Learning Curves: Accuracy in Predicting Future Performance

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Abstract: Many repetitive construction field operations exhibit a phenomenon known as the learning or experience effect. A learning curve is generated when the time or cost required to complete one cycle of an activity is plotted as a function of the cycle number. For practicing construction engineers and managers, the greatest potential value of learning curves lies in their ability to predict future performance, instead of fitting historical data. This paper presents a new method for using learning curves to predict the time or cost to complete the remaining cycles of an activity in progress, to assess the accuracy of this method, and to compare the accuracy of this method with the standard forecasting technique used in construction cost reporting. Using the proposed method, the accuracy of predicting the time or cost required to complete an ongoing activity improves dramatically for about the first 25–30% of the activity and then levels off to within 15–20% of the actual value. Compared to the standard method using the cumulative average, the new learning curve method is shown to be more accurate. The analysis quantifies the trade-off between accuracy of predicting future performance and the timeliness and potential value of such a prediction.

INTRODUCTION

Many repetitive construction activities exhibit a learning or experience effect, where the time or cost required to perform one cycle is less than that required to perform the previous cycle. A learning curve is generated when the time or cost required to complete one cycle is plotted as a function of the cycle number. Learning curve data can be presented in several terms such as man-hours/cycle, dollars/cycle, minutes/cycle, etc. For the remainder of this paper, cost/cycle will be used as the unit of measure, but other units could be used as well. Several factors contribute to the decreasing activity cost with increasing number of repetitions: greater familiarity with the task, better coordination, more effective use of tools and methods, and more attention from management and supervision (Oglesby et al. 1989).

Most research in learning curve theory has focused on developing mathematical models that describe the cost/cycle as a function of the cycle number and fitting the models to historical data. Well-known mathematical models include the linear model (Wright 1936), Stanford B model (*An Improved* 1949), cubic model (Carlson 1973), exponential model (''Effect'' 1965), and piecewise model (Thomas et al. 1986). Detailed descriptions of these models and their abilities to fit historical data can be found in Thomas et al. (1986) and Everett and Farghal (1994).

For practicing construction engineers and managers, the real potential value of learning curves lies not in documentation or fitting of historical data, but in prediction of future performance. Everett and Farghal (1994) described a method to evaluate a learning curve model's ability to predict the cost required to perform future cycles based on completed cycles.

To assess a learning curve model's ability to predict future performance, the following methodology was developed by Everett and Farghal (1994). Data from a construction activity with n cycles is collected. For each learning curve model, the equation of the least-squares best-fit curve for the first $m = n/2$ observations is determined. The learning curve is extrapolated according to the best-fit equation beyond m, out to the nth or last cycle. The first m cycles become the historical data, and the remaining cycles become the future data. The correlation between the extended or extrapolated curve and the future data is calculated. For 60 published construction activities, the correlations for 12 learning curve models (variations of linear, quadratic, and cubic models) were compared to determine which model generally provides the most reliable prediction of future performance.

Mathematically, m = number of cycles to be fitted; k = number of cycles to be predicted; and $n = m + k$ = total number of cycles. Pearson's coefficient of determination, R^2, is not valid for correlating points with best-fit curves outside the range of points used to determine the best-fit curve, so the following procedure is used.

Everett and Farghal (1994) developed a statistic called E_f that gives the average percentage difference between the values $y'_{m+1}, y'_{m+2}, \ldots, y'_{m+k=n}$ found on the extension of the best-fit curve for cycles $m + 1, m + 2, \ldots, m + k = n$, and the actual measured values $y_{m+1}, y_{m+2}, \ldots, y_{m+k=n}$.

$$E_f = \frac{\sum_{i=1}^{k} \frac{|y'_{m+i} - y_{m+i}|}{y_{m+i}}}{k} \times 100 \quad (1)$$

E_f can range from 0%, indicating a perfect correlation between the extended best-fit curve and the actual data, to large positive values, indicating little or no correlation.

Everett and Farghal (1994) showed that Wright's (1936) original linear model

$$\log y = a + b \log x \text{ (or, equivalently: } y = ax^b) \quad (2)$$

where x = cycle number; y = cost required to perform cycle x; a = cost required to perform the first cycle; and b = a constant that reflects the rate of learning.

This model provides the best correlation between actual and predicted performance for the models and activities tested. The cubic models that best correlate to historical data (Thomas et al. 1986, Everett and Farghal 1994) are shown to be poor predictors of future performance.

