

# Linear Model with higher order features

Our error analysis of the toy problem suggested that a straight line was perhaps not the best fit

- positive errors in the extremes
- negative errors in the center

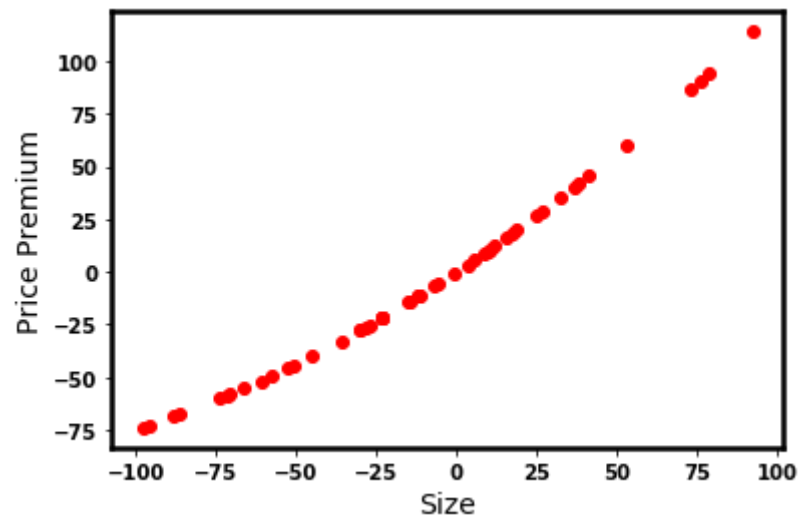
Perhaps a "curve" would be a better hypothesis ? What if our data is not linear ?

Here's what the first dataset looked like

```
In [4]: (xlabel, ylabel) = ("Size", "Price Premium")

# I will give you the data via a function (so I can easily alter the data in sub
sequent examples)
v1, a1 = 1, .005
lin = recipe_helper.Recipe_Helper(v = v1, a = a1)
X_lin, y_lin = lin.gen_data(num=50)
```

```
In [5]: X_orig, y_orig = X_lin, y_lin  
_ = lin.gen_plot(X_orig, y_orig, xlabel, ylabel)
```



```
In [6]: _ = lin.run_regress(X_orig, y_orig)
```

Coefficients:

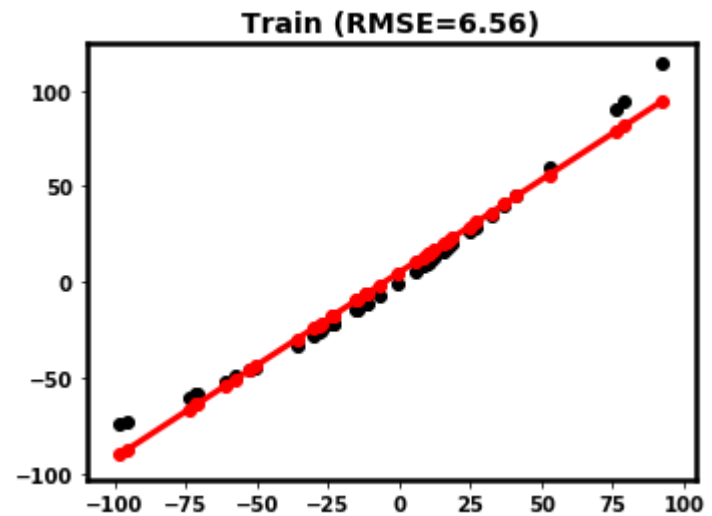
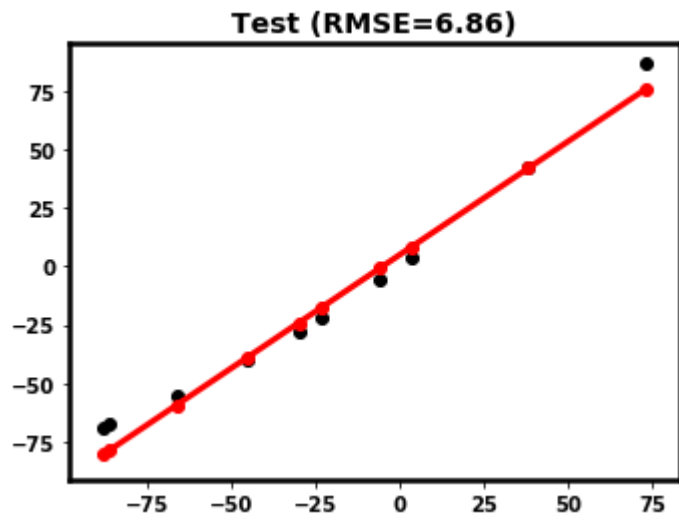
```
[4.93224426] [[0.96836946]]
```

