

# Method to Determine Minimum Contract Bid for $A+B+I/D$ Highway Projects

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**Abstract:** More and more state highway agencies (SHAs) have begun to consider the value of time in highway construction. The “ $A$  (cost) +  $B$  (time cost) +  $I/D$  (incentive/disincentive)” bidding concept is designed to shorten the total contract time by allowing each contractor to bid the number of days in which the work can be accomplished, in addition to the traditional cost bid.  $I/D$  are not only used to provide an incentive to the contractor for earlier completion, but also to provide a disincentive for late completion of a project. Contractors are then presented with the problem of determining the best strategy of bid estimation, including construction cost, time cost, and incentive/disincentive. SHAs are also faced with the problem of placing a maximum and/or minimum on the time bid. To provide users a useful tool to estimate project time more accurately using this advanced method, this study develops a quantified model of the price-time bidding contract. The functional relationship between the construction cost and time duration is developed based on data from the Florida Department of Transportation (FDOT). The contractor’s construction cost “ $A$ ” is then combined with the road user cost and incentive/disincentive to determine the optimum low bid price and time. This optimum can then be used by the SHA to set limits on the range of acceptable time bids. Finally, several projects completed by the FDOT will be used to illustrate the validity of this model.

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## Introduction

The  $A$  (cost) +  $B$  (time cost) +  $I/D$  (incentive/disincentive) bidding concept is designed to shorten the total contract time by allowing each contractor to bid the number of days in which the work can be accomplished. This method of bidding will allow the contractor with the best combination of price and time to win the bid. Thus, the price portion of the bid does not become the only consideration for award. The project is awarded to the contractor with the lowest sum of  $A+B$ .

This cost-plus-time method of bidding enables the contractor to determine a reasonable contract duration required for completion of the project, thus allowing the contractor to control the important element of time. In most cases, it is the contractor who can best determine the most reasonable amount of time necessary to complete the project. This technique encourages a contractor to capitalize on a particular construction method or process that may speed up the construction.

Using the  $A+B$  method, a dollar value for each contract day is established by the state highway agency (SHA) prior to advertisement of the project. The number of days bid by the contractor is multiplied by the daily road user cost (DRUC) established by the SHA to determine the value of the contractor’s time bid. The time bid is added to the “traditional” contract work bid to obtain the  $A+B$  bid for award purposes. The bid is rejected if the contractor’s time bid exceeds the SHA’s maximum number of days noted in the specifications. The work is to be completed within the time bid by the contractor, and there is an incentive/disincentive for early or late completion.

The  $A$  portion of the bid consists of the total of all quantities multiplied by the associated unit price quoted by each contractor. The  $B$  portion consists of the preestablished dollar value for each contract day multiplied by the number of contract days quoted by each contractor. The number of contract days provided by the contractor is a required part of the bid.

To further reduce the actual time used by the contractor, an incentive/disincentive clause is added, with the amount usually being equal to the DRUC amount. The contractor will receive an incentive for each day that work is completed ahead of the contractor’s original contract time bid. Conversely, if the contractor completes the project late, the disincentive will be assessed as well as liquidated damages as per the contract. For purposes of the disincentive, the contract time may be adjusted for weather, unforeseen conditions, and extra work. The incentive/disincentive clause is used when a project is time critical and cost beneficial.

This technique is best used on projects where shortened contract time is critical. The objective of  $A+B+I/D$  is to reduce the inconvenience to the traveling public, to minimize disturbances to adjacent business and property owners, and to reduce safety risks that evolve from road construction and the detouring of existing traffic. Urban reconstruction projects with high average daily traffic and bridge projects are generally good candidates for this

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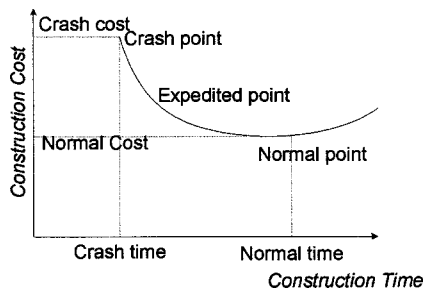


Fig. 1. Interrelationship of construction time and cost

method. The  $A + B + I/D$  bidding focuses the contractor on completing the project in a timely manner (FDOT 1997, 1998).

In the “ $A + B + I/D$ ” contract type, there are three parameters that the contractor needs to consider. Those parameters are the construction cost ( $A$ ), the contract time ( $B$ ), and the incentive/disincentive ( $I/D$ ). To win the bid, bidders need to make an estimate as low as possible. If bidders make the duration estimate and know their functional equations of construction cost versus duration, bidders can consider decreasing the duration in order to win the bid. Use of the incentive to compensate bidders for the extra cost due to shortening the project duration is an adoptable strategy. This study will help contractors determine the optimum combination of cost and time to bid and maximize the chance of being awarded the bid.

