PROJECT SCHEDULING USING FUZZY SET CONCEPTS

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ABSTRACT: Probabilistic methods are being used increasingly in construction engineering. However, when a parameter is expressed in linguistic rather than mathematical terms, classical probability theory fails to incorporate the information. The linguistic variables can be translated into mathematical measures using fuzzy set and system theory. A construction management problem, i.e., estimation of the duration of an activity, is solved using this theory. In order to implement the proposed technique, various membership functions need to be estimated using judgment or with the assistance of experts. The proposed technique is not sensitive to small variations in the membership values. This is a very desirable property. However, the method is sensitive to the choice of the fuzzy relations. The uncertainty in the fuzzy relations can be modeled along with other sources of uncertainty. The mean and variance of the parameters involved in the problem under consideration are estimated here using a new method. The method maximizes the product of the sum of the membership associations for a certain frequency of occurrence and the corresponding frequency of occurrence. One of the main advantages of the proposed technique is that it can be easily implemented in existing computer programs for project scheduling.

INTRODUCTION

The construction phase is one of the most important aspects of a civil engineering structure. The success of a project depends on how well the construction phase is carried out. Efficient and economical construction is particularly important because of the increasing complexity of the structures being built, the availability of improved materials and construction equipment, the high level of competition in the industry, high interest rates, high labor cost, inflation, and regulations—all of which make construction more challenging than ever before. Thus, the economy of a project is highly dependent on accurate and elaborate analysis in the early stages of construction.

Practically every construction project is complex to some extent. In a large, complex project, there are hundreds or even thousands of operations and activities. Construction engineers use several techniques, with varying degrees of complexity, to handle project scheduling. Bar charts were one of the early tools for project scheduling (19–21). While bar charts were improved into sophisticated networks, operation research techniques such as linear programming, simulation, time and motion studies, work study methods, value engineering, statistical quality control, and inventory control were increasingly used in the construction industry. Essentially, the initial function of operation research was the analysis of existing construction operations to find more efficient per-

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Note.—Discussion open until November 1, 1984. To extend the closing date one month, a written request must be filed with the ASCE Manager of Technical and Professional Publications. The manuscript for this paper was submitted for review and possible publication on June 13, 1983, This paper is part of the *Journal of Construction Engineering and Management*, Vol. 110, No. 2, June, 1984. ©ASCE, ISSN 0733-9364/84/0002-0189/$01.00. Paper No. 18889.

formance methods. In 1956, the Critical Path Method (CPM) was first formulated and implemented on a computer to schedule construction projects (1,15,18,20,21). In 1957, a technique called the Program Evaluation and Review Technique (PERT) was developed to integrate and coordinate contractors working on a single project (1,3,13,15,19–21). This method uses probability theory, and it is considered a stochastic Critical Path Method. PERT enables management to plan and control projects by knowing the probabilities of occurrence of events. Recently, a method called Graphical Evaluation and Review Techniques (GERT) was developed. GERT is the simplest way of showing the dovetailed operations in a construction project, and is useful when performance of all the operations is not necessary for the completion of a project. Nowadays, the use of these methods is increasing due to the ease of implementing them on computers (1,15,21).

All these methods can be broadly divided into two groups: deterministic and probabilistic. When the information needed for a particular method is assumed to be known during the analysis, it can be considered deterministic. Bar charts and CPM may be classified as deterministic methods. In reality, however, most of the information used in these methods is nondeterministic in nature. In other words, a particular value of a parameter, such as the duration of an activity, is not known with certainty. The incorporation of uncertainty in the parameters in project scheduling techniques leads to probabilistic methods. In these methods, each parameter is generally expressed in terms of mean, standard deviation, coefficient of variation (COV), and appropriate probability distribution. The mean value indicates the expected or average value of a parameter, e.g., duration. The standard deviation indicates the dispersion or scatter of the data from the mean value. The COV is a nondimensional quantity which is the ratio of mean and standard deviation, and is a measure of uncertainty in the parameter (2,4,14). PERT and GERT can be classified as probabilistic methods.

Basically, whether the method is deterministic or probabilistic, all the parameters need to be estimated. However, some parameters may not be estimated properly, since some of the factors that affect these parameters cannot be quantified. Instead, they are qualified. Good or bad weather can be considered as a factor that influences the duration of an activity. However, future weather conditions can be at best described as good or bad, and there is no standard acceptable numerical value attached to this qualitative statement. Consequently, these factors were not properly incorporated in the past in the estimation of the parameters. For example, PERT requires a subjective data interpretation and estimation of the duration of an activity in the form of most probable, pessimistic, and optimistic values (3,15). This subjective estimation procedure does not properly consider the different factors which affect the duration and may result in an inaccurate estimate, and, consequently, in construction delays and losses. The objective of this paper is to propose a method of incorporating such qualitative factors in the estimation of parameters. In a probabilistic formulation, this is basically to study the effects of such qualitative factors on the statistics (mean, standard deviation, COV, etc.) of the parameter. In this paper, the effects of qualitative factors are evaluated using the fuzzy set concept.

