Design of Tall Formworks by a Finite-Element Model

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Abstract: The design of formworks for holding fresh concrete possesses a difficult engineering challenge. Present standards assume fresh concrete to have a nonviscous fluid behavior when calculating the lateral pressure to which the formwork walls will be the subject. This paper describes a finite-element model to determine these pressures, taking into account the interaction between the fresh concrete and the formwork wall. The use of this numerical model shows that present standards may underestimate the lateral pressures that can be exerted particularly with respect to tall formworks. The paper also discusses the influence of different mechanical variables on the results returned by the proposed model. The proposed model may be of use to practicing engineers and of interest to researchers examining load distributions in formworks.

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Introduction

The use of concrete as a structural material in buildings and civil engineering projects requires the employment of formworks which must remain in place until setting is complete. The cost of these contention structures can reach between 40 and 60% of the total budget of a concrete structure (Hanna and Senouci 1997). From a work safety standpoint, formwork design is of the utmost importance; failures frequently lead to serious accidents (Shapira 1999; Huang et al. 2000).

The shape and mechanical characteristics of formworks depend on the project being undertaken. Generally, their dimensions (as well as those of the structural elements that support them) increase with the complexity of the structure being built. Thus, in civil engineering projects such as the construction of viaducts, bridges, and retaining walls, it is quite common to use formworks of more than 5 m in height.

Traditionally, the pressure exerted by fresh concrete on formwork walls has been calculated assuming that this material behaves hydrostatically as a nonviscous fluid without taking into account the tangential stress derived from the friction forces between the mass of concrete and the formwork encasing it (referred to as the "silo effect"). Although this method of calculation provides a wide safety margin, it only satisfactorily reproduces the behavior of fresh concrete in formworks of low height; as formworks become taller, the pressure exerted by the concrete can become quite seriously overestimated (Arslan 2002; Andriamanantsilavo and Amziane 2004; Arslan et al. 2005). The study of the variables affecting the setting of concrete (ambient temperature, concrete pouring velocity, characteristics of the mix, etc.) has led to the appearance of a number of empirical equations for the determination of the pressure exerted by this material (Deutsches Institut für Normung 1980; Harrison 1983; CIRIA 1985; American Concrete Institute 2004). Nevertheless, the design of tall formworks or with a complex geometry is not adequately addressed in these standards and it would require the use of advanced numerical tools.

The use of advanced calculation techniques, such as those based on the finite-element method (Zienkiewicz and Taylor 1991), allows the analysis of the concrete-formwork system as a whole while taking into account the aforementioned concerns (Kajewski 2005). However, the development of a reliable numerical model requires the adequate selection of a behavior model for fresh concrete, as well as knowledge of the mechanical variables that characterize this material.

During the earliest phase of setting, the behavior of fresh concrete likened to a non-Newtonian fluid, while at the end of the process it behaves like a rock. However, the behavior of fresh concrete is similar to that exhibited by cohesive granular materials for the most part of setting (Gardner 1985).

This work presents a two-dimensional finite-element model (FEM) for simulating the intermediate state of setting of fresh

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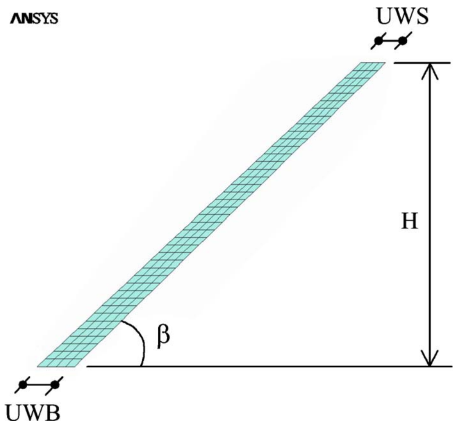

Fig. 1. Formwork used in this paper

concrete. An elastoplastic behavior was assumed to characterize the material during this phase of setting. A preliminary validation of the model was performed using experimental values obtained by Arslan et al. (2005). Given the scarcity of information on the mechanical variables affecting the behavior of fresh concrete, a sensitivity analysis was performed to determine their influence on the distribution of the pressure exerted by this material on formwork walls. Pressures predicted by the model were compared to those calculated using different international standards for formwork design.

Research Methodology

A FEM has been developed by the writers for the determination of lateral pressures in tall formworks. A material model commonly used for cohesive granular materials has been selected for simulating the intermediate state of setting of fresh concrete. The values of the mechanical parameters required have been obtained from the literature. A sensitivity analysis has been conducted to analyze the influence of the mechanical properties in lateral pressures because of the different types of concretes and the wide range of setting conditions. A preliminary validation of the FEM has been conducted by using the experimental results obtained by Arslan et al. (2005). In addition, the numerical results have been compared to lateral pressures predicted by standards (Deutsches Institut für Normung 1980; CIRIA 1985; American Concrete Institute 2004) for different formwork heights: 2.5, 5, 10, and 15 m.

