Time-Cost Trade-Off among Related Activities

By Rehab Reda[1] and Robert I. Carr,[2] Fellow, ASCE

Abstract: This paper describes the typical pragmatic approach that construction planners taken in performing time-cost trade-off (TCT). In general, projects have major dominant characteristics, operations, or resources whose planning affects or dominates planning of other operations and resources. Planning focuses first on the dominant characteristics and is then fine-tuned in its details. Planners typically cycle between plan generation and cost estimating at ever finer levels of detail until they settle on a plan that has an acceptable cost and duration. Computerized TCT methods do not follow this cycle. Instead, they separate the plan into activities, each of which is assumed to have a single time-cost curve in which all points are compatible and independent of all points in other activities' curves and that contains all direct cost differences among its methods. In general, these assumptions are not true for construction. Construction activities are related because they share methods and resources. Crashing activities usually require changes from normal, least-cost methods and resources. Changes in one activity are not independent of changes in related activities. Therefore, normal computerized TCT techniques are conceptually wrong for construction and they are not useful in practice.

Introduction

Construction contractors make time-cost trade-off decisions every day in planning and managing their work. Automated time-cost trade-off (TCT) methods have been available for almost 30 years but have not been of use in construction decisions. This paper describes the typical pragmatic approach that managers take in making time-cost trade-offs. It then describes computerized TCT methods and the assumptions upon which they depend. It is evident that the TCT assumptions do not fit construction projects. Therefore, though TCT methods are conceptually interesting and useful in understanding practical time-cost decisions, current computer models are not applicable to construction work and are not useful for construction planning.

Construction Planning

A facility is constructed by application of resources to construction operations in a sequence. The cost of the resources is the project cost and the time that the sequence of operations requires is the project duration. A particular set of resources, operations, and sequences that a contractor selects to perform construction make up a contractor's plan for construction. This can also be called the contractor's method of construction. An estimate of the time required to perform the project by the plan is the estimated project duration and an estimate of the cost required to perform the project by the plan is the estimated project cost.

[1]Asst. Prof., Civ. Engrg. Dept., King Abdulaziz Univ., P.O. Box 9027, Jeddah-21413, Saudi Arabia.
[2]Prof. of Civ. Engrg., Univ. of Michigan, Ann Arbor, MI 48109.

Note. Discussion open until February 1, 1990. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on May 16, 1988. This paper is part of the ***Journal of Construction Engineering and Management***, Vol. 115, No. 3, September, 1989. ©ASCE, ISSN 0733-9364/89/0003-0475/$1.00 + $.15 per page. Paper No. 23901.

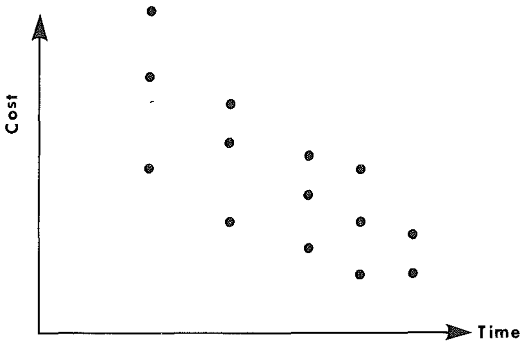

FIG. 1. Project Time-Cost Relationship

There can be any number of sets of resources, operations, and sequences that can be expected to complete a project and each set is an alternative plan method. Each alternative plan will have its estimated project duration and cost. If a contractor creates alternative plans and estimates the duration and cost of each, they could be plotted as shown in Fig. 1.

The challenge to a contractor in planning a project is to determine a plan that will perform the project at or near the least possible total cost. If the project cost and duration are sensitive to the plan and the best plan is not intuitively obvious, then selection of a good plan requires generation of alternative plans, estimation of project cost for each plan, and comparison of estimated costs of alternative plans. This requires a method by which alternative plans can be described or characterized, a method of generating plans that can be constructed near lowest cost, and a method of estimating that is sufficiently accurate that alternative plans can be compared to identify a plan that is close to the minimum cost.

