Model-free Value methods

In the absence of a model

- we can only learn about the dynamics of the Environment
    - by *interacting* with the Environment
    - playing out an entire episode/trajectory
    - gather rewards and successor state information

Each interaction yielding an episode/trajectory

- is a *sample*
- drawn from an unknown Environment Transition Probability distribution $\transp(\state, \act)$

This means multiple episodes are necessary

- to draw a sufficiently diverse sample of $\backslash\mathrm{transp}(\backslash\mathrm{state}, \backslash\mathrm{act})$
  - for each $(\backslash\mathrm{state}, \backslash\mathrm{act})$ combination

Contrast this with model-based methods

- all knowledge of the Environment is available without interacting

Model-free vs Model-based methods: differences

Although the update equations for model-based and model-free methods

- appear similar on the surface
- there are deeper differences

The update equation for both model/model-free

- define $\statevalfun_{k+1}(\state)$ as a function of successor states $\statevalfun_{k}(\state')$

In model-based methods

- $\statevalfun_{k}(\state')$ is **not** a random variable
- it is a biased estimate (incorrect prior to convergence, but deterministic)
- Notation: we use placeholders for $\state$, $\act$, $\rew$, $\state'$

In model-based methods

- $\state = \stateseq_{}, \act = \actseq_{}, \rew = \rewseq_{+1}$,
  $\state' = \stateseq_{+1}$
- are *samples* from one episode/trajectory
  - i.e., not place-holders
- are *random*
  - dependent of the Environment's choice of $\transp(\stateseq_{}, \actseq_{})$ during the episode.

So

- $\statevalfun_{k+1}(\stateseq_)$ is a random variable
- that depends on *another* random variable: $\statevalfun_{k}(\stateseq_{+1})$

We can characterize the estimates by

- the bias
- *variance*
  - since $\statevalfun_{k+1}$ is a random variable
    $(\stateseq_)$

The process of estimated one value based on other estimates

- is call *Bootstrapping*

We will subsequently introduce a family of methods based on bootstrapping

- the *Temporal Difference (TD)* family

Converging to true $\statevalfun(\state)$

In order for a model-free method to converge to the true $\statevalfun(\state)$

- it must sample enough episodes
- to *experience* the full distribution of $\transp(\state, \act)$

Discovering the effect of $\transp(\state, \act)$ without knowing it

Estimating $\statevalfun(\state)$ essentially boils down to computing the *expected* rewards that can be accumulated starting at state $\state$.

To simplify:

- consider the immediate reward received from taking action $\act$ in $\state$.
- each time I take this action, I observe a *sample* of random variable $\hat{\rew}$ that is the reward
    - randomness according to distribution $\transp(\state, \act)$ which I can't observe
- but if I add these samples together
    - I obtain a probability-weighted reward: Expected reward

So sampling repeatedly allows me to compute the Expected Reward from an action without explicitly knowing $\transp(\state, \act)$

Model-free vs Model-based methods: Summary

Aspect	Model-based Bellman backup	Model-free TD backup
Determinism/randomness	Deterministic (no random sampling)	Stochastic (sampled; random variable)
Need for exploration	No	Yes
Bias (before convergence)	Yes, if values are not converged	Yes, for same reason—and also due to bootstrapping
Variance of backup	Zero (averaged over all outcomes)	Nonzero (sample-specific, reduces with averaging)
Source of update target	Full probability model	Single experience sample
Convergence (with enough sweeps)	To true value	To expected value (with sufficient exploration and learning)

Greed is not (always) good

Consider the policy $\backslash\mathrm{act} = \pi(\backslash\mathrm{state})$ used by a method during its learning phase.

Before learning

- each $\backslash\mathrm{act}$ will be equally likely
- symmetric initialization of initial, uninformed policy

If $\pi(\backslash\mathrm{state})$ were completely deterministic

- in every episode during learning in which we arrive at state $\backslash\mathrm{state}$
- we would take the same action $\backslash\mathrm{act}$
- most likely the *first* action chosen in the first episode

But the action chose in the first episode

- may not be the optimal one

A deterministic policy locks us into a potentially sub-optimal choice.

