## How does the NN "learn" the transformations?

The matrix  $\mathbf{W}$  contains the "patterns" that serve to recognize the synthetic features created by each layer

 $oldsymbol{\mathbf{W}}_{(l),j}$  are the weights /pattern for feature  $oldsymbol{\mathbf{y}}_{(l),j}$ 

How are these patterns discovered?

The answer is: exactly as we did in Classical Machine Learning

• Define a loss function that is parameterized by W:

$$\mathcal{L} = L(\hat{\mathbf{y}}, \mathbf{y}; \mathbf{W})$$

- Per example loss  $\mathcal{L}^{(i)}$
- Average loss  $\mathcal{L} = \frac{1}{m} \sum_{i=1}^{m} \mathcal{L}^{(i)}$
- ullet Our goal is to find  $\mathbf{W}^*$  the "best" set of weights

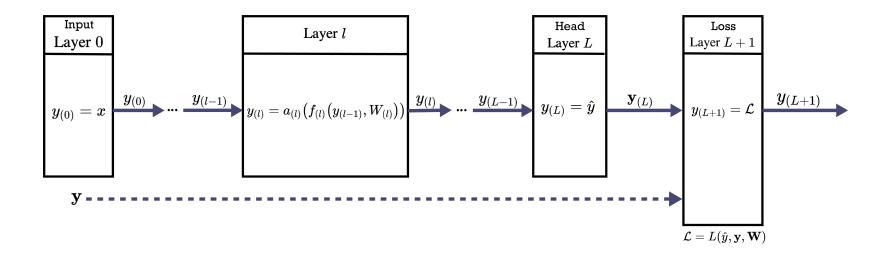
$$\mathbf{W}^* = \operatorname*{argmin}_W L(\hat{\mathbf{y}}, \mathbf{y}; \mathbf{W})$$

• Find  $\mathbf{W}^*$  using Gradient Descent!

Very much in spirit of the multi-layer architecture

• We add a new layer (L+1) to compute the loss  $\mathcal{L}$  !

### Additional Loss Layer (L+1)



## **Gradient Descent review**

Gradient Descent is an iterative method for finding the minimum of a function. <!--EdX: Omit this from EdX: can't refer to prior course

See the <u>Gradient Descent lecture (Gradient Descent.ipynb)</u> in the Classical ML part of the course for more details -->

Let's review Gradient Descent using our current notation

- ullet We start with an initial guess for f W and iteratively improve it.
- ullet Compute the loss  ${\cal L}$  given the current  ${f W}$ 
  - ullet Average loss of the m examples in the training examples
- Compute the gradient

$$\frac{\partial \mathcal{L}}{\partial W}$$

- ullet Update f W in the direction of the *negative* of the gradient
- Scaled by a learning rate lpha

$$\mathbf{W} = \mathbf{W} - lpha * rac{\partial \mathcal{L}}{\partial W}$$

A unit change in  $\mathbf{W}$  increases  $\mathcal{L}$  by  $\frac{\partial \mathcal{L}}{\partial W}$ 

- ullet That's why there is a negative sign: we proceed in the direction *opposite* the one that increases  ${\cal L}$
- ullet We move only a fraction  $lpha \leq 1$  of the (negative) of the gradient
- To avoid the possibility of over-shooting the minimum

 $\mathbf{W}$  is a multi-dimensional vector, not a scalar

- So the gradient is multi-dimensional
- We will formally discuss Matrix Gradients in a later lecture
  - lacktriangledown For now: we compute the derivative with respect to each element of f W and arrange in a matrix

The loss  $\mathcal{L}$  is averaged over all m training examples.

This can be expensive to compute.

We can approximate  $\mathcal L$  by sampling from the m training examples

- ullet Choose a random subset (of size  $m' \leq m$ ) of examples:  $I = \{i_1, \dots, i_{m'}\}$
- ullet Approximate  ${\cal L}$  on I

$$\mathcal{L} pprox rac{1}{|I|} \sum_{i \in I} \mathcal{L}^{(\mathbf{i})}$$

### Minibatch gradient descent

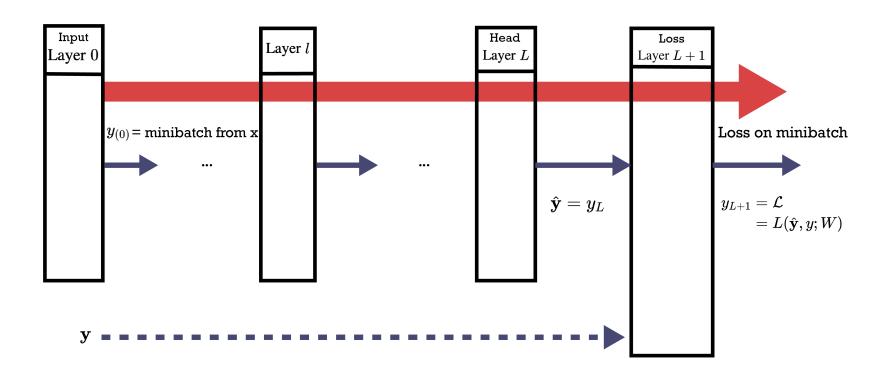
- ullet Divides the m training examples
- ullet Into b=m/m' disjoint batches of  $m'\leq m$  examples each

