Numerical Math HW5

By 段浩东 1500017705

8.3

- 节点与权数数值来源：http://www.pitt.edu/~dejong/dbook/code/gauss_hermite.txt
- 计算结果

```
N=3     Value=1.38203307138804665222    Error=0.00164462434490375564
N=4     Value=1.38032975716125538845    Error=-0.00005868988188750812
N=5     Value=1.38039007593565621335    Error=0.00000162889251331677
N=6     Value=1.38038841005073376067    Error=-0.00000003699240913591
N=7     Value=1.38038844775407842924    Error=0.00000000071093553267
N=8     Value=1.38038844703130036962    Error=-0.00000000001184252696
N=9     Value=1.38038844704331653546    Error=0.00000000000017363888
N=10    Value=1.38038844704314134226    Error=-0.00000000000000155431
N=11    Value=1.38038844704314422884    Error=0.00000000000000133227
N=12    Value=1.38038844704314400680    Error=0.00000000000000111022
N=13    Value=1.38038844704314311862    Error=0.00000000000000022204
N=14    Value=1.38038844704314334066    Error=0.00000000000000044409
N=15    Value=1.38038844704314267453    Error=-0.00000000000000022204
N=16    Value=1.38038844704314445089    Error=0.00000000000000155431
N=17    Value=1.38038844704314311862    Error=0.00000000000000022204
N=18    Value=1.38038844704314223044    Error=-0.00000000000000066613
N=19    Value=1.38038844704314356271    Error=0.00000000000000066613
N=20    Value=1.38038844704314178635    Error=-0.00000000000000111022
```

- 计算代码

```
#include<iostream>
#include<cstdio>
#include<cmath>
using namespace std;
char s[250];int i=3;
int main(){
    freopen("node&weight.txt","r",stdin);
    freopen("out.txt","w",stdout);
    gets(s);gets(s);
    double trueval=sqrt(acos(-1))*exp(-0.25);
    for(int i=3;i<=20;i++){
        gets(s);
        double ans=0;
        for(int j=1;j<=i;j++){
            double node,w;
            cin>>node>>w;
            ans+=w*cos(node);
        }
        gets(s);gets(s);
        printf("N=%d\tValue=%.20lf\tError=%.20lf\n",i,ans,ans-trueval);
    }
}
```

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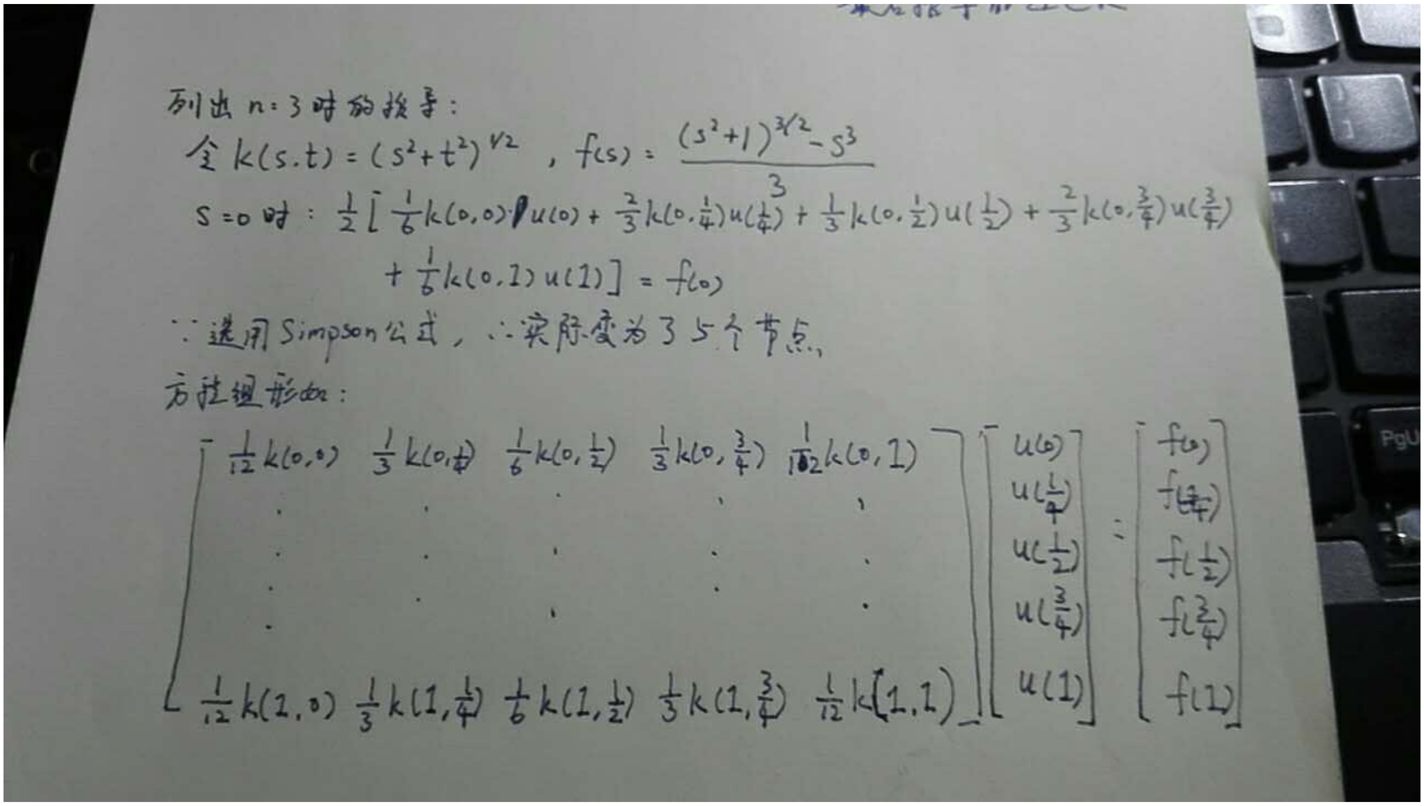
- 推导



