Recap of "Classical" Machine Learning

Canonical Machine Learning tasks

- Classification
    - Given attributes, predict a target value from among a *discrete* set of values
- Regression
    - Given attributes, predict a target value from a *continuous* range of values

To solve the task

- We create a *parameterized* model: mapping from input features to label/target
- Fit the model parameters on a collection of examples (feature/target pairs)

$$\langle \mathbf{X}, \mathbf{y} \rangle = [\mathbf{x}^{(i)}, \mathbf{y}^{(i)} | 1 \leq i \leq m]$$

- Use the fitted model to predict a $\hat{\mathbf{y}}$ from an unseen feature vector $\mathbf{x}$

It is often the case that we can make our model more successful (e.g. more accurate) by

- rather than using raw inputs $\mathbf{x}$
- we transform the raw inputs
- fit our model and make predictions on the transformed feature vectors

To illustrate, consider some features describing a person

- Height
- Weight
- Income

Our goal is to predict the target: some measure of Happiness.

Extremely small differences in Height or Weight probably don't affect Happiness.

We may choose to transform each variable from a continuous value to a discrete value

- Height transformed to Height Bucket: { Short, Medium, Tall }
- Weight transformed to Weight Bucket: { Thin, Just Right, A Little Extra }

It is sometimes the case that each variable individually is a poor predictor, but the combination is powerful.

In such a case, a transformation that creates a new "combination" feature may be helpful

- Create a discrete binary variable Combo indicating "Perfect Height and Perfect Weight"

The absolute level of Income may not be as good a predictor of Happiness as the level relative to one's peers

- We may choose to transform Income into Relative Income: Income/(Median Income)

By fitting/training on

- (Height Bucket, Weight Bucket, Combo, Relative Income)
- rather than (Height, Weight,Income) our model might make better predictions.

This process of transforming/augmenting input features is called *feature engineering*

- In Classical Machine Learning, it is the responsibility of the person fitting the model

In "Classical" Machine Learning (ML) the paradigm is to

- *Manually* create a sequence of transformations from raw input to an alternate representation
    - Feature engineering: create representations corresponding to "concepts" expressed by the data
    - A sequence of transformations a *pipeline*
- The final representation created by the sequence may result in a better prediction than that which could be obtained from the raw representation

We continue to use our Basic Notational standards (ML_Notation.ipynb) from the Classical ML part of the course.

This will be extended to Deep Learning.

Deep Learning: Introduction

We introduce the topic of Deep Learning

- by comparing and contrasting it to Classical Machine Learning

How are tasks defined

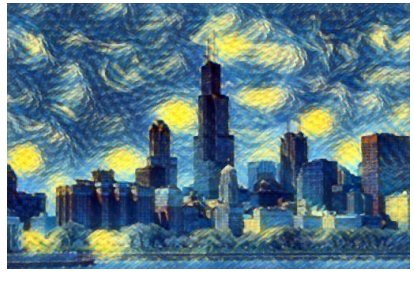
The objectives of Deep Learning are similar to that of Classical ML

- to solve tasks

In Classical ML we confined the tasks to Regression and Classification.

In Deep Learning, the task is *implicitly* defined by the Loss Function

Recall the "Neural Style Transfer" task

Content		Style
	+	
	=	
Content depicted in given Style		
		

What does it mean to "depict the Content image in the style of the Style image" ?

We define a "solution" $\vec{x}$ to this task

- as an image vector $\vec{x}$
- that minimizes a Loss Function

$$\loss = \loss_{\text{content}}(\vec{p}, \vec{x}) + \loss_{\text{style}}(\vec{a}, \vec{x})$$

- that we have created to capture the meaning of Neural Style Transfer

As defined by the 2 part Loss Function, a solution to the Neural Style Transfer task

- is similar to the Content image in appearance
    - the Content Loss
- has a style similar to the Style image

The *task is defined* by the Loss Function.

- we defer the details of $\backslash\text{loss}_{\text{content}}$ and $\backslash\text{loss}_{\text{style}}$

There are no constraints on the Loss Functions of Deep Learning.

In contrast, in Classical ML

- Loss Functions were well-defined, limited mathematical formulae
- often chosen for being amenable to closed-form or simple methods for solution

As a simplification

- In this course, we will (mostly) limit our tasks to Regression and Classification.

