Comparison of Using Mixed-Integer Programming and Genetic Algorithms for Construction Site Facility Layout Planning

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Abstract: The use of modular construction has gained wide acceptance in the industry. For a specific construction facility layout problem such as site precast standardized modular units, it requires the establishment of an on-site precast yard. Arranging the precast facilities within a construction site presents real challenge to site management. This complex task is further augmented with the involvement of several resources and different transport costs. A genetic algorithm (GA) model was developed for the search of a near-optimal layout solution. Another approach using mixed-integer programming (MIP) has been developed to generate optimal facility layout. These two approaches are applied to solve with an example in this paper to demonstrate that the solution quality of MIP outperforms that of GA. Further, another scenario with additional location constraints can also be solved readily by MIP, which, however, if modeled by GA, the solution process would be complicated. The study has highlighted that MIP can perform better than GA in site facility layout problems in which the site facilities and locations can be represented by a set of integer variables.

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Introduction

The use of modular construction has gained wide acceptance in the housing sector in Hong Kong. Standardization enables wider use of precast techniques, as well as facilitating production under a controlled environment. Precasting is one of the most important techniques by which prestressed concrete has been made practicable and economical. It offers all the advantages of prefabrication: quality; mechanized production; speed; and economy. The gain in quality and less reliance on skilled labor help reduce the overall production cost. With a planned annual output of around 35,000–50,000 units, the Hong Kong Housing Authority took the lead to pioneer the use of standardized components about 20 years ago. The scope of application has expanded ever since. The production of precast units within a precast yard involves repeated movements of resources between the essential production facilities. The layout of these facilities directly affects the magnitude of the resources flow and the cost factor. Hence, optimization techniques have been proposed in studying the site facility layout, including genetic algorithms (GAs) and mixed-integer programming (MIP).

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FLP

Facility layout is believed to be the heart of efficient production. Construction-site facility layout planning (FLP), which defines the types, quantities, and positioning of the mechanical plant, storage areas, and fabrication yards, has significant impact on productivity, costs, and duration of construction. Although FLP is such a critical process in construction planning, a systematic analysis of construction-site layout is always difficult because of the existence of the vast number of trades and interrelated planning constraints. Practitioners of the building industry lack a well-defined approach in construction-site layout planning, especially for high-rise building construction.

There is increasing pressure to build faster and cheaper in the construction industry. To address this demand for building faster, contractors are scheduling more activities at the same time increasing the resources used by the activities. Realizing the need for space management, many researchers have focused on developing methodologies to improve space management in construction sites. Riley and Sanvido (1997) stated that numerous material handling paths and storage spaces also contribute to the challenge of finding sufficient space to execute tasks.

Various space scheduling approach focuses on modeling the different types of work spaces required by construction activities. In this study, the optimization-site layout approach captures the spatial knowledge generically in relation to the construction methods being used and automates the generation of specific spaces with respect to the volume of the reference objects that are represented in the GA model (Tommelein et al. 1992; Zouein and Tommelein 1993; Li and Love 1998; Thabet and Beliveau 1994;

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Thabet and Beliveau 1997; Choo and Tommelein 1999; Tam et al. 2002; Deb and Bhattacharyya 2005).

GA

GA is a computational method modeled on biological evolutionary process (Holland 1985). It can be used to find a near-optimal solution to a problem although there may have many near-optimal solutions in the solution terrain. The search process is independent to the problem and the search can be performed under many types of fitness functions, be it discrete or continuous, linear, or nonlinear. Furthermore, wide flexibility is accorded to the construction of the fitness function to suit a wide range of problems (Gen and Cheng 1997).

Applications of GAs

Since the past decade, the application of GA has increased significantly. GA can be applied to a wide range of problems in various industries. For example, in the biochemistry industry, GA has long been used in genetic engineering. Before actual laboratory testing, a conceptual idea represented by mathematical model(s) is encoded. Computerized simulation would then be carried out to obtain preliminary test results. This saves both time and cost for the whole genetic engineering process. In the manufacturing industry, GA has been commonly used in many processes such as design of workshop layout plans (Rao et al. 1999), arrangement of departments with unequal area requirements (Hamamoto et al. 1999), cost optimization in manufacturing process (Dereli and Filiz 1999), and production scheduling of grouped jobs (Lam et al. 1999; Cheng et al. 1999).

Mathematically, GA transforms a population of individual objects, each associated with a fitness value, into a new population using operations patterned after the Darwinian principle of reproduction and survival of the fittest. The following summarizes the essentials of the GA operation:

1. Establishment of a representation of the problem;
2. Setting values for the various parameters that the GA uses;
3. Creation of an initial population of potential solutions;
4. Rating the population in terms of their fitness; and
5. Population evolution through genetic operators.

Al-Tabtabai and Alex (1999) suggested that the use of GA methodology was appropriate in the following circumstances:

1. Conventional statistical and mathematical methods are inadequate;
2. The problem is very complex because the possible solution space is too large to analyze in a finite time;
3. The additional information required by the conventional methods to guide the search is absent or not sufficient;
4. The solution to the problem can be encoded in the form of a string of characters;
5. The problem is large and poorly understood; and
6. There is a need for near-optimal solutions quickly for use as starting points for conventional optimization methods.

Construction problems typically involve multiple optimization objectives under certain constraints. The application of GA in construction has gained popularity in recent years. Notable examples include site layout optimization (Li and Love 2000; Gero and Kazakov 1997), scheduling (Chan et al. 1996), resources allocation (Li and Love 1997), equipment selection (Haidar et al. 1999), and determination of laying sequence for a continuous girder reinforced concrete floor system (Natsuaki et al. 1995), fuzzy rule determination, and fuzzy membership tuning (Ross 1995).