This is an illustration of the *exploration/exploitation* tradeoff

- Choosing the action currently believed to be optimal is *exploitation* (*greedy* choice)
- Choosing a different action is called *exploration* (*non-greedy* choice)

In model-free methods

- we may be forced to make action choices *that are not optimal* with respect to the current policy
- in order to better sample the Environment

An ϵ-greedy policy makes

- the greedy choice with probability $(1 - \epsilon)$
- the non-greedy choice with probability ϵ

Temporal Difference (TD) methods

Temporal Difference (TD) is a family of value-based methods characterized by the use of *bootstrapping*

- creating an estimate that depends on other estimates

We will show two members of this family, that are model-free analogs of previously introduced model-based methods

- Value learning $\statevalfun(\state)$
- Q-learning $\actvalfun(\state, \act)$

To be concrete: we focus on $\statevalfun(\state)$.

The update (backup) for $\statevalfun(\state)$ is usually written

- as an increment δ to the current estimate
- moderated by a learning rate α
  - similar to Gradient Descent

$$\statevalfun_{\pi,k+1}(\stateseq) = \statevalfun_{\pi,k}(\stateseq) + \alpha * \delta$$

where

$$\delta = \statevalfun_{\pi,k+1}(\stateseq) - \statevalfun_{\pi,k}(\stateseq)$$

is called the *Temporal Difference Error*

And

$$\begin{aligned} \statevalfun_{\pi,k+1}(\stateseq) \quad &= \quad \rewseq_{+1} && \text{immedi} \\ &\quad + \\ &\quad \disc \statevalfun_{\pi,k}(\stateseq_{+1}) && \text{current} \end{aligned}$$

$\statevalfun_{\pi,k+1}(\stateseq)$ is called the *target*

Thus, the update equation moves the current value towards the target.

p-step ahead updates

The update above is based on the immediate (one step ahead) reward.

This method is called *1-step TD*

We can generalize this to *p-step TD* which uses

- rewards $\{\rewseq_{<'\leq +\mathtt{p}}\}$
- $\statevalfun_{\pi,k}(\stateseq_{+\mathtt{p}})$

$$\delta = \left[\sum_{i=1}^{p} \gamma^{i-1} R_{t+i} + \gamma^{p} \statevalfun_{\pi,k}(\stateseq_{t+p})\right] - \statevalfun_{\pi,k}(\stateseq)$$

Note that the value used for the end state $\backslash\mathrm{stateseq}_{+\mathrm{p}}$ is

- the pre-update value
  $\backslash\mathrm{statevalfun}_{\pi,k}(\backslash\mathrm{stateseq}_{+\mathrm{p}})$
- rather than the post-update value
  $\backslash\mathrm{statevalfun}_{\pi,k+1}(\backslash\mathrm{stateseq}_{+\mathrm{p}})$

That is

- all backups occur simultaneously
- not sequentially
  - so the update to $\statevalfun_{\pi}(\stateseq_{+1})$ *does not* influence the update to $\statevalfun_{\pi}(\stateseq)$
  - even though we might happen to evaluate it first (bottom up)

To illustrate the backups:

```
State_0 --a--> State_1 --a--> ... --a--> State_T (terminal)

TD:    Update $V(s)$ at every transition using $V(s_{t+1})$
```

Pseudo-code for p-step TD

We give some code below.

The apparent complexity of the code arises

- because p-step TD, for $p > 1$
- involves returns that won't be experienced
- until $(p-1)$ steps after step

So the update for $\statevalfun_{\pi,k}(\stateseq_{)}$ is delayed $p-1$ steps

- until we accumulate the remaining rewards
- at the beginning of update $(k+1)$ we already have the value of $\statevalfun_{\pi}(\stateseq_{\texttt{+p}})$ needed
    - i.e., $\statevalfun_{\pi,k}(\stateseq_{\texttt{+p}})$

```
from collections import defaultdict

gamma = 0.99             # Discount factor
alpha = 0.1              # Learning rate
k = 3                    # Number of steps (set k as needed)
V = defaultdict(float)   # State-value function

def td_k_episode(env, policy, k):
    states = []
    rewards = []

    s = env.reset()
    states.append(s)
    done = False
    t = 0

    while not done:
        a = policy(s)
        s_next, r, done, info = env.step(a)
        rewards.append(r)
```