- Iterates over the batches
  - Approximate the loss on the current batch
  - $\bullet$  Update  $\boldsymbol{W}$  according to

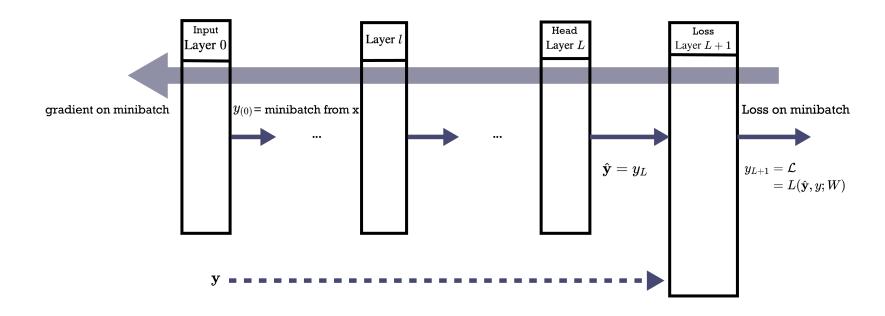
$$\mathbf{W} = \mathbf{W} - \alpha * \frac{\partial \mathcal{L}}{\partial W}$$

Repeat until all the batches have been processed

# Minibatch: Forward Pass From minibatch to Loss



# Minibatch: Backwards Pass From minibatch Loss to Gradient



#### Thus, Minibatch Gradient Descent

- ullet Examines all m training examples
- In batches of size  $m' \leq m$
- Resulting in  $b=m/\overline{m'}$  updates to  ${\bf W}$  for each complete pass through the m training examples

Minibatch Gradient Descent is faster than a single batch of size m

- ullet Update f W b times, rather than once
- ullet A complete pass through the b mini-batches is called an epoch

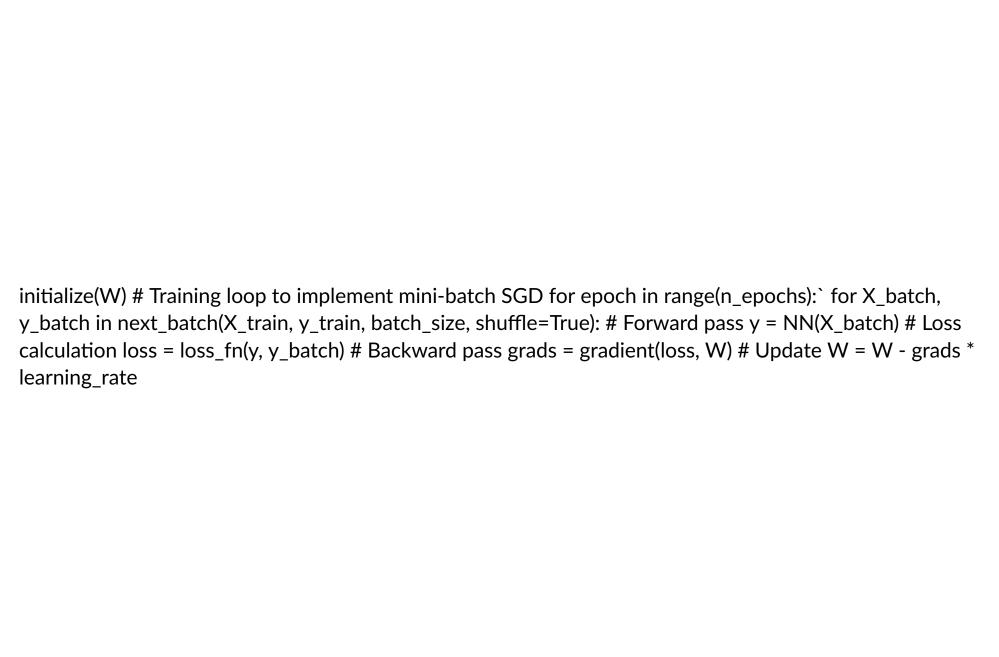
## The Training loop

A single epoch of Gradient Descent encounters all m examples and makes b updates

We may need additional epochs to continue to drive down the Loss.

This iterative process is called the *training loop*.

Here is some pseudo-code:



It used to be the case that this fairly standard training loop was coded for each problem.
Just as sklearn wrapped common code into a high-level API
We will use a toolkit that hides the training loop behind a high level API

# **Scaling the inputs**

Many times in this course we have pointed out that some models are scale sensitive.

Neural Networks are not *mathematically* sensitive but tend to be so *in practice*.

It is *highly recommended* to scale your data so their absolute values are around 1.0 or at least somewhat small.

#### Gradient Descent is the root of the problem:

- Two features on different scales can cause the optimizer to favor one over the other
- Activations can saturate
  - Output of dot product (Dense layer) is in the "flat\* area of the activation"
  - Zero derivative: no learning
- The Cost/Loss may be large in initial epochs when the target values are too different from the dot products
  - Large gradients: unstable learning
  - Weights are typically initialized to values less than 1.0, leading to small dot products

Remember: if you re-scale the inputs, you will need to invert the transformation when
communicating the results.

```
In [4]: print("Done")
```

Done