Problem Description

Construction projects are divided into activities. The relationship and sequence of these activities are presented in the form of a network. Each activity requires a certain amount of resources which may include time, labor, material, or money. The objective of a construction manager is to find the combination of resources which will minimize the total cost of not only one activity but of all the activities involved in the project, and to finish the project on time. In order to estimate the completion time of a project, the time required to finish each activity (duration of activity) needs to be estimated. The nominal duration, or the mean value and the standard deviation of the duration, or the probability distribution and its parameters of the duration of each activity need to be estimated, depending on which scheduling method is being used, i.e., CPM, PERT, GERT, or simulation techniques. Obtaining reasonable activity duration estimates is important because all subsequent calculations and decisions are based on these estimates. There are many factors which affect the duration of an activity, e.g., weather, labor skill (which changes with time because of the learning effect), superintendent experience, type of equipment used, and level of operators' experience. The effect of these factors on the duration of an activity depends on the activity being considered. For example, the pouring of concrete in an open area is highly sensitive to weather conditions compared to other factors. The construction engineer or superintendent estimates the duration of the activities using experience and judgment. The level of experience and judgment will affect the final outcome and result in uncertainties in the durations. These uncertainties need to be modeled mathematically.

The major problem lies with the factors that are expressed in linguistic, rather than mathematical terms. Good or bad weather, long or short experience, etc., fall into this category. Even the sensitivity of the activity's duration to any of these factors is measured in linguistic terms, e.g., highly sensitive, strong influence, etc. Not only are future weather conditions uncertain at the present time, the definition of good or bad weather complicates the problem. Uncertainties in future weather conditions can be modeled mathematically (2,4–6,11,12); however, additional sources of uncertainty due to the qualitative assessment of good or bad weather need to be considered. The linguistic variables can be translated into mathematical measures by fuzzy sets and systems theory. Conventional procedures like PERT can still be used if updated probabilistic input is used to obtain the required information.

In this paper, the concept of fuzzy sets and systems is introduced and applied in construction project scheduling. Weather conditions and labor skill are considered here to help explain the applicability of the fuzzy set concept in construction scheduling. However, any number of similar factors can be modeled accordingly. The proposed method for systematically qualifying the linguistic factors is realistic and simple. It could easily be implemented in the available computer programs of some project scheduling techniques, such as PERT.

Elements of Fuzzy Set Theory

Since fuzzy sets theory was introduced by Zadeh (23), it has been re-

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ceiving more and more attention from researchers in many different fields (7–10,22). This section summarizes the fundamental definitions and operations of the theory of fuzzy sets that will be used in this paper, as proposed by Zadeh and others (17,24–26).

As mentioned earlier, qualitative factors or linguistic variables (terms) are routinely used in construction project scheduling. These linguistic measures add to the overall uncertainty in the final outcome of any decision process. In order to incorporate these uncertainties in the analysis, the linguistic terms (or variables) need to be translated into mathematical measures.

A linguistic variable is defined as a variable, the values of which are words, phrases, or sentences in a given language. For example, labor experience can be considered as a linguistic variable if the values of this variable, such as "long experience" or "short experience," are not clearly defined but are meaningful classifications nonetheless.

Let X be a universe, or a set of elements, x's, and let A be a subset of X. Each element, x, is associated with a membership value to the subset A, $\mu_A(x)$. If A is an ordinary, non-fuzzy, or crisp set, then the membership function is given by

$$\mu_A(x) = \begin{cases} 1 & \text{if} \quad x \text{ belongs to } A \\ 0 & \text{if} \quad x \text{ does not belong to } A \end{cases} \qquad (1)$$

Eq. 1 means that there are only two possibilities for an element x, either being a member of A, i.e., $\mu_A(x) = 1$, or not being a member of A, i.e., $\mu_A(x) = 0$. In this case, A has sharp boundaries. On the other hand, if the membership function is allowed to take values in the interval (0,1), A is called a fuzzy set. Therefore, A does not have sharp boundaries and the membership of x to A is fuzzy. For example, let x be the level of experience of labor which may range from excellent experience, i.e., $x = 1.0$, to "never been to a construction site," i.e., $x = 0$. By dividing the range of labor experience into increments of 0.1, "short experience," A, as a linguistic variable, can be defined as

$$\begin{aligned} &\text{short experience,} \quad A = [x_1 = 1|\mu_A(x_1) = 0,\ x_2 = 0.9|\mu_A(x_2) = 0, \\ &x_3 = 0.8|\mu_A(x_3) = 0,\ x_4 = 0.7|\mu_A(x_4) = 0,\ x_5 = 0.6|\mu_A(x_5) = 0, \\ &x_6 = 0.5|\mu_A(x_6) = 0,\ x_7 = 0.4|\mu_A(x_7) = 0.1,\ x_8 = 0.3|\mu_A(x_8) = 0.5, \\ &x_9 = 0.2|\mu_A(x_9) = 0.7,\ x_{10} = 0.1|\mu_A(x_{10}) = 0.9, \\ &x_{11} = 0|\mu_A(x_{11}) = 1.0] \end{aligned} \qquad (2)$$

