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Prediction Markets in FE

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Contribution Weighted Model Based on Order Data

Introduction

A popular method for aggregating the predictions of individuals is the Wisdom of Crowds method. The idea behind this method is to use the predictions of multiple individuals in order to minimize the noise of individual predictions. My goal in this project is to draw inspiration from the Wisdom of the Crowds method and apply an aggregation method to create a model to formulate predictions and place bets in past prediction markets created by the Good Judgement Project and analyze the results of the model.

The dataset that I analyze comes from the Good Judgement Project. The Good Judgement Project was created to identify individuals that are less influenced by cognitive biases deemed "super forecasters" whose predictions can be aggregated to make more informed predictions. The dataset that I used consists of prediction market data that tracks user orders and trades on contracts regarding prediction events. The contracts involve binary questions whose outcome can be verified in the future. The value of the contracts being traded becomes 100 if the event takes place or the value of the contract becomes 0 if the event does not take place.

The model implemented in my project is inspired by the paper "Identifying Expertise to Extract the Wisdom of Crowds" by David Budescu and Eva Chen. In their paper, the authors devise a Contribution Weighted Model to identify which individuals contribute the most to the successful predictions of the group. They then use the predictions of the individuals with positive contributions to predict future events and weigh the predictions of the positive contributors according to the size of their contributions.

To implement a contribution weighted model to the Good Judgement Project dataset which involves prediction markets data and order data, I had to make several modifications to the model that Budescu and Chen used as well as make several inferences about the Good Judgement Project dataset. One such modification was the use of a scoring function based on profit. Whereas Bundescu and Chen used a quadratic scoring function, I assessed the contribution of individuals based on how much profit they made while trading the contracts belonging to the training dataset. I then removed the individuals with negative profits and only use the individuals with positive profits. I then weigh their contributions in my predictions of test dataset events according to the size of their profits.

Methods

Data Preprocessing

I first started out by processing the data. I first had to determine which types of events I wanted to consider. For simplicity I only use prediction markets on binary questions. An example of such a question is “Will family reunions between South and North Korea begin on or before 25 February 2014”. Another issue with prediction markets is the effect that time has on the information that the traders have access to. When predicting an event in the future, individuals have access to more information and thus are more informed as time progresses. In many of the prediction markets, I observed that by the time trading in the market was suspended, the price of the contracts being traded either went to 1 or to 99 indicating that traders were almost certain of the outcome of the binary question. To avoid these prediction markets, I only consider prediction markets where the price from the first trade to the last trade does not differ by more than 30. I also organize the prediction markets that fit this criteria into a training dataset and test dataset. I use the training dataset to identify which individuals are positive contributors and create a contribution weighted model. I then make trades on the prediction markets in the test dataset using the contribution weighted model to assess how well the model performs.

Calculating Profit

In starting to build the contribution weighted model, I first calculated the profit that each individual made in prediction markets belonging to the training dataset. Similar to the method in the paper by Bundescu and Chen, I only want to consider people with positive profits in my model. The individuals with positive profits have been more successful in determining the true outcome of events. Thus I eliminated the individuals with negative profit from my model. I keep track of the exact profit of each individual which will be used in the model to weigh the beliefs of each individual.

Bayesian Contribution Weighted Model

After calculating the profit of each individual, I start to build the contribution weighted model to predict the true probability of an event. Since the value that I am trying to predict takes on a Bernoulli distribution, I use the Beta distribution as the conjugate prior. I use the last market order that an individual puts out before a market closes to infer their true belief of the probability that an event occurs. Then I use the Beta distribution to weigh an individual's true belief such that the mean of the Beta distribution is equal to their true belief. The variance of the Beta distribution depends on the size of the contribution which was determined by the individual's profit. Individuals with a higher contribution will have a Beta distribution with lower variance. This means that our model's aggregate prediction is more certain of the predictions of these individuals with a higher contribution and thus relies more heavily on these individuals. The point estimate of the model's aggregate prediction is the sum of all positive contributor's true belief multiplied by their contribution. The distribution of the model's aggregate prediction will also follow a Beta distribution. That is, the model's aggregate prediction for the true probability of an event occurring will have the following probability density function:

$P(X = x) = \frac{x^{\alpha-1}(1-x)^{\beta-1}}{B(\alpha,\beta)}$ where $B(\alpha, \beta)$ is a normalizing constant

