# A Fuzzy Decision Framework for Contractor Selection

D. Singh[1] and Robert L. K. Tiong[2]

**Abstract:** Contractor selection is the process of selecting the most appropriate contractor to deliver the project as specified so that the achievement of the best value for money is ensured. Construction clients are becoming more aware of the fact that selection of a contractor based on tender price alone is quite risky and may lead to the failure of the project in terms of time delay and poor quality standards. Evaluation of contractors based on multiple criteria is, therefore, becoming more popular. Contractor selection in a multicriteria environment is, in essence, largely dependent on the uncertainty inherent in the nature of construction projects and subjective judgment of decision makers (DMs). This paper presents a systematic procedure based on fuzzy set theory to evaluate the capability of a contractor to deliver the project as per the owner's requirements. The notion of Shapley value is used to determine the global value or relative importance of each criterion in accomplishing the overall objective of the decision-making process. The research reported upon forms part of a larger study that aims to develop a fuzzy decision model for construction contractor selection involving investigating multiple criteria selection tendencies of construction clients, relationship among decision criteria, and construction clients' preferences of criteria in the contractor selection process. An illustration with a bid evaluation exercise is presented to demonstrate the data requirements and the application of the method in selecting the most appropriate contractor for the project under uncertainty. The proposed model is not intended to supplant the work of decision-making teams in the contractor selection process, but rather to help them make quality evaluations of the available candidate contractors. One major advantage of the proposed method is that it makes the selection process more systematic and realistic as the use of fuzzy set theory allows the DMs to express their assessment of contractors' performance on decision criteria in linguistic terms rather than as crisp values.

**DOI:** 10.1061/(ASCE)0733-9364(2005)131:1(62)

**CE Database subject headings:** Contractors; Fuzzy sets; Evaluation; Decision making; Construction industry.



## Introduction

The construction industry is characterized by cost and duration overruns, serious problems in quality standards and safety measures, and an increased number of claims, counterclaims, and litigation. To minimize or optimize all these risks, selection of an appropriate contractor to deliver the project under consideration as per requirements is the most crucial challenge faced by any construction client. The construction industry is also one of the most dynamic, challenging, rewarding, full of uncertainty and associated risks, and these arise from the nature of the industry itself. Low entry barriers to the industry has also encouraged in many countries mushrooming of construction firms ranging from small-scale firms to large firms. The proliferation of these construction firms, along with the shrinking construction markets in developed and developing countries, has led to a fierce competition for the limited number of construction projects and they usually compete in a high volatile construction environment, full of uncertainties and associated risks. Furthermore, the peculiarity of construction is that no two projects are identical in terms of site conditions, design, use of construction materials, labors requirements, and plants and equipment requirements, construction methods, technical complexity, and level of management skill required. In such a situation, the crucial dilemma faced by all construction clients is which contractor to select for the job.

Owners in different private construction sectors practice different procedures for evaluating tender proposals. They mostly develop their own procedure for selecting the most appropriate contractor for the job. In public sectors, however, the tender price is the main criterion for selecting the contractor (Barrie and Paulson 1992), because clients are publicly accountable and must demonstrate that the best value for their money has been achieved (MPBW 1964; Merna and Smith 1990).

Hatush and Skitmore (1998) opined that the selection of the contractor based on the lowest tender price is one of the major reasons for project delivery problems, as contractors desperately quote low prices by reducing their quality of work and hope to be compensated by submitting claims. Fong and Choi (2000), after review of attitudes cited by researchers since 1967 concerning the influence of the tender price on the final selection of a contractor, summarized: (1) Apart from the acceptance of the lowest tender price, there should be a tradeoff between cost, time and quality in the final selection of contractor. (2) However, in public projects, tender price still dominates over other criteria in tender evaluation exercise. The previous works by Helmer and Taylor (1977), Samuelson and Levitt (1982), Moore (1985a, b), Holt et al. (1995), Kumaraswamy (1996), and Hatush and Skitmore (1997) show that despite a considerable increase in the complexity of projects and client's needs in the last 3 decades, along with an associated

[1]PhD Candidate, School of Civil and Environmental Engineering, Nanyang Technological Univ., Singapore 639798, Singapore.

[2]Associate Professor, School of Civil and Environmental Engineering, Nanyang Technological Univ., Singapore 639798, Singapore (corresponding author). E-mail: clktiong@ntu.edu.sg

Note. Discussion open until June 1, 2005. Separate discussions must be submitted for individual papers. To extend the closing date by one month, a written request must be filed with the ASCE Managing Editor. The manuscript for this paper was submitted for review and possible publication on March 13, 2003; approved on January 12, 2004. This paper is part of the ***Journal of Construction Engineering and Management***, Vol. 131, No. 1, January 1, 2005. ©ASCE, ISSN 0733-9364/2005/1-62–70/$25.00.

increase in alternative forms of project delivery system, contractor evaluation criteria themselves largely remain unchanged. These criteria can, in general, be summarized as tender cost, past performance, financial, technical, managerial, quality, and health and safety aspects.

According to Hipel et al. (1993), a decision problem is said to be complex and difficult, if there exist:

- multiple criteria—both qualitative and quantitative in nature;
- multiple decision makers;
- uncertainty and risk; and
- incomplete information, imprecise data, and vagueness surrounding the decision making

Contractor selection is, in practice, a complex multicriteria decision making (MCDM) problem in which multiple DMs evaluate the contractors' attributes to deliver the project at hand against a large number of the decision criteria. Russell and Skibniewski (1988) also pointed out that contractor selection is a decision-making process that involves the development and consideration of a wide range of necessary and sufficient decision criteria as well as the participation of many decision-making parties.