The proposed model, which was developed using ANSYS 8.1 finite-element software (*ANSYS 8.1 university high option manual* 2004), allows the analysis of two-sided vertical wall formworks of different shapes and sizes. The geometry of a formwork (Fig. 1) is defined by its height (H), the wall inclination angle (β), the upper width of the structure (UWS), and the width of the base (WB).

The numerical analysis was performed assuming the plane stress hypothesis, i.e., considering a wall-type formwork of sufficient length to obviate the need for including a third dimension. The fresh concrete was simulated using eight-node plane elements (which allow plasticity and the effect of shape deformation on stresses to be taken into account) and quadratic interpolation functions.

The interaction between the fresh concrete and the formwork was represented using the surface-to-surface contact procedure, in which both the target and contact surfaces have to be specified (Vidal et al. 2005, 2006). The former is assumed to be the rigid surface and the latter the deformable surface. Hence, the target surface is the formwork wall and the boundary surface of the fresh concrete represents the contact surface. The formwork wall was assumed to be rigid, and the Coulomb model used to determine the friction between the formwork and the fresh concrete [Eq. (1)]. The shear stress (τ) was obtained from the normal pressure (p) and the friction coefficient between the formwork and the fresh concrete (μ_c). The fresh concrete-to-wall adherence (k) was regarded as negligible

$$\tau = k + \mu_c p \qquad (1)$$

Fresh-Concrete Behavior Model

The selection of a theory that adequately describes the behavior of fresh concrete over most of the setting period was the main challenge in developing the model. Tattersall (1976) proposed that during the earliest phases of setting, fresh concrete could be likened to a non-Newtonian fluid. The formulas proposed by Bingham (1922) and Herschel and Bulkley (1926), which take into account the viscosity and yield stress of fresh concrete (Ferraris and de Larrard 1998; Petit et al. 2007; Murata and Kukawa 1992), properly describe the initial moments of setting. However, as setting advances, fresh concrete becomes similar to soil in that both are weakly bonded particulate systems in a fluid medium (Gardner 1985). Therefore, a granular cohesive material behavior was assumed, thus allowing the logic of soil mechanics to predict the lateral pressures developed on the formwork walls (Vanhove et al. 2004).

Accordingly, the behavior of the fresh concrete was taken as being elastoplastic to reflect an intermediate state between the fluid and solid forms of the material. A linear isotropic behavior was assumed for the elastic region of the fresh concrete [Eq. (2)], where the stresses T_{ij} are obtained from the strains E_{ij} via the use of the Lamé λ and μ constants. The values of these constants can be deduced from those of the Poisson coefficient (ν) and the modulus of elasticity of the material (E) [Eqs. (3) and (4)]

$$\mathbf{T}_{ij} = \lambda \cdot \mathbf{D}_{kk} \cdot \delta_{ij} + 2 \cdot \mu \cdot \mathbf{D}_{ij} \qquad (2)$$

$$\mu = E/[2(1+\nu)] \qquad (3)$$

$$\lambda = \nu E/[(1+\nu)(1-2\nu)] \qquad (4)$$

To define the plastic region, the plastification criterion of Drucker and Prager (1952) was used as expressed by Eq. (5), where α and κ are characteristic variables of the material, I_1 is the first invariant stress tensor, and J_2 is the second invariant of the deviatoric stress tensor. The yield function F is also known as the flow function

$$F = \alpha I_1 + (J_2)^{0.5} - \kappa = 0 \qquad (5)$$

The Drucker-Prager plastification criterion is really a modification of the well-known Mohr-Coulomb criterion. Thus, the characteristic variables of the material α and κ expressed in Eq. (5) can be obtained from the two mechanical variable characteristics of the latter criterion: the angle of internal friction (ϕ) and the cohesion of the material (c) (Chen 1982; Chen and Mizuno 1990). The plastification criterion chosen allows the possibility of using a nonassociated flow rule (Lubarda 2002), which requires knowledge of the dilatancy angle (ψ). The dilatancy angle cannot be greater than the angle of internal friction.

