Functional model: the basics

The `Sequential` model

- organizes layers as an ordered list
- restricts the input to layer $(l+1)$ to be the output of layer l.

The computation of a Sequential model is easy to describe and picture

- a graph
- each node represents the computation of a layer
- the nodes are connected sequentially in a straight line
- single input, single output
    - mostly true
    - can have inputs/outputs that are arrays, each element representing a different input/output value

The Functional model

- imposes **no** ordering on layers
- imposes **no** restriction on connect outputs of one layer to the input of another

The computation of a Functional model can be pictured as a general graph

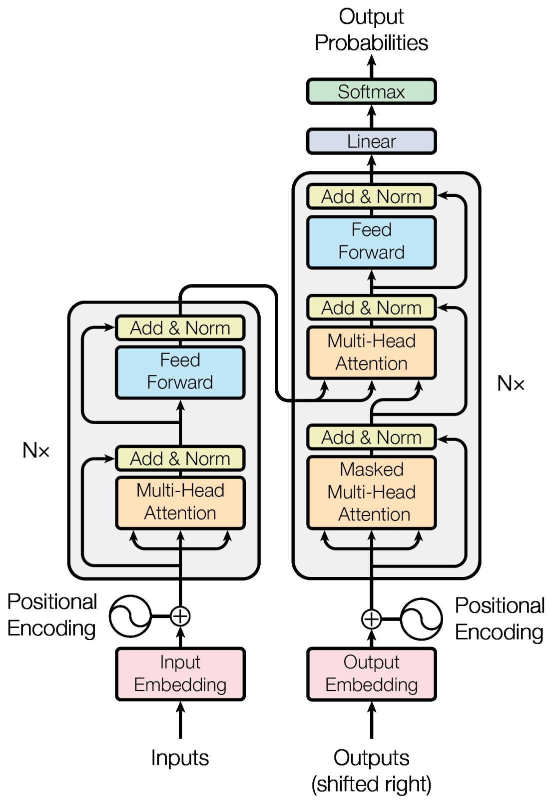
- each node represents a computation
- edges can flow from any node to any other
    - non-cyclic
- multiple inputs, multiple outputs possible

To illustrate the `Functional` model let's take a first look at model implementing a single `Transformer` block

- we will revisit this code later to illustrate other concepts

Here is the picture of a Transformer block

Transformer (Encoder/Decoder)



We can identify some connections that *don't* flow sequentially between adjacent nodes

- the *skip connection* that bypasses
    - the Multi-Head Attention node in the Encoder and the top Multi-Head Attention node in the Decoder
    - the Masked Multi-Head Attention node in the Decoder
- the connection from the output of the Encoder (top left) to the input of the Decoder Multi-Head Attention node

The Functional Model architecture, in code

[reference (https://www.tensorflow.org/guide/keras/functional)](https://www.tensorflow.org/guide/keras/functional)

In the Sequential model, the output of the node representing layer l is *always* fed to the input of the node representing layer $(l+1)$

- So can describe the computation graph as a sequence of nodes (each node a `Layer` type)

In the Functional model a node represents a function that takes one or more inputs and produces an output

- the node does not need to be a `Layer`
    - any TensorFlow `op`
- we connect the output of node $\mathbb{N}_{\mathrm{a}}$ to the input of node $\mathbb{N}_{\mathrm{b}}$
    - by assigning the output of $\mathbb{N}_{\mathrm{a}}$ to a variable (typically denoted as x)
    - calling the computation of $\mathbb{N}_{\mathrm{b}}$ with the variable as actual parameter

Here is an example ([source (https://www.tensorflow.org/api_docs/python/tf/keras/Model)](https://www.tensorflow.org/api_docs/python/tf/keras/Model))

```
import tensorflow as tf

inputs = tf.keras.Input(shape=(3,))
x = tf.keras.layers.Dense(4, activation=tf.nn.relu)(inputs)
outputs = tf.keras.layers.Dense(5, activation=tf.nn.softmax)(x)
model = tf.keras.Model(inputs=inputs, outputs=outputs)
```