Or, in short, it can be expressed as

$$\text{short experience,} \quad A = (0.4|0.1,\ 0.3|0.5,\ 0.2|0.7,\ 0.1|0.9,\ 0.0|1.0) \qquad (3)$$

The fuzziness in the definition of short experience is obvious from Eq. 2 or 3 as opposed to Eq. 1. It is clear from Eq. 2 or 3 that different values of x or grades of experience have different membership values, $\mu_A(x)$, to the subset A, "short experience." The values of x are 0.4, 0.3, 0.2, 0.1, and 0, and the corresponding membership values are 0.1, 0.5, 0.7, 0.9, and 1.0, respectively. Other values of x have zero membership values to the subset A. These membership values are generally assigned based on subjective judgment with the help of experts and can be updated with more applications of the method in various projects. This is examined in detail in the example. If a crisp set were used in this example, the value of x would be 0.0 with a membership value of 1.0. Similarly, long experience, B, can be defined as

$$\text{long experience,} \quad B = (1.0|1.0,\ 0.9|0.9,\ 0.8|0.7,\ 0.7|0.2,\ 0.6|0.1) \quad \text{(4)}$$

In general, any subset A may be represented by m discrete values (or continuous intervals) of x together with membership values (or continuous membership functions), μ_A, as follows:

$$A = [x_1|\mu_A(x_1),\ x_2|\mu_A(x_2),\ \ldots,\ x_m|\mu_A(x_m)] \quad \text{(5)}$$

in which = should be interpreted as "is defined to be;" and | = a delimiter.

In order to use fuzzy sets in practical problems, some operational rules similar to those used in classical set theory need to be defined. Some of the operational rules used in this paper are described here.

Simple Operations.—The union, U, of fuzzy subsets A and B of a universe, X, corresponds to the connective "or," and its membership function is

$$\mu_{A \cup B}(x) = \max\,[\mu_A(x),\ \mu_B(x)] \quad \text{(6)}$$

The intersection, $\cap$, of fuzzy subsets A and B correspond to the connective "and" and its membership function is

$$\mu_{A \cap B}(x) = \min\,[\mu_A(x),\ \mu_B(x)] \quad \text{(7)}$$

For example, consider "superintendent experience" as a linguistic variable, to be expressed by the fuzzy subset

$$C = (1.0|1.0,\ 0.9|0.8,\ 0.8|0.6,\ 0.7|0.4,\ 0.6|0.2) \quad \text{(8)}$$

and "long labor experience" is represented by Eq. 4. Then, the labor or superintendent experience can be expressed by the union of the fuzzy subsets B and C, and is given by

$$B \cup C = (1|1,\ 0.9|0.9,\ 0.8|0.7,\ 0.7|0.4,\ 0.6|0.2) \quad \text{(9)}$$

On the other hand, the labor and superintendent experience can be expressed by the intersection of the fuzzy subsets B and C, and is given by

$$B \cap C = (1|1,\ 0.9|0.8,\ 0.8|0.6,\ 0.7|0.2,\ 0.6|0.1) \quad \text{(10)}$$

The complement of a fuzzy subset A is denoted by $\bar{A}$, and its membership function is

$$\mu_{\bar{A}}(x) = 1 - \mu_A(x) \quad \text{(11)}$$

Fuzzy Relation.—A fuzzy relation, R, or cartesian-product, $A \times B$, between two fuzzy subsets A (subset of a universe X) and B (subset of a universe Y) has the following membership function:

$$\mu_R(x_i, y_j) = \mu_{A \times B}(x_i, y_j) = \min \mu_A(x_i),\ \mu_B(y_j) \quad \text{(12)}$$

The relation is usually expressed in matrix form as

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$$R = A \times B = A \begin{array}{c|cccc} & \multicolumn{4}{c}{\overbrace{\quad y_1 \qquad\qquad y_2 \qquad\qquad \cdots \qquad\qquad y_m \quad}^{B}} \\ \hline \left\{\begin{array}{c} x_1 \\ x_2 \\ \vdots \\ x_n \end{array}\right. & \begin{array}{c} \mu_R(x_1,y_1) \\ \mu_R(x_2,y_1) \\ \vdots \\ \mu_R(x_n,y_1) \end{array} & \begin{array}{c} \mu_R(x_1,y_2) \\ \mu_R(x_2,y_2) \\ \\ \mu_R(x_n,y_2) \end{array} & \begin{array}{c} \cdots \\ \cdots \\ \\ \cdots \end{array} & \begin{array}{c} \mu_R(x_1,y_m) \\ \mu_R(x_2,y_m) \\ \vdots \\ \mu_R(x_n,y_m) \end{array} \end{array} \quad \ldots\ldots \text{(13)}$$

With this notation, $\mu_R(x_i, y_j)$ indicates the support, or membership, value for the ordered pair (x_i, y_j), and is a measure of association between x_i and y_j. It is computed as the minimum value of the membership values $\mu_A(x_i)$ and $\mu_B(y_j)$.

