

Using Gaussian and Hyperbolic Distributions for Quality Improvement in Construction: Case Study Approach

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Abstract: The Gaussian distribution and the 6σ principle have been widely used in the field of construction quality management with great success. This paper proposes a theoretical study on a new hyperbolic distribution using the 6σ principle to improve quality in construction management. The hyperbolic and Gaussian distributions are then numerically compared by estimating their important statistical properties, such as population in range, number of defects, yield percentage, and defects per million opportunities. The impacts of these factors are briefly discussed to give guidance to organizations in the construction industry on how to lower cost and improve project quality by prevention. A case study showing the cost data of a construction consultant company is presented. The data's population in range and defects per million opportunities are estimated using Gaussian and hyperbolic distributions. In this particular case study, the hyperbolic distribution is shown to be more effective in quality improvement by prevention than the Gaussian distribution. This also validates the hyperbolic distribution as a suitable distribution for construction quality management.

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Introduction

Poor quality of activities commonly occurs in the construction industry, e.g., short-piling and unsuitable use of materials (Construction Industry Review Committee 2001). Although the quality management is implemented by many organizations, serious problems can still be found in construction sites (Construction Industry Review Committee 2001). Thus, there is an urgent need to improve the quality performance in the construction industry.

The most commonly adopted quality management system, ISO 9000, which was introduced in 1994, has been implemented and certified by most construction organizations (International Organization of Standardization 2006). However, this does not mean that construction organizations have achieved the required quality standard. On the other hand, the quality performance found in the construction industry is worse than before, as more serious problems continue to happen in the industry.

There are various methods, which have been used by organizations in the construction industry to improve quality, employing mainly two major techniques: (1) Management techniques, such as quality control, quality assurance, and total quality management and (2) statistical techniques, such as cost of quality, cus-

tomers satisfaction, and 6σ principle (Prasad 1998). The 6σ principle has been widely employed by organizations around the world because of its effectiveness in improving quality by prevention and in lowering production costs. A Gaussian distribution's probability density function has been used as the main distribution with great success employing the 6σ principle because of its ideal bell shape. From that, important factors, such as population in range, number defects in the population, yield percentage, and defects per million opportunities can be numerically estimated.

The Gaussian distribution has been widely used in many fields of science such as medicine, civil engineering, construction management, signal processing, astronomy, physics, and optics (Cohen 1989). In the field of signal processing, a hyperbolic kernel, which is used to generate a hyperbolic distribution, has been shown to be more effective and robust than the Gaussian distribution, which has been known as Laplace of Gaussian (LoG) (Cohen 1989) or the kernel study of Choi and Williams (1989). Inspired by encouraging results obtained in the studies of Le et al. (2003a, b), more work toward the use of the hyperbolic distribution in the field of construction management has been carried out to see whether the same success can be achieved in the field of construction management.

This paper therefore focuses on the following main objectives:

- To statistically show the effectiveness of the hyperbolic distribution to the Gaussian distribution in improving quality using the 6σ principle;
- To demonstrate that there exists more than one suitable distribution for use with the 6σ principle to improve quality performance in construction activities; and
- To validate the hyperbolic and Gaussian distributions for quality improvement by using a construction consultant case study.

Hyperbolic versus Gaussian: A 6σ Approach

The Gaussian distribution has been widely and effectively employed using the 6σ principle to estimate important factors such

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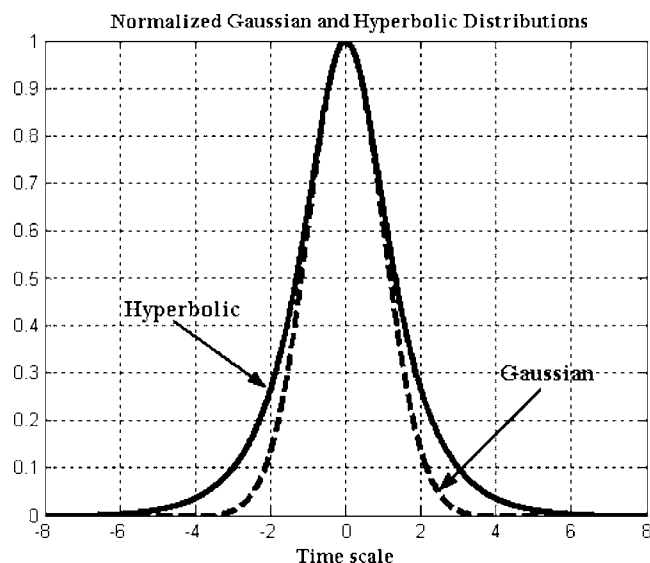