When managers are required to estimate future performance for a series of repetitive operations (e.g., person-hours needed for each floor of a multistory building) they are usually faced with two alternatives: (1) estimate performance before the activity commences; or (2) wait until part of the activity is complete. The disadvantage of the first alternative is that managers must rely on historical data from other, possibly similar, ac-

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tivities. The advantage of this alternative is that the prediction of productivity and performance is done early in the project when it may be most useful or necessary for estimating, scheduling, or other project management purposes.

The advantage of the second alternative is that it more realistically accounts for and includes the actual project conditions. Waiting until the project is partially complete before judging productivity gives managers a more reliable idea of what is really happening on that particular project. The disadvantage of this alternative is that waiting too long before judging performance and productivity can put managers in a situation where it may be too late to make significant changes.

Thus, there is a trade-off between the accuracy of predicting future performance and the timeliness and potential value of such a prediction. It would be of enormous benefit to construction managers to be able to make predictions early enough in the project so that they take actual project conditions into consideration and, yet, still have enough time to control the remaining operations.

Everett and Farghal (1994) used the first half of each activity's set of data as the historical data upon which extended learning curves are based, and the second half of each set as indicators of future performance. Selection of the midpoint of the data set as the boundary between historical and future data is somewhat arbitrary, but it was shown that for the linear log-x log-y model, E_f is not very sensitive around the midpoint of the data set to the number of cycles used to derive the extended best-fit curves.

The objective of this paper is to present a new method for using learning curves to predict the time or cost to complete the remaining cycles of an activity in progress, to assess the accuracy of this method, and to compare the accuracy of this method with the standard forecasting technique used in construction cost reporting. The paper will quantify the trade-off between accuracy of predicting future performance and the timeliness and potential value of such a prediction.

METHODS

Accuracy of New Method for Predicting Total Remaining Cost

Historical data for 60 construction activities were gathered from several published sources (''Effect'' 1965; Everett and Slocum 1993; McClure et al. 1980; Oglesby et al. 1989). The type of activity and the number of cycles is listed in Table 1.

Learning curve data has traditionally been represented in two formats: unit data and cumulative average data. Unit data is the time or cost to complete a given cycle versus the cycle number. Cumulative average data is the average time or cost to complete all cycles up to and including the given cycle versus the cycle number. All of the methods and calculations that follow are based on unit data.

For each activity, the least-squares best-fit curve of the form $\log y = a + b \log x$ (or equivalently $y = ax^b$) is derived for the first $m = 2$ of n points in the data set. These first m points become the historical data. The learning curve is extrapolated out to the final cycle, n.

The extrapolated learning curve is then compared to the remaining points (from $m + 1$ to n) and a new statistic, E_p, is determined

$$E_p(m) = \frac{\left| \sum_{i=m+1}^{n} y_i' - \sum_{i=m+1}^{n} y_i \right|}{\sum_{i=m+1}^{n} y_i} \times 100 \quad (3)$$

In (3), y_i' = predicted cost to perform the ith cycle; and y_i = actual cost. The numerator of the fraction on the right-hand side of (3) is the absolute value of the difference between the predicted total remaining cost and the actual total remaining cost. The denominator is the actual remaining cost. Therefore, $E_p(m)$ gives a percentage measure of how accurately the learning curve predicts future costs based on the actual performance

TABLE 1. Sixty Repetitive Construction Activities

Activity (1)	Number of cycles (2)
Residential construction (*Effect* 1965)	
Erection of pairs of houses (site A)	15
Erection of pairs of houses (site B)	12
Erection of pairs of houses (site C)	16
Erection of pairs of houses (site D)	15
Erection of pairs of houses (site E)	27
Assembling and dismantling formwork (A)	15
Assembling and dismantling formwork (B)	15
Assembling formwork	15
Assembling of framework for hoistways	32
Brickwork operations	10
Building element assembly	439
Concreting (A)	15
Concreting (B)	15
Concreting (C)	15
Concreting of first-floor walls and roof	14
Dismantling formwork	15
Electrical carcassing	9
Erection of 15-storey buildings	15
Erection of one-family houses	45
Erection of non-traditional houses	15
Erection of pairs of traditional houses	15
Erection of plaster board partitions	15
Erection of tunnel formwork	19
Facing of outside wall	44
Fitting of mouldings and window-sills	44
Fixing of wall measurements	600
Floor-boarding	12
Formwork and concreting	9
Formwork element assembly	1,600
Handling of bolts (A)	22,769
Handling of bolts (B)	48,000
Making of roof trusses on site	50
Placing of light concrete insulation	8
Placing of panels (A)	15
Placing of panels (B)	15
Production of dwellings	120
Production of identical dwellings (A)	240
Production of identical dwellings (B)	1,152
Production of raw structure	17
Sawing and erecting structure (A)	44
Sawing and erecting structure (B)	44
Setting up kitchen and bathrooms	44
Support-pole setting	44
Wooden shuttering (A)	9
Wooden shuttering (B)	8
Crane operations (Everett and Slocum 1993)	
Conventional crane operations	17
Using CRANIUM video system	29
Precast concrete fabrication (McClure et al. 1980a)	
Applying epoxy	34
Critical part of mixing epoxy	34
Hookup, mate, and set down	34
Mating after epoxy	34
Removing excess epoxy from ducts	31
Removing excess epoxy from segment joint	30
Temporary posttensioning cycle	34
Tensioning a pair of diagonal bars	44
Tensioning bars	31
Segmental bridge construction (McClure et al. 1980b)	
Casting and finishing operation	13
Form preparation	14
Void form placement and securement	12
Tunnelling (Oglesby et al. 1989)	
Tunnel excavation	14