Interaction of Planning and Estimating

This process is a normal and regular process in the management of construction, performed in one fashion or another every day in estimating, planning, and managing projects. Plans are generated by a combination of experience, intuition, trial and error, and using what one has available to fit the facilities design. One estimates direct costs for a particular plan by estimating quantities and prices of materials, durations and costs of time spent by workers, and duration of use and cost of equipment to perform the selected operations. Indirect costs are added to include costs that are directly related to the project duration. Therefore, one first estimates direct costs and durations of operations for a particular set of resources and sequence of operations. Then one estimates indirect costs for that set of durations.

Time-related indirect costs will decrease as project duration decreases but this may be at the expense of increased direct costs of a faster but less efficient set of resources and sequence of operations. We may therefore prefer a plan that has higher estimated direct costs if it reduces project duration and related indirect costs such that the total project cost is decreased. There can therefore be a trade-off between alternative plans, one of whose esti-

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mated direct costs are significantly higher than the others but whose estimated duration and indirect costs are significantly lower. In the jargon of construction management, this is called time-cost trade-off and, formally or informally, it is a daily part of managing construction.

The general order of planning is to generate a plan, estimate its costs, generate an alternative plan, estimate its costs, etc., until one has a plan with which one is satisfied with regard to its cost and duration. This requires that the estimating methods be sensitive to the differences in plans, so that alternative plans can be compared.

Therefore, when one is creating a plan, one always has costs in mind and when one is estimating costs, one usually is trying to think of possible alternative plans that will have lower costs. Therefore, estimating is a part of planning and planning is a part of estimating, whether the planning is for an entire project or for the next week or the next day.

Dominance of Methods or Resource Decisions

In general, projects have major dominant characteristics, or operations, or resources whose planning affects or dominates planning of many other operations and resources and managers focus their first attention on such dominant features. For example, for a multistory reinforced concrete building, a general contractor will probably focus first on selecting a general formwork, steel, and concrete erection plan and will then fine tune the detail of those plans so that concrete construction can be expected to proceed in an integrated, efficient, continuous manner.

Similarly, in a large earthmoving project, a major focus will be on selection of an equipment spread for the major work. Attention will be given to details of cut and fill operations at specific locations and to details of other activities after the equipment spread has been selected. On a building or industrial site, type and location of crane or cranes must be selected to meet the needs of a variety of activities on the site and those decisions affect each of the activities. Therefore, from start to finish of planning and estimating, a manager keeps the overall picture in mind so the plan and estimate are realistic and include both direct costs of individual work items and the indirect costs that are necessary to perform the project.

The planning proceeds first by creating, estimating, and comparing broad conceptual plans. Then, for several of the preferred conceptual plans, more detailed plans are developed, estimated, and compared. One continues until one has a plan at a level of detail that is sufficient for organizing the work and whose estimated cost is about as low as can be expected to perform the work properly. A manager approaches the planning and managing of a project knowing that one must select from a variety of overall approaches, each of which can be fine-tuned and adjusted until work is completed. The resources and sequences of operations will vary significantly among the overall approaches but within a set of basic resources and operations there are management actions that will affect final cost and duration.

Computerized Time-Cost Trade-Off (TCT)

As an alternative to the pragmatic approach described above, time-cost trade-off optimization techniques that are based upon CPM methods of anal-

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ysis have been available for almost 30 years. These require that all operations in a project be represented in activities, each of whose start is dependent upon completion of other activities. An input time-cost curve is required of each activity that describes the relationship between activity duration and direct cost for alternative plans for performing the activity.

Activity Time-Cost Curves

Common activity time-cost curves are defined by a normal or least-cost point and a crash or least-duration point. An activity's normal point is typically the method that is expected to complete the activity at a lower cost than other methods. This estimated cost is called its normal or least cost and the estimated duration required to perform the activity at its least cost is called the normal or least-cost duration. If more than one method is expected to complete the activity at its lowest estimated cost, the normal point is that which does so at the lower estimated duration. An activity's crash point is typically the method that is expected to complete the activity at a lower duration than any other method. This estimated duration is called the crash or least duration and the estimated cost required to perform the activity at its least duration is called the crash or least-duration cost. If more than one method is expected to complete the activity at this crash duration, the crash point is the method that is expected to produce the crash duration at the lower cost.