How is computation defined

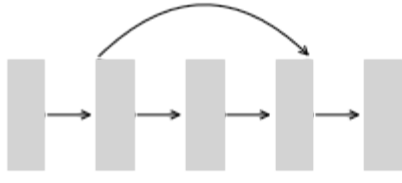
Another difference between Classical ML and Deep Learning

- the computations in Deep Learning are implemented by a network of nodes
    - the *Neural Network*
    - rather than an imperative program
        - sequence of operations (e.g., Python)
- the nodes implement a limited number of computations
    - *simple*: dot-product
    - *parameterized*

The network is described by computation graph, which can be an arbitrary Directed Acyclic Graph.

In [6]:
```
fig_tf_func
```

Out[6]:

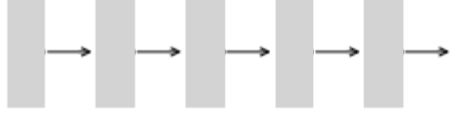


In this course

- we limit the graph to a *linear sequence* of nodes
    - called *layers*

In [7]:
```
fig_tf_seq
```

Out[7]:



This restriction limits graphs to implementing what is called the

- *Sequential* architecture.
- the simplicity mainly has notational advantages
- many layers: "deep" learning

In the Advanced Deep Learning course, we remove this restriction, creates what is know as

- the *Functional* architecture

Aside

This sequential organization restricts the types of functions (and tasks we can solve) that can be implemented

Letting $F_{\llp}$ denote the simple function computed by the node at layer $\ll$ and $\W_{\llp}$ its parameters ("weights")

- the network can compute functions defined by function compositions

$$
\begin{aligned}
\y_{\llp} &= F_{\llp}(\y_{(\ll -1)}; \W_{\llp}) \\
&= F_{\llp}(\ F_{(\ll -1)}(\y_{(\ll -2)};\ \W_{(\ll -1)});\ \W_{\llp}\ ) \\
&= F_{\llp}(\ F_{(\ll -1)}(\ F_{(\ll -2)}(\y_{(\ll -3)};\ \W_{(\ll -2)});\ \W_{(\ll -1)}\ );\W_{\llp}\ ) \\
&= \vdots
\end{aligned}
$$

These limitations in architecture and tasks enables a paradigm very similar to that of Classical Machine Learning

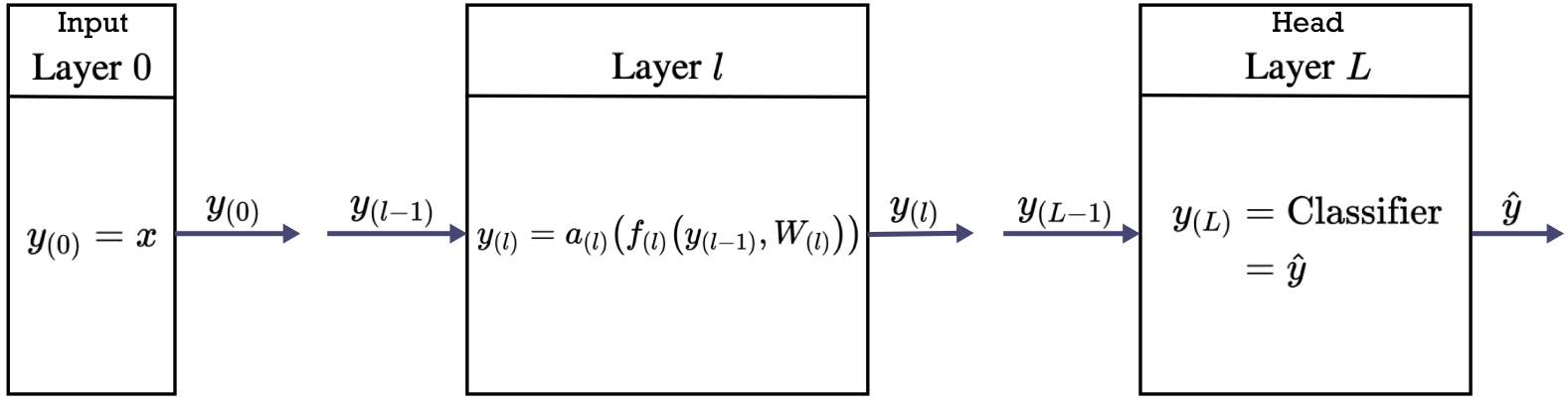
- a sequence of transformations
- that transforms the input
- such that the final representation
- results in better predictions
- by a final layer that implements either Regression or Classification

