Reliability Analysis of Shotcrete Lining during Tunnel Construction

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Abstract: To a tunnel under construction, a prompt assessment of the stability of the lining through in situ measured data is of practical importance both for the observational designing of the tunnel and to guarantee the safety of construction. Among all the parameters measured during tunnel construction, displacement measurement is the most convenient and at the same time, the most credible and cost effective method to track the performance of tunnel lining. The aim of this paper is to develop a displacement-based method suitable for conducting reliability evaluation of the shotcrete lining in the progress of construction of a tunnel. First of all, a performance function at the cross section is put forward. Next, formulas are listed to calculate the internal forces in a lining segment. Then, the variability of the basic random variables is explained. Further, a process to compute the reliability index through measured displacements is presented. After that, two case studies are done. Finally, the nature of the proposed method is observed and some remarks are made on further developments. The proposed method can meet the need of reliability evaluation of shotcrete lining during tunnel construction and also is certainly of theoretical significance to a large extent.

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Introduction

Owing to the complexity of tunneling, it is almost impossible in the design stage to understand the ground conditions and the interaction between support structures and the ground. In response to the need to guarantee the safety of construction and the stability of structures, monitoring, and measuring the mechanical behavior of both the ground and the support structures is a must in the course of construction, which leads to the so-called observational designing in the building of underground tunnels.

The parameters to be monitored or measured in the construction of a tunnel vary from a few numbers up to a few tens according to the scale of the project. Usually the displacement of the ground, the groundwater table, the earth pressure, the stress of reinforce bars (if applicable), the displacement of lining structures, as well as the inclination and settlement of the buildings in the vicinity of the tunnel are among the measured parameters. These data are now used in a deterministic way, whereas most of the factors involved in tunneling are obviously random variables carrying large amounts of uncertainties. Thus, it is of great practical value to make use of the data to quantitatively and objectively evaluate the reliability of the lining within the period of construction.

Before moving further, a choice must be made on what measured quantity is to be used in the analysis. Various quantities are employed by different researchers in order to meet the needs of different objectives in the evaluation of safety during the construction stage. Sakurai et al. (1995) assessed the stability of tunnels by comparing strain occurring in the surrounding rock of a tunnel with the allowable strain of the rock. The resulting strain is calculated from displacement measurements and the allowable shear strain can be attained through laboratory tests. Zhu et al. (1998), and Zhu (2001) estimated the safety of tunnels by comparing displacement measurements with the limiting displacement that the lining is able to bear. The limiting displacements are determined, according to the ground conditions as well as the material and dimension of the lining, through numerical simulations in conjunction with engineering experiences. In this paper, lining displacement is chosen to be the basic quantity through which the internal forces existing in the lining are calculated, whereas the ground conditions and the interaction between ground and shotcrete lining are ignored.

It is well known that the radial displacement of lining plays an important role in tunnel construction. Although the mechanical behaviors of the ground and the interaction of it with the lining are influenced by various factors during the excavation and support process, and are thereby very complicated, the displacement of the lining is the most direct, essential, objective, and credible manifestation of the properties of the ground and the stability of the structures (Mu 1996). Even if little is known about the influencing factors, the dynamics and interactions of the factors, the stability of the ground and the structures can still be reflected through the displacement of the lining. On the other hand, measuring the lining displacement is easy and requires simple instruments. It causes little interruption with construction work, thereby

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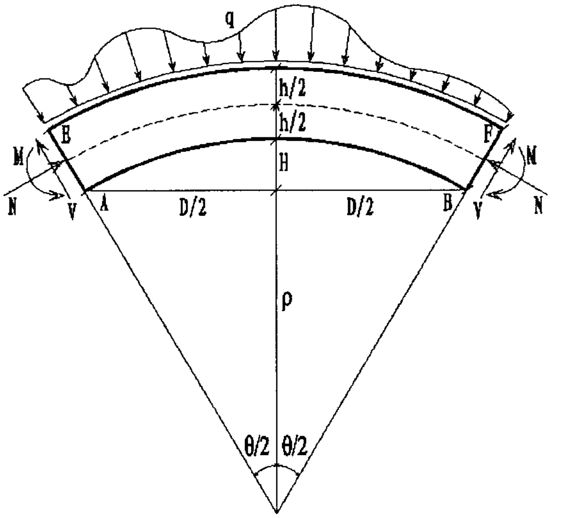

Fig. 1. Lining segment

results in low cost and, as such, is being widely incorporated in practice.