## Formulation of Model

### Interrelationship of Construction Cost and Construction Time

Construction cost and time for undertaking a specific construction project are interrelated. Literature on construction project scheduling (e.g., Callahan et al. 1992) has reported that, for a specific construction company, there is an optimum cost-time balancing point for every construction contract. At this point, the contractor would have the lowest construction cost. In general, the interrelationship between cost and time for a construction project is expressed as the curve shown in Fig. 1 (Cusack 1991). On this curve, the “normal point” represents the construction plan where construction cost (normal cost) is the lowest at a specific construction time (normal time). Theoretically, any variation in time from the normal point, regardless of magnitude, will result in a corresponding increase in construction cost. For example, to shorten construction time will increase project direct cost due to the use of multiple shifts, overtime work, or other costly measures. Crowded work crews or excessive equipment on site will make job supervision more difficult and is likely to result in lower work productivity. Material delivery in a shorter time is normally more expensive. On the other hand, an increase in the construction duration from the normal point will obviously incur an increase in general indirect cost (Shen et al. 1999).

### Model Creation—Process and Solution

Users of this model first need to develop a functional relationship between construction cost and duration and determine the normal point of the project-construction time ( $D_0$ ) and construction cost ( $C_0$ ).

Owners determine the time value rate and  $I/D$  rate. The time

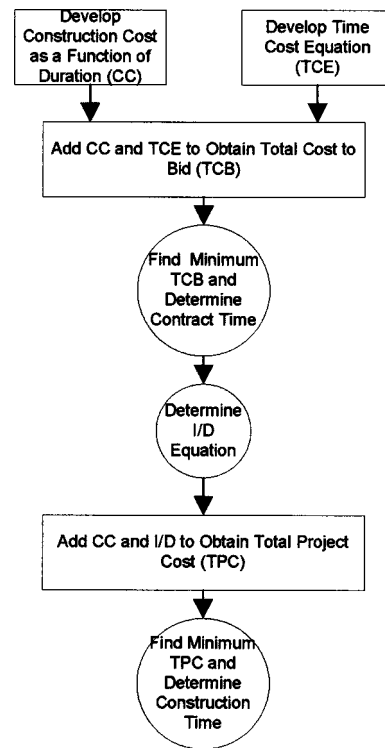


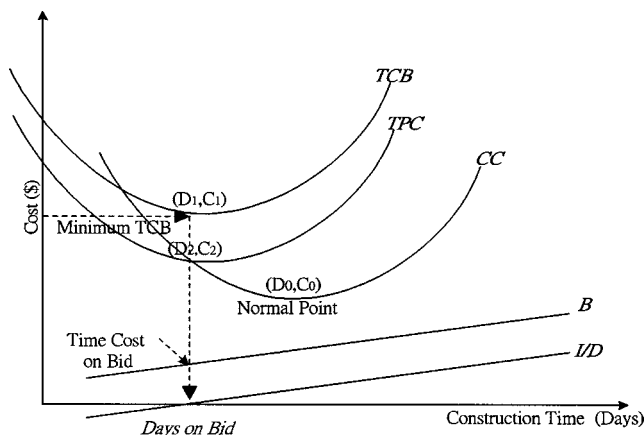
Fig. 2. Process to determine bidder's best strategy on “ $A + B + I/D$ ”

value rate is not necessarily equal to the  $I/D$  rate. Later we will use three kinds of  $I/D$  rate with different slopes to illustrate the difference. The project data analyzed here are from the Florida Department of Transportation (FDOT) and their time value rate is equal to the  $I/D$  rate.

Using the  $A + B + I/D$  method, total cost to bid for contractors is a function of construction cost and time cost. An optimization method can be used to find the minimum bid price and contract time. Once the contractor wins the bid, the contract time is the time contractor used to bid. The  $I/D$  equation is given by the owner. For the contractor to complete the project, what is the best strategy under the consideration of  $I/D$ ? The  $I/D$  and the construction cost can be combined and considered as a function of duration. An optimization method can then be used to find the construction time at the minimum total project cost. This construction time from the optimization will be the best completion time for that project (Fig. 2).