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Table 1. Intervals for the Fresh-Concrete Mechanical Parameters Obtained from the Literature and Considered in the Parametric Study

Mechanical parameter	Symbol	Interval	References	Selected value
Angle of internal friction (degrees)	ϕ	2–40	[a]	30
Cohesion (kPa)	c	2–10	[a]	5
Dilatancy angle (degrees)	ψ	2–40	[a]	20
Modulus of elasticity (kPa)	E	2×10^3–2×10^6	[b]	2×10^4
Poisson coefficient	ν	0.30–0.45	[b]	0.40
Concrete-to-wall friction coefficient	μ_c	0.01–0.20	[c]	0.05

[a]L'Hermite 1949; Ritchie 1962; Olsen 1968; Alexandridis and Gardner 1981.
[b]Krauß and Hariri 2006.
[c]Djelal et al. 2004.

Mechanical Parameters

In agreement with the above, the mechanical variables required by the proposed model include the angle of internal friction (ϕ), the cohesion (c), the dilatancy angle (ψ), the modulus of elasticity (E), the Poisson coefficient for fresh concrete (ν), and the friction coefficient between the concrete and the formwork walls (μ_c). The values for these variables, except the wall friction coefficient values, can be obtained experimentally in triaxial tests.

The first triaxial tests for the determination of the angle of internal friction and the cohesion of fresh concrete mixtures date from the end of the 1940s (L'Hermite 1949). Later, Ritchie (1962), Olsen (1968), and Alexandridis and Gardner (1981) investigated the influence of the composition and consistency of the concrete, the water-cement relationship, and the setting temperature.

Alexandridis and Gardner (1981) obtained values for the angle of internal friction of 34°–41° and cohesion values of 2–6 kPa for samples of fresh concrete at different temperatures (4–20°C) and different stages of setting (40, 80, 120, and 160 min). Ritchie (1962) and Olsen (1968) obtained much lower values for the angle of internal friction (2°–10°) in concretes with high cement contents (aggregate:cement <4). Therefore, in the present parametric study, the angle of internal friction was taken as ranging from 2° to 40° and the cohesion range as 2–10 kPa. The range for the dilatancy angle was assumed to be equal to that of the angle of internal friction since its value cannot exceed that of the angle of internal friction.

Wide ranges were selected for the Poisson coefficient and for the modulus of elasticity of the fresh concrete, in accordance with the variability shown by these variables during setting (Krauß and Hariri 2006). The coefficient of friction between the fresh concrete and the formwork wall was that reported by Djelal et al. (2004). The value of this variable (range of 0.01–0.20) depends, among other factors, on the rate of placement of the concrete, the type of mix used, and the use of products to facilitate the removal of the formwork.

Table 1 shows the intervals for the different mechanical variables taken into account in the parametric study plus the values selected for the comparison of the results obtained with the numerical model and the formwork design standards.

Validation of the FEM

The validation of the FEM requires the comparison of numerical results with lateral pressures measured in experimental prototypes. The writers of the paper are working now in the implementation of an experimental setup in order to achieve this objective. However, a preliminary validation of the model was done by using the experimental results obtained by Arslan et al. (2005) with a formwork 2 m in height, 1 m in length and 0.15 m in width.

The tests undertaken by Arslan et al. (2005) involved a normal concrete made of Type I cement, natural sand (0–3 mm), crushed sand (3–7 mm), and crushed gravel (7–15 mm) in the proportion 1:1:1.4:1.75. The cement content in the mix was 400 $\mathrm{kg/m^3}$; the water/cement ratio was 0.48, and the density was 2,400 $\mathrm{kg/m^3}$. The pressure recorded at the base of the formwork by strain gauge plates varied between 18.00 and 26.96 kPa, depending on the formwork material (*Populus nigra, Pinus silvestris*, plywood, or steel) and whether or not oils were used to facilitate the removal of the formwork, as shown in Table 2.

For a formwork of the above shape and size and when contemplating the fresh concrete mechanical parameters specified in Table 1, the proposed model returned a maximum normal pressure on the formwork walls of 20.17 kPa quite similar to the experimental values reported by Arslan et al. (2005). Fig. 2 shows the pressure distribution on the formwork wall obtained with the FEM.

Numerical Results and Discussion

Comparison of Pressures Obtained by the Numerical Model and the Formwork Design Standards

A comparison of the results obtained with the FEM and the lateral pressures predicted according to standards (DIN 18218, CIRIA

Table 2. Lateral Pressures Measured by Arslan et al. (2005) at the Base of Experimental Formworks 2 m in Height, 1 m in Length, and 0.15 m in Width

	Lateral pressure (kPa)		
Formwork surface	Minimum	Mean	Maximum
Populus nigra	19.94	20.93	21.88
Populus nigra (watered with concrete mold oil)	21.77	22.85	23.74
Pinus silvestris	18.01	19.91	21.15
Pinus silvestris (watered with concrete mold oil)	21.39	23.68	25.11
Plywood	19.22	21.48	22.81
Plywood (watered with concrete mold oil)	22.47	24.55	25.96
Steel	24.70	26.19	26.97
FEM	—	**20.17**	—

