1 Nomenclature

1.1 Indices

t	Index of time-steps (hours).
g	Index of thermal generating units.
h	Index of hydro plants.
b	Index of buses.
l	Index of transmission lines.
ref	Index of reference bus.

1.2 Variables

php_{ht}	Power output of the hydro plant h in time-step t (p.u. of MW).
Q_{ht}	Turbined outflow of the hydro plant h in time-step t (m^3/s).
s_{ht}	Spillage of the hydro plant h in time-step t (m^3/s).
v_{ht}	Volume of the hydro plant h at the beginning in time-step t (m^3/s).
y_{ht}	Binary variable indicating the on/off status of the hydro plant h in time-step t.
x_{gt}	Binary variable indicating the on/off status of the thermal unit g in time-step t.
z_{gt}	Binary variable indicating the startup of the thermal unit g in time-step t.
w_{gt}	Binary variable indicating the shutdown of the thermal unit g in time-step t.
pt_{gt}	Power output of the thermal unit g in time-step t (p.u. of MW).
pl_{lt}	Active power in the transmission line l in time-step t (p.u. of MW).
θ_{bt}	Voltage angle of the bus b in time-step t (radians).

1.3 System Parameters

$\mathbf{NT}$	Number of periods under study (24 h).
$\mathbf{NG}$	Number of thermal generating units.
$\mathbf{C0}_g$	Unitary variable cost of the thermal unit g (\$/p.u. of MW).
$\mathbf{C1}_g$	Fixed cost of the thermal unit g (\$).
$\mathbf{C2}_g$	Startup cost of the thermal unit g (\$).
$\mathbf{C3}_g$	Shutdown cost of the thermal unit g (\$).
$\mathbf{UT}_g$	Minimum uptime of the thermal unit g (h).
$\mathbf{DT}_g$	Minimum downtime of the thermal unit g (h).
$\mathbf{RU}_g$	Maximum ramp-up of the thermal unit g (p.u. of MW/h).
$\mathbf{RD}_g$	Maximum ramp-down of the thermal unit g (p.u. of MW/h).
$\mathbf{NH}$	Number of hydro plants.
$\mathbf{CQ}$	Constant that converts a one hour of water flow (m^3/s) to water volume (hm^3).
$\mathbf{Y}_{ht}$	Incremental inflow of the hydro plant h in time-step t (m^3/s).
Ω_h	Set of hydro plants that are on the upstream of the hydro h.
τ_{ih}	Water traveling time between hydro plants i and h (h).
$\mathbf{U}_b$	Set of thermal units and hydro plants connected on the bus b.
$\mathbf{B}$	Set of buses of the system.
$\mathbf{L}$	Set of transmission lines of the system.
$\mathbf{L}_b$	Set of transmission lines connected on the bus b.
$\mathbf{P}_{bt}$	Demand of active power on the bus b in time-step t (p.u. of MW).
$\mathbf{SR}_t$	System reserve in time-step t (p.u. of MW).
$\mathbf{X}_{ab}$	Reactance of the line which connects the buses a and b (p.u.).
$\mathbf{a}_m$	Coefficients of the HPF for $m = 0, 1, 2, 3$.
$\mathbf{N1}$	Number of hyperplanes used to represent the HPF.

2 Model Description

The Hydrothermal Unit Commitment (HTUC) considered in this study is related to a centralized tight-pool-based model. The thermal operating generation cost composes the objective funcion. The planning horizon is composed of 24 time-steps of one hour, and the transmission line is modeled as a classic DC model. The formulation of this problem is given by:

$$\min \sum_{t=1}^{\mathbf{NT}} \sum_{g=1}^{\mathbf{NG}} \left(\mathbf{C0}_g pt_{gt} + \mathbf{C1}_g x_{gt} + \mathbf{C2}_g z_{gt} + \mathbf{C3}_g w_{gt} \right) \tag{1}$$

subject to:

$$\sum_{g \in \mathbf{U}_b} pt_{gt} + \sum_{h \in \mathbf{U}_b} php_{ht} - \sum_{l \in \mathbf{L}_b} pl_{lt} = \mathbf{P}_{bt} \qquad \forall b \in \mathbf{B}, t \in \{1, \ldots, \mathbf{NT}\} \tag{2}$$

$$\sum_{g=1}^{\mathbf{NG}} \left(\overline{pt}_g - pt_{gt} \right) x_{gt} + \sum_{h=1}^{\mathbf{NH}} \left(\overline{php}_h - php_{ht} \right) \geq \mathbf{SR}_t \qquad t \in \{1, \ldots, \mathbf{NT}\} \tag{3}$$