On the basis of the radial displacement of tunnel lining, which has been chosen as the parameter under study, this paper focuses on plain shotcrete linings, aimed at setting up a reliability analysis method. The study starts with the establishment of a performance function, followed by a list of formulas to compute the length and the curvature of the central axis of a lining segment, as well as the axial force and bending moment in the segment. After investigating the statistical characteristics of the basic random variables, including the lining displacement, the lining thickness and the mechanical properties of shotcrete, the process to compute the reliability index through measured displacements is given. Making use of the aforementioned work, two case studies are conducted and the results of the calculation are compared with in situ situations. The paper ends with a summary on the nature of the proposed method and an outlook on further developments.

Performance Function

The initial failure of a cross section of the shotcrete lining is taken into consideration. The deformation of the lining is assumed to be small, linear, and elastic. To a concrete component without rebars, two types of failures, crushing due to axial compression and cracking due to bending, must be avoided simultaneously. To a section subject to a certain set of internal forces of axial force N, bending moment M, and shear force V, only one type of failure may occur. Therefore, the performance function for the limit state against crushing and cracking can be expressed as (Yang and Zhang 1999; Chinese Railway Standard 2005, Chinese National Standard 2002)

$$g=\begin{cases}\varphi\alpha bhf_c-N & \text{for } e<0.225h\\ 1.75\varphi bh^2f_t+Nh-6M & \text{for } e\geqslant 0.225h\end{cases} \tag{1}$$

In order to evaluate the internal forces through displacements, let us consider a lining segment as shown in Fig. 1. The segment can be a portion cut out of a lining, or the segment itself is a whole piece of lining alone as often constructed when non-full-face construction methods are adopted. The forces exerting on the segment can be represented by arbitrarily distributed loads acting on the extrados (the exterior curve of an arch), as well as axial forces, bending moments and shear forces acting at the two ends. All these external forces applied on the segment will make contribution to the displacement of it (axially compressive deformation and bending deformation). Theoretically speaking, the internal forces within the segment can be found by using the displacements only. To make this a reality, the following assumptions are employed:

1. Deflection of the lining is small and, within this amount of deformation, shotcrete is a linearly elastic material.
2. Cross sections of the segment remain plane and normal to its longitudinal axis, i.e., the assumption of plane cross section holds.
3. Ignored is the deformation in the plane of cross section itself due to Poisson's ratio and the load acting on the extrados of the segment, in other words, the dimension of a cross section and the distance between any two points within the plane stay unchanged during deformation.
4. The neutral axis passes through the centroid of the cross-section area.
5. The presence of internal shear forces is disregarded, which means pure bending is adopted instead of actual nonuniform bending. A detailed investigation shows that, for a rectangular beam with a slenderness ratio no less than 10, the proportion of the deflection made by shear forces will be no more than 2.2% (Gere and Timoshenko 1984; Timoshenko and Goodier 1970).

Then for a lining segment $ABFE$, as drawn in Fig. 1, the changes in the axial force and the bending moment, under the assumption made earlier, can be written as

$$\Delta N_i=E_ibh\Delta\varepsilon_i \tag{2}$$

$$\Delta M_i=E_iI\left(\frac{1}{\rho_i}-\frac{1}{\rho_{i-1}}\right) \tag{3}$$

It is obvious that the segment does not have to always be in the shape of an arc during deformation. When it happens to be a straight line at the time of measurement i, e.g., the term $1/\rho_i$ in Eq. (3) becomes zero. Another noteworthy item concerning Fig. 1 is that the configuration of the arch-like segment is expressed by two values: The arch rise H and arch span D of the intrados (the inner curve of an arch). Doing so does not mean these two values have to be measured directly. If the intrados of the lining is determined by the coordinates of a certain number of measured points, a conversion from the coordinates to the two values is needed to perform, in order to use the following formulas to compute the length and radius of curvature of the central axis.