The sum of project activities' normal costs produces the estimate of the projects' least direct cost and CPM calculations using project activities' normal durations will produce the estimated duration to perform the project at its least direct cost. Similarly, CPM calculations using the activities' crash durations produce the estimate of the least possible duration to perform the project and the sum of the crash costs of the activities produces the estimated cost to perform the project at its least estimated duration, if all activities were crashed. In most cases, a project can be performed at its crash duration at far less direct cost than the sum of its activities' crash costs, because only activities that are critical in determining the crash duration need be performed in other than their normal method. And some critical activities will not require crashing.

TCT Modeling

The purpose of TCT is to identify for the project crash duration, and for other durations between it and the project normal duration, the least estimated direct cost at which the project can be performed. The basis of TCT is that for each feasible duration of a project, there is a set of points on the activities' time-cost curves that will produce the duration at the least estimated cost. TCT techniques require identification of an activity's normal and crash points and some assumption of intermediate points between the two. The simplest and most common assumption is that a line drawn between the normal point and the crash point of an activity describes the relationship between the estimated duration and direct cost for the activity. The slope of the line on the time-cost curve describes the rate at which estimated cost rises as the activity is shortened. Some TCT programs allow input of intermediate points or discontinuous functions for activities whose time-cost relationships are not linear or continuous.

The typical starting point for TCT is with each activity at its normal point.

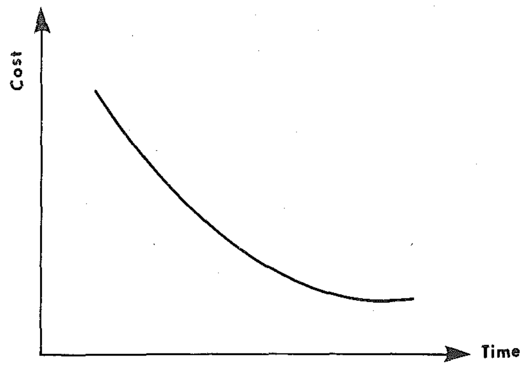

FIG. 2. Project Minimum Direct-Cost Curve

TCT then determines the least direct cost of shortening the duration of the project's critical path by determining the critical activity or activities that can be shortened at least cost. TCT progressively shortens the project duration, at each step shortening the least-cost combination of critical activities. As the critical activities that have the lowest cost slope are shortened to their crash points, further shortening of project duration can only be performed at the increased expense of other critical activities that have steeper rates of cost per day of shortening. In addition, as the project is shortened, float of noncritical activities decreases and additional critical paths form, each of which must be shortened simultaneously if the project duration is to be shortened. Therefore, the project direct-time-cost curve has a generally increasing slope from right to left as it is shortened to its crash duration, as shown in Fig. 2.

The rationale for developing the project time-cost curve is to add the direct

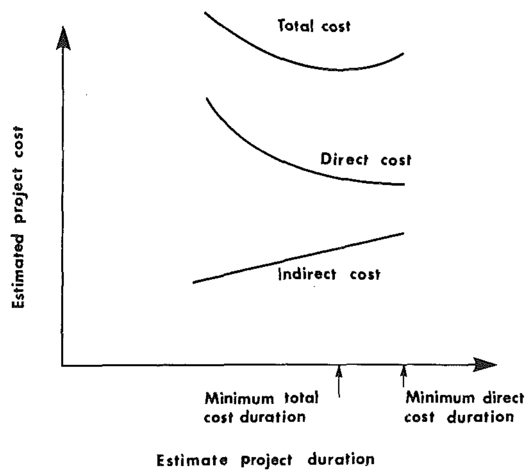

FIG. 3. Relationship Among Estimated Costs and Durations

costs it describes to the project's indirect costs to determine the plan that is estimated to have the lowest total costs. If the estimated indirect costs are an increasing function of estimated duration and the slope of the indirect time-cost function is greater than the slope of direct cost to time line at the minimum direct-cost point, the least total cost will lie at some duration less than the least direct-cost duration, as shown in Fig. 3.