A fuzzy relation can be expressed in a conditional form. For example, let the relation, R, be defined as: if labor experience is short, then the rate of accidents is medium. Defining "short experience" by Eq. 3 and a medium rate of accidents as

medium rate of accidents = (0.7|0.2, 0.6,|0.7, 0.5|1, 0.4|0.7, 0.3|0.2) .. (14)

the fuzzy relation, R, becomes

$$R = \begin{array}{c|c} & \text{short experience} \\ & \begin{array}{ccccc} 0.4 & 0.3 & 0.2 & 0.1 & 0 \end{array} \\ \hline \text{medium rate of accidents} \begin{array}{c} 0.7 \\ 0.6 \\ 0.5 \\ 0.4 \\ 0.3 \end{array} & \begin{bmatrix} 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.1 & 0.5 & 0.7 & 0.7 & 0.7 \\ 0.1 & 0.5 & 0.7 & 0.9 & 1.0 \\ 0.1 & 0.5 & 0.7 & 0.7 & 0.7 \\ 0.1 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots \text{(15)}$$

Note that the fuzzy subsets "short experience" and "medium rate of accidents" are from two different universes, namely, "experience" and "rate of accidents," respectively. The membership values of the first row in Eq. 15 are evaluated as follows:

$\mu_R(0.7, 0.4) = \min(0.2, 0.1) = 0.1$

$\mu_R(0.7, 0.3) = \min(0.2, 0.5) = 0.2$

$\mu_R(0.7, 0.2) = \min(0.2, 0.7) = 0.2$

$\mu_R(0.7, 0.1) = \min(0.2, 0.9) = 0.2$

$\mu_R(0.7, 0) = \min(0.2, 1.0) = 0.2$

The union of two relations, say R and S, is denoted by $R \cup S$ and has the following membership function:

$\mu_{R \cup S}(x_i, y_j) = \max[\mu_R(x_i, y_j), \ \mu_S(x_i, y_j)]$ .... (16)

On the other hand, the intersection of two fuzzy relations, $R \cap S$, has the following membership function:

$\mu_{R \cap S}(x_i, y_j) = \min[\mu_R(x_i, y_j), \ \mu_S(x_i, y_j)]$ .... (17)

More generally, if R_i, for $i = 1, 2, \ldots, n$, are fuzzy relations, then

$$\mu_{\bigcup_{k=1}^{n} R_k}(x_i, y_j) = \max_{k=1}^{n} [\mu_{R_k}(x_i, y_j)] \quad (18)$$

$$\mu_{\bigcap_{k=1}^{n} R_k}(x_i, y_j) = \min_{k=1}^{n} [\mu_{R_k}(x_i, y_j)] \quad (19)$$

The compliment of fuzzy relations has the following membership functions:

$$\mu_{\bar{R}}(x_i, y_j) = 1 - \mu_R(x_i, y_j) \quad (20)$$

Fuzzy Composition.—If R is a fuzzy relation from X to Y, and S is a fuzzy relation from Y to Z, the composition of R and S is a fuzzy relation that is described by the following membership function:

$$\mu_{RoS}(x_i, z_k) = \max_{y_j} \{\min [\mu_R(x_i, y_j), \ \mu_S(y_j, z_k)]\} \quad (21)$$

Eq. 21 basically evaluates a fuzzy relation between the fuzzy subsets X and Z using the fuzzy relations of X and Z to the common fuzzy subset Y.

An interesting case of fuzzy composition is the composition of a fuzzy subset A with a relation R. The membership function is described by

$$\mu_{AoR}(y_j) = \max_{x_i} \{\min [\mu_A(x_i), \ \mu_R(x_i, y_j)]\} \quad (22)$$

Eq. 22 gives the membership values for a fuzzy subset of y's induced by the fuzzy subset of x's, i.e., induced by the fuzzy subset A.

Eq. 15 represents a fuzzy relation between a medium rate of accidents and short experience. Suppose a new fuzzy subset for a medium rate of accidents has been proposed, and is given by

$$A = (0.7|0.3, 0.6|1, 0.5|0.99, 0.4|0.8, 0.3|0.1) \quad (23)$$

Then, the expectation of "short experience" of labor, B, is given by the composition AoR, i.e.

$$B = (0.4|0.1, 0.3|0.5, 0.2|0.7, 0.1|0.9, 0|0.99) \quad (24)$$

The first element in Eq. 24 is obtained by taking the maximum value of [min (0.3,0.1), min (1,0.1), min (0.99,0.1), min (0.8,0.1), min (0.1,0.1)] = max (0.1,0.1,0.1,0.1,0.1) = 0.1.

The fuzzy condition statement, "if A_1 then B_1 else if A_2 then B_2 ... else if A_n then B_n" is defined to be

$$(A_1 \times B_1) \cup (A_2 \times B_2) \ldots \cup (A_n \times B_n) \quad (25)$$

As an illustration of the application of fuzzy sets to construction project scheduling, the following example is presented. For further details of the fuzzy set theory, the reader is referred to Refs. 7–10, 16, and 22.