Q-learning: Action-value function

Just as in the model-based method

- Q-learning creates a function from $(\state, \act)$ pairs to expected returns

$$\actvalfun_\pi : \state \times \act \to \Reals$$

So the action $\act^*$ that leads to maximum $\statevalfun_\pi(\state')$ is easily obtained from the Action-Value function via

$$\pi^*(\state) = \argmax\act\actvalfun(\state, \act)$$

Note: the $\argmax$ results in a *deterministic* policy just as before

The Bellman equation for the Action-Value function is

$$\actvalfun_{\pi}(\state, \act) \quad = \quad \E_{\pi}\left(\rewseq_{+1} + \disc \max a' \actvalfun_{\pi}(\sta\right.$$

The **notable aspect of the model-free** version (contrast to model-based version) of Q-learning:

- the use of a Greedy Policy to balance Exploration and Exploitation

Pseudo code for Q-learning

```
Initialize Q(s, a) arbitrarily for all states s and actions a (often Q(s, a) =
0)

Set learning rate alpha, discount factor gamma, exploration rate epsilon

For episode = 1 to number_of_episodes:
    Initialize state s

    Repeat until s is terminal:
        With probability epsilon:
            Choose a random action a (exploration)
        Otherwise:
            Choose action a = argmax_a Q(s, a) (exploitation)

        Take action a, observe reward r and next state s'

        Update Q(s, a) using:
            Q(s, a) = Q(s, a) + alpha * [r + gamma * max_{a'} Q(s', a') - Q(s,
a)]
```

In the above code

- we build a table Q for the action value function $\actvalfun_\pi$
- You can see the ϵ-greedy strategy

```
With probability epsilon:
    Choose a random action a (exploration)
Otherwise:
    Choose action a = argmax_a Q(s, a) (exploitation)
```

- Update assumes subsequent action choices are greedy

```
max_{a'} Q(s', a')
```

- Model-free: Notice that we don't make use of
  - $\transp(\state', \rew|\state, \act)$
  - or any reward other than the one received by taking the chosen action

Deep Q-learning (DQN)

The method for Q-learning presented involved creating a *table* implementing the mapping $\backslash \mathrm{actvalfun}_{\pi}$.

This is only practical when the size of the table is small

- the number of states for many problems (e.g., games) is extremely large
- not practical

Deep Q-Learning

- treats $\backslash\mathrm{actvalfun}_{\pi}$ as a function
- which is approximated by a Neural Network (the *Deep Q-Network (DQN)*)

The function is *trained*

- to reproduce the value calculated in ordinary Q-learning
- via a Loss function (MSE)
    - that minimizes the difference between
        - the NN output
        - and the mathematical definition of the Q function.

That is:

- We train the NN function $Q_\theta(\stateseq, \actseq)$ to approximate true value $\actvalfun_\pi(\stateseq, \actseq)$
- Using an MSE per-example loss
$$(Q_\theta(\stateseq, \actseq) - \actvalfun_\pi(\stateseq, \actseq))^2$$

The loss is

- calculated on a mini-batch of steps
- gradient descent on the batch loss results in an update to the NN parameters Θ

So one fundamental addition to the ordinary Q-learning algorithm

- record state transitions in a Replay Memory

Mini-batches are sampled from the Replay Memory

The Replay Memory enables *Experience Replay*

- off-line training
    - can train on the *same state transition* without executing the episode again
    - efficient use of transitions (can reuse)
- batch creation
- having previous transitions in a batch
    - prevents forgetting that might occur by only seeing *new* transitions

