Tender Evaluation by Fuzzy Sets

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Abstract: The process of evaluating tenders is considered to be largely dependent on subjective judgment when cost is not the only criterion used. A systematic procedure based on fuzzy set theory and multicriteria modeling is proposed for the selection of bid contracts. The proposed procedure is suitable for a general tender evaluation process that may involve many decision-making parties and noninteractive multiple criteria. Illustrative examples are given for cases involving three major criteria: cost, present bid information, and past experience of tenderers.

Introduction

Tender evaluation is the process of selecting a contractor from a number of tenderers, given that the owner-client has received the bids (or tenders) from these tenderers for a specified project.

Tender evaluation is as important to an owner-client as bidding strategy formulation to contractors. It can be recognized however, that while rational and analytical approaches can be used to formulate bidding strategies (2,9), tender evaluation remains largely an art where subjective judgment based on the engineer's experience becomes an essential element of the tender selection process. Contractors observe quite frequently that the lowest bid does not necessarily win the contract, and owners do not always find tender selection an easy task.

Besides quantitative criteria, like monetary costs and benefits, qualitative and intangible factors such as managerial safety accountability, competence, and efficiency of contractors are usually taken into account in a tender evaluation process.

In many cases, not one but many interest groups are involved in a specified project, and the viewpoints and socio-political necessities of all decision-making parties must be incorporated in the decision process. Contract selection may also be influenced by the economical and environmental constraints that may exist at the time of tender evaluation or planned construction. The process of evaluating tenders is thus a decision-making process involving a wide range of criteria for which the information can be imprecise and subjective. Uncertainty associated with such information is thus not totally suitable for analysis by probabilistic approaches, and the process of selecting tenders can evoke a famous statement given by Zadeh (23): "Our ability to make precise and significant statements concerning a given system diminishes with increasing complexity of the system."

The objective of this paper is to present a procedure based on the theory of fuzzy sets (22) and multi-criteria decision modeling, for deter-

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J. Constr. Eng. Manage., 1985, 111(3): 231-243

mining the more desirable solutions to the problem of selecting contract bids. Through illustrative examples, it is shown that the tender selection process involving multiple criteria in general, can be performed systematically by using basic fuzzy set operations.

Fuzzy Set Theory and Tender Evaluation

Fuzzy Sets.—In classical set theory, a set is defined as a collection of objects having a general property, e.g., a class or a set of concrete mixers, or a group of contractors. A construction engineer works either as a contractor or for some organization. If he works as a contractor, he is said to belong to the group of contractors, or to have a membership of 1 in the class of contractors. If he is not a contractor, he then has no membership in the class or his membership grade is zero. Clearly, the grade of membership in a classical or crisp set, such as the group of contractors, is binary, either one or zero. Suppose we want to extend this classical set concept to embrace another kind of set, say a subset of "very experienced" contractors. We may then ask ourselves, "How experienced is very experienced?". A satisfactory answer to this question may be a difficult one as the class of "very experienced" contractors is not a set in the classical sense, but belongs to a fuzzy, not crisply-defined type. The definition of "very experienced" may involve a spectrum of human perceptions and the class of "very experienced" contractors is therefore said to represent a fuzzy set.

The notion of fuzzy sets, first introduced by Zadeh (22), deals with certain sets which may admit partial membership. A fuzzy set is thus a set with members having a continuum of grades of membership, from 0 to 1. (Fig. 1 illustrates the basic concept of fuzzy sets by their membership definition.)

Let U denote a space of objects or a universe of discourse; a fuzzy set A in U is a set of

$$A = \{[x, \mu_A(x)]\}, \quad x \in A \quad \text{and} \quad A \subset U \quad \quad (1)$$

in which, $\mu_A(x)$ is termed the grade of membership of x in A, and is a real number:

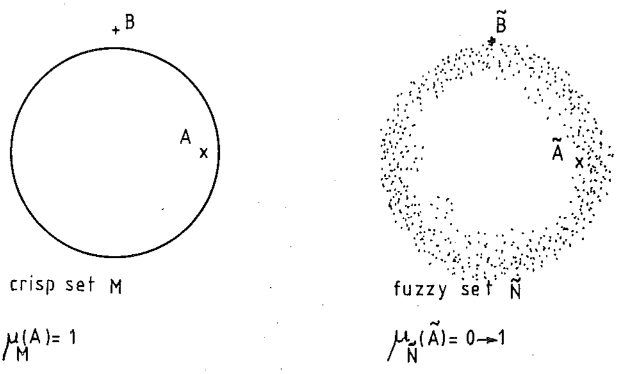

FIG. 1.—Crisp Set and Fuzzy Set

J. Constr. Eng. Manage., 1985, 111(3): 231-243

$0 \leq \mu_A(x) \leq 1$

i.e., $\mu_A(x) = 0$ represents nonmembership of x in A, e.g., *NOT* very experienced contractor; and $\mu_A(x) = 1$ full membership of x in A, e.g., the belief of a contractor being "very experienced" is 1.0. x is a continuous variable, usually a linguistic variable of which each value corresponds to a degree of support or belief of $\mu(x)$.

The theory of fuzzy sets thus involves the interpretation of words and phrases such as "cheap," and "the contractor is competent," by the use of membership functions. For example, "a strong bridge foundation" can be expressed as

$$\text{Strong foundation} = \{0.6|0.1, 0.7|0.5, 0.8|0.7, 0.9|0.9, 1.0|0.6\}$$

in which $\{0.6, 0.7, 0.8, 0.9, 1.0\}$ are the various possible values assumed by the linguistic variable "strong," and $\{0.1, 0.5, 0.7, 0.9, 0.6\}$ are the corresponding grades of membership or strength of belief for these values, subjectively assigned by an observer; and | is a delimeter. It is noted that "strong" has a central value of 0.9 in the set.

"Very strong foundation" can be expressed as the set of $(\text{strong})^2$, in which respective membership grades of all the values of "strong" are squared:

$$\text{Very strong} = \{0.6|0.01, 0.7|0.25, 0.8|0.49, 0.9|0.81, 1.0|0.36\}.$$

The following are some basic mathematical operations of fuzzy set theory:

1. Assume U and V are two fuzzy sets, a fuzzy relation R in $U \times V$ is a fuzzy subset of $U \times V$, each element having a dual grade of membership $\mu_R(x,y)$ with $x \in U$ and $y \in V$.

2. If A and B are both fuzzy subsets of U, then the operation (or requirement) of A "AND" B is a fuzzy subset $A \cap B$ with grade of membership:

$$\mu_{A\cap B}(x) = \text{Min}\,[\mu_A(x), \mu_B(x)] \quad \text{(2)}$$

3. Likewise, A "OR" B is a fuzzy subset with

$$\mu_{A\cup B}(x) = \text{Max}\,[\mu_A(x), \mu_B(x)] \quad \text{(3)}$$

4. "NOT A" (or Complement of A) has grades of membership of:

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x), \quad x \in A \quad \text{(4)}$$

5. Assume A is a fuzzy subset of U, and B is a fuzzy subset of V, then the operation A "AND" B is a binary relation.

$$\mu_R(x,y) = \text{Min}\,[\mu_A(x), \mu_B(y)] \quad \text{(5)}$$

The projection of R on U is a fuzzy subset of

$$\text{Proj. } R(x) = \underset{y\in V}{\text{Max}}\,[\mu_R(x,y)] \quad \text{for all} \quad x \in U \quad \text{(6)}$$

Tender Evaluation Process.—Diekmann (6) presented a study of contractor selection for a cost-plus contract. In the study (6), a wide range of attributes that may be used in selecting a contractor were discussed, and the selection methodology was based on aggregation of scores or rankings on these attributes, as commonly used in multicriterion decision modeling. It is considered, from the literature of opinions aggregation in social sciences (8, 13, and 14), that there is a wide variety of methods to obtain a score for an object based on a number of attributes. However, there are theoretical difficulties surrounding a choice of aggregation criteria, especially when some or all of these attributes are qualitative or subjective. The fuzzy sets aggregation presented in the following, therefore, underlies the importance of aggregation of subjective probability judgments. For the purpose of illustrating the proposed method of aggregation in the context of tender evaluation or contractor selection, it is recalled that the three most important and basic criteria used in contractor selection are cost, experience, and performance potential. For completeness, these three criteria are described as follows.