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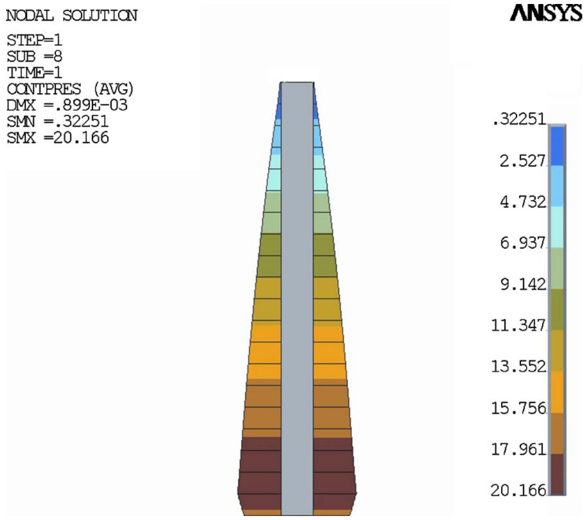

Fig. 2. Distribution of normal pressures exerted by fresh concrete on a vertical formwork 2 m in height and 150 mm thick determined using the proposed model

108, and ACI 347-04) is presented in this section. Different heights for the formwork (2.5, 5, 10, and 15 m) were considered, while the same width (0.5 m) and length (5 m) were assumed for all cases.

The characteristics of the concrete mix and the execution conditions employed in the calculations using both the FEM and the above standards were as follows: Type I cement with neither admixtures nor retarding agents, a slump of 100 mm in the Abrams cone, a density of 2,000 $\mathrm{kg/m^3}$, a setting temperature of 20°C, and a concrete pouring velocity of 3 m/h.

Table 1 shows the values selected for the mechanical parameters required by the numerical model that simulates the behavior of the fresh concrete. Mean values were selected within the range obtained from the literature for these mechanical parameters.

The American standard ACI 347-04 proposes pressures be obtained from Eq. (6), where the maximum lateral pressure on the wall of the formwork ($P_{\max}$) depends on the rate of placement of the concrete (R, ranging from 2.1 to 4.5 m/h), the temperature of the concrete during placement (T), a nondimensional correction factor that refers to the specific weight of the fresh concrete (C_w), and a coefficient related to the type of cement and additives used in the mix (C_c)

$$P_{\max} = C_w C_c[7.2 + 1{,}156/(T + 17.8) + 244R/(T + 17.8)] \quad (6)$$

The German standard DIN 18218 proposes a series of expressions for the calculation of the pressure on formwork walls [Eq. (7) shows the final expression]. In this case, the rate of placement of the concrete (R) is the main variable used to calculate the maximum lateral pressure on the wall of the formwork ($P_{\max}$). In addition, two coefficients (K_1 and K_2) that depend on the consistency of the fresh concrete (determined using the Abrams cone test) are also used. This standard contemplates a 3% increase/reduction in the maximum lateral pressure for every 1°C