Table 1. Example of a Chromosome in a Trial Solution Set

Gene number	Location (*X*-coordinate, *Y*-coordinate)	Types of facility (assigned facility number)
1	L_1 (15, 40)	Side gate (2)
2	L_2 (13, 30)	Formwork storage area (5)
3	L_3 (22, 30)	Main gate (1)
4	L_4 (25, 20)	Casting yard (10)
5	L_5 (20, 10)	Lifting yard (11)
6	L_6 (12, 10)	Curing yard (8)
7	L_7 (40, 10)	Batching plant (3)
8	L_8 (48, 20)	Cement, sand, and aggregate storage area (7)
9	L_9 (48, 35)	Bending yard (6)
10	L_{10} (5, 20)	Refuse dumping area (9)
11	L_{11} (32, 42)	Steel storage area (4)

Cheung et al. (2002) developed a GA model enabling construction planners to search for and identify optimal locations of all temporary facilities on site in order to optimize construction safety and cost simultaneously. These new capabilities have proven useful to construction planners and can lead to significant improvement in construction safety and reduction in the cost of constructed facilities.

The optimization model can provide logistic planners and construction managers with a valuable technique to develop an efficient sequence of work that optimally utilizes the construction facilities' space and minimizes the movement of specific facilities on high-rise building construction. The model has considered the conflicting space demands of all the facilities involved in the project. Optimizing this requirement would provide a number of benefits such as improved safety, decreased conflicts among workers, reduced crew waiting time and work stoppage, improved efficiency and reduced unnecessary movement of materials and equipment, increased productivity, better quality, and reduced project delays.

Study of Construction Site Precast Yard

In the optimization of construction-site precast yard layout, the efficiency of a site precast yard is very much affected by positioning of the various facilities. A GA model is used to obtain a near-optimal location arrangement for the precast facilities through minimization of transport costs of resource flow among the facilities.

It is assumed that the geometric layout of available locations is predetermined and fixed. Each of the locations is assumed to be capable of accommodating the large enough to hold all kinds of facilities. If the number of locations is more than that of facilities, dummy facilities can be added for computation purposes. The coordinates $[L_i = (XL_i, YL_i)]$ identify the locations within the yard area. The facilities listed in Table 1 are to be located within the yard.

The traveling distance (for resources) between the locations i and j is given by D_{ij}. Rectangular distance is used as opposed to diagonal distance as the physical sizes of the facilities prohibit diagonal movement of resources between locations. The use of

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rectangular distance between locations resembles the actual resource movements.

With the coordinates, a matrix of distance D_L can be constructed

$$D_{ij} = |XL_j - XL_i| + |YL_j - YL_i|$$

$$D_{L(q\times q)} = \begin{bmatrix} D_{1,1} & D_{1,2} & \dots & \dots & D_{1,q} \\ D_{2,1} & \dots & \dots & & \vdots \\ \vdots & & & & \\ \vdots & D_{i,j} & & & \\ D_{q,1} & \dots & \dots & & D_{q,q} \end{bmatrix} \quad (1)$$

where D_{ij}=rectangular distance between locations i and j. $L_i = (XL_i, YL_i)$ are the coordinates of the locations within the yard area.

With the daily anticipated output, the frequencies (per day) of trips for resource flow between the facilities can be calculated and presented in a frequency matrix $F_{Mk(q\times q)}$ for all Mk in $[1,n]$, where n is number of types of resource flow

$$F_{Mk(q\times q)} = \begin{bmatrix} F_{Mk1,1} & F_{Mk1,2} & \dots & \dots & F_{Mk1,q} \\ F_{Mk2,1} & \dots & \dots & & \vdots \\ \vdots & & & & \\ \vdots & F_{Mkr,s} & & & \\ F_{Mkq,1} & \dots & \dots & & F_{Mkq,q} \end{bmatrix}$$

The cost per unit distance for the n types of resources flow is given by C_{Mk} for all Mk in $[1,n]$.

Setting Values for the Various Parameters

The setting of population size, probability of crossover, and mutation is a trial and error process.

Initial Population

The type of chromosome used in Evolver (Palisade Corp., New York), the software used in this study, is called the order chromosome. This type of chromosome is normally used in solving sequencing problems. The setting of the precast yard layout can be reduced to a traveling salesman's problem. The facility numbers are arranged into the location numbers, in which each facility is unique. In a similar context, no duplicate is permitted for locations of the facilities within a site precast yard. Hence, order chromosome with unique genes is suitable for the site precast yard study. Table 1 shows an example of an order chromosome. For example, in Table 1 the batching plant (Facility No. 3) is placed at Location 7 and the corresponding gene position within the chromosome is 7. The function of the GA is to find the optimal order of the facilities in the 11 genes within the chromosome. Fifty chromosomes were generated as the initial population.

Rating the Population in Terms of Their Fitness

In this study, the fitness of the chromosome is assessed by the total cost per day for transporting all resources necessary to achieve the anticipated output. The objective function is therefore given by

$$\text{Total\_cost} = \min\left(\sum_{k=1}^{n}\sum_{i=1}^{q}\sum_{j=1}^{q} TCL_{Mk,i,j}\right)$$

$$TCL_{Mk(q\times q)} = \begin{bmatrix} TCL_{Mk1,1} & TCL_{Mk1,2} & \dots & \dots & TCL_{Mk1,q} \\ TCL_{Mk2,1} & \dots & \dots & & \vdots \\ \vdots & & & & \\ \vdots & TCL_{Mki,j} & & & \\ TCL_{Mkq,1} & \dots & \dots & & TCL_{Mkq,q} \end{bmatrix} \quad (2)$$

where

$$TCL_{Mkij} = M^*_{LMkij} C_{Mk} \quad (3)$$

$$M_{LMkij} = FL^*_{Mkij} D_{ij}$$

$$FL_{Mk(q\times q)} = \begin{bmatrix} FL_{Mk1,1} & FL_{Mk1,2} & \dots & \dots & FL_{Mk1,q} \\ FL_{Mk2,1} & \dots & \dots & & \vdots \\ \vdots & & & & \\ \vdots & FL_{Mkr,s} & & & \\ FL_{Mkq,1} & \dots & \dots & & FL_{Mkq,q} \end{bmatrix} \quad (4)$$

