DistilE: Distiling Knowledge Graph Embeddings for Faster and Cheaper Reasoning

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Abstract

Knowledge Graph Embedding (KGE) is a popular method for KG reasoning and usually a higher dimensional one ensures better reasoning capability. However, high-dimensional KGEs pose huge challenges to storage and computing resources and are not suitable for resource-limited or time-constrained applications, for which faster and cheaper reasoning is necessary. To address this problem, we propose DistilE, a knowledge distillation method to build low-dimensional student KGE from pre-trained high-dimensional teacher KGE. We take the original KGE loss as hard label loss and design specific soft label loss for different KGEs in DistilE. We also propose a two-stage distillation approach to make the student and teacher adapt to each other and further improve the reasoning capability of the student. Our DistilE is general enough to be applied to various KGEs. Experimental results of link prediction show that our method successfully distills a good student which performs better than a same dimensional one directly trained, and sometimes even better than the teacher, and it can achieve $2\times -8\times$ embedding compression rate and more than $10 \times$ faster inference speed than the teacher with a small performance loss. We also experimentally prove the effectiveness of our two-stage training proposal via ablation study.

Introduction

Knowledge Graph (KG) is composed of triples representing facts in the form of (head entity, relation, tail entity), abbreviate as (h, r, t). KGs have been proven to be useful for various AI tasks, such as semantic search (Berant et al. 2013; Berant and Liang 2014), information extraction (Hoffmann et al. 2011; Daiber et al. 2013) and question answering (Zhang et al. 2016; Diefenbach, Singh, and Maret 2018). However, it is well known that KGs are usually far from complete and this motivates many researches for knowledge graph completion and reasoning, among which a common and widely used series of methods is Knowledge Graph Embedding (KGE), such as TransE (Bordes et al. 2013), TransH (Wang et al. 2014), ConvE (Dettmers et al. 2018) etc.

To achieve better performance, training KGEs with higher dimension are typically preferred. While the model size, i.e. the number of parameters, and the cost of reasoning times usually increase fast as the embedding dimension goes up. As shown in Figure 1, as the embedding dimension becomes larger, the performance of the model (MRR) grows more and more slowly, while the model size and reasoning cost increase still quickly.

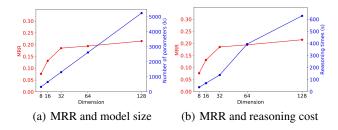


Figure 1: The changes of performance, model size and reasoning cost along the growth of embedding dimensions

However, high-dimensional embeddings are impractical in many real-life scenarios. For example, a pre-trained billion-scale knowledge graph is expected to be fine-tuned to solve downstream tasks and deployed frequently with a cheaper cost. For applications with limited computing resources such as deploying KG on edge computing or mobile devices, or with limited time for reasoning such as online financial predictions, embeddings with lower dimensions provides obvious or even indispensable conveniences.

Although low dimension enables faster deployment and cheaper reasoning, directly training with small embedding size normally performs poorly as shown in Figure 1. Thus we propose a new research question: is it possible to distill low-dimensional KGEs from pre-trained high-dimensional ones so that we could achieve good performance as long as faster and cheaper inference?

Knowledge Distillation (Hinton, Vinyals, and Dean 2015) is a technology to distill knowledge from a large model (teacher) to build a smaller model (student) and has been widely researched in Computer Vision and Natural Language Processing. The student learns from both the hard labels (ground-truth labels) and the soft labels from the teacher. In this work, we propose a novel distillation method for large-scale knowledge graph training, named DistilE, which is capable of distilling essence from a high-dimensional KGE into a smaller embedding size without losing too much accuracy while performs much better than

training directly with the same smaller size.

Conventional distillation methods usually use the logits or softmax output from the teacher to supervise the student. Considering the diversity of loss functions of KGE methods, we design specific soft labels for different KGEs in DistilE. For KGEs based on Margin Loss, such as TransX series (Bordes et al. 2013; Wang et al. 2014; Lin et al. 2015; Ji et al. 2015), whose output does not have probabilistic interpretation, we use the triples' scores from the teacher as soft labels because these scores directly reflect the existence of triples. For KGEs based on Cross Entropy Loss, such as bilinear models (Nickel, Tresp, and Kriegel 2011; Yang et al. 2015), rotation models (Sun et al. 2019b; Zhang et al. 2019) and models based on neural networks (Dettmers et al. 2018; Nguyen et al. 2019), we use the sigmoid function outputs of the teacher as soft labels.