Fig. 1. Normalized hyperbolic and Gaussian distributions with $\beta = \sigma = 1$

as the population in range (PIR), the number of defects and the number of defects per million opportunities (DPMO). These estimations give satisfactory guidance for organizations in the construction industry on how to improve project quality. The main aim of this section is to introduce the hyperbolic distribution and some of its properties, which are compared with those of the Gaussian distribution. From that, the crucial above-mentioned factors can be estimated and compared with those obtained using the Gaussian distribution. The effectiveness of each distribution is then assessed, which enables organizations to determine which one is best for a particular population. It should be noted that not all populations are Gaussian distributed. Thus, by having more available distributions for use under the 6 σ principle, more choices are made available to organizations yielding lower production cost and higher product quality.

The hyperbolic and Gaussian distributions, respectively, are mathematically given by

$$H = \frac{\beta \operatorname{sech}(\beta x)}{\pi} \quad \text{with } \operatorname{var}_H \approx \frac{2.4674}{\beta^2} \quad (1)$$

$$G = \frac{\exp\left(-\frac{x^2}{2\sigma^2}\right)}{\sigma\sqrt{2\pi}}, \quad \text{with } \operatorname{var}_G = \sigma^2 \quad (2)$$

where β and σ = distribution parameters and var represents variance. The normalized Gaussian and hyperbolic distributions are plotted in Fig. 1.

It is clear that both distributions resemble the ideal bell shape, which approximately governs most measured data and populations in the construction industry. It should be noted that, in the literature, the word “distribution” refers to a “probability density function” (pdf). In fact, all numerical results reported in this paper have been obtained using the pdfs of the Gaussian and hyperbolic distributions. At first glance, the hyperbolic distribution may not be that familiar as compared to the well-known Gaussian distribution. In fact, the hyperbolic variance possesses a nonlinear parabolic characteristic. However, the former distribution has been shown to be more effective than the Gaussian distribution in terms of processing information and noise robustness as was shown in the studies of Le et al. (2003a, b). By saying that, there ought to be improvement by using the hyperbolic distribution compared to using the Gaussian distribution in the field of construction management. Thus, the main aim of this paper is to show the effectiveness of the new distribution in the field of construction management as this has been carried out in the field of signal processing.

In this paper, for simplicity, $\beta = 1$ is chosen as one typical candidate of the hyperbolic distribution family. It should be noted that for $\beta = 1$, the hyperbolic variance becomes linear with its value of $\operatorname{var}_H \approx 2.4674$, which can be used to estimate the number of defects in a hyperbolic-distributed population. Other values of β will also be included in this paper, resulting in different numerical values for factors, such as population in range and number of defects per million opportunities. It should also be noted that for the Gaussian distribution family, because its population in range and other factors are independent of σ , it can be suggested that the hyperbolic distribution is more diversified than the Gaussian distribution. The effectiveness of these distributions on how to improve quality will be assessed later.

It should be noted that the distributions are assumed to possess zero means for simplicity and this assumption does not affect the degree of generality. For different values of β and σ , different shapes of the distributions can be obtained, yielding distributions to be dependent on β and σ .

The population in ranges (PIRs) of the Gaussian and hyperbolic distributions are given in Table 1. The PIR is the probability of members of a population existing in the ranges of $(-\sigma, +\sigma)$, $(-2\sigma, +2\sigma)$, ..., and $(-6\sigma, +6\sigma)$. From Table 1, by using the 6 σ principle, it is clear that the hyperbolic distribution outperforms the Gaussian distribution for standard deviation less than 3. For standard deviation greater than 3, the Gaussian distribution is more effective than the hyperbolic distribution. As can be seen in Table 1, the larger the standard deviation, the more effective the Gaussian distribution compared to the hyperbolic distribution. In quality management, because the standard deviation of less than 3 is more common (Schwalbe 2006) than greater than 3, it can be

Table 1. Standard Deviation and Approximate Defective Units with $\beta = 1$

| Standard deviation | | Percent of population within range | | Defective units per billion | |
|--------------------|------------|------------------------------------|-------------|-----------------------------|-------------|
| Gaussian | Hyperbolic | Gaussian | Hyperbolic | Gaussian | Hyperbolic |
| 1 σ | 2.4674 | 68.2689492 | 89.22839568 | 317,300,000 | 107,720,000 |
| 2 σ | 4.9348 | 95.4499736 | 99.08437244 | 45,400,000 | 9,156,300 |
| 3 σ | 7.4022 | 99.7300002039 | 99.92235126 | 2,700,000 | 776,490 |
| 4 σ | 9.8696 | 99.999366575 | 99.9934152 | 63,000 | 65,848 |
| 5 σ | 12.337 | 99.999994267 | 99.99944159 | 57 | 5,584 |
| 6 σ | 14.8044 | 99.99999998 | 99.99995265 | 2 | 473 |

