Infrastructure Development and Expansion under Uncertainty: A Risk-Preference-Based Lattice Approach

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Abstract: Optimal infrastructure development and expansion decision making requires taking into account contingent expansion decisions and underlying uncertainty, such as demand, during the planning period. Recent publications on life cycle management have used lattice models and Markov decision models to evaluate infrastructure development and management policies. In this paper, a risk-preference based lattice model is presented, and it provides the optimal contingent decisions in terms of a risk preference that can be other than risk neutrality. The proposed methodology is demonstrated in an example involving constructing and expanding a parking garage.

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Introduction

There is a growing need, especially in light of homeland security, for maintaining the reliability, sustainability, and robustness of national infrastructure systems, including road, water, electrical, and rail systems. However, many facilities in the United States infrastructure systems are in disrepair and are not able to adapt to new service requirement in a fast changing world. For example, as of December 2001, about 14.2% of highway bridges in the United States were considered structurally deficient, and another 13.8% were deemed functionally obsolete (Federal Highway Administration 2001). One major reason is that the decision making in infrastructure development and operation under uncertainty is far from optimality, reliability, and flexibility.

Comprehensive infrastructure development and management require a life-cycle perspective, or an integrative consideration of planning, design, construction, service, inspection, maintenance, and decommissioning. Analysis based on life-cycle costing has been implemented by various researchers, including Thoft-Christensen and Sorensen (1987), Wirsching and Ortiz (1990), Estes et al. (1997), Hearn and Shim (1998), Abaza (2002), and Zayed et al. (2002). Markov decision processes have been used to model infrastructure management decision making (e.g., Golabi and Thompson 1990; Ben-Akiva et al. 1993; Madanat 1993a,b; Scherer and Glagola 1994; Tao et al. 1995; Smilowitz and Madanat 2000), which yield management policies with least expected costs. Real options approach models infrastructure management processes as contingent decision making and is capable of yielding optimal solutions in light of multiple uncertainties by using least squares Monte Carlo simulation method (Zhao et al. 2004). However, the optimality in decision making is on an expected basis, or a risk-neutral basis. That is, the alternative with the maximum expected benefit or minimum expected cost would be selected without regard to any other factors, such as variance. This may not meet the requirement of reliability, in the sense that system failure has a high probability to prevail, though the decision is viable on an expected value basis. At the same time, risk neutrality may not reflect decision makers' actual risk preference. In fact, decision makers in civil engineering tend to be conservative and prefer alternatives with high reliability.

An improved real-options approach is developed for optimal decision making based on decision maker's risk preference in infrastructure development and management in this paper. Different from traditional real-options approach, the contingent decision making in the proposed approach does not rely on risk neutral optimization, and it not only integrates decision maker's risk preference, but also achieves optimality for reliability and sustainability. The proposed approach contributes to the literature by making a radical shift in the decision process, moreover its practical significance is substantiated by integrating decision making with decision maker's risk preference, and even achieving sustainability

In this paper, the decision-making process is modeled as a multistage stochastic problem with uncertainty modeled by lattice. The decision maker (DM) is assumed to maximize her risk-preference based utility which may be very different from the overall net profit. We have applied the proposed approach to a case study of constructing a parking garage. We demonstrate that the proposed method can select an optimal alternative according to DM's preference. In this paper, we limit our research attention to infrastructure development and expansion decisions, but it can be seen that the proposed method can also be easily extended to incorporate such infrastructure management decisions as inspections, maintenance, repair, and rehabilitation.

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The remainder of this paper is organized as follows: The mathematical formulation for risk-preference based decision making processes under uncertainty and solution method will be first presented. Then the proposed approach is implemented on binomial lattice. Finally, we conclude the paper after demonstrating the proposed approach by a case study of constructing a parking garage.