We also propose a **two-stage distillation approach** to improve the distillation results further. The basic idea is that although the trained teacher is already strong, it could achieve better performance if the teacher could also learn from the student instead of being fixed all the time. Sun (Sun et al. 2019a) also proved that the overall performance also depends on the student's acceptance of the teacher. Therefore, in addition to a standard distillation stage in which the teacher is always static, we devise a second stage distillation in which the teacher is unfrozen and try to adjust itself to become more acceptable for the student.

We evaluate DistilE with several typical KGEs and standard KG reasoning datasets. Results prove the effectiveness of our method, showing that (1) the low-dimensional KGEs distilled by DistilE performs much better than directly training the same sized embeddings without the distillation stage, (2) the low-dimensional KGEs distilled by DistilE significantly infer faster than original high-dimensional KGEs, (3) our two-stage distillation approach works well and could further improve the distillation results.

In summary, our contributions are:

- We propose a novel framework to distill lower dimensional KGEs from higher dimensional ones. To the best of our knowledge, this is the first one to apply knowledge distillation to knowledge graph embedding.
- We propose different soft labels for different kinds of KGEs with special structure and a two-stage distillation to enhance the distillation results.
- We experimentally prove that our proposal can reduce the number of parameters of a KGE by 8 times and increases the inference speed by more than 10 times while retaining good performance.

Related Work

Knowledge Distillation and Model Compression

In the last few years, the acceleration and compression of models has attracted a lot of research works. Common methods include network pruning (Castellano, Fanelli, and Pelillo 1997; Molchanov et al. 2017), quantification (Lin, Talathi, and Annapureddy 2016; Sachan 2020), parameters sharing

(Dehghani et al. 2019; Lan et al. 2020), and knowledge distillation (Hinton, Vinyals, and Dean 2015).

Among them, knowledge distillation has been widely used in Computer Vision and Natural Language Processing. Its core idea is to regard the probability distributions output by the teacher as soft labels to help guide the training of the student. And knowledge distillation has an advantage different from the other model compression methods mentioned above: in addition to the probability distributions, other soft labels can also be designed according to needs, providing more modeling freedom. (Tang et al. 2019) proposes to distill the pre-trained language model BERT (Devlin et al. 2019) into a single-layer bidirectional long and short-term memory network (BiLSTM). (Sun et al. 2019a) proposes to enable students to learn more knowledge by fitting the middle layer output of the teacher, rather than just using the probability distribution from the softmax layer. (Tian, Krishnan, and Isola 2020) believes that there are dependencies between the dimensions of data representation, and proposes maximizing the mutual information of the data representation by the student and the teacher. (Zhao et al. 2019) gives up the transfer of the softmax layer in BERT distillation and directly approximates the corresponding weight matrix in the student and the teacher.

However, many KGEs do not have a deep network structure and probability distribution output, such as KGEs of TransX series, these existing distillation methods are not suitable in our settings. In this work, we introduced the first method of KG embedding compression using knowledge distillation. We designed specific distillation soft labels for different KGEs, and also proposed a two-stage distillation approach to further improve the distillation effect.

Knowledge Graph Embeddings

In recent years, knowledge graph embedding (KGE) technology has been rapidly developed and applied. Its key idea is to transform the entities and relations of KG into a continuous vector space, named embedding. And then the embeddings can be further applied to various KG downstream tasks. RESCAL (Nickel, Tresp, and Kriegel 2011) is the first relation learning method based on tensor decomposition and encodes relations and entities into two-dimensional matrices and vectors respectively. To improve RESCAL, DistMult (Yang et al. 2015) restricts the relation matrix to a diagonal matrix to simplify the model, ComplEx (Trouillon et al. 2016) embeds entities and relations into the complex space to model asymmetric relations better, and HolE (Nickel, Rosasco, and Poggio 2016) combines the expressive power of RESCAL with the simplicity of DistMult. TransE (Bordes et al. 2013) is the first translation-based KGE method and regards the relation as a translation from the head entity to the tail entity. Various variants of TransE are proposed to deal with more complex relations. TransH (Wang et al. 2014) proposes that an entity should have different representations under different relations. TransR (Lin et al. 2015) believes that different relations pay attention to different attributes of entities. TransD (Ji et al. 2015) demonstrates that a relation may represent multiple semantics. With the rise of neural networks, many KGE methods based on neural networks have emerged. ConvE (Dettmers et al. 2018) and ConvKB (Nguyen et al. 2018) use convolutional neural networks (CNN), and CapsE (Nguyen et al. 2019) uses capsule neural network as the score function of triples. In addition, rotation models such as RotatE (Sun et al. 2019b), QuatE (Zhang et al. 2019), DihEdral (Xu and Li 2019), etc. regard the relation as the rotation between the head entity and the tail entity.