The solution process is illustrated in Fig. 2 and described as follows:

- Step 1: Contractor develops the construction cost as a function of duration ( $CC$ ) and determines the normal point—the lowest construction cost ( $C_0$ ) and the time ( $D_0$ ) in which the project can be finished from the minimum point of the  $CC$  curve in Fig. 3.
- Step 2: From the SHA, a time cost equation ( $B$ ) is developed.
- Step 3: Add  $CC$  and  $B$  together to form the total cost to bid (TCB).
- Step 4: Use the optimization method to find the construction time ( $D_1$ ) at the minimum TCB ( $C_1$ ).  $C_1$  will be the total bid to submit and  $D_1$  will be the contract time.
- Step 5: Using the contract time  $D_1$  and the  $I/D$  rate from the SHA, the  $I/D$  equation is determined.
- Step 6: Add  $I/D$  and  $CC$  together to form the total project cost (TPC) to complete the project.



**Fig. 3.** Solution to determine bidder's best strategy on "A+B + I/D;" time value rate = I/D rate

- Step 7: Use an optimization method to find the construction time ( $D_2$ ) at the minimum TPC ( $C_2$ ). The construction time ( $C_2$ ) will be the time in which the contractor tries to complete that project.

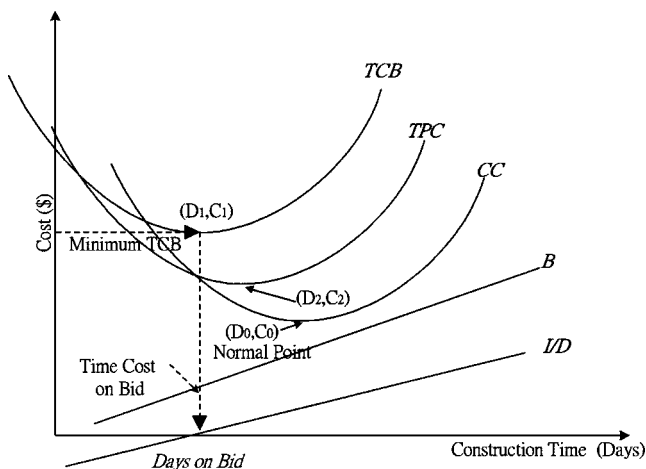
In the FDOT project data analyzed in this paper, the time value rate and  $I/D$  rate are the same. The contract time ( $D_1$ ) and the construction time ( $D_2$ ) will be equal (Fig. 3). If the time value rate is larger than the  $I/D$  rate, the construction time ( $D_2$ ) will be larger than the contract time ( $D_1$ ) (Fig. 4). If the time value rate is less than the  $I/D$  rate, the construction time ( $D_2$ ) will be less than the contract time ( $D_1$ ) (Fig. 5). If the time cost ( $B$ ) increases faster than the  $I/D$ , and these two curves are added to the same  $CC$  curve, then point ( $D_1, C_1$ ) will shift down and to the right ( $D_2, C_2$ ), as illustrated by Fig. 5.

## Model Formulation for Florida Projects

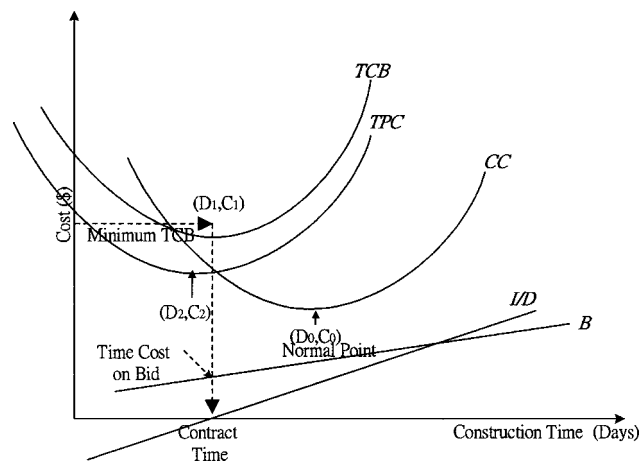
### Process and Equations to Determine Contract Time and Bid Price

#### Construction Cost as Function of Duration

The Florida legislature passed the innovative contracting statute F.S. 337.025, the time-plus-money contracting statute F.S.



**Fig. 4.** Solution to determine bidder's best strategy on "A+B + I/D;" time value rate > I/D rate



**Fig. 5.** Solution to determine bidder's best strategy on "A+B + I/D;" time value rate < I/D rate

337.11(04), and the incentive/disincentive statute F.S. 337.18(4), along with a modified design/build statute, F.F. 337.11(7), in response to the increasing demands on Florida highways by an increasing number of road users. The alternative contracting techniques that the Florida Department of Transportation has used during the past two years have been applied in a variety of ways, either as a single method or in combination with other methods. The following statistical information is from the projects that were awarded by the Florida Department of Transportation using alternative contracting during fiscal years 1996–1997 and 1997–1998. The data was acquired from 07/01/96 to 04/16/99 (FDOT 1997, 1998). This research only considers those projects 100% complete before 04/16/99 (Tables 1–3).