The first criterion in a tender selection process is naturally the sum total offer made by each contractor. It is always important, however, to see whether each contractor is offering the same thing. Some tenders may be submitted with certain conditions or provisos, which are contrary or additional to the conditions laid down in the tender document (20). A meticulous examination of the various items and conditions submitted by tenderers is often required and would constitute the basis for a second set of criteria. The following set of criteria is based on the information contained in each bid, mainly comprising proof of the contractor's:

1. Soundness of approach to perform the work.
2. Understanding of the nature and content of the project.
3. Compliance with the specifications stipulated in the tender document for the project.
4. Specific experience suitable for undertaking the work.
5. Management qualifications.
6. Explanations for variation of pricing, e.g., significant difference in pricing between different items of work, not generally found in other tenders.

In the majority of contracts, prequalification of tenderers is used by the client to exclude contractors that may not have sufficient qualifications and experience for the job. The basic advantage of "prequalification method" is that quite often the client only needs to accept the lowest bid from the short list of selected tenderers. In general, prequalification is a criterion arising from the overall rating of contractors (or subcontractors) established by the client or referees, and thus based principally on the contractors', or subcontractors' past experience, performance records and managerial safety accountability. In more detail, the various components of this criterion may cover the following factors concerning the contractor's past performance:

1. Technical competence and managerial expertise.
2. Compliance with specifications in previous undertakings.
3. Attitude towards correcting faulty or incomplete work.
4. Safety records (e.g., management safety accountability, experience modification ratings, incidence rate; see, for example, Ref. 18).

J. Constr. Eng. Manage., 1985, 111(3): 231-243

5. Ability to meet work schedules.
6. Attitude towards claims and counter-claims.

Thus, besides the cost criterion, evaluating tenders usually requires two other general criteria based on the contractor's previous experience and the predictive judgment on the contractor's likely performance for the present job. These three general criteria do not by and large compensate each other. Conventional methods of assessing a multicriteria situation, such as arithmetic, product mean and probabilistic sum therefore do not appear to be appropriate. [See for example, Dubois and Prade, (7)].

Multicriteria Tender Selection

In a tender evaluation process, be it for prequalification or final selection, the client normally holds meetings with his partners or associates. The basic purpose of these meetings is to exchange individual assessments and opinions on various tenderers. Each partner or interest group presents a rating table for all the factors considered on every tenderer. Aggregation of rating matrices provided by all decision making groups is essential in the first step of a selection process. It should be emphasized that aggregation methods presented in the following place equal importance, or weights, among the partners or interest groups. For incorporation of different weights with various decision parties, the use of power laws on fuzzy membership grades as described in scenarios *B* and C of the following example, could be used.

Aggregation.—A rating given to tenderer i on account of a criterion j is a binary grade of membership μ_{ij}. Aggregation of k rating values of μ_{ij} given by k different decision makers can be performed by either "OR" or "AND" operator. For pessimistic aggregation, e.g., the tender selection panel will be concerned if one member of the panel is unhappy about tenderer i on criterion j, the aggregated rating is then defined as:

$$\mu_{ij} = \mu_{ij}^1 \cap \mu_{ij}^2 \cap \ldots \cap \mu_{ij}^k \quad (\text{"AND" operator})$$
$$= \min(\mu_{ij}^1, \mu_{ij}^2, \ldots, \mu_{ij}^k) \quad (7)$$

where each superscript (1 to k) denotes the entry by a respective decision-making member, 1 to k. For optimistic aggregation, such as the selection panel being satisfied if at least one member is happy with tenderer i on the jth count, the final rating becomes:

$$\mu_{ij} = \mu_{ij}^1 \cup \mu_{ij}^2 \cup \ldots \cup \mu_{ij}^k \quad (\text{"OR" operator})$$
$$= \max(\mu_{ij}^1, \mu_{ij}^2, \ldots, \mu_{ij}^k) \quad (8)$$

Based on a study of waste disposal alternatives, Hipel (10) concluded that pessimistic aggregation attempts to minimize risk while optimistic aggregation may present the best case viewpoint among the interest groups. Znotinas and Hipel (25) however, had doubts on the efficacy of optimistic aggregation for most practical applications. If it were desirable to have a decision from a spectrum of polarized opinions, the modified pessimistic aggregation

$$\mu_{ij} = \frac{1}{2}\left[(\mu_{ij}^1 \cap \mu_{ij}^2 \cap \ldots \cap \mu_{ij}^k) + \left(\frac{1}{k}\sum_{l=1}^{k} \mu_{ij}^l\right)\right] \quad \ldots \ldots \ldots \ldots \quad (9)$$

which is the average of the pessimistic approximation and the arithmetic mean, may prove to be useful (10).