Pseudo code for Deep Q-learning

```
Initialize replay memory D to capacity N
Initialize main Q-network with random weights θ
Initialize target Q-network with weights θ^- = θ

Set exploration rate ε, discount factor γ, learning rate α

For episode = 1 to M:
    Initialize state s

    For each step in episode:
        With probability ε:
            Choose random action a (exploration)
        Else:
            Choose a = argmax_a Q(s, a; θ) (exploitation)

        Take action a, observe reward r and next state s'

        Store transition (s, a, r, s') in replay memory D

        Sample random mini-batch of transitions (s_j, a_j, r_j, s'_j) from
```

Key aspects of the above code:

- ϵ-greedy choice of action
- Replay memory D
    - stores experiences *using the policy that was in effect* when the experience was created
        - not necessarily the current policy (different values of parameter θ)
    - a batch of experiences is sampled from D to train the NN computing `Q_pred`

- Updates in a *batch*
    - in the "basic" value-action method
        - $\backslash\mathrm{actvalfun}_{\pi}$ is updated for *each action* of the Agent
    - in Q-learning
        - Q (analagous to $\backslash\mathrm{actvalfun}_{\pi}$) is updated with *multiple*, *randomly chosen* **prior** actions of the Agent
        - mini-batch: target computed for each example in the batch

```
  For each sample in the batch:
      y_j = r_j + γ * max_{a'} Q(s'_j, a'; θ^-)   # Targ
et Q-value
      # Predicted Q-value
      Q_pred = Q(s_j, a_j; θ)
```

        - per-example loss: compare target `y_j` with prediction `Q_pred`

- Loss function: MSE between `Q_pred` and calculated `y_j`

```
Compute loss = mean squared error between y_j and Q_pred
```

  - `Q_pred` is the value predicted by the NN
    - for a *batch* of examples
  - `y_j` is the "target value":
    - for the batch
    - the true value that the NN is trying to match
    - *defined* by the same calculation as regular Q-learning (Bellman equation)

      r + gamma * max_{a'} Q(s', a')

      but `Q(s', a')` replaced by NN calculated `Q(s', a'; θ^-)`
    - where θ^- are the NN's lagged weights

The advantage of updating on multiple actions rather than just the current one

- smoother updates
    - changes "averaged" over many actions, not just the current one
- avoids catastrophic forgetting
    - emphasizes retention of past knowledge, not just current action

This is all similar to the reason we use Mini-Batch Gradient Descent in Neural Networks.

Why the lagged weights θ^- in computing the target value via `Q(s', a'; θ^-)` ?

To introduce stability in training.

If we don't lag the weights: the targets computed for other mini-batches will be based on different weights

- so we have a moving target as well as a moving function

The lagged weights are periodically synchronized with the most recent weights.

Q-learning (Action-value function) vs V-learning (Value function)

The main difference between the Value function and the Action-Value function

- Value function: the value of a state is averaged over *all* actions
- Action-Value function
    - absent a model: estimate effect of a single action

The averaging in V-learning can make the estimates

- more accurate and less noisy
- particularly when the action chosen is via the "max"

(We will quantify this in the section on Bias and Variance)

Monte Carlo (MC) method (not a TD method)

In the *Monte Carlo* method

- the update δ to *each state*
- is based on the return to go from the state
    - return accumulated *to the end of the episode*
- rather than the return of just the next p experiences

Loosely, it is TD(∞)

Note

Technically: MC is *not* a TD method

- as it does not rely on bootstrapping
    - not based on other estimated values

To highlight the difference between TD an MC:

- consider an episode from initial $\stateseq_1$ to $\stateseq_2$ to terminal state $\stateseq_3$
- earning a reward of $+1$ on the transition from $\stateseq_1$ and $\stateseq_2$
- with initial state values
$$\statevalfun_\pi(\stateseq_{1,0}) = \statevalfun_\pi(\stateseq_{2,0}) = 0$$

After episode 1

- using TD
$$\statevalfun_\pi(\stateseq_{1,1}) = \statevalfun_\pi(\stateseq_{2,1}) = 1$$
- using MC
$$\statevalfun_\pi(\stateseq_{2,1}) = 1 \quad \text{same as TD}$$
$$\statevalfun_\pi(\stateseq_{1,1}) = 2 \quad \text{because it uses } \statevalfun_\pi(\states$$