#### Regression Analysis

The projects examined for construction cost as a function of duration are described in Tables 1–3. These projects from the FDOT were 100% completed before 04/16/99. Six projects in Table 1 used the A+B method, four projects in Table 2 used the incentive/disincentive method, and five projects in Table 3 adopted the no excuse bonus method. From those tables, four columns of data were extracted for a regression analysis. These were "award bid," "present construction cost," "present contract time," and "days used." The award bid is the construction cost that the contractor bids. The present construction cost is the final construction cost. The present contract time is the final contract time that is adjusted for the weather or additional work. The days used is the number of days that the contractor actually used. Because each project scope is different, the regression used the following data: (present construction cost – award bid)/(award bid) and (days used – present contract time)/(present contract time) (Table 4).

After a regression analysis (Table 5), the following equation was developed:

$$\left(\frac{C - C_0}{C_0}\right) = 0.03214 - 0.10481\left(\frac{D - D_0}{D_0}\right) + 0.46572\left(\frac{D - D_0}{D_0}\right)^2 \quad (1)$$

where  $C$  = present construction cost;  $D$  = days used;  $C_0$  = award bid;  $D_0$  = present contract time; and  $R$ -square = 0.7507.

When estimating the bid, the contractor can use construction cost estimate as  $C_0$  and construction time estimate as  $D_0$ . Because the function of construction cost versus duration was based

**Table 1.** Florida Department of Transportation Project Progress Report—A + B Projects [Begin Date: 19960701; End Date: 19990416]

Lead WP item number	Start date	Work mix	Official contract estimate (K)	Award bid (K)	Present construction cost (K)	Bid days	Present contract time	Days used	Maximum days	Dollars per day	I/D paid (K)
2106231	7/17/97	Replace	9,213	9,424	9,718	300	381	311	650	6,000	234.9
2108971	3/1/98	Widen	3,359	3,101	3,151	101	162	145	—	2,694	43.1
2179021	2/3/97	Replace	15,378	14,325	14,612	429	468	460	739	2,200	30.8
2501641	10/2/98	Miscellan	1,775	1,551	1,601	199	199	142	—	2,000	100
2570171	11/1/97	Resurface	3,119	2,945	2,991	120	142	135	135	5,000	0
2570601	7/13/98	Resurface	1,432	1,700	1,800	150	160	97	—	3,000	0

on the data from the FDOT that includes different projects from different contractors from 1996 to 1999, it can reflect a general interrelationship between construction cost and duration. If clients need a more accurate function, he/she can use his/her internal projects to conduct a regression analysis. When bidding, contractors need to consider other contractors' estimates in the  $CC$  equation, which has been generated from projects of varying sizes and complexities. Taking into consideration bids from other contractors can result in a more accurate estimate.

### Curve Shifting

Eq. (1) illustrates the interrelationship between the change in construction cost and the change in construction time. After  $C_0$  and  $D_0$  are decided, the curve is determined. If the contractor's estimate is accurate, the construction cost and construction time should be at the normal point (Fig. 1). This means that the contractor has devised the most efficient balance of time and cost possible. To shift curve (1) to match the normal point ( $D_0, C_0$ ) (Fig. 6), some mathematical adjustment is needed. The scale of the curve is not changed, but the lowest point of the curve moves to the normal point. After the adjustment (Appendix), the equation for curve (2) in Fig. 6 is

$$C = 1.0059C_0 - 0.1048C_0 \left( \frac{D - 0.8875D_0}{D_0} \right) + 0.4657C_0 \left( \frac{D - 0.8875D_0}{D_0} \right)^2 \quad (2)$$

On curve (2), the "normal point" represents the construction plan where construction cost (normal cost  $C_0$ ) is the lowest with a specific construction time (normal time  $D_0$ ).

To explain Eq. (2) more clearly, there are three steps to adopting this equation:

- Step 1: User determines the construction cost and construction time first. This set of cost and time will create an Eq. (1) that illustrates the interrelationship between construction cost and construction time.

- Step 2: In Eq. (1), the minimum point of the curve does not match the normal point ( $D_0, C_0$ ) determined by the contractor's estimate. After shifting of Eq. (1), Eq. (2) is generated.
- Step 3: From Eq. (2), every construction time has a corresponding construction cost. To make Eq. (2) adoptable for projects in Table 6, the terms used in Eq. (2) are revised as follows:

$$C = 1.0059C_0 - 0.1048C_0 \left( \frac{D - 0.8875D_0}{D_0} \right) + 0.4657C_0 \left( \frac{D - 0.8875D_0}{D_0} \right)^2 \quad (3)$$

where  $C$  = construction cost;  $D$  = construction time;  $C_0$  = construction cost at normal point (column 6, Table 6); and  $D_0$  = construction time estimate at normal point (column 4, Table 6).