In order to evaluate the length and the radius of curvature required in Eqs. (2) and (3), it is needed to choose a curve to interpolate the central axis of the segment. It merits attention that different types of the interpolated curves give different values for the length and the radius of curvature. Here a circle is chosen and used to interpolate through three points: Two end points and the apex (mid point) of the central axis of the segment. The interpolation ends up with the formulas to reckon the length and the radius of curvature of the central axis of the segment, as listed in the following:

$$\rho_i=\frac{1}{2}\left(H_i+\frac{D_i^2}{4H_i}\right)+\frac{h}{2} \tag{4}$$

$$L_i=\rho_i\theta_i \tag{5}$$

J. Constr. Eng. Manage., 2007, 133(12): 975-981

Table 1. Probability Characteristics of the Compressive Strength of Shotcrete at Different Ages

Age of shotcrete	5 h	10 h	3 days	7 days	14 days	28 days	90 days	180 days
Mean (MPa)	3.97	6.56	23.79	27.75	30.36	32.31	34.44	35.17
Coefficient of variation	0.208	0.207	0.129	0.131	0.135	0.140	0.149	0.156
Type of distribution					Normal			

Probability Characteristics of Basic Variables

It can be seen from Eqs. (1)–(5) that the performance function is related to variables such as the lining displacement (the rise and the span of the intrados of the lining segment), thickness of the lining and the properties of shotcrete (the compressive strength, the tensile strength and the elastic modulus). Attention is paid to the probability characteristics of these variables hereinafter.

Uncertainties in Displacement of Lining

The main cause of uncertainties in the displacement of a lining is error in measurement. Generally speaking, this error does not vary considerably with the magnitude of the length surveyed. So, two random variables for the errors are introduced here to represent the uncertainties in the measurement of the rise and the span of an arch-like segment, respectively,

$$H = H' + \xi$$

$$D = D' + \zeta \tag{6}$$

There are two ways often used in practice to survey the displacement of the lining. The first way is to measure the relative displacement (convergence) between two points on the intrados of a cross section of the lining. The instrument used in this measurement could be a convergence meter. The second way is to measure the displacement of a point on the lining with reference to a fixing point elsewhere. A level or a theodolite can be used to do this measurement.

The statistical samples for ξ and ζ can be obtained by taking measurements of the distance between two fixed points. Here we take the convergence measurement data of five cross sections from the Shengjie Tunnel on the Jingjiao Express Way, which connects Jingcheng City in Shanxi Province to Jiaozuo City in Henan Province, to make the statistics of the uncertainties in ξ and ζ. The shotcrete lining at the five cross sections remained stable during the whole measuring period. The statistical results show that ξ and ζ are both of normal distributions with zero means and the standard deviations of 0.812 and 0.740 mm, respectively. The difference between the two standard deviations of ξ and ζ is believed to be caused by different measurement methods used; the rise of the arch was measured with a level, whereas the span of the arch was scaled with a convergence meter. It is justifiable to take the same standard deviation for both ξ and ζ if the same measuring instruments are used somewhere else.

Uncertainties in Thickness of Lining

Zhang and Yang (1996) studied the probability characteristics of the thickness of shotcrete linings. Their work came to the conclusion that, owing to overbreak, actual thickness of shotcrete lining is often greater than the designed value. Here on the basis of their statistical results, we recommend that the excess thickness of shotcrete due to overbreak be ignored and take the designed value as the mean of lining thickness, with the coefficients of variation being equal to 0.05, 0.07, and 0.09 for the ground of Classes III, IV, and V, respectively. The type of distribution of lining thickness came out to be normal.

Uncertainties in Properties of Shotcrete

The parameters for properties of shotcrete include compressive strength, tensile strength, and elastic modulus. Compressive strength is often taken as a basic parameter of the material. Tensile strength and elastic modulus are then assumed to have a linear relationship with compressive strength.