Example

In this example, a procedure is presented for estimating the probability mass function of the duration of an activity, which is the basis of any probabilistic project scheduling method. There are many factors which affect the duration of an activity, as was mentioned earlier. For the purpose of illustration, only two factors which affect the duration of a construction activity are considered here in estimating the duration statistics. These two factors are: (1) Weather, which is classified into good, G; medium, M; and bad, B; and (2) skill or labor experience, which is classified into high, H; medium, M; and low, L.

The frequency of occurrence, F, of each classification of the preceding factors, and the adverse consequences of occurrence, C, on the duration of an activity are estimated in linguistic terms, as shown in Table 1. The objective now is to estimate the duration of the activity using the information in Table 1. Then, the impact of these factors on the duration is studied.

The following translation of linguistic variables into fuzzy subsets are assumed. The membership values would vary from project to project. The assumed membership values are selected for illustration purposes only. The selection of a particular set of membership values and its effects on the statistics of the duration are examined later:

$$\text{large} = (0.8|0.5,\ 0.9|0.9,\ 1|1) \quad (26)$$

$$\text{small} = (0|1,\ 0.1|0.9,\ 0.2|0.5) \quad (27)$$

$$\text{medium} = (0.3|0.2,\ 0.4|0.8,\ 0.5|1,\ 0.6|0.8,\ 0.7|0.2) \quad (28)$$

$$\text{very large} = (\text{large})^2 = (0.8|0.25,\ 0.9|0.81,\ 1|1) \quad (29)$$

$$\text{quite small} = (\text{small})^{1.25} = (0|1,\ 0.1|0.88,\ 0.2|0.42) \quad (30)$$

$$\text{very small} = (\text{small})^2 = (0|1,\ 0.1|0.81,\ 0.2|0.25) \quad (31)$$

The concepts of Eqs. 29–31 were proposed by Blockley (8) and Yao (22) and are quite logical from a practical point of view.

Combining the frequency of occurrence, F, and the adverse consequences, C, for each factor $i = 1, \ldots, 6$ and using Eq. 14, the following fuzzy relations can be calculated:

$$F_1 \times C_1 = \begin{array}{c|c} & \text{consequences} = \text{large} \\ & \begin{array}{ccc} 0.8 & 0.9 & 1 \end{array} \\ \hline \text{frequency} = \text{small} \begin{array}{c} 0.0 \\ 0.1 \\ 0.2 \end{array} & \begin{bmatrix} 0.5 & 0.9 & 1 \\ 0.5 & 0.9 & 0.9 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \end{array} \quad (32)$$

$$F_2 \times C_2 = \begin{array}{c|c} & \text{consequences} = \text{medium} \\ & \begin{array}{ccccc} 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \end{array} \\ \hline \text{frequency} = \text{medium} \begin{array}{c} 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.8 & 0.8 & 0.8 & 0.2 \\ 0.2 & 0.8 & 1 & 0.8 & 0.2 \\ 0.2 & 0.8 & 0.8 & 0.8 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array} \quad (33)$$

TABLE 1.—Quantitative Description of Frequency of Occurrences and Consequences

i (1)	Factor (2)	Frequency of occurrence, F_i (3)	Adverse consequences on duration, C_i (4)
1	Weather, B	small	large
2	Weather, M	medium	medium
3	Weather, G	medium	very small
4	Labor, L	large	medium
5	Labor, M	medium	quite small
6	Labor, H	quite small	very small

$$F_3 \times C_3 = \begin{array}{c|c} & \text{consequences = very small} \\ & \begin{array}{ccc} 0 & 0.1 & 0.2 \end{array} \\ \hline \text{frequency = medium} \begin{array}{c} 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.8 & 0.8 & 0.25 \\ 1 & 0.81 & 0.25 \\ 0.8 & 0.8 & 0.25 \\ 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots (34)$$

$$F_4 \times C_4 = \begin{array}{c|c} & \text{consequences = medium} \\ & \begin{array}{ccccc} 0.3 & 0.4 & 0.5 & 0.6 & 0.7 \end{array} \\ \hline \text{frequency = large} \begin{array}{c} 0.8 \\ 0.9 \\ 1 \end{array} & \begin{bmatrix} 0.2 & 0.5 & 0.5 & 0.5 & 0.2 \\ 0.2 & 0.8 & 0.9 & 0.8 & 0.2 \\ 0.2 & 0.8 & 1 & 0.8 & 0.2 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots (35)$$

$$F_5 \times C_5 = \begin{array}{c|c} & \text{consequences = quite small} \\ & \begin{array}{ccc} 0 & 0.1 & 0.2 \end{array} \\ \hline \text{frequency = medium} \begin{array}{c} 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.8 & 0.8 & 0.42 \\ 1 & 0.88 & 0.42 \\ 0.8 & 0.8 & 0.42 \\ 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots (36)$$

$$F_6 \times C_6 = \begin{array}{c|c} & \text{consequences = very small} \\ & \begin{array}{ccc} 0 & 0.1 & 0.2 \end{array} \\ \hline \text{frequency = quite small} \begin{array}{c} 0 \\ 0.1 \\ 0.2 \end{array} & \begin{bmatrix} 1 & 0.81 & 0.25 \\ 0.88 & 0.81 & 0.25 \\ 0.42 & 0.42 & 0.25 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots (37)$$

The total effect of all the factors on the activity duration is obtained by taking the union of Eqs. 32 through 37, using Eq. 18, i.e.