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of the first m cycles. This procedure is repeated for each value of m from $m = 2$ to $m = n - 1$ for each of the 60 activities.

The difference between E_f in (1) and E_p in (3) is that E_p is an indication of correlation between actual and predicted data. E_f is based on the sum of the differences between individual predicted values and their corresponding actual values. E_p is based on the difference between sums of total predicted values and total actual values.

To illustrate the method, a detailed example of the activity erection of tunnel formwork (''Effect'' 1965) follows. Table 2 shows the actual unit data in the first two columns. In this activity, $n = 19$. Column 3 shows the predicted man-hours for cycles 11–19. These are determined by finding the best-fit curve of the form $y = ax^b$ (the linear learning curve) for the first 10 cycles ($m = 10$) and evaluating the curve at $x = 11, 12, 13, \ldots, 19$. In this case the best-fit learning curve is: $y = 21.3x^{-0.466}$. Fig. 1 shows a plot of the data. The black squares are the actual data. The heavy solid curve shows the best-fit learning curve for the first 10 cycles ($m = 10$). The heavy dashed curve shows the extrapolated learning curve.

The actual total remaining cost is 73.7 man-hours. This is

TABLE 2. Data for Activity: Erection of Tunnel Formwork

Cycle number (1)	Actual man-hours (mh) (2)	Predicted mh learning curve at $m = 10$ (3)	Predicted mh cumulative average at $m = 10$ (4)
1	27		
2	13.3		
3	11.5		
4	10.8		
5	8.8		
6	8		
7	8		
8	8.5		
9	8.8		
10	8.8		
11	9	7.0	11.4
12	8.8	6.7	11.4
13	8.5	6.4	11.4
14	8.3	6.2	11.4
15	8	6.0	11.4
16	8	5.8	11.4
17	7.8	5.7	11.4
18	7.5	5.5	11.4
19	7.8	5.4	11.4
[Sum] 11–19	73.7	54.7	102.6

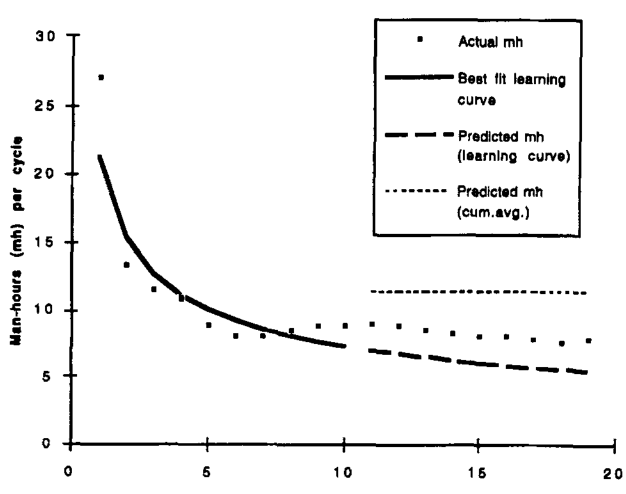

FIG. 1. Erection of Tunnel Formwork, Actual and Predicted Man-Hours to Complete Cycles 11–19