However, although the KGEs are simple and effective, they have an obvious problem that high-dimensional embeddings pose a huge challenge to storage and computing. It is necessary to reduce the dimension of embeddings and still retain a good performance for many practical application scenarios. But now there is little research on KG embedding compression. The only published work of KGE compression is (Sachan 2020), unlike our method, it uses quantitative technology to represent entities as a vector of discrete codes, while we use the knowledge distillation technology.

Method

This section elaborates on our proposal DistilE. Firstly, we identify different distillation objectives with specific distillation soft labels for different types of KGE methods. We then introduce our two-stage distillation approach to continuously adjust the teacher for better distillation results.

Distillation Objective

Knowledge distillation typically involves two models, a larger size *teacher* model with good performance and a small size *student* model. During training of the student, the student is first encouraged to 1) fit the hard labels from data, like the one-hot vector of a sentence's class, with a hard labels loss, and 2) imitate the teacher's behavior via fitting soft labels from the teacher with a soft label loss. Soft labels usually refer to the probability distribution output by the teacher.

Given a KG $\mathcal{K}=\{E,R,T\}$, where E,R and T are the set of entities, relations and triples respectively. A KGE learns to express the relationships between entities in a continuous vector space. Specifically, for a triple (h,r,t), where $h,t\in E,r\in R$, the KGE model could assign a score to it by a score function $f_r(h,t)$, to indicate the existence of (h,r,t). KGEs with different score functions have different training objectives and corresponding loss functions. Two most commonly used loss of KGE include the **Marginal Loss** and **Cross Entropy Loss**. For different types of loss, different types of distillation objectives are required.

Objective for KGEs with Marginal Loss. Marginal loss is often used in translation-based KGEs, including TransE, TransH, TransR, and TransD, etc. They have a score function $f_r(h,t)$ to judge the existence of (h,r,t) based on distance-based metrics. The training goal is to make $f_r(h,t)$ small for positive triples (h,r,t) and large for negative ones, and force the margin between positive and negative ones' score larger than a margin. The teacher is trained through the following loss:

$$L_{hard}^{T} = \sum_{(h,r,t)\in G} [f_r^{T}(h,t) - f_r^{T}(h',t') + \gamma]_+, \quad (1)$$

where $f_r^T(h,t)$ and $f_r^T(h',t')$ is the score for positive triple (h,r,t) and negative triple (h',r,t') given by the teacher. (h',r,t') is generated by randomly replacing h or t in $(h,r,t) \in T$ with h' or t', which could be expressed as:

$$G^{-} = \{ (h', r, t) \notin T | h' \in E \land h' \neq h \}$$

$$\cup \{ (h, r, t') \notin T | t' \in E \land t' \neq t \}.$$
 (2)

Hard Label Loss for Student. The hard label loss of student is identical as the teacher as follows:

$$L_{hard}^{S} = \sum_{(h,r,t)\in G} [f_r^{S}(h,t) - f_r^{S}(h',t') + \gamma]_+, \quad (3)$$

Soft Label Loss for Student. Since these KGEs lack the necessary probability output layers as like conventional knowledge distillation methods, fitting probability distribution is inapplicable here. A natural choice is to make use of teacher's triple score as the soft label for student, since the triple score contains richer information about the truthiness of a triple. We then engage the student to fit itself to the teacher by minimizing the difference between these two scores. Formally, the soft label loss of the student can be expressed as:

$$L_{soft}^{S} = \sum_{(h,r,t)\in G} (\left| f_r^{T}(h,t) - f_r^{S}(h,t) \right| + \sum_{i=1}^{k} \left| f_r^{T}(h_i',t_i') - f_r^{S}(h_i',t_i') \right|),$$
(4)

where (h_i', r, t_i') with $k \in [1, k]$ are negative triples, |x| denotes the absolute value of x. Since the teacher's score of any triple can be regarded as a soft label, we generate multiple negative triples for a positive one during the experiment and make student fit to all their soft labels.

Final Loss. The final distillation loss can be formulated by the weighted sum of the student's soft label loss and hard label loss:

$$L = \alpha L_{soft}^S + (1 - \alpha) L_{hard}^S, \tag{5}$$

where α is a hyperparameter to balance the importance of hard label loss and soft label loss.