Table 2. Population in Range of the Hyperbolic Distribution for $\beta=0.1$ to 1 as a Function of Standard Deviation

| β | Standard deviation | | | | | |
|---------|--------------------|------------------|------------------|------------------|------------------|------------------|
| | 1σ | 2σ | 3σ | 4σ | 5σ | 6σ |
| 0.1 | 0.9999999997541 | ~ 1.00 | ~ 1.00 | ~ 1.00 | ~ 1.00 | ~ 1.00 |
| 0.2 | 0.99999437549544 | 0.999999999754 | ~ 1.00 | ~ 1.00 | ~ 1.00 | ~ 1.00 |
| 0.3 | 0.99965719530112 | 0.9999990770879 | 0.9999999997515 | 0.9999999999999 | ~ 1.00 | ~ 1.00 |
| 0.4 | 0.99731014386575 | 0.99999431771837 | 0.99999998823677 | 0.9999999997515 | 0.9999999999995 | ~ 1.00 |
| 0.5 | 0.99075060209184 | 0.99993281329384 | 0.99999952417588 | 0.9999999654391 | 0.9999999997489 | 0.9999999999982 |
| 0.6 | 0.97910452833918 | 0.99965711818165 | 0.99999437399572 | 0.99999990768803 | 0.9999999848533 | 0.9999999997515 |
| 0.7 | 0.96153567093454 | 0.99887831057654 | 0.99996729612087 | 0.99999901201108 | 0.99999997119859 | 0.9999999916031 |
| 0.8 | 0.94149247151515 | 0.99730906822294 | 0.99987632100692 | 0.99999431544607 | 0.99999973872089 | 0.99999998799061 |
| 0.9 | 0.91442984387436 | 0.99449468381870 | 0.99964633488692 | 0.99997728019414 | 0.99999854043935 | 0.99999990623412 |
| 1.0 | 0.89029330108370 | 0.99050524725722 | 0.99922107857412 | 0.99993277131964 | 0.99999419573946 | 0.99999952387861 |

suggested that the hyperbolic distribution is more effective than the Gaussian distribution in that respect.

It should be noted that all Gaussian distributions, independent of σ , possess the PIR as given in Table 1. For the hyperbolic distribution, as the standard deviation is inversely proportional to β^2 [Eq. (1)], it is clear that its variance can be increased or decreased at a faster rate than that of the Gaussian distribution. Thus, to further improve the PIR of the hyperbolic distribution, decreasing β results in a larger variance, yielding an improved PIR and number of defects. As many practical populations and data are not accurately approximated by the Gaussian distribution, the hyperbolic distribution provides another valuable option to model a non-Gaussian population. Table 2 gives the PIRs of the hyperbolic distribution for $0.1 \leq \beta \leq 1$ using the 6σ principle with numerical results accurately up to 15 decimal digits. It should be noted that the PIR of ~ 1.00 given in Table 2 is due to numerical truncation and therefore is not exactly unity. From Table 2, it is clear that the hyperbolic distribution yields a much higher PIR than the Gaussian distribution for $\beta < 1$. For the hyperbolic distribution, the smaller β , the higher the PIR. For the Gaussian distribution, its PIR remains unchanged as σ varies. Thus, it can be suggested that the hyperbolic distribution provides a more diversified approach to the 6σ principle in which different values of β can result in a different distribution, which improves its effectiveness in approximating or modelling a population.

To further explore the effectiveness of the 6σ principle, Yang (2005) and Schwalbe (2006) converted the standard deviation to a yield percentage (YP), i.e., the percentage of units in a population, which have been correctly handled, and DPMO, which gave more practical understanding on the effects of the 6σ principle (Schwalbe 2006). Table 3 shows these quantities in which it is clear that the hyperbolic distribution gives improved results for

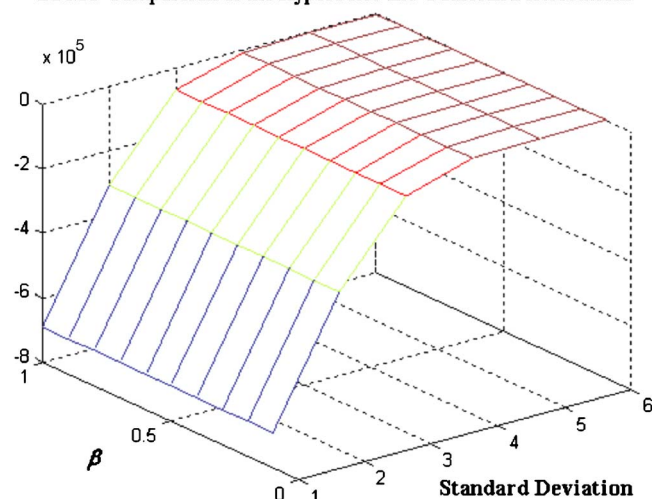
the standard deviation in the range of two and five as shown in the shaded cells.