Example.—Suppose for tender evaluation, we have carried out two stages of aggregation, the first being aggregation on interest group opinions, and the second on decision factors constituting the two principal criteria other than cost. We now proceed with evaluating tenders optimizing the three principal criteria mentioned previously. For the two criteria based on past experience and present bid information, the various components constituting them can be considered to have some interactivity. Their aggregation can be carried out by conventional aggregating schemes such as arithmetic mean or product mean. For the cost criterion, the rating or degree of support can be conveniently scaled from a threshold, as shown in the following:

Scenario A.—Assume we have to choose among five tenders, $A = \{x_1, x_2, x_3, x_4, x_5\}$, a contractor most suitable for the project, satisfying all the three principal criteria, i.e., cost, experience, and performance potential.

Let $B = \{y_1, y_2, y_3\}$ be our set of criteria of equal importance, being:

1. y_1 = the selected contractor should have the lowest bid, or a bid as low as economically acceptable.
2. y_2 = the selected contractor should have a good reputation and excellent performance records. The rating for this criterion is lumped from various factors, and aggregated from individual ratings provided by the selection panel members.
3. y_3 = the selected contractor's tender should have enough information proving that he has sufficient resources, managerial and engineering expertise for a successful undertaking of the project. Aggregation of ratings for this criterion is similar to that for y_2.

For illustration, assume the five contractors offered the following sums in their bids:

$x_1 \rightarrow$ \$316,989;

$x_2 \rightarrow$ \$229,311;

$x_3 \rightarrow$ \$244,946;

$x_4 \rightarrow$ \$276,350; and

$x_5 \rightarrow$ \$222,220.

TABLE 1.—Initial Rating Matrix or Binary Relation $A \times B$

A \ B (1)	y_1 (2)	y_2 (3)	y_3 (4)
x_1	0.68	0.50	0.90
x_2	0.94	0.70	0.67
x_3	0.88	0.90	0.72
x_4	0.78	0.92	0.79
x_5	0.97	0.30	0.93

J. Constr. Eng. Manage., 1985, 111(3): 231-243

We know that if we consider criterion y_1 only, contractor x_5 with the lowest bid must get the job. But we have two more criteria to consider, and the rating matrix for evaluation is set up as Table 1.

It can be seen that Table 1 is just a binary relation $A \times B$ with elements as rating values μ_{xy}.

The values of the ratings in columns y_2 and y_3 were obtained from two-staged aggregation described previously while those in y_1 (cost criterion) have been calculated, in the same manner as that used in work study (15), by arbitrarily assuming:

$$(\text{Bid price}) \times (\mu) = (\text{Basic price}) \quad (10)$$

"Basic price" is that which the panel agreed as the desired lowest price—and in this instance equal to \$215,553, i.e.

$$\$222{,}220 \times 0.97 = \$215{,}553; \quad x_5 \times \mu_{x_5y_1} = \text{Basic price} \quad (11)$$

A quick inspection of Table 1 reveals that the y_1-ratings are not quite compatible in magnitude with those given on y_2 and y_3. The rating matrix thus falls into a situation described by Hipel (10), i.e., one single alternative (or criterion) may dominate others and the completely dominated alternatives will never be selected by the decision makers. By arbitrarily reassigning the upper and lower bounds of membership grades or ratings of y_2 (i.e., 0.92 and 0.30) with values more compatible with those of y_1 and y_3, and by linearly interpolating the ratings between the bounds, we obtain new ratings for y_2 and a decision matrix, shown in Table 2.

Bellman and Zadeh (1) suggested that the best-suited method for multicriteria decision-making processes in a fuzzy framework is a decision subset D, satisfying the pessimistic requirement. For the present example involving three criteria y_1, y_2 and y_3, we have:

$$D = y_1 \cap y_2 \cap y_3;$$

$$D = \{x_1|0.68, x_2|0.67, x_3|0.72, x_4|0.78, x_5|0.77\} \quad (12)$$

In decision subset D, the membership grade of each contractor is obtained by taking the minimum across the respective row in Table 2.