Objective for KGEs with Cross Entropy Loss. Cross Entropy Loss is often used in models whose outputs have probabilistic interpretation, for example, bilinear models such as ComplEx, rotation models such as RotatE, and models based on neural networks. They model link prediction as a classification task and output the probability of truthiness of input triple according to the results from a sigmoid function with triple score as input. Thus the cross entropy loss of training the teacher can be formulated as follows:

$$L_{hard}^{T} = -\sum_{(h,r,t)\in G\cup G^{-}} (y\log p_{(h,r,t)}^{T} + (1-y)\log(1-p_{(h,r,t)}^{T})),$$
(6)

where $p_{(h,r,t)}^T = \frac{\exp f_r^T(h,t)}{1+\exp f_r^T(h,t)}$ is a real number between (0,1) given by teacher, representing the probability that the

| Method | Score function $f_r(h,t)$ | Soft label loss for student ${\cal L}_{soft}^S$ | $oxed{Hard label loss for student L^S_{hard}}$ |
|---------|-------------------------------------|--|--|
| TransE | $-\left\Vert h+r-t\right\Vert _{p}$ | $\Big \sum_{(h,r,t)\in G} (\left f_r^T(h,t)-f_r^S(h,t)\right +$ | $\sum_{(h,r,t)\in G} [f_r^S(h,t) - f_r^S(h',t') + \gamma]_+$ |
| TransH | | $ \sum_{i=1}^{k} \left f_r^T(h_i', t_i') - f_r^S(h_i', t_i') \right) $ | (4,7,7= |
| ComplEx | $Re(h^{\top}diag(r)\bar{t})$ | $ - \sum_{(h,r,t)\in G\cup G^{-}} (p_{(h,r,t)}^{T} \log p_{(h,r,t)}^{S} +$ | $-\sum_{(h,r,t)\in G\cup G^{-}} (y\log p_{(h,r,t)}^{S} + (1 -$ |
| RotatE | $-\left\ hullet r-t ight\ ^2$ | $(1 - p_{(h,r,t)}^T) \log(1 - p_{(h,r,t)}^S))$ | $y)\log(1-p_{(h,r,t)}^S))$ |

Table 1: Score functions, soft label loss and hard label loss for the student of some popular knowledge graph embedding models in DistilE. Here, \bar{x} represents the conjugate of a complex number x, \bullet represents the Hadamard product, $f_r^S(h,t)$ and $f_r^T(h,t)$ represents the score function in the student model and the teacher model respectively, $p_{(h,r,t)}^T = \frac{\exp f_r^T(h,t)}{1+\exp f_r^T(h,t)}$ and $p_{(h,r,t)}^S = \frac{\exp f_r^S(h,t)}{1+\exp f_r^S(h,t)}$.

triple is a true fact. y is the ground-truth label of (h,r,t), and it is 1 for positive triples and 0 for negative ones. **Hard Label Loss for Student.** Similarly, the hard label loss for student is the same as the one of the teacher:

$$L_{hard}^{S} = -\sum_{(h,r,t)\in G\cup G^{-}} (y\log p_{(h,r,t)}^{S}) + (1-y)\log(1-p_{(h,r,t)}^{S})),$$
(7)

Soft Label Loss. Since the outputs of these KGEs have probability interpretation, the soft label loss of the student can be defined as the cross entropy of the probability distribution output by the student and the teacher as in conventional knowledge distillation approach:

$$L_{soft}^{S} = -\sum_{(h,r,t)\in G\cup G^{-}} (p_{(h,r,t)}^{T} \log p_{(h,r,t)}^{S} + (1 - p_{(h,r,t)}^{T}) \log(1 - p_{(h,r,t)}^{S})).$$
(8)

Final Loss. The final loss for these KGEs is the same as Eq. (5).

Table 1 summarizes the score function, soft label loss and hard label loss for the student of some popular knowledge graph embedding models in DistilE.

Two-stage Distillation approach

In the previous part, we introduced how to enable the student to extract knowledge from the KGE teacher, where the student is trained with hard labels and the soft labels generated by a fixed teacher. To obtain a better student, we propose a two-stage distillation approach to improve the student's acceptance of the teacher by unfreezing the teacher and engage it to learn from the student in a second stage of distillation.

The First Stage. The first stage similar to conventional knowledge distillation methods in which the teacher is frozen and unchanged when training the student as introduced in the previous section.

The Second Stage. In this stage, the teacher is unfrozen and tries to adjust itself to improve the acceptance for the student. The basic idea is that we not only train the teacher

with a hard label to guarantee its performance, but also engage it to fit a soft label generated from the student. Essentially, this can be regarded as a process where the teacher also learns from its student in reverse. As a result, the teacher will become more adaptable to the student, thereby improving the distillation effect.