Fig. 2 graphically compares the DPMO using the hyperbolic distribution with that using the Gaussian distribution. From Fig. 2, it can be suggested that for $0.1 \leq \beta < 1$, the resultant hyperbolic distributions yield an identical rate of increasing the difference between the DPMOs of the two distributions. This means that any value of β in the range of $0.1 \leq \beta < 1$ can be used to obtain an improved DPMO and YP compared to those obtained using the Gaussian distribution. Typically, the range of $0.5 < \beta < 1$ is employed because the hyperbolic distributions in this range are more effective and stable than those using a too small β (Le et al. 2003a, b). However, a popular value of $\beta=1$ can always be used as a performance benchmark, as shown earlier.

As can be seen in this subsection, the hyperbolic distribution has been shown to be more effective than the Gaussian distribution using the 6σ principle. This suggests that by using the hyperbolic distribution, it is possible to further lower costs and to improve quality in the construction industry for populations governed by this distribution. In addition, it can be suggested that the Gaussian distribution can be replaced by the hyperbolic distribution for appropriate population of which the latter gives a more accurate approximation than the former distribution.

Table 3. YP and DPMO of the Gaussian and Hyperbolic Distributions with $\beta=1$

| Standard deviation | | YP | | DPMO | |
|--------------------|------------|----------|-------------|----------|------------|
| Gaussian | Hyperbolic | Gaussian | Hyperbolic | Gaussian | Hyperbolic |
| 1σ | 2.4674 | 30.23 | 18.04 | 697,700 | 819,600 |
| 2σ | 4.9348 | 69.13 | 81.96 | 308,700 | 180,400 |
| 3σ | 7.4022 | 93.32 | 98.43 | 66,810 | 15,718.2 |
| 4σ | 9.8696 | 99.379 | 99.87 | 6,210 | 1,333.2 |
| 5σ | 12.337 | 99.9767 | 99.98869401 | 233 | 113 |
| 6σ | 14.8044 | 99.99966 | 99.9904123 | 3.4 | 9.6 |

DPMO Comparison of the Hyperbolic and Gaussian Distributions**Fig. 2.** DPMO comparison of the hyperbolic and Gaussian distributions

Case Study

The following outlines one typical case study using cost data of a construction consultant company in Vietnam from which it is found that the hyperbolic and Gaussian distributions are effective tools for analyzing the company's overall performance and quality. From the estimated PIRs and DPMOs, possible further improvement on the company's performance and quality can be made.

Background on the Company

This research uses a pilot study approach to enhance the quality performance in the Vietnam construction consultant industry. One company is selected to review and to analyze the projects quantitatively. All staff members were also responsible for quality on site through self-monitoring and observation. The supply chain employees, including suppliers, subcontractors, designers, client, project manager, site agent, quantity surveyor and junior staff, were all involved in recording and collecting data, such as being responsible for filing incidents and suggesting possible causes for manifested effects.

Before 1995, the main business of the interviewed company was construction as a family-type small business, and it was a partnership company owned by Vietnamese and Japanese construction experts. In 1998, it started the introduction of construction consultant. Two years later, this company was able to expand to meet the market high demands. As a result, a new business model and business strategy were required. New concepts to manage and to maintain quality were explored by experts around the world. In 2001, this company was awarded an ISO 9002 certificate.

In this research, projects implemented in the financial years from January 2005 to December 2006 are analyzed using the hyperbolic and Gaussian distributions. Costs in adopting hyperbolic and Gaussian distributions are divided into costs of prevention and costs of failure. Costs of prevention mean the costs used to reduce further failures, including monthly review meetings with a foreman to understand the progress and quarter monthly review meetings with workers to assess their working techniques. Except for the review meetings held in this company, major elements of costs of prevention include: (1) training of the engineering clerk and site engineer for the site "information collection of site condition by telephone," which was introduced in December 2002; (2) introduction of the new and advanced machinery in February 2002; (3) upgrading computer systems and introducing an internal e-mail system; (4) using a digital camera system to record all the work and site conditions during the site operation or the site investigation by the engineers; (5) consulting and meeting with professionals by joining seminars and conferences, which can provide up-to-date information on on-site activities; (6) providing related books and magazines in the library to improve employee knowledge; (7) employing one part-time engineer to help and improve the quality; (8) inspection within 100 days upon completion; (9) undertaking digital photos by engineers during inspection to monitor site conditions; and (10) carried out ISO 9001 auditing every six months, which includes checking, inspecting, filing, recording all reference information in the required format.

