Griffiths: Introduction to Electrodynamics

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1 Mathematical Foundations

2 Electrostatics

In electrodynamics, we are often interested in the force a collection of source charges $q_1 \dots q_n$ exerts on a test charge Q.

This problem becomes difficult if our source charge(s) are moving, so we constrain our cases to those where these charges are fixed. Electrostatics is the study of these cases.

2.1 The Electric Field

Definition 2.1. The principle of superposition tells us the interaction between charges are independent from other interactions. If F_1 is the force of q_1 acting on Q, then a collection of $\{q_i\}$ is simply $F = \sum_i q_i$.

Definition 2.2. Coulomb's law gives us the force on Q from q:

$$\frac{1}{4\pi\epsilon_0}\frac{qQ}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}$$

Definition 2.3. We can start to think of the force from $\{q_i\}$ in aggregate by factoring a common Q from a sum of forces:

$$F = Q\boldsymbol{E}$$

$$\boldsymbol{E} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{\boldsymbol{\mathscr{r}}^2}\hat{\boldsymbol{\mathscr{r}}}$$

$\boldsymbol{E}$ is called the **electric field** of the source charges $\{q_i\}$.

This gives machinery if we have a discrete collection of points. However, often our charge is more like a continuous jelly, motivating a continuous analogue to compute a field:

Definition 2.4.

$$\int \rho()$$

Example. 2.2 Calculation for the field distance z from the midpoint of a line of length $2L$ with constant charge λ.

We use Coulomb's law:

$$\int_{-L}^{L} \frac{1}{4\pi\epsilon_o}\frac{\lambda}{\mathscr{r}^2}\hat{\boldsymbol{\mathscr{r}}}\ \mathrm{d}\ell$$

where:

$$\mathscr{r}^2 = z^2 + x^2$$

$$\hat{\boldsymbol{\mathscr{r}}} = \frac{x\hat{\mathbf{x}} + z\hat{\mathbf{z}}}{\sqrt{z^2 + x^2}}$$

Skipping integration steps we are left with our field $\boldsymbol{E}$. Note our field is in general a function of our field point but what we provide here is an evaluation of this function.

$$\boldsymbol{E} = \frac{1}{4\pi\epsilon_o}\frac{2\lambda L}{z\sqrt{z^2+L^2}}\hat{\mathbf{z}}$$

Exercise (2.4)**.** Calculate the electric field distance z away from the midpoint of a square loop of constant charge λ and side a.

Solution (2.4)**.** The field is symmetric with respect to each side of the square, so we can compute a value for each side and add them up using the principal of superposition.

To find the $\boldsymbol{E}'$ contributing from a single side, proceed as in (2.2), but set $z = \sqrt{z^2 + (\frac{a}{2})^2}$. We are interested in the vertical component of this vector, as horizontal components will cancel from opposing sides of the square, so our result is $4\cos(\theta)\boldsymbol{E}'$ where $\cos(\theta) = \frac{z}{\sqrt{z^2+(\frac{a}{2})^2}}$ and 4 comes from each side.