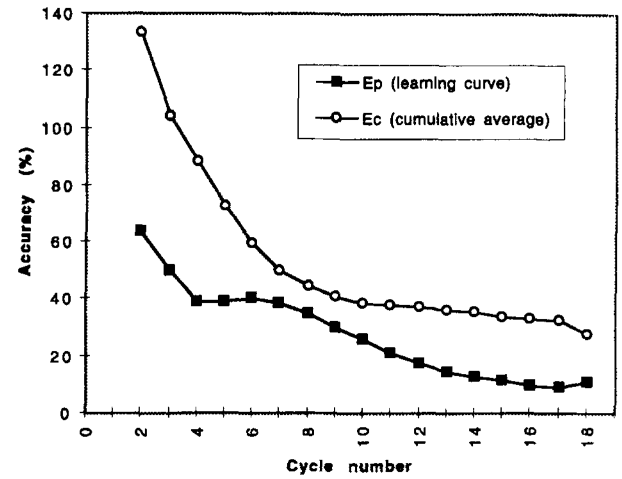

FIG. 2. E_p and E_c versus Cycle Number for Activity: Erection of Tunnel Formwork

the sum of the values in column 2 for cycles 11–19, or $\sum_{i=m+1}^{n} y_i$, where $m = 10$. The predicted total remaining cost using the extrapolated learning curve is 54.7 man-hours. This is the sum of the values in column 3 for cycles 11–19, or $\sum_{i=m+1}^{n} y_i'$, where $m = 10$. These values are then plugged into (3) to get $E_p(10) = 26\%$.

The same procedure is followed for all values of m from 2 to 18. No E_p value is calculated for the first cycle ($m = 1$) because a minimum of two cycles is needed to fit the learning curve. No E_p value is calculated for the final cycle ($m = 19$) because there are no future cycles to predict.

The lower curve in Fig. 2 shows a plot of $E_p(m)$ versus cycle number for the activity erection of tunnel formwork (''Effect'' 1965). In this example, there is a clear trend toward improved ability (lower E_p) to predict the total remaining cost to complete the activity as the activity progresses. In other activities, not shown here, the accuracy of predicting remains relatively constant throughout the activity, and in some activities the accuracy decreases toward the end of the activity. Plots similar to the E_p curve in Fig. 2 for the other 59 activities are published in Everett and Farghal (1995).

Comparison of Accuracy of New Learning Curve Method with Standard Method

To see if the new method presented in this paper offers improved accuracy over standard practice for predicting total remaining costs, the following analysis is offered.

The typical cost report for a construction project has, for each line item, the quantity of work in place, the percent complete, and the cost to date. These are gathered from quantity surveys and accounting records.

One method for calculating the predicted total cost of a line item is to divide the cost to date by the percent complete. To get the predicted total remaining cost, the cost to date is subtracted from the predicted total cost. Thus, m = number of completed cycles, n = total number of cycles, and y_i = actual cost of cycle number i

$$\text{Cost to date} = \sum_{i=1}^{m} y_i \tag{4}$$

$$\text{Percent complete} = m/n \tag{5}$$

$$\text{Predicted total cost} = \frac{\text{cost to date}}{\text{percent complete}} = \frac{\sum_{i=1}^{m} y_i}{(m/n)} \tag{6}$$

Predicted total remaining cost = predicted total cost

$$- \text{cost to date} = \frac{\sum_{i=1}^{m} y_i}{(m/n)} - \sum_{i=1}^{m} y_i = \left(\frac{n-m}{m}\right) \sum_{i=1}^{m} y_i \quad (7)$$

Another method to predict the total remaining cost is to calculate the unit cost to date by dividing the cost to date by the quantity of work in place. The estimate to complete or predicted total remaining cost is derived by multiplying the unit cost to date by the remaining quantity. This is equivalent to multiplying the cumulative average cost to date by the number of remaining cycles.

$$\text{Cumulative average cost to date} = \frac{\sum_{i=1}^{m} y_i}{m} \quad (8)$$

$$\text{Number of remaining cycles} = n - m \quad (9)$$

$$\text{Predicted total remaining cost} = \frac{\sum_{i=1}^{m} y_i}{m} (n - m) = \left(\frac{n-m}{m}\right) \sum_{i=1}^{m} y_i \quad (10)$$

Eq. (10) is identical to (7).

To determine how accurately this method predicts total remaining costs, and to compare it to the new E_p method presented, a procedure analogous to that used for determining $E_p(m)$ is used. For each value of m from 2 to $n - 1$, the predicted total remaining cost is derived using (7) or (10).

The dashed line in Fig. 1 shows the predicted costs for cycles 11–19. The reader will recall that in this example $m = 10$, The line is horizontal, because this method assumes that each remaining cycle will cost the same as the cumulative average cost of the first m cycles. The statistic $E_c(m)$ can be derived to measure how accurately this method predicts total remaining costs.

$$E_c(m) = \frac{\left| \left(\frac{n-m}{m}\right) \sum_{i=1}^{m} y_i - \sum_{i=m+1}^{n} y_i \right|}{\sum_{i=m+1}^{n} y_i} \times 100 \quad (11)$$

Using the example activity erection of tunnel formwork once again, at $m = 10$, the cumulative average cost is 11.4 man-hours (mh). With the nine remaining cycles each costing 11.4 mh, the predicted total remaining cost is 102.6 mh (11.4 mh/cycle × nine cycles). This can be seen in column 4 of Table 2, or calculated using (10). The actual total remaining cost is the same as before, or 73.7 mh. From (11), $E_c(10) = 39\%$. This procedure can be repeated for all values of m from 2 to 18.