We shall frequently say that

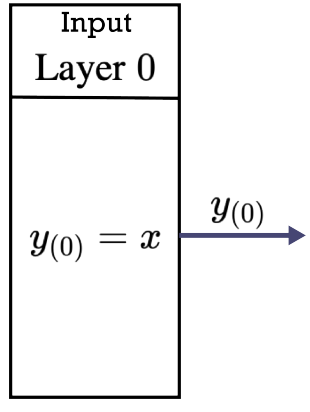
- the purpose of a Layer
- is to create an *alternate representation* of the input
    - no new information not present in the input is created

Here is a "cartoon" diagram of Deep Learning (as applied to the Classification task)

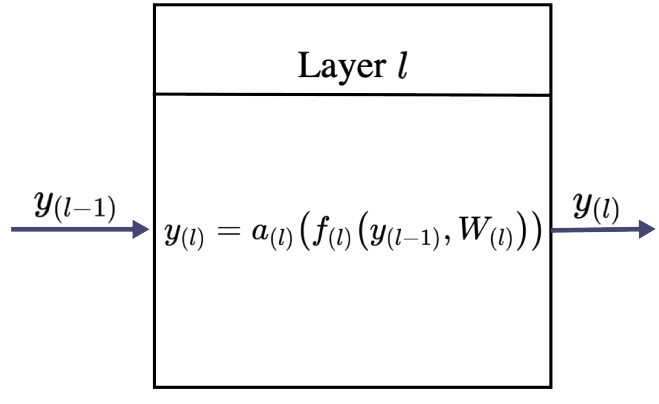
Layers


Input
Layer 0
$y_{(0)} = x$
$y_{(0)}$
$y_{(l-1)}$
Layer l
$y_{(l)} = a_{(l)}(f_{(l)}(y_{(l-1)}, W_{(l)}))$
$y_{(l)}$
$y_{(L-1)}$
Head
Layer L
$y_{(L)} = \text{Classifier}$
$= \hat{y}$
$\hat{y}$

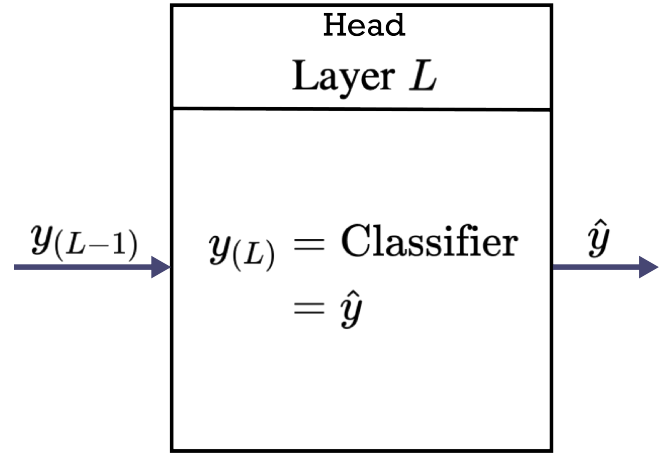
- A sequence of *layers* (vertical boxes)
- Starting with the input layer (the large vertical box on the left)



- Intermediate layer $\ll$ takes as input the output of layer $(\ll -1)$
    - this is the limitation imposed by the Sequential architecture
- Transforming it into an alternate representation
- via a *simple pre-defined* function $f_{\backslash \mathrm{llp}}$