- There is an `Input` layer (a function with no argument) whose output is assigned to variable `inputs`
- There is a `Dense` layer (a function with a single argument and single output)
    - that is called with parameter `inputs`
    - assigns its result to variable `x`

In general, these variables could be used as arguments (i.e., node inputs) anywhere in the computation

- not necessarily the next function appearing sequentially

The collection (not necessarily a sequence) of function calls defines a *Directed Acyclic Graph*

- one or more *root* nodes representing graph inputs
- one of more *leaf* nodes representing graph outputs

The graph encodes a complex function mapping inputs to outputs, composed of simpler functions.

The graph can be used to implement

- a new `Layer`
- a complete `Model`

To turn this collection into a `Model`

- we specify the input nodes
- we specify the output nodes

For example, for the above graph:

```
model = tf.keras.Model(inputs=inputs, outputs=outputs)
```

When `model` is *called*

- the actual parameters are bound to the nodes identified as inputs
    - i.e., `inputs`
- the result of the call are the values associated with the nodes identified as
    - i.e., `outputs``

Since a graph can have multiple inputs and outputs

- the `input` and `output` parameters of the `Model` statement can be *lists*

[`Model` reference (https://www.tensorflow.org/api_docs/python/tf/keras/Model)](https://www.tensorflow.org/api_docs/python/tf/keras/Model)

Example: multiple inputs and outputs

Here is an example (taken from [the reference](https://www.tensorflow.org/guide/keras/functional#models_with_multiple_inputs_and_ou) (https://www.tensorflow.org/guide/keras/functional#models_with_multiple_inputs_and_ou

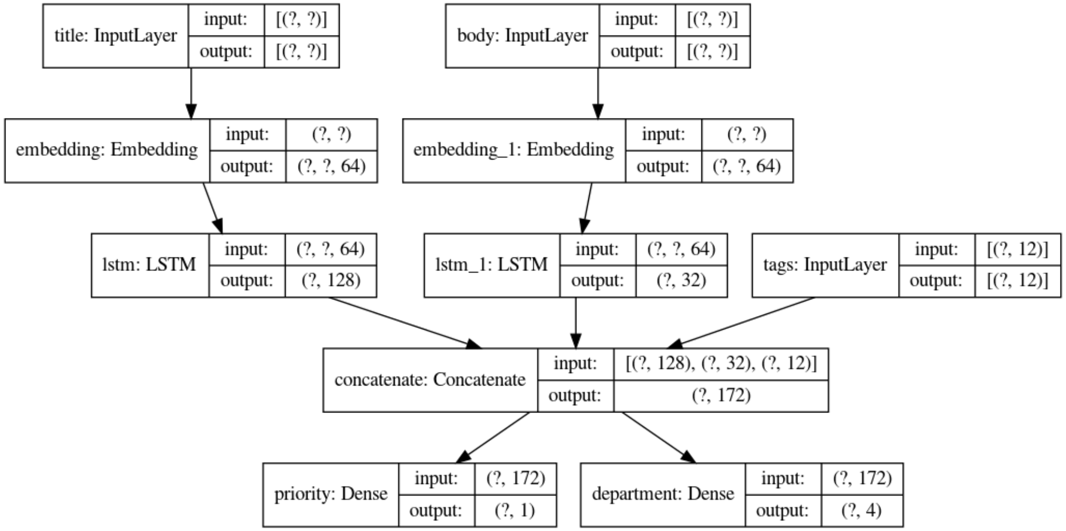
- Takes three inputs: title, body, tags
    - title: sequence of `int`
    - body: sequence of `int`
    - tags: binary vector (of length `num_tags`)
- Reduces each variable length sequence to a fixed length representation (final state of an LSTM)
- Concatenates the fixed length representations of title and body with the tags
- Feeds the concatenated vector to **two separate classifiers**
- Produces two outputs
    - priority: the output of one classifier
    - department: the output of the other classifier