Construction researchers and practitioners have proposed different methods or procedures for contractor selection. To name a few of them: a multiattribute utility model by Diekmann (1981); a fuzzy bid evaluation model by Nguyen (1985); a statistic model by Jaselskis (1988); a dimensional weighing method by Jaselskis and Russell (1991); a performance assessment scoring system by Hong Kong Housing Authority (1994); and analytical hierarchy process (AHP) for contractor selection by Fong and Choi (2000). Most of these researchers have incorporated multicriteria decision analysis methods in their models. However, their models or methodologies are generally based on single principal decision criterion such as time, quality to evaluate the capabilities of the contractor in addition to the price criterion, and on the assumption that the decision is made by a single person rather than multiple decision makers or heavy reliance on historical data in the case of neural network models (Khosrowshahi 1999 and Lam et al. 2001). Mahdi et al. (2002) proposed a model based on AHP that considers the multicriteria approach to contractor selection. But his model also has some shortcomings that are associated with AHP method: (1) it does not take into account the uncertainty associated with the mapping of one's judgment to a number; (2) the subjective judgment and preferences of DMs have great influence on the final decision based on the AHP method; and (3) it is mainly used in nearly crisp decision applications and hence contractor selection is not a perfect case for its application. Pongpeng and Liston (2003) using utility theory also proposed a multicriteria model for tender evaluation. Their model also has some disadvantages: (1) it requires the DMs to give a crisp utility value of a particular criterion to be used in a utility function; and (2) it also does not take into account the uncertainty and risk associated with mapping of one's judgment to a crisp value.

To overcome all these shortcomings, the proposed contractor selection method employs the fuzzy set theory to deal with the uncertainty and vagueness surrounding the subjective nature of the decision making and multiple attributes decision method to cater to the simultaneous consideration of the multiple decision criteria and multiple decision makers. The expected marginal contribution of each of the decision criteria to the overall goal of decision making, that is, to select a contractor who is technically and financially sound enough to deliver the project as specified, is obtained by using the Shapley value formula (Shapley 1953). A hypothetical problem is analyzed to illustrate the data requirements, mechanics, and solution nature of the proposed method. The research reported in this paper forms part of a larger study that aims to design a computer-based fuzzy decision model for contractor selection. The computer model will take into account the incomplete and imprecise information on which the experts' opinions are formed, a more realistic assessment option that uses linguistic variables instead of numerical values to express the experts' opinions, possible difficulty of comparing two alternatives with different level of performance on different decision criteria, and the interaction among decision criteria in order to rank different alternatives on a balanced scale of judgment. The main purpose of this paper is to develop a valid theoretical framework for the future development of a computer-based fuzzy decision model for contractor selection.

## Concept of Fuzzy Set Theory and Tender Evaluation

Zadeh (1965) first introduced the concept of fuzzy set theory. A fuzzy set can be defined mathematically by assigning to each possible individual in the universe of discourse a value representing its grade of membership in the fuzzy set. This grade represents the degree to which that individual is similar or compatible with the concept represented by the fuzzy set. Thus, an individual may belong in the fuzzy set to a greater or lesser degree as indicated by a larger or smaller membership grade. These membership grades are very often represented by real-number values ranging in the closed interval between 0 and 1. The fuzzy set, therefore, introduces vagueness (with the aim of reducing complexity) by eliminating sharp boundary dividing members of the class from nonmembers since the transition of member from nonmember is gradual rather than abrupt (Klir and Folger 1988).

Fuzzy set theory has been used to deal with ill-defined and complex problems due to incomplete and imprecise information that characterized the real-world systems. It uses linguistic variables to model vagueness intrinsic to the human cognitive process. Zadeh (1973) stated that "….as the complexity of a system increases, human ability to make precise yet significant statement about its behaviors diminishes until a threshold is reached beyond which precision and significance become mutually exclusive." Fuzzy set theory does not replace probability theory but rather provides a solution to problems that lack the mathematical rigor required by probability theory (Nguyen 1985). Membership function, linguistic variables, natural language computation, linguistic approximation, fuzzy integrals, and fuzzy weighted sum are main concepts of fuzzy set theory applied to approximate characterization and decision making. A linguistic variable differs from numerical variable in that its values are not numbers but words or sentences in a natural or artificial language. Linguistic variables such as "poor management," "good performance," and "moderate risk" describe the vague concept. Interested readers are referred to Zadeh (1965), Schmucker (1984), and Klir and Folger (1988).

A fuzzy decision-making framework generally consists of the following steps:

1. defining and specifying the types of fuzzy numbers and their membership functions to be used by DMs;
2. establishing the scale of preference structure to be used by DMs;
3. assigning the fuzzy values to attributes based on their performance on the decision criteria;
4. aggregating fuzzy numbers across the DMs;
5. defuzzification;



J. Constr. Eng. Manage., 2005, 131(1): 62-70

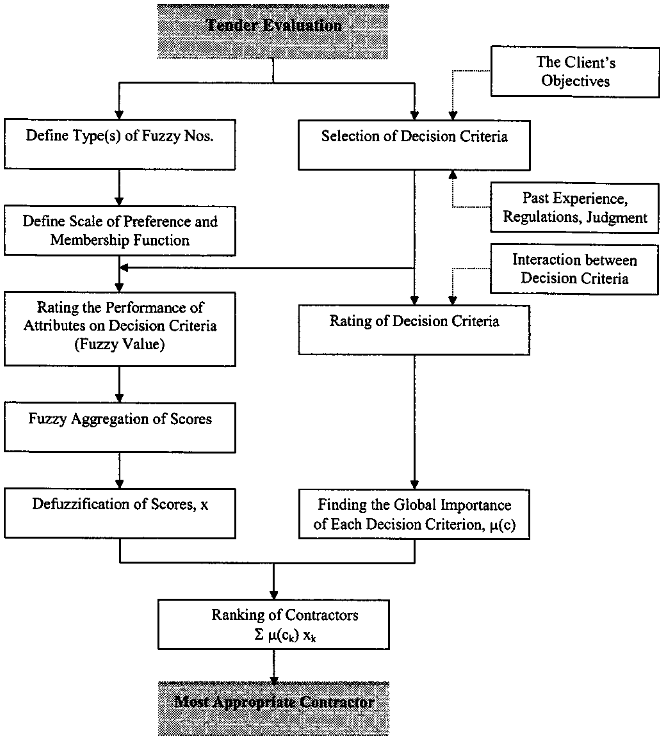


**Fig. 1.** Proposed fuzzy decision framework for contractor selection

6. determination of global importance or overall value of each of the decision criteria; and
7. ranking of alternatives.

Fig. 1 shows an overview of the fuzzy decision framework for contractor selection. For details about different types of fuzzy numbers, membership functions, aggregation, and defuzzification methods, interested readers are referred to Zimmerman (1985), Klir and Folger (1988), and Kaufmann and Gupta (1991).