Risk-Preference Based Decision Making Modeling

The process of infrastructure development and management usually consists of planning, preliminary design, final (detailed) design, and construction. After the infrastructure facility is completed, ongoing operation and service, maintenance, and rehabilitation activities continue until the end of the life cycle, and finally the facility decommissions. Although there are various decisions in the life cycle of an infrastructure facility, we will only focus on infrastructure development and expansion decisions in order to demonstrate the proposed approach in a straightforward way.

In this paper, an infrastructure facility is viewed as a dynamic system. The condition of the infrastructure facility is represented by a set of discrete states, and v_t denotes the system state at time t. The variable representing development and expansion decision at time t is represented by u_t, and $u_t \in U_t$, where U_t is the set of available decisions or real options at time t. The realized value at time t of the underlying uncertainty, such as demand, is denoted by D_t. The timing of event occurrence is as follows. Assume that under state v_t at time t, the uncertainty vector D_t is revealed. Upon observing D_t, the DM: (1) must realize the current system revenue $f_t(v_t;D_t)$, where $f_t(v_t;D_t)$ is revenue function of the infrastructure system in time period t under state v_t, conditioned on the uncertainty realization of D_t at time t; and (2) can strategically utilize available flexibility by making a decision u_t with a cost of $c_t(u_t,v_t)$ incurred, where $c_t(u_t,v_t)$ is cost function for making decision u_t under state v_t at time t.

Let $F_t(v_t;D_t)$ be a function indicating the total expected value (expected profit) of the system for the remaining period at state v_t at time t, and it can be formulated as the following recursive relation:

$$F_t(v_t;D_t) = f_t(v_t;D_t) + e^{-r}E_t[F_{t+1}(v_{t+1};D_{t+1})] - c_t(u_t,v_t) \quad (1)$$

where r=discount rate. In addition, E_t denotes the expectation operator, and the subscript t indicates that the expectation is based on the available information for uncertainty at time t.

In traditional stochastic dynamic programming, the objective is to maximize the expected system value (ESV), then the decision u_t is selected by solving the following optimization problem:

$$\max_{u_t}\{e^{-r}E_t[F_{t+1}(v_{t+1};D_{t+1})] - c_t(u_t,v_t)\} \quad (2)$$

and

$$v_{t+1} = H(v_t,u_t) \quad (3)$$

where $H(\cdot)$=state transition function, and it is assumed that the state transition process can be finished in a time period.

This optimization in Eq. (2) is on a risk-neutral basis, in the sense that decision alternative selection only relies on its expected value, and all the other factors, such as variance, are overlooked. However, this assumption does not reflect the DM's actual risk preference, because in fact, decisions are made conservatively in the field of civil engineering to maintain infrastructure reliability and safety. Even though some researchers impose risk-adjusted discount rate to capture the DM's preference, the DM is still risk-neutral because of the fact that alternative selection only depends on expected value. Therefore, there is a need to model the decision making according to DM's risk preference.

In our proposed methodology, contingent decision u_t is selected by maximizing the DM's utility representing her risk preference. The DM's risk preference is measured by the certainty equivalent (CE) of a random wealth variable (Luenberger 1998). Given a random wealth variable, if its CE of the DM is less than the expected value of this random variable, then the DM is risk-averse; if its CE equals to its expected value, then the DM is risk neutral; otherwise, the DM is risk preferring. The function representing the CE $C(\cdot)$ of a random wealth variable w, subject to normal distribution $N(\mu,\sigma^2)$ is defined as follows:

$$C(w,q) = \sigma\Phi^{-1}(q) + \mu \quad (4)$$

where $\Phi(\cdot)$=normal cumulative density function, and $\Phi^{-1}(\cdot)$=inverse normal cumulative density function. Probability q, defined as risk preference factor, represents the level of the DM's risk preference, and $0<q<1$. It is easy to verify that if $q<0.5$, the DM is risk averse, and with the decrease of q, risk aversion increases; if q=0.5, the DM is risk neutral; otherwise, the DM is risk preferring. For any DM, she would like to select the alternative which brings her the maximum CE of the random wealth. Thus the decision upon observing v_t and D_t at time t can be obtained by solving the following optimization problem:

$$\max_{u_t}\{C[G_t(v_t,u_t;D_t),q]\} \quad (5)$$

Note that the CE can be obtained by directly measuring the DM's utility (Luenberger 1998), thus given the mean and variance of a risky alternative, the risk preference factor q can be reached by the following equation:

$$q = \Phi\left(\frac{C(w,q)-\mu}{\sigma}\right) \quad (6)$$

Next, we will present how to determine parameters for function $C(\cdot)$.