where $D_{L(q\times q):}$=distance matrix between different locations; D_{ij} =distance between locations i and j; $F_{Mk(q\times q)}$=frequency matrix of resource Mk flow between different facilities per unit time; $F_{Mkr,s}$=frequency of resource Mk flow between facilities r and s per unit time; C_{Mk}=cost per unit distance for resources Mk flow; $TCL_{Mk(q\times q)}$=total cost matrix of resource Mk flow between different locations; $TCL_{Mki,j}$=total cost of resource Mk flow between locations i and j; M_{LMkij}=total distance traveled of resource Mk flow per unit time between locations i and j; $FL_{Mk(q\times q)}$ =frequency matrix of resource Mk flow between different locations per unit time; and $FL_{Mki,j}$=frequency of resource Mk flow between locations i and j per unit time.

It is noted that the frequency matrix $F_{Mk(q\times q)}$ of resource Mk flow between different facilities should be mapped into the frequency matrix $FL_{Mk(q\times q)}$ of resource Mk to reflect the flows of resources between the various locations respective to the different combinations of facility locations generated by GA. This process is necessary for each generation. A new set of locations will be assigned for the facilities, with the flow between the facilities being governed by the production logistic; the frequency flow of resources between the locations will be recalculated. It can be done by mapping the row index and column index from the frequency matrix $F_{Mk(q\times q)}$ to the frequency matrix $FL_{Mk(q\times q)}$ through the combinations between facility number and location number generated by the chromosome. The mapping can be implemented by the "index" function in Microsoft Excel.

Population Evolution through Genetic Operators

Crossover and mutation are used as genetic operators for the evolution of the population. Evolver (Evolver, the genetic algorithm solver for Microsoft Excel 1998), the GA package used in this study, employs a steady-state approach. This means that only one organism is replaced at a time rather than an entire "generation" being replaced. To obtain the equivalent number of generations before convergence, it needs to divide the number of individual trials explored by the size of the population.

The order solving method performs crossover using a similar algorithm to the order crossover operator (Fig. 1) described by Davis (1991). This selects items randomly from one parent, finds their place in the other parent, and copies the remaining items into

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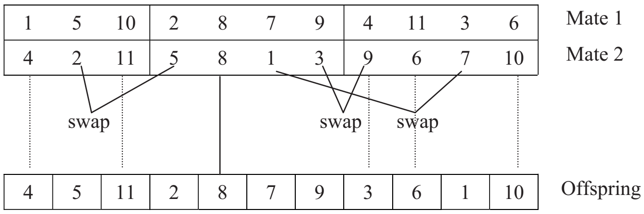

Fig. 1. Crossover of order chromosome

Table 2. Example of the Mutation Operator for 11-Gene Order Chromosome

Before mutation	2	6	3	1	8	10	4	11	7	5	9
After mutation	2	6	3	11	8	10	4	1	7	5	9

the second parent in the same order as they appear in the first parent. This preserves some of the suborderings in the original parents while creating some new suborderings.

To preserve all the original values, the order solving method performs mutation by swapping the positions of some variables in the organism. For example, in Table 2, Genes 1 and 11 swap their positions to form a new chromosome. The number of swaps performed is increased or decreased proportionately to the increase and decrease of the mutation rate setting (from 0 to 1).

In Evolver, parents are chosen with a rank-based mechanism. Instead of other GA systems, where a parent's chance to be selected for reproduction is directly proportional to its fitness, a ranking approach offers a smoother selection probability curve. This prevents good organisms from completely dominating the evolution from an early point. The flows of the GA operations for the site precast yard layout study are presented in Fig. 2.