R-squared (test): 0.98

Root Mean squared error (test): 6.86

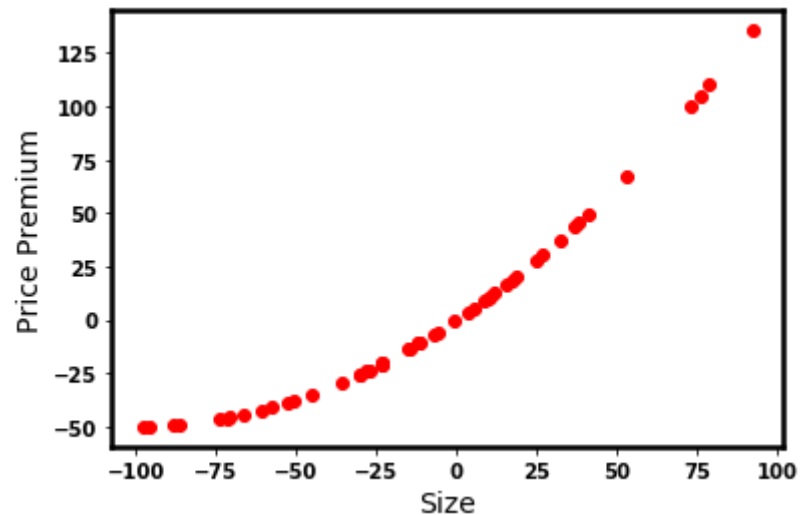
R-squared (train): 0.98

Root Mean squared error (train): 6.56



We will make our point by creating a similar dataset (the "curvy" dataset) that **exaggerates the curvature.**

```
In [7]: v2, a2 = v1, a1*2  
curv = recipe_helper.Recipe_Helper(v = v2, a = a2)  
X_curve, y_curve = curv.gen_data(num=50)  
_ = curv.gen_plot(X_curve,y_curve, xlabel, ylabel)
```



```
In [8]: _= curv.run_regress(X_curve, y_curve)
```

Coefficients:

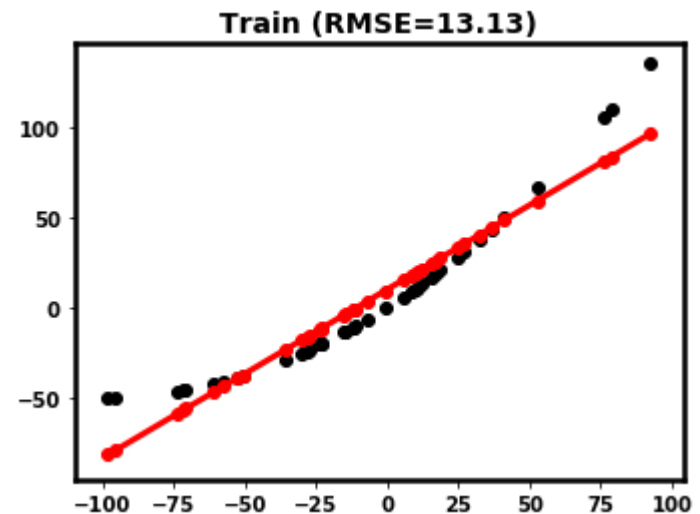
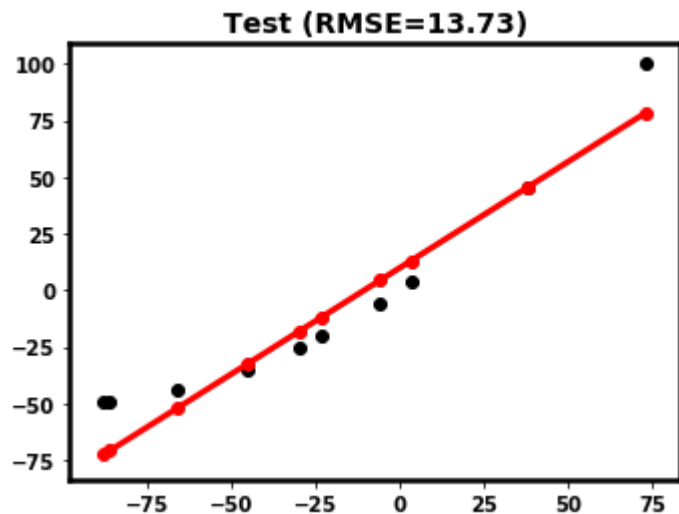
```
[9.86448852] [[0.93673892]]
```

R-squared (test): 0.91

Root Mean squared error (test): 13.73

R-squared (train): 0.91

Root Mean squared error (train): 13.13





Compared to the original, the "curvy" data set has a lot more curvature

- the  $R^2$  is still over 90%
- but the Performance Metric (RMSE) is twice as big