The company's costs of quality failures are divided into two major items: Costs of laboratory and costs of rework. Upon completion of the project, if the customer is not satisfied after 100 days, the clients may orally and/or formally lodge a complaint

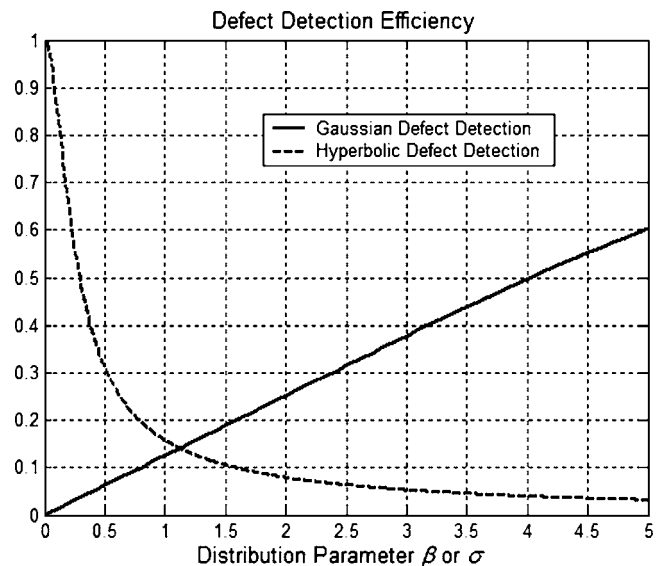


Fig. 3. Defect detection using the hyperbolic and Gaussian distributions

about the situation. The costs used by engineers in understanding and investigating the situations, are classified as costs of laboratory. After the engineers have investigated the situation of failure of part of the project, rework may be required. Therefore, the costs of rework include: (1) Extra time spent by engineers in the office; (2) extra time for site meeting and site investigation in work failures; and (3) material, labor, and machine for the rework. The Appendix shows the company cost of data in terms of prevention and failure over a period of two financial years.

Estimation of PIRs and DPMOs

The procedure of estimating the PIRs and DPMOs is outlined as follows. First, the mean and variance of the data are estimated by using a MATLAB (The mathworks Inc., Natick, Mass.) program. As shown earlier, for extreme cases of small β , it is possible that the hyperbolic distribution can outperform the Gaussian distribution, which is true in this case. With the appropriate values of β and σ obtained from the given data, the PIR and DPMO of each distribution can be approximately worked out up to 15 decimal digits. In this paper, only two decimal digits are shown of simplicity.

The defect detection efficiency of the hyperbolic and Gaussian distributions is plotted in Fig. 3, in which it is clear that the hyperbolic distribution outperforms the Gaussian distribution for $\beta \leq 1$, which is typical for construction data sets. For $\beta > 1$, the Gaussian distribution outperforms the hyperbolic distribution. However, the latter case is usually not typical for construction data sets, which shows the advantage of the hyperbolic distribution to the Gaussian distribution. Fig. 4 shows the degree of accuracy when using the hyperbolic distribution to predict the defect detection theoretically and practically using the data set given in the case study. From Fig. 4, it is clear that the two prediction methods follow closely as β , which validates the effectiveness of the hyperbolic distribution.

It should be noted that the PIR reflects the population percentage, which obeys a particular distribution, however, it does not directly affect the performance of the company. Thus, to correctly assess the company performance, it is necessary to estimate its

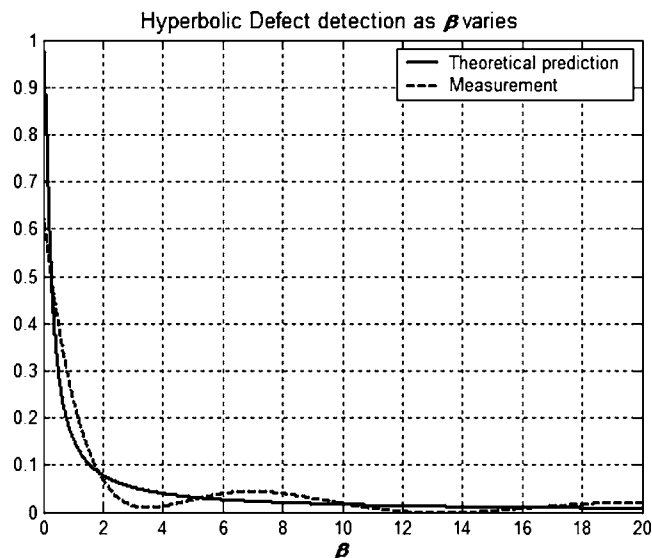


Fig. 4. Theoretical and measured defect detection using the hyperbolic distributions

