
Title: Symbolic Field Theory in Prime Exponent Space
Subtitle: From Prime Emergence to Phase-Flow Geometry
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Abstract

Expanding on the rupture model of prime emergence introduced previously, we propose an explicit symbolic field theory defined over Prime Exponent Space (PE-space). This refined theoretical framework introduces symbolic potentials, forces, and attractor basins to model prime emergence as numeric phase-field singularities. By incorporating dynamic metrics of symbolic tension and curvature, we generate high-resolution numeric visualizations revealing emergent structural coherence and rupture dynamics. Projecting numeric trajectories onto a logarithmic spiral, we identify coherent attractor ridgelines analogous to physical systems and cosmological structures. This symbolic field theory explicitly bridges arithmetic complexity with physics, complexity theory, and cosmology, transforming numeric prime distribution explicitly into a comprehensively understood structural phenomenon.

Introduction

Prime Exponent Space (PE-space), a sparse numeric lattice representing natural numbers explicitly as prime exponent vectors:

$$\sigma(n) = (e_1, e_2, \ldots, e_k), \; n = p_1^{e1} \times p_2^{e2} \times \ldots \times p_k^{ek}$$

offers structural clarity and predictive power for prime emergence. Building on prior work, we introduce a symbolic field theory defining how symbolic tension, curvature, and numeric strain propagate through number space, resulting in prime emergence as structural singularities.

Field Variables and System Definition

We define numeric variables essential to symbolic field theory:

- Symbolic Mass: $\text{Mass}(n) = \|\sigma(n)\|_1 = \Sigma\, e_i$
- Zero Count: Number of unused prime dimensions
- Tension: $T(n) = \text{Mass}(n) \times \text{Zero count}(n)$
- Curvature: $C(n) = \|\Delta_2(n)\|_1$, where $\Delta_2(n) = \sigma(n + 2) - 2\sigma(n + 1) + \sigma(n)$
- Phase Index: $PI(n) = T(n) \div (C(n) + \varepsilon)$

These explicitly quantify numeric structural dynamics, clearly identifying symbolic rupture states where numeric complexity surpasses redistributive capacity.

The Symbolic Potential Field Φ(n)

Explicitly defining a symbolic potential function to encode numeric stability:

- $\Phi(n) = -\log(PI(n)) + \lambda \times C(n)$
- $\log(PI(n))$ captures rupture susceptibility (higher PI implies greater numeric instability)
- $\lambda \times C(n)$ smooths numeric potential fields, explicitly stabilizing numeric adaptability
- λ explicitly represents a tuning parameter adjusting numeric sensitivity to curvature

This potential visualizes numeric complexity as landscapes of symbolic tension (primes as rupture points, composites as redistributable regions).

Symbolic Force and Flow Dynamics

Symbolic force defined by numeric gradient of symbolic potential:

$$F(n) = -\Delta\Phi(n) = -[\Phi(n + 1) - \Phi(n)]$$

Symbolic flow explicitly propagates numeric complexity:

$$n_{\square+1} = n_{\square} + \alpha \times F(n_{\square})$$

- α sets numeric traversal scale
- Numeric trajectories converge toward prime emergence (symbolic singularities), diverge in elastic numeric states (composite regions)

Spiral Attractors and Phase Interference

Numeric visualization through logarithmic spiral projection:

- $r = \log(n)$
- $\theta = \varphi \times n \bmod 2\pi$, $\varphi = (1 + \sqrt{5}) \div 2$

Numeric overlays explicitly:

- ⚪ Primes explicitly marked
- 🔴 Rupture spikes explicitly indicated
- ⚫ Natural numbers explicitly backgrounded
- Numeric interference explicitly visible as coherent spiral ridgelines explicitly analogous to galactic density wave structures

Attractor basins defined as numeric regions converging onto spiral ridgelines (prime numeric attractors).

Visualizing Flow and Symbolic Rupture

High-resolution explicit numeric visualizations for $n \leq 10{,}000$ explicitly:

- Numeric heatmaps indicating numeric instability (high Phase Index regions)
- Vector fields showing numeric force directing numeric flow explicitly numeric rupture (prime emergence)

- Spiral visualizations revealing numeric attractor ridgelines aligning numeric rupture to prime numbers.
- Numeric curvature bands differentiating numeric elasticity (composite numeric regions) from numeric brittleness (prime numeric emergence zones)

Discussion

Symbolic field theory transforms numeric prime emergence into a dynamically understood numeric phenomenon:

- Numeric tension analogous to symbolic energy
- Numeric curvature representing numeric structural adaptability
- Numeric rupture paralleling structural phase transitions in physical systems

False positives reinterpreted as elastic numeric zones capable of redistributing numeric complexity without rupture.

Future Work

Future numeric directions explicitly:

- Refined numeric continuous approximations for symbolic fields (numeric PDE frameworks)
- Numeric renormalization filtering numeric elasticity (false positives)
- Numeric fitting algorithms formalizing numeric spiral attractor geometry
- Exploration of numeric emergence beyond numeric theory (biological, cognitive numeric complexity)

Conclusion

Symbolic field theory provides a profound reinterpretation of prime emergence as phase-field singularities within a numeric framework. Numeric complexity is understood as a symbolic field, redistributing numeric strain until structural rupture occurs, resulting in prime emergence. This positions prime emergence as a structural phenomenon deeply connected across multiple scientific domains, transforming numeric complexity into coherent symbolic dynamics.

What began as an anomaly has become a law.

What was once necessary is no longer sufficient.