Construction cost ( $C$ ) and construction time ( $D$ ) are dependent and independent variables, respectively; and  $C_0$  and  $D_0$  are determined by the users.

### Differences in Bid Cost versus Actual Cost

The data from FDOT does not include the additional cost of the project (actual cost above the bid cost). According to the FDOT report (FDOT 1997, 1998), the increase from award bid to present contract cost is due to contingency supplement agreements and supplement agreements that are based on factors such as plan modification and changed conditions. The increase is 1–5% of the present construction cost.

Another reason is that the bid is based on unit price and not on the lump sum. Thus, the real construction cost and quantity can not be known until the project is finished. The quantity of each item between award bid and present construction cost may be different.

### Time Cost Equation

In determining the total combined bid, time is usually related to cost in terms of a unit time value (UTV). Herbsman et al. (1995)

**Table 2.** Florida Department of Transportation Project Progress Report—Incentive/Disincentive Projects [Begin Date: 19960701; End Date: 19990416]

Lead WP item number	Start date	Work mix	Official contract estimate (K)	Award bid (K)	Present construction cost (K)	Official days	Present contract time	Days used	I/D paid (K)
1955781	4/13/98	Resurface	2,598	2,991	2,972	200	233	301	0
2374531	11/17/97	Add lane	3,356	3,437	3,534	245	327	297	162
2426331	8/6/97	Resurface	13,764	14,136	14,617	440	575	515	475
2586381	8/28/98	Resurface	328	273	290	120	120	81	10



**Table 3.** Florida Department of Transportation Project Progress Report—No Excuse Bonus Projects [Begin Date: 19960701; End Date: 19990416]

Lead WP item number	Start date	Work mix	Official contract estimate (K)	Award bid (K)	Present construction cost (K)	Official days	Present contract time	Days used	I/D paid (K)
2007041	7/7/97	Bridge	1,285	1,172	1,305	185	185	84	100
2408431	3/4/97	Add lane	4,169	4,333	4,415	340	401	401	300
2512401	8/25/97	Add lane	6,676	4,220	4,300	400	400	397	300
2512801	1/29/98	Add lane	4,243	3,177	3,323	400	400	266	400
2570741	5/26/98	Resurface	1,210	1,280	1,330	175	192	172	0

considered the UTV as representing the cost of delays to the owner and suggested that this is made up of both direct costs (e.g., increased use of temporary facilities and increased moving costs) and indirect costs (e.g., losses of business opportunity and reduction of potential profits). The highway construction industry refers to UTV as the “daily road-user cost” and points out that no standard computational procedures have been developed for determining the value of UTV (Shen et al. 1999). From Munzer (1998), the road-user cost includes vehicle-operating cost and time delay cost.

For each project, the UTV is determined by the SHA. The UTV discussed in this paper is a fixed fee for each day. The total time cost ( $C$ ) in Eq. (4) is equal to the UTV multiplied by the construction time ( $D$ ). It follows that

$$C = (\text{UTV}) \times D \quad (4)$$

where UTV=unit time value (cost/day); and  $D$ =construction time.

#### Minimize Total Cost to Bid

Adding Eqs. (3) and (4), the total cost to bid is obtained as follows:

$$C = 1.0059C_0 - 0.1048C_0 \left( \frac{D - 0.8875D_0}{D_0} \right) + 0.4657C_0 \left( \frac{D - 0.8875D_0}{D_0} \right)^2 + \text{UTV} \times D \quad (5)$$

**Table 4.** Data for Regression Analysis

FM number	(Days used – present contract time)/ present contract time	(Present construction cost – award bid)/ award bid
210623	–0.1837	0.0312
210897	–0.1049	0.0161
217902	–0.0171	0.0200
250164	–0.2864	0.0322
257017	–0.0493	0.0156
257060	–0.3938	0.0588
229622	0.0133	0.0683
237453	–0.0917	0.0282
242633	–0.1043	0.0340
258638	–0.3250	0.0623
200704	–0.5459	0.1135
240843	0.0000	0.0189
251240	–0.0075	0.0190
251280	–0.3350	0.0460
257074	–0.1042	0.0391

After solving for  $D$ , the minimum  $C$  occurs when

$$D = D_0 - 1.0736 \left( \frac{(\text{UTV})D_0^2}{C_0} \right) = D_1 \quad (6)$$

For the contractor, the best bid days for  $B$  is  $D$  and this  $D$  is assigned as  $D_1$ . Any time that is less than  $D_1$  will cause more cost to bid. The minimum  $C$  ( $C_1$ ) will be the total bid to submit and  $D_1$  will be the contract time.