In [3]:

```
num_tags = 12  # Number of unique issue tags
num_words = 10000  # Size of vocabulary obtained when preprocessing text data
num_departments = 4  # Number of departments for predictions

title_input = keras.Input(
    shape=(None,), name="title"
)  # Variable-length sequence of ints
body_input = keras.Input(shape=(None,), name="body")  # Variable-length sequence
of ints
tags_input = keras.Input(
    shape=(num_tags,), name="tags"
)  # Binary vectors of size `num_tags`

# Embed each word in the title into a 64-dimensional vector
title_features = layers.Embedding(num_words, 64)(title_input)
# Embed each word in the text into a 64-dimensional vector
body_features = layers.Embedding(num_words, 64)(body_input)

# Reduce sequence of embedded words in the title into a single 128-dimensional v
ector
title_features = layers.LSTM(128)(title_features)
# Reduce sequence of embedded words in the body into a single 32-dimensional vec
tor
body_features = layers.LSTM(32)(body_features)

# Merge all available features into a single large vector via concatenation
x = layers.concatenate([title_features, body_features, tags_input])

# Stick a logistic regression for priority prediction on top of the features
priority_pred = layers.Dense(1, name="priority")(x)
# Stick a department classifier on top of the features
department_pred = layers.Dense(num_departments, name="department")(x)

# Instantiate an end-to-end model predicting both priority and department
model = keras.Model(
    inputs=[title_input, body_input, tags_input],
    outputs=[priority_pred, department_pred],
)
```

In [4]:
```
keras.utils.plot_model(model, os.path.join(tempdir,"multi_input_and_output_mode
l.png"), show_shapes=True)
```

Out[4]:



Observe

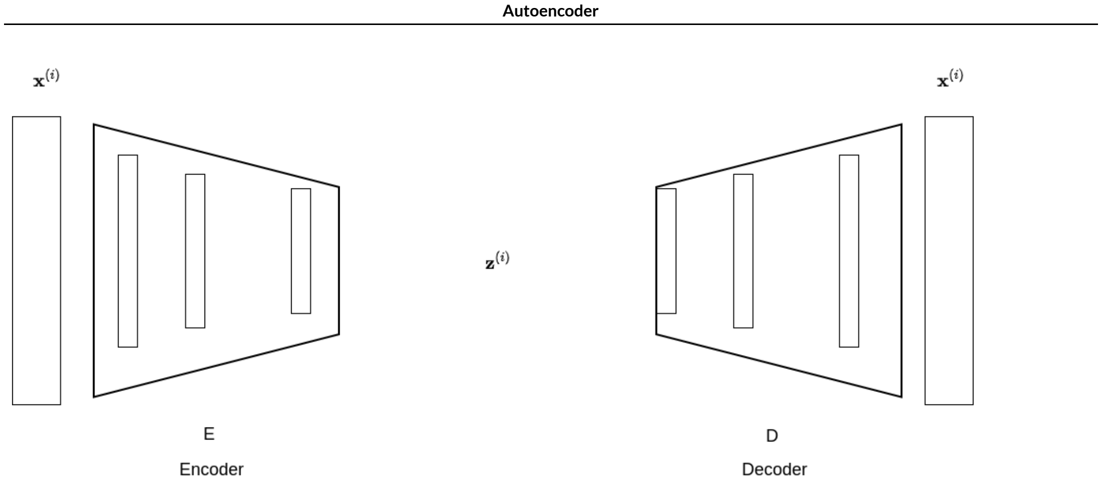
- The 3 input nodes
    - separate processing path
- The 2 output nodes
    - separate classifiers

Example: Nested Models

A `Model` and a `Layer` are very similar

- mapping inputs to outputs

Let's illustrate with an Autoencoder.