Assumptions of Time-Cost Trade-Off Techniques

Optimization is valid only if several assumptions or requirements are met:

1. Each activity's time-cost function can be described by a single time-cost curve that contains its normal point and its crash point. Because the purpose of time-cost trade-off is to determine the plan or method that will achieve the lowest possible estimated cost for each possible duration, each activity time-cost curve must contain its least-cost point and its least duration and all possible points in between must be defined. Multiple-cost curves mean multiple-cost points for a duration, which means that the cost is not completely defined for each possible activity duration.
2. Every point on an activity's time-cost curve must be compatible with every point on every other activity's time-cost curve. That is, the method or plan used for the normal point of activity A must be compatible with the methods used for the crash points of activities B, C, D, etc. Similarly, the estimated cost of performing activity A cannot be dependent upon whether any other activities, B, C, D, etc., use the methods that produce their normal points or whether they use the methods that produce their crash points. We can therefore also describe this assumption by saying the methods and points on the time-cost curve of each activity are independent of whichever point is selected for the other activities.
3. All costs that are dependent upon a particular method or plan of performing an activity must be included in the time-cost curve of the activity. The relative cost of decreasing the durations is the sole guide for selecting which of the critical activities to shorten. If some of the costs of decreasing the duration of activity A are not included in its time-cost curve, then it may be selected for shortening when shortening another critical activity would shorten project duration at less cost.

We can shorten the assumptions by saying that each activity must have a single time-cost curve in which all points are compatible and independent of all points in other activities' curves and which contains all direct cost differences among its methods.

Assessment of Assumptions

When we compare the description of the planning process to the assumptions of time-cost trade-off technologies, we find that the technologies are a useful concept but they are not directly applicable for planning construction projects.

In general, the first two assumptions are in conflict for many activities in most construction projects. Typically, for any activity, we can either describe the activity with a single time-cost curve in which points are not compatible with some points of other activities or we can describe the activity

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with several time-cost curves, for which for any set of points on other activity time-cost curves there is a compatible point on one of the activity's curves. This is because we can expect that the changes in methods of some activities are related to changes in methods of other activities because they share methods in a sequential or concurrent relationship.

Example

A simple example might be construction of two concrete piers for a bridge. Each pier is to be constructed in three sections. The sequence for each section is: (1) Erect formwork; (2) erect steel; (3) place concrete; (4) cure concrete; and (5) remove forms. With two piers in three stages, this produces six cycles. With five activities per cycle, this produces 30 activities that are strongly related. The project manager identifies two basic alternative approaches:

1. Fabricate form panels in small sections that can be moved by a light crane. Tie steel in place. Place concrete with crane and bucket. Balance the resteel crew with the formwork crew so the time to tie steel is about the same as the time to erect and remove formwork. Schedule each cycle to take two weeks to give pier construction duration at single shift straight time of about 12–16 weeks.
2. Fabricate forms into gang forms that can be removed and erected with a heavy crane. Tie steel in cages and erect cages with heavy crane. Place concrete with crane and bucket. Balance tie- and erect-time of steel cages with erecting and removing forms. Schedule each cycle to take one week to give pier construction duration at single shift straight time of about 6–8 weeks. Formwork materials will be more expensive than forms for the first method and fabrication costs will be greater but erection and removal costs will be lower. Steel erection costs will increase because they will need to be tied together more rigidly. Crane costs will be higher per unit of time.

In addition, the project manager can select crew sizes to lengthen or shorten the time frame. He/she can also plan to work overtime or go to multiple shifts with their additional costs. And a rapid curing concrete can decrease curing time, though at some increase in material cost.

Analysis of Separate Methods

A project manager would probably attack this as two different methods and would develop a preliminary plan and cost estimate, including indirect costs of activities, for each. This analysis would recognize that one method used a light crane and the other a heavier crane. It would also recognize other differences between the basic methods. For either or both, the manager could also explore the cost and duration implications of overtime and multiple shifts. This would recognize that multiple shifts would be required on all concurrent activities on the project, and would also have a minimum number of days that it could be used, and it would recognize the resource-cost differences of multiple-shift work. The analysis of the two methods might suggest to the manager that some combination of the two or some other method might be better and the manager could continue the analysis to include them.

