

Correlated features

Consider the following set of examples with 2 features

Two features: perfect correlation

As you can see

- \mathbf{x}_2 is perfectly correlated with \mathbf{x}_1
$$\mathbf{x}_2^{(i)} = 2 * \mathbf{x}_1^{(i)}$$

A way to conceptualize $\mathbf{x}^{(i)}$

- As a point in the space spanned by unit basis vectors parallel to the horizontal and vertical axes.

$$\mathbf{u}_{(1)} = (1, 0)$$

$$\mathbf{u}_{(2)} = (0, 1)$$

- With $\mathbf{x}^{(i)}$ having exposure

$$\mathbf{x}_1^{(i)} \text{ to } \mathbf{u}_{(1)}$$

$$\mathbf{x}_2^{(i)} \text{ to } \mathbf{u}_{(2)}$$

So example $\mathbf{x}^{(i)}$ is

$$\mathbf{x}^{(i)} = \sum_{j'=1}^2 \mathbf{x}_{j'}^{(i)} * \mathbf{u}_{(j')}$$

But because

$$\mathbf{x}_2^{(i)} = 2 * \mathbf{x}_1^{(i)}$$

we can create an *alternate* basis vector (no longer parallel to the axes)

$$\tilde{\mathbf{v}}_{(1)} = (1, 2)$$

such that example $\mathbf{x}^{(i)}$ is

$$\mathbf{x}^{(i)} = \tilde{\mathbf{x}}_1^{(i)} * \tilde{\mathbf{v}}_{(1)}$$

where $\tilde{\mathbf{x}}_1^{(i)} = \mathbf{x}_1^{(i)}$

That is, $\mathbf{x}^{(i)}$ has exposure $\tilde{\mathbf{x}}_1^{(i)}$ to the new, single basis vector.

So

- Rather than representing $\mathbf{x}^{(i)}$ as a vector with 2 features (in the original basis)
- We can represent it as $\tilde{\mathbf{x}}^{(i)}$, a vector with 1 feature (in the new basis)

This is the essence of dimensionality reduction

- Changing bases to one with fewer basis vectors

It is rarely the case for features to be perfectly correlated

Two features: imperfect correlation

In this case

- A second basis vector $\tilde{\mathbf{v}}_{(2)}$

- Orthogonal to the first

$$\tilde{\mathbf{v}}_{(1)} \cdot \tilde{\mathbf{v}}_{(2)} = 0$$

could approximate $\mathbf{x}^{(i)}$

$$\mathbf{x}^{(i)} = \sum_{j'=1}^2 \tilde{\mathbf{x}}_{j'}^{(i)} * \tilde{\mathbf{v}}_{(j')}$$

Two features: imperfect correlation, alternate basis

The black lines represent the alternate basis vectors $\tilde{\mathbf{v}}_{(1)}$, $\tilde{\mathbf{v}}_{(2)}$.

- Alternate, compared to the original basis vectors which were parallel to the axes

As you can see:

- The variation along $\tilde{\mathbf{v}}_{(1)}$ is much greater than that around $\tilde{\mathbf{v}}_{(2)}$
- Capturing the notion that the "main" relationship is along $\tilde{\mathbf{u}}_{(1)}$

In fact, if we dropped $\tilde{\mathbf{v}}_{(2)}$ such that $\|\tilde{\mathbf{x}}\| = 1$

- The examples would be projected onto the line $\tilde{\mathbf{v}}_{(1)}$
- With little information being lost

Subsets of correlated features

It may not be the case that a group of features is correlated across *all* examples

Consider the MNIST digits

- The subset of examples corresponding to the digit "1"
- Have a particular set of correlated features (forming a vertical column of pixels near the middle of the image)
- Which *may not* be correlated with the same features in examples corresponding to *other* digits

Thus, a synthetic feature encodes a "concept" that occurs in many but not all examples

We will present a method to *discover* "concepts"

- It may not necessarily be the pattern of features that corresponds to an entire digit
- It may be a partial pattern common to several digits
 - Vertical band (0, 1, 4, 7)
 - Horizontal band at top (5, 7, 9)

In [5]: `print("Done")`

Done