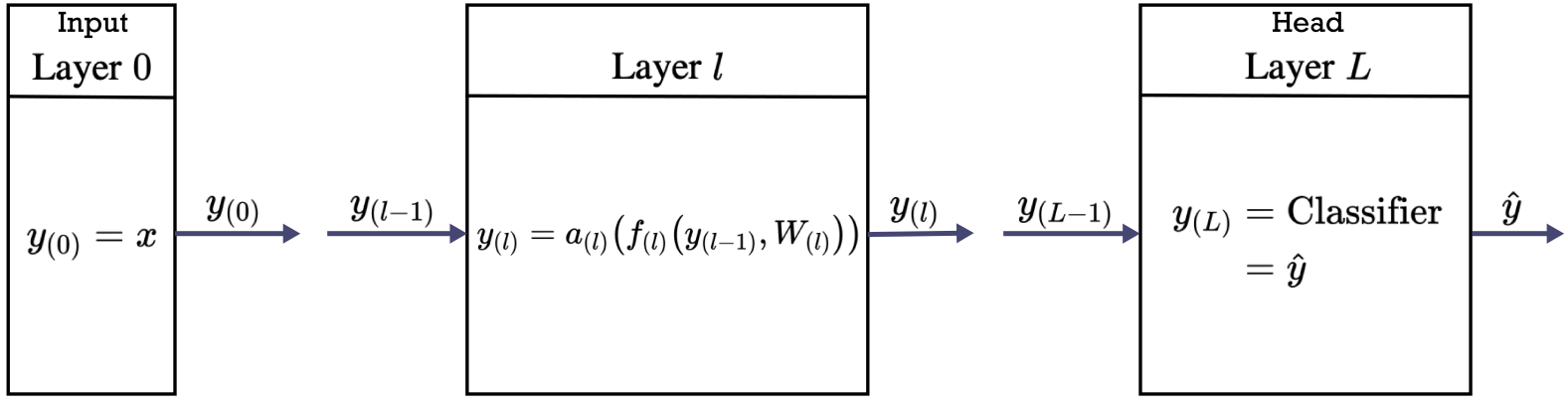


- The output of the penultimate layer $(L-1)$ is used as input to a final layer L that implements either
    - Classification
    - Regression



The process of moving through the layers from Input to penultimate is

- **Successive transformation of the input**
- **Each layer's output is an alternate *representation* of the input**



- **This is similar to the Feature Engineering pipeline of Classical ML**
    - **Implemented in many ML toolkits (e.g., `sklearn`)**
- **Where the final version of the transformed data is fed into a Classifier/Regression model**

Per-layer functions may be non-linear

Another key difference from Classical Machine Learning

- **the functions f_{llp} can be non-linear**

We will see, in a later module, the importance of non-linearity.

The behavior of the per-layer functions is learned, not pre-specified

Finally, a major difference

- **in Classical ML: the Data Scientist explicitly defines the transformations**
- **in Deep Learning**
  - **the transformations are achieved**
  - **by a composition of the per-layer functions of successive layers**
  - ***without pre-specifying* the goal of the transformation !**

The Deep Learning transformations

- **are defined by the parameters of the per-layer functions** f_{llp}
- **which are *learned* by training**

One difficulty with learning the transformations

- **they tend to work**
- **but we don't necessarily know what they mean**

Because the behavior of the model is *learned* from the Training Examples

- **the model behavior is more sensitive to the Training Data**
- **compared to Classical ML**

This is because

- **in Classical ML we can sometimes specify the *functional form* of the model or its transformations**

For example: in our initial examples for Linear Regression (predict House Price from Size)

- **we initially specified a first order polynomial as the functional form**

$$\y = \Theta_0 + \Theta_1 \x$$

When this proved insufficient, we changed the functional form to a second order polynomial

$$\y = \Theta_0 + \Theta_1 \x + \Theta_2 \x^2$$

In the absence of an explicit specification of the functional form

- **the model infers the form from the Training Data**

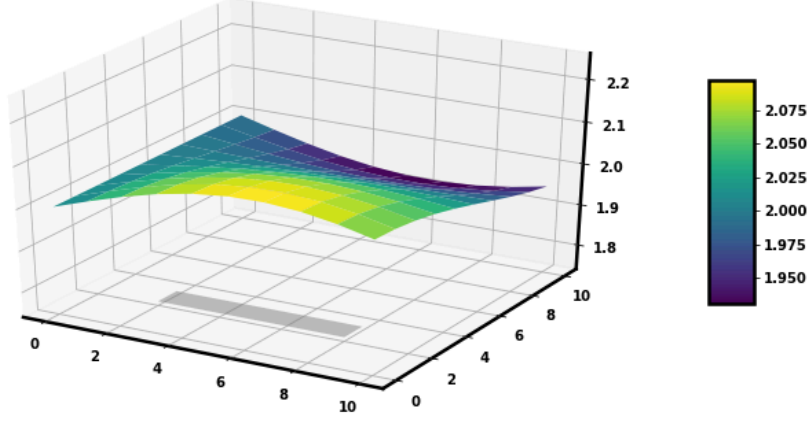
Consider a complex form

In [8]:
```
%matplotlib notebook

fig, ax = nn_ch.draw_surface(visible=True)
```

If the range of the features $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2)$ is not sufficiently varied

