Kenneth Geiler 431388 Homework 3 Question 2

a. The lower bound will not change, no matter which operations are included because each operation has at most 2 outputs. Therefore the tree will grow at a factor of 2, leaving the lower bound unchanged.
b. Again, the lower bound will not change. This time the function has at most 3 outputs and the tree can grow by a factor of 3. The number of leaves is n factorial which means the height of the tree must be $log_3 n! = log\, n! / log\, 3$. This shows that the lower bound does not change.
c. This time, the lower bound can grow at a factor of log n + 1 due to the fact that PopCount returns a number between 0 and log n. The number of leaves is the factorial of n. Therefore the height is $log_{log\, n + 1} n! = log\, n! / log(log n + 1) = n\, log\, n / log\, log\, n$. Therefore the lower bound becomes smaller by log log n.