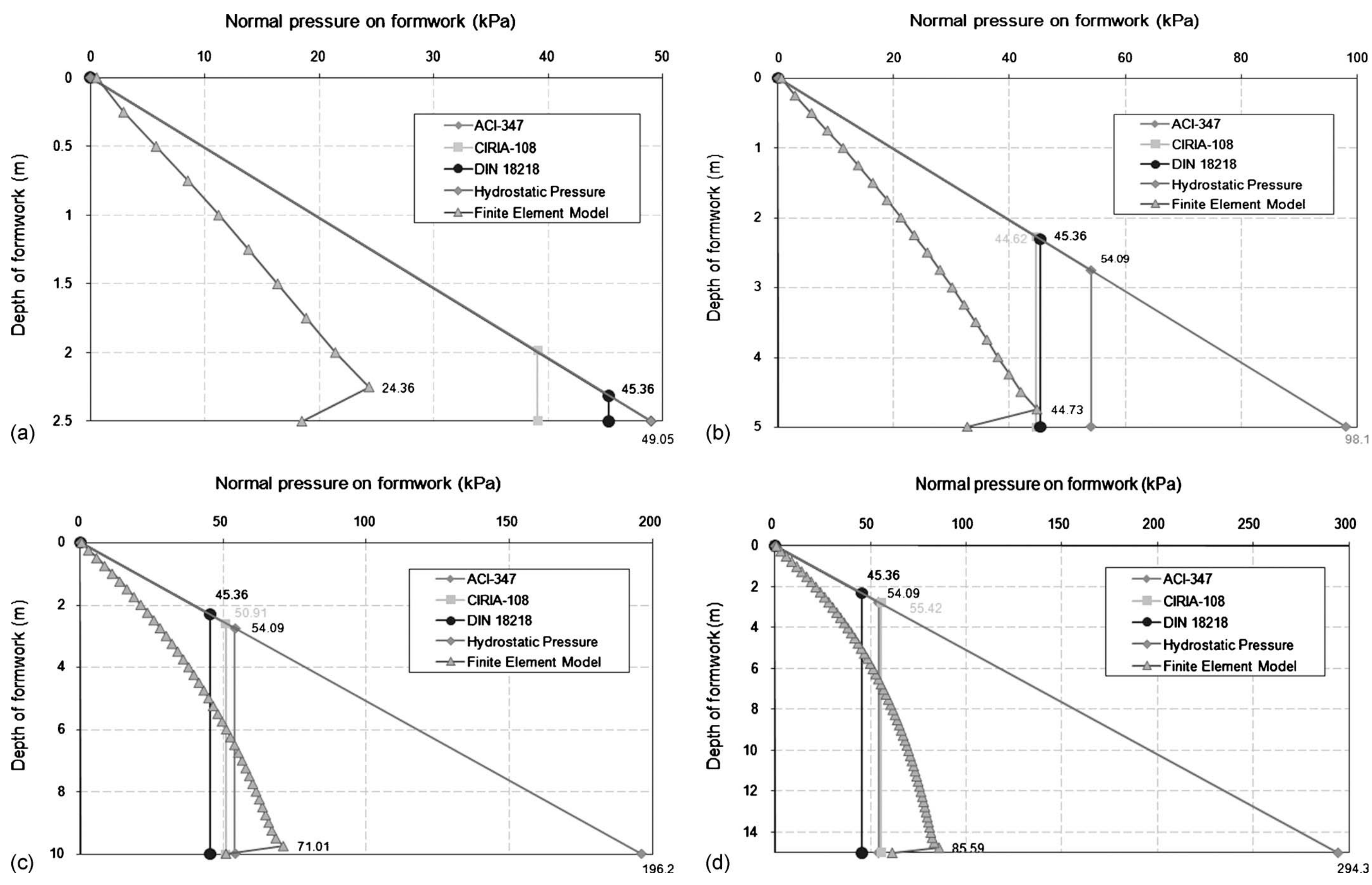

Fig. 3. Distribution of lateral pressures using the proposed model and the different standards for vertical formworks: (a) 2.5; (b) 5; (c) 10; and (d) 15 m in height

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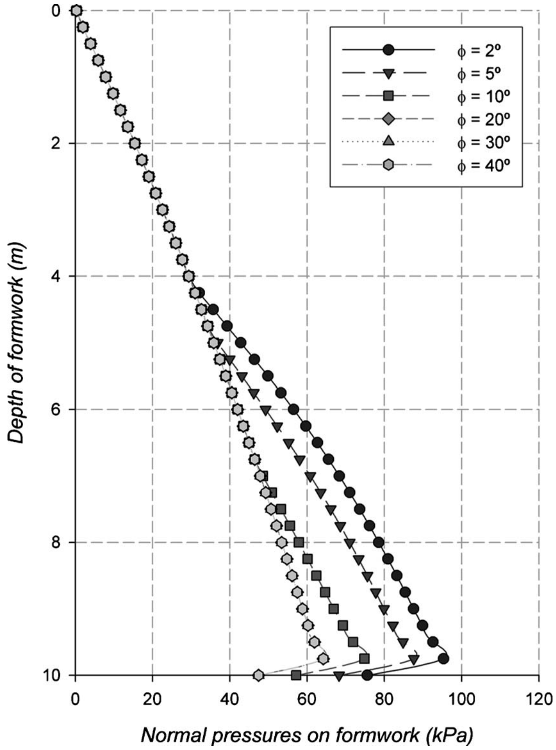

Fig. 4. Influence of the angle of internal friction (ϕ) on the distribution of normal pressures against the formwork walls

difference between ambient temperature and a base temperature of 15°C

$$P_{\max} = K_1 + K_2 R \tag{7}$$

Finally, the British CIRIA Monograph 108 proposes that the pressure be calculated using Eq. (8), where the maximum lateral pressure exerted by the concrete ($P_{\max}$) is obtained by means of the density of the fresh concrete (D), the rate of placement of the concrete (R), the height of the formwork (H), and three coefficients that take into account the size and shape of the formwork (C_1), the composition of the mix (C_2), and the effect of the temperature of the concrete at placing (C_3)

$$P_{\max} = D[C_1 R^{0.5} + C_2 C_3 (H - C_1 R^{0.5})^{0.5}] \tag{8}$$

Fig. 3 shows the pressure distribution curves and the maximum values obtained from the model and the standards in the four studied formwork walls ($H_1 = 2.5$ m, $H_2 = 5$ m, $H_3 = 10$ m, and $H_4 = 15$ m).