Given the state v_t, decision u_t, and the realized uncertainty D_t, the present worth of the net revenue stream from time t until the end of planning horizon is a random variable, denoted as $G_t(v_t,u_t;D_t)$. Suppose μ_τ $(t+1\leqslant\tau\leqslant T)$ is selected by maximizing the DM's utility representing her risk preference. Thus

$$G_t(v_t,u_t;D_t) = f_t(v_t;D_t) + e^{-r}G_{t+1}(v_{t+1},u_{t+1};D_{t+1}) - c_t(u_t,v_t) \quad (7)$$

Note that the relationship between v_t and v_{t+1} in Eq. (7) is subject to Eq. (3), and D_{t+1} is conditional on D_t. Approximately, $G_t(v_t,u_t;D_t)$ is subject to a nonstandard normal distribution $N(\mu,\sigma^2)$. We assume D_t is realized and u_t is selected with regard to the maximization criterion defined in Eq. (5), then it can be easily seen that

$$\mu = E_t[G_t(v_t,u_t;D_t)] = F_t(v_t;D_t) \quad (8)$$

$$\sigma^2 = \mathrm{Var}[G_t(v_t,u_t;D_t)] = E_t[G_t(v_t,u_t;D_t)^2] - \{E_t[G_t(v_t,u_t;D_t)]\}^2 \quad (9)$$

where $E_t[G_t(v_t,u_t;D_t)^2]$ can be derived by the following recursive relation:

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$$E_t[G_t(v_t,u_t;D_t)^2] = E_t\{[f_t(v_t;D_t) + e^{-r}G_{t+1}(v_{t+1},u_{t+1};D_{t+1}) - c_t(u_t,v_t)]^2\} = E_t\{[f_t(v_t;D_t) - c_t(u_t,v_t)]^2\} + E_t\{2e^{-r}[f_t(v_t;D_t) - c_t(u_t,v_t)] \times G_{t+1}(v_{t+1},u_{t+1};D_{t+1})\} + E_t[e^{-2r}G_{t+1}(v_{t+1},u_{t+1};D_{t+1})^2] \quad (10)$$

Since D_t is realized and u_t is selected at time t, it is obvious that $[f_t(v_t;D_t)-c_t(u_t,v_t)]$ is deterministic, or

$$E_t\{[f_t(v_t;D_t) - c_t(u_t,v_t)]^2\} = [f_t(v_t;D_t) - c_t(u_t,v_t)]^2 \quad (11)$$

$$E_t\{[f_t(v_t;D_t) - c_t(u_t,v_t)]G_{t+1}(v_{t+1},u_{t+1};D_{t+1})\} = [f_t(v_t;D_t) - c_t(u_t,v_t)]E_t[G_{t+1}(v_{t+1},u_{t+1};D_{t+1})] \quad (12)$$

Thus Eq. (10) is reduced to

$$E_t[G_t(v_t,u_t;D_t)^2] = [f_t(v_t;D_t) - c_t(u_t,v_t)]^2 + 2e^{-r}[f_t(v_t;D_t) - c_t(u_t,v_t)] \times E_t[G_{t+1}(v_{t+1},u_{t+1};D_{t+1})] + e^{-2r}E_t[G_{t+1}(v_{t+1},u_{t+1};D_{t+1})^2] \quad (13)$$

Once μ and σ are obtained, then the CE of $G_t(v_t,u_t;D_t)$ can be obtained by Eq. (4).

