Problem Set 1

APS110/164 – Engineering Chemistry and Materials Science

Fall 2023

1. A cylinder of radius R and initial length L_0 is under a tensile load F. Its material has a Young's modulus E.

   (a) Elaborate an equation for the cylinder's elongation ΔL in terms of the given variables.

   (b) Say that $R = 1$ mm, $L_0 = 50$ cm, and $F = 20$ kN. Say that the maximum elongation wanted for this cylinder is $\Delta L = 30$ mm. Of the materials presented on the table below, which can be employed?

Table 1: Young's modulus of different materials.

Material	E (GPa)
Aluminum	70
Brass	100
Steel	207

2. Describe the stress distribution through the thickness of a sheet of tempered glass. Explain how these residual stresses increase the strength and improve the safety of tempered glass.

3. A hypothetical ceramic bar is loaded by 960 N in three-point bending. The bar has a span of 70 cm, a width of 4 cm, and a height of 1.5 cm. This material will break at a stress of 515 MPa.

   (a) Does the bar break under this load? If not, what force is needed for it to break?

   (b) Sketch a stress-strain curve for this sample under a test until fracture. Assuming that this material has a Young's modulus of 300 GPa, identify the position of the fracture point on the curve (that is, its values on both axes).

4. A hypothetical alloy has an atomic weight of 43.1 g/mol, an atomic radius of 0.122 nm, and a density of 6.4 g/cm^3. Is its crystal structure FCC or BCC? Justify your answer.

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1a) $\sigma = E\varepsilon$

$\frac{\sigma}{\varepsilon} = E$

$\frac{\frac{F}{A_0}}{\frac{\Delta l}{l_0}} = E$

$\frac{F l_0}{\pi r^2 \Delta l} = E$

$\boxed{\Delta l = \frac{F l_0}{\pi r^2 E}}$ } F, l_0, r, and E are known values ∴ This equation suffices

b) maximum $\Delta l = 30mm$

$R = 1mm$

$l_0 = 50cm$

$F = 20 kN$

$R = 0.001 m$

$l_0 = 0.5m$

$F = 20000 N$

Aluminum: $\Delta l = \frac{F l_0}{\pi r^2 E}$

$\Delta l = \frac{(20000)(0.5)}{(\pi)(0.001)^2(70000000000)}$

$\Delta l = 0.0454 m$

$\boxed{\Delta l = 45.4 mm}$

Brass: $\Delta l = \frac{F l_0}{\pi r^2 E}$

$\Delta l = \frac{(20000)(0.5)}{(\pi)(0.001)^2(100000000000)}$

$\Delta l = 0.0318 m$

$\boxed{\Delta l = 31.8 mm}$

Steel. $\Delta l = \frac{F l_0}{\pi r^2 E}$

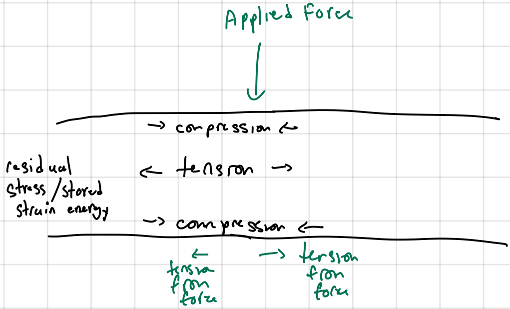
$= \frac{(20000)(0.5)}{(\pi)(0.001)^2(207000000000)}$

$\Delta l = 0.0153m$

$\boxed{\Delta l = 15.3 mm}$

∴ only Steel can be used since $\Delta l < 30mm$

2. Describe the stress distribution through the thickness of a sheet of tempered glass. Explain how these residual stresses increase the strength and improve the safety of tempered glass.



Cross Section

As shown from the diagram, the center of tempered glass experiences tension while the outer surface experiences compression, both as residual stresses. These residual stresses increase the strength of tempered glass because the tension that is caused by an external load is counteracted by the forces of compression. Thus, a greater load is needed to put tempered glass in tension at the bottom. Tempered glass is more safe because at fracture, it will break into small pieces. This is because the residual stress is liberated, causing new surfaces to form. These small pieces are harmless and dull, compared to annealed glass which shatters into large sharp pieces.