- the shape that we are defining *by example*
- is only a partial picture of the true shape




2.2
2.1
2.075

Deep Learning is implemented by special libraries

The computation graphs are defined by a specialized language (really a Python library)

- ***TensorFlow***
- ***PyTorch***

All operations in Deep Learning

- **must be operations of these specialized languages**
- **not pure Python**

As we will see, the reason for restricting the operations

- **the minimization of the Loss Function will be by Gradient Descent**
- **thus, we must be able to compute the Gradient of every operation**
- **operations are chosen and implemented such that**
    - ***Gradients are automatically computed*** **by the language**

Summary

Deep Learning

- tasks defined by arbitrary Loss Functions
- computation via a Graph of nodes
    - nodes implement *simple* pre-defined functions f_{llp}
    - functions f_{llp} are parameterized
    - functions may be non-linear
- the computation is *learned* rather than pre-specified
    - training sets the parameters of the per-layer functions
    - so as to minimize the Loss Function
- specialized language to enable automated computation of Gradients of operations
    - "Learning": find Loss-minimizing values of all parameters, via Gradient Descent

What does a layer do ?

The above was very informal.

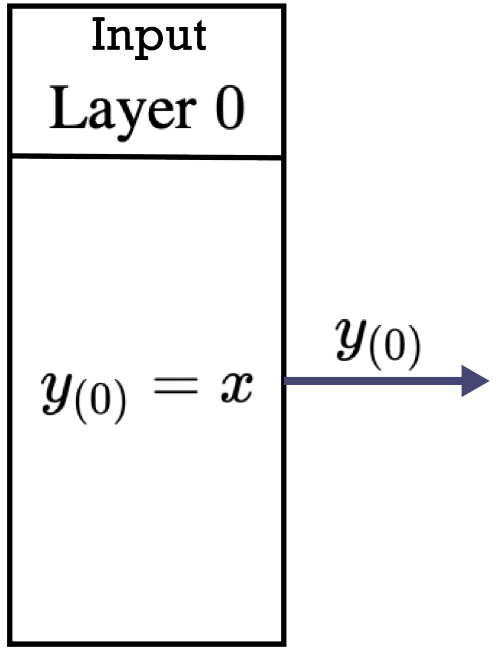
We need to introduce more concepts and, unfortunately, the notation that will carry us through the Deep Learning part of the course.

Time to go under the covers of a layer (Intro_to_Neural_Networks.ipynb)

Intuition

With this pattern/template matching intuition in mind, let's revisit our path through the layers.

Let's start with the inputs to the NN: an example with our original features

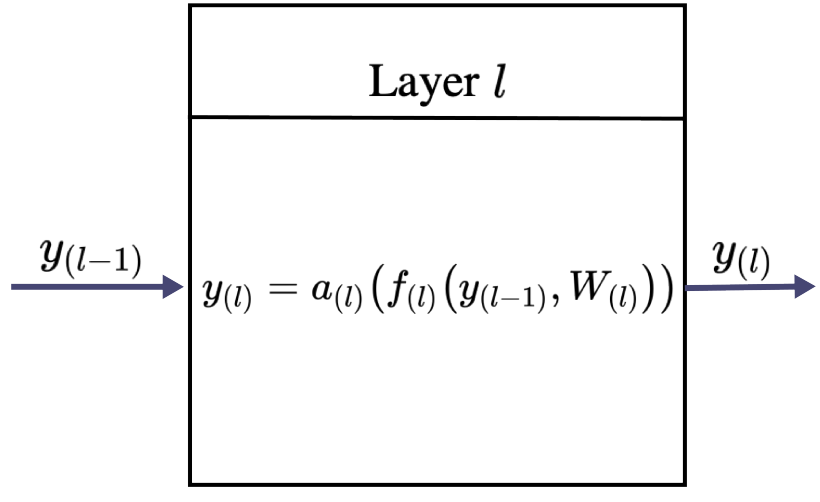


Layer 1 is attempting to match our input $\mathbf{x}$ against $n_{(1)}$ patterns.

So $\mathbf{y}_{(1)}$ is a vector of synthetic features (an alternate representation of $\mathbf{x}$)

- where each feature in $\mathbf{y}_{(1)}$ is an indicator as to whether a particular pattern appears in $\mathbf{x}$.