Here is an example (taken from [the reference (https://www.tensorflow.org/guide/keras/functional#all_models_are_callable_just_like_laye](https://www.tensorflow.org/guide/keras/functional#all_models_are_callable_just_like_laye)

In [5]:

```
encoder_input = keras.Input(shape=(28, 28, 1), name="original_img")
x = layers.Conv2D(16, 3, activation="relu")(encoder_input)
x = layers.Conv2D(32, 3, activation="relu")(x)
x = layers.MaxPooling2D(3)(x)
x = layers.Conv2D(32, 3, activation="relu")(x)
x = layers.Conv2D(16, 3, activation="relu")(x)
encoder_output = layers.GlobalMaxPooling2D()(x)

encoder = keras.Model(encoder_input, encoder_output, name="encoder")
encoder.summary()

decoder_input = keras.Input(shape=(16,), name="encoded_img")
x = layers.Reshape((4, 4, 1))(decoder_input)
x = layers.Conv2DTranspose(16, 3, activation="relu")(x)
x = layers.Conv2DTranspose(32, 3, activation="relu")(x)
x = layers.UpSampling2D(3)(x)
x = layers.Conv2DTranspose(16, 3, activation="relu")(x)
decoder_output = layers.Conv2DTranspose(1, 3, activation="relu")(x)

decoder = keras.Model(decoder_input, decoder_output, name="decoder")
decoder.summary()

autoencoder_input = keras.Input(shape=(28, 28, 1), name="img")
encoded_img = encoder(autoencoder_input)
decoded_img = decoder(encoded_img)
autoencoder = keras.Model(autoencoder_input, decoded_img, name="autoencoder")
autoencoder.summary()
```

```
Model: "encoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
original_img (InputLayer)    [(None, 28, 28, 1)]       0         
_________________________________________________________________
conv2d (Conv2D)              (None, 26, 26, 16)        160       
_________________________________________________________________
conv2d_1 (Conv2D)            (None, 24, 24, 32)        4640      
_________________________________________________________________
max_pooling2d (MaxPooling2D) (None, 8, 8, 32)          0         
_________________________________________________________________
conv2d_2 (Conv2D)            (None, 6, 6, 32)          9248      
_________________________________________________________________
conv2d_3 (Conv2D)            (None, 4, 4, 16)          4624      
_________________________________________________________________
global_max_pooling2d (Global (None, 16)                0         
=================================================================
Total params: 18,672
Trainable params: 18,672
Non-trainable params: 0
_________________________________________________________________
Model: "decoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
encoded_img (InputLayer)     [(None, 16)]              0         
_________________________________________________________________
reshape (Reshape)            (None, 4, 4, 1)           0         
_________________________________________________________________
conv2d_transpose (Conv2DTran (None, 6, 6, 16)          160       
_________________________________________________________________
conv2d_transpose_1 (Conv2DTr (None, 8, 8, 32)          4640      
_________________________________________________________________
up_sampling2d (UpSampling2D) (None, 24, 24, 32)        0         
_________________________________________________________________
conv2d_transpose_2 (Conv2DTr (None, 26, 26, 16)        4624      
_________________________________________________________________
conv2d_transpose_3 (Conv2DTr (None, 28, 28, 1)         145       
=================================================================
Total params: 9,569
Trainable params: 9,569
Non-trainable params: 0
_________________________________________________________________
Model: "autoencoder"
_________________________________________________________________
Layer (type)                 Output Shape              Param #   
=================================================================
img (InputLayer)             [(None, 28, 28, 1)]       0         
_________________________________________________________________
encoder (Model)              (None, 16)                18672     
_________________________________________________________________
decoder (Model)              (None, 28, 28, 1)         9569      
```

```
=================================================================
Total params: 28,241
```

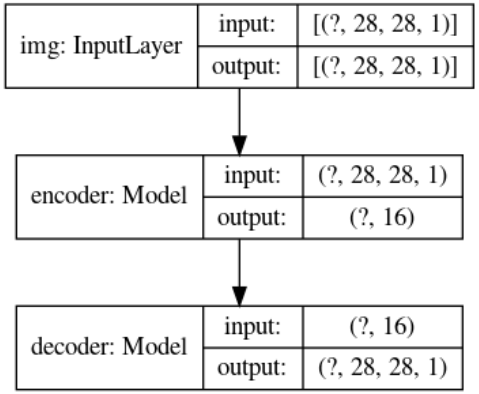
Let's examine the `autoencoder` model

- it is composed of two models
  - `encoder`
  - `decoder`

In [6]:

```
keras.utils.plot_model(autoencoder, os.path.join(tempdir,"autoencoder.png"), show_shapes=True)
```