Given the uncertainty is modeled as lattice, the selection of an appropriate decision making policy can be solved by a stochastic dynamic programming (SDP) approach. This approach applies backwards calculations to determine the optimal system value and alternative selection in the decision making processes.

Implementation on Binomial Lattice

Discrete tree/lattice representations of stochastic processes have been proposed to model uncertainty variables (e.g., Sharpe 1978; Cox et al. 1979; and Rendleman and Barter 1979). Binomial tree/lattice is a simple and versatile model. Assume that the value of the uncertainty variable is known to be D at the beginning of a period. Suppose that it is known that after one time period the value of the uncertainty will be either uD or dD with probabilities p and $1-p$, respectively. The multiples u (>1, for up) and d (<1, for down) are positive constants. Repeating this process to the second period, third period, and so on, results in a (binomial) lattice of the uncertainty, which represents its evolution over time. The term binomial is used because each branching involves only two possible outcomes and that all outcomes at each time period in the lattice follow a binomial distribution. By a binomial model, Trigeorgis and Mason (1987) and Trigeorgis (1996) value real options, such as option to defer, option to expand, and option to contract. The lattice models have been applied in valuing real investment opportunities in research and development (R&D) projects, for example, Herath and Park (1999), Huchzermeier and Loch (2001), and Park and Herath (1999). The trinomial model is similar to the binomial model, which can be seen in an extensive literature (e.g., Luenberger 1998). Zhao and Tseng (2003) model the evolution of parking demand by a trinomial lattice and it is used to evaluate different alternatives and calculated corresponding flexibility values. Since the major contribution of this paper does not lie in lattice construction, we shall use a binomial lattice to illustrate our methodology, and it can be easily extended to another format of lattices, such as trinomial lattices.

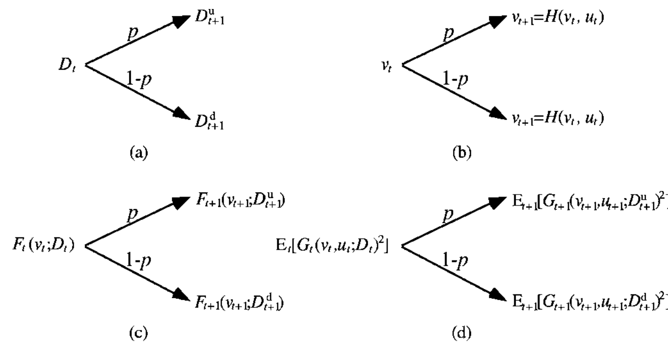

Fig. 1. Binomial branching

In order to implement the risk-preference based approach on lattice model, we first develop the recursive relations for a single-period case. In this paper, the demand is assumed to be the only uncertainty considered. The demand at time t is denoted as D_t. At time $t+1$ the demand will either be D^u_{t+1} with probability p or D^d_{t+1} with probability $1-p$, where $D^u_{t+1}=uD_t$ and $D^d_{t+1}=dD_t$, as shown in Fig. 1(a). We assume that a decision u_t is made at time t upon observing demand uncertainty D_t and system state v_t. After decision u_t is made, the state at $t+1$ will become v_{t+1} when the demand at $t+1$ is either D^u_{t+1} or D^d_{t+1}, as shown in Fig. 1(b). The values, $F_t(\cdot)$, and $E_t[G_t(\cdot)^2]$, corresponding to realized demand shown in Fig. 1(a), are shown in Figs. 1(c and d). From Eqs. (1) and (13), the recursive relations for $F_t(\cdot)$ and $E_t[G_t(\cdot)^2]$ are as follows:

$$F_t(v_t;D_t) = f_t(v_t;D_t) + e^{-r}[pF_{t+1}(v_{t+1};D^u_{t+1}) + (1-p)F_{t+1}(v_{t+1};D^d_{t+1})] - c_t(u_t,v_t) \quad (14)$$