From D preceding, it is seen, therefore, that contractor x_4 is selected on the basis of the highest degree of support (= 0.78) assigned to x_4 by the Bellman-Zadeh procedure (1), even though contractor x_4 by no means tendered with the lowest bid.

TABLE 2.—Rating Matrix Adjusted for y_2-Ratings

A \ B (1)	y_1 (2)	y_2 (3)	y_3 (4)
x_1	0.68	0.83	0.90
x_2	0.94	0.89	0.67
x_3	0.88	0.95	0.72
x_4	0.78	0.96	0.79
x_5	0.97	0.77	0.93

TABLE 3.—Rating Matrix for Scenario B

A \ B (1)	y_1 (2)	y_2 (3)	y_3 (4)
x_1	0.46	0.91	0.90
x_2	0.88	0.94	0.67
x_3	0.77	0.97	0.72
x_4	0.61	0.98	0.79
x_5	0.94	0.88	0.93

Scenario B.—Suppose now the selection panel decides that it is "very important" to accept a very low tender (criterion y_1) and only "more or less" important to satisfy criterion y_2 regarding past performance, but still important to consider criterion y_3.

According to Zadeh (24), the grades of membership, μ_{xy}, for each criterion can be transformed:

$$\begin{array}{llll} \text{for} & y_1, \mu_{xy} & \text{becomes} & \mu^2_{xy}(y = y_1) \\ & y_2, \mu_{xy} & \text{becomes} & \mu^{1/2}_{xy}(y = y_2) \\ \text{and} & y_3, \mu_{xy} & \text{remains} & \mu_{xy}(y = y_3) \end{array}$$

The rating matrix for Scenario B is given in Table 3. The decision subset D becomes

$$D = \{x_1 |0.46, x_2 |0.67, x_3 |0.72, x_4 |0.61, x_5 |0.88\} \quad \text{(13)}$$

And the selection panel would naturally go for contractor x_5.

If now the information contained in the present bid is considered to be "VERY IMPORTANT," the decision matrix of Table 3 can be modified by squaring the y_3-rating column, as shown in Table 4, yielding the decision subset D as

$$D = [x_1 |0.46, x_2 |0.45, x_3 |0.52, x_4 |0.61, x_5 |0.86] \quad \text{(14)}$$

which reveals that contractor x_5 has again won the contract by the highest degree of support (0.86). Contractor x_5 is also the one that tendered with the lowest bid.

Scenario C.—Refer to Table 5 which is taken from Table 2, except that the y_3-rating vector has been replaced with a new one having upper and lower rating bounds of 0.93 and 0.67, respectively, and a linear rating

TABLE 4.—Rating Matrix (Scenario B) with Weights on y_1 and y_3

A \ B (1)	y_1 (2)	y_2 (3)	y_3 (4)
x_1	0.46	0.91	0.81
x_2	0.88	0.94	0.45
x_3	0.77	0.97	0.52
x_4	0.61	0.98	0.62
x_5	0.94	0.88	0.86

J. Constr. Eng. Manage., 1985, 111(3): 231-243

TABLE 5.—Initial Rating Matrix for Scenario C (Equal Weights)

A \ B (1)	y_1 (2)	y_2 (3)	y_3 (4)
x_1	0.68	0.83	0.67
x_2	0.94	0.89	0.73
x_3	0.88	0.95	0.80
x_4	0.78	0.96	0.87
x_5	0.97	0.77	0.93

scale for those between the bounds. We attempt to investigate the sensitivity of our selection by varying the ratings associated with the performance potential (y_3) criterion.

For equal weights among the three criteria, as presented in Table 5, we apply Bellman-Zadeh's min-max scheme (i.e., minima for the rows and maximum of the resulting column), and obtain the highest rating of 0.80 for contractor x_3. It is noted that the final decision subset:

$$D = [x_1|0.67, x_2|0.73, x_3|0.80, x_4|0.78, x_5|0.77] \quad (15)$$

comprises elements drawn from all the three criteria, i.e., 0.67, 0.73, 0.80 from y_3; 0.77 from y_2; and 0.78 from y_1.