#### Process and Equations to Determine Construction Time

The construction cost as a function of duration is the same as Eq. (3).

#### I/D Equation

The  $I/D$  rate (column 15, Table 6) is determined by FDOT. The anticipated incentive/disincentive (AID) is equal to the linear  $I/D$  rate ( $ID$ ; column 15, Table 6) multiplied by the difference in the contract time ( $D_1$ ) and contractor's construction time ( $D$ ). Because the anticipated incentive will reduce the total cost to bid, the anticipated incentive will be negative here. It follows that

$$\text{AID} = (ID) \times (D - D_1) \quad (7)$$

where AID=anticipated incentive/disincentive;  $ID$ =incentive/disincentive rate (column 15, Table 6);  $D_1$ =contract time to bid [Eq. (6)]; and  $D$ =construction time.

#### Minimize Contractor's Total Project Cost

Adding Eqs. (3) and (7), the total project cost is obtained as follows:

$$C = 1.0059C_0 - 0.1048C_0 \left( \frac{D - 0.8875D_0}{D_0} \right) + 0.4657C_0 \left( \frac{D - 0.8875D_0}{D_0} \right)^2 + (ID) \times (D - D_1) \quad (8)$$

where  $C$ =construction cost;  $D$ =construction time;  $C_0$ =construction cost at normal point (column 6, Table 6);  $D_0$ =construction time estimate at normal point (column 4, Table 6);

$$D_1 = D_0 - 1.0736 \left( \frac{(\text{UTV})D_0^2}{C_0} \right)$$

$ID$ =function of the incentive/disincentive (column 15, Table 6); and UTV=unit time value (column 5, Table 6).

After substitution, the minimum  $C$  occurs when

$$D = D_0 - 1.0736 \left[ \frac{(ID)D_0^2}{C_0} \right] \quad (9)$$

**Table 5.** Regression Analysis Report: Dependent Variable  $\text{Cost} = (\text{Present Construction Cost} - \text{Award Bid}) / \text{Award Bid}$ ; Independent Variable  $\text{Day} = (\text{Days Used} - \text{Present Days}) / \text{Present Days}$

Source	DF	Sum of squares	Mean square	F value	Pr > F	Parameter	Estimate	T	Pr >  T	SE
Model	2	0.00739	0.00370	18.07	0.0002	Intercept	0.03214	5.06	0.0003	0.00635
Error	12	0.00246	0.00020			Day	0.10481	1.47	0.1681	0.07144
Corrected total	14	0.00985				Day · day	0.46572	3.23	0.0072	0.14407

Note:  $R$ -square = 0.75071;  $CV$  = 35.56890; root  $MSE$  = 0.01431; Cost mean = 0.04022;  $CV$  = root  $MSE$  / cost mean. Since model  $P$  value 0.0002 is quite small, equation is adaptable.  $R$ -square 0.75071 means 75% of total variation about mean cost explained by regression.

For the contractor, the  $D$  at the minimum  $C$  is the best number of days to finish the project and this  $D$  is assigned as  $D_2$ . Any time that is less than  $D_2$  will cause more total project cost. The minimum  $C$  ( $C_2$ ) will be the lowest total project cost, and  $D_2$  will be the anticipated project completion time.

## Results of Model Analysis

### Project FM 217902

In this project, the FDOT's estimate was higher than the contractors on "A + B." Contractor B, the next to the lowest bid, used the FDOT's time estimate of 739 days (column 4). If Contractor B used the model to adjust the original estimates, which were 739 days (column 4) and \$13,724,000 for the construction cost (column 6), the new estimates would be 645 days (column 9) and \$13,827,000 (column 11). The new "A + B" cost is \$15,246,000 (column 13), which is lower than the lowest bid, \$15,268,000 from Contractor A (column 8). The "days used" (column 14) was 460 days, which is less than the anticipated project completion time computed by our model, which is 645 days (column 10) from Contractor B. From Table 6, it is apparent that Contractor B did not make its own time estimate. It appears that Contractor B focused only on the construction cost estimate and used the Florida DOT's time estimate.

### Project FM 229629

In this project, the "A" (column 6) for Contractor D was lower than the one from the lowest bid, Contractor C, but the time estimate (column 4) was higher. After adjustment, Contractor D's contract time computed by our model (column 9) and "A" adjusted by our model are 289 days and \$3,837,000, respectively. The "A + B" adjusted by our model (column 13) from Contractor D is \$4,617,000, which is less than Contractor C's "A + B" of

\$4,683,000. Compared with Contractor E's time estimation of 300 days, 289 days seems feasible for the contractor to complete the project.