### Fuzzy Membership Function

Membership function of an element represents a degree to which the element belongs to a set. Let $a_i$ be a fuzzy number such that $\forall a_i \in \mathbb{R}$ (set of real numbers) and considered in the form of

$$a_i = \{x_1, x_2, x_3, x_4\}, \text{ for } i = 1, 2, \ldots, m$$

where $x_1 < x_2 < x_3 < x_4$ = scale of preference structure to be used by DMs and $m$ = number of fuzzy number to be used in the analysis. Figs. 2 and 3 show the graphical representation of trapezoidal and triangular membership function $\mu(x)$, respectively.

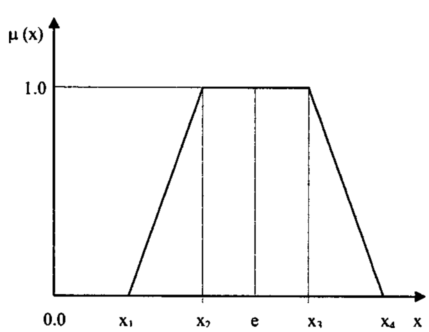


**Fig. 2.** Graphical representation of trapezoidal membership function

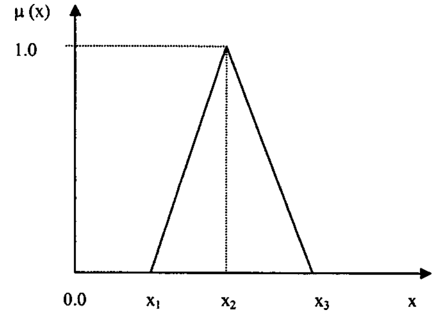


**Fig. 3.** Graphical representation of triangular membership function

The normalized trapezoidal membership function of an alternative $a_i$ can be expressed in the form of

$$\mu_{ai}(x) = \begin{cases} 0, & x < x_1 \\ (x - x_1)/(x_2 - x_1), & x_1 < x < x_2 \\ 1, & x_2 < x < x_3 \\ (x_4 - x)/(x_4 - x_3), & x_3 < x < x_4 \\ 0, & x > x_4 \end{cases}$$

### Operations on Fuzzy Numbers

Let $A = (a_1, a_2, a_3, a_4)$ and $B = (b_1, b_2, b_3, b_4)$ be any two positive trapezoidal fuzzy numbers. Then the operations $[+, -, \times, \div]$ are expressed as (Kaufmann and Gupta 1991)

$$A \oplus B = (a_1, a_2, a_3, a_4) \oplus (b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$$

$$A \odot B = (a_1, a_2, a_3, a_4) \ominus (b_1, b_2, b_3, b_4) = (a_1 - b_4, a_2 - b_3, a_3 - b_2, a_4 - b_1)$$

$$A \otimes B = (a_1, a_2, a_3, a_4) \otimes (b_1, b_2, b_3, b_4) = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$$

$$A \odot B = (a_1, a_2, a_3, a_4) \odot (b_1, b_2, b_3, b_4) = (a_1/b_4, a_2/b_3, a_3/b_2, a_4/b_1)$$

where $\oplus$, $\ominus$, $\otimes$, $\odot$ represent the fuzzy addition, subtraction, multiplication, and division, respectively. Let $a_{ij}^k$ be the fuzzy number (weight) assigned to an alternative $A_i$ by $\mathrm{DM}_j$ for the decision criterion $C_k$, then the average of fuzzy numbers across all the DMs can be expressed as

$$A_{ij}^k = (1/p) \otimes (a_{i1}^k \oplus a_{i2}^k \oplus \ldots \oplus a_{ip}^k) \text{ for } j = 1, 2, \ldots, p \quad (1)$$

where $p$ = numbers of DMs involved in the evaluation process.

### Defuzzification

Defuzzification is an operation that produces a nonfuzzy or crisp value that adequately represents the degree of satisfaction of the aggregated fuzzy number. In this paper, trapezoidal and triangular fuzzy numbers are used to represent the DMs' opinion. Let a trapezoidal fuzzy number be parameterized by $x_1, x_2, x_3, x_4$ as shown in Fig. 2, then its defuzzified value $e$ is given by the following equation (Kaufmann and Gupta 1991):

$$e = (x_1 + x_2 + x_3 + x_4)/4 \quad (2)$$

Similarly, for triangular fuzzy number as represented in Fig. 3



J. Constr. Eng. Manage., 2005, 131(1): 62-70

**Table 1.** Fuzzy Numbers for Linguistic Variables

| Linguistic variables | Fuzzy number |
|---|---|
| **VG/VI** (very good/important) | (0.8,0.9,1.0,1.0) |
| **G/I** (good/important) | (0.6,0.7,0.8,0.9) |
| **AA** (above average) | (0.5,0.6,0.7,0.8) |
| **A** (average) | (0.4,0.5,0.5,0.6) |
| **BA** (below average) | (0.2,0.3,0.4,0.5) |
| **P/LI** (poor/low important) | (0.1,0.2,0.3,0.4) |
| **VP/VLI** (very poor/very low important) | (0.0,0.0,0.1,0.2) |

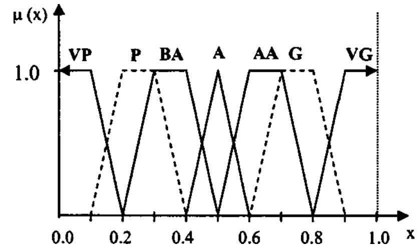


**Fig. 4.** Graphical representation of fuzzy numbers for linguistic variables

$$e = (x_1 + 2x_2 + x_3)/4 \tag{3}$$

For details about different types of fuzzy numbers, membership functions, aggregation, and defuzzification methods, interested readers are referred to Zimmerman (1985), Klir and Folger (1988), and Kaufmann and Gupta (1991).