The pressure exerted on the formwork wall when assuming a hydrostatic behavior for the concrete is greater than that returned by the different standards or the FEM and becomes even greater with the height of the formwork. This occurs because either the frictional effects between the fresh concrete and the formwork wall or the internal frictions of the fresh concrete are not considered by the hydrostatic hypothesis.

For formworks of up to 5 m in height, the pressures obtained by the standards are greater than those obtained by the FEM. However, in taller formworks (10 m or more) the maximum pressure values obtained with the proposed model are considerably greater than those obtained with the standards. These standards make the assumption that, after a certain formwork height is reached, the pressure remains constant; However, according to the pressure distribution obtained with the proposed method, this is not the case. For a formwork of height 10 m, standard ACI 347-04 provides a wider safety margin than its counterparts. However and despite the different variables contemplated by the different standards (such as the rate of placement of the concrete and the temperature, etc.), the maximum pressure predicted by the

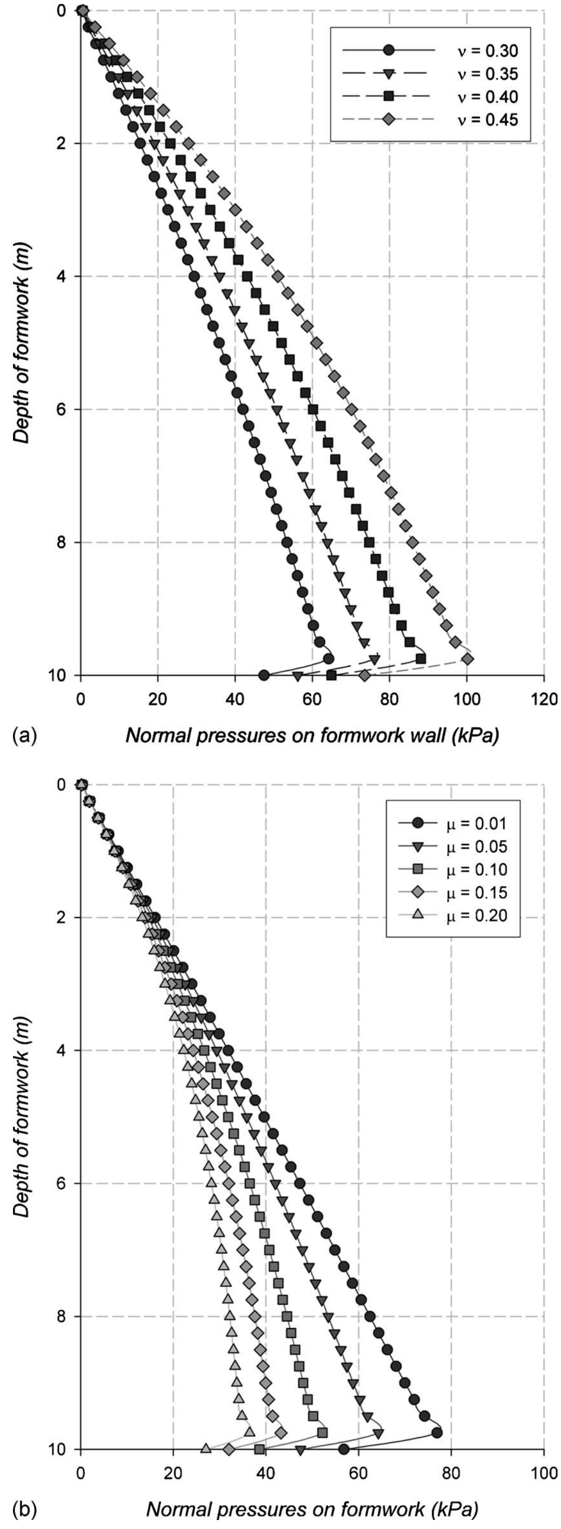

Fig. 5. Influence on lateral pressures exerted by fresh concrete due to (a) the Poisson coefficient; (b) the concrete-to-formwork wall friction coefficient