列出n=3时的推导：

令 $k(s,t)=(s^2+t^2)^{1/2}$，$f(s)=\frac{(s^2+1)^{3/2}-s^3}{3}$

$s=0$时：$\frac{1}{2}\left[\frac{1}{6}k(0,0)u(0)+\frac{2}{3}k(0,\frac{1}{4})u(\frac{1}{4})+\frac{1}{3}k(0,\frac{1}{2})u(\frac{1}{2})+\frac{2}{3}k(0,\frac{3}{4})u(\frac{3}{4})+\frac{1}{6}k(0,1)u(1)\right]=f(0)$

∵选用Simpson公式，∴实际变为了5个节点，

方程组形如：

$$\begin{bmatrix} \frac{1}{12}k(0,0) & \frac{1}{3}k(0,\frac{1}{4}) & \frac{1}{6}k(0,\frac{1}{2}) & \frac{1}{3}k(0,\frac{3}{4}) & \frac{1}{12}k(0,1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{1}{12}k(1,0) & \frac{1}{3}k(1,\frac{1}{4}) & \frac{1}{6}k(1,\frac{1}{2}) & \frac{1}{3}k(1,\frac{3}{4}) & \frac{1}{12}k(1,1) \end{bmatrix} \begin{bmatrix} u(0) \\ u(\frac{1}{4}) \\ u(\frac{1}{2}) \\ u(\frac{3}{4}) \\ u(1) \end{bmatrix} = \begin{bmatrix} f(0) \\ f(\frac{1}{4}) \\ f(\frac{1}{2}) \\ f(\frac{3}{4}) \\ f(1) \end{bmatrix}$$

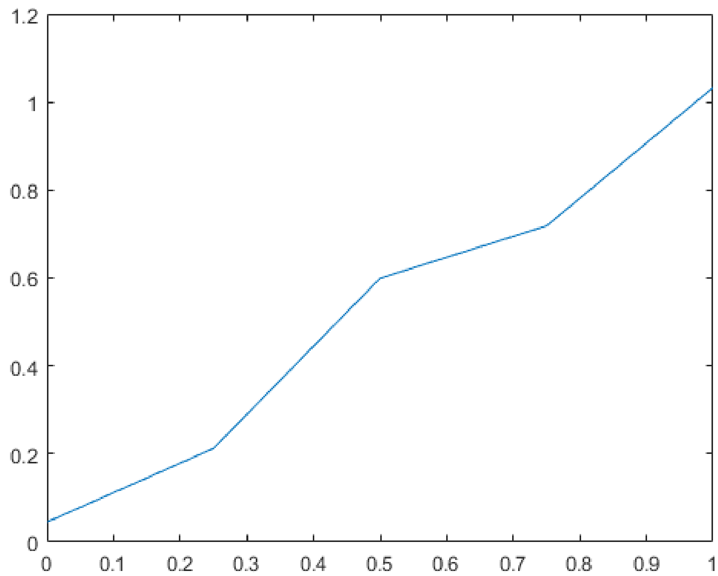
- 计算代码

```
% k函数与f函数如推导过程所定义
% 函数最终画出一图像，即解得的离散点连接为分段线性函数
function []=ex11(m)
mat_k = zeros(2*m-1);
for i = 1:(2*m-1)
    for j = 1:(2*m-1)
        mat_k(i,j) = k((i-1)/(2*m-2),(j-1)/(2*m-2));
        if mod(j,2) == 0
            mat_k(i,j) = mat_k(i,j)*2/3;
        else
            if j==1 | j==(2*m-1)
                mat_k(i,j) = mat_k(i,j)/6;
            else
                mat_k(i,j) = mat_k(i,j)/3;
            end
        end
    end
end
mat_k = mat_k.*(1/(m-1));
cond(mat_k)
vec_f = zeros(2*m-1,1);
for i = 1:2*m-1
    vec_f(i) = f((i-1)/(2*m-2));
end
vec_u = pinv(mat_k)*vec_f;
vec_x = 0:1/(2*m-2):1;
plot(vec_x,vec_u);
```