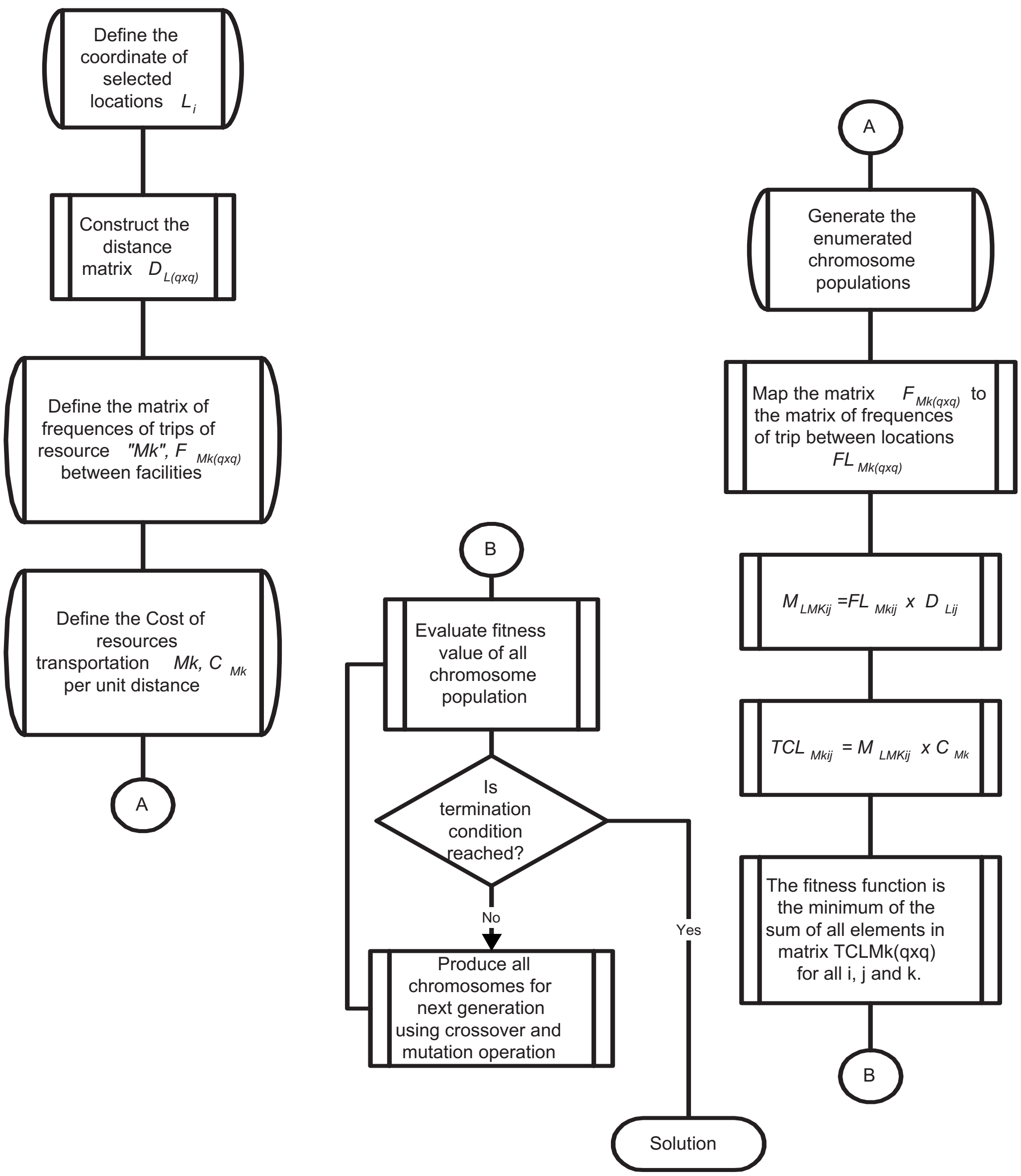

Fig. 2. Flows of the GA operations for site precast layout study

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Table 3. Distance between Locations

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	0	12	17	30	35	33	55	53	38	30	19
2	12	0	9	22	27	21	47	45	40	18	31
3	17	9	0	13	22	30	38	36	31	27	22
4	30	22	13	0	15	23	25	23	38	20	29
5	35	27	22	15	0	8	20	38	53	25	44
6	33	21	30	23	8	0	28	46	61	17	52
7	55	47	38	25	20	28	0	18	33	45	40
8	53	45	36	23	38	46	18	0	15	43	38
9	38	40	31	38	53	61	33	15	0	58	23
10	30	18	27	20	25	17	45	43	58	0	49
11	19	31	22	29	44	52	40	38	23	49	0

Illustration

The GA model so described is applied to a site precast yard sized 50×50 m. The physical size of the site in the numerical example is considered to be small but still quite common in Hong Kong for a single building development. There are always challenges to set up a small scale construction site with all kinds of site facilities in optimized location to support all site construction activities. The facilities to be positioned in the yard together with their designated numbers are given in Table 1. For example, the main gate and the lifting yard are designated as Facilities 1 and 11, respectively. The 11 locations are also determined with the coordinates given in Table 1.

With the coordinates, the rectangular distance matrix $(D_{L(q\times q)})$ for the locations is then calculated and presented as Table 3. The next step is to determine the logistic of resource flow between the facilities. The four types of resource considered are

1. Aggregate, sand, and cement/concrete $(Mk=1)$;
2. Reinforcement bars $(Mk=2)$;
3. Formwork $(Mk=3)$; and
4. Completed precast units $(Mk=4)$.

The flow logistic for the resources is mainly dictated by the production process. Cement, sand, aggregate, reinforcement bars, and formwork materials are stored in their respective storage areas before they are transported to their production units (batching plant, bending yard, and casting yard). Concreting of the precast units is carried out at the casting yard. The concreted units will undergo the curing process in the curing yard before being placed in the lifting yard. The flow pattern is shown in Fig. 3. Tables 4–7 give the trip frequency of the four types of resources between the facilities.

In addition, the cost units per distance for each type of resources (C_{Mk}) are determined and the results are given in Table 8. The initial population is now ready to be generated. Table 9 gives one chromosome of the initial population. With the chromosome as shown in Table 9, the corresponding frequency matrices between locations for the four types of resources, resulting from the

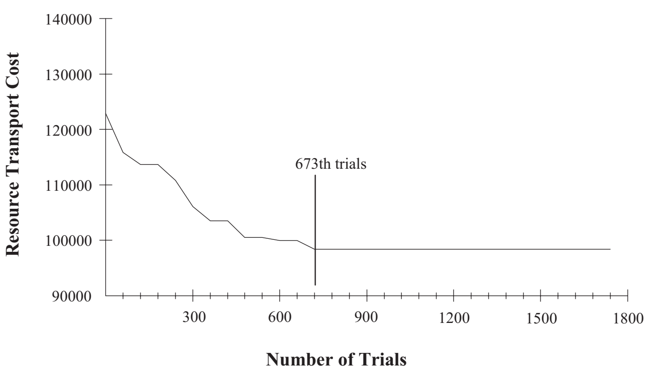

Fig. 3. Fitness of trials

Table 4. Frequency of Resources Flow between Facilities (per Day)—Aggregate, Sand, and Cement

	Facility, s										
Facility, r	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	20	—	—	—	—
2	—	—	—	—	—	—	15	—	—	—	—
3	—	—	—	—	—	—	35	—	—	35	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	—	—	—	—	—
7	20	15	35	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	35	—	—	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

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Table 5. Frequency of Resources Flow between Facilities (per Day)—Reinforcement

	Facility, *s*										
Facility, *r*	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	30	—	—	—	—	—	—	—
2	—	—	—	20	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—	—
4	30	20	—	—	—	50	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	50	—	—	—	—	—	50	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	50	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

assignment of the facilities to gene positions, are calculated and presented in Tables 10–13.