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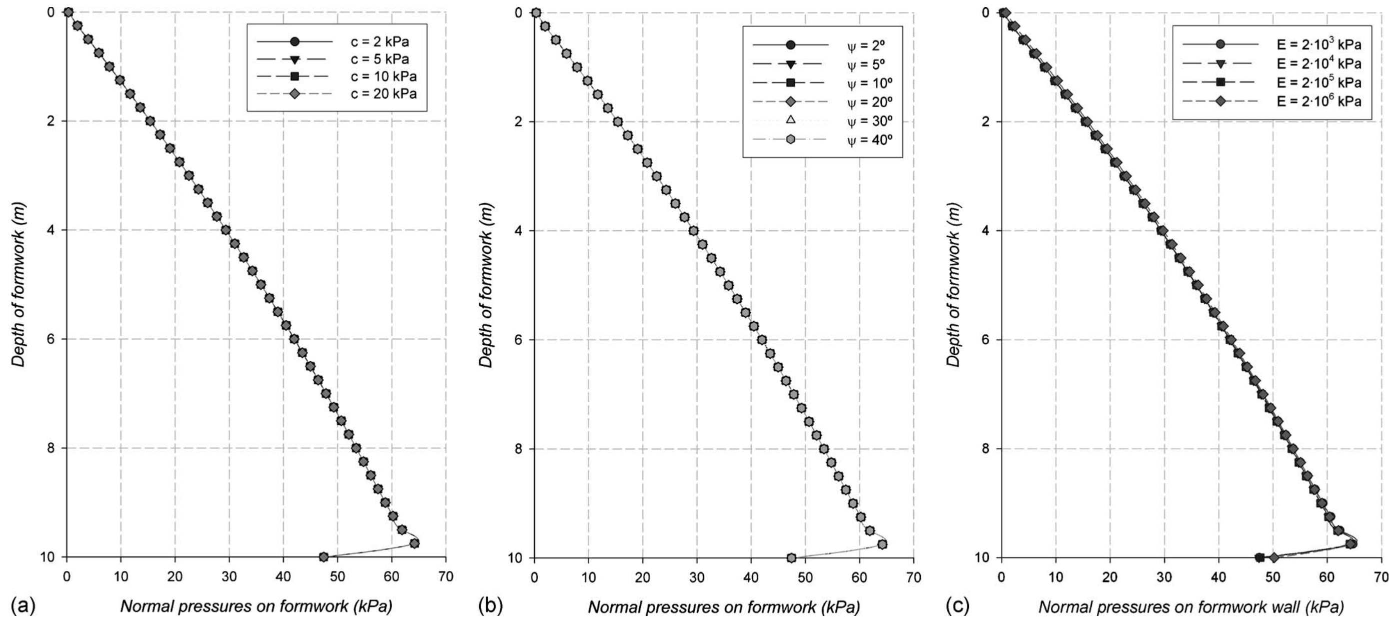

Fig. 6. Influence on lateral pressures exerted by fresh concrete due to (a) cohesion; (b) dilatancy angle; and (c) modulus of elasticity of the fresh concrete

proposed FEM was 37% higher than the highest estimate of any of the standards (71.0 kPa compared to 54.09 kPa).

For formworks greater than 15 m in height, the CIRIA standard calculated a maximum pressure slightly greater than that obtained with ACI 347-04. The maximum pressure determined by the proposed model, however, was 54% greater (85.59 kPa compared to 55.42 kPa).

International standards (DIN 18218, CIRIA 108, and ACI 347-04) propose simplified equations for the design of formworks and their predictions provide accurate results for regular formworks up to 10 m. All of these standards consider that a hydrostatic pattern of pressures appears in most part of the formwork wall. This assumption leads to an overestimation of lateral pressures, which is not very significant for reduced height formworks (Fig. 3).

However, standards consider that lateral pressures remain constant from a certain depth until the bottom in tall formworks (over 10 m). This hypothesis may lead to an underestimation of lateral pressures for high rates of placement of fresh concrete in tall formworks. On the other hand, the assumption of a hydrostatic pattern of pressures leads to a great overestimation of lateral pressures in tall formworks unless self-compacting concretes are used. The use of the proposed FEM allows obtaining a more accurate pattern of lateral pressures in formworks.

Parametric Analysis

The scarcity of information relating to the mechanical parameters necessary for the construction of the proposed model demanded a sensitivity analysis be performed to assess their influence on the distribution of the pressures obtained (Gallego et al. 2007). This was performed taking into account the 10-m-high formwork described above. The value of each variable shown in Table 1 was separately varied over the range shown in the same table while maintaining the selected values for the other variables constant. The results obtained are shown in Figs. 4–6.

The angle of internal friction (ϕ) seemed to have no influence on the distribution of pressures on the formwork walls until a height of 4.25 m was reached (Fig. 4). After this height, however, and when the angle of internal friction was $<20°$, the pressure increased as the angle of internal friction became smaller. The maximum pressure (95.30 kPa) corresponded to an angle of internal friction of 2°. For the most common types of concrete, the literature (Ritchie 1962; Olsen 1968; Alexandridis and Gardner 1981) offers angles of internal friction of $>20°$; in such cases, this variable would have no influence on the pressure distribution patterns obtained by the proposed model.