For KGEs with Marginal Loss. The hard label loss of optimizing the teacher is the same as Eq. (1) and the soft label loss can be formulated as follows:

$$L_{soft}^{T} = \sum_{(h,r,t)\in G} (|f_{r}^{S}(h,t) - f_{r}^{T}(h,t)| + \sum_{i=1}^{k} |f_{r}^{S}(h_{i},t_{i}) - f_{r}^{T}(h_{i},t_{i})|).$$

$$(9)$$

Eq. (4) and (9) are the same because absolute value of the difference between two numbers has commutative property. *For KGEs with Cross Entropy Loss.* The hard label loss of optimizing the teacher is the same as Eq. (6) and the soft label loss can be expressed as:

$$L_{soft}^{T} = -\sum_{(h,r,t)\in G\cup G^{-}} (p_{(h,r,t)}^{S} \log p_{(h,r,t)}^{T} + (1 - p_{(h,r,t)}^{S}) \log(1 - p_{(h,r,t)}^{T})).$$
(10)

Final Loss. It is a weighted sum of the soft label loss and hard label loss of teacher and student:

$$L = \alpha L_{soft}^S + (1 - \alpha) L_{hard}^S + \beta L_{soft}^T + (1 - \beta) L_{hard}^T, \ (11)$$

where α and β are independent weight hyperparameters for different parts.

Experiments

We evaluate DistilE on typical KGE benchmarks, and are particularly interested in the following questions.

 Whether it is capable of distilling a good student from the teacher and performing better than a same dimensional model trained from scratch without distillation;

- How much the inference time is improved after a distillation procedure.;
- Whether and how much the two-stage distillation approach contributes to our proposal.

Datasets and Implementation Details

Datasets. We experiment on two common knowledge graph completion benchmark datasets WN18RR (Toutanova et al. 2015) and FB15k-237 (Dettmers et al. 2018), subsets of WordNet (Bordes et al. 2013) and Freebase (Bordes et al. 2013) with redundant inverse relations eliminated. Table 2 shows the statistics of these two datasets.

| Dataset | #Ent. | #Rel. | #Train | #Valid | #Test |
|-----------|--------|-------|---------|--------|--------|
| WN18RR | 40,943 | 11 | 86,835 | 3,034 | 3,134 |
| FB15k-237 | 14,541 | 237 | 272,115 | 17,535 | 20,466 |

Table 2: Statistics of datasets we used in the experiments.

Evaluation Metrics. We adopt standard metrics MR, MRR, and Hit@k (k = 1, 3, 10). Given a test triple (h, r, t), we first replace the head entity h with each entity $e \in E$ and generate candidate triples (e, r, t). Then we use the score function $f_r(e,t)$ to calculate the scores of all candidate triples and arrange them in descending order, according to which, we obtain the rank of (h, r, t)'s score, $rank_h$ as its head prediction result. For (h, r, t)'s tail prediction, we first replacing t with all $e \in E$ to generate candidate triples (h, r, e), and get the tail prediction rank $rank_t$ in a similar way. We average $rank_h$ and $rank_t$ as the final rank of (h, r, t). Finally, we calculate MR, MRR, and Hit@k via the rank of all test triples. MR is their mean rank. MRR is their mean reciprocal rank. And Hit@k measures the percentage of test triples with rank $\leq k$. We also use the filtered setting (Bordes et al. 2013) by removing all triples in the candidate set that existing in training, validating, and testing sets.

Baselines. We implement DistilE on several teacher KGEs. TransE and TransH are chosen for KGEs with margin loss and ComplEx and RoratE are chosen for KGEs with cross entropy loss.

Implementation Details. For the teacher, we set embedding dimension $d_{teacher} = \{64, 128, 256\}$ and make $d_{teacher} = 64$ for primary experiment, and set $d_{student} =$ $\{32, 16, 8\}$ for the student. We set batch size to 1024 and maximum training epoch to 1000. For other hyperparameters, we follow the setting in original paper of KGEs, setting $\gamma = 1.0$ and L1 as dissimilarity in TransE, $\gamma = 0.5$, soft constrained hyperparameter C = 0.0625 and L1 as dissimilarity in TransH, and $\gamma = 6.0$, $\epsilon = 2.0$ in RotatE. For each positive triple, we generate 5 negative ones in TransE and TransH and 25 in ComplEx and RotatE. We choose Adam (Kingma and Ba 2015) as the optimizer with earning rate decay and trigger decay threshold set to 0.96 and 5 respectively. We perform a grid search on the following hyperparameter combination and report the reults from the best one: learning rate: $\{0.0005, 0.005, 0.05, 0.01\}$, balance hyperparameter for student and teacher $\{\alpha,\beta\} = \{0.3,0.5,0.7\}.$

Q1: Whether our method successfully distills a good student?

To verify whether DistilE successfully distills a good student, we first train a student with a higher dimensional teacher, marked as 'DS', and then train a same dimensional student with only hard label loss, marked as 'no-DS', which is the same as training a same dimensional original KGE model. Then we compare their performance on link prediction. Table 3 and Table 4 shows the results on WN18RR and FB15k-237 with different dimensional setting for student respectively.