$$E_t[G_t(v_t,u_t;D_t)^2] = [f_t(v_t;D_t) - c_t(u_t,v_t)]^2 + 2e^{-r}[f_t(v_t;D_t) - c_t(u_t,v_t)]\{pE_{t+1}[G_{t+1}(v_{t+1},u_{t+1};D^u_{t+1})] + (1-p)E_{t+1}[G_{t+1}(v_{t+1},u_{t+1};D^d_{t+1})]\} + e^{-2r} \times \{pE_{t+1}[G_{t+1}(v_{t+1},u_{t+1};D^u_{t+1})^2] + (1-p)E_{t+1}[G_{t+1}(v_{t+1},u_{t+1};D^d_{t+1})^2]\} = [f_t(v_t;D_t) - c_t(u_t,v_t)]^2 + 2e^{-r}[f_t(v_t;D_t) - c_t(u_t,v_t)] \times [pF_{t+1}(v_{t+1};D^u_{t+1}) + (1-p)F_{t+1}(v_{t+1};D^d_{t+1})] + e^{-2r}\{pE_{t+1}[G_{t+1}(v_{t+1},u_{t+1};D^u_{t+1})^2] + (1-p)E_{t+1}[G_{t+1}(v_{t+1},u_{t+1};D^d_{t+1})^2]\} \quad (15)$$

After plugging Eq. (15) into Eq. (9), we obtain the variance of $G_t(v_t,u_t;D_t)$, and its mean can be obtained by Eq. (14). Then the most preferred alternative can be selected by Eq. (5).

The above single period process can be easily extended to a

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multiperiod case with similar procedures. The single-period process is just repeated at every node of the lattice, starting from the final period and working backward toward the initial time.

Numerical Example

In this section, we present a case study to illustrate the application of the methodology of this paper. First, data for uncertainty modeling and cost parameters need to be obtained. Second, a binomial lattice is established to model the evolution of uncertainty. Third, the available states, the real options (or available alternatives) corresponding to each state, and even state transition relations need to be identified. Finally, the revised stochastic dynamic programming procedures described in previous sections are applied to determine the appropriate alternative and corresponding values.

Zhao and Tseng (2003) address that an enhanced foundation is viewed as a real option for vertical expansion in the future, in the sense that it provides the DM the flexibility to expand the facility vertically whenever it is preferred. Due to the irreversibility of construction, the expansion of a constructed facility requires the foundation to be enhanced beyond immediate needs. They present a case study of determining the optimal size of the foundation and corresponding maximum system value using a real options approach based on risk-neutral optimization. They not only show the value of flexibility in facility expansion due to uncertainty, but also justify the claim in the literature of real options that traditional discount cash flow methods tend to underestimate the alternatives with flexibility. Although we use a case study with similar basic data, we illustrate a different methodology for infrastructure development and expansion, the risk-preference based decision making method; moreover, we also examine the relationship between risk preference and alternative selection.

A case study of constructing a parking garage for illustration purpose is presented below. The cost data are compiled based on a feasibility study done by a public agency in the Washington, D.C. area. Parking demand is the only uncertainty considered in this case study. The initial daily demand when the garage is just built is assumed to be 250 units of parking spaces. The evolution of demand is modeled by a binomial lattice with $u=1.2$, $d=0.9$, and $p=0.6$. Assume that each parking space can generate a net revenue of \$2,000/year from parking fees, denoted by θ (gross revenue minus operation and maintenance cost). The garage will have multiple levels, and each level can accommodate 100 parking spaces, denoted by m. The fixed cost α_f for initial foundation construction is \$400,000, and the variable cost α_v is \$100,000/level. The variable cost α_s of superstructure and miscellaneous for initial construction is \$700,000/level, and expanding an additional superstructure will cost \$750,000, denoted by α_e.