If criterion y_3 is now considered "VERY IMPORTANT," the y_3-rating vector becomes, by squaring all the membership grades:

$$[\mu_{xy_3}] = [x_1|0.45, x_2|0.53, x_3|0.64, x_4|0.76, x_5|0.86] \quad (16)$$

and consequently:

$$D = [x_1|0.45, x_2|0.53, x_3|0.64, x_4|0.76, x_5|0.77] \quad (17)$$

resulting in a maximum degree of support of 0.77 associated with contractor x_5. The membership grade of 0.77 in the final decision subset is also the only one rating drawn from criterion y_2. All other ratings are taken from the y_3-column and the choice is thus biased towards this criterion as a result of importance weighting assigned to y_3.

Finally, if it is considered that criterion y_3 is "VERY, VERY IMPORTANT," we can transform, in the manner of linguistic variable modeling (24), the y_3-rating vector from Table 5 as:

$$[\mu_{xy_3}] \rightarrow [\mu_{xy_3}]^3$$

yielding: $[\mu_{xy_3}] = [x_1|0.30, x_2|0.39, x_3|0.51, x_4|0.66, x_5|0.73]$ (18)

giving an identical decision set, D, after aggregation with criteria y_1 and y_2. Contractor x_5 is selected with a maximum membership grade of 0.73, based on the only criterion y_3 identical with D.

Our choice of contractor becomes totally biased in criterion y_3 when the final decision set, D, comprises rating elements drawn entirely from the rating vector of y_3.

Discussion

In recent civil engineering literature (3, 4, 5, 11, 13 and 19), it has been shown or indicated that fuzzy set theory is particularly suitable for application in the modeling of classes of problems involving fuzzy or imprecise data, and in structural safety analyses for which the information may involve uncertainty of a subjective type, such as vague description, human errors, omissions, and mistakes.

In this paper, a rational tender evaluation procedure based on fuzzy set theory has been presented. Illustration for the proposed procedure has been exemplified by three scenarios involving three basic attributes that may be relied upon in selecting a contractor, i.e., cost, experience, and performance potential. For a more general tender selection process, involving a large number of bids and a wide range of selection criteria, the same set of fuzzy aggregation operations, including most importantly the min-max scheme, will also apply.

The proposed procedure basically searches for a contractor (or subcontractor) that would best satisfy a set of criteria. These criteria may have weights assigned to them in the form of exponents to reflect the order of importance on particular decision parties or emphasis on particular criteria assigned by the decision parties. It is considered, and illustrated in Scenario C, that the final ratings given to all the criteria should be of comparable magnitudes to avoid any bias tendency towards a particular criterion having low ratings. For more details of fuzzy characterization of decision processes, see for example, Refs. 10, 12, 16 and 21.

In the model presented, estimation of cost ratings is based on what is commonly used in work study. If x is used to denote the bid, and a the base cost, the work study rating function (Eq. 10) can be rewritten as:

$$\mu = \frac{1}{1+v} \quad \text{(19)}$$

in which $v = x - a/a$, giving cost ratings in close agreement with an "analytical" formula often used in fuzzy set literature (e.g., Ref. 12), especially for low values of v, i.e., bids being close to the base cost:

$$\mu = e^{-kv^2} \quad \text{(20)}$$

in which k = an arbitrary constant, set equal to 3 in Fig. 2.

The practical estimation of membership functions has always been a common ground for disagreement between various researchers. For a general case, exemplification and statistical methods are frequently used. Exemplification is basically a survey of opinions whereby grade values of certain linguistic variables, such as "efficient," and "economical," can be obtained. Statistical methods involve mainly constructing a set of statistical data in the form of a histogram. The membership grades or ratings are, however, scaled in a different manner to that used in frequency estimation. For membership grading, the histogram is normalized through an affine transformation that brings the highest ordinate to 1, whereas in frequency estimation, the whole area made up by the histogram is brought to 1 (e.g., Ref. 7).