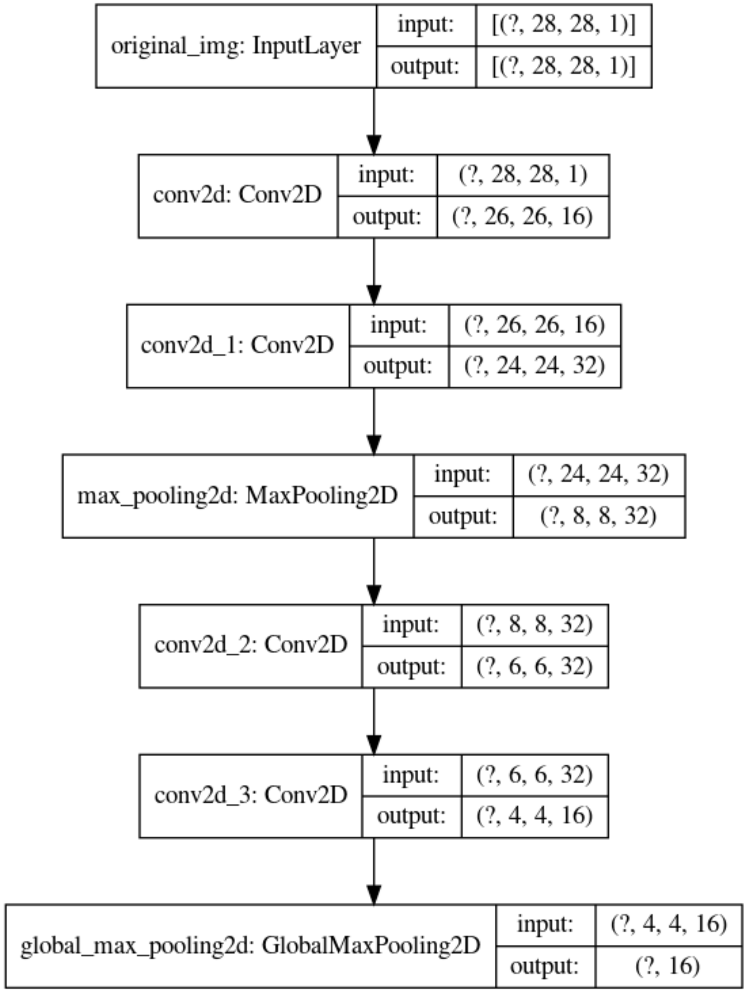
Out[6]:



We can examine the sub-models:

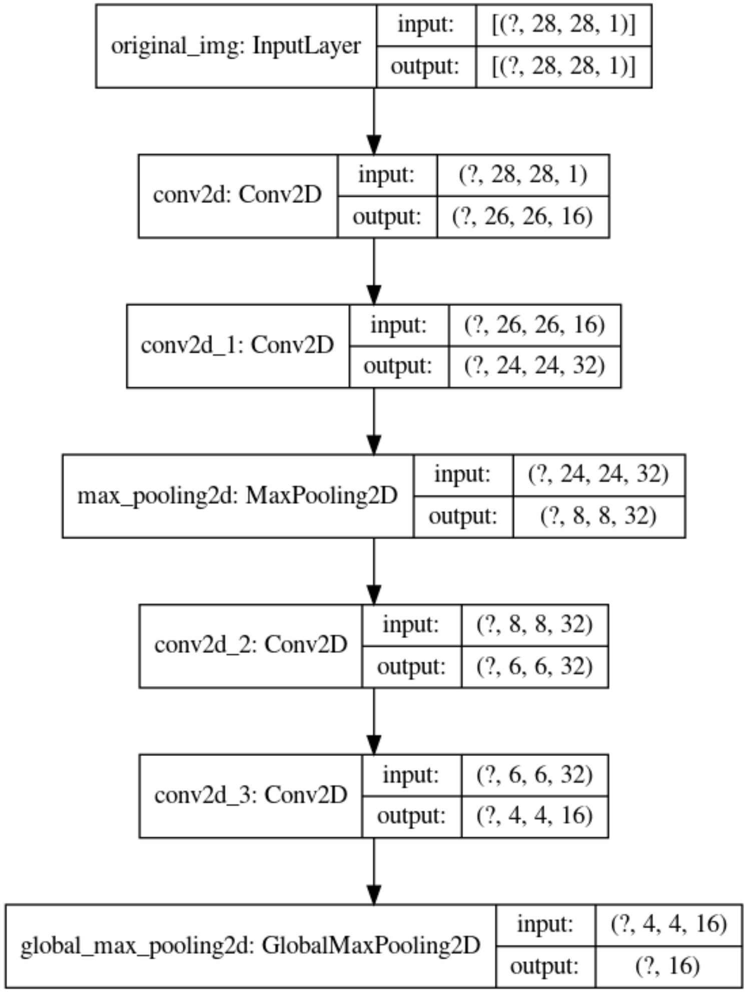
In [7]:
```
keras.utils.plot_model(encoder, os.path.join(tempdir,"encoder.png"), show_shapes
=True)
```

Out[7]:



In [8]:
```
keras.utils.plot_model(encoder, os.path.join(tempdir, "decoder.png"), show_shape
s=True)
```

Out[8]:



Sub-classing models/layers

One can create a new `Model` / `Layer` by sub-classing from the base types `tf.keras.Model` / `tf.keras.layers.Layer`

- can override existing methods of a `Model`
    - e.g., a custom training step (invoked by `fit`)
- can build new `Layer` types

Here is an example (taken from [the reference (https://www.tensorflow.org/guide/keras/functional#functional_api_strengths)](https://www.tensorflow.org/guide/keras/functional#functional_api_strengths))

In [11]:

```
class MLP(keras.Model):

  def __init__(self, **kwargs):
    super(MLP, self).__init__(**kwargs)
    self.dense_1 = layers.Dense(64, activation='relu')
    self.dense_2 = layers.Dense(10)

  def call(self, inputs):
    x = self.dense_1(inputs)
    return self.dense_2(x)

# Instantiate the model.
mlp = MLP()
# Necessary to create the model's state.
# The model doesn't have a state until it's called at least once.
_ = mlp(tf.zeros((1, 32)))
```

Key points:

- Notice that the components of the `Model`
    - are instantiated in the **constructor** (`__init__`)
    - invoked in the `call`
        - the `call` method is invoked when you apply actual parameters to the `Model`

```
_ = mlp(tf.zeros((1, 32)))
```

- What would happen if you instantiated the components in the `call` ?

```
def call(self, inputs):
  x = layers.Dense(64, activation
='relu')(inputs)
  return layers.Dense(10)(x)
```

It would probably **not** be what you expected

- Instantiating the components in `__init__` results in them being defined **once**
- Instantiating the components in `call` results in them being defined separately **each time** the `Model` is called
    - weights **are not shared** between component instances
    - `call` is invoked for each step in training
        - would not learn weights of the component since they would be initialized for each batch of examples

Fitting a model with multiple inputs, multiple outputs

There is a technical question as to how we distinguish among the `Input` s so we can connect it to the desired variable.