Note that the parameters for modeling the evolution of uncertainties can be estimated based on historical data. For example, Zhao and Tseng (2003) demonstrate how to estimate the drift and volatility of the parking demand for a parking facility. More technical issues about parameter estimation, such as data requirement, computation efficiency, and estimation consistency, can be found in Matasov (1998). Given the drift and volatility of an uncertainty variable, u, d, p can be easily obtained. Interested readers please refer to Luenberger (1998).

The foundation size (or strength) of the facility, denoted by N, can be represented in terms of the maximal number of levels of superstructure, which the foundation can support safely (Zhao and Tseng 2003). Now the state variable v_t represents the number of levels of the facility in time period t, and therefore $N-v_t$ represents the number of additional levels the facility can expand safely. The unit of time t is in years, and at the beginning of each year, the DM makes expansion decisions (including the decision that she chooses not to expand the facility) upon observing the realized demand uncertainty, and the decision variable u_t represents the number of levels decided to expand, where $0 \leqslant u_t \leqslant N-v_t$, or $U_t=\{u_t | u_t \in [0, N-v_t]\}$. The problem is to determine the optimal foundation size N and the initial number of levels to construct (v_0) at time 0. In the analysis below, we shall not only demonstrate the optimal design of the facility according to the DM's risk preference, N and v_0, but also examine the relationship between alternative selection and the DM's risk preference.

Table 1. Certainty Equivalent (\$$10^3$) of System Values of Design Alternatives

Initial number of levels n_0	Initial foundation reserve (levels), $N-n_0$						
	0	1	2	3	4	5	6
1	779.1	1,842.4	2,354.6	2,409.4	2,328.5	2,218.1	2,106.7
2	1,892.4	2,404.6	2,459.4	2,378.5	2,268.1	2,156.7	2,050.0
3	2,454.6	2,509.4	2,428.5	2,318.1	2,206.7	2,100.0	1,997.4
4	2,322.3	2,251.8	2,146.8	2,038.3	1,933.0	1,830.8	1,730.2
5	1,836.2	1,733.4	1,626.7	1,522.4	1,420.6	1,320.0	1,220.0
6	1,176.6	1,070.1	966.1	864.5	764.1	664.1	564.1
7	426.1	321.8	220.2	119.7	19.7	−80.3	−180.3

Assume that the demand for parking spaces at time t is denoted by D_t, $t=0,1,2,\ldots,T$. Given the initial foundation size N, the value function $f_t(\cdot)$ and expansion cost function $c_t(\cdot)$ are defined for the case study as follows:

$$f_0(v_0;D_0) = -\alpha_f + \alpha_v N + \alpha_s v_0 \tag{16}$$

$$f_t(v_t,D_t) = \theta \min(D_t, v_t m) \tag{17}$$

$$c_t(v_t,u_t) = \alpha_e u_t \tag{18}$$

where $f_0(\cdot)$, $f_t(\cdot)$, and $c_t(v_t,u_t)$ $(t \in [1,T])$ represent the costs of initial foundation and superstructure construction, net revenue in period t, and expansion cost incurred in period t, respectively.

The planning horizon of this case study is set to 15 years with a salvage value of 0 for the garage. The discount rate in this case study is assumed to be 5%, which corresponds to a discount factor of $e^{-0.05}=0.9512$. We also assume the DM's risk preference factor q is 0.2. By using the proposed approach, we obtain that the optimal initial foundation and superstructure design is $N=4$, $v_0=3$. The foundation is enhanced to support expansion of 1 additional level during the initial construction. The optimal CE is \$2,509.4K, and corresponding ESV is \$3,128.3K. Detailed results for different design alternatives, including CEs, ESVs, and standard deviations of system values are given in Tables 1–3, respectively. Since the objective of optimization is to maximize the CE of system value, not to minimize the variance or maximize the ESV, the optimal alternative $(N,v_0)=(4,3)$ does not necessarily produce a minimal standard deviation or a maximum ESV. If different alternatives are evaluated in terms of expected system value, $(N,v_0)=(5,3)$ would be chosen, in the sense that the foundation would be enhanced to support one more level of superstructure which may be expanded in the future compared to the optimal alternative in terms of the DM's certainty equivalent. Since $q=0.2$, the DM is risk averse, and the decision she makes is