This type of plan development and estimating is normal work for a project manager or superintendent. Each part of analysis is based on an integrated,

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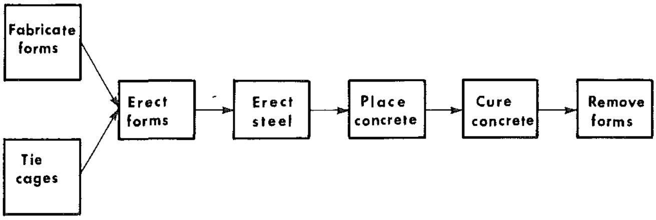

FIG. 4. One Cycle of Pier Concrete Placement

comprehensive plan that is a realistic model of the project. As analysis proceeds, the manager gains more information and insight on the project and can fine tune the plan to save cost and time.

Analysis by TCT

If we try to apply a time-cost optimization algorithm to this reasonably simple example, we run into an impossible situation. It is not difficult to draw an activity diagram for construction. However, the two basic methods have different activities, because tying and erecting steel cages is two activities. This can be handled by inserting the tie cages into both processes but with a duration of zero where steel is tied in place. Such a diagram for a cycle is shown in Fig. 4.

It is impossible to satisfy the three assumptions. Fig. 5 shows time-cost curves for the activities if we look only at their direct costs, which do not include the difference in crane cost. First, we can see that most of the cost functions show one curve for overtime and another for multiple shifts. (In Fig. 5, overtime produces the more steeply sloped curve and multiple shifts produce the less steeply sloped curve.) Second, we see that several of the activities have a different cost function for the first method than for the second method. Therefore, activities can have as many as four time-cost curves (method 1 overtime, method 1 multiple shift, method 2 overtime, method 2 multiple shift). These could be combined into a single curve except for two items:

1. The curves are related with curves of other activities and each point on one is incompatible with many points on others. For example, if multiple shifts are worked on one activity, all concurrent activities will probably also have to work multiple shifts. Therefore, all activities that are concurrent, such as erecting forms on one pier segment and tieing steel for another, are related. If gang forms are fabricated and used for one pier cycle, they will be used for all. Therefore, points on Fabricate Forms [Fig. 5(*a*)], Erect Forms [Fig. 5(*c*)], and Remove Forms [Fig. 5(*e*)] are all related, among all six cycles.

2. The curves would be descending. The right-hand point, called the normal point or least-cost point, is not represented as the least cost. For example, in Erect Forms, the lowest cost estimate is for gang forms and no optimization techniques would consider using the small panels. However, in Fabricate Forms, the lowest cost estimate is for small panels, and no optimization techniques would consider using gang forms. Therefore, time-cost trade-off optimization techniques would never select a compatible set because it would select gang forms for one activity and small panels for the other.