The distance matrix for the four types of sources is given by $M_{LMkij}=FL^{*}_{Mkij}D_{ij}$ [Eq. (4)]. Applying $TCL_{Mkij}=M^{*}_{LMkij}C_{Mk}$ [Eq. (3)], the transport cost units for the four types of resources with location arrangement as shown in Table 9 are obtained and given in Table 8.

Using the layout arrangement in Table 9, the following illustrates the calculation for the transport cost for transporting aggregate, cement, and sand ($Mk=1$) from the storage area (Facility No. 7) to the batching plant (Facility No. 3). From Table 9, the resource movement shall then be between Locations 7 and 8. According to Eqs. (3) and (4), the transport cost between Locations 7 and 8 can be expressed as follows:

Transport cost=distance between Locations 7 and 8 (from Table 3)×frequency of movements between facility Locations 7 and 8 (from Tables 10–13)×transport cost per unit distance from Table 8)=$18\times 35\times 5=3,150$ (Tables 14–17).

Similar operations are applied to all chromosomes within the population. The objective function [Eq. (2)] is used to assess the fitness. Evolutions are performed by the Evolver software. The genetic operators perform crossover and mutation. The probabili-

Table 6. Frequency of Resources Flow between Facilities (per Day)—Formwork

	Facility, *s*										
Facility, *r*	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	48	—
6	—	—	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	48	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

Table 7. Frequency of Resources Flow between Facilities (per Day)—Completed Precast Units

	Facility, *s*										
Facility, *r*	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	28
2	—	—	—	—	—	—	—	—	—	—	20
3	—	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	48	48
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	—	48	—	—	—
11	28	20	—	—	—	—	—	48	—	—	—

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Table 8. Transport Cost per Unit Distance between Facilities of the Four Resource Types

Material type, Mk	Cost per unit, C_{Mk}
Aggregate, sand, and cement	5.0
Reinforcement	4.0
Formwork	8.0
Completed precast unit	8.5

Table 9. One Chromosome in the Initial Population

Location number	Types of facility (assigned facility number)
1	Steel storage yard (6)
2	Bending yard (4)
3	Casting yard (10)
4	Curing yard (8)
5	Lifting yard (11)
6	Main gate (1)
7	Cement, sand, and aggregate storage yard (7)
8	Batching plant (3)
9	Refuse dumping area (9)
10	Side gate (2)
11	Formwork storage yard (5)

ties of crossover and mutation are set at 0.5 and 0.06, respectively. The initial population size is 50. For each generation, the above operations are repeated so that the fitness of all chromosomes within the trials is assessed. The transport cost for resource flow remained constant as from 673rd trials, that is, 13.46 generations (Fig. 3), and the near-optimal solution is shown in Table 18. Tables 19–21 summarize the results from the GA operations performed by Evolver under the computer workstation—Dell Precision with Intel Core Duo E8400 (3.0 GHz). Fig. 4 presents the site precast yard layout as plotted from the results shown in Table 18, and the flow of the resources is also indicated. The best value of the total transport cost found is 99,788 cost units.

MIP Approach

Indeed, available locations for housing different kinds of facilities within a site are always fixed and thus these facilities allocated on different locations can be defined as a set of integer variables. Transportation distances or costs of an activity between any two facilities can be set and calculated in the form of linear constraint sets using the if-then concept. The objective for optimization in this study, i.e., the total induced costs for all the activities within a site, is also linear in nature. Thus, the transportation cost minimization problem in question can be formulated as a mixed-integer program that can be solved by standard solution techniques such as CPLEX solver (Wong and Wong 2006).

Table 10. Frequency of Resources Flow between Locations (per Day)—Aggregate, Sand, and Cement

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	35	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	20	—	—	—	—
7	—	—	—	—	—	20	—	35	—	15	—
8	—	—	35	—	—	—	35	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	15	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

Table 11. Frequency of Resources Flow between Locations (per Day)—Reinforcement

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	50	50	—	—	—	—	—	—	—	—
2	50	—	—	—	—	30	—	—	—	20	—
3	50	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	30	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	20	—	—	—	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

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Table 12. Frequency of Resources Flow between Locations (per Day)—Formwork

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—	48
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	—	—	—	—	—
11	—	—	48	—	—	—	—	—	—	—	—

Mixed-Integer Programming Formulation

See list of symbols in Notation.

Constraint Sets

Establishment of a Single Facility on a Location

In the formulation, we have to ensure that only one facility is allocated on a specific location within the site area; i.e., no duplication is permitted. Mathematically, the binary decision variables $x_{i,j}$ have to be controlled by constraint sets (5) and (6) below. In Eq. (5), for each location $j \in \{1,J\}$ where J is the total number of available locations within the site area, there must be one and only one facility. Similarly, for each facility $i \in \{1,I\}$ where I is the total number of facilities to be allocated within the site area, only one location can be assigned in Eq. (6)

$$\sum_{i=1}^{I} x_{i,j} = 1, \quad \forall\, j \in \{1,J\} \tag{5}$$

Table 13. Frequency of Resources Flow between Locations (per Day)—Completed Precast Units

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	48	—	—	—	—	—	—	—
4	—	—	48	—	48	—	—	—	—	—	—
5	—	—	—	48	—	28	—	—	—	20	—
6	—	—	—	—	28	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	20	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

Table 14. Transport Cost Units for Resources between Locations—Aggregate, Sand, and Cement

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	6,300	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	2,800	—	—	—	—
7	—	—	—	—	—	2,800	—	3,150	—	3,375	—
8	—	—	6,300	—	—	—	3,150	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	3,375	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

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Table 15. Transport Cost Units for Resources between Locations—Reinforcement

