Time
C			1/10				Dist.
D			1/2				Time
E			1/3				Time
F			1/5				Dist.

Activity	10. units/rates						constr.
	1	2	3	4	5	6	
A				1/3			
B				1/2			Time
C				1/8			Dist.
D				1/5			Time
E				1/4			Time
F				1/10			Time

Activity	11. units/rates						constr.
	1	2	3	4	5	6	
A				1/4			
B				1/10			Time
C				1/9			Time
D				1/8			Time
E				1/4			Dist.
F				1/2			Dist.

Activity	12. units/rates						constr.
	1	2	3	4	5	6	
A				1/2			
B				1/4			Dist.
C				1/9			Time
D				1/4			Time
E				1/8			Dist.
F				1/5			Dist.

Activity	13. units/rates						constr.
	1	2	3	4	5	6	
A				1/4			
B				1			Time
C				1/8			Dist.
D				1/10			Dist.
E				1/9			Time
F				1/4			Time

Activity	14. units/rates						constr.
	1	2	3	4	5	6	
A				1/9			
B				1/10			Time
C				1/4			Time
D				1			Time
E				1/5			Time
F				1/8			Time

Activity	15. units/rates						constr.
	1	2	3	4	5	6	
A				1/9			
B				1/5			Dist.
C				1/8			Time
D				1/3			Dist.
E				1			Dist.
F				1/8			Dist.

Activity	16. units/rates						constr.
	1	2	3	4	5	6	
A				1/8			
B				1			Dist.
C	1/2			1/5			Time
D				1/10			Time
E				1/4			Time
F	1/9			1			Time

Activity	17. units/rates						constr.
	1	2	3	4	5	6	
A				1/2			
B				1/4		1/9	Time
C	1/5			1/10			Dist.
D				1/9			Dist.
E	1			1/4			Time
F				1/5			Dist.

Activity	18. units/rates						constr.
	1	2	3	4	5	6	
A	1/2			1/5			
B				1			Time
C				1/8			Time
D	1/10			1/3			Time
E				1/5			Dist.
F				1/3			Time

Activity	19. units/rates						constr.
	1	2	3	4	5	6	
A				1/9			
B				1/5			Time
C				1/8			Dist.
D	1/4			1/10			Dist.
E				1/3			Dist.
F	1/4			1.3			Dist.

Activity	20. units/rates						constr.
	1	2	3	4	5	6	
A				1/10			
B				1/3			Dist.
C	1/5			1/9			Time
D	1/4			1/5			Dist.
E				1/8			Dist.
F				1			Time

Activity	21. units/rates						constr.
	1	2	3	4	5	6	
A				1/10			
B				1/2			Dist.
C				1/8			Time
D				1/5			Dist.
E				1/4			Dist.
F	1/2			1/9			Time

Activity	22. units/rates						constr.
	1	2	3	4	5	6	
A				1/3			
B	1/2			1/5			Dist.
C	1/4			1/3			Time
D				1/10			Time
E	1/4			1/9			Time
F	1/4			1/2			Time

Activity	23. units/rates						constr.
	1	2	3	4	5	6	
A	1/8			1/10			
B				1			Dist.
C	1/4			1/2			Time
D				1/9			Time
E	1/3			1/5			Time
F				1/10			Time

Activity	24. units/rates						constr.
	1	2	3	4	5	6	
A	1/8			1/9			
B	1/4			1/10			Dist.
C				1/8			Time
D	1			1/4			Dist.
E	1/8			1/3			Dist.
F	1/2			1/8			Time

Activity	25. units/rates						constr.
	1	2	3	4	5	6	
A				1/9			
B	1/4			1/10			Time
C				1/3			Dist.
D				1/8			Time
E	1			1/4			Time
F	1/4			1/2			Time



## References

- Ammar, M. A., and Elbeltagi, E. (2001). "Algorithm for determining controlling path considering resource continuity." *J. Comput. Civ. Eng.*, 15(4), 292–298.
- Birrell, G. S. (1980). "Construction planning—Beyond the critical path." *J. Constr. Div.*, 106(3), 389–407.
- El-Rayes, K., and Moselhi, O. (1998). "Resource-driven scheduling of repetitive activities on construction projects." *J. Constr. Management & Economics*, 16(4), 433–466.
- El-Rayes, K., and Moselhi, O. (2001). "Optimizing resource utilization for repetitive construction projects." *J. Constr. Eng. Manage.*, 127(1), 18–27.
- Harmelink, D. J., and Rowings, E. J. (1998). "Linear scheduling model: Development of controlling activity path." *J. Constr. Eng. Manage.*, 124(4), 263–268.
- Harris, R. B., and Ioannou, P. G. (1998). "Scheduling projects with repeating activities." *J. Constr. Eng. Manage.*, 124(4), 269–278.
- Hegazy, T., and Wassef (2001). "Cost optimization in projects with repetitive nonserial activities." *J. Constr. Eng. Manage.*, 127(3), 183–191.
- Ioannou, P. G., and Yang, I.-T. (2004). "Discussion of 'Comparison of linear scheduling model and repetitive scheduling method.'" *J. Compos. Constr.*, 8(3), 461–463.
- Kallantzis, A., and Lambropoulos, S. (2004a). "Linear scheduling and critical path determination using minimum and maximum constraints." *Eng., Constr., Archit. Manage.*, 11(3), 211–222.
- Kallantzis, A., and Lambropoulos, S. (2004b). "Discussion of 'Comparison of linear scheduling model and repetitive scheduling model.'" *J. Compos. Constr.*, 8(3), 463–467.
- Kallantzis, A., and Lambropoulos, S. (2005). "Linear project acceleration without increasing production rates." *Tech. Chron. Sci. J. TCG*, 1, 25(1), 53–63.
- Mattila, K. G., and Park, A. (2003). "Comparison of linear scheduling model and repetitive scheduling method." *J. Constr. Eng. Manage.*, 129(1), 56–64.
- Reda, R. M. (1990). "RPM: repetitive project modeling." *J. Constr. Eng. Manage.*, 116(2), 316–330.
- Yamin, R. A., and Harmelink, D. J. (2001). "Comparison of linear scheduling model (LSM) and critical path method (CPM)." *J. Constr. Eng. Manage.*, 127(5), 374–381.
- Yang, I., and Ioannou, P. G. (2004). "Scheduling system with focus on practical concerns in repetitive projects." *Construction Management and Economics*, 22(6), 619–630.