To illustrate the backups:

```
State_0 --a--> State_1 --a--> ... --a--> State_T (terminal)

MC:    Wait for episode to finish, then update $V(s)$
TD:    Update $V(s)$ at every transition using $V(s_{t+1})$
```

Method	**Update Basis**	**Backup Target**
TD 1-step	After 1 step, immediate reward + next value	$R_{t+1} + \gamma V(S_{t+1})$
TD k-step	After k steps, sum of k rewards + value at $t+k$	$R_{t+1} + \ldots + \gamma^{k-1} R_{t+k} + \gamma^k V(S_{t+k})$
MC (TD(∞) / TD($\lambda = 1$))	End of episode, full sample return	G_t (total return to episode end)

We summarize the comparison of TD and MC:

Feature	Monte Carlo	Temporal Difference
Reliance	Full episode return	Single step + estimated value
Bootstrapping	No	Yes
Update Timing	End of episode	At every step

Bias and Variance

We now have multiple methods for RL

- how can we compare them ?

We will introduce the concepts of Bias and Variance as one metric with which to compare methods.

To be concrete:

What are the advantages/disadvantages of Temporal Difference vs Monte Carlo in updating the Value function ?

In both methods

- the reward that the update to $\statevalfun_{\pi}(\stateseq_{,\mathtt{k}})$ depends on *directly*
  - is the immediate one-step reward $\rew_{+1}$
- subsequent rewards are included via the Value $\statevalfun_{\pi}(\stateseq_{+1})$ of the successor state

Consider iteration k:

In TD:

- the value of the successor state is *not* the true end-of-episode k return
- it is an *estimate* of the state's value based on the *previous* iteration
- updating one estimate using another estimated is called *bootstrapping*

In MC

- the value of the successor state is the *true* end-of-episode k return

Because TD depends on an estimate

- we say that TD is *biased*.

Moreover, let's suppose that rewards are stochastic

- hence, each reward is a random variable

TD 1-step update

- depends on **exactly one** random variable

$$\hat{\mathrm{rew}}_{+1}$$

TD k-step update

- depends on k random variables: rewards for times $+1, \ldots, +k$

MC update

- depends on **at least one**
  - rewards of *all* subsequent states of the episode π

$$\hat{\mathrm{rew}}_{+1}, \ldots,$$

The variance of a sum of k random variables is *at least* k times the variance of each

- assuming the variance of each variable is the same
- independence of random variables

So the variance of the $\statevalfun$ estimated by the various methods

- is smallest for TD 1-step
- increasing with k for TD k-step
- is largest for Monte Carlo

But

- the *bias* decreases in the opposite direction of the increase of the variance
- estimate based on more information

Bias and Variance of the methods presented

Method	Bias	Variance	Notes
DP (Dynamic Programming)	Low	Low	Uses full environment model; updates use true expectations.
MC (Monte Carlo)	None	High	Unbiased (targets equal expected return), but episodes can have widely varying returns.
TD (Temporal Difference)	Moderate	Moderate/Low	Bootstrap introduces bias (approximate next value), but reduces variance compared to MC.
V-learning (State Value)	Moderate	Low	Like TD if bootstrapped; low-variance due to value averaging.
Q-learning	Moderate/High	Moderate/High	Off-policy, can have bias (maximization, bootstrapping); variance may increase due to noise in max operator.

Key Points

DP (Dynamic Programming): Lowest bias and variance, but needs a full model (rarely available in practice).

MC: Unbiased estimates, but high variance due to sampling full returns.

TD: Bootstrapping introduces bias but greatly lowers variance versus MC.

V-learning: Similar profile to TD (if bootstrapped); often lower variance than Q-learning due to value averaging.

Q-learning: Can be more biased/variable due to off-policy backups and maximization over sampled estimates.

