Solve the system of equations.

$$\left[\begin{array}{c} 3x + 6y = -9 \\ -x - 2y = 3 \\ -\frac{1}{2}x - y = \frac{3}{2} \end{array}\right]$$

- ○ $[x = -3, y = 0]$
- ○ $[x = -2a + 3, y = a, \ \ a \textit{ is any real number}]$
- ○ *no solution*
- ○ $[x = -2a - 3, y = a, \ \ a \textit{ is any real number}]$
- ○ $\left[x = 0, y = -\frac{3}{2}\right]$

Give the solution set to the system of equations

$$\left[\begin{array}{c} x - 4y + 2z = -1 \\ -4x + 2y - 2z = 2 \\ -2x + y - z = 2 \end{array}\right]$$

- ○ $\left[x = \frac{3}{2} - \frac{11}{2}s,\ y = \frac{3}{2} - \frac{7}{2}s,\ z = -\frac{9}{2} - \frac{1}{2}s\right]$
- ○ $\left[x = -\frac{1}{2} - \frac{13}{2}s,\ y = -\frac{1}{2} - \frac{1}{2}s,\ z = -\frac{7}{2} - \frac{11}{2}s\right]$
- ○ $\left[x = -\frac{7}{2} - \frac{1}{2}s,\ y = \frac{5}{2} + \frac{3}{2}s,\ z = -\frac{7}{2} - \frac{1}{2}s\right]$
- ○ *The system does not have a solution.*
- ○ $\left[x = \frac{1}{2} - \frac{13}{2}s,\ y = -\frac{1}{2} - \frac{7}{2}s,\ z = -\frac{7}{2} - \frac{1}{2}s\right]$

Which of the following gives the correct matrix for this system of equations in reduced row echelon form?

$$\begin{bmatrix} 3x_1 - 4x_2 + x_3 + 2x_4 = -3 \\ -2x_1 + 2x_2 - 3x_4 = -4 \\ 3x_1 - 2x_2 + 4x_3 + 2x_4 = -2 \end{bmatrix}$$

○ $\begin{bmatrix} 1 & 0 & 0 & \frac{17}{27} & \frac{50}{27} \\ 0 & 1 & 0 & -\frac{47}{54} & -\frac{4}{27} \\ 0 & 0 & 1 & -\frac{11}{27} & -\frac{53}{27} \end{bmatrix}$

○ $\begin{bmatrix} 1 & 0 & 0 & 3 & \frac{38}{5} \\ 0 & 1 & 0 & \frac{3}{2} & \frac{28}{5} \\ 0 & 0 & 1 & -1 & -\frac{17}{5} \end{bmatrix}$

○ $\begin{bmatrix} -3 & -4 & -1 & 2 & -3 \\ 0 & \frac{14}{3} & \frac{2}{3} & -\frac{13}{3} & -2 \\ 0 & 0 & \frac{27}{7} & -\frac{11}{7} & -\frac{53}{7} \end{bmatrix}$

○ $\begin{bmatrix} 3 & -4 & 1 & 2 & -3 \\ 0 & -\frac{2}{3} & \frac{2}{3} & -\frac{5}{3} & -6 \\ 0 & 0 & 5 & -5 & -17 \end{bmatrix}$

○ $\begin{bmatrix} 1 & 0 & 0 & \frac{1}{3} & \frac{10}{21} \\ 0 & 1 & 0 & -\frac{5}{6} & -\frac{22}{21} \\ 0 & 0 & 1 & -\frac{1}{6} & -\frac{29}{21} \end{bmatrix}$

For what values of *a* does the system below have nontrivial solutions?

$$\begin{bmatrix} 3x + 3y - 2z = 0 \\ -6x + ay + 4z = 0 \\ 2x + 4y + 4z = 0 \end{bmatrix}$$

- ○ -2
- ○ 6
- ○ 2
- ○ -6
- ○ 3

Given:

$$A = \begin{bmatrix} -3 & 3 \\ 3 & 5 \\ 5 & 3 \end{bmatrix} \quad B = \begin{bmatrix} -2 & 3 \\ -4 & -1 \\ -1 & 2 \end{bmatrix} \quad C = \begin{bmatrix} -3 & -2 \\ -3 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 5 & 1 \\ 5 & 6 \end{bmatrix}$$

Compute 3*CD* - 2*D*.