## Curvature in a linear model

Our (first-order) linear model was

$$\mathbf{y} = \Theta_0 + \Theta_1 \mathbf{x}$$

We can create a *second order* linear model by adding a feature  $x^2$ :

$$\mathbf{y} = \Theta_0 + \Theta_1 \mathbf{x} + \Theta_2 \mathbf{x}^2$$

$\mathbf{y}$  is a second order polynomial, whose plot is a curve

- but it is linear in features  $\mathbf{x}, \mathbf{x}^2$

In other words, we are performing feature iteration

- in this case: adding the missing feature  $x^2$

Let's modify  $\mathbf{x}^{(i)}$  from a vector of length 1:

$$\mathbf{x}^{(i)} = (\mathbf{x}_1^{(i)})$$

to a vector of length 2:

$$\mathbf{x}^{(i)} = (\mathbf{x}_1^{(i)}, \mathbf{x}_1^{(i)2})$$

by adding a squared term to the vector  $\mathbf{x}^{(i)}$ , for each  $i$ .

The modified  $\mathbf{X}'$  becomes:

$$\mathbf{X} = \begin{pmatrix} 1 & \mathbf{x}_1^{(1)} & (\mathbf{x}_1^{(1)})^2 \\ 1 & \mathbf{x}_1^{(2)} & (\mathbf{x}_1^{(2)})^2 \\ \vdots & \vdots & \\ 1 & \mathbf{x}_1^{(m)} & (\mathbf{x}_1^{(m)})^2 \end{pmatrix}$$

Note that this modified  $\mathbf{X}'$  fits perfectly within our Linear hypothesis

$$\hat{\mathbf{y}} = \mathbf{X}' \Theta$$

The requirement is that the model be linear in its *features*, **not** that the features be linear !

What we have done is added a second feature, that just so happens to be related to the first.

We can now run our linear model with the modified feature vectors

### **A word about our module**

- we add the  $x^2$  column by setting optional parameter `run_transform` to `True`

```
In [9]: _ = curv.run_regress(X_curve, y_curve, run_transforms=True)
```

Coefficients:

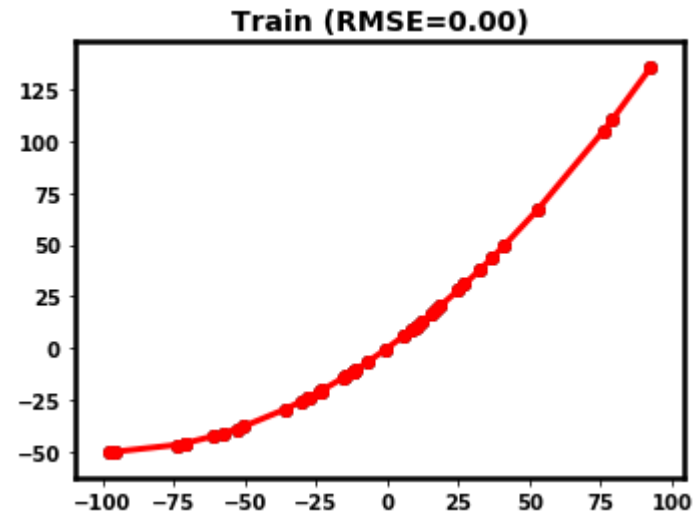
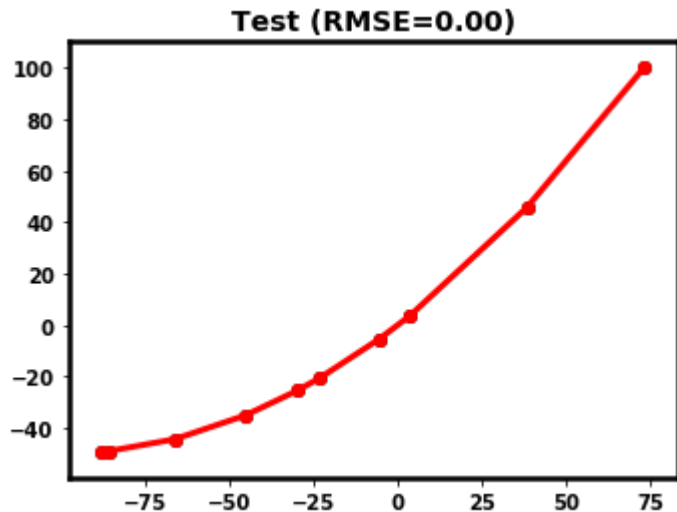
```
[-3.55271368e-15] [[1.    0.005]]
```

R-squared (test): 1.00

Root Mean squared error (test): 0.00

R-squared (train): 1.00

Root Mean squared error (train): 0.00





Perfect fit !

## TIP

- Don't stop just because you scored 91%. And don't give up if the score was awful.
- Examining the errors (residuals) reveals a lot about how to improve your model.
  - Where was the fit good ? Where was it bad ?
  - Is there a pattern to the badly fit observations that points to a missing feature ?

One of the real arts of ML is diagnosing model deficiencies and knowing how to improve them.

We will have a separate module on this topic.

In [10]: `print("Done !")`

Done !