Results Analysis. First we analyze the results on WN18RR in Table 3. Table 3 shows that the performance of 'no-DS' model decreases significantly as the embedding dimension reducing. With TransE as an example, compared with the 64-dimensional teacher, an 8-dimensional 'no-DS' model only achieves 39%, 47%, 29%, and 24% results on MRR, Hit@10, Hit@3, and Hit@1 respectively. And for the MRR results of other methods, TransH's decreases from 0.18 to 0.052 (71%) and ComplEx's decreases from 0.312 to 0.046(85.3%). This illustrates that directly training low dimensional KGEs produce poor results.

Compared with 'no-DS' results, the results of our distilled model with the same dimension achieves better results in most settings. For example, the MRR of TransE improves from 0.076 to 0.125 (64.5%) and Hit@1 improves from 0.009 to 0.032 (255.6%). The MRR, Hit@10, and Hit@1 of ComplEx improves from 0.046 to 0.137 (197.8%), from 0.105 to 0.268 (155.2%), and from 0.022 to 0.068 (209.1%) respectively. The MRR and Hit@1 of RotatE improves from 0.111 to 0.2184 (96.4%) and from 0.047 to 0.107 (127.7%) respectively. t results an **average improvement of 98.8%**, **63.4%**, **69.6%**, and **162.0%** on MRR, Hit@10, Hit@3, and Hit@1 among these four KGEs. On the whole, we could conclude that compared with 'no-DS', our method greatly improves the performance of low dimensional models.

More importantly, compared to the results of 64-dimensional teachers, our 32-dimensional students even surpass in some metrics. For example, with TransH, the MRR, Hit@3, and Hit@1 of our 32-dimensional student surpass teacher by 10%, 10%, and 25% respectively. Our 16-dimensional students, with a 4 times model compression rate of 64-dimensional teacher, achieve similar results to the teacher in many metrics. Take RotatE as an example, our 16-dimensional student achieves 91%, 93%, 94%, and 94% of the teacher's results on MRR, Hit@10, Hit@3, and Hit@1 respectively.

All above analyses are based on results on WN18RR, and with results on FB15k-237 in Table 4 we could find a similar phenomenon. Thus we could conclude that our method does successfully distill a good student.

Higher Dimensional Teachers. Since the teacher's dimension may matters, we also conduct experiments on 128 and 256 dimensional teacher with TransE, to evaluate the influence of the teacher's dimension. Figure 2 shows a

| Dim | Method | TransE | | | TransH | | | ComplEx | | | | RotatE | | | | | |
|-----|----------|--------|------|------|--------|------|------|---------|------|------|------|--------|------|------|------|------|------|
| | | MRR | H10 | Н3 | H1 | MRR | H10 | Н3 | H1 | MRR | H10 | Н3 | H1 | MRR | H10 | Н3 | H1 |
| 64 | Tea. | .194 | .481 | .331 | .037 | .180 | .470 | .326 | .016 | .312 | .444 | .356 | .240 | .478 | .567 | .500 | .431 |
| 32 | no-DS | .185 | .427 | .316 | .030 | .172 | .423 | .307 | .014 | .211 | .368 | .225 | .143 | .467 | .545 | .495 | .419 |
| 32 | DS(ours) | .201 | .486 | .319 | .049 | .198 | .456 | .360 | .020 | .319 | .443 | .361 | .251 | .477 | .562 | .497 | .430 |
| 16 | no-DS | .131 | .311 | .230 | .012 | .131 | .352 | .203 | .013 | .142 | .271 | .156 | .079 | .413 | .479 | .438 | .374 |
| 10 | DS(ours) | .171 | .413 | .274 | .030 | .161 | .415 | .264 | .016 | .262 | .421 | .295 | .187 | .433 | .525 | .468 | .407 |
| 8 | no-DS | .076 | .224 | .096 | .009 | .052 | .133 | .053 | .009 | .046 | .105 | .064 | .022 | .111 | .326 | .245 | .047 |
| | DS(ours) | .125 | .307 | .168 | .032 | .071 | .182 | .071 | .014 | .137 | .268 | .157 | .068 | .218 | .406 | .304 | .107 |

Table 3: Link prediction results on WN18RR. Bold numbers are the better results between 'no-DS' and 'DS(ours)' with same dimension.