### *Tender Evaluation*

Decision making in real-world situations such as tender evaluation is a complex subject shrouded in uncertainty and vagueness. Tender evaluation has traditionally been largely based on experience and subjective judgment of DMs. Therefore, the vague terms are unavoidable in such a situation since it is easier for DMs to express their opinions in terms of more realistic qualitative, linguistic terms, making the use of linguistic approximation more appropriate. Fuzzy set theory can, therefore, be used to model complex decision problems such as contractor selection. In this method, the performance of attributes on each criterion is introduced as a fuzzy number. This comes from the fact that in most cases, the input data cannot be defined with a reasonable degree of accuracy. Other parameters, expressing the opinion of DMs, such as the weighting factors, are considered as regular data with a precise numerical value. In other words, the assumption is made that the performances of attributes on criteria are fuzzy while the performances of the DMs are not.

## Multicriteria Decision Making and Tender Evaluation

A multicriteria decision-making problem can generally be represented in a matrix format as

$$D = \begin{array}{c c} & \begin{array}{cccc} c_1 & c_2 & \dots & c_n \end{array} \\ \begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_m \end{array} & \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{m1} & x_{n2} & \dots & x_{mn} \end{bmatrix} \end{array}$$

$$W = [w_1 \quad w_2 \quad \dots \quad w_n]$$

where $A_1, A_2, \dots, A_m$ = alternatives; $c_1, c_2, \dots, c_n$ = criteria with which performances of alternatives are measured; $x_{ij}$ = rating or score of alternative $Ai$ with respect to criterion $c_j$, and $w_j$ = weight of criterion $c_j$. The main purpose in the MCDM problem is to assess the overall importance values of the alternatives on some permissible scale. Alternatives are generally first evaluated explicitly with respect to each of the decision criteria to obtain some sort of criterion specific priority scores which are then aggregated into overall performance values. Selection of a construction contractor is also a decision-making process by the clients based on their previous experience, judgment, and a set of criteria which might vary between projects and clients. In practice, attributes are usually evaluated from different points of view which correspond to decision criteria. Moreover, in real-life situations, evaluations, based on past data and DMs' subjective judgment, are neither certain nor precise (Roy 1989).

In the contractor selection process, the capability of a contractor to deliver the project on time, within budget, and as specified is evaluated against a number of important decision criteria such as tender price, past performance, and performance potential. Therefore, contractor selection is, in essence, a MCDM problem involving human subjectivity and uncertainty. In fuzzy MCDM problems, the ranking of alternatives must take into account their fuzzy scores on all criteria, the weight assigned to each decision criterion, the possible difficulties of comparing two alternatives when one is significantly better than the other on a subset of criteria, but much worse on at least one criterion from the complementary subset of criteria, and the DMs' attitude towards the risk associated with evaluation. Therefore, the relationships among criteria are crucial for adequate treatment of fuzzy decision making, because they reflect the structure of interaction among the criteria and represent DMs' preferences of the criteria. Thus, the global importance of a particular criterion is not solely determined by the importance of that criterion, but also by the value of all other criteria considered in the evaluation process. Shapley (1953) proposed a method to determine the expected marginal contribution of a particular player (criterion) to the overall goal.

Let us consider a set of decision criteria as $C = \{c_1, c_2, \dots, c_n\}$ and $\mu$ a fuzzy measure on $C = \{c_1, c_2, \dots, c_n\}$ such that

$$\mu(C) = \sum_{i=1}^{n} \mu_i(c_i) \text{ for } i = 1,2, \dots, n$$

$$\mu(\Phi) = 0 \text{ and } \mu(C) = 1$$

where $\phi$ = null set and $\mu(c_i)$ = weight or the importance value of the criterion $c_i$. The importance index or Shapley value of criterion $c_i$ with respect to $\mu$ is defined as (Shapley 1953)

$$\mu(c_i) = \Sigma \frac{(N-A)!(A-1)!}{N!}[\mu(A) - \mu(A - c_i)] \tag{4}$$

where $N$ = number of decision criteria; $A$ = any combination of decision criteria containing criterion $c_i$, and $0! = 1$, as usual. Thus, in addition to the usual weights on criteria taken separately, weights on any combination of criteria must also be defined in multiple criteria decision-making problems.



J. Constr. Eng. Manage., 2005, 131(1): 62-70

## Illustrative Example

An example is designed in a hypothetical manner to illustrate the application of the proposed contractor selection method that involves the selection of the most appropriate contractor among four contractors (A, B, C, and D) with respect to three decision criteria, i.e., tender price ($c_1$), past performance ($c_2$), and performance potential ($c_3$) of the contractors based on the information supplied by the four decision makers $DM_1$, $DM_2$, $DM_3$, and $DM_4$. The past performance criterion *may* consist of a number of subcriteria such as:

1. cost and schedule overruns in the past projects;
2. compliance with specifications and quality standards;
3. attitude towards correcting faulty or incomplete works;
4. attitude towards claims and counterclaims;
5. relationship with past clients and/or subcontractors and suppliers;
6. past failure;
7. safety performance; and
8. scale and type of the projects executed.

The performance potential criterion *may* include:

1. financial soundness;
2. managerial capability;
3. available resources;
4. current workload;
5. technical competence;
6. past experience with the client; and
7. project specific criteria.These subcriteria *may* vary between projects and clients based on their past experience, regulations, and requirements for the project in hand and judgment.

**Table 2.** Ranking of Subcriteria by Decision Makers (DMs)

| Subcriterion | Performance potential ($c_3$) | | | |
|---|---|---|---|---|
| | $DM_1$ | $DM_2$ | $DM_3$ | $DM_4$ |
| $c_{31}$ | VI | VI | I | I |
| $c_{32}$ | I | I | I | VI |
| $c_{33}$ | AA | I | I | I |

Note: VI = very important; I = important; and AA = above average.

Table 1 shows the linguistic variables with their corresponding fuzzy numbers defined by the DMs to be used for the qualitative assessment of the performance of contractors' attributes on the decision criteria. For the proposed contractor selection system only seven fuzzy numbers are chosen to describe the level of performance on decision criteria because it is generally difficult for an expert to distinguish subjectively between more than seven alternatives (Saaty 1977). Fig. 4 shows the graphical representation of the fuzzy numbers for linguistic variables for the DMs to use in the assessment of contractor attributes.

### *Establishing Weights for Decision Criteria*

For simplicity, let us assume that the decision criterion $c_3$ (performance potential) consists of three subcriteria, that is: financial soundness ($c_{31}$), managerial capability ($c_{32}$), and technical competence ($c_{33}$). Table 2 shows the importance weights assigned to each of subcriteria by DMs. For example, VI means that subcriterion is "very important" in assessing the performance of contractors.