- *Not possible.*
- $\begin{bmatrix} -75 & -45 \\ -30 & 9 \end{bmatrix}$
- $\begin{bmatrix} -85 & -48 \\ -41 & -3 \end{bmatrix}$
- $\begin{bmatrix} -85 & -47 \\ -40 & -3 \end{bmatrix}$
- $\begin{bmatrix} -19 & -9 \\ -20 & -9 \end{bmatrix}$

The matrices A and B are given by

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 0 & 2 & -1 \\ 1 & -3 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -3 & -1 \\ -2 & 2 & 1 \\ 2 & -2 & 1 \end{bmatrix}$$

and D = AB. Give the value of $d_{2,1}$.

- -4
- -3
- -6
- -1
- -8

The matrices A, B and C are given by

$$A = \begin{bmatrix} -2 & 3 \\ -2 & 2 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 3 & 0 \\ -2 & -3 & 2 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -3 & 1 \\ -2 & -3 & 2 \\ 0 & -2 & -3 \end{bmatrix}$$

and D = AB - 3C. Give the value of $d_{2,3}$.

- 1
- -2
- 10
- 9
- -4

Let

$$A = \begin{bmatrix} -2 & -3 & 1 \\ 0 & -1 & 2 \\ 3 & 3 & 1 \end{bmatrix}$$

Find A^{-1}, if it exists.

- $\begin{bmatrix} 9 & -8 & 7 \\ -7 & 7 & -3 \\ -3 & 5 & -1 \end{bmatrix}$
- $\begin{bmatrix} 6 & -7 & 4 \\ -4 & 5 & -5 \\ -2 & 3 & -3 \end{bmatrix}$
- *A does not have an inverse.*
- $\begin{bmatrix} 7 & -6 & 5 \\ -6 & 5 & -4 \\ -3 & 3 & -2 \end{bmatrix}$
- $\begin{bmatrix} 6 & -4 & 7 \\ -5 & 5 & -3 \\ -2 & 5 & -2 \end{bmatrix}$

Use Cramer's rule to give the value of y for the solution set to the system of equations

$$\begin{bmatrix} 4x - 5y - 2z = 1 \\ -x + 3y + 2z = -1 \\ -x + 2y + z = -2 \end{bmatrix}$$

- ○ $y = -12$
- ○ $y = -9$
- ○ $y = -8$
- ○ $y = -11$
- ○ *The system does not have a solution.*

Determine the values of λ for which the system below has nontrivial solutions.

$$\left[\begin{array}{l} (4-\lambda)\,x + 5\,y = 0 \\ 6\,x + (5-\lambda)\,y = 0 \end{array}\right]$$

- ○ $\lambda = \{0, 11\}$
- ○ $\lambda = \{-1, 10\}$
- ○ $\lambda = \{-2, 8\}$
- ○ $\lambda = \{4, 5\}$
- ○ $\lambda = \{-11, -1\}$

Find the eigenvalues and number of independent eigenvectors. (Hint: 4 is an eigenvalue.)

$$\begin{bmatrix} 10 & -6 & 0 \\ 12 & -8 & 0 \\ 12 & -5 & -3 \end{bmatrix}$$

- Eigenvalues: 4, 2, -3; Number of independent eigenvectors: 3
- Eigenvalues: 4, 4, -3; Number of independent eigenvectors: 2
- Eigenvalues: 4, -2, -2; Number of independent eigenvectors: 2
- Eigenvalues: 4, -2, 3; Number of independent eigenvectors: 3
- Eigenvalues: 4, -2, -3; Number of independent eigenvectors: 3

Find the eigenvalues and number of independent eigenvectors. (Hint: 3 is an eigenvalue.)

$$\begin{bmatrix} 1 & 0 & 1 \\ -2 & 2 & 2 \\ -2 & -1 & 5 \end{bmatrix}$$

- ○ Eigenvalues: 3, 3, 2; Number of independent eigenvectors: 2
- ○ Eigenvalues: -3, -3, 2; Number of independent eigenvectors: 2
- ○ Eigenvalues: 3, -3, 2; Number of independent eigenvectors: 3
- ○ Eigenvalues: 3, 2, -2; Number of independent eigenvectors: 3
- ○ Eigenvalues: 3, 2, 2; Number of independent eigenvectors: 3

Question number 13. (5.00 points)

Given the linear differential system **x** ' = A **x** with

$$A = \begin{bmatrix} -2 & -3 \\ -8 & 0 \end{bmatrix}$$

Determine if **u**, **v** form a fundamental solution set. If so, give the general solution to the system.

$$u = \begin{bmatrix} -2e^{4t} \\ 4e^{4t} \end{bmatrix}, \quad v = \begin{bmatrix} 4e^{4t} \\ -8e^{4t} \end{bmatrix}$$