We can calculate α and β by taking into account the true beliefs of each trader j, q_j and their profit or contribution, p_j. We use the following equations where there are N traders and c is a constant that corresponds to the certainty of the model and the variance of aggregated Beta distribution. Lower values of c mean the model is less certain. For my model I used a value of .0001 for c.

$$\alpha = \sum_{j=1}^{N} q_j p_j c$$

$$\beta = \sum_{j=1}^{N} p_j c$$

Inferring True Beliefs

In creating the Bayesian Contribution Weighted Model, I needed to elicit the true belief of individuals. I use the price and quantity of individuals' orders to infer their true belief about the probability of an event occurring. To do this, I assume each individual has the same utility function, the Constant Relative Risk Aversion utility function or (CRRA). Using this utility function, demand is a function of an individual's wealth, the market price, the individual's true belief of the probability an event occurs, and a parameter gamma. We set gamma equal to 3.974 as was recommended by a study conducted by G.W. Harrison. We need to find the wealth of each individual and the way it was calculated for this project was by finding the largest transaction of any individual and multiplying this amount by two. Since we now have values for gamma and wealth, and an individual's demand and the market price are known from an individual's order, we can infer the individual's true belief of the probability of an event occurring. The derivation of inferring an individual's true belief from the CRRA utility function is shown below. Here, d is contracts demanded, y is wealth, π is market price, and gamma is a constant representing the level of risk-aversion.

$d = \frac{y}{\pi} \cdot \frac{\pi(a-1)}{1+\pi(a-1)}$ where $a = \frac{q(1-\pi)^{\frac{1}{\gamma}}}{\pi(1-q)}$

$$d\big(1 + \pi(a-1)\big) = y(a-1)$$

$$d + \pi d(a-1) = y(a-1)$$

$$d + y - \pi d = (y - \pi d)a$$

$$a = \frac{d + y - \pi d}{y - \pi d}$$

Plugging in for a, we get:

$$\frac{q(1-\pi)}{\pi(1-q)} = \frac{d + y - \pi d}{y - \pi d}^{\gamma}$$

$q(1-\pi) = \frac{d+y-\pi d}{y-\pi d}^{\gamma}$ π(1-q)

$$q = \frac{\frac{d + y - \pi d}{y - \pi d}^{\gamma} \pi}{1 - \pi + \frac{d + y - \pi d}{y - \pi d}^{\gamma} \pi}$$