### Project FM 238320

In this project, Contractor G adopted the FDOT's time estimate, 485 days. If Contractor G used the model to adjust the time and cost, the contract time by model is 354 days (column 9) and the "A" adjusted by model (column 11) is \$6,991,000. The "A + B" adjusted by model from Contractor G is \$8,230,000 (column 13), which is less than the lowest "A + B," \$8,247,000, from Contractor F (column 8).

Sixty-five days of incentive were earned by Contractor F. This number is generated using column 16 divided by column 15. For Contractor F: (contract time + extended days) – days used = days of incentive; (385 + extended days) – 372 = 65; extended days = 78 days. Therefore, without extra work, Contractor F can complete the project in (days used – extra work) = (372 – 65) = 307 days. Contractor G's anticipated project completion time by model is 354 days, which is greater than 307 days. Thus, this duration is feasible.

### Project FM 210897

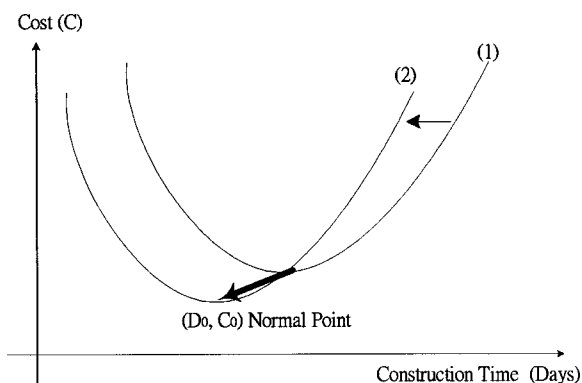
In this project, Contractor H finished the project in 145 days and earned 16 days of incentive. For Contractor H: (contract time + extended days) – days used = days of incentive; (101 + extended days) – 145 = 16; extended days = 60 days. Without extra work, the project could be finished in (days used – extra work) = (145 – 60) = 85 days. After adjustment, Contractor I's anticipated project completion time by model is 87 days. Compared with 85 days, this duration is feasible.

### Project FM 242692

In this project, Contractor J's time and construction cost are all far lower than Contractor K's. After adjustment, Contractor K's anticipated project completion time is 245 days. Compared with Contractor J's 214 days, Contractor K's anticipated project completion time is feasible.

### Project FM 257017

After adjustment, Contractor M's "A + B" adjusted by model (column 13) is very close to the lowest bid, Contractor L's "A + B" (column 8). For Contractor L: (contract time + extended days) – days used = days of incentive; (120 + extended days) – 135 = 7; extended days = 22 days. Without extra work, the project could be finished in (days used – extra work) = (135 – 22) = 113 days. After adjustment, Con-



**Fig. 6.** Equation shift for construction cost as function of duration

**Table 6.** Analysis Result

FM number	Contractor name	Status	Time estimate ( $D_0$ )	Unit time value (K)	"A" (K) ( $C_0$ )	"B" (K)	"A + B" (K)	Contract time by model (days) ( $D_1$ )	Anticipated project completion time by model ( $D_2$ )	"A" adjusted by model (K)	"B" adjusted by model (K)	"A + B" adjusted by model (K) ( $C_1$ )	Days used	I/D rate (K)	I/D paid (K)
217902	FDOT maximum days	DOT's maximum	739	2.2	15,377	1,625	17,003							2.2	
	Contractor A	Lowest bid	429	2.2	14,324	943	15,268						460	2.2	0
	Contractor B	Next to lowest bid	739	2.2	13,724	1,625	15,350	645	645	13,827	1,419	15,246		2.2	
229629	FDOT maximum days	DOT's maximum	485	2.7	3,849	1,340	5,189							2.7	
	Contractor C	Lowest bid	385	2.7	3,643	1,040	4,683						369	2.7	0
	Contractor D	Next to lowest bid	455	2.7	3,613	1,229	4,842	289	289	3,837	780	4,617		2.7	
	Contractor E		300	2.7	4,125	810	4,935							2.7	
238320	FDOT maximum days	DOT's maximum	485	3.5	7,534	1,330	8,864							3.5	
	Contractor F	Lowest bid	385	3.5	6,900	1,437	8,247						372	3.5	227.5
	Contractor G	Next to lowest bid	485	3.5	6,761	1,698	8,459	354	354	6,991	1,239	8,230		3.5	
210897	FDOT maximum days	DOT's maximum	180	2.694	3,359	350	3,709							2.694	
	Contractor H	Lowest bid	101	2.694	3,101	272	3,373						145	2.694	43 (16 days)
	Contractor I	Next to lowest bid	95	2.694	3,155	256	3,411	87	87	3,165	234	3,400		2.694	
242692	FDOT maximum days	DOT's maximum	350	3	5,883	825	6,708							3	
	Contractor J	Lowest bid	214	3	4,938	642	5,580						342	3	0
	Contractor K	Next to lowest bid	300	3	5,291	900	6,191		245	5,374	735	6,109			
257017	FDOT maximum days	DOT's maximum	135	5	3,119	675	3,794							5	
	Contractor L	Lowest bid	120	5	2,945	600	3,545						135	5	35
	Contractor M	Next to lowest bid	135	5	2,959	675	3,634		102	3,041	510	3,551			
257060	FDOT maximum days	DOT's maximum	165	3	1,432	495	1,927							3	
	Contractor N	Lowest bid	150	3	1,700	450	2,150						97	3	150 (50 days)
	Contractor M	Next to lowest bid	130	3	1,937	390	2,327		102	1,979	306	2,285		3	