DPMO based on the given data. Table 4 shows the PIR of the data using the hyperbolic and Gaussian distributions in which it is clear that the PIRs of both distributions are at acceptable levels. This suggests that these distributions can be used to model the data, in other words, they can be used to theoretically assess the performance of the company. It should be noted that the Gaussian distribution has been widely employed to manage quality and to monitor performance. By using the results shown in Table 4, it can be said that the new hyperbolic distribution can be used for quality and performance monitoring as effective as the Gaussian

distribution. This provides a new dimension in the field of construction quality management, which enables managers and organizations to further improve quality.

Table 5 gives the approximate estimated DPMOs by applying the two distributions to the data given in the Appendix in which the hyperbolic DPMO is lower than that of the Gaussian, except when the standard deviation is one. It should be noted that the lower the DPMO, the better the quality. In this case, the company can expect to reduce the number of DPMO from 3.4 to 1.9 using the 6σ principle which is an improved performance. This means that out of one million products, there could theoretically be about 1.9 defects. It is clear that by reducing the DPMO, the company's performance can be theoretically improved by about 1.5 times than before. In practice, its overall performance is also dependent on other factors, such as subcontractor performance, weather conditions and employee skills. As a result, the overall performance may not be as high as theoretically predicted using the hyperbolic and Gaussian distributions. It is important to stress that the actual number of defects is very difficult to measure as there are many external factors, such as weather conditions, different client perceptions on standard and geometrical conditions, which significantly affect this measurement. Therefore it is not possible to compare it with the theoretical values of 3.4 (using the Gaussian distribution) and 1.9 (using the hyperbolic distribution). However, by examining the PIRs and DPMOs given in Tables 4 and 5, it is clear that the company has appropriately invested in its prevention activities with the main aim of avoiding failures. To check on the progress of the company activities and performance, detailed data need to be collected on a regular basis, the PIR and DPMO of the data are then estimated using the hyperbolic and Gaussian distributions. From that, decisions can be made by managers whether an activity or a group of activities should cease to achieve better PIRs and DPMOs. It should also be noted that the process of estimating the PIRs and DPMOs is not time consuming and therefore should be employed by every company in the in-

Table 4. PIR Using the Hyperbolic and Gaussian Distributions for the Total Prevention and Failure Costs

| Standard deviation | Using hyperbolic distribution | | Using Gaussian distribution | |
|--------------------|-------------------------------|--------------------|-----------------------------|--------------------|
| | Total prevention cost | Total failure cost | Total prevention cost | Total failure cost |
| One | 0.7463475 | 0.7472372 | 0.5267245 | 0.5234236 |
| Two | 0.9347832 | 0.9347344 | 0.9352634 | 0.9542434 |
| Three | 0.9862455 | 0.9862345 | 0.9962421 | 0.9972362 |
| Four | 0.9963542 | 0.9962434 | 0.9999634 | 0.9999235 |
| Five | 0.9994277 | 0.9995057 | 0.9999923 | 0.9999994 |
| Six | 0.9997354 | 0.9998253 | 0.9999999 | 0.9999999 |

Table 5. Approximate Defects per Million Opportunities (DPMO) Using the Hyperbolic and Gaussian Distributions for the Total Prevention and Failure Costs

| Standard deviation | Using hyperbolic distribution | | Using Gaussian distribution | |
|--------------------|-------------------------------|--------------------|-----------------------------|--------------------|
| | Total prevention cost | Total failure cost | Total prevention cost | Total failure cost |
| One | 7,354,364.45 | 8,242,354.23 | 7,234,263.42 | 7,243,533.44 |
| Two | 125,236,223.00 | 162,453.43 | 323,452.62 | 323,456.23 |
| Three | 9,345.92 | 9,345.85 | 63,456.40 | 68,234.34 |
| Four | 523.36 | 523.24 | 6,345.36 | 6,324.83 |
| Five | 32.38 | 32.34 | 234.62 | 234.33 |
| Six | 1.73 | 1.74 | 3.40 | 3.40 |

dustry to closely monitor quality and overall performance. As for this case study, more data are currently being collected to show the consistency between the theoretical figures reported in this paper and the real figures obtained in practice. In conclusion, it is crucial that the DPMOs and PIRs of data at different values of the standard deviation should be estimated by using hyperbolic and Gaussian distributions to improve quality and overall performance.

