Correlated features

Consider the following set of examples with 2 features

Two features: perfect correlation

As you can see

- $\mathbf{x}_2$ is perfectly correlated with $\mathbf{x}_1$
$$\mathbf{x}_2^{(\mathbf{i})} = 2 * \mathbf{x}_1^{(\mathbf{i})}$$

A way to conceptualize $\mathbf{x^{(i)}}$

- As a point in the space spanned by unit basis vectors parallel to the horizontal and vertical axes.

$$\mathbf{u}_{(1)} = (1, 0)$$
$$\mathbf{u}_{(2)} = (0, 1)$$

- With $\mathbf{x^{(i)}}$ having exposure

$$\mathbf{x}^{(\mathbf{i})}_1 \text{ to } \mathbf{u}_{(1)}$$
$$\mathbf{x}^{(\mathbf{i})}_2 \text{ to } \mathbf{u}_{(2)}$$

So example $\mathbf{x^{(i)}}$ is

$$\mathbf{x^{(i)}} = \sum_{j'=1}^{2} \mathbf{x}^{(\mathbf{i})}_{j'} * \mathbf{u}_{(j')}$$

But because

$$\mathbf{x}_2^{(\mathbf{i})} = 2 * \mathbf{x}_1^{(\mathbf{i})}$$

we can create an *alternate* basis vector (no longer parallel to the axes)

$$\tilde{\mathbf{v}}_{(1)} = (1, 2)$$

such that example $\mathbf{x}^{(\mathbf{i})}$ is

$$\mathbf{x}^{(\mathbf{i})} = \tilde{\mathbf{x}}_1^{(\mathbf{i})} * \tilde{\mathbf{v}}_{(1)}$$

where $\tilde{\mathbf{x}}_1^{(\mathbf{i})} = \mathbf{x}_1^{(\mathbf{i})}$

That is, $\mathbf{x}^{(\mathbf{i})}$ has exposure $\tilde{\mathbf{x}}^{(\mathbf{i})}_1$ to the new, single basis vector.

So

- Rather than representing $\mathbf{x}^{(\mathbf{i})}$ as a vector with 2 features (in the original basis)
- We can represent it as $\tilde{\mathbf{x}}^{(\mathbf{i})}$, a vector with 1 feature (in the new basis)

This is the essence of dimensionality reduction

- Changing bases to one with fewer basis vectors

It is rarely the case for features to be perfectly correlated

Two features: imperfect correlation

In this case

- A second basis vector $\tilde{\mathbf{v}}_{(2)}$
- Orthogonal to the first
  $$\tilde{\mathbf{v}}_{(1)} \cdot \tilde{\mathbf{v}}_{(2)} = 0$$
  could approximate $\mathbf{x}^{(\mathbf{i})}$

$$\mathbf{x}^{(\mathbf{i})} = \sum_{j'=1}^{2} \tilde{\mathbf{x}}_{j'}^{(\mathbf{i})} * \tilde{\mathbf{v}}_{(j')}$$

Two features: imperfect correlation, alternate basis

The black lines represent the alternate basis vectors $\tilde{\mathbf{v}}_{(1)}, \tilde{\mathbf{v}}_{(2)}$.

- Alternate, compared to the original basis vectors which were parallel to the axes

As you can see:

- The variation along $\tilde{\mathbf{v}}_{(1)}$ is much greater than that around $\tilde{\mathbf{v}}_{(2)}$
- Capturing the notion that the "main" relationship is along $\tilde{\mathbf{u}}_{(1)}$

In fact, if we dropped $\tilde{\mathbf{v}}_{(2)}$ such that $||\tilde{\mathbf{x}}|| = 1$

- The examples would be projected onto the line $\tilde{\mathbf{v}}_{(1)}$
- With little information being lost

Subsets of correlated features

It may not be the case that a group of features is correlated across *all* examples

Consider the MNIST digits

- The subset of examples corresponding to the digit "1"
- Have a particular set of correlated features (forming a vertical column of pixels near the middle of the image)
- Which *may not* be correlated with the same features in examples corresponding to *other* digits

Thus, a synthetic feature encodes a "concept" that occurs in many but not all examples

We will present a method to *discover* "concepts"

- It may not necessarily be the pattern of features that corresponds to an entire digit
- It may be a partial pattern common to several digits
    - Vertical band (0, 1, 4, 7)
    - Horizontal band at top (5, 7, 9)

In [5]:
```python
print("Done")
```

```
Done
```