In our basic introduction to Keras, the `fit` method described its training data simply

- Two `numpy` arrays: one for features, one for labels
    - an element of the first array are features of a single example
    - an element of the second array is the label of a single example (for supervised learning)

A careful examination of the [`fit` method (https://keras.io/api/models/model_training_apis/#fit-method)](https://keras.io/api/models/model_training_apis/#fit-method) describes *multiple* ways to pass train examples (and labels) to a model

- The common `x= .., y=..`
    - In its simplest form:
        - `x` and `y are`numpy` arrays (one element per example)
- More general form
    - both the x and y can be lists
        - Functional models may define multiple *positional* (first, second, etc.) inputs and outputs
        - The x list: one element per input
        - The y list: one element per output
        - models with multiple unnamed inputs or model outputs
    - `x` can be a `dict`
        - A Functional model with multiple *named* inputs
        - the keys of the `dict` are the names of the inputs
- Tensors
    - can pass the Tensor to a *non-`Input`* layer
- `Dataset`
- Generator
    - for (feature, label) pairs when training

Specifying batches

Also: remember that Models process *batches* of examples (in fitting and predicting

- So the variables passed to `Input` layers should be *groups* of examples, not a single example
    - a single example is represented as a group of size 1

Creating batches is **done for you** when using the common `x= .., y=.., batch_size=..` calling method

- The `Dataset` **needs to create the batches** when used as the calling method
    - there is always a "batch" dimension, even if the batch size is 1
    - there is no `batch_size` argument when the inputs are `Dataset`'s
    - we will learn about the `batch` operator for transforming an un-batched `Dataset` into one with batches

Example: Multiple Loss functions from multiple outputs

In discussing multiple outputs, we skipped over an important point

- Loss is associated with an output
- When there are multiple outpus
    - there is a separate loss per output

Technical issue

- How do we specify the loss per output
- How do we combine multiple losses into a single loss, for training

Referring back to our example of multiple inputs/outputs (solving for priority and department)

- we specify a loss for each output
    - with a `dict` that maps a node name to a loss
    - the outputs have been named "priority" and "department"

```
  priority_pred = layers.Dense
(1, name="priority")(x)
  department_pred = layers.Den
se(num_departments, name="depa
rtment")(x)
```

Note how in the `fit` call

- we identify the multiple inputs by the names of their `Input` nodes
- using a `dict` as parameter

In [12]:

```
model.compile(
    optimizer=keras.optimizers.RMSprop(1e-3),
    loss={
        "priority": keras.losses.BinaryCrossentropy(from_logits=True),
        "department": keras.losses.CategoricalCrossentropy(from_logits=True),
    },
    loss_weights={"priority": 1.0, "department": 0.2},
)
```

Note the `loss_weights` parameter

- specifying the relative weight of each loss within the total loss

Here is the call to fit the model:

In [13]:

```
# Dummy input data
title_data = np.random.randint(num_words, size=(1280, 10))
body_data = np.random.randint(num_words, size=(1280, 100))
tags_data = np.random.randint(2, size=(1280, num_tags)).astype("float32")

# Dummy target data
priority_targets = np.random.random(size=(1280, 1))
dept_targets = np.random.randint(2, size=(1280, num_departments))

model.fit(
    {"title": title_data, "body": body_data, "tags": tags_data},
    {"priority": priority_targets, "department": dept_targets},
    epochs=2,
    batch_size=32,
)
```

```
Train on 1280 samples
Epoch 1/2
WARNING:tensorflow:Entity <function Function._initialize_uninitialized_variabl
es.<locals>.initialize_variables at 0x7fcbd06c2c20> could not be transformed a
nd will be executed as-is. Please report this to the AutoGraph team. When fili
ng the bug, set the verbosity to 10 (on Linux, `export AUTOGRAPH_VERBOSITY=10
`) and attach the full output. Cause: No module named 'tensorflow_core.estimat
or'
WARNING: Entity <function Function._initialize_uninitialized_variables.<locals
>.initialize_variables at 0x7fcbd06c2c20> could not be transformed and will be
executed as-is. Please report this to the AutoGraph team. When filing the bug,
set the verbosity to 10 (on Linux, `export AUTOGRAPH_VERBOSITY=10`) and attach
the full output. Cause: No module named 'tensorflow_core.estimator'
1280/1280 [==============================] - 6s 5ms/sample - loss: 1.2775 - pr
iority_loss: 0.7015 - department_loss: 2.8797
Epoch 2/2
1280/1280 [==============================] - 2s 1ms/sample - loss: 1.2766 - pr
iority_loss: 0.6991 - department_loss: 2.8878
```

Out[13]:

```
<tensorflow.python.keras.callbacks.History at 0x7fcbd0071ad0>
```

Gradients

Gradient Descent is the fundamental tool used for optimizing the Loss Function.