- $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C_1 \begin{bmatrix} -2e^{4t} \\ 4e^{4t} \end{bmatrix} + C_2 \begin{bmatrix} 4e^{4t} \\ -8e^{4t} \end{bmatrix}$

- $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C \begin{bmatrix} e^{t} \\ e^{t} \end{bmatrix} + C \begin{bmatrix} e^{\frac{15}{8}t} \\ -e^{\frac{15}{8}t} \end{bmatrix}$

- $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = C_1 \begin{bmatrix} e^{4t} \\ -2e^{4t} \end{bmatrix} + C_2 \begin{bmatrix} -2e^{4t} \\ 4e^{4t} \end{bmatrix}$

- *Not a fundamental solution set.*

- $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2e^{4t} \\ 4e^{4t} \end{bmatrix} + \begin{bmatrix} 4e^{4t} \\ -8e^{4t} \end{bmatrix}$

Find the general solution to the system $\mathbf{x}' = A\mathbf{x}$ where A is the given matrix.

$$A = \begin{bmatrix} 2 & 2 \\ 2 & 5 \end{bmatrix}$$

- $\mathbf{x}(t) = C_1 e^{6t}\begin{bmatrix} -2 \\ 1 \end{bmatrix} + C_2 e^{t}\begin{bmatrix} 1 \\ 2 \end{bmatrix}$
- $\mathbf{x}(t) = C_1 e^{6t}\begin{bmatrix} -2 \\ 1 \end{bmatrix} + C_2 e^{-t}\begin{bmatrix} 2 \\ -2 \end{bmatrix}$
- $\mathbf{x}(t) = C_1 e^{6t}\begin{bmatrix} 1 \\ 2 \end{bmatrix} + C_2 e^{t}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$
- $\mathbf{x}(t) = C_1 e^{-6t}\begin{bmatrix} 2 \\ -2 \end{bmatrix} + C_2 e^{-t}\begin{bmatrix} -2 \\ 1 \end{bmatrix}$
- $\mathbf{x}(t) = C_1 e^{-6t}\begin{bmatrix} 2 \\ -2 \end{bmatrix} + C_2 e^{t}\begin{bmatrix} 2 \\ 5 \end{bmatrix}$

Question number 15. (5.00 points)

Give the value(s) of *a* for which the columns in the matrix

$$A = \begin{bmatrix} 10 & -6 & -9 + a \\ -6 & 18 & 9 \\ -9 + a & 9 & 9 \end{bmatrix}$$

are linearly dependent.

- ○ $\{2, 15\}$
- ○ $\{-1, 13\}$
- ○ $\{3, 14\}$
- ○ $\{2, 11\}$
- ○ $\{0, 12\}$
- ○ None of the above.

Question number 16. (5.00 points)

Find a fundamental set of solution vectors of the system **x** ' = *A* **x** where *A* is the given matrix:

$$A = \begin{bmatrix} 2 & -3 & 4 \\ -2 & 5 & -4 \\ 0 & 3 & -2 \end{bmatrix}$$

and find the solution that satisfies the initial condition:

$$x(0) = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$$

Hint: -1 is an eigenvalue.

○ $\mathbf{x}(t)= \frac{9}{5}e^{-2t}\begin{bmatrix} 4 \\ 3 \\ -2 \end{bmatrix} + \frac{9}{5}e^{4t}\begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} + \frac{1}{15}e^{-t}\begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$

○ $\mathbf{x}(t)= -\frac{4}{3}e^{2t}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + \frac{18}{5}e^{5t}\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} + \frac{1}{15}e^{-t}\begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$

○ $\mathbf{x}(t)= \frac{2}{3}e^{2t}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + \frac{9}{5}e^{-5t}\begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix} - \frac{2}{3}e^{-t}\begin{bmatrix} 2 \\ -4 \\ -3 \end{bmatrix}$

○ $\mathbf{x}(t)= -\frac{4}{3}e^{-2t}\begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} - \frac{4}{3}e^{-4t}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + \frac{1}{15}e^{-t}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

○ $\mathbf{x}(t)= -\frac{2}{3}e^{2t}\begin{bmatrix} 0 \\ 4 \\ 3 \end{bmatrix} + \frac{9}{5}e^{4t}\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + \frac{1}{15}e^{-t}\begin{bmatrix} -3 \\ 1 \\ 3 \end{bmatrix}$

○ None of the above

Find the general solution to the system **x** ' = *A* **x** where *A* is the given matrix.