Total effect, $T = (F_1 \times C_1) \cup (F_2 \times C_2) \ldots \cup (F_6 \times C_6)$ .... (38)

Therefore

$$T = \text{frequency} \begin{array}{c|c} & \text{consequences} \\ & \begin{array}{ccccccccccc} 0 & 0.1 & 0.2 & 0.3 & 0.4 & 0.5 & 0.6 & 0.7 & 0.8 & 0.9 & 1.0 \end{array} \\ \hline \begin{array}{c} 0 \\ 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \\ 0.8 \\ 0.9 \\ 1 \end{array} & \begin{bmatrix} 1 & 0.81 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.9 & 1 \\ 0.88 & 0.81 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.9 & 0.9 \\ 0.42 & 0.42 & 0.25 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0.42 & 0.2 & 0.8 & 0.8 & 0.8 & 0.2 & 0 & 0 & 0 \\ 1 & 0.88 & 0.42 & 0.2 & 0.8 & 1 & 0.8 & 0.2 & 0 & 0 & 0 \\ 0.8 & 0.8 & 0.42 & 0.2 & 0.8 & 0.8 & 0.8 & 0.2 & 0 & 0 & 0 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.5 & 0.5 & 0.5 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 & 0.9 & 0.8 & 0.2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.2 & 0.8 & 1 & 0.8 & 0.2 & 0 & 0 & 0 \end{bmatrix} \end{array} \quad (39)$$

To establish a fuzzy relation, $R(c, d_a)$, between the fuzzy subset of consequences, C, and the fuzzy subset of duration of the activity in days, D_a, let the duration be very large if the consequences are large; let the duration be large if the consequences are medium; and let the duration be small if the consequences are small, i.e.

$R{:}D_a$ = very large = (15|0.04, 18|0.64, 20|1), if C is large;

$R{:}D_a$ = large = (15|0.2, 18|0.8, 20|1), if C is medium;

$R{:}D_a$ = small = (18|0.2, 15|0.5, 10|1), if C is small .... (40)

The components of R can be combined using Eqs. 13, 18, and 25. Therefore

$$R_1 = C_1 \times D_{a1} = \begin{array}{c|c} & \text{duration = very large} \\ & \begin{array}{ccc} 15 & 18 & 20 \end{array} \\ \hline \text{consequences = large} \begin{array}{c} 0.8 \\ 0.9 \\ 1 \end{array} & \begin{bmatrix} 0.04 & 0.5 & 0.5 \\ 0.04 & 0.64 & 0.9 \\ 0.04 & 0.64 & 1 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots (41)$$

$$R_2 = C_2 \times D_{a2} = \begin{array}{c|c} & \text{duration = large} \\ & \begin{array}{ccc} 15 & 18 & 20 \end{array} \\ \hline \text{consequences = medium} \begin{array}{c} 0.3 \\ 0.4 \\ 0.5 \\ 0.6 \\ 0.7 \end{array} & \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 0.2 & 0.8 & 0.8 \\ 0.2 & 0.8 & 1 \\ 0.2 & 0.8 & 0.8 \\ 0.2 & 0.2 & 0.2 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots (42)$$

$$R_3 = C_3 \times D_{a3} = \begin{array}{c|c} & \text{duration = small} \\ & \begin{array}{ccc} 10 & 15 & 18 \end{array} \\ \hline \begin{array}{cc} \text{consequences} & 0 \\ \text{= small} & 0.1 \\ & 0.2 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.9 & 0.5 & 0.2 \\ 0.5 & 0.5 & 0.2 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (43)$$

Taking the union of R_1, R_2 and R_3, the relation, R, is obtained

$$R = R_1 \cup R_2 \cup R_3 = \begin{array}{c|c} & \text{duration} \\ & \begin{array}{cccc} 10 & 15 & 18 & 20 \end{array} \\ \hline \begin{array}{cc} & 0 \\ & 0.1 \\ & 0.2 \\ & 0.3 \\ \text{consequences} & 0.4 \\ & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \\ & 1 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.2 & 0 \\ 0.9 & 0.5 & 0.2 & 0 \\ 0.5 & 0.5 & 0.2 & 0 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.2 & 0.8 & 0.8 \\ 0 & 0.2 & 0.8 & 1 \\ 0 & 0.2 & 0.8 & 0.8 \\ 0 & 0.2 & 0.2 & 0.2 \\ 0 & 0.04 & 0.5 & 0.5 \\ 0 & 0.04 & 0.64 & 0.9 \\ 0 & 0.04 & 0.64 & 1 \end{bmatrix} \end{array} \quad \ldots\ldots\ldots \quad (44)$$