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Table 2. Standard Deviations (10^3) of System Values of Design Alternatives

Initial number of levels n_0	Initial foundation reserve (levels), $N-n_0$: 0	1	2	3	4	5	6
1	0.7	64.1	440.3	735.4	913.3	998.8	1,042.1
2	64.1	440.3	735.4	913.3	998.8	1,042.1	1,061.3
3	440.3	735.4[a]	913.3	998.8	1,042.1	1,061.3	1,067.5
4	921.4	1,086.9	1,166.0	1,205.8	1,223.4	1,229.0	1,230.6
5	1,352.4	1,428.8	1,466.5	1,483.0	1,488.2	1,489.6	1,489.6
6	1,705.3	1,742.7	1,758.8	1,763.8	1,765.1	1,765.1	1,765.1
7	1,985.4	2,001.8	2,006.8	2,008.2	2,008.2	2,008.2	2,008.2

[a]Optimal alternative in term of certainty equivalent.

less aggressive than the alternative selected based on maximization on ESV. However, alternative $(N,v_0)=(4,3)$ is also better than $(N,v_0)=(5,3)$ in terms of standard deviation of system value, which means the risk is reduced for a risk-averse DM. Furthermore, some design alternatives, such as $(N,v_0)=(12,7)$ which are viewed profitable in terms of expected system value, are considered unprofitable by this risk-averse DM.

Next we explore the relation between the optimal ESV and the DM's risk preference. The results are summarized in Table 4. When q is less than 0.5, the ESV increases with the increase of q; when q is greater than 0.5, the ESV decreases with the increase of q; and the highest optimal ESV is reached at $q=0.5$. Although this general trend seems very intuitive, the detailed quantitative relation is not obvious. When $q=0.5$, the DM is risk neutral. All decisions made in the whole planning horizon are in terms of maximization on ESV. If we evaluate the decisions made when q is other than 0.5, it is easy to see that they are suboptimal in this optimization. And the more q deviates from 0.5, the smaller the optimal ESV we obtain. In addition, the alternative corresponding to optimal ESV is not sensitive to the change of q, and keeps at $(N,v_0)=(5,3)$. That is due to the insensitivity of optimal ESV, for example, the difference of optimal expected system values between $q=0.5$ and $q=0.9$ is only -3.95%.

We also examine the relation between the optimal CE and the DM's risk preference. The results are summarized in Table 5, from which we can find that the general trend is very straightforward. When q increases, the optimal CE of system value also increases. And the optimal certainty equivalent is very sensitive to the change of q, for example, the difference of that value between $q=0.5$ and $q=0.9$ is 48.03%. That is because q reflects the DM's

Table 3. Expected System Value (10^3) of Design Alternatives

Initial number of levels n_0	Initial foundation reserve (levels), $N-n_0$: 0	1	2	3	4	5	6
1	779.7	1,896.3	2,725.1	3,028.3	3,097.2	3,058.7	2,983.8
2	1,946.3	2,775.1	3,078.3	3,147.2	3,108.7	3,033.8	2,943.3
3	2,825.1	3,128.3[a]	3,197.2[b]	3,158.7	3,083.8	2,993.3	2,895.8
4	3,097.7	3,166.6	3,128.1	3,053.1	2,962.6	2,865.2	2,765.9
5	2,974.4	2,936.0	2,861.0	2,770.5	2,673.1	2,573.7	2,473.7
6	2,611.9	2,536.9	2,446.4	2,349.0	2,249.6	2,149.6	2,049.6
7	2,097.1	2,006.6	1,909.2	1,809.9	1,709.9	1,609.9	1,509.9

[a]Optimal alternative in term of certainty equivalent.