Layer $\ll$ works similarly except that the patterns it matches are against synthetic feature in $\mathbf{y}_{(\ll -1)}$ rather than $\mathbf{x}$.



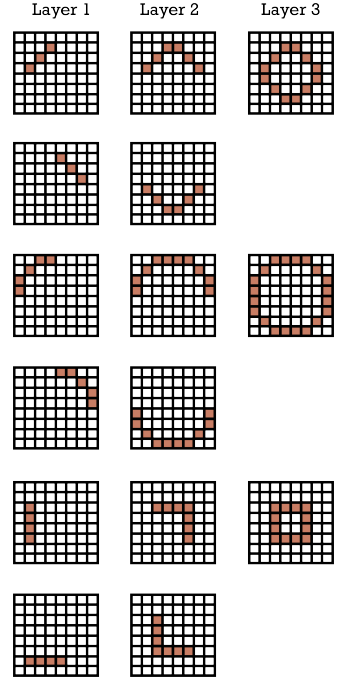
One interpretation/theory of how Deep Learning works

- **the patterns of layer $\ll$ are more complex patterns**
- **composed from the simpler patterns of layer $(\ll -1)$**

So, as we proceed deeper into the network

- **we create *synthetic features* (the alternate representation) of increasing complexity**
- **such that the alternate representation of penultimate layer $(L-1)$**
- **is sufficient for the layer L head (Regression/Classifier) to succeed**

Features by layer



What is $\mathbf{W}_{\mathrm{llp}}$? Where did Θ go ?

Our old friend the dot product is back in the forefront.

But now, the pattern matching is written

$$\mathbf{W}_{\mathrm{llp},j} \cdot \mathbf{y}_{(\ll -1)}$$

rather than the

$$\Theta \cdot \mathbf{x}$$

which was familiar in the Classical Machine Learning part of the course.

$\mathbf{y}_{(\ll -1)}$ appears rather than $\mathbf{x}$

- since, in the Sequential architecture
- the input to layer $\ll$ is the output of layer $(\ll -1)$

But why did the denotation of the "pattern" change from Θ to $\mathbf{W}$ ("weights")

Unfortunately, this is an artifact of two different communities working independently

- The Classical Machine learning community uses the term *parameters* and the Greek letter Θ
- The Computer Science community uses the term *weights* and the letter W

They are *exactly the same thing*.

Size of $\mathbf{W}$: Always count the number of parameters !

When constructing a layer of a Neural Network: always count the number of weights/parameters. They grow quickly !!

For a Fully Connected/Dense layer $\ll$

- Consists of $n_{\text{llp}} = ||\mathbf{y}_{\text{llp}}||$ units, each unit producing a new feature
- Each unit performs the dot product
$$\mathbf{y}_{\text{llp},j} = \mathbf{y}_{(\ll -1)} \cdot \mathbf{W}_{\text{llp},j}$$
- Each dot product thus involves $||\mathbf{y}_{(\ll -1)}|| + 1$ weights in $\mathbf{W}_{\text{llp},j}$
    - the "+ 1" is because of the bias term in each unit
- Thus the number of weights in $\mathbf{W}_{\text{llp}}$ is $||\mathbf{y}_{\text{llp}}|| * (||\mathbf{y}_{(\ll -1)}|| + 1)$

Activation functions

At this point perhaps you have a mechanical understanding of a neural network

- **A sequence of layers**
- **Each layer is creating new features**
- **Subsequent layers creating features of increasing complexity but transforming the prior layer's features**
- **A new feature is created by a linear dot product followed by an non-linear activation**

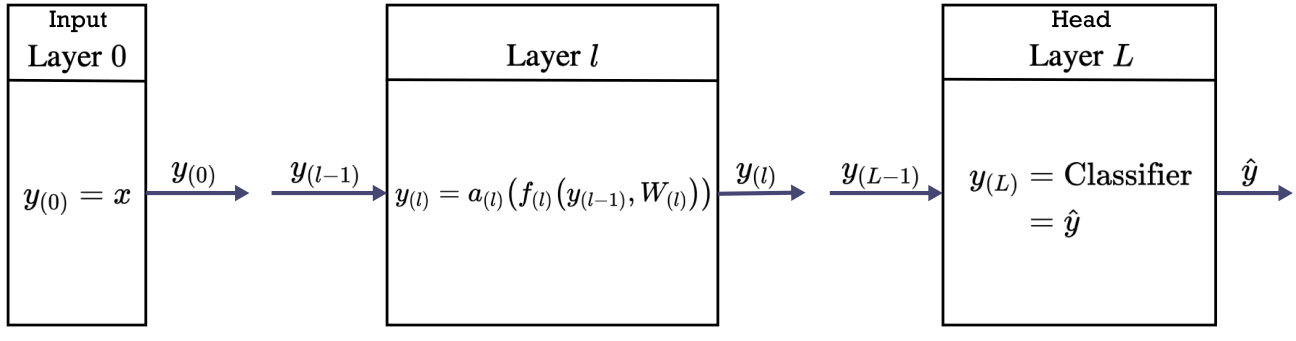
It turns out that the non-linear activation function is *one of the keys* to Neural Networks !