It is also difficult to define the meaning of “defects” from a practical point of view. For example, a company is currently working on N_0 projects, out of these projects, M_0 projects have been satisfactorily completed to customer standards, which means $N_0 - M_0$ projects are resubmitted to the company for further work. It is important to note that the projects that are considered as unsuccessful only with respect to certain client perception, meaning that other clients may consider these projects as “satisfactory.” As a result, different PIRs and DPMOs may be obtained from two different companies working on a similar number of projects in the same discipline. By saying that, it is necessary to closely examine detailed data for each company as client perception standards are always different. To further study the effectiveness of the hyperbolic and Gaussian distributions for quality improvement, more case studies are required which can be used as typical comparison benchmarks for performance and quality prediction, which, in the long run, can generate more profit for organizations.

It should be clear by now that the numerical PIRs and DPMOs using the hyperbolic and Gaussian distributions of the total prevention cost and total failure cost are very similar. This shows that for this particular company, the hyperbolic and Gaussian distributions can be effectively used to estimate its PIRs and DPMOs. From that, it is possible for managers to predict the company performance and to reduce unnecessary cost. Further improvement can be also identified by studying the PIRs and DPMOs. This validates the hyperbolic distribution as a suitable distribution for construction quality management.

From the findings in this section, it is clear that the PIRs and DPMOs using the Gaussian and hyperbolic distributions yield useful information on how the company has performed in the last 2 years, based on the given cost data. This has also validated the 6σ principle and its significant role in the construction industry. It should also be clear that to perform the 6σ principle, it is crucial that real cost data are collected and available for analysis. It is also understood that cost data are usually confidential and hardly released by companies for other purposes, thus making detailed and thorough analyses on the company's performance much harder. From the case study given in this paper, a conclusion can be drawn that any type of construction company can employ the 6σ principle to analyze their performance in the last two financial years. From that, more appropriate and timely decisions can be made by managers to possibly generate positive profits. It should also be noted the more data being used in the 6σ principle, the more accurate the prediction.

Another question that may be asked by technical experts is: Are there other distributions that can be employed to further improve the performance of the Gaussian and hyperbolic distributions? The answer to this question lies in the fact that new distributions must assemble the well-known Gaussian bell shape as the necessary criterion to hopefully outperform the Gaussian and hyperbolic distributions. Given that a new distribution is found, then the next task is to find an appropriate set of data,

which can yield better PIRs and DPMOs than those of the Gaussian and hyperbolic distributions. Research toward this direction is interesting and currently under progress.

Conclusion

The importance of quality has been shown and recognized by researchers in the field of construction management. Although various methods of quality management have been implemented, the quality performance still cannot achieve the required requirements. The 6σ principle therefore has been effectively employed as an effective statistical tool to improve quality in construction. The Gaussian distribution has been extensively used to verify the effectiveness of this principle. This paper has shown that there are other distributions that can be used as effective tools for the design of quality management. These distributions thus help in diversifying the field of quality management in which populations are not always governed by the Gaussian distribution. In addition, with more suitable tools available, managers and manufacturers can further improve quality by minimizing the number of defects and maximizing the yield percentage in a population. As a result, the project cost will be significantly reduced. This paper has mathematically shown that the Gaussian distribution is not always the best distribution for a random population, but rather, other distributions should have a trial period so that the most suitable one is chosen to approximate the population. The performance of the hyperbolic and Gaussian distributions has been assessed by estimating important factors for quality improvement such as population in range, yield percentage and defects per million opportunities.

One case study has been presented in this paper in which cost data from a construction consultant company were analyzed using the hyperbolic and Gaussian distributions. The population in range and defects per million opportunities have been numerically estimated in which it was shown that the hyperbolic distribution theoretically yielded about 1.9 defects per million opportunities compared to 3.4 using the Gaussian distribution. It is evident from this research that the hyperbolic distribution is a valid and suitable tool for construction quality management analyses. Regular data collection has also been shown to be necessary to closely monitor quality and overall performance of organizations.