It should be emphasized that noninteractivity is one of the important features that underlie the fuzzy aggregation procedure. Noninteractivity simply means the ratings assigned to major criteria such as cost and past performance, do not compensate each other. For a discussion of non-

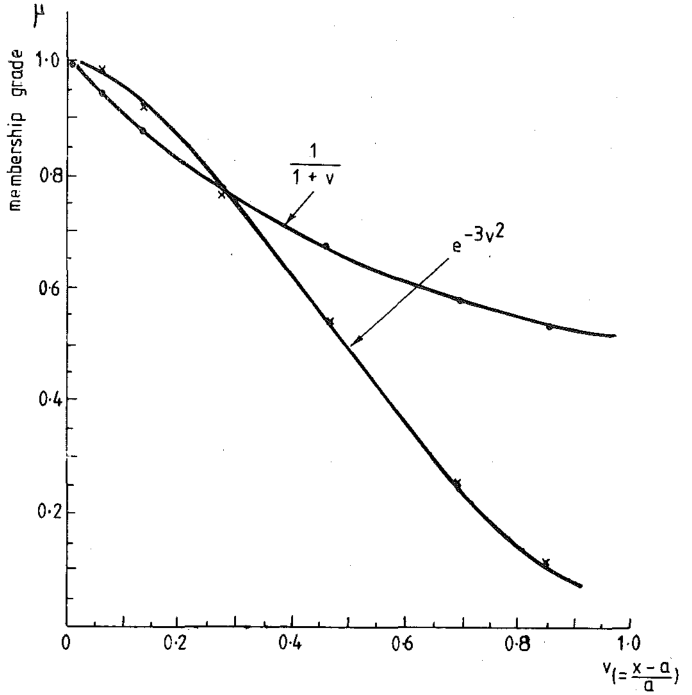

FIG. 2.—Analytical Functions for Membership Grade

interactivity in the fuzzy framework, see, e.g., Refs. 7 and 23. It has been noted that the proposed procedure using fuzzy sets is but an addition to the range of multicriteria decision methodologies. The particular characteristic of the present procedure is its emphasis on the human perception nature, the qualitative or subjective criteria used in tender evaluation, and the theoretical basis for aggregation of these criteria. It is recalled that fuzzy set theory was originally devised to model uncertainty associated with human perception or subjective probability (possibilistic) judgments, whereas statistics and probability theory deal with uncertainty that is of a random or statistical nature and associated with the likely frequency of occurrence. In this respect, fuzzy multicriteria modeling, as proposed for tender selection, appears to be more theoretically appropriate than statistically-based methodologies, such as clustering techniques described in Refs. 13 and 14, which sometimes incorporate the correlation between the variables or criteria concerned. Interactivity among our possibilistic judgments of the mixture of quantitative and qualitative criteria in tender evaluation, however, cannot be totally disregarded. Further research in this area, and particularly a modification of the Bellman-Zadeh scheme to account for criteria interactivity, is therefore suggested.

Conclusions

A decision model for selecting bid contracts based on fuzzy set theory has been proposed. The model is exemplified by simple criteria comprising: cost, experience and performance potential or bid information. Modification of the model to cover other criteria such as safety, environmental compliance, and schedule can be easily incorporated. The same model can be used by general contractors to choose subcontractors, and is theoretically independent of the type of contract considered.

The model emphasizes the multicriteria aspect of tender evaluation, the qualitative, subjective, and noninteractive nature of these criteria, and their aggregation in the selection process.