Inasmuch as shotcrete gains its strength as it cures, the strength growth of the material with time should be taken into account. The uniaxial compressive strength of shotcrete during hardening can be expressed through the following negative exponential equation (Oreste 2003):

$$f_{c,t} = f_{c,0}(1 - e^{-\lambda t}) \tag{7}$$

Supposing that the tensile strength and the elastic modulus have the same hardening rate as the compressive strength does, one finds

$$f_{t,t} = f_{t,0}(1 - e^{-\lambda t}) \tag{8}$$

$$E_t = E_0(1 - e^{-\lambda t}) \tag{9}$$

Up to now the data for the strength of shotcrete at different curing ages are still rare. The stress measuring on shotcrete lining near the cutting face tells that the largest stress-to-strength ratio often appears at the times of 3 to 5 days after the shotcrete lining is cast in place (Celestino and Guimaraes 1995). Cheng and Yang (1998) as well as Lian and Han (2001) summarized the range of variation of the compressive strength of shotcrete at different ages.

The mix design of the shotcrete collected in the statistical data by Cheng and Yang (1998) are as follows: Maximum diameter of gravel=20 mm; sand ratio=50%; type and grade of cement =Portland cement of number 425; mixing ratio=cement: sand: gravel: water: accelerants=1:2:2:0.45:0.03; spray process =dry-mix shotcreting; and rebound loss=25%.

The shotcrete samples used for strength tests in the statistical data were prepared in the following way: (1) Spay the mix into a mode box of 450 mm × 350 mm × 120 mm until the box is filled; (2) cut a specimen of size 100 mm × 100 mm × 100 mm out of the sprayed mix; (3) cure the specimen for 28 days under conditions of temperature 20±3°C, relative humidity no less than 90%; (4) measure the compressive strength of the specimen; and (5) finally, multiply the tested value of the strength by 0.95 which is the dimension-effect subtraction factor.

Presuming that the compressive strength of shotcrete is normally distributed and making use of the method given in Appendix A of a Chinese standard (Chinese National Standard 1995), we take advantage of the data collected by Cheng and Yang (1998) and come up with the statistical parameters of the compressive strength of shotcrete, as shown in Table 1.

J. Constr. Eng. Manage., 2007, 133(12): 975-981

Table 2. Uniaxial Compressive Strength for Three Types of Shotcerte (MPa)

	Hardening time			
Type of shotcrete	1–3 h	3–8 h	1 day	28 days
Shotcrete without accelerants	0.0	0.2	5.2	41.4
Shotcrete with accelerants (3%)	0.69	5.2	10.3	34.5
Shotcrete with regulated hardening	8.27	10.3	13.8	34.5

As can be seen in Table 1, the coefficient of variation changes with curing time, with the largest difference being 0.079. However, considering the scarcity of the data and for the sake of simplicity, it could be reasonable to take one value for each strength grade of shotcrete throughout the time of curing.

The hardening speed changes for different types of shotcrete. Some typical values of uniaxial compressive strength of shotcrete in time are illustrated in Table 2 (Oreste 2003).

By virtue of data in Tables 1 and 2, as well as the regulations pertinent to materials in Chinese codes (Chinese Railway Standard 2002; Chinese National Standard 2002), the mechanical parameters of shotcrete can be figured out as shown in Table 3 with all parameters normally distributed.

Time constant λ in Eqs. (7)–(9) describes the speed of hardening. It changes largely with the types of the shotcrete. Hence it needs to be determined according to the specific shotcrete used in the project under analysis. The typical value of λ ranges from 0.003 to 0.03 (1/h).

Reliability Index Calculation

Referring to Fig. 1, the rise H and the span D of the intrados can be measured directly or be determined through the coordinates of at least three measured points (Points A and B and a midpoint on the intrados). To a specific lining under consideration, how many segments to use and how to divide the lining are on user's choice. In general, the measured points can be placed at the crown, the springs, the middles of the sidewalls, the feet, and the midpoint of the invert. The whole lining can be divided into five segments: One segment of arch, two segments of sidewalls, and one segment of invert. Fig. 2 schematically demonstrates the general layout of measured points and measured lines often incorporated in practice in China. The relevant displacement between two points can be taken with a convergence meter. When higher accuracy is required, more points and more segments are needed, and more precise instruments should be used in the measurement.