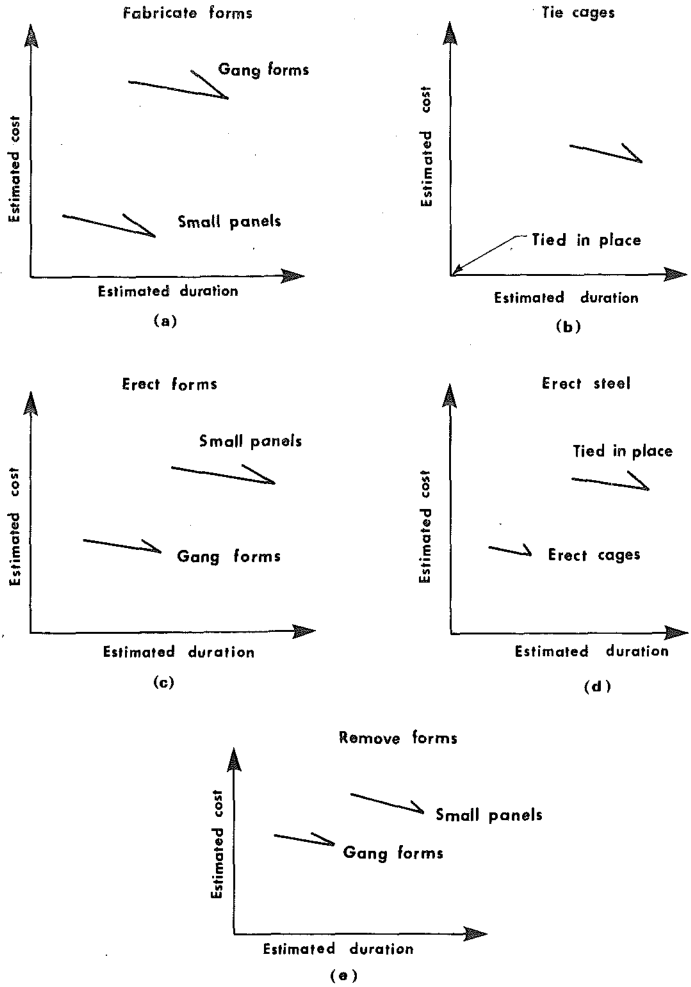

FIG. 5. Time-Cost Curves for (*a*) Fabricate Forms; (*b*) Tie Cages; (*c*) Erect Forms; (*d*) Erect Steel; and (*e*) Remove Forms

In addition, we cannot satisfy the third condition of time-cost trade-off because we cannot represent the cost difference due to change in duration in each activity. This is evident for the difference in cost of a light crane of method 1 and a heavier crane of method 2. The project either uses method 1 with its light crane for all affected activities or method 2 with its heavier crane for all affected activities. There is no method by which we can assign the difference in crane cost to activities such that time-cost trade-off optimization will correctly represent the reduction of time and increase of costs that occurs when managers move from method 1 to method 2. This is because the change from method 1 to method 2 is a discrete change in the overall project that affects multiple activities, not an incremental change in

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a single activity. And TCT has no way of recognizing a change that affects more than one activity.

Assigning Cost of Crane

The difficulty becomes evident when we consider the four possible methods of assigning the cost of the crane in switching from method 1 to method 2 and none of them correctly reflect the actual change. The four methods are to: (1) Assign to each affected activity; (2) spread among affected activities; (3) assign to a single activity; or (4) assign to no activity but put in indirect cost.

In the first case, TCT would count the entire added cost each time any affected activity was selected, which would count the entire cost many times rather than once. This is obviously incorrect. In each of the other three cases, TCT would select an affected critical activity without considering the full cost of the change in cranes. The only one of the three cases that shows the full cost of changing cranes is the next to last (method 3). In that case, the time-cost curves of all affected activities except the one that carries the full burden are meaningless because their shortening does not show the cost impact of the crane in switching methods. And the cost would be so large for the activity that carries the full cost that the activity would not be selected for shortening until the other activities on the critical path, with their artificially low cost of shortening, had been crashed.

Previous Recognition of Problems

Different authors have recognized different problems of TCT. Harris recognized that a single activity's time-cost curve is for a single method and only the quantities of resources should be allowed to vary (Harris 1978). Antill (1982) pointed out that different methods of construction produce different activity time-cost curves. However, this results in activity time-cost curves that are composites of several methods. Neither of these recognizes the interrelatedness of project activities (Antill 1982). Fondahl (1961), however, acknowledged this interdependence by saying that crashing one critical activity may require crashing of other related activities because of industrial or political constraints that are very real in many instances. Moder and Phillips (1970) also pointed out that buying time on a critical activity may automatically require buying time on other activities. Therefore, several authors have indirectly recognized the presence of unrealistic assumptions underlying TCT.

Underlying Basis for Decisions

Though the basic assumptions of TCT are not generally valid for construction projects, the underlying concept of a trade-off between duration and direct cost is valid. We can recognize that different methods produce different estimated project durations and costs. In addition, for each method there can be different possible levels of resources that influence estimated project duration and cost. Therefore, in planning a project, alternative methods and levels of resources should be considered to identify a combination that is estimated to be at or near the least estimated cost for the estimated duration, and a good balance between estimated cost and duration.