The fuzzy decision matrix for subcriteria can be written as

$$X_{C2} = \begin{bmatrix} (0.8,0.9,1.0,1.0) & (0.8,0.9,1.0,1.0) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) \\ (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) & (0.8,0.9,1.0,1.0) \\ (0.5,0.6,0.7,0.8) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) \end{bmatrix}$$

Using Eq. (1) for aggregating and averaging as explained in the previous section, the average fuzzy score matrix for each of the subcriteria is obtained as follows:

$$X_{C2} = \begin{bmatrix} (0.700,0.800,0.900,0.950) \\ (0.650,0.750,0.850,0.925) \\ (0.575,0.675,0.775,0.875) \end{bmatrix}$$

Using Eq. (2), the crisp scores (defuzzified values) for subcriteria are obtained as follows:

$$\text{criterion } c_{31} = (0.700 + 0.800 + 0.900 + 0.950)/4 = 0.8375$$

$$\text{criterion } c_{32} = (0.650 + 0.750 + 0.850 + 0.925)/4 = 0.7938$$

$$\text{criterion } c_{33} = (0.575 + 0.675 + 0.775 + 0.875)/4 = 0.7250$$

The normalized weight for each subcriterion is obtained by dividing scores of each $c_{3i}$ by $\Sigma c_{3i}$ as

$$w(c_{31}) = 0.35$$

$$w(c_{32}) = 0.34$$

$$w(c_{33}) = 0.31$$

Now, based on the weight established for subcriteria DMs are asked to assign the weight to the combinations of subcriteria, for example $w\ (c_{31},c_{32})$ represents the average weight assigned to the combination of subcriteria $c_1$ and $c_2$ ("financial soundness" and "managerial capability") and $w(\phi)$ is the weight to the null set of subcriteria. These three subcriteria, namely: financial soundness, managerial capability, and technical competence, do not compensate each other, that is, a bad score on one criterion cannot be adjusted by a good score on any other complementary criteria as the purpose of the tender evaluation is to select the optimum contractor among a group of tenderers. However, in practice, these subcriteria by and large corporate each other, i.e., an increase (or decrease) in the degree to which one criterion is satisfied often increases (or decreases) the degree to which another criterion is satisfied. Therefore, the weight assigned to the combination of any two criteria must be at least equal to the sum of their individual weight assigned separately (Shapley 1953), that is



J. Constr. Eng. Manage., 2005, 131(1): 62-70

**Table 3.** Weights Assigned by Decision Makers (DMs) to Different Combination of Subcriteria

| Combination of subcriteria | DM[1] | $DM_2$ | $DM_3$ | $DM_4$ |
|---|---|---|---|---|
| $w\ (c_{31}, c_{32})$ | 0.80 | 0.75 | 0.85 | 0.85 |
| $w\ (c_{31}, c_{33})$ | 0.70 | 0.80 | 0.80 | 0.75 |
| $w\ (c_{32}, c_{33})$ | 0.75 | 0.85 | 0.80 | 0.75 |

$$w(c_1,c_2) \geqslant w(c_1) + w(c_2) \tag{5}$$

For example, if the maximum score that can be assigned to a contractor with performances on all decision criteria close to their maxima, i.e., as per or close to the expectation of the DMs is 1, then what score they are willing to assign a contractor whose performances on any combination of criteria, say $c_{31}$ and $c_{32}$, are close to their maxima and very poor on the rest of the criteria, provided Eq. (5) is satisfied. The same process is repeated for all possible combinations of criteria to get the importance values the DMs are willing to assign to all possible combinations of decision criteria. This process is, in fact, to elicit the DMs' preferences of criteria in the selection process and the relationship among them. Let us assume that the weights assigned by DMs to different combinations of subcriteria are as presented in Table 3. The average weight of each combination of subcriteria across all DMs can be worked out as

$$w(c_{31},c_{32}) = (0.80 + 0.75 + 0.85 + 0.85)/4 = 0.8125$$

$$w(c_{31},c_{33}) = (0.70 + 0.80 + 0.80 + 0.75)/4 = 0.7625$$

$$w(c_{32},c_{33}) = (0.75 + 0.85 + 0.80 + 0.75)/4 = 0.7875$$

$$w(c_{31},c_{32},c_{33}) = 1.0$$

$$w(\phi) = 0$$

### Calculation of Shapley Values

Using Eq. (4), an important index or Shapley value for subcriteria can be calculated as follows: when $A$ is $c_{31}$ alone

$$\mu(c_{31})_1 = [\ \{w(c_{31}) - w(\phi)\}\{(3-1)!(1-1)!\}/3!\}]$$
$$= \{0.35 - 0.0\}\{2 \times 1\}/6 = 0.1167$$

when $A$ is the combination of $c_{31}$ and $c_{32}$

$$\mu(c_{31})_2 = [\ \{w(c_{31},c_{32}) - w(c_{32})\}\{(3-2)!(2-1)!\}/3!\}] = \{0.8125 - 0.34\}\{1 \times 1\}/6 = 0.0787$$

when $A$ is the combination of $c_{31}$ and $c_{33}$

$$\mu(c_{31})_3 = [\ \{w(c_{31},c_{33}) - w(c_{33})\}\{(3-2)!(2-1)!\}/3!\}] = \{0.7625 - 0.31\}\{1 \times 1\}/6 = 0.0754$$

when $A$ is the combination of $c_{31}$, $c_{32}$, and $c_{33}$

$$\mu(c_{31})_4 = [\ \{w(c_{31},c_{32},c_{33}) - w(c_{32},c_{33})\}\{(3-3)!(3-1)!\}/3!\}]$$
$$= \{1.0 - 0.7875\}\{1 \times 2\}/6 = 0.0708$$

Now, Shapley value for subcriterion $c_{31}$ can be obtained as

$$\mu(c_{31}) = \mu(c_{31})_1 + \mu(c_{31})_2 + \mu(c_{31})_3 + \mu(c_{31})_4$$
$$= 0.1167 + 0.0787 + 0.0754 + 0.0708 = 0.3416$$

Similarly, for subcriteria $c_{32}$ and $c_{33}$, Shapley value $\mu\ (c_{32})$ and $\mu\ (c_{33})$ can be calculated as 0.3492 and 0.3091.