Table 3. Strength Parameters of Shotcerte (MPa)

	Strength grade of shotcrete				
Parameter	C20	C25	C30	C35	C40
Compressive strength $f_{c,0}$	20	25	30	35	40
Tensile strength $f_{t,0}$	1.8	2.0	2.2	2.4	2.6
Elastic modulus E_0	21,000	23,000	25,000	27,000	28,000
Coefficient of variation	0.18	0.16	0.14	0.13	0.12

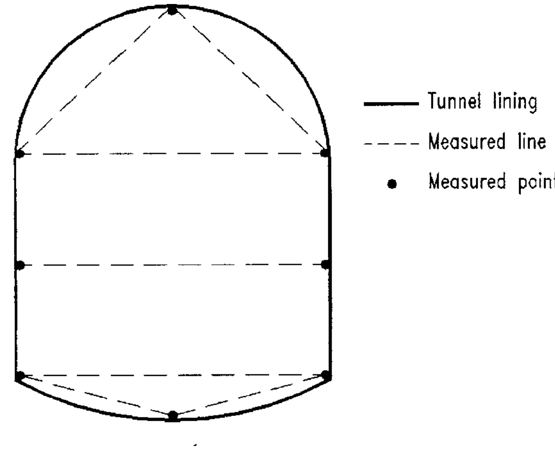

Fig. 2. Arrangement for displacement measurement on tunnel lining

Now arrived at is, on the basis of the displacements and by means of direct Monte Carlo simulation, the process to compute the reliability index of the lining, which can be elaborated as follows:

1. For a segment of the lining under analysis at a typical measurement time i, generate a set of the values of the basic variables.
2. Compute the arc length L_i and the radius of curvature ρ_i of the central axis of the segment at measurement time i.
3. Through Eqs. (2) and (3), calculate the changes in the internal forces from time $i-1$ to i, attaining ΔN_i and ΔM_i.
4. Taking sum of the internal forces at time $i-1$ and the changes gives the axial force N_i and the bending moment M_i at time i, being equal to $N_{i-1}+\Delta N_i$ and $M_{i-1}+\Delta M_i$, respectively.
5. By use of Eq. (1), evaluate the value of the performance function g.
6. Repeat Steps 1–5 by m times (m usually should range from 50,000 to 100,000 times of simulation, in order to meet precision requirement in engineering practice), and count the number of failure n, i.e., the occurrence where $g<0$.
7. Reckon the failure probability $P_f(=n/m)$ and reliability index $\beta(=\Phi^{-1}(1-P_f))$ corresponding to the segment. Here Φ^{-1} stands for the inverse of the standard normal distribution function.
8. Perform Steps 1–7 over all segments at a cross section of tunnel lining.
9. The smallest reliability index among all the segments is picked to represent the reliability of the shotcrete lining at this typical measurement time i.

Case Studies

Pishuangao Tunnel

This example comes from Pishuangao Tunnel on the Jingzhu Express Way which goes from Beijing to Zhouhai in Guangdong province. A length of shotcrete lining was constructed at 10:00 on March 20, 1999 in one of the two unitary tunnels. The class of the ground around the tunnel is IV. The cross section, which is composed of an arch, two vertical sidewalls and an invert, has an inner width of 10.5 m and an inner height of 7.0 m. The thickness of the lining is 15 cm. Shotcrete of strength grade C25 is used.