The upper curve in Fig. 2 shows E_c for the standard cumulative average method versus cycle number for the activity erection of tunnel formwork. Clearly the new learning curve method gives a prediction of future costs that is closer to the actual value than the standard cumulative average method for this example.

Generalization of New Method to 60 Construction Activities

Different activities have different numbers of cycles. To compare the activities with each other, the cycle numbers were normalized to the total number of cycles for each activity to get the percentage of activity completion.

By averaging E_p at specified intervals for all 60 activities,

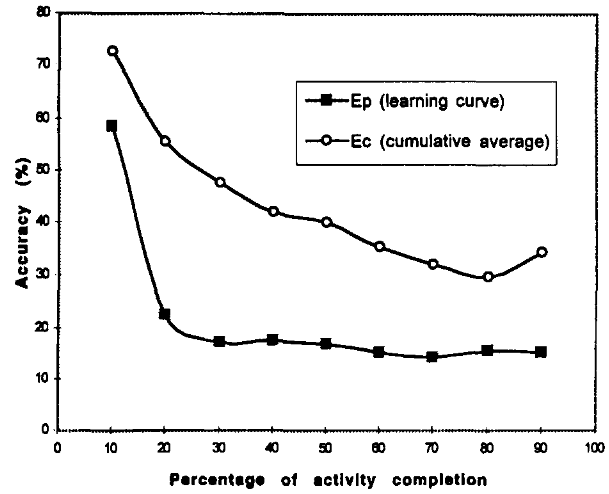

FIG. 3. E_p and E_c versus Percentage of Activity Completion (E_p Based on Average of 60 Activities; E_c Based on Average of 55 Activities)

an average plot of E_p versus percentage of activity completion can be generated. The lower curve in Fig. 3 shows the average E_p for all 60 activities versus the percentage of activity completion at 10% intervals.

For example, at 20% of an average activity's completion, a linear learning curve can predict the total remaining cost of the activity to within about 22% of the actual value. The accuracy of prediction improves dramatically for about the first 25–30% of the activity and then levels off to within about 15% of the actual value for the remainder of the activity.

The curve represents the average of 60 activities. Of course, the E_p values of some activities were better than the average shown, and some were worse. E_p values ranged from 0% (perfect prediction) to about 100% (useless prediction). Because of the wide variety of construction activities, the small number of any one type, and the differences in numbers of cycles, it was not possible to determine which types of activities might lend themselves to this type of analysis and which might not.

A similar procedure was followed to generate an average plot of E_c versus percentage of activity completion. Because some of the original data was incomplete, it was not possible to determine the cumulative averages and, therefore, E_c for five of the 60 activities. The upper curve in Fig. 3 shows the average E_c for 55 activities versus the percentage of activity completion at 10% intervals. Again there was a large range of E_c values that produced the averages shown.

The standard cumulative average method of predicting future costs may work well for construction activities with relatively constant unit costs. As shown in Fig. 3, it does not work nearly as well as the learning curve method for activities with continuously declining unit costs, such as those with significant learning effects. In these cases, the cumulative average cost will always be higher than the current and future unit costs.

It is difficult to tell in advance whether or not learning effects will be present in any given activity. Construction managers concerned about accurately predicting future costs should plot the unit data. If it appears that learning effects are taking place, the managers may wish to use the methods presented here rather than the standard methods. If no learning effect is apparent, either method could be used.

CONCLUSION

For the construction manager who decides to use the method presented in this paper to predict future performance, the accuracy of predicting future performance gets about as good as

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it is going to get at about 25–30% of activity completion. After this point the difference between the predicted total remaining cost and the actual total remaining cost is within plus or minus 15–20%. There is no point in waiting any longer for more accuracy.

For some applications, 15–20% accuracy may not appear very good, but these results are based solely on the actual results of performing the first few cycles of a repetitive activity. Experience with utilizing learning curves and the use of historical data from previous activities may help the manager better estimate the coefficients of the linear learning curve model to improve its usefulness. In conjunction with other cost and schedule control techniques, the learning curve can be a useful tool in predicting future performance of repetitive activities where learning effects are present.

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