A subjective estimation of the duration can be calculated by taking the composition of T and R, which are given in Eqs. 39 and 44, respectively. Therefore, by using Eq. 21, T_oR becomes

$$T_oR = \begin{array}{c|cc} & \text{duration} & \\ & \begin{array}{cccc} 10 & 15 & 18 & 20 \end{array} & \text{; row summation} \\ \hline \begin{array}{cc} & 0 \\ & 0.1 \\ & 0.2 \\ & 0.3 \\ & 0.4 \\ \text{frequency} & 0.5 \\ & 0.6 \\ & 0.7 \\ & 0.8 \\ & 0.9 \\ & 1 \end{array} & \begin{bmatrix} 1 & 0.5 & 0.64 & 1 \\ 0.88 & 0.5 & 0.64 & 0.9 \\ 0.42 & 0.42 & 0.5 & 0.9 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0.8 & 0.5 & 0.8 & 0.8 \\ 1 & 0.5 & 0.8 & 1 \\ 0.8 & 0.5 & 0.8 & 0.8 \\ 0.2 & 0.2 & 0.2 & 0.2 \\ 0 & 0.2 & 0.5 & 0.5 \\ 0 & 0.2 & 0.8 & 0.9 \\ 0 & 0.2 & 0.8 & 1 \end{bmatrix} & \begin{array}{c} 3.14 \\ 2.92 \\ 2.24 \\ 0.80 \\ 2.90 \\ 3.30 \\ 2.90 \\ 0.80 \\ 1.20 \\ 1.90 \\ 2.00 \end{array} \end{array} \quad \ldots\ldots \quad (45)$$

Eq. 45 gives the membership values for different durations of the activity and frequencies of occurrence considering the total effect of the factors given in Table 1. According to Yao (22), a subset, D_a, can be chosen from Eq. 45 as a fuzzy representation of the duration of the activity. The membership value for each value of the duration of the activity in D_a is equal to the largest membership value in the corresponding column for that duration in Eq. 45. However, if Yao's method is used for this case, some of the information given in Eq. 45 would not be considered, i.e., the frequency of occurrence of the total effect of the factors. Therefore, the writers suggest the following method: choose from Eq. 45 a row (subset) which maximizes the product of the row summation given in Eq. 45 and the corresponding frequency. The last row of Eq. 45 gives the maximum value of this product for the problem under consideration. Therefore, the following fuzzy subset of the activity duration, D_a, is chosen:

$$D_a = (10|0.0, 15|0.2, 18|0.8, 20|1) \quad \text{(46)}$$

According to Zadah (25), the probability mass function of the duration activity can be calculated as follows:

$$P(d_a = 10) = \frac{0}{0.2 + 0.8 + 1} = 0 \quad P(d_a = 15) = \frac{0}{0.2 + 0.8 + 1} = 0.1$$

$$P(d_a = 18) = \frac{0.8}{0.2 + 0.8 + 1} = 0.4 \quad P(d_a = 20) = \frac{1}{0.2 + 0.8 + 1} = 0.5 \quad \text{(47)}$$

Therefore, estimates of the mean value, $\bar{d}_a$, and standard deviation of the duration, σ_{D_a}, of the activity duration and calculated as follows:

$$\bar{d}_a = 15 \times 0.1 + 18 \times 0.4 + 20 \times 0.5 = 18.7 \text{ days}$$

$$\sigma^2_{D_a} = 15^2 \times 0.1 + 18^2 \times 0.4 + 20^2 \times 0.5 - (18.7)^2 = 2.41$$

$$\sigma_{D_a} = 1.552 \text{ days}; \quad \mathrm{COV}(D_a) = 0.083 \quad \text{(48)}$$

Similarly, for each activity in the construction project, the probability mass function, mean value and standard deviation of the duration can be calculated. Using the PERT method and the statistics of the activity duration as obtained by the proposed techniques, a project schedule can be determined. It is also possible to generate the duration of each activity as a random variable from the probability mass function calculated by the proposed technique for use in any simulation technique for project scheduling.

If Yao's method (22) is used, the following fuzzy subset of the activity duration, D_a, would result (refer to Eq. 45):

$$D_a = (10|1, 15|0.5, 18|0.8, 20|1) \quad \text{(49)}$$

It should be noted that $d_a = 10$ has a membership value equal to 1 although it is taken from a row corresponding to zero, or 0.5 frequency of occurrence. The probability mass function and the statistics of the duration for Eq. 49 would be

$$P(d_a = 10) = 0.303; \quad P(d_a = 15) = 0.152; \quad P(d_a = 18) = 0.242$$

$$P(d_a = 20) = 0.303; \quad \bar{d}_a = 15.726 \text{ days}; \quad \sigma_{D_a} = 4.1 \text{ days};$$

and $\text{COV}(D_a) = 0.261$ .... (50)

By comparing Eqs. 47 and 50, some important observations can be made. The duration of the activity considered here is expected to be between 10 and 20 days. According to Eq. 47, the probability mass function of the duration is a unimodal function with the modal value (the most probable value) of 20 days. However, according to Eq. 50 (Yao's method), the most probable values are 10 and 20 days which are at the ends of the range. It has a bimodal probability mass function. Any value within the range is less likely. From a practical point of view, this is very unlikely. In probabilistic project scheduling, it is common to model the duration of an activity by a unimodal probability mass function. Therefore, the proposed method might be more realistic than Yao's method.