### Rating of Performance of Contractors

Now, contractors' performances on each of the subcriteria are to be rated by DMs. Table 4 shows the linguistic assessment of the contractors' performances on the subcriterion ($c_{31}$) "financial soundness."

The fuzzy decision matrix for subcriterion $c_{31}$ can be written as

$$X_{C31} = \begin{bmatrix} (0.6,0.7,0.8,0.9) & (0.8,0.9,1.0,1.0) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) \\ (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) & (0.5,0.6,0.7,0.8) \\ (0.8,0.9,1.0,1.0) & (0.8,0.9,1.0,1.0) & (0.6,0.7,0.8,0.9) & (0.6,0.7,0.8,0.9) \\ (0.4,0.5,0.5,0.6) & (0.4,0.5,0.5,0.6) & (0.5,0.6,0.7,0.8) & (0.5,0.6,0.7,0.8) \end{bmatrix}$$

Using Eq. (1) for aggregating and averaging, the average fuzzy score matrix for the contractors is obtained as follows:

$$X_{C31} = \begin{bmatrix} (0.650,0.750,0.850,0.925) \\ (0.575,0.675,0.775,0.875) \\ (0.700,0.800,0.900,0.950) \\ (0.450,0.550,0.600,0.700) \end{bmatrix}$$

The crisp scores on subcriterion $c_{31}$ for the contractors are obtained using Eq. (2) as follows:

$$\text{contractor } A = (0.650 + 0.750 + 0.850 + 0.925)/4 = 0.794$$

$$\text{contractor } B = (0.700 + 0.800 + 0.900 + 0.950)/4 = 0.725$$

$$\text{contractor } C = (0.375 + 0.450 + 0.525 + 0.625)/4 = 0.838$$

$$\text{contractor } D = (0.450 + 0.550 + 0.600 + 0.700)/4 = 0.575$$

Similarly, scores for different contractors against criterion $c_{32}$ and $c_{33}$ are obtained and provided in Table 5.

Using the simple additive weighting method (Hwang and Yoon 1981), the total score (TS) for each contractor can be calculated as follows:

$$\text{TS} = \sum x_k \,.\, \mu(c_k) \quad \text{for } k = 1,2,3 \tag{6}$$

where $\mu\ (c_k)$ = weight (importance index or calculated Shapley value) of subcriterion $k$, and $x_k$ = aggregated score of the contrac-



J. Constr. Eng. Manage., 2005, 131(1): 62-70

**Table 4.** Linguistic Assessment of Contractors' Performance on Subcriterion $c_{31}$

| Contractor | Financial soundness ($c_{31}$) | | | |
|---|---|---|---|---|
| | $DM_1$ | $DM_2$ | $DM_3$ | $DM_4$ |
| A | G | VG | G | G |
| B | G | G | G | AA |
| C | VG | VG | G | G |
| D | A | A | AA | AA |

Note: DM=decision maker; G=good; VG=very good; A=average; and AA=above average.

tor against the subcriterion $k$ as shown in Table 5. In matrix form, they can be represented as

$$TS = \begin{array}{c} \begin{array}{cccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{array} \\ \begin{bmatrix} 0.794 & 0.725 & 0.838 & 0.575 \\ 0.857 & 0.894 & 0.598 & 0.565 \\ 0.879 & 0.864 & 0.725 & 0.625 \end{bmatrix} \end{array} \begin{bmatrix} 0.3016 \\ 0.3492 \\ 0.3091 \end{bmatrix} \begin{array}{l} \mathbf{c}_{31} \\ \mathbf{c}_{32} \\ \mathbf{c}_{33} \end{array}$$

Total scores for contractor A on "performance potential" criterion can be calculated as $(0.794\times0.3016+0.857\times0.3492+0.879\times0.3091)$ 0.810. Hence, total scores for contractors on criterion "performance potential" are

$$\text{contractor A} = 0.810$$

$$\text{contractor B} = 0.798$$

$$\text{contractor C} = 0.686$$

$$\text{contractor D} = 0.564$$

For this illustrative evaluation problem, let us assume that the four contractors quoted the following prices in their tender proposals:

$$\text{contractor A} = \$21{,}250$$

$$\text{contractor B} = \$22{,}780$$

$$\text{contractor C} = \$23{,}160$$

$$\text{contractor D} = \$19{,}500$$

And base tender price for the project is, say, \$19,580. Base tender price is the amount which, in the opinion of DMs is economically low enough to deliver the project without compromising the quality standards and health and safety aspects of the facility or end product. Table 6 shows the scores for different contractors against each of three decision criteria. Scores for tender price (row 2) are obtained by dividing the base tender price by the proposed tender price. For example, the contractor A offers a tender price of \$21,250 and his score against the tender price criterion is obtained as $19{,}580\div21{,}250=0.921$. Scores in row 3 and 4 represent the average defuzzified value scored on criteria $c_2$ and $c_3$ which are obtained as illustrated in the previous section.

**Table 5.** Rating of Contractors' Performance on Subcriteria

| Subcriteria | Contractor | | | |
|---|---|---|---|---|
| | A | B | C | D |
| Financial soundness ($c_{31}$) | 0.794 | 0.725 | 0.838 | 0.575 |
| Management capability ($c_{32}$) | 0.857 | 0.894 | 0.598 | 0.565 |
| Technical competence ($c_{33}$) | 0.879 | 0.864 | 0.725 | 0.625 |

**Table 6.** Score Matrix for Contractors

| Criteria | Contractor | | | |
|---|---|---|---|---|
| | A | B | C | D |
| $c_1$ (tender price) | 0.921 | 0.860 | 0.845 | 1.004 |
| $c_2$ (past performance) | 0.725 | 0.836 | 0.706 | 0.656 |
| $c_3$ (performance potential) | 0.810 | 0.798 | 0.686 | 0.564 |

Let us assume that the average weight assigned by DMs to each of the decision criteria and combinations of them for this evaluation process are as follows:

$$w(c_1) = 0.4$$

$$w(c_2) = 0.3$$

$$w(c_3) = 0.3$$

$$w(c_1,c_2) = 0.7$$

$$w(c_1,c_3) = 0.85$$

$$w(c_2,c_3) = 0.9$$

$$w(c_1,c_2,c_3) = 1.0$$

$$w(\phi) = 0$$

The importance index or the Shapley value of each decision criterion calculated using Eq. (4) are as shown in Table 7. These values indicate the expected marginal contribution or global importance of each criterion to the overall goal. It is convenient to scale these values by a factor $N$ (in the example $N=3$), so that an importance index greater than 1 indicates a criterion more important than the average. From the table, it is clear that the DMs attach more weight to "performance potential," as the scaled Shapley value (Shapley value$\times$3) is greater than 1 for $c_3$, than any other criterion in the selection process, that is, the attitude of the DMs is such that they are likely to accept a contractor with good performance potential and high tender price than a contractor with low tender price and poor performance potential, with past performance being considered equal.