Appendix I.—References

1. Bellman, R. E., and Zadeh, L. A., "Decision-Making in a Fuzzy Environment," *Management Science*, Vol. 17, No. 4, 1970, pp. 141–154.
2. Benjamin, N. B. H., and Meador, R. C., "Comparison of Friedman and Gates Competitive Bidding Models," *Journal of the Construction Division*, ASCE, Vol. 105, No. CO1, Mar., 1979, pp. 25–40.
3. Blockley, D., "The Calculation of Uncertainty in Civil Engineering," *Proceedings*, Institute of Civil Engineers, Part 2, Vol. 67, 1979, pp. 313–326.
4. Brown, C. B., "The Merging of Fuzzy and Crisp Information," *Journal of the Engineering Mechanics Division*, ASCE, Vol. 106, No. EM1, 1980, pp. 123–133.
5. Brown, C. B., and Yao, J. T. P., "Fuzzy Sets and Structural Engineering," *Journal of Structural Engineering*, ASCE, Vol. 109, No. 5, May, 1983, pp. 1211–1225.
6. Diekmann, J. E., "Cost-Plus Contractor Selection," *Journal of the Technical Councils*, ASCE, Vol. 107, No. TC1, April, 1981, pp. 13–25.
7. Dubois, D., and Prade, H., "Fuzzy Sets and Systems: Theory and Applications," Academic Press, 1980.
8. Fishbein, M., and Ajzen, I., "Belief, Attitude, Intention and Behavior: An Introduction to Theory and Research," Addison-Wesley, Reading, Mass., 1975.
9. Gates, M., "Bidding Strategies and Probabilities," *Journal of the Construction Division*, ASCE, Vol. 93, No. CO1, March, 1967, pp. 75–107.
10. Hipel, K. W., "Fuzzy Set Methodologies in Multicriteria Modelling," *Fuzzy Information and Decision Processes*, North Holland, 1982, pp. 279–288.
11. Kam, T. Y., and Brown, C. B., "Updating Parameters with Fuzzy Entropies," *Journal of Engineering Mechanics*, ASCE, Vol. 109, No. 6, 1983, pp. 1334–1343.
12. Leung, Y., "A Fuzzy Set Procedure for Project Selection with Hierarchical Objectives," *Fuzzy Sets: Theory and Applications to Policy Analysis and Information Systems*, Plenum Press, 1980, pp. 329–340.
13. Lorr, M., "Cluster Analysis for Social Scientists," *Techniques for Analysing and Simplifying Complex Blocks of Data*, Jossey-Bass Inc., Publishers, San Francisco, Calif., 1983.
14. Michaud, P., "Opinions Aggregation," *New Trends in Data Analysis and Applications*, Elsevier Science Publishers, B.V., North Holland, 1983, pp. 5–27.
15. National Coal Board, *Work Study in Mines*, NCB—U.K., 1979.
16. Neitzel, L. A., and Hoffman, L. J., "Fuzzy Cost and Benefit Analysis," *Fuzzy Sets: Theory and Applications to Policy Analysis and Information Systems*, Plenum Press, 1980, pp. 275–290.
17. Okuda, T., Tanaka, H., and Asai, K., "A Formulation of Fuzzy Decision Problems with Fuzzy Information using Probability Measures of Fuzzy Events," *Information and Control*, Vol. 38, 1978, pp. 135–147.
18. Samelson, N. M., and Levitt, R. E., "Owner's Guidelines for Selecting Safe Contractors," *Journal of the Construction Division*, ASCE, Vol. 108, No. CO4, 1982, pp. 617–623.
19. Shiraishi, N., and Furuta, H., "Reliability Based on Fuzzy Probability," *Journal of Engineering Mechanics*, ASCE, Vol. 109, No. 6, Dec., 1983, pp. 1445–1459.

J. Constr. Eng. Manage., 1985, 111(3): 231-243

20. Twort, A. C., *Civil Engineering—Supervision and Management*, Edward Arnold, 1972.
21. Yager, R. R., "Satisfaction and Fuzzy Decision Function," *Fuzzy Sets: Theory and Applications to Policy Analysis and Information Systems*, Plenum Press, 1980, pp. 171–194.
22. Zadeh, L. A., "Fuzzy Sets," *Information and Control*, Vol. 8, 1965, pp. 338–353.
23. Zadeh, L. A., "Outline of a New Approach to the Analysis of Complex Systems and Decision Processes," *Systems, Man and Cybernetics*, Institute of Electrical and Electronics Engineers, Vol. SMC-3, No. 1, 1978, pp. 28–44.
24. Zadeh, L. A., "A Fuzzy Algorithmic Approach to the Definition of Complex or Imprecise Concepts," *International Journal Man-Machine Studies*, Vol. 8, 1976, pp. 249–291.
25. Znotinas, N. M., and Hipel, K. W., "Comparison of Alternative Engineering Design," *Water Resources Bulletin*, Vol. 15, No. 1, 1979, pp. 44–59.

Appendix II.—Notation

The following symbols are used in this paper:

$\bar{A}$ = complement of fuzzy set, A;
D = decision matrix of aggregated ratings;
$\mu_A(x)$ = grade of membership, or degree of belief, of x in fuzzy set, A;
μ_{ij} (or $\mu_{x_i y_j}$) = rating given to tenderer, i, for criterion, j;
U = fuzzy union, or "maximum of," or linguistic "OR"; and
$\cap$ = joint fuzzy operator, or "minimum of," or linguistic "AND."

J. Constr. Eng. Manage., 1985, 111(3): 231-243