The success in incorporating the impact of weather and labor skill depends on the assumptions used in translating the linguistic variables into fuzzy sets, i.e. Eqs. 26–31 and 40. The more this technique is used and compared with the actual impact of these factors on activity duration, the higher the level of success will be in choosing the proper membership values in the definition of the linguistic variable. It is possible to define different grades of any linguistic variable, e.g.

Large 1 = $(0.8|0.2, 0.9|0.8, 1|1)$

Large 2 = $(0.8|0.3, 0.9|0.85, 1|1)$

Large 3 = $(0.8|0.4, 0.9|0.9, 1|1)$

Large 4 = $(0.8|0.5, 0.9|0.95, 1|1)$

Large 5 = $(0.8|0.7, 0.9|0.99, 1|1)$ .... (51)

or Large 1 = $(0.8|0.2, 0.9|0.8, 1|1)$

Large 2 = $(\text{Large } 1)^{0.9}$

Large 3 = $(\text{Large } 1)^{0.8}$

⋮

Large 5 = $(\text{Large } 1)^{0.6}$ .... (52)

or any other appropriate definition.

In order to evaluate the sensitivity of the proposed techniques to membership values in the definition of the linguistic variables, the example is solved again using different membership values in defining the fuzzy relation, R, between the consequences and the duration, i.e., Eq. 40. If Eq 40 is changed to

$R: D_a$ = very large = $(15|0.02, 18|0.81, 20|1)$, if C is large;

$R: D_a$ = large = $(15|0.15, 18|0.9, 20|1)$, if C is medium;

$R: D_a$ = small = $(18|0.2, 15|0.5, 10|1)$, if C is small .... (53)

then, according to the proposed method, D_a would be as follows:

$D_a = (10|0, 15|0.15, 18|0.81, 20|1)$ .... (54)

and $$P(d_a = 15) = \frac{0.15}{0.15 + 0.81 + 1} = 0.077$$

$$P(d_a = 18) = 0.413 \quad P(d_a = 20) = 0.510 \quad \ldots \quad (55)$$

The statistics of D_a can be shown to be: $\bar{d}_a = 18.791$ days, $\sigma_{D_a} = 2.035$, and $\mathrm{COV}(D_a) = 0.108$. By comparing Eqs. 46 and 54 and the corresponding statistics, it is clear that the proposed technique is not sensitive to small variations in the membership values.

If a different fuzzy relation, R, between the consequences and the duration is used, e.g.

$$R{:}D_a = \text{large} = (15|0.2, 18|0.8, 20|1) \quad \text{if} \quad C \text{ is large;}$$

$$R{:}D_a = \text{medium} = (10|0.5, 15|0.9, 18|0.9, 20|0.5) \quad \text{if} \quad C \text{ is medium;}$$

$$R{:}D_a = \text{small} = (18|0.2, 15|0.5, 10|1) \quad \text{if} \quad C \text{ is small} \quad \ldots \quad (56)$$

then, the following subset, D_a, can be obtained

$$D_a = (10|0.5, 15|0.9, 18|0.9, 20|0.5) \quad \ldots \quad (57)$$

and $P(d_a = 10) = 0.179$, $P(d_a = 20) = 0.179$, $P(d_a = 15) = 0.321$ ... (58)

and $P(d_a = 18) = 0.321$. The statistics of D_a corresponding to Eq. 58 can be estimated as: $\bar{d}_a = 15.964$ days; $\sigma_{D_a} = 3.30$ days, and $\mathrm{COV}(D_a) = 0.2068$.

Eq. 57 is quite different than Eqs. 46 and 54. Therefore, the proposed technique is sensitive to the choice of the fuzzy relation, R, between the consequence and the duration of the activity.

Conclusions

Different probabilistic methods with various degrees of complexity are being used in construction engineering. However, when a parameter is expressed in linguistic rather than mathematical terms, classical probability theory fails to incorporate the information. The linguistic variables can be translated into mathematical measures by fuzzy sets and systems theory. A construction management problem is solved in this paper using this concept. It is expected that this concept will be used in similar construction management problems in the future. In order to implement the proposed technique, various membership functions need to be estimated, which could be difficult in some cases. However, they could be estimated with the assistance of experts, and the information can be refined as this method is used more frequently.

It is observed here that the proposed technique is not sensitive to small variations in the membership values. This is a very desirable property. However, the method is sensitive to the choice of the fuzzy relation between the consequences and the duration of an activity. This is expected. This relation could be modified with more applications in various projects.

The uncertainty in the fuzzy relations needs to be transformed in such a way that it can be used with other sources of uncertainties obtained by using classical statistical methods. Several methods can be used for this purpose. A new method is proposed for the problem under consideration that maximizes the product of the sum of the membership associations for a certain frequency of occurrence and the corresponding frequency of occurrence. It is expected to be superior to other available methods since it utilizes all the available information. One of the main advantages of the proposed technique is that it can be easily implemented in existing computer programs for project scheduling.

Appendix.—References

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