The Poisson coefficient (ν) was found to have a strong influence on the pressure exerted on the formwork walls, increasing it by up to 56% (64.2–100.2 kPa) as the coefficient increased from 0.30 to 0.45 (Fig. 5). The friction coefficient between the fresh concrete and the formwork wall (μ_c) also seems to exert a significant influence, especially at heights of over 2 m. Fig. 5 shows the increase in maximum pressure to be 111% (36.5–76.9 kPa) over the interval of values selected for this variable (0.01–0.2).

The cohesion of the fresh concrete (c), the dilatancy angle (ψ), and the modulus of elasticity for fresh concrete (E) appear to have no influence on the distribution of pressure, as determined by the proposed model, when within the intervals indicated in Table 1 (Fig. 6). The values adopted for the mechanical parameters that represent the behavior of the fresh concrete influence on the magnitude of lateral pressures predicted by the developed FEM (Figs. 4–6). The parametric study shows that for regular concretes without retarders or fluidizers only the Poisson ratio (ν), the friction coefficient between the fresh concrete and the formwork wall (μ_c), and the angle of internal friction (ϕ) influence on the lateral pressures exerted over the formwork wall. Experimental tests should be carried out for determining the values of these mechanical parameters for each specific project.

Summary and Conclusions

The present work shows the possibilities offered by numerical calculation methods in the determination of the pressure exerted

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Table 3. Influence of the Fresh-Concrete Mechanical Parameters on the Pressure Exerted on Formwork Walls

Parameter	E	ν	c	ϕ	ψ	μ_c
Influence	No	Yes	No	Yes[a]	No	Yes
Interval of variation of the maximum pressure (kPa)	65	65–100.2	65	65–95.3	65	36.5–76.9
Maximum increase (%)	0	56	0	48	0	111

[a]Influence only exerted when ϕ is greater than 20°.

by fresh concrete on formwork walls; the proposed method allows the study of the concrete/formwork system as a whole and can be used to simulate different situations.

Based on the results obtained with the proposed model, the following conclusions can be drawn:

1. The experimental formulas in international standards for formwork design are quite safe for formworks smaller than 5 m in height but may underestimate the pressures exerted by fresh concrete on the walls of larger formworks.
2. The use of numerical models requires knowledge of the values of different fresh concrete mechanical variables and of others describing the contact between the concrete and the formwork walls. The literature, however, contains only scant information in this regard.
3. The Poisson coefficient (ν) and the angle of internal friction (ϕ) were found to influence the pressures predicted by the proposed model, while the modulus of elasticity (E), the cohesion (c), and the dilatancy angle of the fresh concrete (ψ) had no influence over the range tested. Table 3 shows a summary of the influence of the different mechanical variables contemplated on the pressure distribution of fresh concrete on formwork walls.
4. The variable with the strongest influence on pressure distribution was the friction coefficient between the fresh concrete and the formwork wall (μ_c). The value of this variable depends, among other factors, on the material of the formwork wall and whether or not oils are used to facilitate the removal of the formwork; maximum pressure variations of over 111% may be seen.

Notation

The following symbols are used in this technical note:

C_c = coefficient related to the type of cement and additives used in the mix;
C_w = nondimensional correction factor referring to the specific weight of the fresh concrete;
C_1 = coefficient that takes into account the size and shape of the formwork;
C_2 = coefficient that takes into account the composition of the mix;
C_3 = coefficient that takes into account the effect of concrete temperature at placing;
c = cohesion;
D = density of the fresh concrete;
E = modulus of elasticity, in kPa;
$\mathbf{E}_{ij}$ = strain tensor;
F = yield function;
H = height of the formwork;
I_1 = first invariant of the stress tensor;
J_2 = second invariant of the deviatoric stress tensor;
K_1 = coefficient that depends on the consistency of the fresh concrete;
K_2 = coefficient that depends on the consistency of the fresh concrete;
k = fresh concrete-to-wall adherence;
P = normal pressure, in kPa;
$P_{\max}$ = maximum lateral pressure on the wall of the formwork, in kPa;
R = concrete rate of placement, in m/h;
T = temperature of the concrete during placement;
$\mathbf{T}_{ij}$ = stress tensor;
α = characteristic variable of the material;
κ = characteristic variable of the material;
λ = Lamé constant;
μ = Lamé constant;
μ_c = friction coefficient between the formwork and the fresh concrete;
ν = Poisson's coefficient;
τ = shear stress, in kPa;
ϕ = angle of internal friction; and
Ψ = dilatancy angle.

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