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	2,400	3,400	—	—	—	—	—	—	—	—
2	2,400	—	—	—	—	2,520	—	—	—	1,440	—
3	3,400	—	—	—	—	—	—	—	—	—	—
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	2,520	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	1,440	—	—	—	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

$$\sum_{j=1}^{J} x_{i,j} = 1, \quad \forall\, i \in \{1,I\} \tag{6}$$

Linkage of a Pair of Facilities in Different Locations

In the study, the total transport costs of material movements for performing specific construction procedures within the site area are optimized. These kinds of material movements usually involve the interactions of two facilities on two different locations. For example, raw steel reinforcement has to be moved from a steel storage yard to a bending yard for casting into suitable shape and size. An auxiliary binary-type variable $\delta_{i,j,m,n}$ is constructed by constraint set (7) based on the decision variables $x_{i,j}$ $\forall i,j$ to represent the linkage between facility i (steel storage yard) on location j and facility m (bending yard) on location n where $i \neq m$ and $j \neq n$. Numerically, if $x_{i,j}=1$ and $x_{m,n}=1$, then the linkage must be established, that is, $\delta_{i,j,m,n}=1$

Table 16. Transport Cost Units for Resources between Locations—Formwork

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	—	—	—	—	—	—	—	8,448
4	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—
6	—	—	—	—	—	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	—	—	—	—	—	—	—
11	—	—	8,448	—	—	—	—	—	—	—	—

Table 17. Transport Cost Units for Resources between Locations—Completed Precast Units

	Location, j										
Location, i	1	2	3	4	5	6	7	8	9	10	11
1	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—
3	—	—	—	5,304	—	—	—	—	—	—	—
4	—	—	5,304	—	6,120	—	—	—	—	—	—
5	—	—	—	6,120	—	1,904	—	—	—	4,250	—
6	—	—	—	—	1,904	—	—	—	—	—	—
7	—	—	—	—	—	—	—	—	—	—	—
8	—	—	—	—	—	—	—	—	—	—	—
9	—	—	—	—	—	—	—	—	—	—	—
10	—	—	—	—	4,250	—	—	—	—	—	—
11	—	—	—	—	—	—	—	—	—	—	—

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Table 18. Near Optimal Layout (Using GA Approach)

Location number	Location coordination X	Y	Types of facility (facility number)
1	15	40	Main gate (1)
2	13	30	Lifting yard (11)
3	22	30	Curing yard (8)
4	25	20	Casting yard (10)
5	20	10	Steel storage yard (6)
6	12	10	Bending yard (4)
7	40	10	Refuse dumping area (9)
8	48	20	Formwork storage yard (5)
9	48	35	Batching plant (3)
10	5	20	Side gate (2)
11	32	42	Cement, sand, and aggregate storage yard (7)
Total transport cost: 99,788			

$$M(2-x_{i,j}-x_{m,n}) \geq (1-\delta_{i,j,m,n}), \quad \forall \{(i \neq m) \in I; (j \neq n) \in J\} \tag{7}$$

Total Cost for Transporting Materials between a Pair of Facilities

With the auxiliary variable $\delta_{i,j,m,n}$ expressing the actual linkages of facilities within the site area, the total cost of material movements across the facilities within the site area can be calculated using Eq. (8). The total cost C is simply the sum of all transport costs for the materials involved according to the facility linkage $\delta_{i,j,m,n}$. In Eq. (8), $f_{i,m,\mu}$ is the daily frequency (amount) of resource material μ between facilities i and m; ζ_μ is the unit cost of transporting material type μ; and $D_{j,n}$ is the actual transport distance between locations j and n.

$$C = \sum_i \sum_j \sum_m \sum_n \sum_\mu \delta_{i,j,m,n} \times f_{i,m,\mu} \times \zeta_\mu \times D_{j,n} \tag{8}$$

The problem is formulated as a binary-mix-integer-linear-programming problem to minimize the total cost C subject to constraint sets (5)–(8) and can effectively be solved by standard branch-and-bound technique. The total cost obtained is 98,424 cost units.

Table 19. True Optimal Layout by MIP

Location	Types of facility (facility number): Using GA approach	Using MIP approach
1	Main gate (1)	Main gate (1)
2	Lifting yard (11)	Casting yard (10)
3	Curing yard (8)	Curing yard (8)
4	Casting yard (10)	Lifting yard (11)
5	Steel storage yard (6)	Steel storage yard (6)
6	Bending yard (4)	Bending yard (4)
7	Refuse dumping area (9)	Formwork storage yard (5)
8	Formwork storage yard (5)	Batching plant (3)
9	Batching plant (3)	Cement, sand, and aggregate storage yard (7)
10	Side gate (2)	Side gate (2)
11	Cement, sand, and aggregate storage yard (7)	Refuse dumping area (9)
Optimized total cost	99,788	98,424

Table 20. Result from Evolver (GA Approach)

Results	
Trials	1,673
Recalcs	1,673
Original value	122,362
+soft constraint penalties	0
=result	122,362
Best value found	99,788
+soft constraint penalties	0
=result	99,788
Occurred on trial number	673
Time to find this value	00:01:43
Stopped because	No improvement for 1,000 trials
Optimization started at	12:30:30 p.m.
Optimization finished at	12:34:22 p.m.
Total optimization time	00:03:52

New Features for Facility Location Problem Using MIP

On top of the above, the MIP can also be used to enhance the solution quality and incorporate new modeling features in which the probabilistic solution routines such as the GAs may not be able to tackle directly. In this section, highlights will be given to enhance the MIP formulation for solving the facility location problem with more design requirements/constraints. It should be noted that the GAs are powerful optimization techniques that are only able to search a solution point within a very well-defined solution region. However, the search process will become clumsy if the solution space is disintegrated by hard design constraints. In the present facility locations within a construction site, for example, raw materials are required to be transported between two facilities to have a value-added process in which steel bars are needed to be shaped in a bending yard serving as special reinforcement so that the raw materials have to be transported from a steel storage yard to a bending yard and probably returned to the storage yard after shaping. It indeed imposes design requirements to locate the bending yard by making sure that the steel material before and after the shaping process can be transported freely between locations. Due to the construction-site limitation, the connecting paths for some specific locations may be too narrow for transporting the shaped reinforcement that may be larger in size or odd in shape. For that, this new modeling feature should be considered in the optimization process.