tractor M's anticipated project completion time by model (column 10) is 102 days. Compared with 113 days, this duration is feasible.

### Project FM 257060

After adjustment, Contractor M's "A+B" adjusted by model (column 13) is lower than the original "A+B" (column 8). For Contractor N: (contract time+extended days)−days used=days of incentive; (150+extended days)−97=50; extended days=−3 days. Without extra work, the project could be finished in (days used−extra work)=(97−(−3))=100 days. After adjustment, Contractor M's anticipated project completion time by model (column 10) is 102 days. Compared with 100 days, this duration is feasible.

Based on the projects studied, using data from FDOT, the model presented here appears to be a useful tool for determining the optimal project duration to bid for an A+B+I/D type of highway construction project.

### Conclusions

A+B+I/D contracting is gradually being adopted by more and more SHAs. This research offers a quantifying model and analysis process to contractors or estimators. Using this model, a best bid strategy including the lowest A+B, the contract time, and the project completion time can be determined. The functional relationship between the construction cost and time duration is developed based on data from the Florida Department of Transportation (FDOT). The contractor's construction cost "A" is then combined with the road-user cost and incentive/disincentive to determine the optimum low bid price and time. The Florida Department of Transportation's projects were used to illustrate the validity of this model.

From this model, SHAs also can adjust their UTV and I/D to motivate contractors more to accelerate the schedule. The I/D rate for those FDOT projects analyzed here are all equal to the unit time value (Fig. 3). If the FDOT can raise the I/D rate higher than the UTV (Fig. 5), contractors will have more motivation to shorten the construction time.

### Appendix. Equation Shift for Construction Cost as Function of Duration

$$\left[ \frac{C - C_0}{C_0} \right] = 0.03214 - 0.10481 \left[ \frac{D - D_0}{D_0} \right] + 0.46572 \left[ \frac{D - D_0}{D_0} \right]^2$$

$$C = 1.03214C_0 + 0.10481C_0 \left( \frac{D - D_0}{D_0} \right) + 0.46572C_0 \left( \frac{D - D_0}{D_0} \right)^2$$

$$\frac{\partial C}{\partial D} = 0.10481 \left( \frac{C_0}{D} \right) + 0.93144C_0 \left( \frac{D - D_0}{D_0^2} \right) = 0$$

$$D = 0.887475D_0$$

$$C = 1.026246C_0$$

Minimum C happens at (0.887475D<sub>0</sub>, 1.026246C<sub>0</sub>).  
Distance from (D<sub>0</sub>, C<sub>0</sub>) to (0.887475D<sub>0</sub>, 1.026246C<sub>0</sub>)  
= (−0.11252D<sub>0</sub>, 0.026256C<sub>0</sub>).  
Shift Eq. (1) with distance from (0.887475D<sub>0</sub>, 1.026246C<sub>0</sub>) to (D<sub>0</sub>, C<sub>0</sub>):

$$C + 1.026256C_0 = 1.03214C_0$$

$$- 0.10481C_0 \left( \frac{D - 0.11252D_0 - D_0}{D_0} \right)$$

$$+ 0.46572C_0 \left( \frac{D - 0.11252D_0 - D_0}{D_0} \right)^2$$

$$C = 1.0059C_0 - 0.1048C_0 \left( \frac{D - 0.8875D_0}{D_0} \right)$$

$$+ 0.4657C_0 \left( \frac{D - 0.8875D_0}{D_0} \right)^2$$

### Notation

The following symbols are used in this paper:

- B = function of time cost;
- C = cost; construction cost;
- CC = function of construction cost;
- C<sub>0</sub> = construction cost estimate at normal point;
- C<sub>1</sub> = minimum construction cost from function of total cost to bid (TCB);
- D = days used, construction time;
- D<sub>0</sub> = construction time estimate at normal point;
- D<sub>1</sub> = contract time to bid correspond to C<sub>1</sub>; and
- I/D = function of incentive/disincentive.

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