Each combination of methods or resources shared can have its own optimum project time-cost curve for different levels of resources or different

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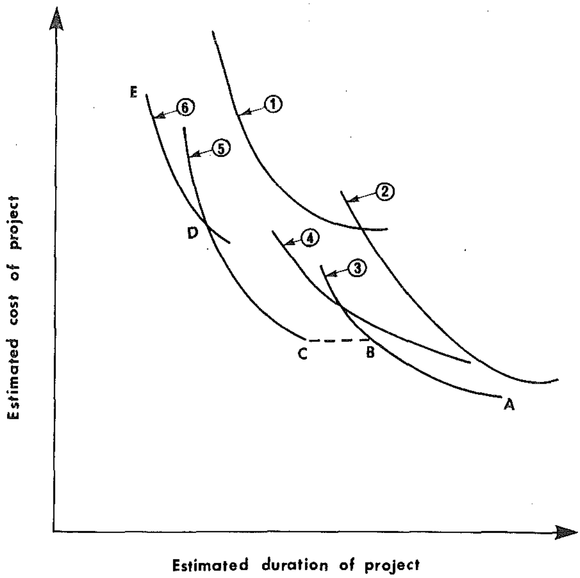

FIG. 6. Project Time-Cost Curves

methods for individual activities. Characteristic curves for direct costs might be as shown in Fig. 6 for six overall project methods. These curves can be compared, if we can assume that they all share the same relationship of indirect cost to project duration. The figure shows that curve 1 is dominated by curve 6, because for any point on curve 1 there is a point on curve 6 that will provide a shorter duration at a lesser cost. Similarly, curve 3 dominates curve 2. The project optimum time-cost curve is a composite of the individual method's time-cost curves, in this case a combination of portions of curve 3, curve 5, and curve 6, described by points A, B, C, D, and E. The portion B–C is actually undefined, because point C would be preferred to any point to the left of B on curve 3 or curve 4 because of its lower cost for equal (or greater) duration. Though curve 4 is not dominated by any single curve, it is never as good as some point on curve 3 or curve 5 and it is therefore not on the optimum curve.

In practice, it is not practical to determine time-cost curves for every possible method. A useful approach can be to identify various overall project methods and compare a reasonable configuration of activity resources and individual methods for each of them to get a comparative cost. Discard the overall methods that do not compare favorably and fine tune those that look good. Continue fine tuning and comparing until a method has been identified that one feels one cannot significantly beat. Then perform detailed planning for that method. The preferred methods for widely different estimated durations will probably differ. One does whatever level of planning and estimating one needs to develop enough information to select among them. However, one does not expect to calculate more than a few of the points that would lie on the optimum project time-cost curve.

An optimization approach to the problem is to define time-cost curves for each activity for each overall project method, perform TCT for each method, draw the comparative curves, and select the plan that has the lowest curve. The TCT for each method is subject to the same assumptions that were shown earlier for TCT. Another method of solution is to formulate the TCT as a set of integer and linear constraints and to optimize it by mixed integer programming. This has been done by the writers, and it will be described in a later paper.

Appendix. References

Antill, J. M. (1982). *Critical path methods in construction practice,* 3rd Ed., John Wiley and Sons, New York, N.Y.

Fondahl, J. W. (1961). "A non-computer approach to the critical path method for the construction industry." *Technical Report No. 9,* The Construction Inst., Dept. of Civ. Engrg., Stanford Univ., Stanford, Calif.

Fulkerson, D. R. (1961). "A network flow computation for project cost curves." *Manage. Sci.*, 7(2), 167–179.

Fulkerson, D. R., and Ford, C. R. (1962). "Flows in networks." The Rand Corporation.

Harris, R. B. (1978). *Precedence and arrow networking techniques for construction.* John Wiley and Sons, New York, N.Y.

Kelley, J. E. (1961). "Critical path planning and scheduling: Mathematical basis." *Oper. Res.*, 9(3), 296–320.

Moder, J. J., and Phillips, C. R. (1970). *Project management with CPM and PERT,* 2nd Ed., Van Nostrand Reinhold Co., New York, N.Y.

Reda, R. M. (1986). "Time-cost trade-off modeling of related activities by using mixed integer programming," thesis submitted to the University of Michigan, at Ann Arbor, Mich., in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

J. Constr. Eng. Manage., 1989, 115(3): 475-486