In a form of a linear constraint, it is however relatively easier to be handled in the MIP formulation by setting the design variable $x_{s,t}=0$, meaning that facility s cannot be located at site location t. In the GA formulation, it involves probabilistic solution procedures such as crossover and mutation that will allow $x_{s,t}=1$ in the trial solution set. Since it is not a feasible solution point, additional steps must be added to eliminate these genes from evolution, which will of course complicate the GA implementation. This is only one possible scenario in the facility location problem and the number of this kind of limitation will grow, to which GA may not be as efficient as the MIP formulation in the identification of the optimal facility locations.

For further illustration of the MIP capability to handle practical site limitations, we consider in our numerical example that the site Location 6 (the original results of GA and MIP are to estab-

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Table 21. Sequence of Results Generated by MIP

From		To						
Location number	Facility number	Location number	Facility number	Frequency	Transport cost	Distance	Total cost	Resources
1	1	2	11	28	8.5	12	2,856	Precast
1	1	6	4	30	4	33	3,960	Reinforcement
1	1	9	7	20	5	38	3,800	Aggregate
2	11	1	1	28	8.5	12	2,856	Precast
2	11	3	8	48	8.5	9	3,672	Precast
2	11	10	2	20	8.5	18	3,060	Precast
3	8	2	11	48	8.5	9	3,672	Precast
3	8	4	10	48	8.5	13	5,304	Precast
4	10	3	8	48	8.5	13	5,304	Precast
4	10	5	6	50	4	15	3,000	Reinforcement
4	10	7	5	48	8	25	9,600	Formwork
4	10	8	3	35	5	23	4,025	Aggregate
5	6	4	10	50	4	15	3,000	Reinforcement
5	6	6	4	50	4	8	1,600	Reinforcement
6	4	1	1	30	4	33	3,960	Reinforcement
6	4	5	6	50	4	8	1,600	Reinforcement
6	4	10	2	20	4	17	1,360	Reinforcement
7	5	4	10	48	8	25	9,600	Formwork
8	3	4	10	35	5	23	4,025	Aggregate
8	3	9	7	35	5	15	2,625	Aggregate
9	7	1	1	20	5	38	3,800	Aggregate
9	7	8	3	35	5	15	2,625	Aggregate
9	7	10	2	15	5	58	4,350	Aggregate
10	2	2	11	20	8.5	18	3,060	Precast
10	2	6	4	20	4	17	1,360	Reinforcement
10	2	9	7	15	5	58	4,350	Aggregate

Optimized total cost: 98,424

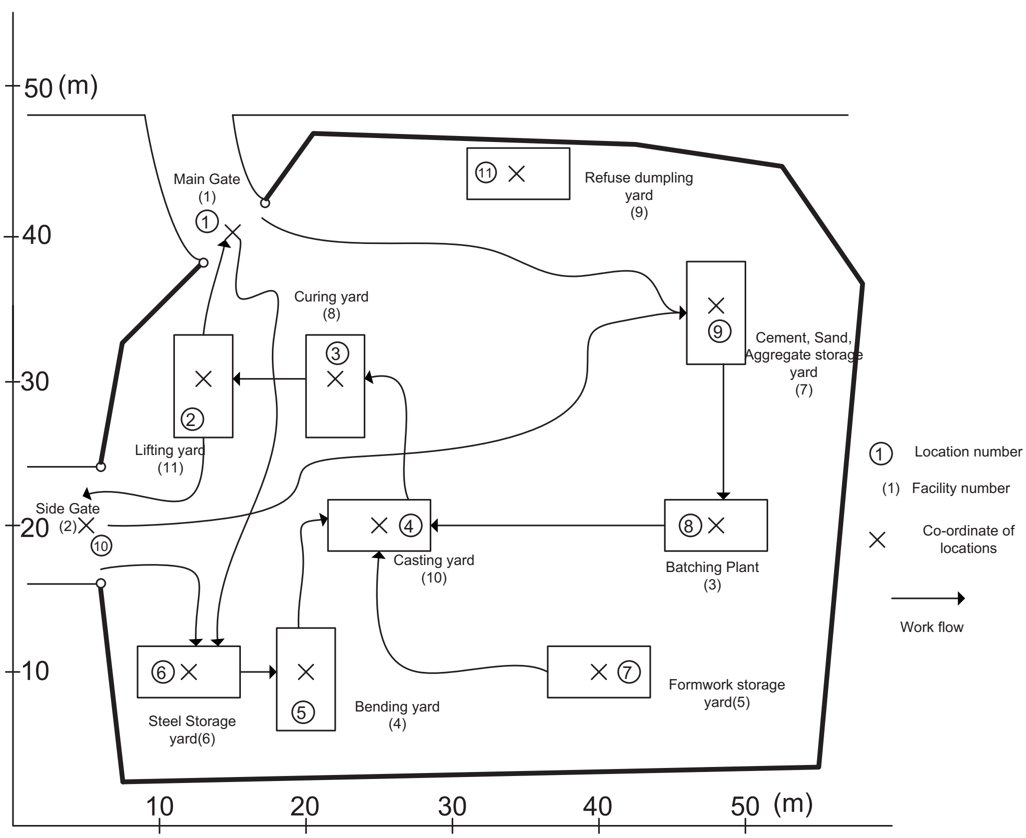

Fig. 4. Site precast yard layout arrangement plan for the MIP

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Table 22. Optimal Layout by MIP with Location Constraint