Let's explore Activation functions (Neural_Networks_Activations.ipynb) in more depth.

Final layer: Regression/Classification

The Head Layer (layer L)

- takes the final representation of the data
- applies a layer determined by the task
    - Regression
    - Classification



- Regression is nothing more than a dot product
    - $\mathbf{y}_L = \mathbf{y}_{(L-1)} \cdot \mathbf{W}_L$
    - Implemented by a Fully Connected layer with no activation
- Classification is nothing more than a dot product followed by a sigmoid activation
    - $\mathbf{y}_L = \sigma(\mathbf{y}_{(L-1)} \cdot \mathbf{W}_L)$
    - Exactly the same way as discussed in our "Classical" Machine Learning lecture
    - Implemented by a Fully Connected layer with a sigmoid activation

We will see this explicitly when we show the code for each of these tasks.

Questions to consider

Some natural questions to ask at this point

- **How many layers should we have (What is the right value for L) ?**
- **How many units $n_{\llp}$ should I have for each layer $1 \leq \ll \leq (L-1)$?**
- **What activation function should I use for each unit?**

We will address each of these in the future.

Perhaps the biggest question

- $\backslash \mathbf{W}_{\backslash \mathrm{llp}, j}$ is the pattern used to recognize the feature created by unit j of layer $\ll$
- How does $\backslash \mathbf{W}_{\backslash \mathrm{llp}, j}$ get set ?
    - i.e., how do we "learn" $\backslash \mathbf{W}_{\backslash \mathrm{llp}, j}$

This will be the topic of the next section.

Training a Neural Network

We will start to answer the question of how $\mathbf{W}$ is determined.

The short answer:

- **we find the weights that minimize the Loss Function**
- **using Gradient Descent**

We will briefly introduce training a Neural Network (Neural_Networks_Intro_to_Training.ipynb).

- **our initial focus will be on the mechanics**
- **the process by which the gradient is calculated will be the subject of a separate module.**

Tensorflow: A toolkit for Neural Networks

Why do we need a dedicated toolkit (Tensorflow) to aid the programming of Neural Networks ?

It's mainly about the use of Gradient Descent in training the network.

Recall that a Neural Net (including one augmented by a Loss Layer) is doing nothing more than compute a function.

Gradient Descent needs to take the gradient of this function (evaluated on a mini batch of examples) in order to update the weights $\mathbf{W}$.

There are at least two ways to obtain the Gradient

- **Numerically**
- **Analytically**

Numerical differentiation applies the mathematical definition of the gradient

$$\frac{\partial f(x)}{\partial x} = \frac{f(x+\epsilon) - f(x)}{\epsilon}$$

- It evaluates the function twice: at $f(x)$ and $f(x+\epsilon)$

- This can get expensive, especially since
  - $\mathbf{x}$ is a vector
  - Potentially very long (e.g., many features)
  - We need to evaluate the derivative of $f(\mathbf{x})$ with respect to each $\mathbf{x}_j$

Analytic derivatives are how you learned differentiation in school

- **As a collection of rules, e.g.,**
$$\frac{\partial(a+b)}{\partial x} = \frac{\partial a}{\partial x} + \frac{\partial b}{\partial x}$$

This is very efficient.

The issue in ordinary code

- The expression `(a + b) * c`
- Is evaluated `tmp = a + b`
- And the result passed to the next step of the computation, e.g, `tmp * c`
- Losing the connection between `tmp` (a value) and the operation (plus) and addends (`a`, `b`)

There is no information recorded in ordinary code that would allow the application of analytic rules of differentiation.