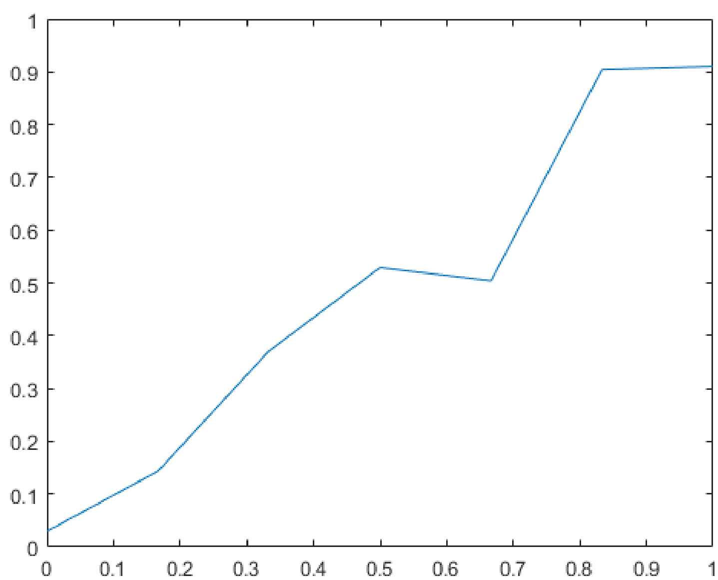
- 经计算发现，n=3时求得的效果最好，猜测可能是因为n较大时矩阵的条件数过大，影响了求解的准确度：

部分图像：

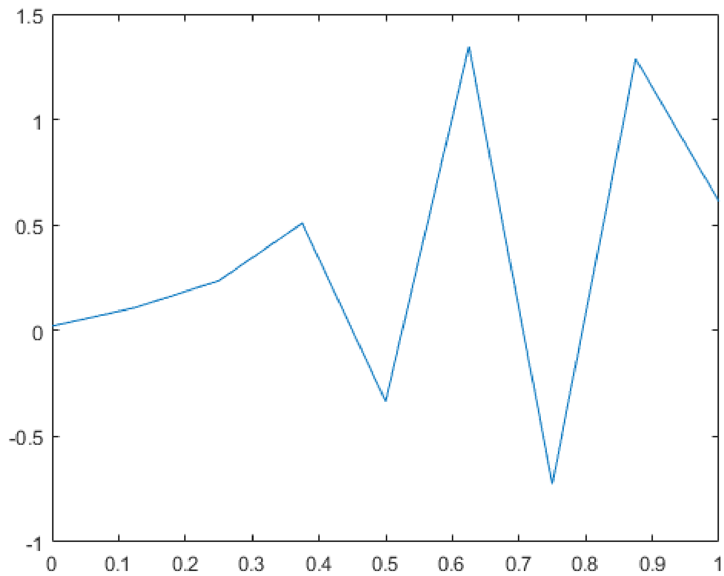
n=3


1.2
1
0.8
0.6
0.4
0.2
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

n=4


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0 0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 1

n=5



- N与条件数

```
N=3 cond=1.8334e+04
N=4 cond=3.3103e+06
N=5 cond=5.3249e+08
N=6 cond=8.1512e+10
N=7 cond=1.2158e+13
```

- 条件数与n约为一指数函数的关系