When the `fit` method of a `Model` object is called, it runs a *training step* of the `Model` on a mini-batch of training examples.

The default training step of a `Model`

- Runs the *forward* calculation:
    - presenting an input example to the NN inputs
    - calculating the NN outputs by Forward Propagation through the network
    - computing the Loss
    - computing the gradients of the Loss with respect to the NN weights
    - updating the weights in the negative direction of the Gradient

Here is an example (from the notebook on [VAE (https://colab.research.google.com/github/keras-team/keras-io/blob/master/examples/generative/ipynb/vae.ipynb)](https://colab.research.google.com/github/keras-team/keras-io/blob/master/examples/generative/ipynb/vae.ipynb) that we will study in depth in the future).

It defines a `Model` sub-type, with its own training step.

```
    def train_step(self, data):
        with tf.GradientTape() as tape:
            z_mean, z_log_var, z = self.encoder(data)
            reconstruction = self.decoder(z)
            reconstruction_loss = tf.reduce_mean(
                tf.reduce_sum(
                    keras.losses.binary_crossentropy(data, reconstruction), axi
s=(1, 2)
                )
            )
            kl_loss = -0.5 * (1 + z_log_var - tf.square(z_mean) - tf.exp(z_log_
var))
            kl_loss = tf.reduce_mean(tf.reduce_sum(kl_loss, axis=1))
            total_loss = reconstruction_loss + kl_loss
        grads = tape.gradient(total_loss, self.trainable_weights)
        self.optimizer.apply_gradients(zip(grads, self.trainable_weights))
        self.total_loss_tracker.update_state(total_loss)
        self.reconstruction_loss_tracker.update_state(reconstruction_loss)
        self.kl_loss_tracker.update_state(kl_loss)
        return {
```

In the above example, we override the default training step

- How to override a `Model`'s methods will be a future topic
- The mathematics of the VAE will be a future topic

For now, we focus on the code of the custom training step.

Note

We **didn't** create a `call` method for the `Model`

- we won't ever "call" the VAE model
    - only its `encoder` and `decoder` sub-components

The Loss (`total_loss`) consists of two parts

- `kl_loss`
- `reconstruction_loss`

We manually invoke the computation of the Gradient of the Loss

- with respect to the model's weights (`self.trainable_weights`)

    ```
    grads = tape.gradient(total_loss,
    self.trainable_weights)
    ```

- In order to signal to TensorFlow that gradients are to be calculated for an expression
    - the expression must occur within the scope of a `tf.GradientTape` block

        ```
        with tf.GradientTape() as tape:
        ```

We manually update the weights in the negative direction of the gradients

```
self.optimizer.apply_gradients(zip(grads,
self.trainable_weights))
```

We track the total loss as well as it's subparts

```
self.total_loss_tracker.update_state(total_loss)
        self.reconstruction_loss_tracker.update_state(reconstruction_loss)
        self.kl_loss_tracker.update_state(kl_loss)
```

We return 3 losses

```
return {
            "loss": self.total_loss_tracker.result(),
            "reconstruction_loss": self.reconstruction_loss_tracker.result(),
            "kl_loss": self.kl_loss_tracker.result(),
        }
```

But the calculation of gradients is powerful apart from deriving a model's weights.

TensorFlow allows you to compute the gradient of any expression with respect to any value on which the expression depends.

Let's visit this notebook on [Gradient Ascent (Gradient_ascent.ipynb)] to see how gradients can be used to visualize which inputs the various layers of a NN respond to most highly.

In [14]:
```
print("Done")
```

```
Done
```