Location	Types of facility (facility number): Using MIP without location constraint	Types of facility (facility number): Using MIP with location constraint
1	Main gate (1)	Formwork storage yard (1)
2	Casting yard (11)	Casting yard (11)
3	Curing yard (8)	Curing yard (8)
4	Lifting yard (10)	Lifting yard (10)
5	Steel storage yard (6)	Batching plant (3)
6	Bending yard (4)	Cement, sand, and aggregate storage yard (7)
7	Formwork storage yard (5)	Refuse dumping area (9)
8	Batching plant (3)	Formwork storage yard (5)
9	Cement, sand, and aggregate storage yard (7)	Steel storage yard (6)
10	Side gate (2)	Side gate (2)
11	Refuse dumping area (9)	Bending yard (4)
Optimized total cost	98,424	101,448

lish the bending yard) is no longer suitable for housing the bending yard (facility number 4) due to the limited road space for the material transportation. This site limitation is then converted into a linear mathematical constraint in the form of ($x_{4,6}=0$) and combined to the original constraint set for considerations. The new optimization results are given in Table 22. The original MIP result without the location constraint and the new MIP result with the location constraint are tabulated. It is found that the bending yard is relocated to Location 11. With the change of the bending yard location, batching plant, cement and sand and aggregate storage yard, refuse dumping area, formwork storage yard, and steel storage yard all need to be relocated. The optimized total transportation cost becomes 101,448 cost units which is a little bit higher than the one without location constraint.

Using standard CPLEX solver, the mixed-integer program can solve the problem effectively taking the various facilities and their assigned locations into consideration. The following integer solution in Table 22 can be found. Fig. 5 demonstrates the site precast yard layout arrangement plan and its work flow using the MIP.

Concluding Remarks

The illustration example demonstrates the searching ability of GA. Timewise, with an initial population of 50, the near-optimal solution was obtained in 1 min and 43 s. The search space is calculated by

$$P_{k,n}=\frac{n!}{(n-k)!} \tag{9}$$

where $P_{k,n}$=number of permutations for k facilities to be located in n locations.

In the example, $n=k=11$, hence the possible layout arrangements are given by 11! (39,916,800). Attainment of the near-optimal solution at the 673rd trial represents only a coverage of 0.001 686% of the search space.

As for the transport cost, the daily transport for resource movements to achieve the planned output reduced from 122,362 cost units (based on the best layout within the initial population) to 99,788 cost units (based on the near-optimal solution). Reduction of 18.45% was achieved through the use of GA.

GAs are suitable for tackling combinatorial problems involving large search space. Near-optimal solution for this type of problem is often obtained through evolution. Order chromosome representation neatly fills this need. This research takes full advantage of this salient feature of the order chromosome and applies to a site precast yard layout problem. The methodology so described in this paper, through the use of an illustration example, is shown to be an efficient method to obtain a near "optimal" solution. Efficiency is achieved in terms of the small population size and relatively short convergence process.

On the other hand, the site facility layout problem can also be solved using the MIP approach. The optimization result obtained using MIP is found to be slightly better than that derived from GA although the difference is small. It should be well noted that solution algorithm of MIP always produces true optimal solution while GA only gives suboptimal solution. Further, some new features with more design requirements and constraints can be easily managed by MIP while the same may complicate the GA operations. This concludes that MIP can perform better than GA in this construction-site facility layout problem in which the facilities

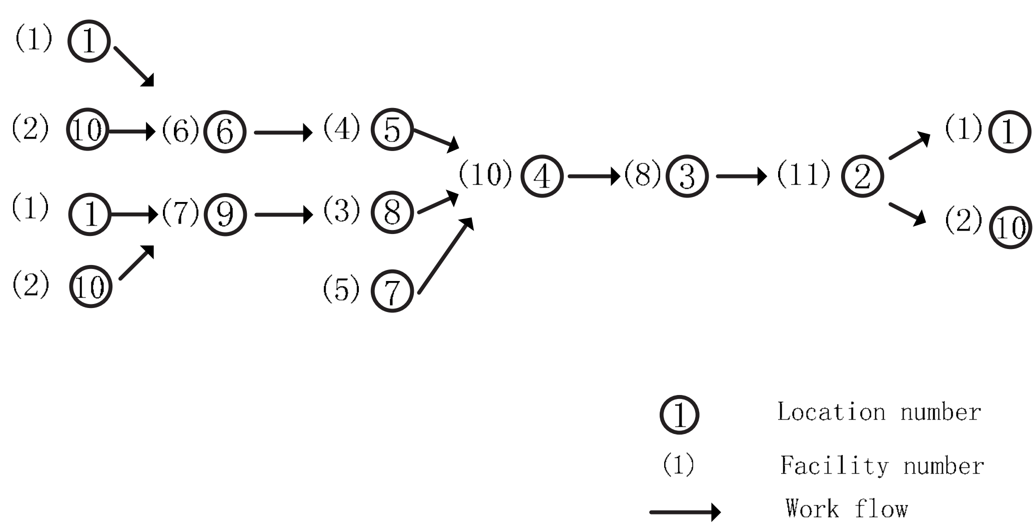

Fig. 5. Site precast yard layout arrangement plan work flow for the MIP

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and locations can be represented using a series of integer variables.

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Notation

The following symbols are used in this paper:

C = total cost in the construction-site setting;
$D_{j,n}$ = travel distance between locations j and n;
$f_{i,m,\mu}$ = frequency of daily flow between facilities i and m of material type μ;
I = total number of facilities to be allocated within the site area;
i,m = facilities i and m;
J = total number of available locations for facilities within the site area;
j,n = locations j and n within the site area;
M = arbitrary large number;
$x_{i,j}$ = decision of establishing facility i on location j where "1" is yes but "0" is not;
$\delta_{i,j,m,n}$ = auxiliary binary-type variable where 1 means facility i establishing on location j and facility m establishing on location n but "0" otherwise;
ζ_{μ} = unit cost of transporting material type μ;
μ = material type; and
$\bar{\mu}$ = total number of material types.

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