The aggregated scores for contractor against decision criteria can be represented in the matrix form as

$$TS = \begin{array}{c} \begin{array}{cccc} \mathbf{A} & \mathbf{B} & \mathbf{C} & \mathbf{D} \end{array} \\ \begin{bmatrix} 0.921 & 0.860 & 0.845 & 1.004 \\ 0.725 & 0.836 & 0.706 & 0.656 \\ 0.810 & 0.798 & 0.688 & 0.564 \end{bmatrix} \end{array} \cdot \begin{bmatrix} 0.325 \\ 0.300 \\ 0.375 \end{bmatrix} \begin{array}{l} \mathbf{c}_{1} \\ \mathbf{c}_{2} \\ \mathbf{c}_{3} \end{array}$$

**Table 7.** Importance Index for Decision Criteria

| Criteria | Tender price ($c_1$) | Past performance ($c_2$) | Performance potential ($c_3$) |
|---|---|---|---|
| Shapley value, $\mu(c)$ | 0.325 | 0.300 | 0.375 |
| Scaled shapley value $[\mu(c)\times3]$ | 0.975 | 0.900 | 1.125 |



J. Constr. Eng. Manage., 2005, 131(1): 62-70

**Table 8.** Overall Performance Scores and Final Ranking of Contractors

| Contractor | A | B | C | D |
|---|---|---|---|---|
| Overall score | 0.8206 | 0.8296 | 0.7444 | 0.7346 |
| Rank | 2 | 1 | 3 | 4 |

Using Eq. (6), the overall score for each contractor can be calculated. For example, the overall score for contractor A is $(0.921\times0.325+0.725\times0.300+0.810\times0.375)0.8206$. Hence, the overall performance score for contractors and their ranking are as shown in Table 8.

The purpose of contractor selection exercise is to choose the contractor whose performance attributes on all or more important decision criteria are closer to their maxima. It is evident from the example that the contractor B, which offers the second highest tender price, scores approximately better than other contractors on past performance and performance potential criteria and is ranked first in preference order when all criteria are considered simultaneously. On the other hand, even though contractor D offers the lowest tender price it is ranked only the last in preference order because of his low scores on other decision criteria.

## Conclusion

The success level of any construction project may well be argued to depend significantly on the basic philosophy of "the right contractor for the right project." Therefore, the selection of the most appropriate contractor for the project under consideration is a crucial challenge faced by every construction client to derive the best value for the money. In this paper, the authors present a fuzzy decision framework for contractor selection in a multicriteria environment. The proposed method allows DMs to express their opinions about the performances of attributes on decision criteria in the more realistic manner as the use of fuzzy set theory facilitates assessment to be made in qualitative and linguistic or approximate terms which better correspond to real-world situations. In developing the framework, it is assumed that the performances of attributes on decision criteria are fuzzy, whereas the performances of the decision makers are not. However, the proposed system also facilitates DMs to express their opinions using linguistic terms. The interaction among the decision criteria is taken into consideration for adequate treatment of fuzzy decision making. In order to avoid the overestimation of the importance of a particular criterion, the global importance of the criterion is not solely determined by the importance of that criterion, but also by the value of all other criteria considered in the evaluation process. For this purpose, the marginal contribution of each of the decision criteria to the overall goal is determined by calculating the Shapley value of each criterion. Using the concept of the Shapley value, the method presented produces such an evaluation that reflects the relationships among the decision criteria and DMs' preferences and concerns in the decision-making process.

In an actual contractor selection process, in addition to tender price, a large number of decision criteria and subcriteria need to be considered simultaneously and in most cases the DMs are less reluctant to handle the uncertainty associated with decision making directly in the scores of performance on particular criteria by using approximate values than by using crisp values and this makes the use of a fuzzy linguistic variable for the proposed contractor selection system more appropriate. Therefore, the use of the proposed method, even though it is no panacea for all troubles of decision making regarding the contractor selection process, will assist the construction clients in performing more realistic, linguistic assessment of the contractors to select "the right contractor for the right project" so that the risk to the client of project failure due to the selection of an inappropriate contractor is reduced and more efficient utilization of resources by all parties associated with the selection process is ensured. However, it is recommended that the final ranking of the contractors by the proposed method be simply used as a guide for viewing the relative differences and similarities between candidate contractors and the final decision should be made on engineering judgment of the clients or their representatives considering all the uncertainty and associated risks in the decision process.

The challenges in using the proposed decision framework would be defining and specifying the types of fuzzy numbers for linguistic variables and establishing the scale of preference structure to be used by DMs. When there are many stakeholders with different interests in the outcomes of the project, it would be more difficult and complicated to establish the preference of scale structure as each of them may have different ideas about the importance of decision criteria and how they should be evaluated making group decision making much more complicated and fuzzier. One of the simple and effective ways to address this issue is use of the fuzzy Delphi method (Kaufmann and Gupta 1988) to achieve a group consensus. In this method, a number of industry experts are first asked to express their opinions about the fuzzy numbers for linguistic variables and scale of preference structure for them and results are then aggregated. These aggregated results are sent back to them so that they can change their opinions based on the aggregated results. This process continues until a level of general agreement is achieved. The major disadvantage of the proposed method is that the exhaustive establishment of weights for different combinations of criteria, if there are many, requires consistency and is time consuming.