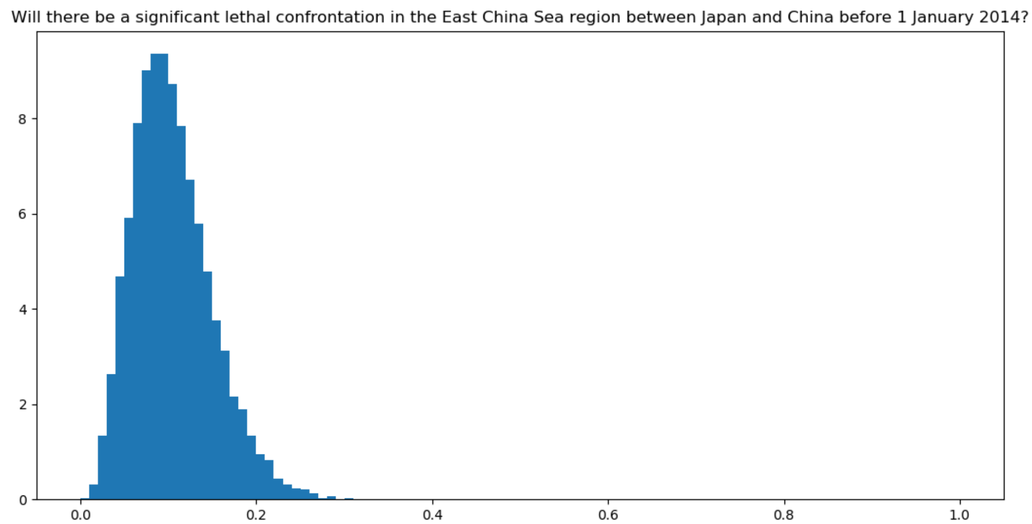
Trading Strategy

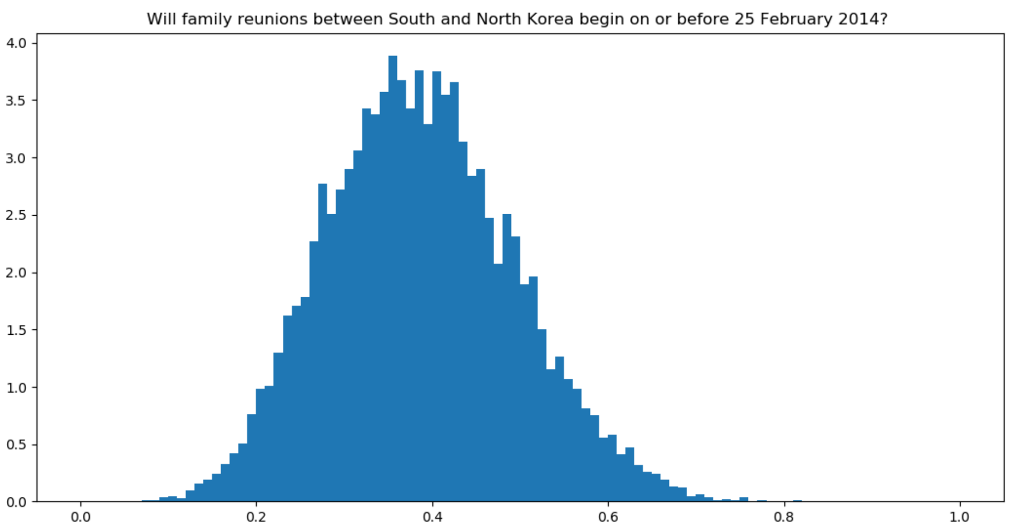
Now that we have constructed a model that generates a point estimate for our prediction of the probability of an event occurring, we can use the Constant Relative Risk Aversion (CRRA) utility function to determine our demand for contracts. To test this strategy, we assume we can only trade at the average of the last three prices of a prediction market in the test dataset. After implementing this trading strategy, we compare the profits using this strategy to the profits of a Naive strategy where we randomly buy and sell contracts.

Results

To test the effectiveness of the model, I calculated the profit using 80% of the prediction markets as test data to set the parameters of the model to make predictions on the remaining 20% of prediction markets in my test data. To avoid outliers in calculating profit, I ran 50 iterations and in each iteration I randomly chose which markets would be used in the training data and which would be use test data. To calculate profit, I set the maximum amount the model can bet on a single prediction market at $10,000. I also ran a control experiment where I randomly choose to buy or sell contracts with an exposure of $5,000 for each prediction market in the test data. After running 50 iterations, I found the average profit of an iteration using the Contribution Weighted Model was $3,919. The average profit of an iteration from randomly choosing to buy or sell contracts was -$689.

Depicted below are two markets that the Contribution Weighted Model made a prediction about the true probability an event occurring. The x-axis represents the model's estimate of the true probability of an event taking place and the y-axis represents the how likely the model thinks the estimated probability is true. The model predicted that neither event in the graphs below would come true but was more certain that the first event would not come true. In reality, the first event did not occur while the second event did occur.





Discussion

While running many iterations and comparing the average profit of the model to the average profit of randomly buying or selling showed that implementing a trading strategy according to the model's predictions consistently resulted in positive profits, there were many assumptions used in the model that need to be addressed. One of the assumptions made in the model that would not hold in real prediction markets is assuming infinite liquidity. The model assumes it can trade up to $10,000 worth of contracts at the average price of the last three trades of the market. However, in reality if a trader wants to buy or sell a large number of contracts, they will likely be affected by slippage, the concept of the execution price being different from the expected price that was requested for the trade. Slippage in real world markets would result in worse prices and lower profits.

Another issue that was not accounted for was the time at which traders submit orders. The model used the last order a trader submitted before the market closed to infer their true belief. However, as more time progresses in prediction markets, traders become more knowledgeable from either new information being released or by accounting for the price and order book of the prediction market. Therefore, a more realistic model would only take into account the open orders at one instant of time and only be able to trade at the price at that time. With this method however, the model would use the opinions of fewer traders and the model's prediction would be heavily skewed towards the traders who happen to have an open order at that instant of time.

Another aspect of the model that could be improved is the assumption that each trader has the same level of risk aversion in eliciting the traders' true beliefs. Rather than using the same value of the risk aversion parameter for each trader, a more accurate method would be to set the risk aversion parameter for each trader according to the total dollar amount of contracts the trader bought or sold. Using this method, traders who traded higher amounts would be considered less risk averse and those who traded less often or in smaller values would be considered more risk averse.

Works Cited

Budescu, David V., and Eva Chen. “Identifying expertise to extract the wisdom of crowds.” Management Science 61.2 (2015): 267-280.

G. W. Harrison et al. Risk aversion in experiments. Emerald Group Publishing, 2008.