On-policy versus Off-policy

For many methods (including Q-learning above) the method uses two sub-policies

- the *behavior policy*: the one that chooses an action. For Q-learning this is

```
With probability ε:
    Choose random action a (exploration)
Else:
    Choose a = argmax_a Q(s, a; θ) (exploitation)
```

- the *target policy*: the one that we are trying to learn
  - target value

```
y_j = r_j + γ * max_{a'} Q(s'_j, a'; θ^-)   # Target Q-value
```

  - the value to which Gradient Descent will guide the NN
  - in minimizing Loss
    - compares target `y_j` with prediction `Q_pred = Q(s_j, a_j; θ)`

When the Behavior and Target policies are identical

- we call the method *On-policy*

If the Behavior and Target polices are potentially different

- we call the method *Off-policy*

Q-learning is an **off-policy** method for several reasons.

The primary reasons are

- target policy is always greedy
- behavior policy
    - ϵ-greedy
    - example from replay buffer may have been conduced with *an older behavior policy* (different NN weights)
        - even if the example's ϵ-greedy choice was from the "greedy" side of the choice

Reason	**Behavior Policy (used to collect experience)**	**Target Policy (used in Q-value update)**
Greedy backup (max operator in update)	Behavior policy may choose non-greedy actions due to exploration	Update always targets the action with highest Q-value (optimal policy estimate)
Experience replay buffer	Policy used during sampling in past episodes (possibly older ϵ-greedy or random)	Greedy policy from current network: $a^* = \argmax_a Q(s, a; \theta^-)$
Policy/parameter mismatch over time	Behavior policy determined by weights θ at time of sampling	Target policy determined by weights θ^- at time of update (can differ)

SARSA: On-Policy Q-learning

We present another member of the TD family.

There is a method called *Deep SARSA* that modified Q-learning to be on-policy.

- does not uses Experience Replay buffer
- changes the target from

```
y_j = r_j + γ * max_{a'} Q(s'_j, a'; θ^-)
```

- to

```
y_j = r_j + γ * Q(s'_j, a'_j; θ^-)
```

  - target uses the action choice `a'_j` of current behavior policy

    ```
    Q(s'_j, a'_j; θ^-)
    ```

  - rather than the action that **maximizes** value

    ```
    max_{a'} Q(s'_j, a'; θ^-)
    ```

Deep SARSA is more stable than DQN

- it is "risk-aware"
    - target is same as behavior when the "exploratory" choice is made for policy
    - so SARSA can learn to avoid risky choices
        - exploratory choices with extreme (negative) rewards
- it is more conservative
    - DQN always makes the greedy choice

Compared to DQN, these characteristics make it

- lower variance
- more likely to converge

But SARSA is higher bias compared to DQN

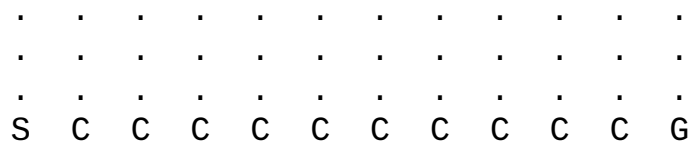
- the exploratory choice is biased away from the true (optimal) policy

Grid-world: comparing conservative SARSA to risk-loving DQN

We need to navigate in a (4×12) grid

- from start cell S
- to goal cell G
- without "falling off the cliff" (large negative reward: -100) by navigating to cliff cells C

There is a negative reward (-1) for each time step so the reward is maximized by getting to the goal quickly.


. . . .
. . . .
. . . .
S C C C C C C C C C C G

DQN will favor a path

- that hugs the edge of the cliff
    - faster route to goal G
- but that will fall off the cliff during exploration

SARSA will avoid the cliff

- slower route

In [2]:
```
from IPython.display import Image

Image(filename='images/cliffwalking_paths.gif')
```

Out[2]:
```
<IPython.core.display.Image object>
```

Each time a SARSA episode results in falling off a cliff

- a large negative value for $Q(s', a')$ is learned when a' leads to falling off the cliff from state s'
- the large negative reward propagates back through all the preceding (safe) states and actions along the path
    - any path that *could* lead to s' with some probability of a' being chosen *becomes deprecated*
- so policy becomes biased toward moving far from the Cliff
- *even if* the action a' was an *exploratory* action (that won't be repeated)

DQN always assumes the continuation of an episode from the current state is optimal