Tensorflow is different in that `(a + b) * c`

- **Is a symbolic expression (i.e., recorded as operation and arguments)**
- **That is saved**
- **Facilitating the application of analytic rules of differentiation**

We still write `(a + b) * c` but it really results in something like:

- `tf.math.mult( tf.math.add(a, b), c)`

The expression

`tf.math.add(a, b)`

can be differentiated analytically because it records

- the arguments are `a, b`
- the operation is addition
- we know the derivative of an addition operation

So Tensorflow facilitates analytic function differentiation while hiding the details from the user.

- **We will see some pseudo-code that shows how this is done**
- **Check out the Deeper Dive on Computation Graphs (Computation_Graphs.ipynb) if you want to know more**

By the way: what is a Tensor ? It is an object with an arbitrary number of dimensions.

We use special cases all the time:

- **A scalar is a tensor of dimension 0**
- **A vector is a tensor of dimension 1**
- **A matrix is a tensor of dimension 2**

As you've seen, we are already dealing with higher dimensional objects.

Consider $\mathbf{y}$:

- $\mathbf{y}_{\ell,j,j'}$
  - Output of layer ℓ: $\mathbf{y}_{\ell}$
  - Unit j of layer ℓ: $\mathbf{y}_{\ell,j}$
  - Element j' of the output of unit j of layer ℓ: $\mathbf{y}_{\ell,j,j'}$

In the future we will talk about *sequences* of $\y$, thus adding another dimension: time.

And, don't forget, the "batch index" dimension

- Tensorflow processes *mini-batches* of examples, not singeltons
- $\y^{\ip}_{\llp,j,j'}$
    - Element j' of the output (given input examples i) of unit j of layer $\ll$: $\y_{\llp,j,j'}$

The notation will become a little heavy but hopefully understandable as a way of indexing a high dimension object.

Some ”Why’s ?”

What took so long: Preview

An historical perspective:

- **Perceptron invented 1957**
- **mid-1970's: First "AI Winter"**
- **Late 1980's: second "AI Winter"**
- **2010: Re-emergence of AI**

The promise of AI led to great expectations, that were ultimately unfulfilled. The difficulty was the inability to train networks.

We will defer a fuller answer to a later lecture.

For now: seemingly minor choices were more impactful than imagined

- **Sigmoid as activation function turned out to be a problematic choice**
- **Initializing $\mathbf{W}$ properly was more important than imagined**

- **Vanishing/Exploding Gradients**
    - **problems arise when the gradient is effectively 0**
    - **problems also occurs when they are effectively infinite**

- Computational limits
    - It turns out to be quite important to make your NN big; bigger/faster machines help
    - Actually: bigger than it needs to be
        - many weights wind up near 0, which renders the neurons useless
        - The Lottery Ticket Hypothesis (https://arxiv.org/abs/1803.03635)
            - within a large network is a smaller, easily trained network
            - increasing network side increases the chance of large network containing a trainable subset
            - summary (https://towardsdatascience.com/how-the-lottery-ticket-hypothesis-is-challenging-everything-we-knew-about-training-neural-networks-e56da4b0da27)

Why do GPU's matter ?

GPU (Graphics Processing Unit): specially designed hardware to perform repeated vector multiplications (a typical calculation in graphics processing).

- **It is not general purpose (like a CPU) but does what it does extremely quickly, and using many more cores than a CPU (typically several thousand).**
- **As matrix multiplication is a fundamental operation of Deep Learning, GPU's have the ability to greatly speed up training (and inference).**

Google has a further enhancement called a TPU (https://cloud.google.com/tpu/docs/tpus) (Tensor Processing Unit) to speed both training and inference.

- highly specialized to eliminate bottlenecks (e.g., memory access) in fundamental Deep Learning matrix multiplication.

Both GPU's and TPU's

- Incur an overhead (a "set up" step is needed before calculation).
- So speedup only for sufficiently large matrices, or long "calculation pipelines" (multiplying different examples by the same weights).

DL involves

- **Multiplying large matrices (each example)**
- **By large matrices (weights, which are same for each example in batch)**

- **Both GPU's and TPU's offer the possibility of large speed ups.**

- **GPU's are not necessary**
    - **but they are a lot faster**
    - **life changing experience**
        - **30x faster means your 10 minute run (that ended in a bug) now only takes 20 seconds**
            - **increases your ambition by faster iteration of experimental cycle**

In [9]:
```
print("Done")
```

```
Done
```