| Dim | Method | TransE | | | TransH | | | ComplEx | | | | RotatE | | | | | |
|-----|----------|--------|------|------|--------|------|------|---------|------|------|------|--------|------|------|------|------|------|
| | | MRR | H10 | Н3 | H1 | MRR | H10 | Н3 | H1 | MRR | H10 | Н3 | H1 | MRR | H10 | Н3 | H1 |
| 64 | Tea. | .288 | .468 | .319 | .196 | .365 | .556 | .406 | .266 | .274 | .471 | .307 | .178 | .420 | .613 | .463 | .322 |
| 32 | no-DS | .263 | .433 | .289 | .176 | .333 | .521 | .367 | .236 | .197 | .349 | .221 | .115 | .365 | .557 | .405 | .268 |
| 32 | DS(ours) | .286 | .463 | .317 | .196 | .351 | .537 | .393 | .253 | .221 | .430 | .258 | .139 | .410 | .599 | .456 | .312 |
| 16 | no-DS | .252 | .410 | .259 | .154 | .295 | .458 | .324 | .209 | .118 | .270 | .125 | .057 | .297 | .484 | .337 | .203 |
| 10 | DS(ours) | .267 | .445 | .299 | .176 | .337 | .516 | .369 | .246 | .179 | .360 | .176 | .096 | .365 | .563 | .423 | .262 |
| 8 | no-DS | .205 | .377 | .221 | .104 | .250 | .423 | .288 | .156 | .102 | .220 | .097 | .043 | .256 | .419 | .269 | .135 |
| | DS(ours) | .238 | .425 | .256 | .147 | .296 | .462 | .331 | .203 | .183 | .368 | .212 | .099 | .297 | .485 | .322 | .178 |

Table 4: Link prediction results on FB15k-237. Bold numbers are the better results between 'no-DS' and 'DS(ours)' with same dimension.

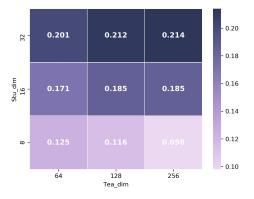


Figure 2: Students' test MRR distilled by teachers with different dimensions on the WN18RR dataset for TransE.

heatmap of MRR results of the student distilled from different dimensional teachers on WN18RR. It shows that (1) for 32-dimensional students, higher dimensional teacher achieve slightly better results, (2) for 16-dimensional students, a higher dimensional teacher does not achieve better results, and (3) for 8-dimensional students, higher dimensional teacher achieves worse results. This indicate that our method's compression capability is about 8 times. Therefor it is not always necessary to distill from a bigger teacher. An intuition is that although a bigger teacher is more expressive, a overly high compression ratio may prevent the student from absorbing important information from the teacher. This analysis reveals that for an application where an especially low-dimensional student is required and suppose the required dimension is d, instead of choosing a very highdimensional teacher with fantastic performance, it is better to choose a teacher with dimension $\leq 8 \times d$ which might not achieve the best pretraining performance.

Q2: Whether the distilled student successfully accelerates training and inference speed?



Figure 3: Test MRR for 32-Dim student as training proceeds on FB15k-237 dataset and RotatE with and without 64-Dim teacher guidance.

Training Speed. Figure 3 shows the convergence of 32-dimensional students with or without distillation. We observed that with distillation, our method converge significantly faster and more stably than 'no-DS' since the beginning and finally achieves better results. After the second stage distillation (S2, the right half of the red line separated by the black dashed line) begin, MRR slightly fluctuates and quickly converges to an even better result. The reason for fluctuation is that at the beginning of S2, the teacher begins to adapt according to the soft labels from the student.

| Dim | Tra | ansE | Tra | nsH | Con | nplEx | RotatE | | |
|----------|-------------|-----------------------|-------------|-----------------------|-------------|-----------------------|-------------|-----------------------|--|
| | # Params(K) | $Inf. \ Time(s)$ | # Params(K) | $Inf. \ Time(s)$ | # Params(K) | Inf. Time(s) | # Params(K) | $Inf. \ Time(s)$ | |
| 64(Tea.) | 2621.056 | 394.3 (1×) | 2621.76 | 472.9 (1×) | 5242.112 | 569.4 (1×) | 5241.408 | 617.2 (1×) | |
| 32 | 1310.528 | $137.4 (2.87 \times)$ | 1310.88 | $173.2 (2.73 \times)$ | 2621.056 | $200.6 (2.84 \times)$ | 2620.704 | $237.2 (2.60 \times)$ | |
| 16 | 655.264 | $69.7 (5.66 \times)$ | 655.44 | $93.2 (5.07 \times)$ | 1310.528 | 117.7 (4.84×) | 1310.352 | 134.7 (4.58×) | |
| 8 | 327.632 | 35.8 (11.01×) | 327.72 | 46.4 (10.19×) | 655.264 | 75.3 (7.56×) | 655.176 | 82.6 (7.47×) | |

Table 5: The number of parameters and inference times for TransE, TransH, ComplEx and RotatE.

Although S2 optimizing student and teacher together introduces additional training, it converges very quickly and does not increase the total training time significantly. As shown in Figure 3, only about 50 epochs are enough for convergence during S2.