J. Constr. Eng. Manage., 2007, 133(12): 975-981

Table 4. Displacement-Based Reliability Analysis for Pishuangao Tunnel

	Date in March 1999					
Date of measurement	22	23	24	25	26	27
Age of shotcrete (day)	2	3	4	5	6	7
Measured span of the arch (mm)	9,852.10	9,851.50	9,851.30	9,851.20	9,851.00	9,851.00
Measured rise of the arch (mm)	3,913.00	3,909.00	3,907.50	3,906.80	3,906.50	3,906.50
Failure probability	—	0.0145	0.0370	0.0512	0.0551	0.0432
Reliability index	—	2.184	1.786	1.633	1.597	1.714

Parameters brought into the calculation involve:

- Measurement error in rise of the arch: mean 0.0 mm, standard deviation 0.740 mm;
- Measurement error in span of the arch: mean 0.0 mm, standard deviation 0.740 mm;
- Thickness of lining: mean 15 cm, coefficient of deviation 0.07;
- Compressive strength of shotcrete: mean 25 MPa, coefficient of deviation 0.16;
- Tensile strength of shotcrete: mean 2.0 MPa, coefficient of deviation 0.16;
- Elastic modulus of shotcrete: mean 23,000 MPa, coefficient of deviation 0.16; and
- Time constant: 0.015 $(1/\mathrm{h})$.

A computer program has been developed to accomplish the calculation. The displacement data from the second day through seventh day are picked out to do the analysis. Before and after this period no apparent displacement was measured.

The measured data, namely convergence of the springs and sag of the crown, are listed in Table 4. Calculation gives the failure probabilities and the reliability indices of the lining at different measurement times, as shown in Table 4 also.

Shengjie Tunnel

This instance goes to Shengjie Tunnel on the Jinjiao Express Way. A length of shotcrete lining was built at 10:30 on December 12, 1998 in one of the twin tunnels. The class of the ground is IV. The cross section is of horseshoe shape, with an inner width of 11.5 m and an inner height of 8.5 m. The thickness of the lining is 25 cm. Shotcrete of strength grade C30 is cast.

Parameters put into the computation contain:

- Measurement error in rise of the arch: mean 0.0 mm, standard deviation 0.812 mm;
- Measurement error in span of the arch: mean 0.0 mm, standard deviation 0.740 mm;
- Thickness of lining: mean 25 cm, coefficient of deviation 0.07;
- Compressive strength of shotcrete: mean 30 MPa, coefficient of deviation 0.14;
- Tensile strength of shotcrete: mean 2.2 MPa, coefficient of deviation 0.14;
- Elastic modulus of shotcrete: mean 25,000 MPa, coefficient of deviation 0.14; and
- Time constant: 0.01 $(1/\mathrm{h})$.

The displacement data from the fourth day through seventh day are taken out for the analysis. Before this period no apparent displacement was detected. On seventh day, the lining at this section cracked, which was caused by an abnormal blast in another of the twin tunnels. Then the measuring work was interrupted.

The measured convergence of the springs and sag of the crown, together with the computed failure probabilities and the reliability indices at different measurement times, are tabulated in Table 5.

Discussions

Now let us take a look at the reliability indices given in Tables 4 and 5. It can be seen that the tunnel lining was stable (at the period from the second to seventh day at the Pishuangao Tunnel and from the fourth to sixth day at the Shengjie Tunnel) when reliability indexes were positive, whereas cracking occurred (on the seventh day at the Shengjie Tunnel) when the index took minus value. This proved the calculated results go consistently with the real situation. In addition, the calculated magnitudes of reliability indexes have a good agreement with the required values prescribed in the present design standards of China (Chinese National Standard 1995).

Conclusions and Remarks

This paper puts forward a displacement-based method which can be used to evaluate the reliability of shotcrete lining in tunnel construction. Contents of this work consist of establishment of the performance function, investigation of statistical characteristics of basic random variables, and development of a computer program and two case studies. The proposed method is applicable to different construction methods, such as full face method, full arch method, bench method, and pocket method. The method relies only upon the displacements occurred of the lining. There is no need to know the behavior of the ground and the interaction between the support structure and the ground. The method is suitable to conduct safety control and observational designing of underground tunnels with shotcrete lining. The work of this paper

Table 5. Displacement-Based Reliability Analysis for Shengjie Tunnel

	Date in December 1998			
Date of measurement	16	17	18	19
Age of shotcrete (day)	4	5	6	7
Measured span of the arch (mm)	9,157.29	9,156.60	9,156.18	9,144.47
Measured rise of the arch (mm)	3,399.96	3,397.65	3,396.12	3,392.24
Failure probability	—	0.0006	0.0051	0.9726
Reliability index	—	3.249	2.568	−1.921[a]

[a]Cracking occurred of the shotcrete lining.