Since, each construction project has its distinct characteristics and requirements future work will address the need of incorporating into the proposed framework a facility that permits the users to add or remove the decision criteria/subcriteria and to modify weighting factors according to the specific requirements of the project under consideration, and an explanation facility capable of providing reasoning why a particular contractor is being selected. Furthermore, an attempt would also be made to incorporate a more comprehensive and rigorous methodology capable of indicating the changes in the financial health of the company, how the management is trying to improve the financial stability of the company, and how well a company is performing relative to its competitors in the industry.

## References


Barrie, D., and Paulson, B. (1992). *Professional construction management*, 3rd Ed., McGraw-Hill, New York.

Diekmann, J. E. (1981). "Cost-plus contractor selection: A case study." *J. Technical Councils ASCE*, 107(TC1), 13–25.

Fong, P. S.-W., and Choi, S. K.-Y. (2000). "Final contractor selection using analytical hierarchy process." *Constr. Manage. Econom.*, 18, 547–557.

Hatush, Z., and Skitmore, M. (1997). "Criteria for contractor selection." *Constr. Manage. Econom.*, 15, 19–38.

Hatush, Z., and Skitmore, M. (1998). "Contractor selection using multicriteria utility theory: An additive model." *Build. Environ.*, 33(2), 148–164.

Helmer, T. F., and Taylor, R. L. (1977). "The evaluation of contractor





J. Constr. Eng. Manage., 2005, 131(1): 62-70

management during source selection." *Proc., AIIE Spring Annual Conf.*, Dept. of Economics, Geography and Management, USAF Academy, Color.

Hipel, K. W., Radford, K. J., and Fang, L. (1993). "Multiple participant-multiple criteria decision making." *IEEE Trans. Syst. Man Cybern.* 23, 1184–1189.

Holt, G. D., Olomolaiye, P. O., and Harris, F. C. (1995). "Applying multi-attribute analysis to contractor selection decisions." *Eur. J. Purch. Supply Manage.*, 1(3), 139–148.

Hong Kong Housing Authority. (1994). "PASS: Performance assessment scoring system." Housing Dept., Hong Kong.

Hwang, C.-L., and Yoon, K. (1981). *Multiple attribute decision making: methods and applications*, Springer, New York.

Jaselskis, E. J. (1988). "Achieving construction projects through predictive discrete choice model." PhD thesis, Univ. of Texas at Austin, Austin, Tex.

Jaselskisl, E. J., and Russell, J. S. (1991). "An efficient structured approach for selection of most promising construction contractors." *Proj. Manage. J.*, 12(4), 31–39.

Kaufmann, A., and Gupta, M. M. (1988). *Fuzzy mathematical models in engineering and management science*, North-Holland, Amsterdam, The Netherlands.

Kaufmann, A., and Gupta, M. M. (1991). *Introduction to fuzzy arithmetic theory and application*, Van Nostrand Reinhold, New York.

Khosrowshahi, F. (1999). "Neural network model for contractor's prequalification for local authority projects." *Eng., Constr., Archit. Manage.*, 6(3), 315–328.

Klir, G. J., and Folger, T. A. (1988). *Fuzzy sets, uncertainty and information*, Prentice-Hall, Englewood Cliffs, N.J.

Kumaraswamy, M. M. (1996). "Contractor evaluation and selection: A Hong Kong perceptive." *Build. Environ.*, 31(3), 273–282.

Lam, K. C., Hu, T., Ng, S. T., Skitmore, M., and Cheung, S. O. (2001). "A fuzzy neural network approach for contractor prequalification." *Constr. Manage. Econom.*, 19, 175–188.

Mahdi, I. M., Riley, M. J., Fereig, S. M., and Alex, A. P. (2002). "A multi-criteria approach to contractor selection." *Eng., Constr., Archit. Manage.*, 9(1), 29–37.

Merna, A., and Smith, N. J. (1990). "Bid evaluation for the U.K public sector construction contracts." *Proc., Inst. of Civil Engineers Part I: Design and Construction*, Vol. 88, Institution of Civil Engineers, London, 91–105.

Ministry of Public Building and Works (MPBW). (1964). "The placing and management of contractor for building and civil engineering works." *Banwell Rep.*, Her Majesty's Stationery Office, London.

Moore, M. J. (1985a). "Selecting a contractor for fast track projects. Part 1: Principles of contractor evaluation." *Plant Eng.*, 39(12), 74–75.

Moore, M. J. (1985b). "Selecting a contractor for fast track projects. Part 2: Quantitative evaluation method." *Plant Eng.*, 39(18), 54–56.

Nguyen, V. U. (1985). "Tender evaluation by fuzzy sets." *Constr. Eng. Manage.*, 111(3), 231–243.

Pongpeng, J., and Liston, J. (2003). "TenSeM: A multicriteria and multidecision-makers' model in tender evaluation." *Constr. Manage. Econom.*, 21(1), 21–30.

Roy, B. (1989). "Main sources of inaccurate determination, uncertainty and imprecision in decision models." *Math. Comput. Modell.*, 12, 1245–1254.

Russell, J. S., and Skibniewski, M. J. (1988). "Decision criteria in contractor prequalification." *J. Manage. Eng.*, 4(2), 148–164.

Saaty, T. L. (1977). "A scaling method for priorities in hierarchical structures." *J. Math. Psychol.*, 15(1), 234–281.

Samelson, N. M., and Levitt, R. E. (1982). "Owner's guidelines for selecting safe contracts." *J. Constr. Div., Am. Soc. Civ. Eng.*, 108(4), 617–623.

Schmucker, K. J. (1984). *Fuzzy sets, natural language computation, and risk analysis*, Computer Science Press, New York.

Shapley, L. S. (1953). "A value for n-person games." *Contribution to the theory of games*, Vol. II, H. W. Kuhn and A. W. Tucker, eds., *Annals of mathematics studies*, Princeton University Press, Princeton, N.J., 307–317.

Zadeh, L. A. (1965). "Fuzzy sets." *Inf. Control.*, 8, 338–353.

Zadeh, L. A. (1973). "The concept of a linguistic variable and its application to approximate reasoning." *ERL-M 411*, Berkeley, Calif.

Zimmerman, H.-J. (1985). *Fuzzy set theory—and its application*, Kluwer-Nijhoff, Hingham, Mass.



J. Constr. Eng. Manage., 2005, 131(1): 62-70