$$pl_{lt} = \left(\theta_{at} - \theta_{bt} \right) / \mathbf{X}_{ab} \qquad \forall l = (a,b) \in \mathbf{L}, t \in \{1, \ldots, \mathbf{NT}\} \tag{4}$$

$$\underline{pl}_l \leq pl_{lt} \leq \overline{pl}_l \qquad \forall l \in \mathbf{L}, t \in \{1, \ldots, \mathbf{NT}\} \tag{5}$$

$$\underline{\theta}_b \leq \theta_{bt} \leq \overline{\theta}_b \qquad \forall b \in \mathbf{B}, t \in \{1, \ldots, \mathbf{NT}\} \tag{6}$$

$$\theta_{\text{ref}} = 0 \qquad t \in \{1, \ldots, \mathbf{NT}\} \tag{7}$$

$$z_{gt} - w_{g,t} = x_{gt} - x_{g,t-1} \qquad g \in \{1, \ldots, \mathbf{NG}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{8}$$

$$\sum_{k=t-\mathbf{UT}_g+1}^{t} z_{gk} \leq x_{gt} \qquad g \in \{1, \ldots, \mathbf{NG}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{9}$$

$$\sum_{k=t-\mathbf{DT}_g+1}^{t} w_{gk} \leq 1 - x_{gt} \qquad g \in \{1, \ldots, \mathbf{NG}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{10}$$

$$pt_{gt} - pt_{g,t-1} \leq \mathbf{RU}_g x_{g,t-1} + \underline{pt}_g z_{gt} \qquad g \in \{1, \ldots, \mathbf{NG}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{11}$$

$$pt_{g,t-1} - pt_{gt} \leq \mathbf{RD}_g x_{gt} + \underline{pt}_g w_{gt} \qquad g \in \{1, \ldots, \mathbf{NG}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{12}$$

$$\underline{pt}_g x_{gt} \leq pt_{gt} \leq \overline{pt}_g x_{gt} \qquad g \in \{1, \ldots, \mathbf{NG}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{13}$$

$$php_{ht} \leq \sum_{k=1}^{\mathbf{N1}} \left(\mathbf{a}_{0k} v_{ht} \cdot y_{ht} + \mathbf{a}_{1k} Q_{ht} + \mathbf{a}_{2k} s_{ht} + \mathbf{a}_{3k} y_{ht} \right) \qquad h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{14}$$

$$v_{ht} + \mathbf{CQ} \left(Q_{ht} + s_{ht} - \sum_{i \in \Omega_h} \left(Q_{i,t-\tau_{ih}} + s_{i,t-\tau_{ih}} \right) \right) = v_{h,t-1} + \mathbf{CQ} \cdot \mathbf{Y}_{ht} \qquad h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{15}$$

$$v_{h,\mathbf{NT}} \geq v_h^* \qquad h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{16}$$

$$\underline{v}_h \leq v_{ht} \leq \overline{v}_h \qquad h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{17}$$

$$\underline{Q}_h y_{ht} \leq Q_{ht} \leq \overline{Q}_h y_{ht} \qquad h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{18}$$

$$0 \leq s_{ht} \leq \overline{s}_{ht} \qquad h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{19}$$

$$y_{ht}, z_{gt}, w_{gt}, x_{gt} \in \{0, 1\} \qquad g \in \{1, \ldots, \mathbf{NG}\}, h \in \{1, \ldots, \mathbf{NH}\}, t \in \{1, \ldots, \mathbf{NT}\} \tag{20}$$

The objective function is composed of the thermal costs. For the hydro plants, we are not considering the operational costs. Nevertheless, centralized tight-pool-based models commonly employ the piecewise convex linear functions to estimate future water value obtained through long-term operational planning models [2].

Equations (2)-(7) are related to the DC network model with reserve constraints, where the transmission line l connects the buses a and b has reactance $\mathbf{X}_{ab}$. The thermal modeling is represented by Equations (8)-(13), based on model presented in [1]. Equation (14) represents the Hydropower Production Function (HPF) model. Equation (15) is the water balance equation, and Equation (16) imposes volume targets at the end of the planning horizon. Equations (13), (17)-(19) are limits on variables, and Equation (20) contains the binary variables of the model.

References

[1] Hedman, K. W., Ferris, M. C., O'Neill, R. P., Fisher, E. B., and Oren, S. S. Co-optimization of generation unit commitment and transmission switching with n-1 reliability. *IEEE Transactions on Power Systems 25*, 2 (2010), 1052–1063.

[2] Wallace, S. W., and Fleten, S.-E. Stochastic programming models in energy. In *Stochastic Programming*, vol. 10 of *Handbooks in Operations Research and Management Science*. Elsevier, 2003, pp. 637–677.