The contributions of this paper are: (1) To introduce a new hyperbolic distribution with fine detailed features for the design of quality management; (2) to show that the hyperbolic distribution is more effective than the well-known Gaussian distribution with much lower defects per million opportunities and much higher yield percentage; (3) to show that there exist suitable distributions, which can be employed using the 6σ principle; and (4) to show that the hyperbolic and Gaussian distributions are effective tools for quality management analyses of cost data in a construction consultant company. Possible extension of this work to other types of activities, such as environmental and safety improvement, general staff training and productivity improvement, is also possible.

Appendix

See Table 6.

Table 6. Survey Results

| Date | Prevention cost (thousand of VND\$) | | | | | | | Failure cost (thousand of VND\$) | | |
|----------------|-------------------------------------|---------------------------|----------------------|------------|---------|----------|--------|----------------------------------|-------------|--------|
| | Maintenance and training | Professional consultation | Professional service | Inspection | On-cost | Audition | Total | Laboratory cost | Rework cost | Total |
| January 2005 | 4,453 | 4,633 | 6,250 | 3,400 | 0 | 5,848 | 24,584 | 9,800 | 24,601 | 34,401 |
| February 2005 | 1,158 | 4,300 | 6,250 | 350 | 10 | 5,848 | 17,916 | 3,100 | 6,389 | 9,489 |
| March 2005 | 5,649 | 4,444 | 6,250 | 0 | 3,754 | 5,848 | 25,945 | 2,200 | 6,610 | 8,810 |
| April 2005 | 2,827 | 8,200 | 6,250 | 2,950 | 0 | 5,848 | 26,074 | 5,000 | 2,800 | 7,800 |
| May 2005 | 7,794 | 79 | 6,250 | 6,450 | 0 | 5,848 | 26,421 | 4,600 | 4,335 | 8,935 |
| June 2005 | 2,803 | 14,575 | 6,250 | 450 | 0 | 5,848 | 29,926 | 2,200 | 100 | 2,300 |
| July 2005 | 11,189 | 276 | 6,250 | 0 | 0 | 5,848 | 23,562 | 0 | 110 | 110 |
| August 2005 | 82,24 | 224 | 6,250 | 3,750 | 0 | 5,848 | 24,295 | 5,400 | 200 | 5,600 |
| September 2005 | 6,451 | 10,067 | 6,250 | 0 | 140 | 5,848 | 28,756 | 5,200 | 300 | 5,500 |
| October 2005 | 15,739 | 52 | 6,250 | 1,900 | 0 | 5,848 | 29,789 | 3,200 | 1,250 | 4,450 |
| November 2005 | 9,230 | 6,020 | 6,250 | 0 | 0 | 5,848 | 27,348 | 7,000 | 1,550 | 8,550 |
| December 2005 | 11,169 | 6,871 | 6,250 | 0 | 91 | 5,848 | 30,230 | 5,000 | 2,900 | 7,900 |
| January 2006 | 22,764 | 811 | 6,250 | 0 | 0 | 5,848 | 35,674 | 6,200 | 1,850 | 8,050 |
| February 2006 | 14,848 | 6,178 | 6,250 | 0 | 567 | 5,848 | 33,691 | 3,200 | 3,350 | 6,550 |
| March 2006 | 6,911 | 441 | 6,250 | 0 | 1,994 | 5,848 | 21,444 | 3,000 | 500 | 3,500 |
| April 2006 | 1,512 | 3,750 | 6,250 | 0 | 118 | 5,848 | 17,477 | 1,000 | 500 | 1,500 |
| May 2006 | 3,27 | 5,007 | 6,250 | 0 | 342 | 5,848 | 17,774 | 7,600 | 19,475 | 27,075 |
| June 2006 | 3,120 | 396 | 6,250 | 0 | 80 | 5,848 | 15,694 | 8,800 | 56,134 | 64,934 |
| July 2006 | 38 | 3,200 | 6,250 | 0 | 0 | 5,848 | 15,335 | 4,600 | 9,050 | 13,650 |
| August 2006 | 231 | 242 | 6,250 | 3,400 | 135 | 5,848 | 16,106 | 4,600 | 9,050 | 13,650 |
| September 2006 | 878 | 5,406 | 6,250 | 0 | 5,118 | 5,848 | 23,500 | 9,900 | 21,222 | 31,122 |
| October 2006 | 2,200 | 19,803 | 6,250 | 0 | 86 | 5,848 | 34,187 | 5,600 | 10,100 | 15,700 |
| November 2006 | 733 | 4,486 | 6,250 | 3,700 | 0 | 5,848 | 21,016 | 5,400 | 4,900 | 10,300 |
| December 2006 | 955 | 7,157 | 6,250 | 700 | 0 | 5,848 | 20,909 | 5,400 | 7,320 | 12,720 |

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