- the `max` prevents a' from being chosen in state s'
- so any path that leads to s' *does not become deprecated*

SARSA vs DQN: Summary

Feature	SARSA (On-Policy)	DQN (Off-Policy)
Policy Updated	Behavior policy (ϵ-greedy)	Greedy (optimal) policy
Update Target	$r + \gamma Q(s', a')$ (actual next action)	$r + \gamma \max a' Q(s', a')$ (max over all actions)
Bias/Variance	Higher bias, lower variance	Lower bias, higher variance (risk of instability/overestimation)
Risk Awareness	Accounts for exploration risk (safer in risky environments)	Does not account for risk of exploratory actions
Performance in Large Spaces	Limited without function approx.	Handles large/continuous spaces with neural networks
Experience Replay	Possible but less common	Standard in DQN
Stability	Often more stable, especially in risky/confusing environments	Can be less stable, especially with bad hyperparameters

Learning via Code

Here (RL_OnPolicy_vs_OffPolicy_code_examples.ipynb) is a notebook that demonstrates

- Q-Learning
    - with and without replay memor
- SARSA

How Each Example Highlights a Salient Point

1. **Q-learning (Off-policy):**
    - **Purpose:** Demonstrates how the behavior policy (epsilon-greedy; explores randomly) differs from the target policy (greedy; always chooses highest Q-value).
    - **Salient Point:** Shows that updates use the greedy maximum Q-value (off-policy), even when the sampled action was exploratory.
2. **SARSA (On-policy):**
    - **Purpose:** Demonstrates on-policy learning where both the behavior and target policies are epsilon-greedy and always match.
    - **Salient Point:** The Q update uses the value of the exact next action chosen by the behavior policy, making sampling and updating fully aligned.
3. **Q-learning with Experience Replay Buffer:**
    - **Purpose:** Shows how experience transitions are stored in a replay buffer and reused for updates.
    - **Salient Point:** Illustrates how off-policy Q-learning leverages exploratory samples and batch updates from the buffer to improve sample efficiency and stabilize learning.

Sources of Variance in RL

We introduced Variance as a way of comparing TD and MC for value-based methods.

But variance is present in many RL methods and is considered a potential impediment to Learning via RL.

- bigger updates in
    - Value function for Value-based methods
    - Parameters for Policy-based methods
- can cause large changes in Policy
- which can lead to unstable training

We list the types of methods affected by each cause.

There multiple sources of Variance, which we summarize below

- stochastic environment
    - stochastic rewards and state transitions
- stochastic policy
- sparse rewards
    - per-episode vs per-step rewards
    - reward estimates become more noisy
- bootstrapping

Source of Variance	Description	Methods Most Affected
Environmental Stochasticity	Random rewards and transitions cause unpredictable outcomes	Monte Carlo, TD, Policy Gradients
Policy Stochasticity	Probabilistic (non-deterministic) action selection	MC, On-policy, Policy Gradients
Long Trajectory Aggregation	Returns summed over many steps compound the randomness	Monte Carlo, n-step methods
Sparse or Delayed Rewards	Few positive signals lead to noisy estimates	Monte Carlo, Value-based
Bootstrapping Error	Using own predictions as targets introduces bias, but lowers variance	TD, Q-Learning
Credit Assignment Difficulty	Uncertainty in linking actions to future rewards increases variance	Policy Gradients, MC
Sample Size & Exploration	Insufficient samples or aggressive exploration cause wide fluctuations	All methods
Model/Optimization Instability	Neural network or optimizer issues amplify variance	Deep RL, Policy Gradient methods
Non-stationarity	Changing environment or policy during training	All methods, esp. online learning

Key Points:

- Variance comes from randomness in data, policy, training procedure, and modeling choices.
- Some sources can be controlled with design choices (e.g., baselines, bootstrapping, averaging), while others are intrinsic to RL problems.

As we introduce new RL methods

- one motivation if to reduce variance

Some potential ways to reduce variance

- $n > 1$-step ahead Temporal Difference
- estimating updates in mini-batches
    - as in Gradient Descent

In [3]:
```
print("Done")
```

```
Done
```