3. A hypothetical ceramic bar is loaded by 960 N in three-point bending. The bar has a span of 70 cm, a width of 4 cm, and a height of 1.5 cm. This material will break at a stress of 515 MPa.

(a) Does the bar break under this load? If not, what force is needed for it to break?

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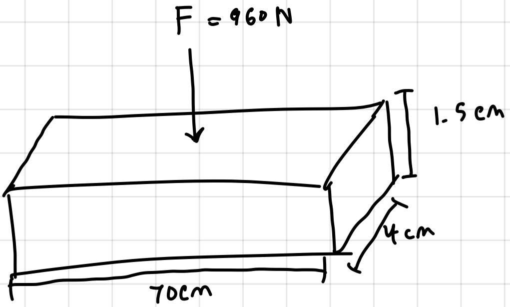


a) $\sigma = \dfrac{3FL}{2wh^2}$

$= \dfrac{(3)(960)(0.7)}{(2)(0.04)(0.015)^2}$ $\boxed{= 112000000 \text{ Pa}}$

112 000 000 Pa < 515 000 000 Pa

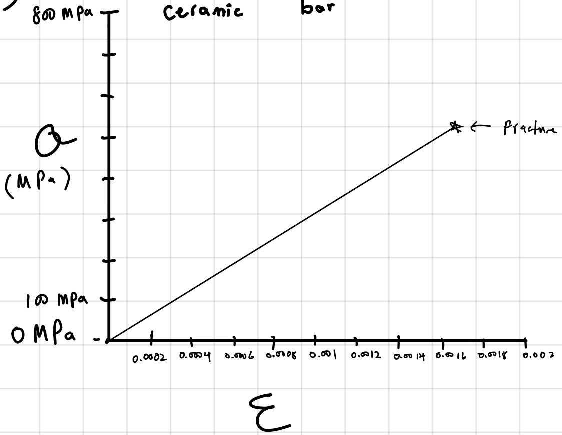
$515\,000\,000 = \dfrac{(3)(F)(0.7)}{(2)(0.04)(0.015)^2}$

F = 4414.29 N

$\boxed{F = 4414 \text{ N}}$

∴ The bar does not break and a force of 4414 N is required for the bar to break

b) Stress vs strain curve for ceramic bar



Ceramics have a linear relationship between stress and strain until fracture

Strain at fracture

$\sigma = E\varepsilon$

$\varepsilon = \dfrac{\sigma}{E}$

$\varepsilon = \dfrac{515\,000\,000}{300\,000\,000\,000}$

$\varepsilon = 0.00172$

4. A hypothetical alloy has an atomic weight of 43.1 g/mol, an atomic radius of 0.122 nm, and a density of 6.4 g/cm^3. Is its crystal structure FCC or BCC? Justify your answer.

atomic radius : 0.0122×10^{-7} cm

Density = 6.4g /cm^3

Let a represent the side of a unit cell cube, let r be the value of the alloy's atomic radius.

FCC

$$a^2 + a^2 = (4R)^2$$
$$2a^2 = 16R^2$$
$$a^2 = 8R^2$$
$$a = 2\sqrt{2}R$$

BCC

$$a^2 + a^2 + a^2 = (4R)^2$$
$$3a^2 = 16R^2$$
$$a^2 = \frac{16}{3}R^2$$
$$a = \frac{4}{\sqrt{3}}R$$

From $\rho = \frac{nA}{V_c N_A}$

$$\rho = \frac{(4)(43.1)}{[(2\sqrt{2})(0.122\times10^{-7})]^3(6.02\times10^{23})}$$

$$\boxed{\rho = 6.97\,g/cm^3}$$

$$\rho = \frac{(2)(43.1)}{[(\frac{4}{\sqrt{3}})(0.122\times10^{-7})]^3(6.02\times10^{23})}$$

$$\boxed{\rho = 6.4\,g/cm^3}$$

The alloy's crystal structure is BCC because the density given other conditions is closer