$$A = \begin{bmatrix} 4 & 3 \\ -3 & -2 \end{bmatrix}$$

○ **x**(t)= $C_1 e^{-t}\begin{bmatrix} -1 \\ 0 \end{bmatrix} + C_2\left(e^{-t}\begin{bmatrix} -1 \\ 0 \end{bmatrix} + te^{-t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2\left(e^{t}\begin{bmatrix} -1 \\ 0 \end{bmatrix} + te^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + C_2\left(e^{t}\begin{bmatrix} -1 \\ 1 \end{bmatrix} + te^{t}\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{-t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 te^{t}\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

○ **x**(t)= $C_1 e^{t}\begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 te^{t}\begin{bmatrix} -1 \\ 0 \end{bmatrix}$

Find the general solution to the system **x** ' = A **x** where A is the given matrix.

$$A = \begin{bmatrix} 3 & 3 \\ -3 & 3 \end{bmatrix}$$

○ **x**(t)= $C_1 e^{3t}\left(\cos(3t)\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \sin(3t)\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) + C_2 e^{-3t}\left(\cos(3t)\begin{bmatrix} -1 \\ 0 \end{bmatrix} + \sin(3t)\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{3t}\left(\cos(3t)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \sin(3t)\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right) + C_2 e^{3t}\left(\cos(3t)\begin{bmatrix} -1 \\ 0 \end{bmatrix} - \sin(3t)\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{3t}\left(\cos(3t)\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \sin(3t)\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) + C_2 e^{3t}\left(\cos(3t)\begin{bmatrix} -1 \\ 0 \end{bmatrix} + \sin(3t)\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{3t}\left(\cos(3t)\begin{bmatrix} 1 \\ 0 \end{bmatrix} - \sin(3t)\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right) + C_2 e^{3t}\left(\cos(3t)\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \sin(3t)\begin{bmatrix} 0 \\ -1 \end{bmatrix}\right)$

○ **x**(t)= $C_1 e^{3t}\left(\cos(3t)\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \sin(3t)\begin{bmatrix} 1 \\ 2 \end{bmatrix}\right) + C_2 e^{3t}\left(\cos(3t)\begin{bmatrix} 1 \\ 2 \end{bmatrix} + \sin(3t)\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)$

Find the general solution to the system $\mathbf{x}' = A\,\mathbf{x}$ where A is the given matrix.

$$A = \begin{bmatrix} 2 & 2 & 4 \\ 0 & 0 & -4 \\ 0 & 2 & 6 \end{bmatrix}$$

○ $\mathbf{x}(t) = C_1 e^{2t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + C_2 e^{2t}\begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix} + C_3 e^{-4t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

○ $\mathbf{x}(t) = C_1 e^{2t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + C_2 e^{2t}\begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix} + C_3 e^{4t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

○ $\mathbf{x}(t) = C_1 e^{2t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 e^{2t}\begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix} + C_3 e^{4t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

○ $\mathbf{x}(t) = C_1 e^{-2t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + C_2 e^{-2t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_3 e^{4t}\begin{bmatrix} 0 \\ 4 \\ -2 \end{bmatrix}$

○ $\mathbf{x}(t) = C_1 e^{-2t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + C_2 e^{-2t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + C_3 e^{4t}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$

Find the general solution to the system **x** ' = A **x** where A is the given matrix.

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -12 & -16 & -7 \end{bmatrix}$$

○ **x**(t)= $C_1 e^{-2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + C_2\left(e^{-2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + t e^{-2t}\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix}\right) + C_3 e^{-3t}\begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$

○ **x**(t)= $C_1 e^{-2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + C_2\left(e^{-2t}\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + t e^{-2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}\right) + C_3 e^{-3t}\begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$

○ **x**(t)= $C_1 e^{-2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + C_2 e^{-2t}\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + C_3 e^{-3t}\begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$

○ **x**(t)= $C_1 e^{2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix} + C_2\left(e^{2t}\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + t e^{2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}\right) + C_3 e^{3t}\begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$

○ **x**(t)= $C_1 e^{-2t}\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + C_2\left(e^{-2t}\begin{bmatrix} 0 \\ 1 \\ -4 \end{bmatrix} + t e^{-2t}\begin{bmatrix} 1 \\ -2 \\ 4 \end{bmatrix}\right) + C_3 e^{-3t}\begin{bmatrix} 1 \\ -3 \\ 9 \end{bmatrix}$