Other decompositions of $\mathbf{X}$

Eigen decomposition of covariance matrix of $\boldsymbol{X}$

There is another matrix factorization method known as Eigen Decomposition.

Eigen decomposition, unlike SVD, only works on symmetric matrices M:

$$M = W\Lambda W^T$$

where $WW^T = I$

We can obtain the PCA from the Eigen Decomposition of $\mathbf{X}\mathbf{X}^T$

- the covariance matrix of $\mathbf{X}$ (i.e., original feature covariance)
- the covariance matrix is symmetric, as required

We can relate the SVD of $\mathbf{X}$ to the Eigen decomposition of $\mathbf{X}\mathbf{X}^T$ as follows:

$$\begin{aligned}
\mathbf{X}^T\mathbf{X} &= V\Sigma U^T U\Sigma V^T && \text{from SVD } \mathbf{X} = U\Sigma V^T \\
\mathbf{X}^T\mathbf{X} &= V\Sigma\Sigma^T V^T && \text{since } U^T U = I
\end{aligned}$$

Similarly, we can show

$$\mathbf{X}\mathbf{X}^T = U\Sigma\Sigma^T U^T \text{since } VV^T = I$$

Setting

- $\Lambda = \Sigma\Sigma^T$
- $W = U = V$ we get $\mathbf{X} = W\Lambda W^T$, the Eigen Decomposition of $\mathbf{X}\mathbf{X}^T$.

The V that transforms $\mathbf{X}$ (original features) to $\tilde{\mathbf{X}} = XV$ (synthetic features)

- Can be computed directly from SVD
- Or by creating covariance matrix $\mathbf{X}\mathbf{X}^T$ and using Eigen decomposition.

SVD is more commonly used

- There are many fast implementations of SVD
- There is no need to compute the big covariance matrix $\mathbf{X}\mathbf{X}^T$

Other factorization methods

- CUR method
  $$\mathrm{CUR}(\mathbf{X}) = C \cdot U \cdot R$$
- C chosen from Columns of $\mathbf{X}$
- R chosen from Rows of $\mathbf{X}$

In [4]:

```
print("Done")
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Done