J. Constr. Eng. Manage., 2007, 133(12): 975-981

helps to improve the assessment method of the safety of tunnels during construction from a deterministic way to a probabilistic one.

Nevertheless, this paper only initiates the first step of this kind of study. Some aspects of the method still need to be further improved, including:

1. The sensitivity of the method to different types of curves of interpolation is subjected to investigation at the next step.
2. Only shotcrete has been discussed. Reinforced shotcrete is also widely used nowadays and should be studied in future approaches.
3. Only the failure of the cross section of the lining has been taken into account. The failure of the support structure system needs to be considered.
4. Deformation of the lining is assumed to be small, whereas the large deformation of the lining during its strength growing period might happen and need to be tackled.
5. Only the initial failure of the shotcrete lining has been dealt with. The deformation behavior of the lining after the initial failure deserves further examination.
6. Deformation caused by shrinkage, temperature, and creep (Hellmich et al. 2000) needs to be analyzed.
7. The method mentioned in this paper is an analytical solution. In order to incorporate the nonlinear behavior and time dependent deformation of shotcrete, numerical solution is waiting to serve in finding internal forces of the lining through displacements.
8. More work could be done in generating the method beyond tunnels to other underground works.

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Notation

The following symbols are used in this paper:

b = width of a cross section (m);
E_i = elastic modulus of shotcrete at time i (kPa);
E_0 = asymptotic elastic modulus of shotcrete for time $t=\infty$ (MPa);
E_t = elastic modulus of shotcrete at time t (MPa);
e = eccentricity$=M/N$ (m);
f_c = compressive strength of shotcrete (kPa);
$f_{c,t}$ = uniaxial compressive strength of shotcrete at time t (MPa);
$f_{c,0}$ = asymptotic uniaxial compressive strength of shotcrete for time $t=\infty$ (MPa);
f_t = tensile strength of shotcrete (kPa);
$f_{t,t}$ = tensile strength of shotcrete at time t (MPa);
$f_{t,0}$ = asymptotic tensile strength of shotcrete for time $t=\infty$ (MPa);
H,D = rise and span of the intrados of a lining segment (m);
H',D' = measured values of H and D, respectively (m);
H_i,D_i = rise and span of the arc AB (refer to Fig. 1) at time i, respectively (m);
h = thickness of a cross section (m);
I = moment of inertia of a cross section, $I=bh^3/12$ (m^4);
i = times of measurement;
L_i,L_{i-1} = arc length of the central axis of a lining segment at time i and $i-1$, respectively (m);
M = bending moment (kN m);
N = axial force (kN);
t = time (h);
α = eccentricity influence factor, which is a function of the ratio of eccentricity of axial force over the section thickness of a component. $\alpha=1+0.648(e/h)-12.569(e/h)^2+15.444(e/h)^3$ (Chinese Railway Standard 2005);
ΔM_i = change in bending moment from time $i-1$ to i (kN m);
ΔN_i = change in axial force from time $i-1$ to i(kN);
$\Delta\varepsilon_i$ = change in strain of the central axis of a lining segment from time $i-1$ to i, $\Delta\varepsilon_i=(L_i-L_{i-1})/L_{i-1}$;
θ_i = central angle of the arc AB (refer to Fig. 1) at time i, $\theta_i=2\arcsin(0.5D_i/(\rho_i-0.5h))$ (rad);
λ = time constant (1/h);
ξ,ζ = random variables for the measurement errors in H and D, separately (m)
ρ_i,ρ_{i-1} = radius of curvature of the central axis of a lining segment at time i and $i-1$, separately (m); and
φ = stability factor for plain concrete component, which is a function of the slenderness ratio of the component. $\varphi=1$ is often a suitable default setting for tunnel linings.

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