Inference Speed. To test the inference speed of the teacher and the student, we conduct link prediction experiments on 93,003 triple sampled from WN18RR. The inference is performed on a single GeForce GTX GPU, and the batch size is set to 1024. In order to avoid accidental factors, we repeat the experiment 3 times and report the average time. Table 5 shows the time cost as long as the number of parameters. It shows that the reduction of parameter numbers is proportional to the compression rate, thus the machine memory for saving 32-, 16- and 8-dimensional students will be saved by 2 times, 4 times, and 8 times respectively, compared to a 64-dimensional one. It also shows that our distillation method achieves almost linear acceleration for inference. The inference time of a 64-dimensional teacher is about 5 times of the 16-dimensional student, and nearly or even more than 10 times that of the 8-dimensional student.

We observe the same phenomenon on FB15k-237, and we do not show it due to the limitation of space. These results support that our distilled student successfully accelerates training and inference speed.

Q3: Whether and how much does the two-stage distillation approach contribute to the result?

To study the impact of the two-stage distillation approach, we conduct an ablation study to compare the performance of our method with two stages (DS) to removing the first stage (-S1) and removing the second stage (-S2). Table 6 summarizes the MRR and Hit@10 results on WN18RR.

After removing *S1* with only *S2* preserved (refer to -*S1*), the performance is overall lower than that of DS. Presumably, the reason is that both the teacher and the student will adapt to each other in *S2*. With a randomly initialized student, the student conveys mostly useless information to the teacher which may be misleading and will crash the teacher.

In addition, the performance of '-S1' is very unstable. With '-S1' setting, 32-dimensional students obtain results only slightly worse than DS, while 16-dimensional and especially 8-dimensional students perform obviously very poor. Taking the 8-dimensional student of RotatE as an example, the MRR and Hit@ 10 of '-S1' are only 23% and 28% compared with DS. This is even worse than directly training the same sized student without distillation, showing that the first

| | M | Trai | nsE | Tra | nsH | Com | plEx | RotatE | | |
|----|-----|------|------|------|------|------|------|--------|------|--|
| υ | | MRR | H10 | MRR | H10 | MRR | H10 | MRR | H10 | |
| | DS | .201 | .486 | .198 | .467 | .319 | .443 | .477 | .562 | |
| 32 | -S1 | .196 | .483 | .179 | .463 | .317 | .442 | .419 | .458 | |
| | -S2 | .193 | .473 | .169 | .456 | .318 | .442 | .474 | .561 | |
| | DS | .171 | .413 | .161 | .415 | .262 | .421 | .432 | .525 | |
| 16 | -S1 | .137 | .352 | .101 | .258 | .133 | .275 | .291 | .342 | |
| | -S2 | .163 | .395 | .134 | .391 | .250 | .412 | .433 | .525 | |
| | DS | .125 | .307 | .071 | .182 | .137 | .268 | .218 | .406 | |
| 8 | -S1 | .081 | .183 | .037 | .082 | .051 | .097 | .050 | .115 | |
| | -S2 | .113 | .284 | .069 | .177 | .136 | .269 | .209 | .403 | |

Table 6: Ablation study. $\bf D$ refers to dimension and $\bf M$ refers to method.

stage is necessary for distillation.

After removing S2 with only S1 preserved (refer to S2), the performance decreases in almost all setting. Taking TransE as an example, compared with DS, the MRR of 32-, 16- and 8-dimensional student of '-S2' drop by 4%, 5% and 10% respectively, indicating that the second stage can indeed make teacher and student adapt to each other, and further improve the result.

We also observe the same phenomenon on FB15k-237, and we do not show it due to space limitation. These results support the effectiveness of our two-stage distillation that first train the student in S1 converging to a certain performance and then co-optimize the teach and student in S2.

Conclusion and Future Work

Too many embedding parameters of the knowledge graph will bring huge storage and calculation challenges to actual application scenarios. In this work, we propose a novel KGE distillation method to compress KGEs to lower dimensional space. In order to successfully apply the knowledge distillation technology to KGEs with special structure, we design specific soft label loss for different KGEs. In order to enable the student to fully accept the rich information from the teacher, our method encourages the teacher and student to adapt to each other through a two-stage distillation approach. We have evaluated our method through link prediction task on several different KGEs and benchmark datasets. Results show that our method can effectively reduce model parameters and greatly improve the inference speed without too much loss in performance compared to the teacher and has better reasoning capability than a directly trained one with the same dimension.

In this work, we only considered transmitting knowledge through the final output of KGEs. In the future, we would like to firstly explore multi-layer distillation from other network layers of KGEs and secondly study the knowledge distillation of KGEs in more complex environments, such as adversarial learning and ensemble learning.

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