[b]Optimal alternative in term of expected system value.

Table 4. Relationship between Expected System Value and Risk Preference

q	Optimal ESV (10^3)	Corresponding (N,v_0)
0.1	3,146.6	(5,3)
0.2	3,197.2	(5,3)
0.3	3,218.3	(5,3)
0.4	3,235.1	(5,3)
0.5	3,245.8	(5,3)
0.6	3,244.5	(5,3)
0.7	3,216.4	(5,3)
0.8	3,181.5	(5,3)
0.9	3,117.6	(5,3)

risk preference directly, and the higher q is, the more risk-preferring the DM is. Accordingly, with the increase of q, the DM tends to build a larger size of foundation and more levels of superstructure, or in other words, she becomes more aggressive in infrastructure investment and construction. In addition, the ESV corresponding to the recommended alternative shows the similar property to the optimal ESV, as illustrated in Table 4.

The proposed methodology can also be used to evaluate expansion decisions after the facility is constructed. We assume that parking garage has been built and used for a number of years, thus $f_0(v_0;D_0)=0$. Currently the garage has two levels ($v_0=2$) and the foundation can support two additional levels ($N=4$). Suppose the risk performance factor q is still 0.2 and a planning period of 15 years is used. As shown in Table 6, the optimal expansion decision is to expand one additional level of superstructure. This alternative provides the optimal CE, \$4659.4K, and coincidentally, the expected system value, \$5278.3K, also achieves optimality. The relation between alternative selection and the DM's risk preference for facility expansion is similar to that for facility development, and can be interpreted similarly.

Conclusive Remarks

This paper presents a new mythological approach for infrastructure development and expansion. This approach not only explicitly recognizes the uncertainty in the life cycle of infrastructure facilities, but also incorporates it in a decision making model reflecting the DM's risk preference. Moreover, decision making in the model achieves optimality for the DM's risk preference. In fact, infrastructure development and expansion are made conser-

Table 5. The Relationship between Certainty Equivalent and Risk Preference

q	Optimal CE (10^3)	Corresponding (N,v_0)	Corresponding ESV (10^3)
0.1	2,260.8	(3, 3)	2,825.1
0.2	2,509.4	(4, 3)	3,128.3
0.3	2,749.4	(4, 3)	3,134.3
0.4	2,992.4	(5, 3)	3,235.1
0.5	3,245.8	(5, 3)	3,245.8
0.6	3,521.7	(6, 3)	3,223.7
0.7	3,827.6	(6, 3)	3,198.7
0.8	4,198.8	(6, 4)	3,090.8
0.9	4,804.8	(6, 4)	2,911.7

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Table 6. Alternatives for Expansion

$u_0 = v_1 - v_0$	Optimal CE ($\$10^3$)	Corresponding std. dev.	Corresponding ESV ($\$10^3$)
0	4,592.8	680.8	5165.8
1	**4,659.4**	735.4	5278.3
2	4,422.3	921.4	5197.7

vatively to maintain the viability and achieve success, and this paper provides a methodology to easily find the optimal alternative the DM desires.

Several refinements on the methodologies presented in this paper are possible. Some of these are listed below:

1. Linking the risk-preference based optimization to the latent performance evolution processes of infrastructure facilities described in Ben-Akiva et al. (1993), Jiang et al. (2000), and Smilowitz and Madanat (2000). The infrastructure management decisions, including maintenance, repair, and rehabilitation, will be made optimally under uncertainties, embedded in both performance evolution and measurement, according to the DM's risk preference.
2. Applying the risk-preference based optimization to a decision making process embedded with multiple uncertainties and real options, such as the highway development and management process described in Zhao et al. (2004). To model the uncertainties, multifactor lattice or Monte Carlo simulation processes will need to be employed. Interested readers may refer to Tseng and Lin (2004) for establishing a lattice with more than one factor, and Zhao et al. (2004) for simulating multiple uncertainties.

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